Volatility, Intermediaries, and Exchange Rates

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Abstract

We study exchange rate determination through financial intermediaries. We propose a model in which the participants in the FX market are intermediaries subject to value-at-risk constraints. Higher volatility translates into tighter leverage constraints. Therefore, intermediaries require higher returns to hold foreign assets and the foreign currency is expected to appreciate. Estimated by the simulated method of moments, our model quantitatively resolves the Backus-Smith puzzle, the forward premium puzzle, and the exchange rate volatility puzzle and explains deviations from covered interest rate parity. The model generates new implications for exchange rates and capital flows consistent with the data.

Keywords: Volatility, Financial Intermediaries, Exchange Rates, Currency Risk, Value-at-Risk

JEL classification: G15, G20, F31

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1 Introduction

Exchange rates are puzzling in many respects. First, exchange rates are disconnected from economic fundamentals, especially the relative consumption growth rate, which is in sharp contrast to the implications of most international macro-finance models (Backus and Smith, 1993). Second, contrary to the uncovered interest rate parity (UIP), high-interest-rate currencies do not depreciate. Instead, they tend to appreciate in subsequent periods (Hansen and Hodrick, 1980; Fama, 1984), and this is known as the "forward premium puzzle". Third, it is difficult to obtain exchange rate volatility that is close to actual data in standard international macro-finance models (Chari et al., 2002; Brandt et al., 2006). Finally, covered interest rate parity (CIP), a classic no-arbitrage condition in currency markets, was violated for a decade following the global financial crisis (Du et al., 2018). In this paper, we quantitatively resolve these puzzles by introducing leveraged financial intermediaries subject to value-at-risk (VaR) constraints into an international asset pricing model.

Financial intermediaries are major participants in the foreign exchange (FX) market. More than 85% of turnover in the FX market involves financial institutions according to recent BIS triennial surveys. Among the advanced economies, BIS reporting banks hold more than half of these countries' external claims and liabilities. Therefore, it is natural for us to explore how exchange rates are driven by intermediaries' behaviors.

In light of the dominant role of financial intermediaries in the FX market, Gabaix and Maggiori (2015) (GM hereafter) develop an exchange rate theory based on frictional intermediaries that are constrained from taking leverage. In this paper, we introduce the feature of time-varying leverage constraints into an otherwise standard international asset pricing model for estimation and quantitative study. The model has two ex ante identical countries, home and foreign. Both countries have representative households and intermediaries. Households only have access to a risk-free money market account with the local intermediaries. The intermediaries take deposits and invest in a local risky asset, a foreign risky asset and an international bond with an endogenous home bias. Both intermediaries face VaR constraints such that the size of the balance sheet cannot exceed a fraction of their market values (Gertler and Kiyotaki, 2010). The fraction increases with the volatility in the economy (Adrian and Shin, 2014). In equilibrium, constrained from taking leverage, intermediaries cannot make investments despite the presence of positive risk-adjusted excess returns. When the home country has higher volatility, its intermediaries' leverage constraints tighten. Therefore, home intermediaries require a higher expected excess return than foreign intermediaries on the same asset. An expected foreign appreciation accommodates this return difference.

We solve the model using a global projection method and then estimate it using the simulated method of moments (SMM). The estimated model can quantitatively resolve the four exchange rate puzzles referenced above. We resolve the Backus-Smith puzzle by replacing the standard consumption-based Euler equations with intermediary-based versions such that consumption and exchange rates are not necessarily related. Regarding the forward premium puzzle, the increase in home volatility simultaneously results in a lower home risk-free rate and subsequent foreign appreciation. The exchange rate volatility better matches the data, as we introduce a new source of exchange rate fluctuations, i.e., time-varying leverage constraints. Finally, the tightened banking regulations after the global financial crises constrain the intermediaries from engaging in arbitrage in the FX swap market and produce deviations from CIP. Moreover, the model generates cyclicality in CIP deviations consistent with empirical evidence that the deviations are large when the home currency (US dollar) is strong and when volatility is high. We assess the values of our estimated parameters by showing that the parameter values are consistent with our external validation measures.

Our model has several new implications regarding exchange rates and capital flows that are consistent with the data. First, despite being disconnected from consumption, our model implies that exchange rates should be correlated with measures of leverage constraints. We use the relative TED spread between the US and other G10 countries as such a measure and find a negative correlation of -0.46. Second, our model implies that the relative tightness of leverage constraints drives capital flows. When the home country has a tighter leverage constraint, capital flows from the rest of the world. We document this pattern in the data. Moreover, our model implies some predictive relationships with respect to exchange rates. We show in both the model and the data that exchange rates can be predicted by the quantity of bank wholesale funding and average dollar exchange rate volatility.

Related Literature

There is a vast theoretical literature on exchange rate puzzles in international macroeconomics and finance. Consumption-based resolutions include habit formation (Verdelhan, 2010; Heyerdahl-Larsen, 2014; Stathopoulos, 2016), long-run risks (Colacito and Croce, 2011, 2013; Bansal and Shaliastovich, 2013), and disaster risks (Farhi and Gabaix, 2016). Maurer and Tran (2016), Favilukis et al. (2015), and Lustig and Verdelhan (2019) explore the exchange rate implications of incomplete financial market. Alvarez et al. (2009) and Malamud and Schrimpf (2018) show that models with market segmentation are able to bring exchange rate models closer to the data.

The literature at the nexus of macroeconomics and finance highlights the effect of intermediary leverage constraints (Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Li, 2013). Jermann and Quadrini (2012) uncover that the time variation of financial constraint is an important source of aggregate fluctuations and financial flows. Adrian et al. (2014), He et al. (2017), and Haddad and Muir (2017) provide empirical evidence in favor of intermediary asset pricing.

In the context of the FX market, Gabaix and Maggiori (2015) propose a theory of exchange rate determination based on constrained intermediaries. Fanelli and Straub (2016) study FX interventions in a similar environment. Bocola and Lorenzoni (2017) examine the currency risk premium and debt denomination with financial frictions. Itskhoki and Mukhin (2017) find that financial shocks are the key to explaining exchange rate puzzles. Empirically, Sandulescu et al. (2018) use a model-free approach to estimate international stochastic discount factors (SDFs) and show strong links between model-free international SDFs and intermediary balance sheets as well as volatility. More broadly, financial constraint plays an important role in international financial crises (Mendoza, 2010; Bocola, 2016; Maggiori, 2017; Perri and Quadrini, 2018).

Deviation from CIP has attracted considerable attention in recent years. Du et al. (2018) document large and persistent deviations from CIP after the financial crisis. Avdjiev et al. (2019) study the dynamic properties of CIP deviation. Amador et al. (2017) show CIP deviation as a consequence of exchange rate intervention policy and interest rate zero lower bound. Cenedese et al. (2019) use contract-level data to provide evidence on the causal relationship between regulatory changes and CIP deviations. Rime et al. (2019) provide evidence that segmented money markets prevent other unregulated participants from engaging in arbitrage. Borio et al. (2018) study the demand factors that drive CIP deviations. Our paper does not provide a new explanation for CIP deviations. Instead, we study CIP deviation, its dynamics and other traditional exchange rate puzzles within a unified quantitative model.

The theoretical mechanism of our model is built on Gabaix and Maggiori (2015)(GM hereafter). In their model, intermediaries require an excess return to intermediate cross-border capital flows, and the required excess return increases with the quantity of capital flows and is related to global financial conditions. Our model is different from GM in several respects. The GM model features one global intermediary that only intermediates international flows, while the two countries in our model each have their own intermediaries that face different domestic conditions. As a result, our model implies that country-specific (rather than global) financial conditions drive exchange rates. In our model, a tighter leverage constraint faced by home intermediaries triggers capital inflows from abroad. In GM, the direction of capital flows does not depend on global financial conditions. We empirically document that capital inflows are indeed associated with tighter leverage constraints in the home country. Finally, GM propose a theoretical framework, while we fit the model to data to quantitatively resolve multiple exchange rate puzzles.

Our work is also related to the literature on the risk-taking effect of global banks and global liquidity, such as Bruno and Shin (2014) and Bruno and Shin (2015). They argue that the dollar exchange rate affects global banks' balance sheets and alters their risk capacity through VaR constraints. They study how the dollar affects intermediaries' leverage-taking behavior when these intermediaries have dollar-denominated debt. Their framework is extended by Avdjiev et al. (2019) to show that dollar appreciations dampen the risk capacity of intermediaries and widen CIP deviations. In our model, it is the tightening of the leverage constraint that simultaneously drives dollar appreciation and CIP deviation. Chien et al. (2019) provide a quantitative model in which a small fraction of active traders' pricing kernels determine exchange rates. Our model differs from theirs by focusing on the effect of time-varying leverage constraints for intermediaries instead of the consumption-based pricing kernel of a specific group of investors.

The remainder of the paper is structured as follows. Section 2 provides some institutional background on the FX market and argues that financial institutions are crucial players in the market. Section 3 describes the model and section 4 discusses its quantitative performance. Section 5 studies why our model helps resolve the exchange rate puzzles. In section 6, we empirically test several new implications of our model. Section 7 concludes the paper.

2 The Relevance of Financial Intermediaries

This section sketches the basic structure of the FX market. The key takeaway of this section is that financial intermediaries play a significant role in the FX market[1].

The FX market is the largest financial market in the world. It has a two-tiered structure: the inter-dealer market and the dealer-customer market. Most inter-dealer transactions are high-frequency market-making transactions. These high-frequency transactions are not our focus since dealers usually end the day with small inventories (Bjønnes and Rime, 2005), and these activities matter little at lower (monthly, quarterly or annual) frequencies. Occasionally, dealers take speculative positions in propriety trading with horizons of up to three months (Sager and Taylor, 2006). Such longer horizon speculation is of interest in this paper.

Non-dealing financial intermediaries are important in determining exchange rates at lower frequencies. The main categories of customers include financial customers, corporate customers,[2] and

[1]Though there have been tremendous changes in the foreign exchange market in the recent decades, we describe the common features of the market across time. The new changes include the use of electronic trading systems, the increase of foreign exchange transactions between financial institutions, etc. For more institutional details of the foreign exchange market, see Osler (2008) and King et al. (2011).

[2]Corporate customers trade for real purposes, such as production, investment, and dividend payout. The size of corporate transactions is small relative to financial transactions.

retail customers.[3] Financial customers can be classified into two groups: real money investors[4] and levered investors.[5]

According to the BIS triennial survey, the turnover associated with nondealer financial institutions increased and had risen to 51% by 2016. Starting in 2013, the survey includes a detailed split of nondealer financial institutions into nonreporting banks (24%, 22%), institutional investors (11%, 16%), hedge funds and PTFs (11%, 8%), the official sector (1%, 1%), and other institutions (6%, 4%). Nonreporting banks, hedge funds and PTFs, and a segment of institutional investors are considered levered investors. Meanwhile, nonfinancial transactions account for no more than 20% of all turnover and have been declining in recent decades.

Generally, levered financial intermediaries are constrained from taking leverage, as are FX market participants. Speculative positions are constrained for various reasons, such as regulation, risk management and avoidance of excess risk taking by traders. Bank regulation is one example of such constraints. Banks in different countries are subject to the Basel regulatory capital adequacy framework with a minimum risk-weighted capital ratio of 8 percent and a non-risk-weighted leverage ratio of 3 percent.

Beyond regulation, FX market participants face market-driven balance sheet management restrictions, usually in the form of VaR constraints (Sager and Taylor, 2006). In practice, most intermediaries adopt VaR as their portfolio risk management tools. It calculates the worst possible loss that will not exceed a given probability over a certain period. The Basel Committee on Banking Supervision allowed banks to use their internal VaR models. Generally, Therefore, we argue that the volatility-driven variation in intermediaries' leverage constraints is the distinct feature of levered institutions and can provide us with novel insights into exchange rate studies.

In summary, financial intermediaries (banks) play a preeminent role in the international financial market and their distinct feature of leverage constraints is related to volatility.

[3] Retail customers, accounting for a very small fraction, are not studied in this paper.

[4] Real money investors include mutual funds, pensions funds, endowments, and so on, which do not take leverage and infrequently adjust their portfolios.

[5] Levered investors include non-dealer commercial banks, hedge funds, and commodity trading advisors, and so on. They take high leverages and actively manage their portfolios.

3 The Model

We depict the structure of the model in a circular flow diagram in Figure 1. There are two ex ante identical countries in the economy, home and foreign, each populated with representative households. They are endowed with different goods and trade with one another in the goods market. In both countries, households own the intermediaries and send managers to operate them. Households make deposits in local intermediaries. Intermediaries combine deposits and their net worth to invest in risky assets. There are three available risky assets, a claim to the local tree, a claim to the foreign tree, and an international bond. Intermediation is imperfect, which is modeled by the intermediaries in each country facing a leverage constraint that depends on the volatility in the local economy. The structure of the domestic economy is similar to that in Gertler and Kiyotaki (2010), and the setting of the international economy is similar to that in Gabaix and Maggiori (2015).

We describe the behavior of households and intermediaries in detail in the following subsections.

3.1 Households

Households in the home and foreign countries are endowed with Lucas trees that deliver different goods, X for home and Y for foreign. The two goods aggregate into a consumption basket $C_t = [(1-\alpha)C_{x,t}^{\frac{\sigma-1}{\sigma}} + \alpha C_{y,t}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$ that exhibits constant elasticity of substitution. $C_{x,t}$ and $C_{y,t}$ are home households' consumption of X and Y. Analogously, the foreign consumption aggregator is $C_t^* = [\alpha C_{x,t}^{*\ \frac{\sigma-1}{\sigma}} + (1-\alpha)C_{y,t}^{*\ \frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$, with $C_{x,t}^*$ and $C_{y,t}^*$ being the consumption of X and Y by foreign households. Households in the home and foreign countries place different weights on X and Y with consumption home bias, i.e., $\alpha < \frac{1}{2}$. σ is the price elasticity of substitution between X and Y. All households have identical constant relative risk aversion (CRRA) preferences over their country-specific consumption basket with risk aversion γ. In the remaining part of model description, we only set up the problems of the home households and intermediaries.

The household solves the following optimization problem:

$$\max_{C_{x,t},C_{y,t},D_t} E\sum_{t=0}^{\infty}\frac{C_t^{1-\gamma}-1}{1-\gamma}$$

$$s.t.: P_{x,t}C_{x,t}+P_{y,t}C_{y,t}+D_t = R_{f,t-1}D_{t-1}+\Pi_t$$

We choose the home composite good as the numeraire. $P_{x,t}$ and $P_{y,t}$ are the price of X and Y, respectively.

In our model, households do not directly hold risky financial assets. They only have access to money market accounts offered by the intermediaries that pay consumption baskets risklessly in the subsequent period. D_t is the deposit by households into intermediaries at time t and $R_{f,t-1}$ is the risk-free rate realized at time t. Π_t is the net payout from the intermediaries. Our assumption of market participation follows the literature on intermediary asset pricing. Because of the households' lack of investment expertise, they rarely trade sophisticated assets such as corporate bonds, bank loans, currencies and derivatives (He and Krishnamurthy, 2013). These assets are usually held and traded over-the-counter by intermediaries. Therefore, intermediaries are likely marginal investors. Even in stock markets, there is evidence of limited participation and passive portfolio behaviors without rebalancing for a significant fraction of households (Chien et al., 2012).

3.2 Intermediaries

Intermediaries are homogeneous and live for two periods, t and $t+1$. In period t, an intermediary is endowed with net worth N_t to operate, and it makes optimal portfolio decisions. In period $t+1$, all the asset returns are realized, and the intermediary rebates all profits and the initial net worth to the shareholder. The household sets up a new intermediary with net worth N_{t+1}. We assume that home and foreign intermediaries aggregate net worth's is a share of their endowment, i.e., $N_t = \eta X_t$ and $N_t^* = \eta Y_t$. The overlapping-generations structure of the banking sector is a simplifying assumption made for tractability, similarly adopted by Coimbra and Rey (2019). They show that the bank equity-to-GDP ratio is stable, while assets and liabilities are actively adjusted over the financial cycle. Therefore, we consider the model structure to be a reasonable approximation of the data.

Intermediaries can take deposits from households and invest in three risky assets: a home risky asset, a foreign risky asset and an international bond. The two risky assets are claims to the Lucas trees and interpreted as corporate bonds, bank loans and other intermediated assets. $P_{s,t}$ and $P^*_{s,t}$ are the price of home and foreign risky assets in units of their domestic consumption basket. There is an international bond that pays fixed $R_{b,t}$ units of both home consumption consumption baskets and foreign consumption baskets. This payoff structure is adopted to preserve symmetry between the two countries, as in Heathcote and Perri (2016), and both countries face exchange rate risks. The home-currency-denominated return on the international bond is equal to $R_{I,t+1} = R_{b,t}(1+Q_{t+1})/(1+Q_t)$. The real exchange rate Q_t is defined as the relative price of foreign consumption basket. An increase in Q_t means a foreign appreciation. The home representative intermediary optimally chooses the holdings of home and foreign risky assets, $S_{x,t}$ and $S_{y,t}$, the international bond $D_{I,t}$, and the amount of deposits D_t. When $D_{I,t} < 0$, home intermediaries are effectively borrowing from foreign intermediaries. An intermediary solves the following portfolio choice problem to maximize its market value $V(N_t)$. The foreign representative intermediary solves an analogous problem.

$$V(N_t) = \max_{S_{x,t},S_{y,t},D_t,D_{I,t}} E_t M_{t+1}(P_{s,t}R_{s,t+1}S_{x,t} + P^*_{s,t}Q_{t+1}R^*_{s,t+1}S_{y,t} + D_{I,t}R_{I,t+1} - D_t R_{f,t})$$

$$s.t.: P_{s,t}S_{x,t} + P^*_{s,t}Q_t S_{y,t} + D_{I,t} = N_t + D_t \tag{1}$$

$$V_t \geq \theta_t(P_{s,t}S_{x,t} + P^*_{s,t}Q_t S_{y,t} + D_{I,t}), \tag{2}$$

where $M_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$ is the home households' SDF. $R_{s,t+1} = (P_{s,t+1} + P_{x,t+1}X_{t+1})/P_{s,t}$ and $R^*_{s,t+1} = (P^*_{s,t+1} + P_{y,t+1}Y_{t+1}/Q_{t+1})/P^*_{s,t}$ are the returns on home and foreign risky assets in local currencies. Equation 1 is the balance sheet of the intermediary. The left hand side is the risky position of the intermediary, while the right hand side includes two sources of funding: intermediary net worth and deposits.

Equation 2 characterizes the leverage constraint faced by the intermediary. V_t is the market value of an intermediary, and $P_{s,t}S_{x,t} + P^*_{s,t}Q_t S_{y,t} + D_{I,t}$ is the total risky position of the intermediary. The constraint states that the market value of an intermediary cannot be smaller than a fraction θ_t of its risky position. These constraints model the distinct feature of intermediaries, the value-at-risk (VaR) constraint, as discussed in section 2. VaR is defined as "the worst-case loss" such that a loss larger than VaR is a low-probability event (usually lower than 1 percent). The constraints impose a limit on the amount that the households are willing to lend the intermediaries, which exists for various reasons, such as the avoidance of excessive risk taking, default, and other regulatory concerns. The constraint also implies that a higher excess return on assets can loosen the leverage constraint by raising the market value of the intermediary.

According to the VaR rule proposed by Adrian and Shin (2014), the VaR equals the equity capital of the intermediary. If we approximate the equity capital by its market value, θ_t can be interpreted as the VaR per unit of asset. θ_t and θ_t^* measure the tightness of leverage constraints and follow

$$\log\theta_t = \theta_0 + \theta_1 log(\sigma_{x,t}), \log\theta_t^* = \theta_0 + \theta_1 log(\sigma_{y,t}). \tag{3}$$

The constraint features two components: θ_0 captures leverage restrictions caused by time-invariant frictions, such as the capital requirement ratio. Naturally, the VaR varies with the risks in the economy, captured by the parameter θ_1. When the home economy has higher volatility, the intermediaries' balance sheets become riskier and they need to delever.

Our intermediary sector is based on Gabaix and Maggiori (2015). They model a risk-neutral global intermediary taking deposits from the trade-surplus country and lending to the trade-deficit country. In our model, each country's intermediary takes deposits from its local households. The intermediaries intermediate not only international bonds but also the home and foreign risky assets. They make portfolio choices, facing leverage constraints that depend on local financial conditions. Therefore, country-specific financial conditions affect equilibrium exchange rates and quantities.

We characterize the solution to the intermediary's problem. Denote by κ_t the Lagrange multiplier of the leverage constraint, and the optimality conditions of the intermediary are

$$E_t M_{t+1} R_{S,t+1} = E_t M_{t+1} R^*_{S,t+1} \frac{Q_{t+1}}{Q_t} = E_t M_{t+1} R_{I,t+1} = 1 + \theta_t \kappa_t \tag{4}$$

$$E_t M_{t+1} R_{f,t} = 1 \tag{5}$$

Derived from the envelope condition, the market value of the intermediary is equal to $V_t = \frac{1}{1-\kappa_t} n_t$, and the marginal value of net worth is $\frac{1}{1-\kappa_t}$. Given the constraint, i.e., $\kappa_t > 0$, intermediaries cannot borrow as much as they would prefer, despite that investment opportunities with excess returns are available. When the constraint becomes tighter, the marginal value of net worth and excess returns increase. Without the constraint, i.e., $\kappa_t = 0$, intermediaries frictionlessly manage the assets on behalf of households. The intermediary sector is a veil, and the equilibrium objects are the same as in an economy where households directly hold the assets.

3.3 Exogenous Processes

The two countries have co-integrated endowment processes:

$$\Delta \log X_{t+1} = \tau(\log Y_t - \log X_t) + \bar{\sigma}\sigma_{x,t}\varepsilon_{x,t+1}$$
$$\Delta \log Y_{t+1} = -\tau(\log Y_t - \log X_t) + \bar{\sigma}\sigma_{y,t}\varepsilon_{y,t+1}. \tag{6}$$

Volatilities are stochastic as follows:

$$\log(\sigma_{x,t+1}) = \rho_\sigma log(\sigma_{x,t}) + \sigma_\sigma \eta_{x,t+1}$$
$$\log(\sigma_{y,t+1}) = \rho_\sigma log(\sigma_{y,t}) + \sigma_\sigma \eta_{y,t+1}. \tag{7}$$

In equilibrium, both the goods market and the asset markets clear. For the sake of space, we only present the essential equations for understanding the model in the main text. The equilibrium is fully characterized by a system of conditions detailed in Appendix A.

3.4 Exchange Rates and Intermediaries

In this section, we study the key theoretical relationship between exchange rates and intermediaries' leverage constraints before obtaining a quantitative analysis. Our model's main prediction is that a country's exchange rate appreciates when its leverage constraint is tighter.

Proposition 1. Assume that $m_{t,t+1}$, $m^*_{t,t+1}$ and Δq_{t+1} follow log-normal distributions. The expected exchange rate change follows:

$$E_t \Delta q_{t+1} = r_{f,t} - r^*_{f,t} - \frac{1}{2} cov_t(m_{t+1} + m^*_{t+1}, \Delta q_{t+1}) + \log(1 + \kappa_t \theta_t) - \log(1 + \kappa^*_t \theta^*_t) \tag{8}$$

The proof is shown in the Appendix B.

Proposition 1 characterizes the relation between the expected currency appreciation rate $E_t\Delta q_{t+1}$ and the risk-free rate difference $r_{f,t}-r^*_{f,t}$, the covariance of the SDF and exchange rate $cov_t(m_{t+1}+m^*_{t+1},\Delta q_{t+1})$, and intermediary variables $\log(1+\kappa_t\theta_t)-\log(1+\kappa^*_t\theta^*_t)$. Without leverage constraints ($\kappa_t=\kappa^*_t=0$) and ignoring the covariance term, UIP holds, i.e., $E_t\Delta q_{t+1}+r^*_{f,t}-r_{f,t}=0$. If we keep the covariance term while letting $\kappa_t=\kappa^*_t=0$, the currency risk premium equals the covariance of the SDFs and the exchange rate. With complete financial markets and $\Delta q_{t+1}=m^*_{t+1}-m_{t+1}$, the currency risk premium is equal to $r^*_{f,t}-r_{f,t}+E_t\Delta q_{t+1}=-\frac{1}{2}[var(m^*_{t+1})-var(m_{t+1})]$. Understanding the determinants and dynamics of the two variance terms has been the focus of a large literature (Backus et al., 2001; Verdelhan, 2010; Bansal and Shaliastovich, 2013). In contrast, our model's key message is that the relative tightness of the leverage constraint in the two countries drives exchange rates and expected currency returns. Suppose that the home country has a tighter leverage constraint; the marginal value of net worth for home intermediaries then increases, and thus, $\log(1+\kappa_t\theta_t)$ increases relative to $\log(1+\kappa^*_t\theta^*_t)$. As home intermediaries require higher expected returns on the international bond than foreign intermediaries, there must be an expected foreign appreciation to accommodate the increased required return.

The next proposition relates the intermediaries' leverage constraints to the exchange rate level.

Proposition 2. Assume that the real exchange rate is stationary and that its long-run value is $\bar{q}$. The level of the exchange rate follows

$$\begin{aligned} q_t = & -E_t\sum_{k=0}^{\infty}(r_{f,t+k}-r^*_{f,t+k})-E_t\sum_{k=0}^{\infty}[\log(1+\kappa_{t+k}\theta_{t+k})-\log(1+\kappa^*_{t+k}\theta^*_{t+k})] \\ & +\frac{1}{2}E_t\sum_{k=0}^{\infty}cov_{t+k}(m_{t+k+1}+m^*_{t+k+1},\Delta q_{t+k+1})+\bar{q}. \end{aligned} \tag{9}$$

Proposition 2 can be directly derived from Proposition 1 by iterating equation 8 forward. When the home country has tighter leverage constraints than the foreign country, $\log(1+\kappa_t\theta_t)$ is higher than $\log(1+\kappa^*_t\theta^*_t)$. As the leverage constraint tightness is persistent, the present value of future $\log(1+\kappa\theta)$ is higher than that of $\log(1+\kappa^*\theta^*)$. Therefore, domestic currency has a higher value in this case. Since exchange rate fluctuations are associated with leverage constraints, this channel can potentially break the counterfactual perfect correlation of exchange rates and consumption growth differentials predicted by complete market models and many incomplete market models.

3.5 Deviation from Covered Interest Rate Parity

We use the model to price the FX forward and swap (FX swap) contracts and discuss the implied deviation from CIP. An FX swap is a combination of a spot transaction and a forward transaction of the foreign currency. For example, an FX swap trader can buy foreign currency in the spot market at the spot exchange rate Q_t and sell the foreign currency after a quarter at the forward rate F_t. The return on the swap contract is

$$R_{swap,t} = \frac{Q_t}{F_t}(1 + r^*_{f,t}).$$

There are two components of the return that the trader earns: the forward discount $\frac{Q_t}{F_t}$ and the risk-free foreign interest rate $1 + r^*_{f,t}$. In the absence of arbitrage, the swap return should be equal to the risk-free rate in the home currency, which leads to CIP:

$$q_t + r^*_{f,t} - f_t - r_{f,t} = 0. \tag{10}$$

where $q_t = \log Q_t$ and $f_t = \log F_t$. CIP is the best-established and the most robust no-arbitrage conditions in international finance. It held very well before the global financial crisis in 2007 (Akram et al., 2008). However, after the crisis, deviations from CIP are large and persistent (Du et al., 2018).

3.5.1 CIP Deviations

Now, we consider an FX swap contract that swaps dollars for foreign currency in our model. Following our assumption on the regarding financial intermediation, only intermediaries have access to the FX swap contract.[6] Assume that the FX swap contract is in zero net supply and that only the home (US) intermediaries have access to dollar funding at the rate $r_{f,t}$ and invest in the swap

[6] According to the recent BIS triennial survey, more than 90% trading in forwards, FX swaps and currency swaps are by financial intermediaries, so our assumption is consistent with data.

contract.[7] Therefore, the home (US) intermediaries' pricing kernel is used to price the FX swaps.

Denote by $S_{s,t}$ the position on the FX swap, the augmented US intermediary's optimization problem is:

$$V(N_t) = \max_{S_{x,t},S_{y,t},D_t,D_{I,t},S_{s,t}} E_t M_{t+1}(P_{s,t}R_{s,t+1}S_{x,t} + P^*_{s,t}Q_{t+1}R^*_{s,t+1}S_{y,t} + D_{I,t}R_{I,t+1} - D_t R_{f,t} + S_{s,t}R_{swap,t})$$

$$V_t \geq \theta_t(P_{s,t}S_{x,t} + P^*_{s,t}Q_t S_{y,t} + D_{I,t} + \psi S_{s,t})$$

where ψ is the parameter that governs the relative tightness of the constraint for the riskless swap position. If $\psi = 0$, the swap position is not constrained. If $\psi = 1$, the swap position has the same constraint as other risky assets. If $0 < \psi < 1$, the swap position faces a looser constraint than the other risky assets. We assume that the constraint is at the bank level, so the level of tightness of constraints for different assets comoves. The first-order condition for $S_{s,t}$ for the augmented optimization problem is:

$$E_t M_{t+1} R_{swap,t} = 1 + \psi \kappa_t \theta_t$$

The CIP deviation (currency basis), denoted by $r_{cip,t}$, is as follows:

$$r_{cip,t} \equiv r_{f,t} - r_{swap,t} = -\log(1 + \psi \kappa_t \theta_t). \tag{11}$$

While risk-free FX swaps are subject to a supplementary leverage ratio constraint, other risky assets are subject to additional regulations, such as a risk-weighted capital ratio. In the quantitative section, we directly compare the value of ψ with documented regulatory ratios.

In our model, large and persistent CIP deviations exist because of the changed regulatory environment after the financial crisis (Du et al., 2018; Boyarchenko et al., 2018). Before the crisis, banks were subject to requirements regarding the risk-weighted capital ratio, the ratio of Tier-1 capital and risk-weighted assets. For the FX swap position, the risk weight is zero, meaning that the FX swap trade is essentially unconstrained, i.e., $\psi = 0$, and CIP holds. However, after the financial crisis, Basel III imposes requirements on an additional supplementary leverage ratio, the

[7] A salient feature of CIP deviation is that the dollar is cheaper in the spot market than in the swap market, which can be explained by the preference for dollar safe assets by global investors. As our model is a symmetric two-country model, we do not explicitly model the specialty of US dollar. Instead, we assume the existence of the swap contract of dollar into the foreign currency. As this contract is in zero net supply, it does not affect the real decisions of households and intermediaries. We can take the pricing kernel solved from our model to price the FX swap contract.

ratio of Tier-1 capital and total on-balance-sheet and specific off-balance-sheet assets, including the FX swap positions. Although an FX swap has zero risk, banks are still required to hold a portion of capital against their FX swap positions. In other words, the FX swap position became constrained after the financial crisis, i.e., $\psi > 0$. As $\kappa_t, \theta_t > 0$, the currency basis $r_{cip,t}$ became negative after the crisis. This is consistent with the empirical finding by Du et al. (2018) that US dollars are cheaper in the cash market than in the swap market. Du et al. (2018) and Cenedese et al. (2019) identify a causal link between bank regulation change and CIP deviation. Our paper complements their work by jointly studying CIP deviation and other exchange rate puzzles in a unified quantitative model. To the best of our knowledge, our paper is the first such attempt.

4 Quantitative Analysis

In this section, we numerically solve the model using a global solution method. Then, we bring the model to the data and examine the quantitative explanatory power of our model in resolving the exchange rate puzzles. We estimate the parameters of our model using the SMM, minimizing the distance between the model and data for moments related to intermediaries, macroeconomic dynamics, the balance of payments, and exchange rates. We also use various external validations to assess the estimated values of the parameters.

4.1 Model Solution

We solve the model using a global projection method similar to that of Rabitsch et al. (2015) and Dou and Verdelhan (2015). Since our model features heteroskedasticity, leverage constraints and heterogenous agents' portfolio choices, local perturbation methods can be inaccurate.

Naturally, the portfolio position of each asset $S_{x,t}$, $S_{y,t}$ and $D_{I,t}$ are state variables. We define a state variable ω_t as the home share of total wealth (cum dividend).

$$\omega_t = \frac{P_{s,t-1}R_{s,t}S_{x,t-1} + P^*_{s,t-1}R^*_{s,t}Q_tS_{y,t-1} + D_{I,t-1}R_{I,t}}{P_{s,t-1}R_{s,t} + P^*_{s,t-1}R^*_{s,t}Q_t}$$

This wealth share summarizes information on each asset position and reduces the dimension of the state space.[8] There are four state variables in total. The other three states are the relative size of endowment $\log Y_t - \log X_t$, volatility in home and foreign country $\sigma_{X,t}$ and $\sigma_{Y,t}$. Facing the curse of dimensionality, we use Smolyak polynomials on sparse grids as the basis functions to approximate the policy and pricing functions $\{C_{x,t}, S_{x,t}, S_{y,t}, P_{s,t}, P^*_{s,t}\}$ (Fernández-Villaverde et al., 2016). The other endogenous variables can be solved accordingly. We then compute the system of equilibrium conditions and minimize the approximation errors over the grids. The expectations are computed using monomial integration methods. The solution method is detailed in Appendix C.

Our method is reasonably accurate with an average of Euler equation errors of approximately 10^{-5}. In the literature, very few papers estimate equilibrium models with international portfolio choice in incomplete markets, since global methods introduce large computational burden. Obtaining convergence is challenging and not guaranteed because the equilibrium is a joint determination of multiple allocations, goods prices, portfolios and asset prices. Notably, our method is fast and stable enough that we are able to solve the model repeatedly and this facilitates the estimation.

4.2 Model Estimation

In this subsection, we discuss the parameters to be estimated and the moments used to identify them.

We estimate the model with the simulated method of moments (SMM). The model is at a quarterly frequency. Since the model does not focus on risk channels, we set risk aversion γ at a conservative value of 2. We select a small value for co-integration coefficient $\tau = 0.005$ to keep the world economy on the balanced growth path. The remaining ten parameters are estimated: parameters related to preference $\{\beta, \sigma, \alpha\}$, endowment dynamics $\{\bar{\sigma}, \rho_\sigma, \sigma_\sigma\}$, and intermediary characteristics $\{\theta_0, \theta_1, \eta, \psi\}$.

We target 13 moments, listed in Table 1, to estimate these 10 parameters. As the number of moments is greater than the number of parameters, the model is overidentified. We weight

[8] This technique is useful at some additional costs. We have to solve the law of motion of ω_t, which is not predetermined and depend on $R_{s,t}$, $R^*_{s,t}$, $R_{I,t}$. In contrast, usually the state variables are predetermined (e.g., $S_{x,t}$, $S_{y,t}$ and $D_{I,t}$).

the squared differences by the inverse of the square of the sample moments so that the unit and magnitude of each moment do not play a role. The SMM searches the parameter space to minimize the distance between the sample moments in the data and those implied by the model. Details on the estimation procedure are provided in Appendix C.

We select the following moments that are a priori informative about the parameters. The average leverage ratio (ϕ), the excess return the intermediaries earn ($r_s - r_f$) and the mean and standard deviation of CIP deviation (r_{cip}, $sd(r_{cip})$) are informative about θ_0, θ_1, η and ψ. The time discount rate β can be inferred from the average risk-free rate (r_f). The volatility of the net-export-to-GDP ratio ($sd(NX/GDP)$), the import-to-consumption ratio ($P_y C_y / C$) and the home share of home risky assets (S_x) are informative about σ and α. The volatility of consumption growth ($sd(\Delta c)$) and of log volatility ($sd(\log(\sigma_{X,t+1}))$) captures the dynamics of endowments $\bar{\sigma}$, ρ_σ and σ_σ. Finally, we utilize three exchange rate moments in estimation: the correlation between exchange rate growth and consumption growth differentials ($corr(\Delta q,\ \Delta c - \Delta c^*)$), the regression coefficient of future currency excess return on interest rate differentials (β_{fp}), and the standard deviation of exchange rates ($sd(\Delta q)$).

We measure the leverage ratio of intermediary sector (ϕ) as the ratio of equity to total assets for the "broadly defined banking sector" in the Financial Account of the United States reported by the Federal Reserve Board.[9] We follow Krishnamurthy and Vissing-Jorgensen (2015) to include types of institutions that have a substantial fraction of their funding from short-term debt financing and thus are prone to leverage constraint changes. We measure the excess return ($r_s - r_f$) by US banks' return on asset (ROA). The measure of other moments are standard and detailed in Appendix D.

[9]The definition of "broadly-defined banking sector" includes: US-Chartered Depository Institutions, Foreign Banking Offices in the US, Banks in US-Affiliated Areas, Credit Unions, Issuers of Asset-Backed Securities, Finance Companies, Mortgage Real Estate Investment Trusts, Security Brokers and Dealers, Holding Companies, Money Market Mutual Funds. Quarterly assets and liabilities data for each type of financial institution are from the Financial Account of the United States, from 1973Q1 to 2019 Q2.

4.3 Model Fit

Table 1 reports the sample moments in the data and the implied population moments in the model using the estimated parameters. Overall, our model fits the data reasonably well. Panel A of the table includes all the moments we use for estimation. The volatility of consumption growth, the import ratio, the volatility of net exports to GDP, the volatility of log volatility, the risk-free rate, the excess return, the intermediary leverage ratio, and the magnitude and volatility of CIP deviation are close to their counterparts in the data. The share of domestic risky assets held by domestic intermediaries is 0.89, consistent with our empirical observation of substantial equity home bias. Our model is able to produce the "equity home bias" because the domestic risky asset is a good hedge against real exchange rate fluctuations (Heathcote and Perri, 2013). While we use both the Backus-Smith correlation $corr(\Delta q, \Delta c - \Delta c^{*})$ and the UIP coefficient β_{fp} as target moments in the estimation, our model can quantitatively resolve these exchange rate puzzles while the parameters are tightly tied to other observed moments in the data. We understate the exchange rate volatility by approximately 3 percent per annum.

In Panel B of Table 1, we report a set of non-targeted moments. The autocorrelation of net exports to GDP and intermediaries' leverage ratio are close to data. The leverage ratio is less volatile. In the model, risk-free rate are stable as in the data, but the banks' ROA are excessively volatile. Finally, we examine the characteristics of the dollar carry return, defined as the signed foreign currency return, i.e., $r_{dollar,t+1} = (r_t^* - r_t + \Delta q_{t+1}) \times sign(r_t^* - r_t)$. With a UIP coefficient of larger than 1, the dollar carry return is positive on average. In the data, the average dollar carry return is 5.3 percent per annum with a Sharpe ratio of 0.64, while in the model, it is 2.19 percent per annum with a Sharpe ratio of 0.41. The model undershoot the magnitude and Sharpe ratio of dollar carry. Lustig et al. (2014) show that the results obtained with the dollar as the base currency have a sizable excess return and Sharpe ratio, while the return is much lower for other base currencies. Since our model is fully symmetric without a special role for the US, we do not obtain a dollar carry return as impressive as that observed in the data.

4.4 Parameter Values

Table 2 reports the estimated parameter values. We discuss the identification of these parameters and the sensitivity of model moments to parameter changes. We also provide an additional validation assessment of the appropriateness of these parameter values.

Average constraint tightness θ_0. θ_0 measures the average tightness of the leverage constraint faced by intermediaries. If θ_0 is larger, intermediaries are limited to borrowing given their equity capital. Therefore, a higher value of θ_0 implies a tighter leverage constraint on average (regardless of the volatility fluctuations) and thus a higher leverage ratio. In the model, the leverage ratio is defined as $\phi_t \equiv \frac{N_t}{P_{s,t}S_{x,t}+P^*_{s,t}Q_tS_{y,t}+D_{I,t-1}}$. Our estimate $\theta_0 = 0.118$ matches the average leverage ratio of 0.12 for the US broadly-defined banking sector. In the sensitivity analysis, we change θ_0 to 0.10 and 0.15 while holding other parameters fixed. Table 3 shows a clear mapping between θ_0 and the model-generated leverage ratios.

Intermediary net worth η. Intermediary net worth ratio $\eta = N_t/X_t$ determines a bank's risk-taking capacity. If η is low, intermediaries have less capital and borrowing capacity to invest in the risky assets. The constraint is tighter and the marginal value of net worth $\frac{1}{1-\kappa_t}$ is high. According to Equation 4, intermediaries earn higher excess returns on assets. Excess returns are sensitive to η and θ_0, as Table 3 shows. After θ_0 is identified by the leverage ratio, the excess return can be used to identify η.

VaR per unit of asset θ_1. θ_1 controls the extent to which the tightness of the leverage constraint varies with the volatility in the economy. As CIP deviations arises from the limits on arbitrage, the volatility of CIP deviations $r_{cip,t}$ is directly affected by θ_1. In Table 3, $sd(r_{cip})$ strongly increases with θ_1. θ_1 also contributes to the volatility of consumption by introducing an additional source of variation. Our estimate of θ_1 equals 0.392.

Recall that we interpret θ_t as unit asset VaR. We examine the sensitivity of major financial institutions' unit asset VaR to equity market volatility. Specifically, we follow Adrian et al. (2014)

and collect the VaR per unit of asset reported by major banks: Goldman Sachs, Citibank, JP Morgan, Morgan Stanley, Bank of America, Merrill Lynch, and Lehman Brothers in their 10-Q filings. Regressing log VaR per unit asset on log realized volatility of aggregate US stock returns in the last month of the contemporaneous quarter, we get an estimate of 0.46 ($t = 3.99$), which is statistically significant.[10] We also regress the currency log VaR per unit asset of these banks on the log realized dollar exchange rate volatility and get a regression coefficient of 0.34 ($t = 3.14$). These additional results confirm that banks largely respond to changing volatility in the financial market when calculating their VaR. Our regression coefficients are both close to the estimated value of θ_1, which further supports the appropriateness of our estimated value.

From Proposition 2, the exchange rate dynamics are driven by the time variation in the leverage constraint. The value of θ_1 is crucial in explaining the exchange rate puzzles. Notably, our estimated value is able to fit the all three moments of standard deviation of CIP deviation, Backus Smith correlation $corr(\Delta q, \Delta c - \Delta c^*)$, and the UIP regression coefficient β_{fp}.

Relative constraint tightness for FX swap ψ. The relative constraint tightness for FX swap ψ is pinned down by equation 11. The banks' ROA is approximately 0.98 percent, which is roughly equal to $\log(1+\kappa_t\theta_t)$, while the average CIP deviation is approximately 25 bps, which equals $\log(1+\psi\kappa_t\theta_t)$. The estimate of the relative constraint tightness is equal to 0.199.

This estimate is consistent with the changing regulatory environment. Under the new Basel III regulatory framework, global banks are required to hold at least 3% of equity capital against all assets, regardless of their riskiness, while the required leverage ratio did not exist before the crisis. The global banks (especially the G-SIBs) face a required Tier-1 capital ratio of 9.5% to 13% and a total capital ratio of 11.5% to 15% after the crisis.[11] We take 15% to be banks' capital ratio for risky assets. The relative tightness of the leverage constraint for the risk-free FX swap position is equal to $3/15 = 0.2$, close to what we estimate from the magnitude of CIP deviation and banks'

[10] Admittedly, different banks have different procedures of calculating their VaR based on their asset holdings. The volatility of stock returns is an proxy of common financial market volatility. After taking log on both sides, we estimate the elasticity of unit-asset VaR on volatility.

[11] For more details about the changes of the regulatory environment after the crisis, see Du et al. (2018); Cenedese et al. (2019); Boyarchenko et al. (2018).

ROA.

Other parameters. The discount factor β is pinned down by the average risk-free rate of 0.74%. The elasticity of substitution between goods σ is estimated to match the volatility of net export share of GDP. If σ is lower, consumers are less willing to substitute one good for another, so the intertemporal adjustment through net exports is weaker. The volatility of net exports as a share of GDP in the US was 1.72%, which gives us as estimate of $\sigma = 0.452$. Consumption home bias α is estimated to fit the average import ratio of 0.17. As an approximation, at the steady state when the two goods have identical prices, a lower consumption home bias implies more imports of the good produced in the foreign country. Our estimates of σ and α are similar to the estimates in the literature (Stockman and Tesar, 1995). The volatility process drives the heteroskedasticity of the endowment and the fluctuations in the tightness of the leverage constraint. Naturally, σ_σ determines the volatility of volatility of exchange rates and consumption. We estimate the volatility of the log realized volatility of the average dollar exchange rate change with respect to other G10 currencies to be 0.23. The volatility of log realized volatility of G10 stock returns is approximately 0.3. Schorfheide et al. (2018) carefully estimate the consumption process and account for measurement errors and obtain a volatility of log volatility ranging between 0.24 and 0.46. We choose to use a more conservative value of 0.23 to pin down σ_σ and ensure that the volatility channel is not overstated. The volatility is highly persistent, as in other data sources. The monthly autocorrelation is 0.981 for the log realized volatility of the average exchange rate and stock returns and is 0.976 for consumption growth volatility.

4.5 Impulse Response Analysis

In this section, we illustrate the key economic mechanism of our model by examining the responses of endogenous variables to volatility and endowment shocks. We begin with the impulse responses to a positive volatility shock in the home country. Figure 2 plots the responses of the marginal value of net worth κ_t, expected excess return $E_t[R_{I,t+1} - R_{f,t}]$, aggregate consumption C_t, the risk-free rate $r_{f,t}$, and the net export NX_t for both countries and the real exchange rate.

When the home country's volatility is higher, intermediaries' balance sheets become riskier. According to equation 3, the leverage constraints tighten and risky assets have higher expected returns. Therefore, the marginal value of net worth $\frac{1}{1-\kappa_t}$ is higher for home intermediaries. Since higher volatility makes intermediaries' balance sheets riskier, households are willing to lend less and consume more. The expected lower consumption growth, together with higher volatility-induced precautionary saving, drive down the home risk-free rate. As illustrated in Proposition 2, the home exchange rate appreciates in response to a tighter home leverage constraint (higher $\log(1+\kappa_t\theta_t)$). From a goods market perspective, home households increase their consumption, implying declining net exports. From a financial market perspective, although home intermediaries are constrained from taking deposits, they can still borrow through the international bond market. Higher expected returns induce capital to flow into the home country.

Now let us examine the responses of foreign variables. Since the total endowments do not change, higher home consumption implies lower foreign consumption. The foreign risk-free rate increases to accommodate the reduced consumption. The increased cost of borrowing lowers the expected excess return. Thus, the marginal value of net worth $\frac{1}{1-\kappa_t^*}$ decreases.

Next, we analyze the impulse responses to a positive endowment shock in the home country, as shown in Figure 3. We use identical scales in Figures 2 and 3 for comparison purposes. When the home country has a higher endowment, both home and foreign consumption increase. This is the standard "risk-sharing" mechanism. The home risk-free rate decreases due to the lower expected consumption growth rate. The lower risk-free rate reduces the funding cost of home intermediaries and increases the excess return and thus the marginal value of net worth κ_t. Note that these responses to an endowment shock are smaller than the responses to a volatility shock. The reason is that the shock is nearly permanent with a very low rate of mean reversion. Regarding exchange rate, when the home country has a positive endowment shock, the foreign currency appreciates. The movements of the foreign variables follow a similar logic. The foreign country has a higher expected consumption growth rate; thus, the foreign risk-free rate increases. As a result, all foreign variables move in the opposite direction as the home variables.

5 Exchange Rate Puzzles

In this section, we explore the underlying mechanism of our model that helps resolve the exchange rate puzzles.

5.1 Backus-Smith Puzzle

The Backus-Smith puzzle states that standard international finance models imply a very high correlation between the consumption growth differential and exchange rate change, while in the data the correlation is fairly low. Under the assumption of complete financial markets and a CRRA utility function, standard models have

$$\Delta q_{t+1} = \gamma(\Delta c_{t+1} - \Delta c^*_{t+1}). \tag{12}$$

Therefore, the correlation between the exchange rate change Δq_{t+1} and consumption growth differential $\Delta c_{t+1} - \Delta c^*_{t+1}$ should be one. Even when financial market is incomplete and equation 12 does not hold state by state, this correlation is typically close to one, in sharp contrast to the weak correlation we observe in the data (Chari et al., 2002).

In our model, fluctuations in leverage constraints introduce a new source of exchange rate variation. Proposition 2 explicitly displays this new source, which is separate from consumption. As a result, the model breaks the tight link between exchange rates and consumption and generates a low correlation of -0.05.

The impulse responses of both countries' consumption and the exchange rate to a home volatility shock also help us understand the disconnect. When the home country exhibits heightened volatility, home consumption increases and the home currency appreciates, leading to a negative comovement between the exchange rate Δq_{t+1} and consumption differentials $\Delta c_{t+1} - \Delta c^*_{t+1}$. On the other hand, a positive home endowment shock is associated with increased home consumption and a depreciated home currency. These two forces offset one another and generate a weak correlation between consumption and the exchange rate.

5.2 Forward Premium Puzzle

UIP suggests that when the home country has a higher risk-free rate than the foreign country, the home currency is expected to depreciate, and thus, investing in home and foreign deliver the same payoffs in expectation. Specifically, the regression coefficient of currency excess return on the interest rate differentials β_{fp} should be zero.

$$\Delta q_{t+1} + r^*_{f,t} - r_{f,t} = \beta_0 + \beta_{fp}(r^*_{f,t} - r_{f,t}) + u_{t+1}.$$

However, this parity condition is rejected by the data (Hansen and Hodrick, 1980; Fama, 1984). Typically, a currency with higher interest rate tends to further appreciate in the subsequent period, i.e., $\beta_{fp} > 1$. In our sample, β_{fp} equals 2.20. This puzzle is also called the "forward premium puzzle".

Our model resolves this puzzle and generates a coefficient of 1.63. As we see from Proposition 1, the expected currency return is associated with the tightness of the two countries' leverage constraints. Absent these two terms and the covariance, UIP holds. With the constraint, intermediaries cannot invest freely in foreign assets, despite the presence of positive risk-adjusted excess returns. The excess return is larger when the home country experiences high volatility, as we explain in section 4.5. The home risk-free rate is lower than the foreign risk-free rate. Moreover, home currency appreciates contemporaneously with an expected depreciation. Therefore, higher interest differentials $(r^*_{f,t} - r_{f,t})$ indicates a tighter home constraint $\log(1+\kappa_t\theta_t) - \log(1+\kappa^*_t\theta^*_t)$ and a higher excess return on foreign assets. The foreign currency, albeit having higher interest rate, tends to further appreciate, generating a β_{fp} larger than 1. In Table 3, β_{fp} is larger under a large θ_1, illustrating that of the volatility-driven leverage constraint helps to account for the magnitude of forward premium puzzle.

5.3 Exchange Rate Volatility

In our model, there are two sources of exchange rate fluctuations, one originating from the endowment shocks and the other originating from the volatility shocks. Volatility shocks move exchange rates through the variation in leverage constraint tightness, raising exchange rate volatility compared to other standard models with endowment (or productivity) shocks only. Without the time varying constraint tightness ($\theta_1 = 0$), exchange rate volatility decreases to 3.82 percent.

5.4 CIP Deviation

Our discussion of CIP deviation is based on equation 11. Naturally, CIP is violated when intermediaries are constrained in taking FX swap positions. As in shown in Table 1, the CIP basis has a mean of -25 bps and a standard deviation of 27 bps in the model that matches the data counterparts very well.

Dynamically, Avdjiev et al. (2019) show that $r_{cip,t}$ is larger in absolute value when the dollar is strong and when dollar exchange rate volatility is high. This is exactly what our model implies. The regression coefficients of the growth of CIP based on exchange rate growth $\beta_{cip,-\Delta q}$ and change in volatility $\beta_{cip,\sigma}$ are both negative as in the data.

In the model, CIP deviation $r_{cip,t}$ is equal to $\log(1+\psi\kappa_t\theta_t)$, which is in turn determined by the tightness of the leverage constraint. When the US economy experiences higher volatility, US intermediaries face a tighter constraint, the deviation from CIP is larger and the dollar is strong, as we show in Proposition 2. Since the dollar exchange rate volatility is driven mainly by the volatility in the US, a higher US volatility leads to a higher dollar exchange rate volatility.[12]

Our model implies that the single common source of high US volatility drives up the dollar exchange rate, dollar exchange rate volatility, and the CIP deviation. This interpretation is complementary to that in Avdjiev et al. (2019). In their interpretation, it is the dollar appreciation that dampens the balance sheets of global banks with net dollar liabilities and thus widens CIP devia-

[12] In our model, dollar exchange rate volatility is affected by both US and UK volatility. This is a result of a two-country model setting. When there are more than 2 countries, the volatility of average US exchange rate is mainly driven by the US fundamental volatility, but not others.

tion. In our view, both mechanisms at play are important. However, our model has two symmetric countries, meaning that no country systematically has dollar liabilities, and their mechanism is abstracted from our model. A model that incorporates both mechanisms for quantitative assessment of their relevant importance is left for future research.

Du et al. (2018) show striking causal evidence that CIP deviation spikes at the quarter-end when banks report their leverage ratio to the regulatory authority. Our model is at the quarterly frequency and cannot be directly used to quantify the quarter-end spikes. Qualitatively, we interpret "quarter-end" as a tightening of the leverage constraint and CIP deviation spike follows. Our model is useful to infer the difference in effective regulation within a quarter and at its end from the quarter-end evidence. According to the model, the CIP deviation is equal to $\psi\theta_t\kappa_t$, which is linear in ψ. Assume θ_t and κ_t does not change significantly at quarter end, while ψ changes at the reporting dates of quarter end. We can directly infer the effectiveness of regulation on the CIP arbitrage or any other similar contract by the ratio of CIP deviation. For example, as shown in the regression in Du et al. (2018), quarter-end CIP deviation is 9.7 bp higher than that within the quarter. Suppose the average CIP deviation is 25 bp, that implies the regulation is 38.8 percent more stringent at quarter end than within the quarter.

Cenedese et al. (2019) exploit the introduction of UK leverage ratio framework and identify that the CIP basis increase by 25 bps for banks subject to the regulation. In the model, introducing leverage ratio constraint ψ would generate the same magnitude.

6 New Implications of the Model

In the previous section, we show that our model can resolve the four outstanding exchange rate puzzles in the literature. In this section, we discuss the new implications of our model: exchange rates and capital flows are related to intermediaries' constraint tightness, and exchange rates can be predicted by the quantity of wholesale funding and exchange rate volatility. We examine these implications in the data and compare the empirical relationships with their counterparts in our quantitative model.

6.1 Exchange Rates and Intermediaries

While our model generates exchange rates disconnected from consumption, it does imply some exchange rate "connect". Proposition 2 states that exchange rates are correlated with the relative tightness of the leverage constraints in two countries. The TED spread is a common measure of funding liquidity adopted by practitioners and academics (Rösch et al., 2016). Following the literature, we measure the constraint tightness by the TED spread (the difference between the LIBOR rate and a treasury bill of the same maturity of one year).

In Panel A of Table 4 we report the regression coefficients and correlation of average dollar real exchange rates on the average TED spread difference between the US and other G10 countries. The coefficient of relative spread is highly significant and negative, which is consistent with our model. Economically, a one-basis-point increase in the relative TED spread is associated with a foreign depreciation of 20 basis points. The correlation between the two variables is -0.38. In the second row, we include controls for consumption and the output growth differential and obtain similar results.

For comparison, we report in Panel C of Table 1 that the model-implied correlation is -0.27. In the model, the TED spread is constructed as follows. Assume that the relative leverage constraint tightness for interbank lending is ψ_{ted}. Similar to our analysis in section 3.5, the TED spread is equal to $\log(1+\psi_{ted}\kappa_t\theta_t)$ and ψ_{ted} can be inferred from the average TED spread of 0.60%.

In Panel B, we repeat the exercise in Panel A with the first difference of the exchange rate and relative TED spread.[13] All results are similar, and the correlation is as high as -0.46 in the data and -0.61 in the model.

In Panel C, we separate the relative TED spread into two components: the TED spread in the

[13] Our result on the correlation between exchange rate and relative TED spread is related to Jiang et al. (2018). They find that the CIP deviation based on treasuries (treasury basis) is correlated with the exchange rate in first difference. While treasury basis is correlated with the relative TED spread, our result is not driven out by the inclusion of the treasury basis as a control.

US and the average foreign TED spread. The results indicate that TED spreads in both the US and foreign countries are significantly correlated with the average dollar exchange rate. To visualize the high correlation, Figure 4 plots the time-series of relative TED spread and log dollar exchange rate from 1988Q1 to 2017Q2. The dollar exchange rate is computed by cumulating the average dollar exchange rate changes with the 1988Q1 log exchange rate normalized to 0.

6.2 Capital Flows and Intermediaries

In this section, we examine our model's implications for capital flows (and international trade). In contrast to standard models in which capital flows are purely driven by shocks in the real economy, our model uniquely implies that a difference in leverage constraints across countries triggers capital flows. When the home country has higher volatility and tighter leverage constraint, capital flows from the foreign country. In Panel C of Table 1, we show that the regression coefficient of capital flow on relative TED spread $\beta_{cf,Ted_{us}-Ted_f}$ is -0.31.

By balance of payment identity, the net exports of a country are equal to the total net purchase of foreign securities, so net exports and international capital flows are simply two sides of the same coin. To empirically test the capital flow implication, we measure capital flows using the net exports for the G10 countries. In Panel A of Table 5, we regress the G10 countries' quarterly net export ratio $\frac{NetExport}{NetExport+Consumption}$ on their TED spreads minus the average of the other nine G10 countries, i.e., relative TED spreads, both in first-differences. Row 1 shows the pooled time-series regression coefficient. Economically, it means that a 1-basis-point increase in the relative TED spread is associated with a 0.84 percent lower net export ratio or capital inflow. In row 2, we control for country fixed effects and obtain a largely identical result. In rows 3 and 4, we control for output and consumption growth. The additional controls do not significantly change the coefficient of the TED spread.

The implication regarding capital flows and intermediaries is useful to distinguish our model from Gabaix and Maggiori (2015). In their model, intermediaries impede capital flows by requiring an expected excess return, while the direction of the capital flow does not depend on the intermediaries' constraint. In our model, the difference in leverage constraint tightness triggers the flow of capital from one country to the other due to the difference in the expected returns of risky assets across countries. Our empirical exercise does show such features in the data.

6.3 Predictive Relation between Exchange Rate Volatility and the Exchange Rate

Next, we explore predictive relations with exchange rates. We first examine the relationship between dollar exchange rate volatility and the future change in the dollar exchange rate and currency excess returns. In the model, high home volatility implies higher foreign currency returns. In Panel C of Table 1, the regression coefficient of log volatility on future currency return $\beta_{rx,\sigma}$ is positive.

This relationship is consistent with the data. We use the average log dollar exchange rate volatility as the predictor, as all investors that trade foreign currencies are subject to dollar exchange rate fluctuations. The results of the predictive regressions are shown in Table 6. Panel A shows the results for exchange rate changes, and Panel B shows the results for currency excess returns. Standard errors are robust to heteroskedasticity, serial autocorrelation, and overlapping observations (Hodrick, 1992). The univariate regression results in rows 1 to 3 show that higher dollar exchange rate volatility predicts foreign currency appreciation and a higher currency return. A one-percent increase in dollar exchange rate volatility predicts a 20-basis-point average foreign currency appreciation and 25-basis-point currency excess returns per annum at horizons of 1 month, 3 months, and 12 months. In the model, the regression coefficient of currency return on US volatility is equal to 0.10. Although the magnitude of the predictive power is less than that in the data, our model obtains the correct direction of this predictive relationship. The empirical results are robust to including various controls, including average forward discount, US price dividend ratio, and US industrial production growth. The average forward discount and industrial production growth are considered drivers of countercyclical currency risk premiums (Lustig et al., 2014). Moreover, we find that the predictive power of average forward discounts on both exchange rate changes and currency returns are weakened after controlling for exchange rate volatility. The upper panel of Figure 5 reports the regression coefficients and confidence intervals of exchange rate predictability at the 3-month horizon for each currency pair. All point estimates are positive and similar to our results in Table 6, and most coefficients are statistically significant.

While the predictive relations between currency excess returns and exchange rate volatility are novel and consistent with our proposed model, we are not able to distinguish our model from other alternative theories. The relationship between volatility and the currency risk premium is well studied. Backus et al. (2001) show that in a complete market setting with affine linear SDFs, stochastic volatility is necessary to generate a time-varying currency premium. Bansal and Shaliastovich (2013) attribute time variation in the currency risk premium to volatility fluctuations in a structural model. Lustig et al. (2014) also empirically document the link between consumption and inflation volatility and currency returns.

6.4 Predictive Relation between Wholesale Funding Quantity and the Exchange Rate

While our model implies that the tightness of leverage constraints predicts the exchange rate, we examine the predictive power of another widely used measure, the amount of financial commercial paper (CP) outstanding. The empirical results are similar to those of Adrian et al. (2015). Financial CP outstanding is known as the quantity of wholesale funding, which is indicative of the funding conditions in the market. When there is ample financial CP outstanding, the intermediaries have high borrowing capacity, and the leverage constraint is loose. In the context of our model, suppose that the US is the home country; a lower level of financial CP in the US is associated with a tighter home leverage constraint such that the US dollar is expected to depreciate. In our model, we interpret the amount of borrowing D_t as the financial CP outstanding. Using simulated data, we run a predictive regression of currency excess returns on financial CP outstanding and obtain a negative coefficient of -0.15, which validates our intuition. In Table 7, we report the predictive regression of financial CP on the average dollar exchange rate (Panel A) and currency excess returns of a US investor investing in all G10 foreign currencies at a monthly frequency. The first two columns report the key coefficient on financial CP. For all different horizons of 1, 3, and 12 months, higher financial CP growth predicts foreign currency depreciation. In magnitude, using the 1-month horizon as an example, a one-percent increase in CP growth predicts a 0.3 percent foreign currency depreciation in the subsequent month. We also add controls for the average forward discount, price-dividend ratio, growth of industrial production, and exchange rate volatility; the regression coefficient on CP is little affected. In Panel B, we see similar results for excess returns. A 1-percent increase in CP growth predicts a 0.37 percent lower foreign return in the subsequent period. The magnitude is similar to that of the exchange rate change. In the lower panel of Figure 5, we depict the univariate predictive regression coefficients and confidence intervals of exchange rate predictability at the 3-month horizon for each currency pair. All point estimates are negative, and most coefficients are statistically significant.

The amount of CP contains rich information about the status of the economy, including both supply and demand factors. There are numerous other potential channels through which exchange rates are affected. We do not claim to distinguish our model from other theories by using the empirical results in this section. Instead, we argue that these empirical results do not contradict our model.

7 Conclusion

Financial intermediaries are major participants in the foreign exchange (FX) market. In light of the dominance of intermediaries in the FX market and the constraints they face, we introduce these features into an otherwise standard international asset pricing model. An essential feature of financial intermediaries is the constraint on taking leverage. The financial constraint is tightly linked to the volatility in the economy because of the value-at-risk (VaR) rule adopted by major financial intermediaries. We estimate the model using the simulated method of moments (SMM) and show that the model can quantitatively resolve four exchange rate puzzles. We resolve the Backus-Smith puzzle by replacing the standard consumption Euler equation with an intermediary

Euler equation such that consumption and exchange rates are disconnected. Regarding the forward premium puzzle, when volatility increases in the home country, its interest rate declines. Moreover, because of the higher excess return required by home intermediaries, there is an expected appreciation of the foreign currency. The exchange rate volatility better approximates the data as we introduce another source of exchange rate fluctuations. Tightened banking regulations after the global financial crises constrain the intermediaries from engaging in arbitrage in the currency forward market and generate deviations from covered interest rate parity (CIP). Moreover, the model generates cyclicality in CIP deviations consistent with empirical evidence. The deviations are large when the home currency is strong and when volatility is large. Several new model implications are consistent with the data. The relative TED spread, a common measure of the relative tightness of leverage constraints, is highly correlated with the exchange rates and it drives capital flows in the same direction as the model prediction. In terms of predictive relations, exchange rate volatility and financial commercial paper both significantly predict exchange rates and currency excess returns at various horizons. These two variables are also considered measures of leverage constraint tightness.

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Table 1: Estimation Results

The table shows the sample moments in the data and implied population moments in the model. Panel A reports targeted moments in the SMM estimation. Panel B reports additional untargeted moments. Panel C reports moments related to new implications of the model.

Moments	Data	Model
Panel A. SMM target moments		
$sd(\Delta c)$	1.83	2.01
P_yC_y/C	0.17	0.16
S_x	0.85	0.89
$sd(NX/GDP)$	1.72	1.98
$sd(\log(\sigma_{X,t+1}))$	0.23	0.21
r_f	0.74	0.73
$r_s - r_f$	0.98	0.99
ϕ	0.12	0.12
$corr(\Delta q,\ \Delta c - \Delta c^*)$	-0.05	-0.05
β_{fp}	2.20	1.63
$sd(\Delta q)$	8.13	5.27
r_{cip}	-0.25	-0.25
$sd(r_{cip})$	0.27	0.23
Panel B. Additional moments		
$sd(r_f)$	1.16	0.42
$sd(r_s - r_f)$	0.71	3.69
$corr(NX/GDP_t, NX/GDP_{t-1})$	0.99	0.99
$sd(\phi)$	0.03	0.01
$corr(\phi_t, \phi_{t-1})$	0.98	0.95
$r_{dolloar}$	5.30	2.19
SR_{dollor}	0.64	0.41
Panel C. New implications		
$\beta_{cip,-\Delta q}$	-2.02	-1.47
$\beta_{cip,\sigma}$	-0.21	-1.01
$corr(q, Ted_{us} - Ted_f)$	-0.38	-0.27
$corr(\Delta q, \Delta(Ted_{us} - Ted_f))$	-0.46	-0.61
$\beta_{cf,Ted_{us}-Ted_f}$	-0.84	-0.31
$\beta_{rx,\sigma}$	0.25	0.10
$\beta_{rx,cp}$	-0.37	-0.15

Table 2: Parameter Estimates

The table reports the parameters estimates from the simulated methods of moments.

Parameter	Description	Value
	Preference	
β	Discount factor	0.998
α	Home bias	0.029
σ	Trade elasticity	0.452
	Endowment dynamics	
$\bar{\sigma}$	Volatility of endowment shock	0.010
ρ_σ	Volatility persistence	0.953
σ_σ	Volatility of volatility shock	0.065
	Intermediaries	
θ_0	VaR constant	0.118
θ_1	VaR volatility elasticity	0.392
η	Net worth	19.854
ψ	CIP constraint	0.199

Table 3: Sensitivity Analysis

The table shows the sample moments in the data and model implied population moments. The third column show the benchmark model with the parameter estimated from SMM. $\theta_0 = 0.118$, $\theta_1 = 0.392$ and $\eta = 19.854$. The other columns report models taking different parameters values shown in the table while fixing the other parameters at the benchmark.

Moments	Data	Benchmark	$\theta_0 = 0.10$	$\theta_0 = 0.15$	$\theta_1 = 0$	$\theta_1 = 0.6$	$\eta = 15$	$\eta = 25$
$sd(\Delta c)$	1.83	2.01	1.96	2.07	1.68	2.33	2.00	2.01
$P_y C_y / C$	0.17	0.16	0.16	0.17	0.17	0.16	0.17	0.16
S_x	0.85	0.89	0.90	0.87	0.88	0.89	0.87	0.91
$sd(NX/GDP)$	1.72	1.98	1.96	2.01	1.95	1.98	2.01	1.95
$sd(\log(\sigma_{X,t+1}))$	0.23	0.21	0.21	0.21	0.21	0.21	0.21	0.21
r_f	0.74	0.73	0.74	0.72	0.81	0.66	0.74	0.73
$r_s - r_f$	0.98	0.99	0.73	1.46	0.90	1.09	1.51	0.66
ϕ	0.12	0.12	0.10	0.15	0.12	0.12	0.11	0.12
$corr(\Delta q,\ \Delta c - \Delta c^*)$	-0.05	-0.05	0.03	-0.15	1.00	-0.43	-0.04	-0.06
β_{fp}	2.20	1.63	1.59	1.67	0.30	1.75	1.63	1.62
$sd(\Delta q)$	8.13	5.27	5.08	5.56	3.82	6.67	5.25	5.28
r_{cip}	-0.25	-0.25	-0.20	-0.34	-0.18	-0.32	-0.33	-0.20
$sd(r_{cip})$	0.27	0.23	0.20	0.27	0.02	0.31	0.25	0.21

Table 4: Intermediaries and Exchange Rates

The table reports estimates from OLS regressions of average dollar real exchange rate and the average relative ted spread between US and other G10 countries. In Panel A, the first row reports the regression coefficients for exchange rate and relative ted spread both in levels. Row 2 reports the regression coefficients with controls of consumption and output growth differential. In Panel B, we repeat the exercise for exchange rate and relative ted spread in first difference. In the last column of rows with univariate regression (row 1 in Panel A and B), we report the correlation between the regressor and the regressand. In Panel C, we separate the relative ted spread into average ted spread in other G10 countries and the ted spread in the US. All t statistics are calculated based on heteroskedasticity and autocorrelation adjusted standard errors.

Panel A. Relative Ted Spread and Exchange Rates: Level								
	$Ted_{us} - Ted_f$	(t-stat)	$\Delta c_{us} - \Delta c_f$	(t-stat)	$\Delta y_{us} - \Delta y_f$	(t-stat)	R^2	$corr$
q	-0.20	(-2.67)					0.15	-0.38
q	-0.20	(-2.64)	-0.12	(-0.05)	0.35	(0.13)	0.15	
Panel B. Relative Ted Spread and Exchange Rates: First Difference								
	$\Delta(Ted_{us} - Ted_f)$	(t-stat)	$\Delta c_{us} - \Delta c_f$	(t-stat)	$\Delta y_{us} - \Delta y_f$	(t-stat)	R^2	$corr$
Δq	-0.12	(-4.05)					0.21	-0.46
Δq	-0.12	(-4.04)	0.29	(0.33)	0.85	(1.06)	0.22	
Panel C. Ted Spreads and Exchange Rates								
	Ted_{us}	(t-stat)	Ted_f	(t-stat)			R^2	
q	-0.14	(-1.86)	0.29	(3.80)			0.23	
Panel D. Ted Spread and Exchange Rates: First Difference								
	ΔTed_{us}	(t-stat)	ΔTed_f	(t-stat)			R^2	
Δq	-0.12	(-4.15)	0.12	(2.83)			0.21	

Table 5: Intermediaries and Capital Flows

This table reports the coefficients for pooled-time-series regression of country level net capital flows and ted spread. G10 countries are included in the regression. Net capital flow is measured as each country's $\frac{NetExport}{Consumption+NetExport}$. Both ted spread and the net capital flow are in first-differences. Row 1 does not control for country fixed effect while row 2 control for country fixed effect. In row 3 and 4, additional controls of output and consumption growth are included. Data span 1988Q1 to 2017Q2, with t-statistics in the parentheses calculated using clustered standard errors.

Country Level Net Capital Flow							
Ted	(t-stat)	Δy	(t-stat)	Δc	t-stat	Country FE	R^2
-0.84	(-3.55)					N	0.01
-0.85	(-3.56)					Y	0.02
-0.84	(-3.57)	0.43	(4.23)			Y	0.06
-0.88	(-4.07)			-0.26	(-3.15)	Y	0.03

Table 6: Volatility and Exchange Rates

The table reports estimates from OLS regressions of future exchange rate changes and currency excess returns on currency volatility and other controls. $\Sigma_{i=1}^{h}\Delta y_{t+i} = \beta_0 + cp_t\beta_1 + AFD_t\beta_2 + pd_t\beta_3 + \Delta ip_t\beta_4 + vol\beta_5 + u_{t+h}$. Δy_t is either exchange rate changes or currency excess returns. cp_t is the annual growth rate of commercial paper outstanding. AFD is the average forward discount. pd is price-to-dividend ratio. Δip_t is the annual growth of industrial production. the average dollar realized volatility. h shows the predictive horizon on month. The t-statistics are based on heteroscedasticity and autocorrelation consistent (HAC) standard errors (Hodrick, 1992). Data are monthly from 1980M1 to 2015M12.

h	vol	(t-stat)	AFD	(t-stat)	pd	(t-stat)	Δip	(t-stat)	R^2
				Panel A. Exchange Rate Regressions					
1	0.20	(2.43)							0.02
3	0.21	(2.59)							0.05
12	0.21	(3.22)							0.16
1			1.26	(1.43)					0.01
3			1.06	(1.41)					0.01
12			1.03	(1.76)					0.04
1	0.17	(1.96)	0.68	(0.71)					0.02
3	0.19	(2.26)	0.40	(0.51)					0.05
12	0.20	(2.83)	0.35	(0.56)					0.17
1	0.22	(2.32)	0.94	(0.91)	0.04	(1.02)	0.55	(1.16)	0.02
3	0.21	(2.33)	0.72	(0.85)	0.05	(1.27)	0.36	(0.82)	0.06
12	0.19	(2.39)	0.70	(1.08)	0.06	(1.50)	0.09	(0.25)	0.21
				Panel B. Currency Excess Return Regressions					
1	0.24	(2.93)							0.02
3	0.25	(3.04)							0.07
12	0.24	(3.62)							0.20
1			2.59	(2.93)					0.03
3			2.20	(2.89)					0.06
12			1.80	(3.05)					0.12
1	0.17	(1.94)	2.02	(2.15)					0.04
3	0.19	(2.24)	1.55	(1.96)					0.09
12	0.20	(2.84)	1.12	(1.80)					0.24
1	0.22	(2.31)	2.16	(2.10)	0.02	(0.51)	0.51	(1.08)	0.04
3	0.22	(2.36)	1.73	(2.01)	0.03	(0.65)	0.35	(0.79)	0.10
12	0.20	(2.52)	1.28	(1.98)	0.03	(0.66)	0.12	(0.31)	0.25

Table 7: Commercial Paper Outstanding and Exchange Rates

The table reports estimates from OLS regressions of future exchange rate changes and currency excess returns on commercial paper outstanding and other controls. $\Sigma_{i=1}^{h}\Delta y_{t+i} = \beta_0 + cp_t\beta_1 + AFD_t\beta_2 + pd_t\beta_3 + \Delta ip_t\beta_4 + vol\beta_5 + u_{t+h}$. Δy is either exchange rate changes or currency excess returns. cp is the annual growth rate of commercial paper outstanding. AFD is the average forward discount. pd is price-to-dividend ratio. Δip_t is the annual growth of industrial production. vol is the average dollar realized volatility. h shows the predictive horizon of months. The t-statistics are based on heteroscedasticity and autocorrelation consistent (HAC) standard errors (Hodrick, 1992). Data are monthly from 1991M1 to 2015M12.

h	cp	(t-stat)	AFD	(t-stat)	pd	(t-stat)	Δip	(t-stat)	vol	(t-stat)	R^2
					Panel A. Exchange Rate						
1	-0.30	(-2.18)									0.02
3	-0.30	(-2.35)									0.05
12	-0.19	(-2.01)									0.08
1	-0.39	(-2.17)	-1.56	(-0.95)							0.02
3	-0.40	(-2.57)	-1.68	(-1.28)							0.07
12	-0.24	(-2.38)	-0.83	(-1.14)							0.09
1	-0.53	(-2.54)	-1.66	(-0.91)	0.02	(0.18)	0.66	(0.98)			0.02
3	-0.48	(-2.60)	-1.68	(-1.18)	0.02	(0.23)	0.38	(0.62)			0.07
12	-0.22	(-1.81)	-0.55	(-0.67)	0.04	(0.52)	-0.20	(-0.45)			0.10
1	-0.32	(-2.06)							-0.02	(-0.19)	0.02
3	-0.28	(-2.00)							0.02	(0.16)	0.05
12	-0.12	(-1.11)							0.09	(1.03)	0.10
					Panel B. Currency Excess Return						
1	-0.38	(-2.73)									0.03
3	-0.37	(-2.87)									0.08
12	-0.26	(-2.65)									0.14
1	-0.41	(-2.32)	-0.55	(-0.37)							0.03
3	-0.41	(-2.67)	-0.71	(-0.57)							0.08
12	-0.26	(-2.52)	-0.01	(-0.02)							0.14
1	-0.53	(-2.61)	-0.74	(-0.44)	0.00	(-0.01)	0.63	(0.95)			0.03
3	-0.47	(-2.65)	-0.76	(-0.57)	0.01	(0.08)	0.33	(0.54)			0.08
12	-0.22	(-1.83)	0.28	(0.34)	0.04	(0.49)	-0.28	(-0.63)			0.15
1	-0.40	(-2.60)							-0.03	(-0.25)	0.03
3	-0.36	(-2.54)							0.01	(0.08)	0.08
12	-0.19	(-1.77)							0.08	(0.93)	0.16

Figure 1: Model Structure

The figure shows the structure of the model in a circular flow diagram.

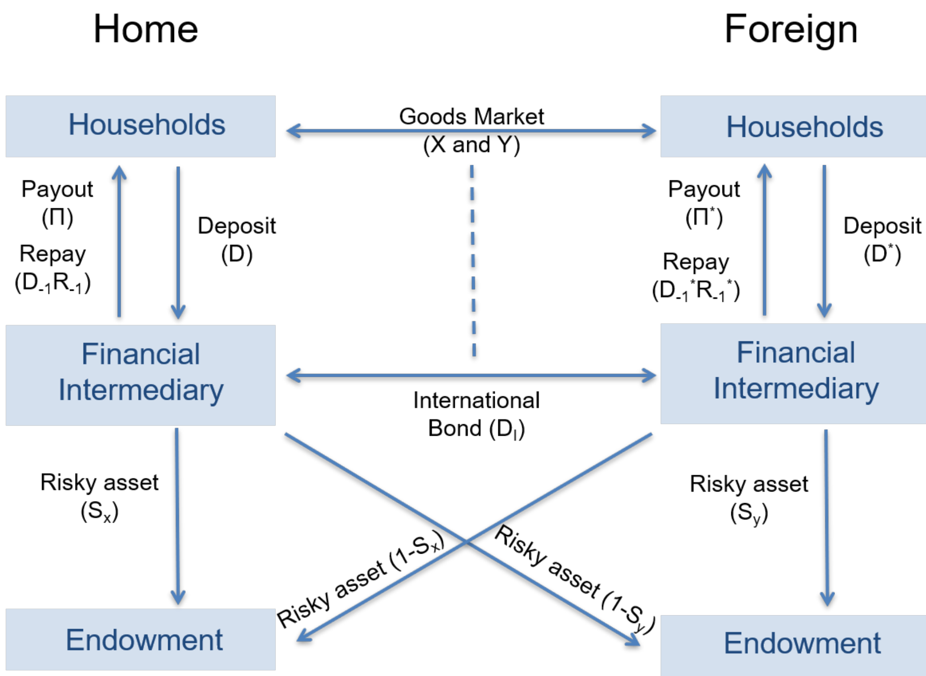


Figure 2: Impulse Response Functions to a Positive Home Volatility Shock

The figure reports the impulse responses to a positive one-standard-deviation home volatility shock. Variables include the expected excess return on international bond return ($E[R_{I,t+1} - R_{f,t}]$), the constraint multiplier (κ_t), the exchange rate (q_t), consumption (C_t), the risk-free rate ($R_{f,t}$), and net export (NX_t). Impulse responses of both home and foreign variables are shown in the same figure. The ranges of y-axis are the same as Figure 3 for comparison.

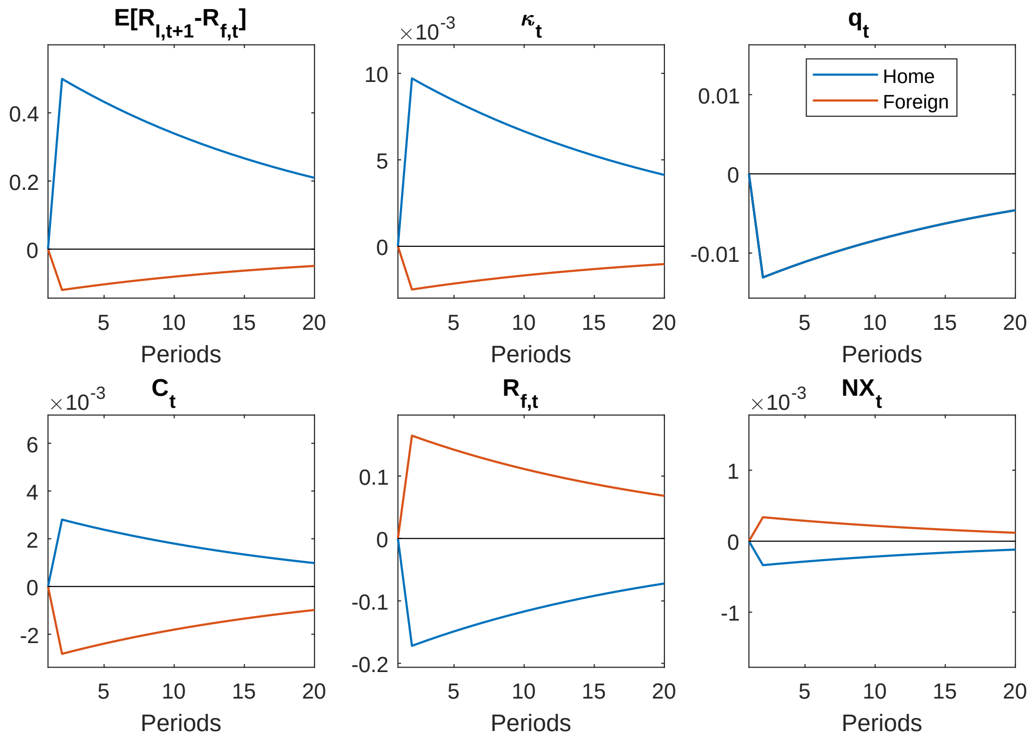


Figure 3: Impulse Response Functions to a Positive Home Endowment Shock

The figure reports the impulse responses to a positive one-standard-deviation home endowment shock. Variables include the expected excess return on international bond return ($E[R_{I,t+1} - R_{f,t}]$), the constraint multiplier (κ_t), the exchange rate (q_t), consumption (C_t), the risk-free rate ($R_{f,t}$), and net export (NX_t). Impulse responses of both home and foreign variables are shown in the same figure. The ranges of y-axis are the same as Figure 2 for comparison.

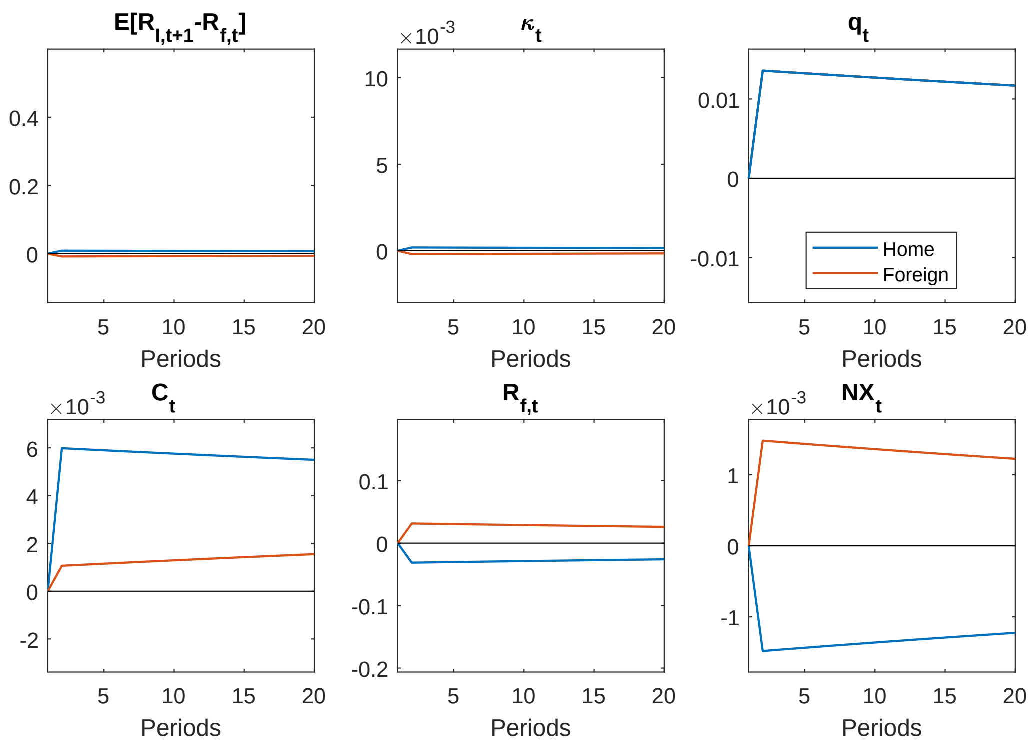


Figure 4: Exchange Rate and Ted Spread

This figure plots the time series of average ted spread difference (US-foreign) and the log of dollar real exchange rate from 1988Q1 to 2017Q2. Log real dollar exchange rate is obtained by cumulating average dollar exchange rate change every quarter, with 1988Q1 normalized to 0. Dollar real exchange rate is defined as the price of foreign currency in dollars after CPI adjustment. An increase in the real exchange rate value means a dollar depreciation.

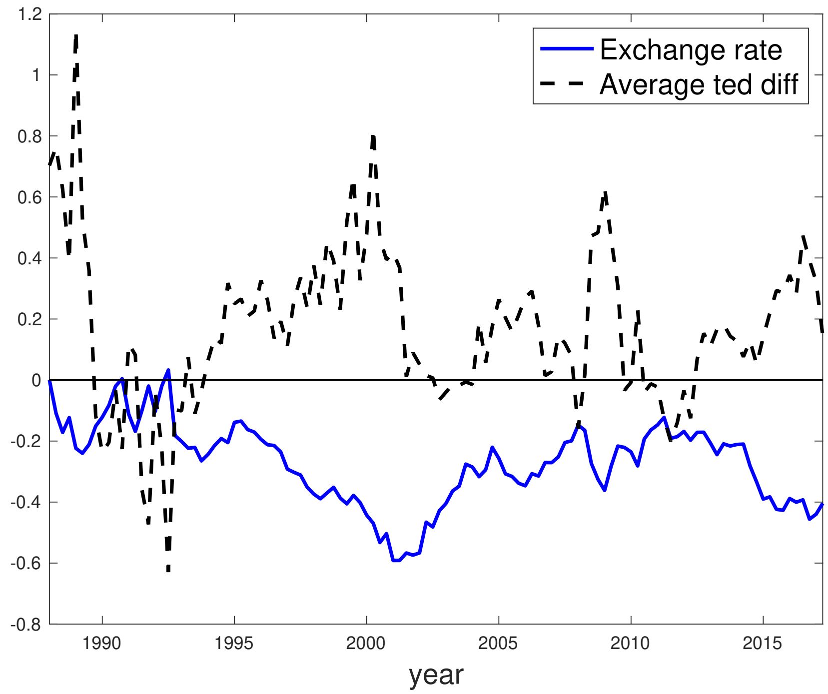
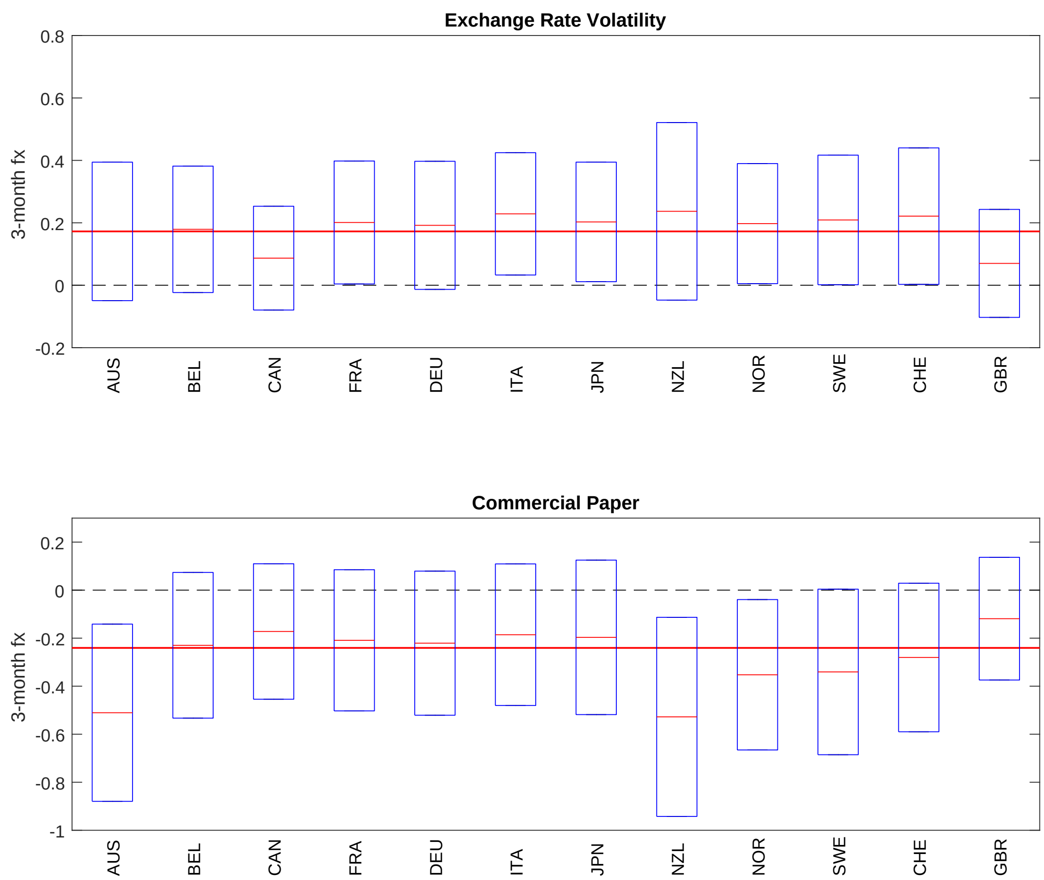


Figure 5: Exchange Rate Predictability: Individual Countries
The figures present the univariate regression evidence of predictability of future dollar value by volatility and growth of commercial paper outstanding. The dependent variables are the log changes in real dollar values against individual currencies. The Figure shows the OLS coefficients on exchange rate volatility (upper panel) and commercial paper outstanding (lower panel) and the associated HAC 95% confidence intervals. Data are monthly from 1980M1 to 2015M12.



A Equilibrium Conditions Characterization

In this appendix, we characterize all the equilibrium conditions of our model.

Home and foreign households' consumption aggregation

$$C_t^{\frac{\sigma-1}{\sigma}} = (1-\alpha)C_{x,t}^{\frac{\sigma-1}{\sigma}} + \alpha C_{y,t}^{\frac{\sigma-1}{\sigma}} \tag{13}$$

$$(C_t^*)^{\frac{\sigma-1}{\sigma}} = \alpha(C_{x,t}^*)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(C_{y,t}^*)^{\frac{\sigma-1}{\sigma}} \tag{14}$$

Home and foreign households' optimality conditions for relative consumption and relative price:

$$\frac{C_{y,t}}{C_{x,t}} = (\frac{P_{x,t}}{P_{y,t}}\frac{\alpha}{1-\alpha})^{\sigma} \tag{15}$$

$$\frac{C_{y,t}^*}{C_{x,t}^*} = (\frac{P_{x,t}}{P_{y,t}}\frac{1-\alpha}{\alpha})^{\sigma} \tag{16}$$

Home and foreign households' period budget constraint:

$$P_{x,t}C_{x,t} + P_{y,t}C_{y,t} = C_t \tag{17}$$

$$P_{x,t}C_{x,t}^* + P_{y,t}C_{y,t}^* = Q_tC_t^* \tag{18}$$

Home and foreign households' Euler equations:

$$E_tM_{t+1}R_{f,t} = 1 \tag{19}$$

$$E_tM_{t+1}^*R_{f,t}^* = 1 \tag{20}$$

Home consolidated budget constraint for households and intermediaries:

$$\begin{aligned} &C_t + S_{x,t}P_{s,t} + S_{y,t}P_{s,t}^*Q_t + D_{I,t} \\ =&S_{x,t-1}(P_{s,t} + P_{x,t}X_t) + S_{y,t-1}(P_{s.t}^*Q_t + P_{y,t}Y_t) + D_{I,t-1}R_{b,t-1}\frac{1+Q_t}{1+Q_{t-1}} \end{aligned} \tag{21}$$

Foreign consolidated budget constraint for households and intermediaries can be implied by Walras' law.

Return on home and foreign trees (both denominated in the home consumption basket):

$$R_{s,t} = \frac{P_{x,t}X_t + P_{s,t}}{P_{s,t-1}} \tag{22}$$

$$R_{s,t}^* = \frac{P_{y,t}Y_t + Q_tP_{s,t}^*}{Q_{t-1}P_{s,t-1}^*} \tag{23}$$

Home and foreign intermediaries' Euler equations:

$$E_t M_{t+1} R_{s,t+1} = 1 + \kappa_t \theta_t \tag{24}$$

$$E_t M_{t+1} R^*_{s,t+1} = 1 + \kappa_t \theta_t \tag{25}$$

$$E_t M^*_{t+1} \frac{Q_t}{Q_{t+1}} R_{s,t+1} = 1 + \kappa^*_t \theta^*_t \tag{26}$$

$$E_t M^*_{t+1} \frac{Q_t}{Q_{t+1}} R^*_{s,t+1} = 1 + \kappa^*_t \theta^*_t \tag{27}$$

$$E_t M_{t+1} R_{b,t} \frac{1+Q_{t+1}}{1+Q_t} = 1 + \kappa_t \theta_t \tag{28}$$

$$E_t M^*_{t+1} R_{b,t} \frac{1+Q_{t+1}}{1+Q_t} \frac{Q_t}{Q_{t+1}} = 1 + \kappa^*_t \theta^*_t \tag{29}$$

Intermediaries' value functions:

$$\theta_t (S_{x,t} P_{s,t} + S_{y,t} P^*_{s,t} Q_t + D_{I,t}) = \frac{1}{1-\kappa_t} N_t \tag{30}$$

$$\theta^*_t ((1-S_{x,t}) \frac{P_{s,t}}{Q_t} + (1-S_{y,t}) P^*_{s,t} + D^*_{I,t}) = \frac{1}{1-\kappa^*_t} = N^*_t \tag{31}$$

Market clearing conditions:

$$C_{x,t} + C^*_{x,t} = X_t \tag{32}$$

$$C_{y,t} + C^*_{y,t} = Y_t \tag{33}$$

$$D_{I,t} + D^*_{I,t} Q_t = 0 \tag{34}$$

Exogenous home and foreign intermediaries' net worth:

$$N_t = \eta X_t \tag{35}$$

$$N^*_t = \eta Y_t \tag{36}$$

Exogenous processes:

$$\log X_t - \log X_{t-1} = \tau(\log Y_{t-1} - \log X_{t-1}) + \bar{\sigma} \sigma_{x,t-1} \varepsilon_{x,t} \tag{37}$$

$$\log Y_t - \log Y_{t-1} = -\tau(\log Y_{t-1} - \log X_{t-1}) + \bar{\sigma} \sigma_{y,t-1} \varepsilon_{y,t} \tag{38}$$

$$\log \sigma_{x,t} = \rho_\sigma \log \sigma_{x,t-1} + \sigma_\sigma \varepsilon_{\sigma_x,t} \tag{39}$$

$$\log \sigma_{y,t} = \rho_\sigma \log \sigma_{y,t-1} + \sigma_\sigma \varepsilon_{\sigma_y,t} \tag{40}$$

$$\log \theta_t = \log \theta_0 + \theta_1 \log \sigma_{x,t} \tag{41}$$

$$\log \theta_t^* = \log \theta_0 + \theta_1 \log \sigma_{y,t}^* \tag{42}$$

The intermediaries' net payout to the households have two components. First, they rebate the initial net worth and profits to the households. Second, households endow these intermediaries net worth to set up new intermediaries.

$$\Pi_{t+1} = P_{s,t}R_{s,t+1}S_{x,t} + P_{s,t}^* R_{s,t+1}^* Q_{t+1} S_{y,t} + D_{I,t}R_{I,t+1} - D_t R_{f,t} - N_{t+1} \tag{43}$$

B Proof of Proposition 1

We start from the Euler equation for the international bond for both home and foreign intermediaries:

$$E_t M_{t,t+1} R_{b,t} \frac{1+Q_{t+1}}{1+Q_t} = 1 + \kappa_t \theta_t, \quad E_t M_{t,t+1}^* R_{b,t} \frac{1+Q_{t+1}}{1+Q_t} \frac{Q_t}{Q_{t+1}} = 1 + \kappa_t^* \theta_t^*$$

We use lower case letters to denote variables in log. Under the log-normal assumption, we can write the two Euler equations as:

$$E_t m_{t,t+1} + r_{b,t} + E_t \log \frac{1+Q_{t+1}}{1+Q_t} + \frac{1}{2} var_t(m_{t,t+1}, \log \frac{1+Q_{t+1}}{1+Q_t}) = \log(1+\kappa_t \theta_t)$$

$$E_t m_{t,t+1}^* + r_{b,t} + E_t \log \frac{1+Q_{t+1}}{1+Q_t} - E_t \Delta q_{t+1} + \frac{1}{2} var_t(m_{t,t+1}^*, \log \frac{1+Q_{t+1}}{1+Q_t} - \Delta q_{t+1}) = \log(1+\kappa_t^* \theta_t^*)$$

Take the difference of the two equations:

$$E_t \Delta q_{t+1} = -(E_t m_{t,t+1} - E_t m_{t,t+1}^*) - \frac{1}{2} var_t(m_{t,t+1} + \log \frac{1+Q_{t+1}}{1+Q_t}) + \frac{1}{2} var_t(m_{t+1}^* + \log \frac{1+Q_{t+1}}{1+Q_t} - \Delta q_{t+1})$$
$$+ \log(1+\kappa_t \theta_t) - \log(1+\kappa_t^* \theta_t^*)$$

Further, as in our model Q_t fluctuates around 1, we approximate the function $\log(1+Q_t)$ as follows:

$$\log(1+Q_t) = \log[2 \times (1 + \frac{Q_t - 1}{2})] \approx \log 2 + \frac{q_t}{2}$$

The expression for expected exchange rate change becomes:

$$
\begin{aligned}
E_t \Delta q_{t+1} = & -(E_t m_{t,t+1} - E_t m^*_{t,t+1}) - \frac{1}{2} var_t(m_{t,t+1} + \frac{1}{2}\Delta q_{t+1}) + \frac{1}{2} var_t(m^*_{t+1} - \frac{1}{2}\Delta q_{t+1}) \\
& + \log(1+\kappa_t \theta_t) - \log(1+\kappa^*_t \theta^*_t) \\
= & \, r_{f,t} - r^*_{f,t} - \frac{1}{2} cov_t(m_{t,t+1} + m^*_{t,t+1}, q_{t+1} - q_t) + \log(1+\kappa_t \theta_t) - \log(1+\kappa^*_t \theta^*_t)
\end{aligned}
$$

C Model Solution and Estimation Method

C.1 Solution Method

The model is solved numerically using a global projection method (Fernández-Villaverde et al., 2016). The model has four state variables, the home share of total wealth ω_t, the relative size of endowment $\log Y_t - \log X_t$, the volatility of home and foreign country $\sigma_{X,t}$ and $\sigma_{Y,t}$. We use Smolyak polynomials on sparse grids as the basis functions to approximate the policy and pricing functions $\{C_{x,t},\, S_{x,t},\, S_{y,t},\, P_{S,t},\, P^*_{S,t}\}$.

Denote the state variables as $\mathscr{X}_t = [\omega_t, \log Y_t - \log X_t, \sigma_{X,t}, \sigma_{Y,t}]$. Use rescale function $\Phi : R^4 \rightarrow [-1,1]^4$ to rescale the state variables between -1 and 1. For example,

$$
\Phi(\omega_t) = -1 + 2\frac{\omega_t - \omega_{min}}{\omega_{max} - \omega_{min}}
$$

Each policy and pricing function is approximated as $\hat{f}(\mathscr{X}_t; b) = \sum_{n=1}^{N_p} b_n \Psi_n(\Phi(\mathscr{X}_t))$ where b is the approximation parameter. Given the state variables and the approximated $\{C_{x,t},\, S_{x,t},\, S_{y,t},\, P_{S,t},\, P^*_{S,t}\}$, the other endogenous variables can be solved accordingly following the equilibrium conditions. The expectations are computed using monomial integration methods. We compute the state variables at each monomial nodes in the next period. The law of motion of ω_{t+1} is unknown and we approximate it with Smolyak polynomials. Given the state variables, we solve the other endogenous variables and compute errors of the five Euler equations. The algorithm can be briefly structured as follows.

Given the approximation parameters b, at each grid point,

1. Approximate the policy and pricing functions.

2. Solve the other endogenous variables.

3. Solve the state and endogenous variables next period.

4. Compute the Euler equation approximation errors.

Iterate on parameter b to minimize the approximation errors over the grids.

We rely on a Fortran-based numerical optimizer. A crucial element is the initial guess of the solution. We use second-order perturbation to compute the average portfolio holdings and use them as the initial guess. We gradually increase the grids and polynomials to facilitate the convergence and increase the accuracy.

C.2 Estimation Method

The equilibrium model is estimate by simulated method of moments (SMM). Estimation methods are detailed in Fernández-Villaverde et al. (2016).

Denote the moment from data sample by $\hat{m}_T(Y)$ and mode-implied moments $\hat{m}_T(Y;\theta_0,\ M_1)$ under model $M_1$and parameter θ_0. Define the discrepancy

$$G_T(\theta|Y) = \hat{m}_T(Y) - \hat{m}_T(Y;\theta_0,\ M_1)$$

Our estimator $\hat{\theta}_{smm}$ minimizes the criterion function of weighted discrepancy

$$\hat{\theta}_{smm} = argmin_{\theta} G_T(\theta|Y)' W G_T(\theta|Y)$$

Suppose these is a unique θ_0 that $G_T(\theta|Y) \rightarrow 0$ almost surely, then the estimator is consistent. We weight the squared differences by the inverse of the square of the sample moments. The off diagonal elements of the weighting matrix are zero.

$$W_{i,j} = \begin{cases} \frac{1}{\hat{m}_T(Y_i)^2} & i = j \\ 0 & i \neq j \end{cases}$$

In this case, the weight matrix adjust for the difference of the units. When computing the model-implied moments, we simulated the model for 10000 periods.

D Data Sources

In this appendix, we report the details of data that are used in our paper, including measurement, data source, and time coverage.

Consumption, output, import and export data. Source: Quarterly National Accounts of OECD Database. Time coverage: 1973-2018 quarterly.

Share of home assets. Measurement: $\frac{MarketCap - ExternalLiabilities}{MarketCap}$. Source: Market cap data are from the World Bank. External Liabilities in stocks are from the database compiled by Lane and Milesi-Ferretti (2007), the most updated version is extended to 2015. Time coverage: 1985-2015 annual.

Banks' ROA. Source: FRED. Time coverage: 1984-2018 quarterly.

Banks' leverage ratio. Measurement: $\frac{TotalAssets - TotalLiabilities}{TotalAssets}$. Source: the Financial Account Database of the US, reported by the Federal Reserve Board. Time coverage: 1973-2019 quarterly.

Libor and Tbill rate at 1-year maturity. Source: Bloomberg and Datastream. Time coverage: 1988-2017 quarterly.

Stock return and exchange rate. Source: Datastream. Time coverage: 1973-2015 daily. The daily data is used to calculate the realized volatility in each month.

Financial commercial paper outstanding. Source: Federal Reserve. Time coverage: 1991-2015 monthly.

Value-at-risk and total assets of banks. Source: Bloomberg. Time coverage: 1998-2019 quarterly.