Uncertainty and Contracting: A Theory of Consensus and Envy in Organizations*

David L. Dicks
Hankamer School of Business
Baylor University

Paolo Fulghieri
Kenan-Flagler Business School
University of North Carolina
CEPR and ECGI

December 19, 2019

Abstract

We explore the impact of Knightian uncertainty on contracting within a multi-layered firm. We study a setting where, absent uncertainty, division managers should be paid based on their division performance, but not other divisions' performance. As uncertainty increases, division managers become more conservative (than company headquarters) about the prospects of their own division, diminishing effort. Correspondingly, division managers become also relatively more positive about the prospects of others divisions within the firm, generating envy and discord in the organization. When uncertainty is large enough, headquarters grants a division manager a share of other divisions' payoff to hedge uncertainty, thus instilling confidence and promoting a shared view (i.e., consensus) within the organization. Our model can explain the prevalence of equity-based incentive contracts in (young) firms with uncertain cash-flow prospects, and the prevalence of performance-based contracts in more mature and well-established firms.

*We would like to thank David Hofmann, Paige Ouimet, Jacob Sagi, and seminar participants at the University of Amsterdam, Erasmus, Iowa, Tilburg, George Mason, Purdue, UNC, the Third Annual CEPR Spring Symposium in Financial Economics at Imperial College, the Workshop on Corporate Finance, Lancaster University, and the SFS Cavalcade Asia-Pacific 2019 for very helpful comments. All errors are only our own. We can be reached at David_Dicks@baylor.edu and Paolo_Fulghieri@kenan-flagler.unc.edu

A classical theme in the theory of incentive contracts is determining the appropriate performance measures to be used as a base of incentive pay in organizations.[1] A key question is whether firms should use incentive contracts based on overall equity performance, or contracts where pay is based on division-specific performance. The distinction between equity-based contracts and division-specific pay, such as pay-for-performance compensation, is particularly important for lower-level managers.[2] The case for equity-based contracts for top managers is rather strong, as they are responsible for the performance of the overall firm. More puzzling is the widespread use of equity-based compensation for division and rank-and-file managers who are deeper down in an organization. This is because, for such lower-level managers, equity-based contracts reduce the responsiveness of their pay to their actions, thus "diluting" their incentives. In addition, adding non-informative risk (that is, "noise") to compensation contracts for risk-averse agents reduces their expected utility with no benefit to the principal (Holmstrom, 1979).

This paper proposes a novel explanation of the optimal incentive contracts in organizations based on uncertainty aversion. We consider a multi-divisional firm endowed with company headquarters and (two) uncertainty-averse division managers. Company headquarters must design optimal incentive contracts for the division managers. If pay is contingent only on a division's performance, as uncertainty increases division managers become relatively more conservative on the prospects of their divisions, causing incentives to be less effective at inducing effort. Division managers' more conservative beliefs on future expected cash flow of their division is due to their greater exposure to division-specific risk under uncertainty aversion.[3] When uncertainty is sufficiently large, company headquarters find it desirable to offer division managers compensation contracts with cross-pay with the aim of making them more confident about their own division and inducing more effort. Thus, optimal incentive contracts will be a combination of equity-based and division-specific compensation when there is sufficient uncertainty.

The benefit of equity based-contracts for division managers in multidivisional firms is to align division managers' expectations on future cash flows with those held by the company headquar-

[1] See Murphy (1999, 2013), Frydman and Jenter (2010) and Oyer and Schaefer (2011) for extensive surveys.

[2] An example of pay-for-performance compensation is a contract based on Economic Value Added, or EVA.

[3] This property is an implication of the fact that, under uncertainty aversion, probabilistic assessments (or "beliefs" in the sense of de Finetti, 1974) held by uncertainty-averse agents are not uniquely determined by a single prior but, rather, are determined endogenously as the solution of a minimization problem (see Dicks and Fulghieri, 2019a, for further details).

ters. By providing cross pay, which hedges the division managers' exposure to division cash-flow uncertainty, company headquarters induce division managers to hold more favorable expectations on their divisions, with a positive impact on their effort. Cross-pay compensation, however, is costly to company headquarters since it provides pay that is not linked to the specific divisional performance (which is directly affected by effort) but, rather, is beneficial only through its effect on division managers' beliefs. The optimal cross-pay component in a compensation contract for a division manager will thus depend on its beneficial effect on managerial expectations.

We show that the structure of the optimal incentive contracts for division managers depends on the level of uncertainty characterizing a firm's future profitability and on the attitude of headquarters toward such uncertainty. For low levels of uncertainty, optimal contracts have only traditional pay-for-performance features, where a division manager's pay depends only on the performance of their own division. If uncertainty is sufficiently large, the optimal contracts for division managers also depend on headquarters attitude toward uncertainty. When headquarters are uncertainty neutral, they grant cross pay to induce the division manager to agree with their own (exogenous) expectations on future cash flows. Because headquarters designs optimal incentive contracts to induce agreement of beliefs within the organization with their own (exogenous) beliefs, we will denote this case as one of *"visionary"* leadership. In contrast, when headquarters are themselves uncertainty-averse, the optimal contracts induce again division managers to agree with firm headquarters, but now equilibrium beliefs enacted in the firm (that is, the point of agreement) are endogenous and depend on firm characteristics. Because, in this case, beliefs promoted by headquarters for their organization are determined endogenously, we will refer to uncertainty-averse headquarters as *"pragmatic"* leadership. This type of leadership is pragmatic in that it will determine the beliefs that it wishes to induce (in equilibrium) in the organization endogenously, depending on firm characteristics.

Our model predicts that the relative importance of the equity-based and division-based (or pay-for-performance) compensation depends critically on firms' characteristics. Specifically, division-based compensation contracts are optimal in more mature firms, which are less affected by uncertainty on future divisional cash flow. In contrast, at younger firms, especially those involved in the development of new products and technologies that are more exposed to uncertainty on firm's fundamentals, optimal incentive contracts have equity-based components. In the absence of such equity-based compensation, division managers, by being more conservative on the future prospects of their own division than for other divisions in the organization, will value their compensation packages less than the ones offered to other division managers, which can create "envy" in the organization.

Our paper provides a new decision-theoretic foundation for the use of equity-based compensation in organizations. In particular, we show that equity-based compensation can be used by the company HQ to promoted a "shared view" within the organization. The necessity of promoting a shared view stems from the disagreement that emerges endogenously within an organization because of agents' uncertainty aversion. Uncertainty-averse agents will assess the prospects of their own divisions more conservatively than company HQ. In addition, divisional managers will also become more positive about other divisions' prospects and compensation in the firm, generating envy across division managers. Such disagreement is detrimental because it affects adversely incentives and promotes discord in the organization. Our paper argues that equity-based compensation can play an important role in realigning expectations (i.e., beliefs) within an organization and, thus, produce a shared view of the company's prospects, promoting internal consensus.

Our paper is linked to several streams of literature. The first one is the traditional principal-agent theory and the theory of optimal contracts design within organizations. Contract theory builds on the seminal work by Mirrlees (1976) and (1999), Holmstrom (1979) and Grossman and Hart (1983).[4] One of the key results of the early stages of this literature is that, to address a moral hazard (or "hidden action") problem, optimal contracts can be thought as the solution of an inference problem where contractual compensation should be a function of all and only observable variables that are informative on the hidden action selected by the agent.[5]

Closer to our paper is the research belongs to the emerging literature on contract theory under uncertainty. Lee and Rajan (2018) consider the model in the spirit of Innes (1990) and study the optimal incentive contract between a principal and an agent where both parties are uncertainty-averse. The source of uncertainty in the model is the exact probability distribution of the random

[4]See Hart and Holmstrom (1987) and Prendergast (1999) for surveys of the earlier literature on contract theory.

[5]Ravid and Spiegel (1997) show that incentive contracts more directly tailored to shareholder value, such as shareholder equity, are optimal when agents can choose their hidden action from rich sets of possible action-profiles.

cash flow. The paper shows that, contrary to basic case of uncertainty-neutrality of Innes (1990), the optimal contract has equity-like components. Szydlowski (2019) considers a dynamic contracting model where an ambiguity-neutral principal designs an optimal dynamic contract for an ambiguity-averse agent, where the source of ambiguity is the agent's cost of effort. In that setting, uncertainty (specifically, the worst-case scenario) evolves over time depending on firm performance, inducing dynamic contracts with over- and under-compensation with respect to the case where the agent is uncertainty-neutral. Miao and Rivera (2016) consider the optimal contract between uncertainty-averse principal and an uncertainty-neutral but risk-averse agent: they show that the principal's preference for robustness can cause the incentive compatibility constraint to be lax. Carroll (2015) shows that a principal who is uncertain about the actions of an agent will optimally grant the agent a linear contract, to align their payoffs. The main feature of these papers is to consider principal-agents problems in isolation. In contrast, in our paper we consider the problem of incentive contracting within organizations, where the principal (company HQ) design contracts with multiple agents who are exposed to multiple sources of uncertainty.[6]

Our paper is strictly related to the literature on the optimal employee compensation structure and, particularly, on the use of equity-based compensation. Oyer (2004) suggests that equity-based compensation (for example, through stock option plans) have the advantage of adjusting employees' compensation to their outside options, which may be correlated to firm value. Oyer and Schaefer (2005) document that broad-based stock option plans are more common at smaller and riskier firms, and argue that (in calibrations) option-based compensation provide weak incentives to middle-level managers. Rather, option plans seem to be more effective as tools to attract and retain more optimistic employees. Bergman and Jenter (2007) argue that firms adopt option-based compensation to attract (and under-pay) over-optimistic employees. Duchin et al. (2018) document that a change in industry pay in one division of a conglomerate generates spillovers on divisional managerial pay in other divisions of the same firm.

Other papers test the more traditional theory that equity-based compensation has a positive effect on incentives, resulting in better performance ex-post. For example, Hochberg and Lindsey (2010) argue that firms with option-plans that have higher implied incentives exhibit higher sub-

[6] An exception is Garlappi, Giammarino and Lazrak (2017), which shows that group decision-making by individuals with heterogeneous beliefs may generate decisions that have ambiguity-like features.

sequent operating performance, a feature that is concentrated in firms with fewer employees and with higher growth opportunities. Baker, Jensen and Murphy (1988) suggests that non-executive option plans may induce cooperation among employees helping to overcome the free-rider problem implicit in large organizations, resulting in better firm performance. Our paper provides a novel explanation for the seemingly "cooperative" outcome due to equity-based compensation based on the better coordination of beliefs in the organization that is induced by such plans.

Our approach differ from theories where wage structure and equity-based compensation is a way for cash constrained firms to raise "cheap" capital. Core and Guay (2001) argue that firms use stock option compensation plans when facing greater capital requirements and stronger financing constraints. Guiso, Pistaferri, and Schivardi (2013) argue that credit constrained firms offer lower initial wages, but offer a steeper wage progression over time. Kim and Ouimet (2014) argue that ESOP plans allows (financially constrained) firms to save cash by substituting wages with equity compensation, with a smaller effect on incentives (especially in larger firms) due to the free-rider problem. Our approach can explain the optimality of equity-based contracts also for larger firms (such as, for example, Microsoft or Google) which are presumably less affected by financing constraints. In addition, our paper can explain features of the compensation structure in large organizations, such as the use of bonuses, where the value of the year-end bonus for division managers is tied not only to the performance of their division, but also to the performance of the entire organization.[7]

More generally, our paper is linked to a growing literature on the determination of organization styles. Rotemberg and Saloner (1993) argue that, when contracts are incomplete, promotion of a participatory corporate culture by firm leadership can be beneficial in innovative environments. Rotemberg and Saloner (2000) show that a "visionary" CEO who is committed to a certain strategy (or product line) can promote innovation by middle managers in situations where internal competition for product implementation may stifle innovation. Dessein and Santos (2006) examine the trade-off between the benefits of decentralized decision making in adaptive organizations, where agents can make best use of local information, and the benefits of centralized coordination. More recently, Bolton, Brunnermeier, and Veldkamp (2013) argue that "resolute" leaders are able

[7] For example, Ma, Tang, and Gomez (2019) find that in the mutual fund industry it is rather common that fund managers' bonuses are directly linked to the overall profitability of the mutual fund family.

to commit ex-ante to a corporate strategy, avoiding a time-inconsistency problem that otherwise would undermine coordination in the organization.

Our work also contributes to the emerging literature on uncertainty aversion in financial decision making and asset pricing. Uncertainty aversion has been proposed as an alternative to Subjective Expected Utility (SEU) to describe decision making in cases where agents have limited information on probability distributions. This stream of research was motivated by a large body of work documenting important deviations from SEU and the classic Bayesian paradigm (see Etner, Jeleva, and Tallon, 2012). While the degree of ambiguity aversion may vary across treatments and subjects, the presence of ambiguity aversion appears to be a robust experimental regularity. Interestingly, Chew, Ratchford, and Sagi (2018) document that ambiguity-averse behavior is particularly relevant among more educated (and analytically sophisticated) subjects.

Uncertainty aversion has also been shown to be an important driver of asset pricing, providing an explanation for observed behavior that would otherwise be puzzling in the context of SEU. For example, Anderson, Ghysels, and Juergens (2009) find stronger empirical evidence for uncertainty than for traditional risk aversion as a driver of cross-sectional expected returns. Jeong, Kim, and Park (2015) estimate that ambiguity aversion is economically significant and explains up to 45% of the observed equity premium. Boyarchenko (2012) shows that the sudden increase in credit spreads during the financial crisis can be explained by a surge in uncertainty faced by uncertainty-averse market participants. Dimmock et al. (2016) show that ambiguity aversion helps explain several household portfolio choice puzzles, such as low stock market participation, low foreign stock ownership, and high own-company stock ownership.

The paper is organized as follows. Section 1 presents the model. Section 2 discusses the case of a visionary leadership. Section 3 examines the case of a pragmatic leadership. Section 4 presents empirical implications of our paper, and Section 5 concludes with directions for further ongoing research. All proofs are in the appendix.

1 The Model

We consider a firm composed by two divisions, $\tau \in \{A, B\}$. Divisions are run by a division manager, DM_τ, $\tau \in \{A, B\}$, and are supervised by the company headquarters, HQ. Each division is endowed with a single investment project which represents the divisional assets. The investment project of division τ requires that at the beginning of the period, $t = 1$, division manager DM_τ exerts effort a_τ; after that, the project generates a payoff at the end of the period, $t = 2$. Divisional projects can either succeed, S, or fail, F. If successful, division-τ project generates a contractible payoff, Y_τ, which accrues to HQ. If the investment project fails, it generates zero payoff, 0. Let $\tilde{Y}_\tau$ be the random variable representing the (random) payoff from division τ taking values in $\{0, R_\tau\}$. Let q be the (joint) probability distribution of $\tilde{Y}_A$ and $\tilde{Y}_B$.

1.1 Modeling Uncertainty

A key feature of our model is to acknowledge that most corporate decisions are taken without full knowledge of the probability distributions involved, a situation that is characterized as uncertainty (Knight, 1921). Accordingly, and differently from traditional contract theory, we assume that division managers are uncertainty averse. While we will initially assume that company HQ are uncertainty neutral, later we will allow for an uncertainty-averse HQ.

We model uncertainty (or "ambiguity") aversion by adopting the minimum expected utility (MEU) approach of Gilboa and Schmeidler (1989).[8] A key feature of this approach is that economic agents do not have a single prior on future events but, rather, they believe that the probability distribution of future events belongs to a given set M, denoted as the "core beliefs set."[9] Thus,

[8] MEU was originally derived by Gilboa and Schmeidler (1989). An alternative approach is "smooth ambiguity" developed by Klibanoff, Marinacci, and Mukerji (2005). In their model, agents maximize expected felicity of expected utility. Agents are uncertainty averse if the felicity function is concave. Our results follow also in that framework if the felicity function is sufficiently concave. The main results of our paper will hold also in this latter approach, but at the cost of requiring a substantially greater analytical complexity.

[9] Note that in our paper we take the core-beliefs set of agents as a representation of their primitive preferences. The core-beliefs set, however, could be obtained as the outcome of a "micro-foundation" that builds directly on uncertainty on economic fundamentals. In Appendix B, we present a model specification that generates qualitatively identical results, where the source of uncertainty is consumer demand (formally, the proportion of consumers that exhibit a relatively stronger preference for each good produced by the two divisions).

uncertainty-averse agents maximize their MEU utility

$$\mathcal{U} = \min_{\mu \in M} E_\mu \left[u\left(\cdot \right) \right], \tag{1}$$

where μ is a probability distribution over future events, and $u(\cdot)$ is a von-Neumann Morgenstern (vNM) utility function.[10] In addition, following Epstein and Schneider (2010), we assume that uncertainty-averse agents are sophisticated with consistent planning. This means that agents correctly anticipate their future uncertainty aversion and, thus, correctly take into account how they will behave at future dates in different states of the world.[11]

An important property of uncertainty aversion, one which plays a critical role in our paper, is *uncertainty hedging*. Given our two random variables $\tilde{Y}_\tau$, with $\tau \in \{A, B\}$, representing the random returns on the two divisions, we have that for any $\beta \in [0,1]$,we have

$$\beta \min_{\mu \in M} E_\mu \left[u\left(\tilde{Y}_1\right)\right] + (1-\beta) \min_{\mu \in M} E_\mu \left[u\left(\tilde{Y}_2\right)\right] \leq \min_{\mu \in M} \{\beta E_\mu \left[u\left(\tilde{Y}_1\right)\right] + (1-\beta) E_\mu \left[u\left(\tilde{Y}_2\right)\right]\}. \tag{2}$$

Uncertainty hedging is a consequence of the *min* operator in (2), and is a direct implication of the "uncertainty aversion axiom" of Gilboa and Schmeidler (1989). It implies that uncertainty-averse agents prefer to hold a portfolio of uncertain assets (rather than a single uncertain asset). This property can be best understood as the analog of the benefits of diversification in the standard portfolio theory, but when agents are uncertainty averse. It derives from the property that, by holding a portfolio of assets, investors can lower their exposure to underlying uncertainty. We will show that uncertainty hedging also implies that an investor will be more "optimistic" about the returns on a portfolio of assets rather than any single asset in the portfolio.

We model investor uncertainty aversion by assuming that investors are uncertain on the probability distribution of the return of the assets of the two divisions. Following Hansen and Sargent (2001), (2007) and Hansen et al. (2006), we characterize the core beliefs set by using the notion of relative entropy. For given pair of (discrete) probability distributions $(q, \hat{q})$, the *relative entropy* of

[10] In the traditional SEU framework, players have a single prior μ and maximize their expected utility $E_\mu [u(\cdot)]$.

[11] Siniscalchi (2011) describes this framework as preferences over trees.

q with respect to $\hat{q}$ is defined as the Kullback-Leibler divergence of q from $\hat{q}$, and is given by

$$R(q|\hat{q}) \equiv \sum_{i=1}^{N} q^i \ln \frac{q^i}{\hat{q}^i}. \tag{3}$$

The core beliefs set for the uncertainty-averse investors in our economy is then given by

$$M \ \equiv \{q : R(q|\hat{q}) \leq \eta\}, \tag{4}$$

where q is the joint distribution of the returns of the assets of the two divisions, and $\hat{q}$ is a certain, exogenously given "reference" probability distribution (for example, an "uninformed prior" where $\hat{q}_i = 1/N$). Thus, the core beliefs set M is the set of distributions q with a divergence not greater than η with respect to the reference distribution q. The parameter η can be interpreted as representing the extent of uncertainty that is faced by the division managers.[12] From (3), it is easy to see that the relative entropy of q with respect to $\hat{q}$ represents the (expected) log-likelihood ratio of the pairs of distributions $(q, \hat{q})$, when the "true" probability distribution is q. Thus, the core beliefs set M includes the set of probability distributions, q, with the property that, if true, the investor would expect not to reject the ("null") hypothesis $\hat{q}$ in a likelihood-ratio test.[13]

Intuitively, the core belief set M can be interpreted as the set of probability distributions that are not "too unlikely" to be the true (joint) probability distribution that characterizes the two technologies, given the reference distribution $\hat{q}$. Note that a small value of η represents situations where agents have more confidence that the probability distribution $\hat{q}$ is a good representation of the success probability of the two divisions, while a large value of η corresponds to situations where there is great uncertainty on the true probabilities underlying the two investments.[14] It is immediate to verify the following property of the core beliefs set M.

Lemma 1 *Let $\eta < \underline{\eta}\,(\hat{q})$, defined in the appendix. The core beliefs set M is a strictly convex set with smooth boundary.*

[12] As in Epstein and Schneider (2010), Hansen and Sargent (2005), (2007), and (2008), relative entropy can also be interpreted as characterizing the extent of "misspecification error" that affects decision makers.

[13] Note that our results will go through, more generally, as long as the core belief set $\mathcal{M}$ is a strictly convex set with smooth boundaries.

[14] As in Hansen and Sargent (2001), (2007), (2008), and Epstein and Schneider (2010), relative entropy can be interpreted as characterizing the extent of "misspecification error" that affects investors.

Note that Lemma 1 is an implication of the fact that relative entropy $R(q|\hat{q})$ is a strictly convex function of q.[15] Lemma 1 also implies that, for uncertainty-averse agents with positive endowment of the underlying risky assets, the relevant part of the core beliefs set M is a smooth, decreasing and convex function. This property is an implication of the fact that uncertainty-averse agents solving problem (1), will select their probability assessments that lie in the "lower-left" boundary of the core beliefs set M (see Figure 1 for a numerical example).

It is easy to see that restricting investors' beliefs to belong to the core beliefs set (4) has the effect of ruling out probability distributions that are "too far" from the reference probability $\hat{q}$. In other words, the maximum entropy criterion implied by (4) excludes from the core-belief set probability distributions that give "too much" weight to extreme events. In addition, because from Lemma 1 uncertainty-averse investors are essentially concerned about "left-tail" events, we denote this property as "trimming pessimism."

Because there is no closed-form solution for the level set of relative entropy for binomial distributions in (4), for ease of exposition we model the relevant portion of the core beliefs set (namely, the decreasing and convex "lower-left" boundary) by using a lower-dimensional parametrization, as follows. Following Dicks and Fulghieri (2019a), we assume that the success probability of the investment project of a type-τ division depends on the value of an underlying parameter θ_τ, and is denoted by $q_\tau(\theta_\tau)$, with $\theta_\tau \in [\theta_L, \theta_H]$, where $\theta_L \leq \theta_H < \theta_M$. For analytical tractability, we assume that $q_\tau(\theta_\tau) = \phi_\tau p_\tau(\theta_\tau)$, with $\tau \in \{A, B\}$, where $p_\tau(\theta_\tau) \equiv e^{\theta_\tau - \theta_M}$ and ϕ_τ is a scaling factor that will be specified below. Uncertainty-averse agents treat the vector $\vec{\theta} \equiv (\theta_A, \theta_B)$ as ambiguous and assess that $\vec{\theta} \in C^\theta \subset \{(\theta_A, \theta_B) : (\theta_A, \theta_B) \in [\theta_L, \theta_H]^2\}$.[16] We interpret the parameter combination $\vec{\theta}$ as describing the state of the economy at $t = 2$.

Different from Dicks and Fulghieri (2019a), we assume that the returns on investments in the two technologies are potentially correlated. Specifically, we assume that the probability that both project are successful is equal to $q_{SS} \equiv r q_A(\theta_A) q_B(\theta_B)$, while the probability that only project A is successful is $q_{SF} \equiv q_A(\theta_A)(1 - r q_B(\theta_B))$ and the probability that only project B is successful is

[15] For a general discussion, see Theorem 2.5.3 and 2.7.2 of Cover and Thomas (2006). Our results hold, generally, when the core-belief set M is a strictly convex set with smooth boundaries. Note that "rectangular" core-belief sets do not satisfy such condition, thus defeating the benefits of uncertainty hedging of the uncertainty-aversion axiom.

[16] In Figure 2, compare our lower-dimensional approximation of the core-belief set, based on (5), with the one based on relative entropy (4).

$q_{FS} \equiv q_B(\theta_B)(1 - rq_A(\theta_A))$. Agents believe that also the correlation of the returns on investments in the two technologies is also uncertain and that $r \in C^r \equiv [\underline{r}, \bar{r}]$, where $\underline{r} < \frac{e^{-\alpha}}{1+e^{-2\alpha}}$ and $\bar{r} > e^{\alpha}$. Finally, we denote $C \equiv C^\theta \times C^r$ as the set of "core beliefs" of our uncertainty-averse investors. In light of Lemma 1 and subsequent discussion, we assume that for $\vec{\theta} \in C^\theta$ we have that

$$(\theta_A + \theta_B)/2 = \theta_T, \tag{5}$$

where $\theta_T \equiv (\theta_H + \theta_L)/2$.[17] In addition, we set $\alpha \equiv \theta_T - \theta_L$, where the parameter α characterizes the level of uncertainty affecting division investment projects, as perceived by the division managers. We assume, for simplicity, that the level of uncertainty perceived by the two managers, that is, the value of α, is the same for the two divisions, an assumption that can be relaxed.

We will at times benchmark the behavior of uncertainty-averse agents with the behavior of an uncertainty-neutral SEU agent, and we will assume that uncertainty-neutral investors has $\theta_L = \theta_H$, so that she assesses $\theta_\tau = \theta_T$. This assumption guarantees that the uncertainty-neutral investor has the same probability assessment on the return on the two divisions as a well-diversified uncertainty-averse investor (and thus there is no "hard-wired" difference between the to type of investors).

We will assume throughout our paper that all agents, company HQ and both division managers, are risk neutral. The assumption of risk neutrality will allow us to separate the effects of uncertainty aversion, which are the primary focus of our paper, from those of traditional risk aversion.[18] We will also assume that division managers are uncertainty averse, and we will consider two types of HQ. For reasons that will become apparent below, we denote an uncertainty-neutral HQ as *visionary leadership*, while we denote an uncertainty-averse HQ as *pragmatic leadership*.

[17] We note that correlation of returns across divisions is introduced for the specific reason of avoiding the desirability option-like incentive contracts that pay in state SS only. Given our assumption that $(\theta_A+\theta_B)/2 = \theta_T$, these contracts would shield division managers from uncertainty. The adoption of a similar specification of the set C^θ, such as L^1, as in Dicks and Fulghieri (2019b) would make such contracts suboptimal even in the absence of correlation uncertainty.

[18] The presence of risk-aversion for divisional managers will induce a risk sharing element in the optimal compensation structure. The effect of the interaction of risk and uncertainty aversion is important, and is currently under investigation by the authors.

1.2 Uncertainty aversion and incentive contracts

The success probability of a type-τ division project, q_τ, depends on the level of effort, a_τ, exerted by its division manager

$$q_\tau = z_\tau a_\tau p_\tau(\theta_\tau), \tag{6}$$

where z_τ represents the productivity of division τ, and θ_τ is parameter that characterizes the state of the economy at $t = 2$. We assume that exerting effort is costly, and that DM_τ must pay a personal non-pecuniary cost equal to

$$c_\tau(a_\tau) = \frac{a_\tau^{1+\frac{1}{\gamma}}}{k_\tau\left(1+\frac{1}{\gamma}\right)} \tag{7}$$

to implement effort level a_τ (note $k_\tau, \gamma > 0$). Company HQ cannot directly contract on the level of effort exerted by the division managers, creating a moral hazard problem that must be addressed with optimal incentive contracts.

Optimal contract design is constrained by the fact company HQ can only observe the outcome of each division project, $\tilde{Y}_\tau$, $\tau \in \{A, B\}$. We will assume (and later verify) that parameters value are such that the division manager's participation constraint is lax, which implies that under the optimal contract division managers earn (strictly) positive rents. This implies that, because the division manager is risk neutral and protected by limited liability, optimal contracts have no base pay, that is, payments in the case of failure of division projects. Thus, an incentive contract $\hat{\omega}_\tau$ for division manager τ is a triplet $\hat{\omega}_\tau \equiv \{w_\tau, x_\tau, b_\tau\}$, such that HQ pays DM_τ the sum w_τ if his division has a successful project, x_τ if the other division has a successful project, and b_τ if both projects are successful.

A key property of uncertainty aversion is that probability assessment held by decision makers, that is, their "beliefs," are endogenous, and depend on the decision maker's overall exposure to uncertainty. In our setting, this implies that incentive contracts affect a division manager effort through two distinct channels. The first one is the traditional effect of inducing effort by rewarding division managers in the case of success of their investment projects. The second channel depends on the impact of incentive contracts on managerial assessment of the success probability of their projects. Specifically, incentive contract may be used by company HQ to lead uncertainty averse division managers to hold more favorable assessment of the success probability of their division, thus generating a positive effect on effort. This is a new channel and the key driver of our paper.

We now establish a preliminary property of optimal contracts that will allow us to simplify the exposition of our results. It is useful to note that ambiguity by division managers on the degree of correlation between their division projects, r, implies that company HQ will never find it desirable to make payments conditional on the success of both projects.

Lemma 2 *HQ sets $b_\tau = 0$ in the optimal contract.*

Lemma 2 shows that it is always optimal for company HQ to grant division managers only contracts that are linear in projects' payoffs. This property derives from the fact that, because of uncertainty about correlation, payments contingent on both projects succeeding, b_τ, will make division managers more conservative about the joint success probability of both divisions, with an adverse impact on their incentives to exert effort. Thus, company HQ will always find it optimal to set $b_\tau = 0$ and not to expose division managers to uncertainty on the correlation between project outcomes. On the basis of Lemma 2, we can restrict optimal contracts between HQ and division manager τ to consist of a share of their division, w_τ, the pay-for-performance component, and potentially, a share of the other division, x_τ, the cross-pay component, and we set $\omega_\tau \equiv \{w_\tau, x_\tau\}$.

Given an incentive contract ω_τ, division manager τ continuation utility is

$$\hat{u}_\tau \left(a, \omega_\tau; \overrightarrow{\theta}\right) = z_\tau a_\tau p_\tau \left(\theta_\tau\right) w_\tau + z_{\tau'} a_{\tau'} p_{\tau'} \left(\theta_{\tau'}\right) x_\tau, \tag{8}$$

where $a = \{a_\tau, a_{\tau'}\}$ is the action profile selected by the managers of both divisions. Because of uncertainty aversion, the beliefs held by DM_τ, $\overrightarrow{\theta^a}$, are endogenous and are the solution to the minimization problem

$$\overrightarrow{\theta^a}(\omega_\tau, a) = \arg \min_{\overrightarrow{\theta} \in C} \hat{u}_\tau \left(a, \omega_\tau; \overrightarrow{\theta}\right)$$

and are characterized in the following Lemma.

Lemma 3 *Let*

$$\check{\theta}^a_\tau \equiv \theta_T + \frac{1}{2} \ln \frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau a_\tau w_\tau}. \tag{9}$$

The assessment held by an uncertainty-averse division manager endowed with an incentive contract toward his project is

$$\theta_\tau^a(\omega_\tau) = \begin{cases} \theta_L & \check{\theta}_\tau^\tau \leq \theta_L \\ \check{\theta}_\tau^\tau & \check{\theta}_\tau^\tau \in (\theta_L, \theta_H) \\ \theta_H & \check{\theta}_\tau^\tau \geq \theta_H \end{cases} \quad \textit{for} \quad \tau \in \{A, B\}. \tag{10}$$

Lemma 3 shows that the probability assessment, or beliefs, that a division manager holds toward the success of their own project, θ_τ^a, is endogenous and it depends on the incentive contract offered by company HQ, ω_τ. We will say that the agent has *interior beliefs* when $\theta_\tau^a \in (\theta_L, \theta_H)$, in which case, the agent's assessments are equal to $\check{\theta}_\tau^a$ in (9). Otherwise, we will say that the division manager holds *corner beliefs.*

Several important features are emerge from Lemma 3. First, note that if headquarters grants to a division manager only division pay, that is $\omega_\tau \equiv \{w_\tau, 0\}$, the division manager will assess the prospects of his division conservatively and set $\theta_\tau^a = \theta_L$. It is useful to note that this implies that $p_\tau(\theta_L) = e^{-\alpha} p_\tau(\theta_T)$, where α is the uncertainty of the firm and $p_\tau(\theta_T)$ is the belief held by an uncertainty-neutral agent. Second, note that probability assessment held by a divisional manager toward his own division, θ_τ^a, is a (weakly) decreasing function of the pay-for-performance component, w_τ. This property reflects the fact that, holding everything else constant, an increase of the pay-for-performance component, w_τ, in the incentive contract increases the divisional manager's exposure to the uncertainty affecting his own division project, generating a more conservative assessment of its success probability.

In contrast, and symmetrically, the division manager probability assessment θ_τ^a is increasing function of the cross pay x_τ. This property is the consequence of uncertainty-hedging, and its reflects the fact that increasing the exposure to the *other* division uncertainty, will make the division manager more "conservative" about the other division and, consequently, more "optimistic" about its own division. Thus, Lemma 3 shows the key benefit of granting cross-pay: it makes a division manager more confident about their division's prospects when they have a share of the other division. Granting cross pay, however, is costly and provides no *direct* incentive to effort. We will show later that, when there is sufficiently great uncertainty surrounding divisional investment projects (that is, when α is sufficiently large), it is optimal for company HQ to grant cross pay.

Incentive contracts affect the choice of managerial effort as follows. Division manager τ chooses his optimal level of effort a_τ by setting

$$a_\tau(\omega_\tau, a_{\tau'}) = arg \max_{a_\tau} \; U_\tau\left(a, \omega_\tau\right) \equiv z_\tau a_\tau p_\tau\left(\theta_\tau^a\right) w_\tau + z_{\tau'} a_{\tau'} p_{\tau'}\left(\theta_{\tau'}^a\right) x_\tau - \frac{a_\tau^{1+\frac{1}{\gamma}}}{k_\tau\left(1+\frac{1}{\gamma}\right)}, \tag{11}$$

where θ_τ^a is determined in Lemma 3. The optimal level of effort $a_\tau(\omega_\tau, a_{\tau'})$ depends on the composition of his own incentive contract $\{w_\tau, x_\tau\}$, the productivity of his own division, z_τ, as well as level of effort exerted by the manager of the other division, $a_{\tau'}$, and its productivity, $z_{\tau'}$. We have the following.

Theorem 1 *Division manager τ optimally exerts effort $a_\tau(\omega_\tau, a_{\tau'}) = [z_\tau k_\tau p_\tau(\theta_\tau^a) w_\tau]^\gamma$, which is increasing in pay-for-performance pay, w_τ, cross-division pay, x_τ, and the effort exerted by the other division manager, $a_{\tau'}$.*

Theorem 1 provides one of the key drivers of our paper. If division managers are uncertainty neutral, $p_\tau(\theta_\tau^a) = p_\tau(\theta_T)$, giving that their optimal level of effort is equal to

$$a_\tau^N = \left[z_\tau k_\tau p_\tau\left(\theta_T\right) w_\tau\right]^\gamma, \tag{12}$$

where beliefs for uncertainty-neutral division managers are equal to θ_T. It is immediate to verify that a division manager's effort, a_τ^N, is an increasing function of his own division-based pay, w_τ, but is affected by neither their cross-division pay, x_τ, nor the action of the other division manager, $a_{\tau'}$. This means that it is never optimal for company HQ to offer cross pay, and thus the two divisions are effectively operating independently.

In contrast, when divisional managers are uncertainty averse, from Theorem 1 it can immediately be verified that optimal divisional effort it is equal to

$$a_\tau^a = a_\tau^N \left[\frac{p_\tau(\theta_\tau^a)}{p_\tau\left(\theta_T\right)}\right]^\gamma,$$

where divisional managers' beliefs $p_\tau(\theta_\tau^a)$ are determined in Lemma 3 The presence of uncertainty aversion introduces now a link across division managers' optimal effort levels, which is driven by beliefs. This happens because, when division managers are uncertainty averse, Lemma 3 implies that their assessment of their division's success probabilities are endogenous. Specifically, a division manager is more confident about the success probability of his project if he is also granted pay that depends on the payoff from the other division's project, that is, cross pay. In addition, the presence of cross pay makes a division manager's beliefs also an increasing function of both the effort level and the productivity of the other division. This means that effort levels by division managers are strategic complements.

Finally, note that if HQ grants a small level of cross pay, division managers will be conservative, believing $\theta_\tau^\tau = \theta_L$, and will exert very low levels of effort, $a_\tau^a << a_\tau^N$, reflecting their pessimism. If HQ grants a moderate level of cross pay, the division manager will have an interior solution for θ_τ^τ in Lemma 3, leading to an intermediate level of effort. Note that in this case, $a_\tau^a > a_\tau^N$ iff $p_\tau(\theta_\tau^a) > p_\tau(\theta_T)$. Finally, if company HQ grants a very large level of cross pay, division managers will be very optimistic, $\theta_\tau^\tau = \theta_H$, and will be willing to exert a high effort level, with $a_\tau^a >> a_\tau^N$.

The main implication of this section is that a division manager's level of effort depends on both the pay-for-performance pay, that is division-specific pay, and a component that depends on the performance of the other division, the cross pay. Standard principal-agents models suggest that, with uncertainty-neutral agents, is never optimal to make incentive contracts for a division manager contingent on other divisions' performance. Under uncertainty aversion, in contrast, company HQ can use incentive contracts and, specifically, cross pay to motivate division manager effort by inducing more favorable beliefs on project outcomes. In other words, a new and desirable effect of incentive contracts is to determine the level of "optimism" that is prevalent in the organization. The desirable belief structure that company HQs would like to generate in their company will, in turn depend on their beliefs structure and whether they are uncertainty neutral or uncertainty averse themselves, that is whether they have a strong or a pragmatic leadership, situations that which we study next.

2 Visionary Leadership

We consider now the ex-ante problem faced by the company HQ in selecting the optimal incentive pay for the two divisions. We start with the case in which the company HQ is uncertainty neutral and holds a belief $\theta_\tau = \theta_T$ for both divisions. Given the set of incentive contracts offered by HQ, $\omega \equiv \{\omega_A, \omega_B\}$, the division managers choose simultaneously the optimal level of effort exerted, $a_\tau(\omega)$, with $\tau \in \{A, B\}$. Given the (optimal) action profile selected by the two division managers, $\tilde{a}_\tau(\omega)$, we can express the payoff of company HQ, as

$$\pi\left(a, \overrightarrow{\theta}\right) = z_A \tilde{a}_A(\omega) p\left(\theta_T\right)\left(R_A - w_A - x_B\right) + z_B \tilde{a}_B(\omega) p\left(\theta_T\right)\left(R_B - w_B - x_A\right). \quad (13)$$

As a benchmark, it is useful to note that if division managers are uncertainty neutral, division manager effort is given by (12). By direct substitution in (13), it can immediately be determined that the optimal incentive contract for uncertainty neutral managers is equal to $\omega_\tau^N \equiv \{w_\tau^N, 0\}$, where

$$w_\tau^N = \frac{\gamma}{1+\gamma} R_\tau.$$

In addition, the optimal incentive contract w_τ^N will induce each division manager to exert an effort level equal to

$$a_\tau^N = \left[z_\tau k_\tau p_\tau\left(\theta_T\right) \frac{\gamma}{(1+\gamma)} R_\tau\right]^\gamma.$$

If division managers are uncertainty averse, company HQ may choose one of three possible kinds of incentive contracts: it can offer (i) a contract with no cross-pay to any the division manager, (ii) a contract with cross-pay to one division manager, but not the other, or (iii) a contract with cross-pay to both division managers. The optimal incentive contract depends on the level of uncertainty faced by division managers, α, and is characterized as follows.

Theorem 2 *For low levels of uncertainty, $\alpha < \ln 2$, the optimal contract grants only pay-for-performance compensation to division managers. For higher levels of uncertainty, $\alpha > \ln 2$, the optimal contract grants cross pay to division manager. Further, if HQ grants only pay-for-performance compensation, division managers will be relatively conservative: $\theta_\tau^\tau < \theta_T$. If HQ grants division managers cross pay, the optimal contract induces agreement: $\theta_\tau^\tau = \theta_T$*

Theorem 2 shows that when the level of uncertainty faced by division managers is low, specifically for $\alpha < \ln 2$, company HQ grants only pay-for-performance compensation and does not offer cross pay, $x^a_\tau = 0$. Furthermore, in this case, company HQ allows division managers to be conservative, $\theta^\tau_\tau = \theta_T - \alpha$, even if this results in a diminished effort. This happens because, when division managers face low levels of uncertainty, and α is small, company HQ and division managers beliefs are sufficiently aligned so that division manager's uncertainty aversion does not reduce their effort significantly (with respect to the uncertainty-neutral case). This implies that it not worthwhile for the company HQ to improve division manager beliefs by also offering cross pay, which would be costly to them (and with the sole effect of improving effort through its impact on division manager's beliefs). Finally, in this case, the optimal pay-for-performance compensation is the same as the one that would be offered in the case the division manager is uncertainty neutral

$$w^{a,\ell}_\tau = w^N_\tau = \frac{\gamma}{1+\gamma} R_\tau.$$

Note that this incentive contract induces a level of effort by the division manager, a^a_τ, that is lower than that would prevail under uncertainty neutrality, a^N_τ, as follows:

$$a^{a,\ell}_\tau = e^{-\alpha\gamma} a^N_\tau.$$

When the level of uncertainty faced by division managers is sufficiently large, $\alpha \geq \ln 2$, company HQ grants cross pay. This happens because, when uncertainty is sufficiently large, beliefs by held by division managers and company HQ will diverge significantly, and the division manager's pessimism will result in too little effort. In this case, it is worthwhile for the company HQ to elicit greater effort by offering cross pay, through the positive effect of cross pay on the division manager's assessment of the success probability of his division. The optimal incentive contract is now given by

$$w^{a,h}_\tau = \frac{1}{2}\frac{\gamma}{1+\gamma} R_\tau = \frac{1}{2} w^N_\tau, \quad \text{and} \quad x^{a,h}_\tau = \Theta_\tau \frac{1}{2}\frac{\gamma}{1+\gamma} R_\tau = \Theta_\tau \frac{1}{2} w^N_\tau, \tag{14}$$

where $\Theta_\tau \equiv (z_\tau/z_{\tau'})^{1+\gamma}(k_\tau R_\tau/k_{\tau'}R_{\tau'})^\gamma$, and it will induce an effort level equal to

$$a_\tau^h = \frac{1}{2^\gamma} a_\tau^N.$$

It is easy to verify that $a_\tau^h > a_\tau^{a,\ell}$ for $\alpha > \ln 2$, which means that granting cross pay increases division managers' effort levels. Finally, note that by offering an incentive contract with cross pay, HQ induces agreement from the division manager, $\theta_\tau^\tau = \theta_T$. When uncertainty is low, HQ grants only pay-for-performance compensation and accepts divisional manager's pessimism. At greater levels of uncertainty, company HQ find it optimal to boost division managers beliefs through cross pay and align them to their own beliefs. Thus, when headquarters optimally contracts with division managers, their vision spreads through the organization.

Corollary 1 *If divisions are symmetric (same z, k, and R) and there is sufficient uncertainty,* $\alpha > \ln 2$*, the optimal contract is equity:* $\Theta_\tau = 1$.

When divisions are symmetric and the uncertainty is large, we have that the optimal incentive contract allows for cross-pay with $\Theta_\tau = 1$. This implies that $w_\tau^{a,h} = x_\tau^{a,h}$ and the optimal incentive contact is one where each division manager is granted an equal proportions of the payoff of both divisions, that is, an equity contract.

3 Pragmatic Leadership

In this section we consider the optimal incentive contract for division managers when company HQ is uncertainty averse as well. Different from the case of visionary leadership, where company HQ was uncertainty neutral and held "strong" beliefs θ_T for their company, beliefs for an uncertainty averse company HQ are not fixed, but they will be determined endogenously as well. Specifically, beliefs held by uncertainty-averse company HQ will be pragmatically adapted to the company characteristics. Thus, we will identify this type of leadership as the "pragmatic leadership."

Given the action profile selected by the two division managers, $a(\omega) \equiv (a_A(\omega), a_B(\omega))$,we can express the payoff of a uncertainty averse company HQ, as

$$\pi\left(a,\omega,\overrightarrow{\theta}^{HQ}\right)=z_Aa_A(\omega)p\left(\theta_A^{HQ}\right)(R_A-w_A-x_B)+z_Ba_B(\omega)p\left(\theta_B^{HQ}\right)(R_B-w_B-x_A). \quad (15)$$

where $\overrightarrow{\theta}^{HQ}\equiv\{\theta_A^{HQ},\theta_B^{HQ}\}$ represents the company HQ beliefs regarding the success probabilities of the two divisions. We still assume that company HQ and division managers face the same level of uncertainty, α. Similar to the division managers' case, company HQ beliefs are determined by setting

$$\overrightarrow{\theta}^{a}(\omega_\tau,a)=\underset{\overrightarrow{\theta}\in C^{HQ}}{\arg\min}\ \pi\left(a,\omega,\overrightarrow{\theta}^{HQ}\right), \quad (16)$$

where now the HQ core beliefs set, C^{HQ}, is given by

$$C_{HQ}=\left\{\overrightarrow{\theta}:\frac{1}{2}(\theta_A+\theta_B)=\theta_T,\ \text{where } \theta_A,\theta_B\in[\theta_L,\theta_H]\right\}.$$

Similar to Lemma 3, HQ beliefs solve (16) and are given by

$$\theta_\tau^{HQ}(\omega_\tau)=\begin{cases}\theta_L & \check{\theta}_\tau^{HQ}\le\theta_L\\ \check{\theta}_\tau^{HQ} & \check{\theta}_\tau^{HQ}\in(\theta_L,\theta_H)\\ \theta_H & \check{\theta}_\tau^{HQ}\ge\theta_H\end{cases}\quad \text{for}\quad \tau\in\{A,B\}. \quad (17)$$

where now

$$\check{\theta}_\tau^{HQ}\equiv\theta_T+\frac{1}{2}\ln\frac{z_{\tau'}a_{\tau'}(R_{\tau'}-w_{\tau'}-x_\tau)}{z_\tau a_\tau(R_\tau-w_\tau-x_{\tau'})}. \quad (18)$$

Similar to the case examined in the previous section, company HQ can offer their division managers one of three possible contracts. It can offer (i) a contract with no cross-pay to any the division manager, (ii) a contract with cross-pay to one division manager, but not the other, or (iii) a contract with cross-pay to both division managers. Different from the previous section, the optimal incentive contract depends now both on the level of uncertainty faced by division managers, α, and the relative sizes and productivity of the two divisions, and is characterized as follows.

Theorem 3 *For low levels of uncertainty, $\alpha<\ln 2$, the optimal contract grants only pay-for-performance compensation to division managers. For higher levels of uncertainty, $\alpha\ge\ln 2$, the* *optimal contract grants cross pay to at least one division managers, and to both if the divisions are not too dissimilar. In addition, if headquarters grants only pay-for-performance compensation to a manager, that division manager will be relatively conservative: $\theta_\tau^\tau < \theta_\tau^{HQ}$. If headquarters grants both division managers cross pay, headquarters will induce agreement: $\theta_\tau^a = \theta_\tau^{HQ}$.*

If the level of uncertainty is low, $\alpha < \ln 2$, optimal incentive contracts include again only pay-for-performance compensation without cross pay. In this case, optimal incentive contract and divisional effort will be the same as in the case of visionary leaders:

$$w_\tau^{a,\ell} = w_\tau^N = \frac{\gamma}{1+\gamma} R_\tau, \quad a_\tau^{a,\ell} = e^{-\alpha\gamma} a_\tau^N.$$

The difference with the case of visionary leaders is that now company HQ beliefs are endogenous and are given by (17). It is easy to see that company HQ will be relatively more optimistic on the less productive (lower z_τ) division; if the two divisions are equally productive, $z_\tau = z_{\tau'}$, company HQ will hold the same beliefs as the one held by visionary leaders, θ_T. In addition, managers of both divisions will be (weakly) more conservative than company HQ: $\theta_\tau^a = \theta_L \leq \check{\theta}_\tau^{HQ}$ (with an equality only if division τ has sufficiently greater productivity than division τ', so that HQ holds corner beliefs at θ_L).

If uncertainty is sufficiently large, $\alpha \geq \ln 2$, granting a division manager cross pay can be part of an optimal incentive contract. This will have an impact on the beliefs in equilibrium. The properties of the optimal contract depend on relative characteristics of the two divisions. If the divisions are sufficiently similar, the optimal contract grants each manager equity share such that $w_\tau = \frac{\gamma}{2(1+\gamma)} R_\tau$ and $x_\tau = \frac{\gamma}{2(1+\gamma)} R_{\tau'}$.

In contrast, if one division is sufficiently stronger than the other one, it is optimal to grant only pay-for-performance compensation to the stronger division and cross-pay to the weaker division. Specifically, if the expected payoff of division τ, $z_\tau a_\tau R_\tau$, is sufficiently larger than the expected payoff of division τ', $z_{\tau'} a_{\tau'} R_{\tau'}$, it will be optimal to set $w_\tau = \frac{\gamma}{1+\gamma} R_\tau$, $x_\tau = 0$ for the stronger division, and to set $w_{\tau'} = \frac{\gamma}{2(1+\gamma)} R_{\tau'}$ as the pay-for-performance component, and to set the cross-pay component $x_{\tau'}$ so as to induce agreement with HQ. This means that managers of the stronger division will have beliefs (weakly) more conservative that company HQ, while managers of the weaker division will have the same beliefs as company HQ.

Corollary 2 *If the headquarters grants both division managers cross pay, the optimal contract is straight-equity with consistent beliefs in the organization:* $\theta_\tau^\tau = \theta_\tau^{HQ}$.

Note that, at large levels of uncertainty, equity compensation in firms with adaptable leadership is optimal whenever the two divisions are not too dissimilar. This feature differs from the case of visionary leadership, where equity based compensation occurs only when the two divisions are identical. In addition, when the optimal contract is equity-based compensation, the optimal incentive contract equalizes beliefs in the organization: $\theta_\tau^\tau = \theta_\tau^{HQ} \equiv \theta_\tau^{Firm}$. This common level of beliefs in a firm with pragmatic leadership, θ_τ^{Firm}, is endogenous and it depends on the firm's fundamentals, as follows

$$\theta_\tau^{Firm} = \theta_T + \frac{1}{2} \ln \frac{z_{\tau'} a_{\tau'} R_{\tau'}}{z_\tau a_\tau R_\tau}.$$

In a firm with visionary leadership, if the optimal contract grants cross pay, it leads division managers to agree with headquarters' vision for the company: $\theta_\tau^\tau = \theta_T$, where θ_T is exogenous. Thus, in a firm with visionary leadership, the uncertainty-neutral company HQ design incentive contracts to hedge division managers uncertainty and induce them to agree with their uncertainty-neutral beliefs. In contrast, in firms with pragmatic leadership, company HQ share the exposure to uncertainty with division managers. This allows the vision of the firm to be flexible and to adapt to the firm fundamentals.

We conclude this section by pointing out that incentive contracts with cross pay, by introducing a positive externality among division managers, make a division manager's welfare to depend on the behavior an characteristics of other divisions in their firm. In particular, a division manager derives a benefit from exposure to positive attributes of the other division, such as its productivity and the effort level chosen by its division manager. In addition, this positive externality improves managerial incentives to exert effort, through its impact on beliefs, leading to greater divisional managers' welfare. In the spirit of Scharfstein and Stein (2000), we denote this feature of the positive spillover across divison managers' welfare as "socialism."

Corollary 3 *Visionary leaders do not exhibit socialism in their organization. Pragmatic leaders exhibit socialism if there is sufficient uncertainty.*

Because of uncertainty neutrality, a visionary leader will accept any amount of uncertainty, and insulate division managers from their exposure to uncertainty, providing division managers with full insurance from uncertainty. Thus, he will write contracts to optimally limit the uncertainty that his division managers must face. In contrast, a pragmatic leader dislikes uncertainty, so he writes contracts to optimally share uncertainty with his division managers. This optimal uncertainty sharing between company HQ and division managers results in socialistic behavior.

4 Empirical Implications

Our paper has several empirical implications that can help explaining some otherwise puzzling features of the compensation policies adopted by corporations.

1. Firms characterized by high uncertainty, such as young firms, adopt compensation contracts with an equity component. A puzzling feature of the compensation structure of many young firms is the widespread use of equity-based compensation throughout the organization. While equity-based compensation appears to be justified for the members of the top management, such as the CEO, it is less clear why lower-level managers should receive equity-based compensation. This is because equity-based compensation reduces the sensitivity of managerial pay to their action, and thus reduces its effectiveness as an incentive. In other words, equity-based compensation makes divisional managers lose line of sight between their actions and their impact on compensation. This practice is even less justifiable for low-level employees, where the connection between an employee's actions and equity value is even more tenuous.

Our paper provides an explanation for the common occurrence of equity-based compensation. Our paper suggests that equity-based compensation plays two important roles. The first one is to better align the beliefs of members of the organization with the one held by the top management. In particular, absent the equity component in pay, individuals would hold more conservative beliefs than the top management on the expected performance of their unit. Inclusion of the equity-based compensation would have the benefit of better aligning their expectations with the ones held by the top management, improving the overall disposition of the organization. The second benefit is that, because of the improvement of expectations, employees will exert greater effort, improving firm value.

2. Mature firms adopt compensation contracts primarily based on pay-for-performance measures. As firms mature, the level of uncertainty surrounding their business activities decrease, reducing (or even eliminating) the need of equity-based compensation. For these firms, effort levels in the organization is better elicited by the use of pay-for-performance incentive contract, making equity-based compensation redundant. This means that firms should first start, when they are young, with incentive contracts heavily skewed toward equity-based compensation, and then move toward pay-for-performance based contracts as they mature.

3. Optimal compensation in business groups. Our paper has also implications for the compensation structure in business groups. Consider an executive manager in a subsidiary of a business (or family group). Traditional theory would suggest compensation for that this type of managers should depend only on the performance of their subsidiary or business unit. In contrast, compensation for such managers is often tied to the performance of the entire business group. For example, Ma, Tang, and Gomez (2018) study the compensation structure for the mutual funds industry, and find that in about half of their sample, managers' bonuses are directly linked to the overall profitability of the advisor. A similar practice is common in the investment bank industry, where individual bonuses depend also on the overall performance of the intermediary. Such features, which would be difficult to be justified on the basis of risk-aversion only, are consistent with the findings of our paper.

4. Managerial (over)optimism. Our model predicts that managers in the upper echelon of corporate ladders tend to be more optimistic about their firm's future performance. This implies that, rank-and-file managers perceive members of the top management team of a firm (such as CEOs and CFOs) as overconfident and unrealistically optimistic. The role of managerial overconfidence in corporations has been extensively documented (see, for example, Heaton, 2002, and Malmendier and Tate, 2005, among others). Goel and Thakor (2008) suggest that managerial optimism can be the outcome of the managerial selection process, whereby lucky and overconfident managers are more likely to rise to the top positions of companies. Our paper suggests that top managers' optimism is the consequence of uncertainty hedging, and not necessarily the sign of a negative behavioral bias.

5. Entrepreneur CEOs and family wealth. Entrepreneurship is commonly associated with family wealth (Hurst and Lusardi, 2004), and access to family wealth is a primary determinant of entrepreneurship (Levine and Rubinstein, 2017). There are several reasons why family wealth may be associated with greater incentives to become entrepreneurs and, thus, CEOs. These include relaxation of financial constraints and greater diversification opportunities (lower cost of capital). Note that traditional risk-diversification rationales would imply the wealthy families invest in industries with low (or negative) correlation with the bulk of family money. Our paper adds a novel rationale for the association between family wealth and entrepreneurship. Individuals in wealthy families, by virtue of their broad portfolio, benefit more from uncertainty hedging, giving them a comparative advantage in investing in business surrounded by greater uncertainty. As a consequence, owners/CEOs belonging to wealthy families would (endogenously) be characterized by more optimistic views of their companies. These are new and testable implications.

5 Conclusions

In this paper we have examined the impact of uncertainty aversion on the design of optimal incentive contracts in an organization. We have studied the problem faced by a multi-divisional firms, for simplicity with two divisions, where agents may be uncertainty averse. Divisional managers exert unobservable effort that affects the success probability of their division, creating moral hazard. The contracting problem is further complicated by the fact that division managers are uncertainty averse, which makes them unduly conservative (in the eyes of the company HQ) on the success probability of the investments in their divisions. We showed that the structure of optimal incentive contracts depends on the level of uncertainty that affects the firms. For firms with low uncertainty, incentive contracts exhibit only pay-for-performance compensation. For firms characterized by high levels of uncertainty, optimal incentive contracts have also a cross pay compensation or are straight-equity contracts. Our paper can explain how young firms award equity compensation to their employees and then switch to pay-for-performance compensation as they mature.

Our approach can be extended in a number of ways. Specifically, it is interesting to examine the case where division managers are not only uncertainty averse, but also risk averse. Risk aversion by division managers has two distinct effects. First, it creates a welfare cost for awarding cross pay. This welfare cost is due to the traditional negative effect of exposing risk-averse agents to an additional source of risk, and it will offset the benefit of cross pay examined in our paper. This implies that managerial risk aversion produces a trade-off between risk bearing and uncertainty hedging. Second, the presence of risk aversion creates a benefit from having a managerial division pay that is based on relative performance. Following current theory (see, for example, the discussion in DeMarzo and Kaniel, 2017) relative performance would make the pay of a division manager to depend negatively on the performance of the other division, in contrast to the results in our paper. This feature suggests that the presence of uncertainty aversion can help explaining the lack of use of such contracts in reality, due to their effect on beliefs (confidence). These issues are currently under investigation by the authors.

References

[1] Anderson, E., E. Ghysels, and J. Juergens (2009) "The Impact of Risk and Uncertainty on Expected Returns," *Journal of Financial Economics*, **94**: 233-263.

[2] Baker, G., M. Jensen and K. Murphy (1988) "Compensation and Incentives: Practice vs. Theory," *Journal of Finance* **43**: 593–616.

[3] Bergman, N. and D. Jenter (2007) "Employee sentiment and stock option compensation," *Journal of Financial Economics*, **84**: 667-712.

[4] Bolton, P., M. Brunnermeier, and L. Veldkamp (2013) "Leadership, Coordination, and Corporate Culture," *Review of Economic Studies*, **80**: 512-537.

[5] Boyarchenko, N. (2012) "Ambiguity Shifts and the 2007-2008 Financial Crisis," *Journal of Monetary Economics*, **59**: 493-507.

[6] Carroll, G. (2015) "Robustness and Linear Contracts," *American Economic Review*, **105**: 536-563.

[7] Chew, S., M. Ratchford, and J. Sagi (2018) "You Need to Recognize Ambiguity to Avoid It," *The Economic Journal*, **128**: 2480-2506.

[8] Core, J. and W. Guay (2001) "Stock Option Plans for Non-Executive Employees" *Journal of Financial Economics,* **61:** 253–287.

[9] Cover, T. and Thomas, J. (2006) *Elements of Information Theory* (second edition). Hoboken, New Jersey: Wiley-Interscience, John Wiley & Sons, Inc.

[10] de Finetti, B. (1974) *Theory of Probability: A Critical Introductory Treatment.* John Wiley & Sons Ltd.

[11] DeMarzo, P. and R. Kaniel (2017) "Relative Pay for Non-Relative Performance: Keeping Up with the Joneses with Optimal Contracts" CEPR Discussion Paper No. DP11538.

[12] Dessein, W. and T. Santos (2006) "Adaptive Organizations," *Journal of Political Economy*, **114**: 956-995.

[13] Dicks, D. and P. Fulghieri (2019a) "Uncertainty and Systemic Risk," *Journal of Political Economy*, **127**: 1118-1155.

[14] Dicks, D. and P. Fulghieri (2019b) "Uncertainty, Investor Sentiment, and Innovation" SSRN Working Paper 2676854.

[15] Dimmock, S., R. Kouwenberg, O. Mitchell, and K. Peijnenburg (2016) "Ambiguity Aversion and Household Portfolio Choice Puzzles: Empirical Evidence," *Journal of Financial Economics*, **119**: 559-577.

[16] Duchin, R., A. Goldberg, and D. Sosyura (2017) "Spillovers inside Conglomerates: Incentives and Capital," *Review of Financial Studies*, **30**: 1696-1743.

[17] Epstein, L. and M. Schneider (2010) "Ambiguity and Asset Markets," *Annual Review of Financial Economics*, **2**: 315-346.

[18] Etner, J., M. Jeleva, and J. Tallon (2012) "Decision Theory under Ambiguity," *Journal of Economic Surveys*, **26**: 234-270.

[19] Frydman, C. and D. Jenter (2010) "CEO Compensation," *Annual Review of Financial Economics*, **2**: 75-102.

[20] Garlappi, L., R. Giammarino, and A. Lazrak, (2017) "Ambiguity and the Corporation: Group Disagreement and Underinvestment," *Journal of Financial Economics*, **125**: 417-433.

[21] Gilboa, I. and D. Schmeidler (1989), "Maxmin Expected Utility with Non-Unique Prior" *Journal of Mathematical Economics* **18**: 141-153.

[22] Goel, A. and A. Thakor (2008) "Overconfidence, CEO Selection, and Corporate Governance," *Journal of Finance* **63**: 2737-2784.

[23] Grossman, S., and O. Hart, (1983) "An Analysis of the Principal-Agent Problem," *Econometrica,* **51**:7-45

[24] Guiso, L., L. Pistaferri, and F. Schivardi (2013) "Credit within the Firm," *Review of Economic Studies* **80**: 211–247

[25] Hansen, L. and T. Sargent (2001) "Robust Control and Model Uncertainty," *American Economic Review*, **91**: 60-66.

[26] Hansen, L. and T. Sargent (2005) "Robust Estimation and Control under Commitment," *Journal of Economic Theory* **124**: 258-301.

[27] Hansen, L. and T. Sargent (2007) "Recursive Robust Estimation and Control without Commitment," *Journal of Economic Theory*, **136**: 1-27.

[28] Hansen, L. and T. Sargent (2008) *Robustness.* Princeton University Press.

[29] Hansen, L., T. Sargent, G. Turmuhambetova, and N. Williams (2006) "Robust Control and Model Misspecification," *Journal of Economic Theory*, **128:** 45–90.

[30] Hart, O. and B. Holmstrom (1987) "The Theory of Contracts," in: T. Bewley, ed., *Advances in Economic Theory*, Fifth World Congress (Cambridge University Press, Cambridge, UK).

[31] Heaton, J. (2002) "Managerial Optimism and Corporate Finance," *Financial Management* **31**: 33-45

[32] Hochberg, and L. Lindsey (2010) "Incentives, Targeting, and Firm Performance: An Analysis of Non-executive Stock Options," *Review of Financial Studies*, **23:** 4148–4186.

[33] Holmstrom, B. (1979) "Moral Hazard and Observability," *Bell Journal of Economics*, **10:** 74-91.

[34] Hurst, E. and A. Lusardi (2004) "Liquidity Constraints, Household Wealth, and Entrepreneurship," *Journal of Political Economy*, **112**: 319-347.

[35] Innes, R. (1990), "Limited Liability and Incentive Contracting with Ex-Ante Action Choices," *Journal of Economic Theory*, **52:** 45–67.

[36] Jeong, D., H. Kim, and J. Park (2015) "Does Ambiguity Matter: Estimating Asset Pricing Models with a Multiple-Priors Recursive Utility," *Journal of Financial Economics*, **115**: 361-382.

[37] Kim, H. and P. Ouimet (2014) "Broad-Based Employee Stock Ownership: Motives and Outcomes" *Journal of Finance*, **69**: 1273-1319.

[38] Klibanoff, P., M. Marinacci, and S. Mukerji (2005) "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, **73**: 1849–1892.

[39] Knight, F. (1921) *Risk, Uncertainty, and Profit.* Boston: Houghton Mifflin.

[40] Lee, S. and U. Rajan (2018) "Robust Security Design," SSRN Working Paper 2898462.

[41] Levine, R. and Y. Rubinstein (2017) "Smart and Illicit: Who Becomes an Entrepreneur and Do They Earn More?" *Quarterly Journal of Economics*, **132**: 963-1018.

[42] Ma, L., Y. Tang, and J. Gomez (2019) "Portfolio Manager Compensation and in the U.S. Mutual Fund Industry," *Journal of Finance*, **74**: 587-638.

[43] Malmendier, U. and G. Tate (2005) "CEO Overoptimism and Corporate Investment," *Journal of Finance*, **60**: 2661-2700.

[44] Miao, J. and A. Rivera (2016) "Robust Contracts in Continuous Time," *Econometrica*, **84**: 1405-1440.

[45] Mirrlees, J. (1976) "The optimal structure of incentives and authority within an organization," *Bell Journal of Economics*, **7**: 105–131.

[46] Mirrlees, J (1999) "The Theory of Moral Hazard and Unobservable Behaviour: Part 1," *Review of Economic Studies*, **66**: 3–21,

[47] Murphy, K. (1999) "Executive Compensation," *Handbook of Labor Economics*, Vol. 3, Elsevier Science, 2485-2563.

[48] Murphy, K. (2013) "Executive Compensation: Where We Are, and How We Got There" *Handbook of Economics and Finance*, Vol. 2, Part A, Ch. 4, 211-356.

[49] Oyer, P. (2004) "Why Do Firms Use Incentives That Have No Incentive Effects?" *Journal of Finance* ***59**:* 1619-1650.

[50] Oyer, P. and S. Schaefer (2005) "Why Do Some Firms Give Stock Options to All Employees?: An Empirical Examination of Alternative Theories" *Journal of Financial Economics,* **76:** 99-133.

[51] Oyer, P. and S. Schaefer (2011) "Personnel economics: Hiring and incentives," in *Handbook of Labor Economics*, ed. Orley Ashenfelter and David Card, vol. 4, pt. B, chap. 20, 1769–1823. Amsterdam: Elsevier.

[52] Prendergast, C. (1999) "The Provision of Incentives in Firms," *Journal of Economic Literature,* **37**: 7-63.

[53] Ravid, A. and M. Spiegel (1997) "Optimal Financial Contracts for a Start-Up with Unlimited Operating Discretion," *Journal of Financial and Quantitative Analysis*, **32**: 269-286.

[54] Rotemberg, J. and Saloner, G. (1993) "Leadership Styles and Incentives," *Management Science*, **39**: 1299-1318.

[55] Rotemberg, J. and Saloner G. (2000) "Visionaries, Managers, and Strategic Direction," *RAND Journal of Economics*, **31**: 693-716.

[56] Scharfstein, D. and J. Stein (2000) "The Dark Side of Internal Capital Markets: Divisional Rent-Seeking and Inefficient Investment" *Journal of Finance* **55**: 2537-2564.

[57] Siniscalchi, M. (2011) "Dynamic Choice under Ambiguity," *Theoretical Economics*, **6**: 379–421.

[58] Szydlowski, M. (2019) "Ambiguity in Dynamic Contracts," SSRN Working Paper 1986186

A Appendix: Proofs

Proof of Lemma 1. Let $x = \{x_A, x_B\}$ be a vector of indicator variables for success of type A and B projects: $x \in \{0,1\}^2$. If the probability of success is $q = \{q_A, q_B\}$ the probability of x is $q_A^{x_A} q_B^{x_B} (1-q_A)^{1-x_A} (1-q_B)^{1-x_B}$. Thus, the relative entropy of q w.r.t. $\hat{q}$ is

$$R(q|\hat{q}) = \sum_{x\in\{0,1\}^2} q_A^{x_A} q_B^{x_B} (1-q_A)^{1-x_A} (1-q_B)^{1-x_B} \ln \frac{q_A^{x_A} q_B^{x_B} (1-q_A)^{1-x_A} (1-q_B)^{1-x_B}}{\hat{q}_A^{x_A} \hat{q}_B^{x_B} (1-\hat{q}_A)^{1-x_A} (1-\hat{q}_B)^{1-x_B}}.$$

Because the log of a product is the sum of the logs, and probabilities sum to one, we can express this as

$$R(q|\hat{q}) = R(q_A|\hat{q}_A) + R(q_B|\hat{q}_B)$$

where $R(q_\tau|\hat{q}_\tau) = q_\tau \ln \frac{q_\tau}{\hat{q}_\tau} + (1-q_\tau)\ln\frac{1-q_\tau}{1-\hat{q}_\tau}$. Because $\frac{\partial^2 R}{\partial q_\tau^2} = \frac{\hat{q}_\tau}{q_\tau} + \frac{1-\hat{q}_\tau}{1-q_\tau}$, $R(q_\tau|\hat{q}_\tau)$ is strictly convex in q_τ. Thus, $R(q|\hat{q})$ is strictly convex in $q = \{q_A, q_B\}$. Also, $\lim_{q_\tau\to 0^+} R(q_\tau|\hat{q}_\tau) = \ln\frac{1}{1-\hat{q}_\tau}$ and $\lim_{q_\tau\to 1^-} R(q_\tau|\hat{q}_\tau) = \ln\frac{1}{\hat{q}_\tau}$. Define $\underline{\eta}(\hat{q}) = \min_{\chi\in Q}\ln\frac{1}{\chi}$, where $Q = \{\hat{q}_A, 1-\hat{q}_A, \hat{q}_B, 1-\hat{q}_B\}$. Therefore, if $\eta < \underline{\eta}(\hat{q})$, $\mathcal{M}$, as the lower level set of a strictly convex function, is strictly convex. Note this generalizes: Theorem 2.5.3 of Cover and Thomas (2006) shows relative entropy is additively separable in independent variables, and Theorem 2.7.2 shows it is strictly convex. ■

Proof of Lemma 2. Suppose first HQ has visionary leadership: HQ is uncertainty neutral and believes that $r = 1$ and $\theta = \theta_T$, so we can express HQ's objective as

$$\Pi = z_A a_A e^{\theta_T - \theta_M} \left(R_A - w_A - x_B\right) + z_B a_B e^{\theta_T - \theta_M} \left(R_B - w_B - x_A\right) - z_A a_A z_B a_B e^{2(\theta_T - \theta_M)} \left(b_A + b_B\right).$$

DM$_\tau$'s objective is $U_\tau = \min_{\overrightarrow{\theta} \in C, r \in [\underline{r}, \bar{r}]} u_\tau$, where

$$u_\tau = z_\tau a_\tau e^{\theta_\tau - \theta_M} w_\tau + z_{\tau'} a_{\tau'} e^{\theta_\tau - \theta_M} x_\tau + r z_\tau z_{\tau'} a_\tau a_{\tau'} e^{2(\theta_T - \theta_M)} b_\tau - \frac{1}{k_\tau \left(1 + \frac{1}{\gamma}\right)} a_\tau^{1+\frac{1}{\gamma}}.$$

We will later show that both division managers exert effort in equilibrium, so $a_{\tau'} > 0$. Off-equilibrium, if $a_{\tau'} = 0$, joint success would have zero probability, so b_τ does not affect the objective or constraints, and thus $b_\tau = 0$ is WLOG optimal. Thus, consider the case when $a_{\tau'} > 0$. The worst-case scenario for DM$_\tau$ is $r_\tau = \underline{r}$ if $b_\tau > 0$ but $r_\tau = \bar{r}$ if $b_\tau < 0$. Thus, HQ solves

$$\begin{aligned} \max \quad & \Pi \\ s.t. \quad & a_\tau \in \arg\max_a U_\tau, \ \overrightarrow{\theta}^\tau \in \arg\min_{\overrightarrow{\theta} \in C} u_\tau \end{aligned}$$

Let L be the Lagrangian function, λ_τ be the multiplier for the incentive compatibility constraint for DM τ, and let κ_τ be the multiplier for the worst-case scenario for DM$_\tau$. Because w_τ and x_τ are positive, u is strictly convex-concave in (θ, a), so FOCs are sufficient for a minimum on θ and for a maximum on a. Because each DM's participation constraints is lax, ICs will bind, so $\lambda_\tau > 0$. The worst-case scenario will either have a corner solution, $\kappa_\tau = 0$, or an interior solution, $\kappa_\tau > 0$.

If $\kappa_\tau = 0$, DM$_\tau$ has corner beliefs, so either $\theta^\tau_\tau = \theta_L$ or θ_H. $\frac{\partial u_\tau}{\partial a_\tau} = \left(w_\tau + r z_{\tau'} a_{\tau'} e^{\theta^\tau_{\tau'} - \theta_M} b_\tau\right) z_\tau e^{\theta^\tau_\tau - \theta_M} - \frac{1}{k_\tau} a_\tau^{\frac{1}{\gamma}}$, and the spanning condition of Jewitt (1988) is satisfied, so FOC for the IC is sufficient: $\frac{\partial u_\tau}{\partial a_\tau} = 0$. Thus, $L = \Pi + \sum_{\tau \in \{A,B\}} \lambda_\tau \frac{\partial u_\tau}{\partial a_\tau}$. Therefore,

$$\frac{\partial L}{\partial w_\tau} = -z_\tau a_\tau e^{\theta_T - \theta_M} + \lambda_\tau z_\tau e^{\theta^\tau_\tau - \theta_M},$$

and

$$\frac{\partial L}{\partial b_\tau} = -z_\tau z_{\tau'} a_\tau a_{\tau'} e^{2(\theta_T - \theta_M)} + \lambda_\tau r_\tau z_\tau z_{\tau'} a_{\tau'} e^{2(\theta_T - \theta_M)},$$

because $\theta_A + \theta_B = 2\theta_T$, which implies $p\left(\theta^\tau_\tau\right) p\left(\theta^\tau_{\tau'}\right) = e^{2(\theta_T - \theta_M)}$. Thus,

$$\frac{\partial L}{\partial b_\tau} - z_{\tau'} a_{\tau'} e^{\theta_T - \theta_M} \frac{\partial L}{\partial w_\tau} = \lambda_\tau z_\tau z_{\tau'} a_{\tau'} e^{2(\theta_T - \theta_M)} \left[r_\tau - e^{\theta^\tau_\tau - \theta_T}\right]$$

Suppose to the contrary that $b_\tau > 0$, which implies $r = \underline{r} < e^{-\alpha} \leq e^{\theta^\tau_\tau - \theta_T}$, so $\frac{\partial L}{\partial b_\tau} < z_{\tau'} a_{\tau'} e^{\theta_T - \theta_M} \frac{\partial L}{\partial w_\tau}$. Because w_τ has no upper bound, $\frac{\partial L}{\partial w_\tau} \leq 0$, so $\frac{\partial L}{\partial b_\tau} < 0$ for all $b_\tau > 0$. Therefore, $b_\tau \leq 0$. Suppose to the contrary that $b_\tau < 0$, which by limited liability requires that $w_\tau > 0$, so $\frac{\partial L}{\partial w_\tau} = 0$. This implies that $r = \bar{r} > e^\alpha \geq e^{\theta^\tau_\tau - \theta_T}$, so $\frac{\partial L}{\partial b_\tau} > 0$ for all $b_\tau < 0$. Therefore, $b_\tau = 0$.

If $\kappa_\tau > 0$, DM$_\tau$ has interior beliefs. Substituting in the constraint that $\theta^\tau_\tau + \theta^\tau_{\tau'} = 2\theta_T$ and differentiating, $\frac{\partial u_\tau}{\partial \theta^\tau_\tau} = z_\tau a_\tau e^{\theta^\tau_\tau - \theta_M} w_\tau - z_{\tau'} a_{\tau'} e^{\theta^\tau_{\tau'} - \theta_M} x_\tau$, and the worst case scenario satisfies $\frac{\partial u_\tau}{\partial \theta^\tau_\tau} = 0$. Therefore, the $L = \Pi + \sum_{\tau \in \{A,B\}} \left\{\lambda_\tau \frac{\partial u_\tau}{\partial a_\tau} - \kappa_\tau \frac{\partial u_\tau}{\partial \theta^\tau_\tau}\right\}$. Note

$$\frac{\partial L}{\partial x_\tau} = -z_{\tau'} a_{\tau'} e^{\theta_T - \theta_M} + \kappa_\tau z_{\tau'} a_{\tau'} e^{\theta^\tau_{\tau'} - \theta_M},$$

so $\frac{\partial L}{\partial x_\tau} = 0$ iff $\kappa_\tau = e^{\theta_\tau^\tau - \theta_T}$. This implies

$$\begin{aligned}\frac{\partial L}{\partial w_\tau} &= -z_\tau a_\tau e^{\theta_T - \theta_M} + \lambda_\tau z_\tau e^{\theta_\tau^\tau - \theta_M} - \kappa_\tau z_\tau a_\tau e^{\theta_\tau^\tau - \theta_M} \\ &= -z_\tau a_\tau e^{\theta_T - \theta_M}\left[1 + e^{2\left(\theta_\tau^\tau - \theta_T\right)}\right] + \lambda_\tau z_\tau e^{\theta_\tau^\tau - \theta_M}\end{aligned}$$

and

$$\frac{\partial L}{\partial b_\tau} = -z_\tau z_{\tau'} a_\tau a_{\tau'} e^{2(\theta_T - \theta_M)} + \lambda_\tau r_\tau z_\tau z_{\tau'} a_{\tau'} e^{2(\theta_T - \theta_M)},$$

so

$$\frac{\partial L}{\partial b_\tau} - z_{\tau'} a_{\tau'} \frac{e^{\theta_T - \theta_M}}{1 + e^{2\left(\theta_\tau^\tau - \theta_T\right)}} \frac{\partial L}{\partial w_\tau} = \lambda_\tau z_\tau z_{\tau'} a_{\tau'} e^{2(\theta_T - \theta_M)}\left[r_\tau - \frac{e^{\theta_\tau^\tau - \theta_T}}{1 + e^{2\left(\theta_\tau^\tau - \theta_T\right)}}\right]$$

Because $\frac{x}{1+x^2}$ is maximized at $x = 1$, $\frac{e^{\theta_\tau^\tau - \theta_T}}{1+e^{2(\theta_\tau^\tau - \theta_T)}} \leq \frac{1}{2}$ for all θ_τ^τ. Because $r_\tau = \bar{r} > e^\alpha > 1$, and $\frac{\partial L}{\partial w_\tau} = 0$, $\frac{\partial L}{\partial b_\tau} > 0$ for all $b_\tau < 0$. Similarly, $\frac{\partial L}{\partial b_\tau} < 0$ for all $b_\tau > 0$ because $r = \underline{r} < \frac{e^{-\alpha}}{1+e^{-2\alpha}}$. Therefore, it is optimal to set $b_\tau = 0$.

Therefore, correlation uncertainty makes it inefficient for HQ to grant DMs payment contingent on both projects succeeding. If HQ is uncertainty averse as well, it becomes even more costly to grant this kind of pay. Finally, if both HQ and DMs were uncertainty neutral, then HQ would be indifferent between using b and w, so the claim follows WLOG. ■

Proof of Lemma 3. By Lemma 2, DM$_\tau$ has the objective $U_\tau(a) = \min_{\overrightarrow{\theta} \in C} u_\tau(a; \theta)$, where

$$u_\tau\left(a; \overrightarrow{\theta}\right) = z_\tau a_\tau e^{\theta_\tau - \theta_M} w_\tau + z_{\tau'} a_{\tau'} e^{\theta_{\tau'} - \theta_M} x_\tau - \frac{1}{k_\tau\left(1 + \frac{1}{\gamma}\right)} a_\tau^{1+\frac{1}{\gamma}}.$$

Also, for all $\overrightarrow{\theta} \in C$, $\frac{1}{2}(\theta_A + \theta_B) = \theta_T$. Define L_τ as the Lagrangian for the worst-case scenario of DM$_\tau$, and λ as the multiplier for the constraint on θ. Thus, $\frac{\partial L_\tau}{\partial \theta_\tau} = -z_\tau a_\tau e^{\theta_\tau - \theta_M} w_\tau + \frac{\lambda}{2}$ and $\frac{\partial L_\tau}{\partial \theta_{\tau'}} = -z_{\tau'} a_{\tau'} e^{\theta_{\tau'} - \theta_M} x_\tau + \frac{\lambda}{2}$, so $\frac{\partial L_\tau}{\partial \theta_\tau} = \frac{\partial L_\tau}{\partial \theta_{\tau'}} = 0$ iff $z_\tau a_\tau e^{\theta_\tau - \theta_M} w_\tau = z_{\tau'} a_{\tau'} e^{\theta_{\tau'} - \theta_M} x_\tau$, or equivalently, $\theta_\tau - \theta_{\tau'} = \ln \frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau a_\tau w_\tau}$. Because $\theta_\tau + \theta_{\tau'} = 2\theta_T$, this holds iff $\theta_\tau = \check{\theta}_\tau^\tau$, where

$$\check{\theta}_\tau^\tau = \theta_T + \frac{1}{2} \ln \frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau a_\tau w_\tau}.$$

u_τ is strictly convex in θ, so FOCs are sufficient for a minimum. Thus, if $\check{\theta}_\tau^\tau \in [\theta_L, \theta_H]$, $\theta_\tau^\tau = \check{\theta}_\tau^\tau$. If $\check{\theta}_\tau^\tau < \theta_L$, $\frac{\partial L_\tau}{\partial \theta_\tau} < 0$ for all $\theta \in [\theta_L, \theta_H]$, so $\theta_\tau^\tau = \theta_L$. If $\check{\theta}_\tau^\tau > \theta_H$, $\frac{\partial L_\tau}{\partial \theta_\tau} > 0$ for all $\theta \in [\theta_L, \theta_H]$, so $\theta_\tau^\tau = \theta_H$. Therefore, (10) corresponds to the worst-case scenario. ■

Proof of Theorem 1. DM$_\tau$ has the objective $U_\tau(a) = \min_{\overrightarrow{\theta} \in C} u_\tau(a; \theta)$, where

$$u_\tau\left(a; \overrightarrow{\theta}\right) = z_\tau a_\tau e^{\theta_\tau - \theta_M} w_\tau + z_{\tau'} a_{\tau'} e^{\theta_{\tau'} - \theta_M} x_\tau - \frac{1}{k_\tau\left(1 + \frac{1}{\gamma}\right)} a_\tau^{1+\frac{1}{\gamma}}.$$

Applying the minimax theorem, $\frac{\partial u}{\partial \theta}|_{\overrightarrow{\theta} = \overrightarrow{\theta}^\tau} \frac{d\overrightarrow{\theta}^\tau}{da} = 0$, so $\frac{dU}{da} = \frac{\partial u}{\partial a} = z_\tau e^{\theta_\tau - \theta_M} w_\tau - \frac{1}{k_\tau} a_\tau^{\frac{1}{\gamma}}$. Because U is strictly concave, FOCs are sufficient for a maximum, so $a_\tau = \left[z_\tau k_\tau e^{\theta_\tau^\tau - \theta_M} w_\tau\right]^\gamma$, where θ_τ^τ is given by Lemma 3. By Lemma 3, the belief of DM$_\tau$ depends on his action. For small a_τ, $a_\tau < e^{-2\alpha} \frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau w_\tau}$, DM$_\tau$ believes $\theta_\tau = \theta_H$, so $a_\tau = \left[z_\tau k_\tau e^{\theta_H - \theta_M} w_\tau\right]^\gamma$. Note we are in this region iff $k_\tau^\gamma e^{\gamma(\theta_T - \theta_M)} z_\tau^{\gamma+1} w_\tau^{\gamma+1} < e^{-\alpha(\gamma+2)} z_{\tau'} a_{\tau'} x_\tau$. For moderate a_τ, $a_\tau \in \left[e^{-2\alpha} \frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau w_\tau}, e^{2\alpha} \frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau w_\tau}\right]$, $\theta_\tau^\tau = \check{\theta}_\tau^\tau$, so $e^{\theta_\tau^\tau - \theta_M} = e^{\theta_T - \theta_M}\left[\frac{z_{\tau'} a_{\tau'} x_\tau}{z_\tau a_\tau w_\tau}\right]^{\frac{1}{2}}$. Because $a_\tau = \left[z_\tau k_\tau e^{\theta_\tau^\tau - \theta_M} w_\tau\right]^\gamma$, this implies

$$a_\tau = \left[z_\tau z_{\tau'} k_\tau^2 e^{2(\theta_T - \theta_M)} a_{\tau'} x_\tau w_\tau\right]^{\frac{\gamma}{\gamma+2}}.$$

Note we are in this region iff $e^{-\alpha(\gamma+2)} z_{\tau'} a_{\tau'} x_\tau \leq k_\tau^\gamma e^{\gamma(\theta_T - \theta_M)} z_\tau^{\gamma+1} w_\tau^{\gamma+1} \leq e^{\alpha(\gamma+2)} z_{\tau'} a_{\tau'} x_\tau$. Finally, for big a_τ, $a_\tau > e^{2\alpha} \frac{z'_\tau a_{\tau'} x_\tau}{z_\tau w_\tau}$, DM$_\tau$ believes $\theta_\tau = \theta_L$, so $a_\tau = \left[z_\tau k_\tau e^{\theta_L - \theta_M} w_\tau\right]^\gamma$. We are on this region iff $k_\tau^\gamma e^{\gamma(\theta_T - \theta_M)} z_\tau^{\gamma+1} w_\tau^{\gamma+1} > e^{\alpha(\gamma+2)} z_{\tau'} a_{\tau'} x_\tau$. Therefore, a_τ is strictly increasing in w_τ and weakly increasing in x_τ (strictly increasing for interior θ). When the division manager is uncertainty neutral, $\theta_H = \theta_T = \theta_L$ and $\alpha = 0$, so $a_\tau = \left[z_\tau k_\tau e^{\theta_T - \theta_M} w_\tau\right]^\gamma$. ■

The following Lemma will be helpful for the proofs of Theorems 2 and 3.

Lemma 4 *If HQ grants cross pay, setting $x_\tau > 0$, DM_τ will have interior beliefs.*

Proof of Lemma 4. HQ has the objective $\Pi = \min_{\overrightarrow{\theta} \in C_{HQ}} \pi$, where

$$\pi = z_A a_A e^{\theta_A - \theta_M} (R_A - w_A - x_B) + z_B a_B e^{\theta_B - \theta_M} (R_B - w_B - x_A),$$

where $C_{HQ} = \{(\theta_T, \theta_T)\}$ if HQ is uncertainty neutral, while $C_{HQ} = \left\{ \overrightarrow{\theta} \,|\, \frac{1}{2} (\theta_A + \theta_B) = \theta_T, \text{ where } \theta_A, \theta_B \in [\theta_L, \theta_H] \right\}$ if HQ is uncertainty averse. Optimal actions a_τ are from Theorem 1.

$$\frac{d\Pi}{dx_\tau} = \frac{\partial \Pi}{\partial x_\tau} + \left[\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{da_{\tau'}}{da_\tau} \right] \frac{da_\tau}{dx_\tau}.$$

Applying the minimax theorem, $\frac{\partial \Pi}{\partial x_\tau} = -z_{\tau'} a_{\tau'} e^{\theta_\tau^{HQ} - \theta_M} < 0$. Suppose to the contrary it is optimal to grant very large x_τ, so $z_{\tau'} a_{\tau'} x_\tau > e^{2\alpha} z_\tau w_\tau a_\tau$. For such a large x_τ, $\theta_\tau^\tau = \theta_H$ and $\frac{da_\tau}{dx_\tau} = 0$, so $\frac{d\Pi}{dx_\tau} = \frac{\partial \Pi}{\partial x_\tau} = -a_{\tau'} e^{\theta_\tau^{HQ} - \theta_M} < 0$, so it would be optimal to lower x_τ. Thus, it must be that $z_{\tau'} a_{\tau'} x_\tau \leq e^{2\alpha} z_\tau w_\tau a_\tau$. Similarly, it cannot be that $z_{\tau'} a_{\tau'} x_\tau \in \left[0, e^{-2\alpha} z_\tau w_\tau a_\tau\right)$, because $\frac{da_\tau}{dx_\tau} = 0$ on that region as well, so $\frac{d\Pi}{dx_\tau} < 0$ on that region. Therefore, we can restrict attention to contracts that set $x_\tau = 0$ and those that induce interior beliefs: $z_{\tau'} a_{\tau'} x_\tau \in \left[e^{-2\alpha} z_\tau w_\tau a_\tau, e^{2\alpha} z_\tau w_\tau a_\tau\right]$. ■

Proof of Theorem 2. By Lemma 4, there are three types of contracts to consider: those that grant only division pay to both managers, those that grant both division and cross pay to both managers, and those that grant only division pay to one manager but both division and cross pay to the other.

If HQ grants only division pay to both DMs: $x_\tau = 0$ and $a_\tau = \left[z_\tau k_\tau e^{\theta_L - \theta_M} w_\tau\right]^\gamma$, so

$$\Pi = z_A^{1+\gamma} k_A^\gamma e^{-\alpha\gamma} \left[p(\theta_T)\right]^{\gamma+1} w_A^\gamma (R_A - w_A) + z_B^{1+\gamma} k_B^\gamma e^{-\alpha\gamma} \left[p(\theta_T)\right]^{\gamma+1} w_B^\gamma (R_B - w_B),$$

Note $\frac{\partial \Pi}{\partial w_\tau} = z_\tau^{1+\gamma} k_\tau^\gamma e^{-\alpha\gamma} w_\tau^{\gamma-1} \left[p(\theta_T)\right]^{\gamma+1} (\gamma R_\tau - (\gamma+1) w_\tau)$, so $w_\tau = \frac{\gamma}{\gamma+1} R_\tau$. This gives HQ payoff

$$\Pi = e^{-\alpha\gamma} \left[p(\theta_T)\right]^{\gamma+1} \frac{\gamma^\gamma}{(\gamma+1)^{\gamma+1}} \left[z_A^{\gamma+1} k_A^\gamma R_A^{\gamma+1} + z_B^{\gamma+1} k_B^\gamma R_B^{\gamma+1}\right].$$

Suppose instead that uncertainty-neutral HQ sets $x_\tau > 0$ for both DMs. From Lemma 4, DMs have interior beliefs: $e^{-2\alpha} z_\tau w_\tau a_\tau \leq z_{\tau'} a_{\tau'} x_\tau \leq e^{2\alpha} z_\tau w_\tau a_\tau$. From Theorem 1, DMs set $a_\tau = \left[z_\tau z_{\tau'} k_\tau^2 e^{2(\theta_T - \theta_M)} a_{\tau'} x_\tau w_\tau\right]^{\frac{\gamma}{\gamma+2}}$. Thus, HQ maximizes

$$\Pi = z_A a_A p(\theta_T) (R_A - w_A - x_B) + z_B a_B p(\theta_T) (R_B - w_B - x_A).$$

Thus,

$$\frac{d\Pi}{dw_\tau} = \frac{\partial \Pi}{\partial w_\tau} + \left(\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{\partial a_{\tau'}}{\partial a_\tau} \right) \frac{\partial a_\tau}{\partial w_\tau}$$
$$\frac{d\Pi}{dx_\tau} = \frac{\partial \Pi}{\partial x_\tau} + \left(\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{\partial a_{\tau'}}{\partial a_\tau} \right) \frac{\partial a_\tau}{\partial x_\tau}$$

Note that $\frac{\partial \Pi}{\partial a_\tau} > 0$ and $\frac{\partial a_{\tau'}}{\partial a_\tau} > 0$, so $\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{\partial a_{\tau'}}{\partial a_\tau} > 0$. Because HQ sets both $w_\tau > 0$ and $x_\tau > 0$, $\frac{d\Pi}{dw_\tau} = \frac{d\Pi}{dx_\tau} = 0$, which implies $\frac{\frac{\partial a_\tau}{\partial x_\tau}}{\frac{\partial a_\tau}{\partial w_\tau}} = \frac{\frac{\partial \Pi}{\partial x_\tau}}{\frac{\partial \Pi}{\partial w_\tau}}$. Because $\frac{\partial \Pi}{\partial w_\tau} = -z_\tau a_\tau p(\theta_T)$, $\frac{\partial \Pi}{\partial x_\tau} = -z_{\tau'} a_{\tau'} p(\theta_T)$, $\frac{\partial a_\tau}{\partial w_\tau} = \frac{\gamma}{\gamma+2} \frac{a_\tau}{w_\tau}$, and $\frac{\partial a_\tau}{\partial x_\tau} = \frac{\gamma}{\gamma+2} \frac{a_\tau}{x_\tau}$, this implies $x_\tau = \frac{z_\tau a_\tau}{z_{\tau'} a_{\tau'}} w_\tau$. Thus, $a_\tau = \left[z_\tau k_\tau e^{\theta_T - \theta_M} w_\tau\right]^\gamma$. Substituting in x_τ and a_τ, HQ's objective becomes

$$\Pi = z_A^{1+\gamma} k_A^\gamma \left[p(\theta_T)\right]^{1+\gamma} w_A^\gamma (R_A - 2w_A) + z_B^{1+\gamma} k_B^\gamma \left[p(\theta_T)\right]^{1+\gamma} w_B^\gamma (R_B - 2w_B),$$

so

$$\frac{\partial \Pi}{\partial w_\tau} = z_\tau^{1+\gamma} k_\tau^\gamma \left[p(\theta_T)\right]^{1+\gamma} w_\tau^{\gamma-1} \left[\gamma R_\tau - 2(1+\gamma) w_\tau\right].$$

Therefore, $\frac{\partial \Pi}{\partial w_\tau}=0$ iff $w_\tau=\frac{1}{2}\frac{\gamma}{1+\gamma}R_\tau$. By substitution, $a_\tau=z_\tau^\gamma k_\tau^\gamma\left[p\left(\theta_T\right)\right]^\gamma\left[\frac{1}{2}\right]^\gamma\left[\frac{\gamma}{1+\gamma}\right]^\gamma R_\tau^\gamma$, $x_\tau=\frac{z_\tau^{1+\gamma}k_\tau^\gamma R_\tau^\gamma}{z_{\tau'}^{1+\gamma}k_{\tau'}^\gamma R_{\tau'}^\gamma}\frac{1}{2}\frac{\gamma}{1+\gamma}R_\tau$, and HQ earns

$$\Pi=\left[\frac{1}{2}\right]^\gamma\frac{\gamma^\gamma}{(1+\gamma)^{\gamma+1}}\left[p\left(\theta_T\right)\right]^{1+\gamma}\left\{z_A^{1+\gamma}k_A^\gamma R_A^{1+\gamma}+z_B^{1+\gamma}k_B^\gamma R_B^{1+\gamma}\right\}$$

Note that it is better to induce interior beliefs iff $\frac{1}{2^\gamma}>e^{-\alpha\gamma}$, or equivalently, iff $\alpha>\ln 2$.

Suppose to the contrary that HQ grants cross pay to DM$_A$ but not DM$_B$: $x_A>0$ but $x_B=0$. Because $a_A=\left[z_A z_B k_A^2 e^{2(\theta_T-\theta_M)}a_B x_A w_A\right]^{\frac{\gamma}{\gamma+2}}$ and $a_B=\left[z_B k_B e^{\theta_L-\theta_M}w_B\right]^\gamma$, by same argument as above, $x_A=\frac{z_A a_A}{z_B a_B}w_A$, so the HQ maximizes it payoff, which can be expressed as

$$\Pi=z_A^{1+\gamma}k_A^\gamma\left[p\left(\theta_T\right)\right]^{1+\gamma}w_A^\gamma\left(R_A-2w_A\right)+e^{-\alpha\gamma}z_B^{1+\gamma}k_B^\gamma\left[p\left(\theta_T\right)\right]^{1+\gamma}w_B^\gamma\left(R_B-w_B\right),$$

by offering the optimal cross pay contract to DM$_A$, $w_A=\frac{1}{2}\frac{\gamma}{1+\gamma}R_A$, $x_A=\frac{z_A^{1+\gamma}k_A^\gamma R_A^\gamma}{z_B^{1+\gamma}k_B^\gamma R_B^\gamma}\frac{1}{2}\frac{\gamma}{1+\gamma}R_A$, and the optimal division-based pay contract to DM$_B$, $w_B=\frac{\gamma}{\gamma+1}R_B$ and $x_B=0$. This gives HQ payoff

$$\Pi=\frac{\gamma^\gamma}{(1+\gamma)^{\gamma+1}}\left[p\left(\theta_T\right)\right]^{1+\gamma}\left\{\left[\frac{1}{2}\right]^\gamma z_A^{1+\gamma}k_A^\gamma R_A^{1+\gamma}+e^{-\alpha\gamma}z_B^{\gamma+1}k_B^\gamma R_B^{\gamma+1}\right\}.$$

Note that the payoff from division A is the same as if HQ induces interior beliefs, while the payoff from division B is the same as if HQ induces corner beliefs, so it is optimal to provide the same type of contract to both. Formally, if $\alpha<\ln 2$, it is better to offer both DMs only division pay, while if $\alpha>\ln 2$, it is better to offer both DMs cross pay.

Finally, if HQ grants only division-based pay, $x_\tau=0$, so by Lemma 3, $\theta_\tau^\tau=\theta_L<\theta_T$, the belief of HQ. Thus, the division manager will be conservative. If HQ grants cross pay, Theorem 2 showed that the optimal cross pay contract sets $x_\tau=\frac{z_\tau a_\tau}{z_{\tau'}a_{\tau'}}w$, so $\theta_\tau^\tau=\theta_T$ by Lemma 3. ■

Proof of Corollary 1. If $\alpha>\ln 2$, Theorem 2 showed $w_\tau=\frac{1}{2}\frac{\gamma}{1+\gamma}R_\tau$ and $x_\tau=\frac{z_\tau^{1+\gamma}k_\tau^\gamma R_\tau^\gamma}{z_{\tau'}^{1+\gamma}k_{\tau'}^\gamma R_{\tau'}^\gamma}\frac{1}{2}\frac{\gamma}{1+\gamma}R_\tau$. If the divisions are symmetric, $z_A=z_B$, $k_A=k_B$, and $R_A=R_B=R$, $w_\tau=x_\tau$: each division manager receives $\frac{1}{2}\frac{\gamma}{1+\gamma}$ share of the firm. ■

Proof of Theorem 3. HQ has flexible leadership – they are uncertainty averse. Thus, their objective is $\Pi=\min_{\vec{\theta}\in C_{HQ}}\pi$, where

$$\pi=z_A a_A e^{\theta_A-\theta_M}\left(R_A-w_A-x_B\right)+z_B a_B e^{\theta_B-\theta_M}\left(R_B-w_B-x_A\right),$$

and $C_{HQ}=\left\{\vec{\theta}\,|\,\frac{1}{2}\left(\theta_A+\theta_B\right)=\theta_T,\text{ where }\theta_A,\theta_B\in\left[\theta_L,\theta_H\right]\right\}$.[19] Beliefs of HQ are similar to Lemma 3, except that $\check{\theta}_\tau^{HQ}\equiv\theta_T+\frac{1}{2}\ln\frac{z_{\tau'}a_{\tau'}\left(R_{\tau'}-w_{\tau'}-x_\tau\right)}{z_\tau a_\tau\left(R_\tau-w_\tau-x_{\tau'}\right)}$. Theorem 1 gives the optimal actions of DMs given contracts, and Lemma 3 gives beliefs of DMs.

Suppose HQ has interior beliefs: $\theta_\tau^{HQ}=\check{\theta}_\tau^{HQ}$, so

$$\Pi=2p\left(\theta_T\right)\left[z_A a_A\left(R_A-w_A-x_B\right)z_B a_B\left(R_B-w_B-x_A\right)\right]^{\frac{1}{2}}.$$

By Lemma 4, we only need to consider contracts that set $x_\tau=0$ or induce DM$_\tau$ to have interior beliefs. If HQ grants only division pay to both DMs, $x_\tau=0$, by Theorem 1 $a_\tau=\left[z_\tau k_\tau e^{\theta_L-\theta_M}w_\tau\right]^\gamma$, which implies

$$\Pi=2e^{-\alpha\gamma}\left[p\left(\theta_T\right)\right]^{\gamma+1}z_A^{\frac{\gamma+1}{2}}z_B^{\frac{\gamma+1}{2}}k_A^{\frac{\gamma}{2}}k_B^{\frac{\gamma}{2}}w_A^{\frac{\gamma}{2}}\left(R_A-w_A\right)^{\frac{1}{2}}w_B^{\frac{\gamma}{2}}\left(R_B-w_B\right)^{\frac{1}{2}}.$$

Thus,

$$\frac{\partial\Pi}{\partial w_\tau}=\frac{\Pi}{w_\tau\left(R_\tau-w_\tau\right)}\left\{\gamma R_\tau-\left(\gamma+1\right)w_\tau\right\}$$

so $\frac{\partial\Pi}{\partial w_\tau}=0$ iff $w_\tau=\frac{\gamma}{1+\gamma}R_\tau$. As the minimum of strictly concave functions, the objective is strictly concave, FOCs

[19] If HQ had a different α, results are similar.

are sufficient for a maximum, so this it the optimal contract that grants only division pay, giving HQ payoff

$$\Pi = 2e^{-\alpha\gamma} \frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}} \left[p\left(\theta_T\right)\right]^{\gamma+1} z_A^{\frac{\gamma+1}{2}} z_B^{\frac{\gamma+1}{2}} k_A^{\frac{\gamma}{2}} k_B^{\frac{\gamma}{2}} R_A^{\frac{\gamma+1}{2}} R_B^{\frac{\gamma+1}{2}}.$$

Suppose that HQ wants to give both DMs interior beliefs: $x_\tau > 0$. Note that

$$\frac{d\Pi}{dw_\tau} = \frac{\partial \Pi}{\partial w_\tau} + \left[\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{\partial a_{\tau'}}{\partial a_\tau}\right] \frac{\partial a_\tau}{\partial w_\tau},$$

because the minimax theorem implies that θ terms disappear.[20] Similarly,

$$\frac{d\Pi}{dx_\tau} = \frac{\partial \Pi}{\partial x_\tau} + \left[\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{\partial a_{\tau'}}{\partial a_\tau}\right] \frac{\partial a_\tau}{\partial x_\tau}.$$

Because Π is increasing in effort, $\frac{\partial \Pi}{\partial a_\tau}$ and $\frac{\partial \Pi}{\partial a_{\tau'}}$ are strictly positive, and because there are strategic complementarities in effort, $\frac{\partial a_{\tau'}}{\partial a_\tau} \geq 0$. Therefore, $\left[\frac{\partial \Pi}{\partial a_\tau} + \frac{\partial \Pi}{\partial a_{\tau'}} \frac{\partial a_{\tau'}}{\partial a_\tau}\right] > 0$. Therefore, $\frac{\frac{\partial a_\tau}{\partial x_\tau}}{\frac{\partial a_\tau}{\partial w_\tau}} = \frac{\frac{\partial \Pi}{\partial x_\tau}}{\frac{\partial \Pi}{\partial w_\tau}}$. We can express HQ's payoff

$$\Pi = z_A a_A p\left(\theta_A^{HQ}\right)\left(R_A - w_A - x_B\right) + z_B a_B p\left(\theta_B^{HQ}\right)\left(R_B - w_B - x_A\right),$$

Thus, $\frac{\partial \Pi}{\partial w_\tau} = -z_\tau a_\tau p\left(\theta_\tau^{HQ}\right)$ and $\frac{\partial \Pi}{\partial x_\tau} = -z_{\tau'} a_{\tau'} p\left(\theta_{\tau'}^{HQ}\right)$. Therefore, $\frac{\frac{\partial \Pi}{\partial x_\tau}}{\frac{\partial \Pi}{\partial w_\tau}} = \frac{z_{\tau'} a_{\tau'} p\left(\theta_{\tau'}^{HQ}\right)}{z_\tau a_\tau p\left(\theta_\tau^{HQ}\right)}$. Because DMs' problems are identical to those in Theorem 2, their optimal effort given a contract is the same, so $\frac{\partial a_\tau}{\partial w_\tau} = \frac{\gamma}{\gamma+2} \frac{a_\tau}{w_\tau}$ and $\frac{\partial a_\tau}{\partial x_\tau} = \frac{\gamma}{\gamma+2} \frac{a_\tau}{x_\tau}$, which implies $\frac{\frac{\partial a_\tau}{\partial x_\tau}}{\frac{\partial a_\tau}{\partial w_\tau}} = \frac{w_\tau}{x_\tau}$. Therefore, $x_\tau = \frac{z_\tau a_\tau}{z_{\tau'} a_{\tau'}} \frac{p\left(\theta_\tau^{HQ}\right)}{p\left(\theta_{\tau'}^{HQ}\right)} w_\tau$. By Lemma 3, this implies $\theta_\tau^\tau = \theta_\tau^{HQ}$, and that $\frac{w_A}{x_A} = \frac{R_A - w_A - x_B}{R_B - w_B - x_A} = \frac{x_B}{w_B}$. Define $T_\tau \equiv R_\tau - w_\tau - x_{\tau'}$ and $m_\tau \equiv \frac{w_\tau}{T_\tau}$. Thus, $x_\tau = m_\tau T_{\tau'}$. Substituting back into T_τ, this implies $T_\tau = \frac{1}{1+m_A+m_B} R_\tau$. Therefore, $w_\tau = \frac{m_\tau}{1+m_A+m_B} R_\tau$ and $x_\tau = \frac{m_\tau}{1+m_A+m_B} R_{\tau'}$.

Thus, HQ optimal grants DM$_\tau$ equity share $\beta_\tau = \frac{m_\tau}{1+m_A+m_B}$ of the firm: $w_\tau = \beta_\tau R_\tau$ and $x_\tau = \beta_\tau R_{\tau'}$. By Theorem 1, $a_\tau = \left[z_\tau z_{\tau'} k_\tau^2 e^{2(\theta_T - \theta_M)} a_{\tau'} x_\tau w_\tau\right]^{\frac{\gamma}{\gamma+2}}$, which implies (both DMs select effort optimally)

$$a_\tau = e^{\gamma(\theta_T - \theta_M)} z_\tau^{\frac{\gamma}{2}} z_{\tau'}^{\frac{\gamma}{2}} k_\tau^{\frac{\gamma}{2}\frac{(\gamma+2)}{\gamma+1}} k_{\tau'}^{\frac{\gamma}{2}\frac{\gamma}{\gamma+1}} x_{\tau'}^{\frac{\gamma}{4}\frac{\gamma}{\gamma+1}} w_{\tau'}^{\frac{\gamma}{4}\frac{\gamma}{\gamma+1}} x_\tau^{\frac{\gamma}{4}\frac{(\gamma+2)}{\gamma+1}} w_\tau^{\frac{\gamma}{4}\frac{(\gamma+2)}{\gamma+1}}$$

Because HQ pays straight equity, $w_\tau = \beta_\tau R_\tau$ and $x_\tau = \beta_\tau R_{\tau'}$,

$$a_\tau = \beta_{\tau'}^{\frac{\gamma}{2}\frac{\gamma}{\gamma+1}} \beta_\tau^{\frac{\gamma}{2}\frac{\gamma+2}{\gamma+1}} e^{\gamma(\theta_T - \theta_M)} z_\tau^{\frac{\gamma}{2}} z_{\tau'}^{\frac{\gamma}{2}} k_\tau^{\frac{\gamma}{2}\frac{\gamma+2}{\gamma+1}} k_{\tau'}^{\frac{\gamma}{2}\frac{\gamma}{\gamma+1}} R_\tau^{\frac{\gamma}{2}} R_{\tau'}^{\frac{\gamma}{2}}.$$

Substituting in, this gives HQ

$$\Pi = 2p\left(\theta_T\right)\left[e^{2\gamma(\theta_T - \theta_M)} z_A^{\gamma+1} k_A^{\gamma} R_A^{\gamma+1} z_B^{\gamma+1} k_B^{\gamma} R_B^{\gamma+1}\right]^{\frac{1}{2}} \beta_A^{\frac{\gamma}{2}} \beta_B^{\frac{\gamma}{2}} \left(1 - \beta_A - \beta_B\right).$$

Therefore, HQ maximizes its payoff by choosing equity shares to maximize $\Upsilon = \beta_A^{\frac{\gamma}{2}} \beta_B^{\frac{\gamma}{2}} \left(1 - \beta_A - \beta_B\right)$. Note $\frac{\partial \Upsilon}{\partial \beta_\tau} = \beta_\tau^{\frac{\gamma}{2}-1} \beta_{\tau'}^{\frac{\gamma}{2}} \left[\frac{\gamma}{2}\left(1 - \beta_\tau - \beta_{\tau'}\right) - \beta_\tau\right]$, so $\frac{\partial \Upsilon}{\partial \beta_\tau} = 0$ iff $(2+\gamma)\beta_\tau + \gamma \beta_{\tau'} = \gamma$. Because this holds for both divisions, $\frac{\partial \Upsilon}{\partial \beta_{\tau'}} = 0$ as well, $\beta_A = \beta_B = \frac{\gamma}{2(1+\gamma)}$. Therefore, HQ receives payoff

$$\Pi = 2\left[\frac{1}{2}\right]^{\gamma} \frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}} \left[p\left(\theta_T\right)\right]^{1+\gamma} z_A^{\frac{\gamma+1}{2}} z_B^{\frac{\gamma+1}{2}} k_A^{\frac{\gamma}{2}} k_B^{\frac{\gamma}{2}} R_A^{\frac{\gamma+1}{2}} R_B^{\frac{\gamma+1}{2}}.$$

Therefore, this is better for HQ iff $\alpha > \ln 2$. Proof of asymmetric contract case available on request.

[20] If we have corner beliefs, $\frac{\partial \theta_\tau^{HQ}}{\partial w_\tau} = \frac{\partial \theta_{\tau'}^{HQ}}{\partial w_\tau} = 0$, because $\overrightarrow{\theta}^{HQ}$ is constant under corner beliefs. If beliefs are interior, by identical logic of Lemma 3, $\frac{\partial \Pi}{\partial \theta_\tau^{HQ}} = \frac{\partial \Pi}{\partial \theta_{\tau'}^{HQ}}$, and $\frac{\partial \theta_\tau^{HQ}}{\partial w_\tau} + \frac{\partial \theta_{\tau'}^{HQ}}{\partial w_\tau} = \frac{\partial}{\partial w_\tau}\left[2\theta_T\right] = 0$.

If HQ grants only division-based pay to a division manager, the division manager will be conservative, believing the worst about their division: $\theta_\tau^\tau = \theta_L$. When HQ grants cross-pay, $\theta_\tau^\tau = \theta_\tau^{HQ}$, so HQ contracts to produce agreement with that division manager. ■

Proof of Corollary 2. As shown in the proof of Theorem 3, HQ will grant cross pay to both DMs only if α is large enough and HQ has interior beliefs. In this case, each DM receives share $\frac{\gamma}{2(\gamma+1)}$ of the firm. When HQ grants cross-pay, the proof of Theorem 3 showed that HQ always contracted to produce agreement with that DM. ■

Proof of Corollary 3. If HQ has a visionary leader, Theorem 2 showed that there are two types of contracts offered in equilibrium. If α is small, $w_\tau = \frac{\gamma}{\gamma+1} R_\tau$, $x_\tau = 0$ and $a_\tau = \left[z_\tau k_\tau e^{\theta_L - \theta_M} \frac{\gamma}{\gamma+1} R_\tau\right]^\gamma$, so DM$_\tau$ earns payoff $U_\tau = z_\tau^{1+\gamma} k_\tau^\gamma e^{(\gamma+1)(\theta_L-\theta_M)} R_\tau^{\gamma+1} \frac{\gamma^{\gamma+1}}{(\gamma+1)^{\gamma+2}}$. If α is big, HQ grants $w_\tau = \frac{\gamma}{2(\gamma+1)} R_\tau$ and $x_\tau = \frac{z_\tau^{1+\gamma} k_\tau^\gamma R_\tau^\gamma}{z_{\tau'}^{1+\gamma} k_{\tau'}^\gamma R_{\tau'}^\gamma} \frac{\gamma}{2(\gamma+1)} R_\tau$, inducing DM$_\tau$ to exert effort $a_\tau = \left[z_\tau k_\tau e^{\theta_T - \theta_M} \frac{\gamma}{2(\gamma+1)} R_\tau\right]^\gamma$, giving utility $U_\tau = e^{(\gamma+1)(\theta_T-\theta_M)} z_\tau^{\gamma+1} k_\tau^\gamma \left[\frac{\gamma}{2(\gamma+1)}\right]^{(\gamma+1)} R_\tau^{\gamma+1} \frac{\gamma+2}{\gamma+1}$. In both cases, U_τ is increasing in z_τ, k_τ, and R_τ, but is not affected by $z_{\tau'}$, $R_{\tau'}$, or $k_{\tau'}$.

Suppose HQ has a flexible leader. By Theorem 3, if HQ grants no cross pay, $w_\tau = \frac{\gamma}{\gamma+1} R_\tau$, $x_\tau = 0$ and $a_\tau = \left[z_\tau k_\tau e^{\theta_L - \theta_M} \frac{\gamma}{\gamma+1} R_\tau\right]^\gamma$, so DM$_\tau$ earns payoff $U_\tau = z_\tau^{1+\gamma} k_\tau^\gamma e^{(\gamma+1)(\theta_L-\theta_M)} R_\tau^{\gamma+1} \frac{\gamma^{\gamma+1}}{(\gamma+1)^{\gamma+2}}$. This depends only on division τ characteristics, so this firm does not exhibit socialism. In contrast, if HQ grants cross pay to both DMs, $w_\tau = x_{\tau'} = \frac{\gamma}{2(\gamma+1)} R_\tau$, so

$$a_\tau = \left[\frac{\gamma}{2(\gamma+1)}\right]^\gamma e^{\gamma(\theta_T-\theta_M)} z_\tau^{\frac{\gamma}{2}} z_{\tau'}^{\frac{\gamma}{2}} k_\tau^{\frac{\gamma}{2}\frac{\gamma+2}{\gamma+1}} k_{\tau'}^{\frac{\gamma}{2}\frac{\gamma}{\gamma+1}} R_\tau^{\frac{\gamma}{2}} R_{\tau'}^{\frac{\gamma}{2}}.$$

HQ only writes this contract when they have interior beliefs and induce agreement with DMs, DMs have interior beliefs, so

$$U_\tau = \left[z_\tau^{1+\gamma} k_\tau^\gamma z_{\tau'}^{1+\gamma} k_{\tau'}^\gamma R_\tau^{\gamma+1} R_{\tau'}^{\gamma+1}\right]^{\frac{1}{2}} e^{(\gamma+1)(\theta_T-\theta_M)} \left[\frac{\gamma}{2(\gamma+1)}\right]^{\gamma+1} \frac{\gamma+2}{\gamma+1},$$

which is increasing in each element of $\{z_\tau, k_\tau, R_\tau\}_{\tau\in\{A,B\}}$. Thus, this firm will exhibit socialism. ■

B Appendix: Demand Uncertainty

A key ingredient of our paper is that program (1) is a strictly convex programming problem which generates "interior beliefs" for well-diversified portfolios. In the main body of the paper, the possibility of such interior beliefs is a consequence of (strict) convexity of the relative entropy function $R(\cdot)$, which produces a strictly convex core beliefs set M (see Figure 1). Thus, no specific parametric restriction on the joint probability p is needed to generate our results. In this appendix, we present an alternative "micro-foundation" where interior beliefs are the outcome of uncertainty about consumer demand. All results in our paper remain qualitatively the same in this specification.

Consider a simple extension of our model. Each division specializes in the production of goods of type $\tau \in \{A, B\}$. At $t = 1$, division managers exert effort, a_τ, to improve productivity, a decision made under demand uncertainty (as described below). At $t = 2$, consumer demand is revealed and production decisions of firms are made. For tractability, we assume that division τ has production costs $c_\tau(Q_\tau) = K_\tau Q_\tau$, and that the division manager's effort, a_τ, lowers, at a cost $\xi(a_\tau) = \frac{\kappa}{2} a_\tau^2$, the per-unit production cost: $K_\tau = K_0 - K_1 a_\tau$. For interior solutions, assume $\kappa > \frac{K_1^2}{2\delta}$. For simplicity, we will assume linear contracts: division manager is compensated with s_τ base pay, ω_τ of their own division, and χ_τ of the other division.

There are two types of consumers, type A and type B, with a total mass of 1. Consumers value both goods, as well as the numeraire (which represent consumption outside the firm), but each consumer values one good more that the other, which determines their type. The price of the numeraire is fixed to 1, while the price of type τ good, P_τ, is determined in equilibrium. For simplicity, we assume quadratic utility for each type of consumer. Thus

$$U^\tau(q_\tau^\tau, q_{\tau'}^\tau) = (D + \Delta) q_\tau^\tau - \frac{\delta}{2}(q_\tau^\tau)^2 + D q_{\tau'}^\tau - \frac{\delta}{2}(q_{\tau'}^\tau)^2 + w - P_\tau q_\tau^\tau - P_{\tau'} q_{\tau'}^\tau,$$

where D, Δ, and δ are strictly positive parameters. For simplicity, we assume that w and D large enough so that consumers (in equilibrium) always consume a positive amount from both divisions. It is easy to verify that the consumer τ's demand function for good τ is $q_\tau^\tau = \frac{1}{\delta}(D + \Delta - P_\tau)$, and for good τ' is $q_{\tau'}^\tau = \frac{1}{\delta}(D - P_{\tau'})$. Let $m_\tau \in [m_L, m_H]$ be the proportion of consumers of type τ, with $m_A + m_B = 1$. Market clearing condition for good τ requires that $m_\tau q_\tau^\tau + m_{\tau'} q_\tau^{\tau'} = Q_\tau$, where Q_τ is the output of a division τ. Thus, market clearing requires that

$$P_\tau(Q_\tau) = D + m_\tau \Delta - \delta Q_\tau,$$

and the price of type-τ goods is increasing in m_τ. Because divisions know m_τ when making their production decisions Q_τ, they maximize

$$\pi_\tau(Q_\tau) = P_\tau(Q_\tau) Q_\tau - K_\tau Q_\tau,$$

which gives

$$Q_\tau = \frac{D + m_\tau \Delta - K_\tau}{2\delta}.$$

Letting $\Pi_\tau = \max_{Q_\tau} \pi(Q_\tau)$, ex post profits for division τ is

$$\Pi_\tau = \frac{[D + m_\tau \Delta - K_\tau]^2}{4\delta}.$$

Thus, the division manager's payoff is

$$\begin{aligned} \min_{\{m_\tau, m_{\tau'}\}} \mathcal{U} \quad &\equiv \quad \omega_\tau \frac{[D + m_\tau \Delta - K_\tau(a_\tau)]^2}{4\delta} + \chi_\tau \left[\frac{[D + m_{\tau'} \Delta - K_{\tau'}(a_{\tau'})]^2}{4\delta} \right] + s_\tau \\ s.t. \quad & \quad m_\tau + m_{\tau'} = 1, \end{aligned}$$

which is a (strictly) convex programming problem, with the same qualitative properties as (1). Further, it can quickly be verified that the optimal effort is increasing in both pay-performance sensitivity, ω_τ, and cross pay, $\chi_{\tau'}$.

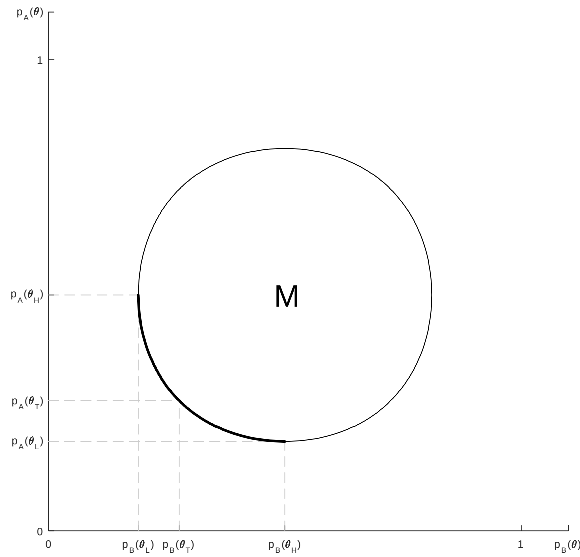

Figure 1: Core belief set under relative entropy.

This figure displays the core of belief set $\mathcal{M}$ when the maximum relative entropy criteria (4) is applied. It shows the set of probability distributions $p = (p_A, p_B)$ that satisfy $\{p|R(p|\hat{p}) \leq \tilde{\eta}\}$ when $\hat{p}_A = \hat{p}_B = \frac{1}{2}$ and $\tilde{\eta} = \frac{3}{10}\ln 2$. If $p_B = \hat{p}_B = \frac{1}{2}$, the relative entropy criteria implies $p_A \in [0.189, 0.811]$. In contrast, if $p_A = p_B$, then the relative entropy criterion implies $p_A \in [0.276, 0.724]$. For division managers with positive exposure on both divisions, $(w_\tau, x_\tau) \geq 0$, the relevant portion of the core beliefs set is given by the lower left boundary, which is bolded in the figure. If an uncertainty-averse division manager had only pay-for-performance, he would assess that his effort is productive with probability 18.9%. In contrast, if the division manager has full equity compensation, $w_\tau = x_\tau$, he would believe that his effort is productive with probability 27.6%.

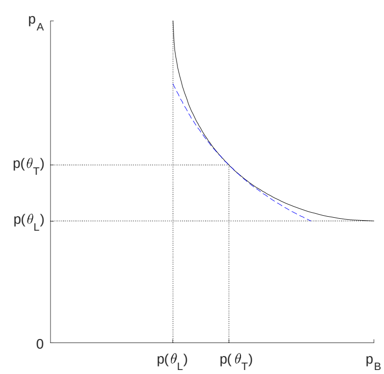

Figure 2: Relative Entropy (solid line) vs. Simplified Specification (dashed line)

The figure compares the lower left portion of the core beliefs set, M, that is obtained under the relative entropy criterion in eq. (4), the solid line, vs. the lower-dimensional specification of eq. (5), the dashed line.