What Do We Learn From Cross-Sectional Empirical Estimates in Macroeconomics?

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Abstract

Recent empirical work uses variation across cities or regions to identify the effects of economic shocks of interest to macroeconomists. The interpretation of such estimates is complicated by the fact that they reflect both partial equilibrium and local general equilibrium effects of the shocks. We present a simple method for recovering estimates of partial equilibrium effects from these cross-sectional empirical estimates. This approach makes use of structural assumptions and additional regression estimates to assess the strength of local general equilibrium effects. We apply our method to recent estimates of housing wealth effects based on city-level variation. For this case, we derive conditions under which the partial equilibrium effect of changes in house prices on consumption is equal to the city-level estimate divided by an estimate of the local fiscal multiplier, which measures the amplification from local general equilibrium. We evaluate the accuracy of this calculation in a multi-region, dynamic model of consumption and housing. The paper also reconciles the positive cross-sectional correlation between house price growth and construction with the notion that cities with larger price volatility have lower housing supply elasticities using a model in which housing supply elasticities are more dispersed in the long run than in the short run.

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1 Introduction

A growing literature uses variation across cities or regions to identify the effects of economic shocks of interest to macroeconomists. What exactly these estimates identify is often complicated by the fact that city- or region-level outcomes reflect both the partial equilibrium effect of the shock and the local general equilibrium response to that shock. In this paper we propose an approach by which applied researchers can isolate the partial equilibrium effect of the shock. The partial equilibrium effect has a clear theoretical interpretation and can be easily calculated in standard one-region macro models for calibration purposes. Our method therefore allows researchers to avoid formulating and solving a multi-region macro model to be able to compare their empirical results to analogous concepts in a model.

We apply this method to the analysis of so-called housing wealth effects. The US housing boom and bust in the early 2000s focused the attention of economists on the implications of home prices changes for consumer spending. Prominent recent papers that estimate this housing wealth effect use regional data to relate outcomes such as spending, car registrations, and employment to changes in home prices (e.g. Mian et al., 2013; Mian and Sufi, 2014). The appropriate interpretation of these empirical results is not straightforward. House prices are endogenous at the level of a city and a shock that changes home prices surely alters consumption through other channels such as the equilibrium response of wages and interest rates. In particular, suppose an increase in home prices triggers some additional spending in a city through a housing wealth effect. The extra spending will raise wages and incomes locally, which will lead to more local spending setting in motion a consumption multiplier. For these reasons, it is not immediately clear what we can learn from the change in city-level consumption in response to a change in house prices in the city.

We show how existing empirical estimates of the housing wealth effect on consumption can be mapped into the partial-equilibrium effect of house prices on consumption. We start from the point of view that some prices are determined nationally and some are determined locally. For example, financial markets are highly integrated at a national level while labor markets and non-tradeable goods are quite local in nature. We lay out the conditions under which the constant in a cross-sectional regression or the time fixed effect in a panel regression will absorb the variation in national prices. Turning to the local general equilibrium effects, we show how the local fiscal multiplier can be used to gauge the effect of local general equilibrium multipliers on the response of consumption to changes in house prices. Under certain assumptions that we discuss in more detail below, we show that one can remove local general equilibrium effects from a city-level estimate of the housing wealth effect simply by dividing that estimate by an estimate of the local fiscal multiplier. The logic underlying this result is that the equilibrium response to an increase in local demand will be the same whether that demand comes from private consumption as a result of the housing wealth effect or from a fiscal shock. The end result is an estimate of the partial equilibrium housing wealth effect and marginal propensity to consume out of housing wealth (MPCH) that corresponds to a change in home prices holding fixed wages other non-housing prices.

In recent complementary work (Guren et al., 2018), we estimate the housing wealth effect based on city-level variation in house prices and using retail employment as a proxy for local consumption. We estimate an elasticity of retail employment with respect to house prices of 0.072. In Guren et al. (2018) we show that retail employment has approximately a unit elasticity with respect to consumption in the aggregate and across cities, which allows us to interpret the retail employment response in terms of a consumption response. To convert the elasticity to an MPCH we divide by the housing-consumption ratio, which averaged 2.17 from 1985 to 2016 leading to an MPCH of 3.3 cents on the dollar. Nakamura and Steinsson (2014) estimate the local fiscal multiplier to be approximately 1.5. Dividing our housing wealth effect estimate by Nakamura and Steinsson's local fiscal multiplier estimate yields a partial equilibrium MPCH of 2.2 cents on the dollar.

The simple approach of dividing by the local fiscal multiplier makes sense if a change in house prices only directly stimulates the local economy through a housing wealth effect on consumption, which is then amplified by a consumption multiplier. However, as we have just discussed, an increase in house prices may also stimulate the local economy by increasing construction activity. The increased income of construction workers may have a separate effect on consumption. This is a separate channel from the standard consumption multiplier because the initial partial equilibrium increase in demand is not consumption but residential investment. We show theoretically how to use to account for this effect of construction using our estimate of the construction employment response to home prices and get back to a partial equilibrium housing wealth effect on consumption. When we do this, we arrive at a partial-equilibrium housing wealth elasticity of 0.040 or an MPCH of 1.8 cents on the dollar.

We introduce our methods in the context of a static economy, but they extend to dynamic economies as well as we show in the context of our application. In a dynamic context, there is no single fiscal multiplier but an impulse response of output to a fiscal shock. For example, for the simple case the partial equilibrium effect of home prices on consumption is $F^{-1}E$ where E is a matrix where the (i, j) element gives the housing wealth effect of a price change at date j on consumption at date i and F is a matrix that gives the effect of a fiscal shock at date j on output at date i. Using simulations, we explore how well our static formula performs when the data are generated by a dynamic model. Under some conditions it holds almost exactly and in general it performs well yielding an implied partial equilibrium housing wealth effect that is substantially more accurate than the raw measured effect.

In our analysis we address Davidoff's (2016) critique of the use of supply constraints—such as those capture by the Saiz (2010) estimates—as instruments for home prices. Davidoff points out that if housing markets experience a common demand shock but move along different supply curves, then prices and quantities should be negatively correlated. However, in the data there is a positive relationship between price growth and the growth of housing units. Using the annual change in construction employment as a proxy for quantity growth and using the empirical research design that we developed in Guren et al. (2018), we confirm Davidoff's critique applies at business cycle frequencies.[1]

Distinguishing between short-run and long-run supply elasticities, however, offers a reconciliation of the view that differences in home price dynamics across cities reflect supply constraints as per the usual application of the Saiz instrument with the observed positive quantity response to home prices at business cycle frequencies. We illustrate this point in the context of a regime-switching version of our model in which supply elasticities are more heterogeneous in the long run than in the short run. The distinction between short-run and long-run elasticities could certainly reflect the time it takes to plan and develop new housing units (short-run elasticities are uniformly low in the short-run), but more generally it could reflect differences in constraints on land supply that are not binding in the short run but may bind in the long run as in Nathanson and Zwick (2018). Home prices, like other asset prices, are forward-looking in nature, and as a result, even the short-run dynamics of home prices are primarily affected by the long-run elasticity of supply. The short-run construction response, on the other hand, reflects the short-run constraints faced by developers which shapes the short-run elasticity. Consider two cities that have the same short-run supply elasticity but differ in the long-run supply elasticity. A common, persistent demand shock can move prices differently across the two cities as they move along different long-run supply curves.

[1] Davidoff (2016) shows that quantity growth was negatively correlated with supply elasticity over the 1980-2010 period. The long-horizon quantity response is less worrisome than the business cycle quantity response because differential demand trends across cities can be absorbed by city-level fixed effects in a panel specification (see Guren et al., 2018).

In terms of construction, both cities move along the same short-run supply curve, but by different amounts. The upward-sloping short-run supply curve yields a positive correlation between prices and quantities even though the changes in prices are generated primarily by a common demand shock moving the cities along different (long-run) supply curves.

A key idea in our approach is that the general equilibrium adjustment to a change in private consumption is equivalent to the general equilibrium adjustment to a government spending shock. This demand equivalence has also been explored by several contemporaneous papers. In the context of a two-period model of the stock market wealth effect, Chodorow-Reich et al. (2019) derive a demand equivalence result that links the direct spending response to the change in the local wage bill. Wolf (2019a) puts forward conditions under which demand equivalence holds exactly for the impulse responses of a dynamic model and Wolf (2019b) applies those results to cross-region comparisons and local general equilibrium. In our analysis, the relationship between impulse responses is expressed in terms of the matrix equation $F^{-1}E$.[2] Applying this result directly is challenging because it requires that the researcher observe the full dynamic response to the shock of interest and a fiscal spending shock that has the same dynamics as the (as yet unknown) partial equilibrium response of interest. We show that the simpler adjustment justified in the static context works fairly well across several alternative specifications of a fully dynamic model. As each of these papers considers a different application, taken together, they demonstrate that the demand equivalence logic that is common among them is useful in a variety of contexts.

The paper is organized as follows. Section 2 lays out the general method. Section 3 applies the method to the housing wealth effect. Section 4 provides a fully structural, multi-region macro model of the housing wealth, which we use to assess and motivate the identification assumptions used in Section 3. Section 5 shows how the method can be applied within the context of the full dynamic model. Section 6 discusses empirical regression specifications through the lens of the model. Section 7 conducts a Monte Carlo analysis of the fully structural model to evaluate the accuracy and robustness of the identification scheme used in Section 3. Section 8 concludes.

[2] Wolf describes his results in terms of addition and subtraction of impulse response functions. To understand the connection, a simplified version of our result is $C_{PE} = F^{-1}E$ while Wolf expresses his result as $C_{PE} + (F - I)C_{PE} = E$ where $(F - I)C_{PE}$ is the private consumption response to the fiscal shock that has the same dynamic profile as the partial-equilibrium housing wealth effect.

2 A Framework for Interpreting Cross-Sectional Regressions

We start by laying out an abstract framework that captures the core ideas behind our approach. In the next section we will apply this framework to the analysis of the housing wealth effect.

We consider a static economy composed of J regions that each produce and consume $\ell + L$ goods. The first ℓ goods are produced and consumed locally. The remaining L goods are traded in integrated markets. In total there are $J\ell + L$ goods. We normalize the price of the first good to 1. To fix ideas, the numeraire good could be the final good in the "home" region. Let p_j be the vector of local prices in region j and let P be the vector of national prices. Let x_j be a set of local shocks to region j and let X be a vector of aggregate shocks. Let the vector-valued functions $D(p_j, P, x_j, X)$ and $S(p_j, P, x_j, X)$ be the demand and supply curves for local goods in j. In the local equilibrium of region j we have $D(p_j, P, x_j, X) = S(p_j, P, x_j, X)$. If we assume linearity we can write:

$$(D_p - S_p)\, p_j + (D_x - S_x)\, x_j + (D_P - S_P)\, P + (D_X - S_X)\, X = 0,$$

where D_p is the partial derivative of D with respect to p and so on. In this formulation the outcomes of the regions differ only because they experience different idiosyncratic shocks. In Section 4, we allow the economies to also differ in their housing supply curves.

We assume that the aggregate prices are insensitive to the idiosyncratic shocks. Using the implicit function theorem, the comparative static of the effect of x_j on p_j is given by:

$$\frac{dp_j}{dx_j} = -\left[D_p - S_p\right]^{-1} (D_x - S_x)\,. \tag{1}$$

Let y_j be the equilibrium quantity of local goods produced in j so that in equilibrium:

$$y_j = D(p_j, P, x_j, X) = S(p_j, P, x_j, X).$$

Differentiating, we have

$$\frac{dy_j}{dx_j} = D_p \frac{dp_j}{dx_j} + D_x. \tag{2}$$

Suppose we observe some elements of $\frac{dy_j}{dx_j}$ and some elements of $\frac{dp_j}{dx_j}$, and we wish to identify some of the elements of D_p, S_p, D_x, and/or S_x. We cannot simply interpret $\frac{dy_j}{dx_j}$ as D_x because $\frac{dy_j}{dx_j}$ also reflects the local general equilibrium effects of price adjustments $D_p \frac{dp_j}{dx_j}$. Our approach is to impose some structural assumptions on these unknown matrices, for example zero restrictions on some elements, and then manipulate (1) and (2) to solve for the objects of interest in terms of the observed responses.

Notice that the aggregate shocks and prices play no role in the analysis. Neither do the matrices D_P, S_P, D_X, or S_X. These aggregate disturbances shift quantities and prices uniformly across regions and are absorbed by regression constants or time fixed effects. This outcome relies on two assumptions: linearity and equal incidence of the aggregate shocks and prices (i.e. D_P, S_P, D_X, and S_X are common across regions). These assumptions can be interpreted as a first-order approximation of a non-linear model around a steady state in which all regions are symmetric.

3 Application to the Housing Wealth Effect

We now present a few examples to illustrate the method presented in section 2. We start with a simple example that involves strong assumptions to facilitate the exposition before turning to successively richer examples.

3.1 The Fiscal Multiplier as a Measure of Local GE Effects

The central idea in this application is that the local fiscal multiplier can be used to gauge the extent to which a partial equilibrium effect of home prices on non-durable consumption is amplified by local general equilibrium effects. We start with a simple, static model that demonstrates this logic. As we discuss above, aggregate shocks and national prices do not affect the analysis. We, therefore, start with the behavior of a single region taking aggregate prices as given. Later in this section we incorporate multiple regions and trade linkages. The region has three markets: a goods market, a housing market, and a labor market.

The economy produces goods using labor. The production function is $Y = N$, where Y is goods produced and N is labor supply. There is a labor supply function $N(w, p)$ that depends on the wage, w, and potentially also the price of housing, p. Both prices are expressed in terms of goods. Household demand for goods is given by $C(w, p)$. In addition to this private consumption demand, goods are used for public consumption in amount g, where g is exogenous. The aggregate resource constraint is $Y = C(w, p) + g$. While this resource constraint is very standard, it embeds an assumption that is crucial to our approach: an increase in demand from private consumption requires the exact same supply response as an increase in demand from the government. For housing, we specify an excess demand function $H(w, p, s)$, where s is a shock. Since we only use data on the price of housing and not on the quantity of housing that is traded, we do not need to specify housing supply and demand separately.

In this simple model there are no interest rates or taxes. We assume these variables are determined at the national level and do not differentially affect the regions so their consequences are absorbed by time fixed effects. We discuss these national prices in more detail in the context of the dynamic model presented in Section 4.

The equilibrium level of wages and house prices in this model is given by the solution to the following two equations:

$$C(w, p) + g = N(w, p)$$
$$H(w, p, s) = 0.$$

Using the implicit function theorem as above, comparative statics for the price vector are:

$$\frac{d(w,p)}{d(g,s)} = -\begin{pmatrix} C_w - N_w & C_p - N_p \\ H_w & H_p \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & H_s \end{pmatrix},$$

which we can rearrange as

$$\frac{d(w,p)}{d(g,s)} = \underbrace{\left[N_w - C_w + (C_p - N_p)\frac{H_w}{H_p}\right]^{-1}}_{\equiv M} \begin{pmatrix} 1 & \frac{H_s}{H_p}(N_p - C_p) \\ -\frac{H_w}{H_p} & \frac{H_s}{H_p}(C_w - N_w)\,. \end{pmatrix} \tag{3}$$

Using the logic behind equation (2), the response of local output to the two shocks is given by the row vector:

$$\frac{dY}{d(g,s)} = \begin{pmatrix} C_w & C_p \end{pmatrix} \frac{d(w,p)}{d(g,s)} + \begin{pmatrix} 1 & 0 \end{pmatrix}. \tag{4}$$

Our goal is to estimate C_p, the partial equilibrium effect of home prices on consumption. Suppose we observe an instrumental variables (IV) estimate of the housing wealth effect based on regional variation. In our notation, this is $dY/dp = (dY/ds)/(dp/ds)$, where s is the shock (instrument) used to estimate dY/dp. The trouble is that our IV estimate is the total derivative of consumption with respect to house prices, not the partial derivative C_p. The total derivative includes local general equilibrium effects; for instance, the initial shock may raise wages and lead to an increase in local consumption, which will further raise wages and increase local consumption, and so on.

We make two additional structural assumptions. First, there are no wealth effects on labor supply, $N_p = 0$. We view as likely being a reasonable approximation to reality at least in the short run. This assumption implies that changes in housing wealth are not supply shocks as well as demand shocks. Second, we assume house prices are independent of income, $H_w = 0$. This assumption is not so plausible and we will relax it shortly in section 3.2, but it is helpful to simplify the problem for the sake of exposition. These two assumptions simplify equation (3) to:

$$\frac{d(w,p)}{d(g,s)} = \underbrace{[N_w - C_w]^{-1}}_{=M} \begin{pmatrix} 1 & -\frac{H_s}{H_p} C_p \\ 0 & \frac{H_s}{H_p}\left(C_w - N_w\right). \end{pmatrix}$$

Combining this equation with equation (4) then yields

$$\begin{aligned} \frac{dY}{dg} &= 1 + MC_w \\ \frac{dp}{ds} &= M\frac{H_s}{H_p}\left(C_w - N_w\right) \\ \frac{dY}{ds} &= -MC_p\frac{H_s}{H_p}N_w, \end{aligned}$$

which imply

$$\frac{dY/ds}{dp/ds} = C_p\frac{N_w}{N_w - C_w} \tag{5}$$

$$\frac{dY}{dg} = \frac{N_w}{N_w - C_w}, \tag{6}$$

where the second line uses the definition of M from equation 3 above. Finally, dividing the first of these two equations by the second yields:

$$C_p = \frac{(dY/ds)/(dp/ds)}{dY/dg} = \frac{dY/dp}{dY/dg}. \tag{7}$$

Equation (7) implies that given the two simplifying assumptions we made above, the partial equilibrium effect of house prices on consumption, C_p, is equal to the cross-region IV estimate of the housing wealth effect, dY/dp, divided by the local fiscal multiplier, dY/dg. Looking back at equations (5) and (6), we see that the the partial equilibrium effect differs from the cross-region

IV estimate by a local general equilibrium multiplier ($N_w/(N_w - C_w)$) and that the local fiscal multiplier provides an estimate of this local general equilibrium multiplier. An estimate of the local fiscal multiplier can therefore be used to convert a cross-region IV estimate of the housing wealth effect into an estimate of the partial equilibrium effect of house prices on consumption. Intuitively, an increase in home prices of one unit spurs an extra C_p of spending, which then triggers local adjustments in wages with accompanying consumption effects. These same local adjustments would occur if the initial spending were due to a government spending shock.

In Guren et al. (2018) we estimate a marginal propensity to consume out of housing wealth (MPCH) of 3.3 cents on the dollar. This estimate corresponds to the total effect captured by $(dY/ds)/(dp/ds)$. Nakamura and Steinsson (2014) estimate a local fiscal multiplier of about 1.5.[3] Equation (7) then implies that the partial equilibrium MPCH is 2.2 cents on the dollar.

3.2 Allowing for Residential Investment and Income Effects on Housing

We now add two elements to the analysis. First, we allow residential investment to respond to home price changes, and, second, we allow housing demand to respond to income (i.e. we do not restrict $H_w = 0$). In this case, the equilibrium level of wages and house prices are given by the solution to the following two equations:

$$C(w,p) + I(w,p) + g = N(w,p) \tag{8}$$

$$H(w,p,s) = 0. \tag{9}$$

Relative to the previous example, we have now added demand for local goods coming from residential investment, $I(w,p)$. The resource constraint implies that an increase in residential investment leads to a supply response and general equilibrium response of wages and incomes that unfolds in the same was a changes in demand coming from private or public consumption. A response of residential investment to home prices will raise labor demand, wages, and incomes, and raise consumption through an income channel that will show up in the general equilibrium response of consumption to home prices. We will have to account for this mechanism in order to recover the partial equilibrium response of consumption to home prices. As with government purchases,

With this richer model, the conversion of the regional estimate of the housing wealth effect into

[3] Nakamura and Steinsson find larger multipliers in regional data than in state data. As the analysis of the housing wealth effect is undertaken at the city (CBSA) level, it may be appropriate to use a fiscal multiplier somewhat below 1.5.

the partial equilibrium effect of house prices on consumption will require the econometrician to being able to observe several additional pieces of information. In particular, suppose we observe estimates of the response of house prices to government spending shocks, dp/dg, and the response of residential investment to a change in house prices, dI/dp, in addition to the estimates we assumed we had above.

As before, we assume that there is no (short-run) wealth effect on labor supply. In this case, we assume in addition that residential investment is independent of income, $I_w = 0$.[4] Under these assumptions, the partial-equilibrium effect of home prices on consumption is:

$$C_p = \frac{E}{\frac{dY}{dg}\left(1 - EZ\right)} - \frac{dI}{dp}, \tag{10}$$

where E is the housing wealth effect on total spending $E = \frac{(dC/ds)+(dI/ds)}{dp/ds}$ and $Z = \frac{dp/dg}{dY/dg}$ is the income effect on home prices. The derivation of equation (10) appears in Appendix A.

In equation (10), there are two considerations in addition to the fiscal multiplier intuition from the previous example. When house prices are affected by income, $H_w \neq 0$, a house price shock differs from the government spending shock because part of the response to the government spending shock comes through home prices. In computing the housing wealth effect we compute the change in spending relative to the observed total change in home prices. It is not appropriate to apply the full local fiscal multiplier to the initial spending, but rather we only want to apply the part of the local fiscal multiplier that operates through wages. Second, some part of the initial spending response to home prices comes from residential investment. We subtract this component to isolate the part coming from consumption.

We can now use these results to generate a new estimate of the partial equilibrium effect of house prices on consumption by plugging empirical estimates into equation (10). For this purpose, it is to rewrite equation (10) in terms of elasticities:

$$\frac{p}{C}C_p = \frac{\frac{p}{C}E}{\frac{dY}{dg}\left(1 - \frac{C}{Y}\frac{p}{C}E\frac{Y}{p}Z\right)} - \frac{I}{C}\frac{p}{I}\frac{dI}{dp}. \tag{11}$$

We have estimated the elasticity of construction and real estate employment to home prices to be 0.362. This estimate is based on an analogous specification to the full-sample housing wealth effect estimate we present in Column 2 of Table 1 of Guren et al. (2018) with construction and

[4] We could relax the $I_w = 0$ assumption if we observed dC/dg or dI/dg.

real estate employment as the outcome variable. We use this as our estimate of the elasticity of residential investment to home prices. Lamont and Stein (1999) provide estimates of the short-run income elasticity of house prices, which imply it is less than 0.8 and more likely near 0.3. We use 0.3 as our estimate, but our conclusions are little changed by using 0.8. Our estimate of the housing wealth effect as an elasticity is 0.072. Between 1985 and 2016, the average ratio of residential investment to personal consumption was 0.077. We use this for I/C. Over the same period, the C/Y ratio averaged 0.56 (we exclude housing services from consumption). We will again use the Nakamura and Steinsson (2014) estimate of the local fiscal multiplier of 1.5. We can rewrite $(p/C)E$ as:

$$\frac{p}{C}E = \frac{p}{C}\frac{dC/ds}{dp/ds} + \frac{I}{C}\frac{p}{I}\frac{dI/ds}{dp/ds} = 0.072 + 0.077 \times 0.362 = 0.100.$$

Plugging these values into equation (11) we get:

$$\frac{p}{C}C_p = \frac{0.100}{1.5\,(1 - 0.56 \times 0.100 \times 0.3)} - 0.077 \times 0.362 = 0.040. \tag{12}$$

In terms of an MPCH, we divide by a ratio of housing wealth to consumption of 2.17 to arrive at 1.8 cents on the dollar.[5] The implied partial housing wealth effect is lower when we account for the response of construction because some of the measured response of consumption to home prices is attributed to the response of the construction sector bidding up wages, which raises consumption independently of the direct response of consumption to home prices. The strength of the consumption response to this change in demand is identified by the local fiscal multiplier.

3.3 Trade Linkages

Now suppose there are two regions that trade goods with one another. Let ϕ be the expenditure share on local goods so $\phi > 1/2$ implies home bias in goods demand. For this example, we will assume the real exchange rate is fixed and equal to one. Building on (8)-(9), we have the following

[5] We take the value of owner-occupied housing from the flow of funds and divide it by total PCE less PCE on housing services and utilities and then take the average over 1985 to 2016 to arrive at a ratio of 2.17.

system:

$$
\begin{aligned}
\phi\left[C(w,p)+I(w,p)\right]+(1-\phi)\left[C(w^*,p^*)+I(w^*,p^*)\right]+g=N(w,p)=Y\\
(1-\phi)\left[C(w,p)+I(w,p)\right]+\phi\left[C(w^*,p^*)+I(w^*,p^*)\right]+g^*=N(w^*,p^*)=Y^*\\
H(w,p,s)=0\\
H(w^*,p^*,s^*)=0,
\end{aligned}
$$

where a star denotes variables in the foreign region and variables without stars refer to the home region. If we linearize the system, take the difference across regions, and impose the structural assumptions from the previous subsection ($N_p = I_w = 0$) we have the following system of two equations:

$$
\begin{aligned}
(2\phi-1)\left[C_w\hat{w}+(C_p+I_p)\,\hat{p}\right]+\hat{g}=N_w\hat{w}=\hat{Y}\\
H_w\hat{w}+H_p\hat{p}+H_s\hat{s}=0,
\end{aligned}
$$

where hats denote cross-region differences, e.g. $\hat{w} \equiv w - w^*$. This is the same as the linearized system we obtain in Section 3.2 except for two differences. First, the variables in the system are the cross-region differences. And second, some of the components of the first equation are scaled by $(2\phi - 1)$, which reflects the attenuation of cross-region differences in expenditure due to trade linkages. The main result of Section 3.2 , Equation (10) in Seciton 3.2, is now slightly modified:

$$
C_p = \frac{E}{\frac{dY}{dg}\left(1 - E(2\phi-1)Z\right)} - \frac{dI}{dp}. \tag{13}
$$

The derivation appears in Appendix A. The only difference relative to equation 10 is that the adjustment to the fiscal multiplier for the effects of government spending on home prices is now attenuated due to trade linkages.

To apply the result to our empirical estimates we require a value of ϕ. We choose 0.69 following Nakamura and Steinsson (2014).[6] Adjusting for trade linkages has a quantitatively negligible effect on the implied partial equilibrium housing wealth effect. We arrive at an elasticity of 0.039 essentially the same as in Section 3.2.

[6]Nakamura and Steinsson estimate the local fiscal multiplier based on state-level data, while GMNS estimate the housing wealth effect on consumption at the CBSA level. Incorporating trade linkages into our analysis is accounting for the fact that the local fiscal multiplier is attenuated by trade linkages so it makes sense to use a value of ϕ consistent with the geographic unit used to estimate the fiscal multiplier.

Trade linkages attenuate the differences in activity across regions because some of the extra spending in the home region spills over onto the foreign region. This attenuation has essentially no impact on our analysis because the fiscal multiplier and the measured housing wealth effect are attenuated to the same degree. There is actually some subtlety to this outcome. The housing wealth effect is measured in terms of a spending response while the fiscal multiplier is measured in terms of a production response and normally one would think that production is more attenuated than spending. However, in specifying the model we assumed that the government buys a purely local good not a mix of home and foreign goods so the production response to a government spending shock is no more attenuated than the spending response to home prices.

4 Fully Structural Model

We now present a fully-microfounded, dynamic, macro model of multiple regions. After presenting the model we show how our method can be applied in the context of the full model. In subsequent sections we will use numeric solutions of this model to assess the accuracy of the formulae we derived using the simple model we presented in Section 3.

4.1 Model Assumptions

Demographics There are two regions, "home" and "foreign." The population of the entire economy is 1 with a share n in the home region. All variables are expressed in per capita terms.

Preferences Households maximize:

$$\sum_{t=0} \beta^t u(C_t, L_t, Q_t; \Omega_t),$$

where the arguments are consumption, labor supply, units of housing Q_t chosen at date t and held to date $t+1$, and Ω_t is an aggregate housing demand shock. The period utility function is given by:

$$u(C, L, Q; \Omega) = \frac{1}{1-\sigma}\left[\left(C - \frac{L^{1+\nu}}{1+\nu}\right)^{\kappa}(Q - \Omega_t)^{1-\kappa}\right]^{1-\sigma}.$$

Note that consumption and leisure are substitutable in style of Greenwood et al. (1988), which eliminates wealth effects on labor supply, an assumption we maintained in Section 3. We model the housing demand shock using a Stone-Geary formulation, but this exact specification is unimportant.

What matters is that there is a shock that changes the marginal rate of substitution between housing and non-durables.

Commodities and technology Consumption C_t is a Cobb-Douglas bundle of final goods produced in home and foreign:

$$C_t = \phi^{-\phi} (1-\phi)^{-(1-\phi)} C_{H,t}^{\phi} C_{F,t}^{1-\phi},$$

where $C_{F,t}$ is the consumption in home of the good produced in foreign.[7] We will use $*$ to denote foreign variables so $C^*_{H,t}$ is the consumption in foreign of the good produced in home and we assume:

$$C_t^* = \phi^{*-\phi^*} (1-\phi^*)^{-(1-\phi^*)} C_{F,t}^{*\phi^*} C_{H,t}^{*(1-\phi^*)}.$$

$\phi > n$ and $\phi^* > 1-n$ capture the home-bias in demand for goods.

Each region produces a final good out of intermediate inputs. The production of the final good satisfies:

$$Y_t = \left(\int_0^1 y_t(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}}.$$

Each intermediate good is produced linearly out of labor $y_t(z) = L_t(z)$.

Housing supply The supply of housing satisfies:

$$Q_t = (1-\delta) Q_{t-1} + I_t^{\alpha} M_t^{1-\alpha}, \tag{14}$$

where I_t is resources devoted to residential investment and M_t is units of construction permits sold by the federal government. Residential investment requires a mix of local and imported inputs like the one used for consumption:

$$I_t = \phi^{-\phi} (1-\phi)^{-(1-\phi)} I_{H,t}^{\phi} I_{F,t}^{1-\phi}.$$

Markets The two regions share the same money, which serves as the numeraire. Intermediate goods markets are competitive and completely integrated across regions. Intermediate variety firms face Calvo price-setting frictions with probability of adjusting their price of $1-\chi$. The labor markets

[7] Including the term $\phi^{-\phi} (1-\phi)^{-(1-\phi)}$ in the definition of the bundle simplifies the expression for the price index.

are local to each region and competitive with real wages denoted w_t. Units of housing trade at relative price J_t. Households trade trade a nominal bond that pays interest i_t between t and $t+1$. Let $P_t B_t$ be the nominal value of bond holdings in the home region at the end of period t and held between periods t and $t+1$, where P_t is the price level. We consider two cases for asset markets. In the "incomplete markets" economy, there is only trade in bonds. In the "complete markets" economy, the regions also trade state-contingent assets in quantities A_t at prices $\Xi_{t,t+1}$. In the complete markets economy, the bond is redundant, but it can still be priced and this price will enter our monetary policy rule.

Intermediate goods firms produce profits, which are rebated to the households in the region. We use D_t to denote the real profits received. We impose a portfolio holding cost in the style of Schmitt-Grohé and Uribe (2003) whereby holding bond position B_t incurs a flow cost ζB_t^2. The role of this portfolio cost is to lead to a determinant steady state level of wealth holdings in each region, which can be viewed as a crude approximation to precautionary savings motives that decline with wealth.

Government The government purchases goods, sells construction permits, and sets monetary policy. Let G_t and G_t^* be per capita spending in home and foreign, respectively. The government buys local goods in each region. The exogenous process for G_t is:

$$G_t = (1-\rho_G)\bar{G} + \rho_G G_{t-1} + \epsilon_{G,t}. \tag{15}$$

There is a monetary policy rule that sets the nominal interest rate:

$$1 + i_t = \frac{1}{\beta} + \varphi_\pi \left(\bar{\pi} - 1\right) + \varphi_y \bar{Y}_t, \tag{16}$$

where $\bar{\pi} \equiv (\pi)^n (1-\pi)^{1-n}$ and $\bar{Y}_t \equiv n\hat{Y}_t + (1-n)\hat{Y}_t^*$ is the average of log deviations of output from steady state.

The government sells construction permits according to the rule:

$$M_t = \bar{M} J_t^\gamma. \tag{17}$$

The parameter γ reflects the elasticity of supply of vacant land. The government sets the price of a permit, P_t^M equal to the marginal product of land in the construction sector. It is fairly standard to model housing supply as combining a flow of new land or permits with residential investment, but we assume that the supply of land is price elastic while the literature typically assumes it is constant (Davis and Heathcote, 2005; Favilukis et al., 2017; Kaplan et al., 2017). Later we will allow the regions to differ in their land supply elasticities, i.e. $\gamma \neq \gamma^*$, in the spirit of identification schemes that follow Saiz (2010).

The government imposes lump sum taxes in nominal amounts $P_t T_t$ and $P_t^* T_t^*$. The national government budget constraint is:

$$nP_{H,t}G_t + (1-n)P_{F,t}^*G_t^* = nP_tT_t + (1-n)P_t^*T_t^* + nP_t^M M_t + (1-n)P_t^{M*}M_t^*.$$

We assume that the government taxes each region equally in nominal terms.

Market-clearing The market for home goods clears if:

$$Y_t = \phi E_t^{\phi-1}\left(C_t + I_t\right) + \frac{1-n}{n}(1-\phi^*)E_t^{-\phi^*}\left(C_t^* + I_t^*\right) + G_t, \tag{18}$$

where $E_t \equiv P_{H,t}/P_{F,t}$ is the real exchange rate. This expression involves local and home expenditure on the bundles of home and foreign produced goods. The terms $\phi E_t^{\phi-1}$ and $(1-\phi^*)E_t^{-\phi^*}$ show that the cost-minimizing bundle depends on the degree of home bias and the real exchange rate. Similarly, the market for foreign goods clears if:

$$Y_t^* = \frac{n}{1-n}(1-\phi)E_t^{\phi}\left(C_t + I_t\right) + \phi^* E_t^{1-\phi^*}\left(C_t^* + I_t^*\right) + G_t^*. \tag{19}$$

Bond market clearing requires:

$$nB_t + (1-n)B_t^* = 0.$$

Decision problems. Define:

$$\begin{aligned}
\tilde{C}_t &\equiv C_t - \frac{L_t^{1+\nu}}{1+\nu} \\
\tilde{Q}_t &\equiv Q_t - \Omega_t \\
x_t &\equiv \frac{1-\kappa}{\kappa}\left(J_t - \mathbb{E}_t\left[J_{t+1}(1-\delta)\beta\frac{u_{C,t+1}}{u_{C,t}}\right]\right)^{-1}.
\end{aligned}$$

Utility maximization implies:

$$\tilde{Q}_t = x_t \tilde{C}_t$$
$$L_t^{\nu} = w_t.$$

Under incomplete markets, assuming certainty equivalence[8] so that the real return on bonds is treated as known $R_t \equiv (1+i_t)/\pi_{t+1}$ and future home prices are discounted at the safe interest rate, and abstracting from the portfolio holding cost, we have:

$$\tilde{C}_t = \kappa \frac{R_{t-1}B_{t-1} + Q_{t-1}J_t\left(1-\delta\right) + \sum_{\tau=t}^{\infty} R_{t,\tau}^{-1}\left[w_\tau L_\tau - \frac{1-\kappa}{\kappa x_\tau}\Omega_\tau - \frac{L_\tau^{1+\nu}}{1+\nu}\right]}{\sum_{\tau=t}^{\infty} R_{t,\tau}^{-1} X_{t,\tau}}, \tag{20}$$

where

$$R_{t,\tau} \equiv R_{t,\tau-1}R_{\tau-1} \quad \forall \tau > t$$
$$X_{t,\tau} \equiv \left[\beta^t R_{t,\tau}\left(\frac{x_\tau}{x_t}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma},$$

and $R_{t,t} = 1$. See Appendix C for the proof.

Turning to construction, a representative competitive real estate developer maximizes revenue from new homes less material and permit costs:

$$\max_{I_t, M_t} J_t I_t^{\alpha} M_t^{1-\alpha} - I_t - \frac{P_t^M}{P_t} M_t.$$

The first order condition of this problem with respect to I_t and equation (17) imply:

$$I_t = \alpha^{\frac{1}{1-\alpha}} \bar{M} J_t^{\gamma + \frac{1}{1-\alpha}}, \tag{21}$$

so the supply of new housing is:

$$I^{\alpha} M_t^{1-\alpha} = \left(\alpha J_t\right)^{\frac{\alpha}{1-\alpha}} M_t.$$

[8] This case applies to the steady state, perfect foresight transitions, and first-order accurate solutions to stochastic economies.

5 Application in the Full Model

We now show how our method can be applied in the context of the full model. We begin by demonstrating that the local fiscal spending response accounts for the local general equilibrium effects in the measured housing wealth effect when one accounts for dynamics. We then show how estimated empirical relationships relate to the structural model.

5.1 Dynamic Spending Responses

We consider perfect foresight transitions lasting $\mathcal{T}$ periods. We will assume that the two regions are equally open to trade and given their unequal sizes this implies $1-\phi^* = \frac{n}{1-n}(1-\phi)$. And we define $\Phi \equiv \phi + \phi^* - 1$. Furthermore, we assume that prices are perfectly rigid in this section.

Differencing the market clearing conditions across regions gives:

$$\hat{Y} = \Phi\left(\hat{C} + \hat{I}\right) + \hat{G}, \tag{22}$$

where $\hat{Y}$ is a column vector of length $\mathcal{T}$ that gives values of $Y_t - Y_t^*$ for all $t \in \{1, ..., \mathcal{T}\}$. $\hat{C}$, $\hat{I}$, and $\hat{G}$ are defined similarly. Linearize the consumption function (20) around a symmetric steady state and take the difference across regions to arrive at:

$$\hat{C} = \mathbf{C}_J \hat{J} + \mathbf{C}_W \hat{W} + \mathbf{C}_D \hat{D}, \tag{23}$$

where $\mathbf{C}_J$ is a $\mathcal{T} \times \mathcal{T}$ matrix where the $[t, s]$ element gives the coefficient of the response of C_t to J_s and $\mathbf{C}_W$ and $\mathbf{C}_D$ are defined similarly. Here we have omitted the dependence of consumption on Ω and interest rates because these variables are common across regions and drop out when we take the difference. The linearized residential investment response is:

$$\hat{I} = \mathbf{I}_{r,J} \hat{J}, \tag{24}$$

where for now we abstract from regional heterogeneity in land supply.

We can combine the labor supply curve and the aggregate production function, $Y = L$, to write wages as a function of output: $w_t = Y_t^{\nu}$. While this is a static relationship between output at date

t and wages at date t we can express the linearized mapping in the same vector notation as above:

$$\hat{W} = \mathbf{W}\hat{Y}, \tag{25}$$

where $\mathbf{W} \equiv \nu Y^{SS\nu-1}$ and Y^{SS} is steady state output. Similarly, dividends are defined as output net of payments to labor so using the aggregate production function we can write:

$$\hat{D} = \left(1 - w^{SS}\right)\hat{Y} - Y^{SS}\hat{W} = \left(1 - w^{SS}\right)\hat{Y} - Y^{SS}\mathbf{W}\hat{Y} \equiv \mathbf{D}\hat{Y}. \tag{26}$$

Combining (22), (23), (24), (25), and (26) yields:

$$\hat{Y} = \Phi\mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_J\right)\hat{J} + \mathbf{M}\hat{G} \tag{27}$$

where $\mathbf{m} \equiv \mathbf{C}_W\mathbf{W} + \mathbf{C}_D\mathbf{D}$ and $\mathbf{M} \equiv [I - \Phi\mathbf{m}]^{-1}$. Using (23), (24), and (27), the local expenditure is given by:

$$\hat{C} + \hat{I} = \mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_J\right)\hat{J} + \mathbf{m}\mathbf{M}\hat{G}, \tag{28}$$

where we have used the definition of $\mathbf{M}$ to note that $I + \mathbf{m}\Phi\mathbf{M} = \mathbf{M}$.

To get the fiscal multiplier we need to account for the endogenous response of home prices to shocks to G. Linearizing housing demand, $Q(J, W, D, T, i, \Omega)$, and housing supply, $Q^S(J)$, and equating them yields:

$$\mathbf{Q}_J J + \mathbf{Q}_i i + \mathbf{Q}_W W + \mathbf{Q}_D D + \mathbf{Q}_T T + \mathbf{Q}_\Omega \Omega = \mathbf{Q}_J^S J.$$

Differencing across regions and rearranging yields:

$$\hat{J} = \mathbf{J}_Y\hat{Y}, \tag{29}$$

where:

$$\mathbf{J}_Y \equiv \left(\mathbf{Q}_J^S - \mathbf{Q}_J\right)^{-1}\left(\mathbf{Q}_W\mathbf{W} + \mathbf{Q}_D\mathbf{D}\right).$$

Substituting this into (27) and rearranging we then have:

$$\hat{Y} = \left[I - \Phi\mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_J\right)\mathbf{J}_Y\right]^{-1}\mathbf{M}\hat{G}. \tag{30}$$

From equation (28), the impulse response equivalent of our IV estimate of home prices on expenditure is:

$$E \equiv \mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_J\right) \tag{31}$$

and from (30) the fiscal multiplier is:

$$F \equiv \left[I - \Phi\mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_J\right)\mathbf{J}_Y\right]^{-1}\mathbf{M}. \tag{32}$$

And the response of house prices to income is $Z \equiv \mathbf{J}_Y$. Rearranging (32) we have:

$$\mathbf{M} = \left[I - \Phi E Z\right] F$$

so from (32) we have.

$$\mathbf{C}_J = F^{-1}\left[I - \Phi E Z\right]^{-1} E - \mathbf{I}_J. \tag{33}$$

This result is very similar to what we found in the static economy in Section 3.3 (see equation 13), however, here the components E, F, and so on are matrices rather scalars. To fix ideas, E is a $\mathcal{T} \times \mathcal{T}$ matrix in which the (i, j) element gives the response of output in period i to a change in home prices in period j. Similarly, the (i, j) element of F gives the response of output at period i to a change in government spending at date j. If these matrices have important off-diagonal elements, then the logic of our static examples is complicated by dynamic responses of the economy. On the other hand, if the contemporaneous responses are large relative to the dynamic responses (i.e. the matrices are close to diagonal), then the logic of the static economy goes through because (33) reduces to the same scalar relationship as (13).

6 Empirical Specifications Through the Lens of the Model

Suppose the data are generated by the model presented in Section 4 and one applies a standard cross-sectional regression estimate, what model object does one recover?

An estimate of the housing wealth effect might regress the change in consumption on the change in home prices and a constant or time fixed effect. In this discussion, we will work with the equations of the full model, but we will assume that the dynamic relationships between variables are dominated by the static relationships so the matrices $\mathbf{m}$ and $\mathbf{C}_J$ are (approximately) diagonal.[9]

The regression specification is easier to describe in terms of demeaned variables rather than cross-region differences. Using similar steps as in the previous subsection yields:

$$Y_r - \bar{Y} = \Phi\left(C_r - \bar{C} + I_r - \bar{I}\right) + G_r - \bar{G},$$

where Y_r is income in region r and $\bar{Y}$ is population-weighted average income across regions. As above $\Phi \equiv \phi + \phi^* - 1$. Using the linearized consumption function and the equation above we can write:

$$C_r - \bar{C} = \mathbf{M}\mathbf{C}_J\left(J_r - \bar{J}\right) + \mathbf{m}\Phi\left(I_r - \bar{I}\right) + \mathbf{m}\left(G_r - \bar{G}\right). \tag{34}$$

In the model, residential investment is increasing in home prices, but with a different slope in each region due to heterogeneous housing supply elasticities. We can write:

$$I_r - \bar{I} = \mathbf{I}_{r,J}J_r - \mathbf{I}_J\bar{J} = \mathbf{I}_J(J_r - \bar{J}) + (\mathbf{I}_{r,J} - \mathbf{I}_J)J_r,$$

where $\mathbf{I}_{r,J}$ is the slope of the residential investment response to home prices in region r and $\mathbf{I}_J$ is defined so that $\bar{I} = \mathbf{I}_J\bar{J}$. Equation (34) can then be written as:

$$C_r = \underbrace{\left(\mathbf{M}\mathbf{C}_J + \mathbf{m}\Phi\mathbf{I}_J\right)}_{\text{coef. of interest}}\left(J_r - \bar{J}\right) + \underbrace{\mathbf{m}\Phi\left(\mathbf{I}_{r,J} - \mathbf{I}_J\right)J_r + \mathbf{m}\left(G_r - \bar{G}\right)}_{\text{error}} + \underbrace{\bar{C}}_{\text{time fixed effect}}. \tag{35}$$

Changes in aggregate variables (i, Ω, T) affect all regions equally and are absorbed by the time fixed effect. The response of residential investment to home prices can be written:

$$I_r = \underbrace{\bar{\mathbf{I}}_J}_{\text{coef. of interest}}\left(J_r - \bar{J}\right) + \underbrace{\left(\mathbf{I}_{r,J} - \bar{\mathbf{I}}_{r,J}\right)J_r}_{\text{error}} + \underbrace{\bar{I}}_{\text{time fixed effect}}. \tag{36}$$

Equations (35) and (36) show a potential source of bias in the housing wealth effect regression: To the extent that cities differ in their housing supply elasticities they will differ in the response of residential investment to home prices and cities with larger price changes will have smaller elasticities of residential investment. The treatment effects are heterogeneous and the treatment (price

[9] A diagonal $\mathbf{m}$ implies $\mathbf{M}$ is diagonal. $\mathbf{I}_J$ is already diagonal as residential investment only depends on the current home price (see equation 21).

changes) are negatively correlated with the treatment effect (cities with less responsive residential investment have larger price changes). Therefore the estimated average treatment effect is not the population average effect.[10] This bias affects both the measured housing wealth effect and the construction regressions.

A benefit of the adjustment we put forward here is that the bias in the two regressions cancels out when we compute the partial equilibrium housing wealth effect. To see this, when we estimate equation (35) we obtain a coefficient of interest of (see Appendix B):

$$\hat{\gamma}^C = \bar{\gamma}^C + \frac{E\left[\left(\gamma_r^C - \bar{\gamma}^C\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]},$$

where $\gamma_r^C \equiv \mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\Phi\mathbf{I}_{r,J}\right)$ and $\tilde{J}_{r,t} \equiv J_{r,t} - \bar{J}_t$. When we estimate (36) we obtain:

$$\hat{\gamma}^I = \bar{\gamma}^I + \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]},$$

where $\gamma_r^I \equiv \mathbf{I}_{r,J}$. Crucially, note that the regional variation in γ_r^C comes only from $\mathbf{I}_{r,J}$ so we have $\gamma_r^C - \bar{\gamma}^C = \mathbf{M}\mathbf{m}\Phi\left(\gamma_r^I - \bar{\gamma}^I\right)$.

To put the pieces together, we form $E \equiv \mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_J\right)$ by summing the coefficients of interest in in (35) and (36).[11] This gives:

$$\hat{E} = \bar{\gamma}^C + \bar{\gamma}^I + \mathbf{M}\frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]}.$$

Now applying our adjustment:

$$\begin{aligned}
\hat{\mathbf{C}}_J &= \frac{\hat{E}}{\mathbf{M}} - \hat{\gamma}^I \\
&= \frac{\bar{\gamma}^C + \bar{\gamma}^I}{\mathbf{M}} + \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]} - \bar{\gamma}^I - \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]} \\
&= \frac{\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\Phi\bar{\mathbf{I}}_{r,J}\right) + \bar{\mathbf{I}}_{r,J}}{\mathbf{M}} - \bar{\mathbf{I}}_{r,J} \\
&= \mathbf{C}_J,
\end{aligned}$$

[10] IV strategies that use supply constraints as instruments for home prices will not overcome this bias because the price variation the isolate is still correlated with the treatment effects.

[11] Recall $\mathbf{M} = 1 + \mathbf{m}\Phi\mathbf{M}$.

where $\bar{\mathbf{I}}_{r,J}$ is the average $\mathbf{I}_{r,J}$ over r. In the second line the bias to the housing wealth effect E cancels with the bias in the residential investment response. Underlying this result is the fact that the heterogeneity in treatment effects in the two regressions has the same underlying source (the heterogeneity in housing supply curves). When we remove the estimated residential investment response from the estimated housing wealth effect we end up removing the bias.

7 Monte Carlo Analysis

This section presents a series of Monte Carlo studies to explore the accuracy of equation (13) in a variety of contexts. After calibrating the model, we start from a complete-markets version of the model presented in Section 4 in which the approach holds almost exactly and then we add features of the model.

We will work with equation (13) rewritten in terms of elasticities:

$$\frac{\partial \log C}{\partial \log J} = \frac{e}{\frac{dY}{dG}(1 - \Phi e \frac{\bar{C}}{\bar{Y}} \frac{d \log J}{d \log Y})} - \frac{I}{C}\frac{d \log I}{d \log J} \tag{37}$$
$$e \equiv \frac{d \log C}{d \log J} + \frac{\bar{I}}{\bar{C}}\frac{d \log I}{d \log J}.$$

The left-hand side of equation (37) is the partial equilibrium response of consumption to home prices, which we compute from the analytical consumption function and the exact dynamics of home prices implied by the general equilibrium model (see Appendix C). We simulate the full model response to a series of aggregate housing demand shocks to generate a path for home prices (and the full state vector of the model) and at each date we compute the difference in log consumption between the home and foreign regions and regress this difference on the contemporaneous difference in log home prices.

To construct the right-hand side of equation (13), we need various reduced form estimates. $\frac{d \log C}{d \log J}$ is the elasticity of local consumption with respect to local home prices. To estimate this, we simulate the model with only aggregate housing demand shocks and then perform an OLS regression of the change in local consumption on the change in local home prices including time fixed effects. By limiting the shocks to aggregate housing demand shocks we are estimating using the same variation that would be isolated with the sensitivity instrument of Guren et al. (2018). $\frac{d \log I}{d \log J}$ is the elasticity of residential investment with respect to local home prices, which is computed in the same way. $\frac{dY}{dG}$ is the local fiscal multiplier, which we compute by simulating the model with idiosyncratic shocks to local government purchases. We then regress the change in local output on the change in local purchases using OLS. $\frac{d\log J}{d\log Y}$ is the elasticity of local home prices with respect to local output. We compute this using local government purchases as an instrument for local output. For all of these calculations, we simulate the model quarterly, time aggregate to annual data, and then take two-year time differences. This time-aggregation and differencing scheme matches the two-year differences of annual observations format of the data used by Nakamura and Steinsson (2014) although the results are little changed if we simply use quarterly differences. Finally, $\bar{C}/\bar{Y}$ and $\bar{I}/\bar{C}$ are steady state ratios and $\Phi \equiv \phi + \phi^* - 1$ is a measure of home bias all of which we assume are known to the analyst.

7.1 Model calibration

We calibrate the model as follows. We set the population of the home region to one tenth the size of the foreign region. We set the home-bias parameter $\phi = 0.4$ based on the share of shipments in the CFS that go to the same metro area. The elasticity of substitution between varieties is $\eta = 6$.

We normalize the steady state supply of construction permits, $\bar{M}$, so that the steady state relative price of a unit of housing is 1. We set the depreciation rate on housing to 3% annually. We target a 4.4% share of residential investment to GDP, which is the average ratio over the period 1970-2019. This implies the material share in the construction of new houses is $\alpha = 0.38$. The home and foreign regions differ in their land (construction permit) supply elasticities, γ. We set them to match the 10th and 90th percentiles of the elasticity estimates from Saiz (2010), which are 1.05 and 4.39. The Saiz estimates reflect the response of housing units, which we interpret as the change in Q_t. The (long-run) price elasticity of housing supply in the model is $\alpha/(1-\alpha)+\gamma$. Therefore we set $\gamma = 0.45$ and $\gamma^* = 3.78$.

Turning to preferences, we set $\beta = 0.99$, and we set $\kappa = 0.58$ to target a 25% expenditure share on housing, which is the average housing expenditure in the CEX in 2018. We set the labor supply elasticity to 1 and the coefficient of risk aversion to 2. We set a steady state G/Y ratio of 20% and we use standard interest rate rule parameters $\varphi_\pi = 1.5$ and $\varphi_y = 0.125$.

The regional government purchases follow independent AR(1) processes with quarterly persistence of 0.95. The housing demand shock follows an AR(1) with the same persistence. We set the scale of the portfolio holding cost to $\zeta = 10^{-4}$. We set the quarterly Calvo adjustment probability to 11% in order to target the point estimate of 0.030 of the inflation response to local government spending shocks reported by Nakamura and Steinsson (2014). We consider the robustness of our results to our parameter choices in Section 7.7.

7.2 Complete markets

As a starting point, we consider a complete-markets version of the model in which the consumption response to home prices is a function of the current user cost only. We start with a specification in which prices are fully rigid ($\chi = 1$) and houses are produced entirely from construction permits with no material inputs ($\alpha = 0$). The actual partial equilibrium housing wealth effect is particularly simple to compute in this case. Equating the marginal utility of consumption between regions yields:

$$\frac{C_t - \psi \frac{L_t^{1+\nu}}{1+\nu}}{C_t^* - \psi \frac{L_t^{*1+\nu}}{1+\nu}} = \left(\frac{J_t - \mathbb{E}_t \left[J_{t+1} \frac{1-\delta}{1+i_t} \right]}{J_t^* - \mathbb{E}_t \left[J_{t+1}^* \frac{1-\delta}{1+i_t} \right]} \right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma}}.$$

The region with a higher user cost will consume more non-durables and less housing. With mean-reverting home price dynamics, a higher price of housing is associated with a larger user cost, which induces a positive relationship between non-durable consumption and home prices.

Results for this version of the model are shown in the first column of Table 1. The first row of the table shows that the measured housing wealth effect is 0.021. By "measured" we mean regressing the simulated consumption data on the simulated home price data. This consumption response includes both the partial equilibrium response to home prices, but also general equilibrium effects. Importantly it includes local, general equilibrium effects, while the aggregate general equilibrium responses are differenced out across regions (i.e. absorbed by time fixed effects). The second row of the table shows the local fiscal multiplier is 1.384,[12] which we have argued is a gauge of the strength of the the local general equilibrium effects. Row (4) shows that the model produces a small response of home prices to changes in income. Putting the pieces together in row (6), our formula implies a housing wealth elasticity of 0.015. Row (7) shows the actual partial equilibrium housing wealth effect is also 0.015. As above, the adjustment to the fiscal multiplier to account for the endogenous response of home prices has little effect on the interpretation of the results. Therefore, without a (differential) response of construction to housing demand shocks, our formula boils down to dividing the measured housing wealth effect by the local fiscal multiplier and indeed $0.021/1.384 = 0.015$. Finally, row (8) reports the magnitude of the error associated with the

[12]This value is lower than the estimates in Nakamura and Steinsson (2014) primarily because we calibrate the model to reflect the trade linkages across cities and cities are more than regions.

Table 1: Monte Carlo Analysis of Housing Wealth Elasticity

		(i)	(ii)	(iii)	(iv)	(v)
	Complete Markets	✓				
	Rigid Prices	✓	✓			
	Construction				✓	✓
	Long-Run Housing Supply Het.					✓
(1)	Measured Housing Wealth Effect	0.021	0.112	0.152	-0.090	0.104
(2)	Local Fiscal Multiplier	1.384	1.399	1.283	1.296	1.289
(3)	Construction Response	–	–	–	-12.293	1.613
(4)	Income Elasticity of Home Prices	-0.026	0.016	0.013	0.031	0.055
(5)	Inflation Response	–	–	0.030	0.030	0.030
(6)	Implied P.E. Housing Wealth Effect	0.015	0.080	0.118	0.087	0.062
(7)	Actual P.E. Housing Wealth Effect	0.015	0.063	0.074	0.052	0.040
(8)	Relative Error	0.001	0.343	0.573	0.241	0.332

implied housing wealth effect relative to the error associated with the measured housing wealth effect defined as $|\text{Row } 6 - \text{Row } 7|/|\text{Row } 1 - \text{Row } 7|$.

7.3 Incomplete Markets

The second column of Table 1 assumes markets are incomplete while continuing to assume prices are rigid and that no resources are used in constructing houses. Under incomplete markets there is a more complicated determination of the consumption response to home prices that is affected by the expectations of future incomes and user costs (see equation 20). In this case, while the relationship between home prices and consumption is determined by the full dynamic response across dates as in equation (33), it need not be the case that these dynamic relationships can be summarized by simple regressions. The measured housing wealth effect is 0.112, adjusting for the local spending response yields an implied partial equilibrium housing wealth effect of 0.080. The actual partial equilibrium housing wealth effect is a bit lower at 0.063. In this case, the adjustment implied by our formula goes in the right direction, but does not go far enough, which will be true of most of the specifications we consider. Here the relative error is about 1/3 as larger after applying our formula.

7.4 Incorporating Price Responses

Under rigid prices, both regions face the same real interest rate. When prices respond differentially in the two regions, the real interest rates differ across regions. Suppose, the home region experiences a larger increase in activity. It will then also experience a larger increase in inflation, which reduces the real interest rate and further stimulates demand in the home region. On the other hand, the differential price response changes the real exchange rate, which reduces the demand for goods from the home region and increases the demand for goods from the foreign region. These differential price responses affect the fiscal multiplier, so in principle, adjusting the measured housing wealth effect by the fiscal multiplier may fully account for these effects. On the other hand, the price response further complicates the dynamics of the responses in ways that may not be fully captured by our approach.

Column (iii) allows for some degree of price flexibility. Specifically we set the quarterly Calvo adjustment probability to 11% in order to target the point estimate of 0.030 of the inflation response to local government spending shocks reported by Nakamura and Steinsson (2014). In this case, we find that the measured housing wealth effect rises substantially to 0.152. The local fiscal multiplier is reduced due to the expenditure switching effect. The combination of these two changes raises the implied housing wealth effect to 0.118. Even though real interest rates do not change in the partial equilibrium calculation, the partial equilibrium housing wealth effect depends on the particular dynamics of home prices that we feed into the calculation and the general equilibrium home price dynamics have changed as a result of price adjustments. As a result, the actual partial equilibrium housing wealth effect is somewhat larger in this case rising to 0.074. The differential price responses to a housing demand shock are not fully accounted for by the local fiscal multiplier so the adjustment works somewhat less well in this case.

7.5 Adding Construction

We now allow for resources to be used in the construction of housing i.e. we set α equal to its calibrated value of $\alpha = 0.38$. We find that there is a strong negative response of construction to home prices with a coefficient of -12.3 (see row 3). For comparison, using an IV estimate using the sensitivity instrument specification in Guren et al. (2018) yields an elasticity of 0.362. This disconnect between the observed and theoretical responses of construction to aggregate housing demand shifts is related to Davidoff's (2016) critique of the Saiz instrument identification scheme for shocks to local house prices.

In response to an aggregate housing demand shock, both regions move along their own housing supply curves. As the housing supply curve is steeper in the home region, there is a larger response of prices and a smaller response of quantities. In our model, the response of residential investment to home prices is given by (21). As the land supply elasticity, γ, is larger in the foreign region, residential investment responds more strongly to a given change in price. At the same time, the larger supply elasticity leads to a smaller price response in the foreign region. In our simulations, the foreign region has a larger construction response and a smaller price response leading to a strong negative relationship between residential investment and home prices.

The construction response turns the measured housing wealth effect on consumption negative in this specification. To understand the logic of this result, suppose there is a positive aggregate housing demand shock that raises home prices in both regions, but more so in the home region due to the less elastic supply of land. The larger construction response in the foreign region pushes up labor demand and wages in the foreign region relative to the home region, which increases consumption in the foreign region relative to the home region. This force is strong enough to dominate the actual housing wealth effect so in total consumption rises more in the foreign region than in the home region despite the smaller home price change in the foreign region resulting in a negative measured housing wealth effect.

Our formula in equation (37) adjusts for the construction response to home prices so even with a negative measured housing wealth effect, the implied housing wealth effect is positive at 0.087 while the actual housing wealth effect is 0.052.

7.6 Long-run Differences in Housing Supply Elasticities

Allowing for differences between short-run and long-run housing supply elasticities can reconcile the view that city-level differences in home price volatility reflect differences in housing supply curves with the observed positive response of construction to home prices. The equilibrium price of housing is forward-looking as the current housing demand depends on expectations of all future capital gains on housing. As a result, the equilibrium housing price is largely determined by long-run forces in the housing market. On the other hand, the incentives to construct and sell new homes depend on the current availability of inputs to construction and the current price of houses. Therefore the construction response depends much more on short-run forces in the housing market. We now show how this logic can generate a positive response of construction to home prices even though regional home price fluctuations reflect differences in (long-run) land supply elasticities.

We use a regime-switching formulation to model short-run and long-run differences. The economy is currently in the short-run and is expected to switch to the long-run regime with 2% probability each period after which it will remain in the long-run regime. In the short-run, the supply of land available for construction is fixed in both regions ($\gamma = \gamma^* = 0$). In the long-run, the supply of land responds to home prices, but differentially in the two regions (γ and γ^* take their calibrated values). When we simulate the economy we assume that the economy is always in the short-run regime and the long-run never materializes. Construction in both regions reflects movements along the same short-run supply curve while home prices are differentially affected by aggregate changes in housing demand that move expectations along the heterogeneous long-run housing supply curves. In terms of the short-run equilibrium in the housing market, the possibility of changing to the long-run creates different expected capital gains/losses in the two regions that lead aggregate shocks to housing demand to have different effects on short-run housing demand.

Column (v) shows the results. The measured housing wealth effect is 0.104 and the construction response is positive with an elasticity of 1.613. Because the two regions have the same short-run housing supply curve, aggregate shocks to housing demand move both regions along the same housing supply curve, which has a positive slope due to the increase in material inputs even though the supply of land is fixed. The short-run-long-run distinction is able to reverse the sign of the construction response even though the heterogeneous response of home prices still reflects differences in supply curves across regions. Taking account of the construction response and the local general equilibrium effects on prices, the implied housing wealth effect is 0.062 while the actual partial equilibrium housing wealth effect is 0.040. The error associated with the implied housing wealth effect is 1/3 as large as with the measured housing wealth effect.

7.7 Robustness

Table 2 shows several variants of the model specification in column (v) of Table 1, which is the model with long-run differences in supply heterogeneity as in Section 7.6. We focus our robustness analysis on this model specification because it is the richest one in Table 1 and the one that comes closest to the magnitudes of the measured housing wealth effect and construction response. Each column reports a separate variation leaving the remaining parameters at their baseline values. Column (i) reduces the quarterly persistence of the government spending shocks from the baseline of 0.95 to 0.90, which raises the multiplier slightly and reduces the implied housing wealth effect.

Table 2: Monte Carlo Analysis of Housing Wealth Elasticity: Robustness

	(i)	(ii)	(iii)	(iv)	(v)
	$\rho_G = 0.9$	$\rho_\Omega = 0.9$	$\chi = 0.75$ $\phi = 0.7$	Regime prob. 0.1	$\zeta = 10^{-5}$
Measured Housing Wealth Effect	0.104	0.124	0.216	0.108	0.088
Local Fiscal Multiplier	1.307	1.289	1.648	1.289	1.288
Construction Response	1.613	1.613	1.613	1.613	1.613
Income Elasticity of Home Prices	0.024	0.055	0.046	0.045	0.051
Inflation Response	0.025	0.030	0.221	0.030	0.030
Implied P.E. Housing Wealth Effect	0.059	0.077	0.097	0.065	0.049
Actual P.E. Housing Wealth Effect	0.040	0.044	0.032	0.038	0.036
Relative Error	0.298	0.411	0.357	0.375	0.249

Column (ii) reduces the persistence of the aggregate housing demand shock from the baseline of 0.95 to 0.90. This change raises the measured and implied housing wealth effects while only slightly increasing the actual housing wealth effect. Overall, Columns (i) and (ii) are reassuring that the exact details of the dynamics of the changes in home prices and changes in government spending are not crucial to the performance of our adjustment.

Column (iii) of Table 2 looks at a case with more price flexibility and less open economies. Making prices more flexible leads to a smaller fiscal multiplier due to expenditure switching after a government purchases shock while making the economies less open raises the fiscal multiplier. The combination of parameters considered here is close to those used by Nakamura and Steinsson (2014). There is a large increase in the measured housing wealth effect and a small decline in the actual housing wealth effect. Our adjustment still yields an implied housing wealth effect with an error that is about 1/3 as large as associated with the measured effect.

Columns (iv) and (v) explore the roles of two parameters that do not have clear empirical targets. Column (iv) raises the regime-switching probability from 2% per quarter to 10% per quarter. This has little effect on our results. Column (v) reduces the scale of the portfolio holding cost from 10^{-4} to 10^{-5}. This change reduces the actual, measured and implied housing wealth effects, but if anything the performance of our adjustment improves in the sense that the relative error is about 25%.

7.8 Summary

Our Monte Carlo analysis shows that our adjustment for local general equilibrium effects yields a housing wealth effect that is closer to the actual partial equilibrium housing wealth effect than the measured housing wealth effect. In our richest specification, column (v) of Table 1, the adjustment yields an estimate of the housing wealth effect that has an error that is 1/3 as large as that associated with the measured housing wealth effect. While in principle, one could potentially do better by more fully accounting for the dynamics of the responses to the two shocks as in Section 5, the simpler approach of using the static logic from Section 3 accounts for two thirds of the local general equilibrium effects.

8 Conclusion

Cross-sectional empirical estimates have become part of the macroeconomist toolkit, but the appropriate interpretation of these estimates can be difficult as they often blend together partial-equilibrium responses with local general equilibrium effects. We have presented a method that allows the researcher to isolate the partial equilibrium effect, which we view as more useful as it is more easily compared to the predictions of standard one-region models.

We applied this method to compute the partial equilibrium housing wealth effect. The key step in the application is to use an estimate of the local fiscal multiplier to gauge the strength of the local general equilibrium effects. Accounting for local general equilibrium reduces the housing wealth effect almost by half.

A Proof for Sections 3.2 and 3.3

The proof for Section 3.2 is a special case of the proof for Section 3.3. Let $\Phi \equiv 1$ for Section 3.2 and let $\Phi \equiv 2\phi - 1$ for 3.3.

We proceed as in the first example to arrive at:

$$\frac{d(w,p)}{d(g,s)} = [1 - \Phi EZ]^{-1} \begin{pmatrix} N_w^{-1}M & \Phi\frac{H_s}{H_w}ZE \\ ZM & -\frac{H_s}{H_p} \end{pmatrix},$$

where we have defined:

$$\begin{aligned} m &\equiv \frac{C_w}{N_w} \\ M &\equiv (1 - \Phi m)^{-1} \\ Z &\equiv -\frac{H_w}{N_w H_p} \\ E &\equiv M\left(C_p + I_p\right). \end{aligned}$$

Using :

$$\frac{dY}{d(g,s)} = \Phi \begin{pmatrix} C_w & C_p + I_p \end{pmatrix} \frac{d(w,p)}{d(g,s)} + \begin{pmatrix} 1 & 0 \end{pmatrix},$$

the observables are:

$$\frac{dY}{dg} = 1 + (1 - \Phi EZ)^{-1}\Phi\left[mM + EZ\right] = (1 - \Phi EZ)^{-1}M \tag{38}$$

$$\frac{dp}{ds} = -(1 - \Phi EZ)^{-1}\frac{H_s}{H_p} \tag{39}$$

$$\frac{dC}{ds} = (1 - \Phi EZ)^{-1}\left(C_w\Phi\frac{H_s}{H_w}ZE - C_p\frac{H_s}{H_p}\right) \tag{40}$$

$$\frac{dp}{dg} = (1 - \Phi EZ)^{-1}ZM \tag{41}$$

$$\frac{dI}{ds} = -I_p(1 - \Phi EZ)^{-1}\frac{H_s}{H_p}. \tag{42}$$

Below we show that we can construct E, M, Z, and I_p from observables. We then have:

$$C_p = \frac{E}{M} - I_p.$$

To measure E we note that:

$$\begin{aligned}\frac{dC/ds + dI/ds}{dp/ds} &= C_p + I_p - C_w \Phi \frac{H_p}{H_w} ZE \\ &= C_p + I_p + \Phi m E \\ &= M^{-1}E + \left(1 - M^{-1}\right) E \\ &= E.\end{aligned}$$

To measure Z we note that:

$$Z = \frac{dp/dg}{dY/dg}.$$

To measure E we note that:

$$M = (1 - \Phi E Z)\frac{dY}{dg}.$$

To measure I_p we note that:

$$I_p = \frac{dI/ds}{dp/ds}.$$

B Bias in Estimating Equations (35) and (36)

Consider the data generating process:

$$y_{r,t} = f_t + \gamma_r J_{r,t} + \varepsilon_{r,t},$$

where t indexes time and r regions. Note that each region has its own γ_r. However, when we estimate the housing wealth effect we estimate a single γ. We do not recover the average γ across regions if, say, regions with larger γ's tend to have smaller fluctuations in home prices $J_{r,t}$, which is the implication of regions with more elastic housing supply having residential investment respond more to home prices but home prices fluctuate less.

We estimate with demeaned variables to eliminate f_t. Let $\tilde{y}_{r,t} = y_{r,t} - \bar{y}_t$. We then have:

$$\tilde{y}_{r,t} = \gamma_r J_{r,t} - \bar{\gamma J}_t + \tilde{\varepsilon}_{r,t}$$
$$\tilde{y}_{r,t} = \bar{\gamma} J_{r,t} - \bar{\gamma} J_{r,t} + \gamma_r J_{r,t} - \bar{\gamma}\bar{J}_t - cov_t + \tilde{\varepsilon}_{r,t}$$
$$\tilde{y}_{r,t} = \bar{\gamma}\tilde{J}_{r,t} + (\gamma_r - \bar{\gamma}) J_{r,t} - cov_t + \tilde{\varepsilon}_{r,t},$$

where $cov_t = E_r\left[(\gamma_r - \bar{\gamma})\left(J_{r,t} - \bar{J}_t\right)\right]$.

We regress $\tilde{y}_{r,t} = \hat{\gamma}\tilde{J}_{r,t} + \nu_{r,t}$. The least squares moment condition is

$$E\left[\tilde{J}_{r,t}\left(\tilde{y}_{r,t} - \hat{\gamma}\tilde{J}_{r,t}\right)\right] = 0$$

. Substituting in:

$$E\left[\tilde{J}_{r,t}\left((\bar{\gamma} - \hat{\gamma})\tilde{J}_{r,t} + (\gamma_r - \bar{\gamma}) J_{r,t} - cov_t + \tilde{\varepsilon}_{r,t}\right)\right] = 0$$
$$(\bar{\gamma} - \hat{\gamma}) E\left[\tilde{J}^2_{r,t}\right] + E\left[(\gamma_r - \bar{\gamma}) J_{r,t}\tilde{J}_{r,t} - cov_t\tilde{J}_{r,t}\right] = 0.$$

Note that cov_t has no variation over r and $\tilde{J}_{r,t}$ has no time-series variation. So we end up with:

$$\hat{\gamma} = \bar{\gamma} + \frac{E\left[(\gamma_r - \bar{\gamma}) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}^2_{r,t}\right]}$$

.

C Partial equilibrium housing wealth effect

The household maximizes

$$\sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t, Q_t\right)$$

, where C_t, L_t, Q_t refer to consumption, labor supply and housing, respectively. The budget constraint is:

$$J_t Q_t + C_t + B_t = W_t L_t + D_t + R_{t-1} B_{t-1} + J_t Q_{t-1}(1-\delta),$$

where R_t is the gross real interest rate between t and $t+1$. The first order conditions are:

$$\begin{aligned}
u_{C,t} &= \lambda_t \\
u_{Q,t} &= \lambda_t J_t - \beta\lambda_{t+1} J_{t+1}(1-\delta) \\
\lambda_t &= \beta R_t \lambda_{t+1} \\
u_{L,t} &= -\lambda_t W_t
\end{aligned}$$

Combining we have:

$$\begin{aligned}
u_{C,t} &= \beta R_t u_{C,t+1} \\
\frac{u_{Q,t}}{u_{C,t}} &= J_t - R_t^{-1} J_{t+1}(1-\delta) \\
-\frac{u_{L,t}}{u_{C,t}} &= W_t.
\end{aligned}$$

Using the period utility function and rearranging yields:

$$\begin{aligned}
L_t &= \left(\frac{W_t}{\psi}\right)^{1/\nu} \\
\tilde{C}_t &\equiv C_t - \psi \frac{L_t^{1+\nu}}{1+\nu} \\
\tilde{Q}_t &\equiv Q_t - \Omega_t \\
x_t &\equiv \frac{1-\kappa}{\kappa}\left[J_t - R_t^{-1} J_{t+1}(1-\delta)\right]^{-1} \\
\tilde{Q}_t &= x_t \tilde{C}_t \\
u_{C,t} &= \kappa x_t^{(1-\kappa)(1-\sigma)} \tilde{C}_t^{-\sigma} \\
\tilde{C}_{t+1} &= \left[\beta R_t \left(\frac{x_{t+1}}{x_t}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma} \tilde{C}_t \\
\tilde{C}_t &= X_{0,t} \tilde{C}_0,
\end{aligned}$$

where:

$$\begin{aligned}
X_{0,t} &= \prod_{s=1}^{t} \left[\beta R_{s-1} \left(\frac{x_s}{x_{s-1}}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma} \\
X_{0,t} &= \left[\beta^t R_{0,t} \left(\frac{x_t}{x_0}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma}.
\end{aligned}$$

Now using the present value budget constraint:

$$\sum_{t=0}^{\infty} R_{0,t}^{-1} \left[J_t \left(Q_t - (1-\delta)Q_{t-1}\right) + C_t - W_t L_t - D_t\right] = R_{-1}B_{-1},$$

substituting in for C_t and Q_t and rearranging gives:

$$\tilde{C}_0 = \kappa \frac{J_0(1-\delta)\tilde{Q}_{-1} + R_{-1}B_{-1} + \sum_{t=0}^{\infty} R_{0,t}^{-1}\left[W_t L_t - \psi \frac{L_t^{1+\nu}}{1+\nu} - \Omega_t \frac{1-\kappa}{\kappa x_t}\right]}{\sum_{t=0}^{\infty} R_{0,t}^{-1} X_{0,t}}. \tag{43}$$

As we are focussing on partial equilibrium fluctuations in home prices, the sum in the numerator is constant. The sum in the denominator depends on all future home prices. To a first order approximation around a steady state with $\beta R = 1$, this sum can be written as:

$$\sum_{t=0}^{\infty} R_{0,t}^{-1} \left[\beta^t R_{0,t} \left(\frac{x_t}{x_0}\right)^{(1-\kappa)(1-\sigma)}\right]^{1/\sigma} = \sum_{t=0}^{\infty} R^{-t} \left(\frac{RJ_0 - J_1(1-\delta)}{RJ_t - J_{t+1}(1-\delta)}\right)^{\frac{(1-\kappa)(1-\sigma)}{\sigma}}$$
$$\approx \frac{(1-\kappa)(1-\sigma)}{\sigma} \frac{R}{(R-1+\delta)\bar{J}} \left[\frac{R}{R-1}\left(J_0 - \frac{1-\delta}{R}J_1\right) - \sum_{t=0}^{\infty} \bar{R}^{-t}\left(J_t - \frac{1-\delta}{R}J_{t+1}\right)\right]. \tag{44}$$

To compute this recursively, suppose that the state vector of the economy is $\mathcal{X}_t$ and the equilibrium dynamics are governed by matrice $\mathcal{P}$ and $\mathcal{Q}$ such that $\mathcal{X}_t = \mathcal{P}\mathcal{X}_{t-1} + \mathcal{Q}\epsilon_t$. We then have $\mathbb{E}\left[\mathcal{X}_t\right] = \mathcal{P}^t \mathcal{X}_0$. Moreover, using $\mathcal{I}$ as a column vector that gives the linear mapping from $\mathcal{X}_t$ to J_t we have:

$$\begin{aligned}
\mathbb{E}\sum_{t=0}^{\infty} R^{-t} J_t &= \mathbb{E}\sum_{t=0}^{\infty} R^{-t}\mathcal{I}\mathcal{X}_t \\
&= \mathcal{I}\sum_{t=0}^{\infty} R^{-t}\mathcal{P}^t\mathcal{X}_0 \\
&= \mathcal{I}\left(I - R^{-1}\mathcal{P}\right)^{-1}\mathcal{X}_0.
\end{aligned}$$

Using this, equation (44) becomes:

$$\approx \frac{(1-\kappa)(1-\sigma)}{\sigma} \frac{R}{(R-1+\delta)\bar{J}} \mathcal{I} \left[\frac{R}{R-1}\left(I - \frac{1-\delta}{R}\mathcal{P}\right) - \left(I - R^{-1}\mathcal{P}\right)^{-1}\left(I - \frac{1-\delta}{R}\mathcal{P}\right)\right]\mathcal{X}_0. \tag{45}$$

This approach can be applied to the regime switching model by expanding the state vector so that

$$\tilde{\mathcal{X}}_0 = \begin{bmatrix} \mathcal{X}_t \\ \mathbf{0} \end{bmatrix}$$

and the state transition matrix is

$$\tilde{\mathcal{P}} = \begin{bmatrix} (1-\omega)P_{\text{short}} & 0 \\ \omega P_{\text{long}} & P_{\text{long}} \end{bmatrix},$$

where ω is the regime-switching probability, P_{short} is the state transition matrix when staying in the short-run regime and P_{long} is the state transition matrix in the long-run regime. One would also set

$$\tilde{\mathcal{I}} = \begin{bmatrix} \mathcal{I}_{\text{short}} & \mathcal{I}_{\text{long}} \end{bmatrix}.$$

In calculating the partial equilibrium housing wealth effect, we simulate the general equilibrium model in response to aggregate housing demand shocks and at each date in the simulation we record $\mathcal{X}_t$, Q_{t-1}, B_{t-1} and J_t. We then plug these values into (43) with the denominator computed by (45). This gives us a time-series of "partial equilibrium" consumption for each region to go along with the time series of home prices. We then regress the difference in log consumption across regions on the difference in log home prices. Notice that the partial equilibrium consumption series will vary over time with the bond positions the two regions have inherited from the past. In the absence of a portfolio holding cost, these bond positions are non-stationary and the partial equilibrium housing wealth effect becomes unstable.

References

Chodorow-Reich, G., Nenov, P. T., and Simsek, A. (2019). Stock market wealth and the real economy: A local labor market approach. National Bureau of Economic Research Working Paper.

Davidoff, T. (2016). Supply constraints are not valid instrumental variables for home prices because they are correlated with many demand factors. *Critical Finance Review*, 5(2):177–206.

Davis, M. A. and Heathcote, J. (2005). Housing and the business cycle. *International Economic Review*, 46(3):751–784.

Favilukis, J., Ludvigson, S. C., and Van Nieuwerburgh, S. (2017). The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy*, 125(1):140–223.

Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, pages 402–417.

Guren, A. M., McKay, A., Nakamura, E., and Steinsson, J. (2018). Housing wealth effects: The long view. Technical report, National Bureau of Economic Research.

Kaplan, G., Mitman, K., and Violante, G. L. (2017). The housing boom and bust: Model meets evidence. Technical report, National Bureau of Economic Research.

Lamont, O. and Stein, J. (1999). Leverage and house-price dynamics in us cities. *RAND Journal of Economics*, 30(3):498–514.

Mian, A., Rao, K., and Sufi, A. (2013). Household balance sheets, consumption, and the economic slump. *The Quarterly Journal of Economics*, 128(4):1687–1726.

Mian, A. and Sufi, A. (2014). What explains the 2007–2009 drop in employment? *Econometrica*, 82(6):2197–2223.

Nakamura, E. and Steinsson, J. (2014). Fiscal stimulus in a monetary union: Evidence from us regions. *American Economic Review*, 104(3):753–92.

Nathanson, C. G. and Zwick, E. (2018). Arrested development: Theory and evidence of supply-side speculation in the housing market. *The Journal of Finance*, 73(6):2587–2633.

Saiz, A. (2010). The geographic determinants of housing supply. *The Quarterly Journal of Economics*, 125(3):1253–1296.

Schmitt-Grohé, S. and Uribe, M. (2003). Closing small open economy models. *Journal of international Economics*, 61(1):163–185.

Wolf, C. (2019a). The missing intercept: A demand equivalence approach. Princeton University Working Paper.

Wolf, C. (2019b). The missing intercept in cross-regional regressions. Princeton University Working Paper.