Collateral Runs*

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Abstract: This paper models an unexplored source of liquidity risk faced by large broker-dealers: *collateral runs*. By setting different contracting terms on repurchase agreements with cash borrowers and lenders, dealers can source funds for their own activities. Cash borrowers internalize the risk of losing their collateral in case their dealer defaults, prompting them to withdraw it. This incentive creates strategic complementarities for counterparties to withdraw their collateral, reducing a dealer's liquidity position and compromising their solvency. Collateral runs are markedly different than traditional wholesale funding runs because they are triggered by a contraction in dealers' assets. Mitigating these risks involve different policy recommendations.

JEL classification: G23, G33, G01, C72

Keywords: runs, repo, rehypothecation, dealer, liquidity, default, collateral

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1 Introduction

This paper presents a theoretical model that formally characterizes a relatively unexplored risk that can affect large broker-dealers: a run from their collateral providers (cash borrowers). In the context of secured wholesale funding markets, much of the existing literature has focused on liquidity risks that arise from a withdrawal of cash lenders. For example, Gorton and Metrick, 2012; Krishnamurthy, Nagel, and Orlov, 2014; and Copeland, Martin, and Walker, 2014 have noted that during the 2007–09 financial crisis, financial firms faced run risk from their secured cash lenders. However, the very nature of broker-dealers' business involves intermediating cash and collateral between cash lenders and cash borrowers. In particular, dealers extend short-term credit against collateral, which is simultaneously used to raise funds for said credit, a process known as rehypothecation. In this paper we argue that this type of intermediation can be an important source of liquidity for a broker-dealer, as different contracting terms between borrowers and lenders can result in a liquidity windfall, which can evaporate with the withdrawal of cash borrowers.

The main set-up of the model considers a dealer providing short-term secured financing, interpreted as repurchase agreements (repos), to a large number of counterparties, called hedge funds.[1] The dealer is able to extend said financing by reusing the collateral they receive to raise secured debt from cash lenders, called money market funds, in the form of another repo. Following Infante (2019), Gottardi et al. (2017), and current market practice, we assume that hedge funds cannot contact money market funds directly, that is, dealers are hedge funds' main source of secured financing.[2] We say that whenever a firm has the ability to rehypothecate its posted collateral they effectively create a *collateral liability*, that is, an obligation to return collateral that they may not have access to, which is the source of fragility in this model

[1]Repos are secured loans backed by financial assets, where the ownership of the collateral is transferred to the cash lender for the duration of the contract. In the initial leg the cash lender "purchases" the security and the cash borrower promises to "repurchase" the same security at fixed price in the closing leg. Reverse repos are repos from the point of view of the cash lender.

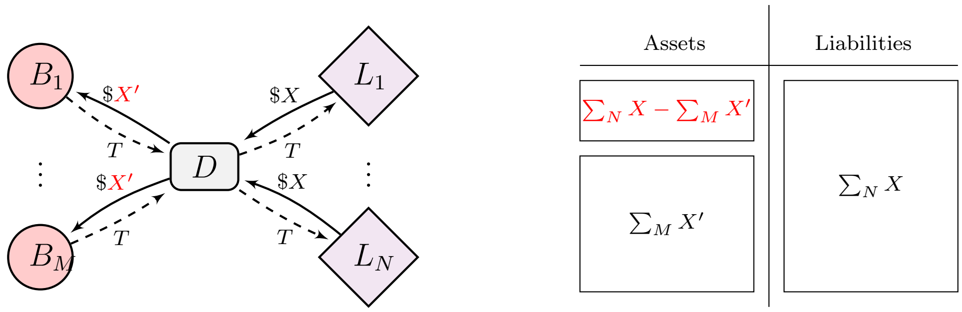
[2]This assumption captures the idea that many hedge funds are small and relatively opaque firms, which wholesale cash lenders will not –or cannot in the case of unrated hedge funds—interact with directly. In effect, in the United States, money market funds can only lend to firms with high credit ratings, which excludes the types of end investors we have in mind. Note that we will use the terms re-use and rehypothecation interchangeably. Strictly speaking, the difference between the two terms is whether the counterparty posting the collateral is a client or just a counterparty, a detail we abstract from.

Figure 1 illustrates a stylized example of this type of dealer intermediation, and how the creation of a collateral liability can result in fragility from cash borrowers. In the figure the dealer receives funds from lenders ($L_1...L_N$), denoted by X, and extends funds to borrowers ($B_1...B_M$), denoted by X'. At the same time the dealer receives and rehypothecates collateral, denoted by T, from borrowers to lenders. The dealer may have incentives to distribute only a fraction of the cash they raise from lenders and use the difference to finance profitable, higher yielding, risky projects, which may be illiquid. In other words, the dealer may wan to set $X' < X$ and invest the liquidity windfall it reaps $\sum_N X - \sum_M X'$ for their own benefit. This funding difference comes from the over-collateralization of the hedge funds' repo relative to the money funds' reverse repo. In case the dealer defaults, money funds have immediate access to the collateral and can sell it to make their claims whole, essentially insulating them from the dealer.[3][4] In contrast, if the dealer defaults, hedge funds risk losing their collateral altogether which is more valuable than the initial loan they received. Specifically, each hedge fund risks losing the amount of over-collateralization on their repo, which becomes an unsecured claim on the dealer's illiquid asset holdings. Each hedge fund's unsecured claim is pooled with the unsecured claims of others, potentially creating a first mover advantage for hedge funds to withdraw their collateral. In order to return the collateral to withdrawing hedge funds, the dealer may have to sell a fraction of its illiquid position to fully repay the money funds holding said collateral. If the amount of hedge fund withdrawals is large enough, dealers may not have enough cash to return all of the demanded collateral, resulting in a *collateral run*.

The intermediation described above can lead to fragility if three conditions are met. First, the collateral must be rehypothecated to create a collateral liability. Without rehypothecation bankruptcy regimes that earmark the underlying collateral upon default would eliminate the pooling of claims and the possibility of creating strategic complementarities from withdrawing them. Second, the dealer must be able to to set different contracting terms between borrowers and lenders. This enables the dealer to reap a cash windfall from

[3]Although an abrupt withdraw of cash lenders is an important consideration for repo market stability, we purposefully shut down that channel to focus on fragility stemming from an abrupt withdrawal of collateral. In the model this comes naturally because the underlying collateral completely insures the cash lender from any loss.

[4]Money funds have immediate access to the underlying collateral because under U.S. law repos are exempt from automatic stay.



(a) Dealer intermediation **(b)** Dealer's balance sheet

Figure 1: Dealer intermediation and Dealer's balance sheet

the first leg of the intermediation chain, effectively monetizing a fraction of the hedge funds' collateral for themselves. And finally, the dealer must have discretion to use any excess funds for their own activities. The investment in risky and illiquid projects financed with pooled funds creates a first mover advantage amongst collateral providers. These three conditions leave hedge funds exposed to lose of the overcollateralization of their initial repo, creating incentives to withdraw their collateral.

More formally, the incentive to withdraw collateral creates strategic complementarities amongst hedge funds' actions because each hedge fund's optimal action and payoff can depend on what other hedge funds do, leading to a multiplicity of equilibria. For example, if all other hedge funds roll over their repo positions, the dealer does not need to liquidate any of their illiquid assets, making it optimal for an individual hedge fund to roll over as well. On the contrary, if all other hedge funds withdraw their collateral, then the dealer may need to sell all of their illiquid assets at a loss, making it optimal for an individual hedge fund to withdraw their collateral. Therefore, an individual hedge fund's payoff not only depends on the dealer's solvency, but also on *its beliefs about the actions/beliefs of other hedge funds.*

To resolve the multiplicity of equilibria we use a global game refinement, akin to coordination problems studied by Morris and Shin (1998). Establishing a unique equilibrium is important because is resolves the uncertainty around the occurrence of a run, allowing agents to determine the equilibrium contracting terms ex-ante. We model an incomplete information game (global game), similar to Goldstein and Pauzner (2005), where hedge funds receive noisy signals about the (fundamental) expected value of the dealer's risky investment. Yet, we extend their framework by introducing a stochastic liquidation value for the risky investment, which is proportional to its fundamental value. This extension allows us to endogenously determine the region of fundamentals where a coordination failure is possible and where it is not, in contrast to many applications of the Goldstein-Paunzer global game where part of that region is set exogenously. To that extent, our framework is close to Kashyap, Tsomocos and Vardoulakis (2017), who introduce stochastic liquidation values in incomplete information games with one-sided strategic complementarities à la Goldstein-Paunzer. Though, in this model, the source of stochastic liquidation values comes from the expected value of the dealer's risky investment.[5]

It is important to note that the underlying collateral pledged and re-used can differ significantly from the risky project purchased by the dealer. In particular, the underlying collateral can be completely riskless, yet there can still be a collateral run. The risk that collateral providers face does not come from their own assets, but rather from the dealer's use of the excess funds they raise with them. Duffie (2013) recognizes that an important source of liquidity for dealers stems from their levered counterparties' assets pledged as collateral, while Infante (2019) characterizes the optimal contracting terms that lead to a liquidity windfall whenever a dealer intermediates repos from one cash lender to one cash borrower. In this paper we formalize how such liquidity windfalls can introduce dealer illiquidity, coordination failures, and run risk.

At first sight, it seems that the mechanism underlying collateral runs is very similar to the one underlying traditional bank runs. In both cases, the agents who may choose to run have an unsecured claim on a financial intermediary and the liquidation value of the assets, which can be less valuable than the total amount owed to all agents. We believe that this similarity is an important feature of our analysis given that the same deep economic sources can generate fragility due to otherwise seemingly unrelated intermediation activities. However, there are important differences some more conceptual and others more relevant for the economic interpretation, policy implications, and monitoring of risks.

First, on a conceptual level, a collateral run implies the withdrawal of an intermediarys assets, not in its liabilities. Despite being counterintuitive, we derive conditions under which cash borrowers—who close their positions—can be the source of fragility rather than cash

[5]Moreover, a stochastic liquidation value enables the endogenous derivation of the regions for fundamentals where individual actions are independent of other funds' actions. These are known as upper and lower dominance regions and are essential for the existence of equilibrium. We derive these regions in Section 4.

lenders. We connect this behavior to the creation of a collateral liability and contrast it with a *deposit liability*. We believe that this concept may be useful to study fragility also in other environments, such as prime brokerage internalization, where the traditional way of thinking about runs may be less useful. The concept of a collateral liability highlights the difference between collateral runs and traditional repo runs. The latter are induced by a desire to withdraw cash because of low collateral valuations. The former are induced by the desire to withdraw one's collateral because it is deemed valuable. Indeed, collateral runs can materialize even under safe collateral whereas repo runs cannot.

Second, with respect to their economic relevance, collateral runs are a side effect of an underlying desire of cash borrowers to take leverage, while traditional bank runs accrue from the liquidity provision and risk sharing role of intermediaries. Indeed, in our model, there is no maturity mismatch between reverse repos and repos. In fact some positive mismatch may mitigate, rather than exacerbate, collateral-run risk as dealers can "lock-in" the collateral, allowing them to roll over their position without fearing a withdrwal from collateral providers.

Third, collateral runs and traditional bank runs differ in terms of their policy implications. The vast majority of regulatory efforts since the financial crisis have focused on fragility from the withdrawal of unsecured and/or secured funding. For example, a proposed solution has been to introduce minimum haircut requirements, also known as haircut floors, which reduces cash lenders incentive to run because the value of pledged collateral is high enough to cover their claims even in stressed times. Our analysis cautious that this policy prescription may in fact *increase* the probability of a collateral run. The intuition is that collateral providers' internalize that they may access as cash providers overcollateralization sooner, increasing their incentives to withdraw their collateral. We discuss other policy interventions are targeted to address the instability from a collateral run, such as limits to rehypothecation which restricts the creation of the collateral liability or limits to reinvesting any liquidity windfall from rehypothecation (see detailed discussion in Section 6). We also show that from a practical basis, policy makers would need more information to monitor the potential risks from traditional bank/repo runs vs. collateral runs.

Finally, it may be hard to disentangle repo runs from collateral runs in the data given that the both materialize at the same time. However, we argue that the dynamics leading to a collateral run are starkly different from those leading to a repo run. Namely, for collateral runs, we would expect to see stable margins for cash lenders and increasing margins for cash borrowers even for collateral deemed to be very safe such as U.S. Treasuries, implying a higher liquidity windfall for the dealer. On the contrary, for repo runs, we would expect cash lenders to require increased margins accompanied by a deterioration in the quality of collateral.

Literature Review. The coordination problem in this paper is akin to coordination problems in currency attacks (Morris and Shin, 1998), risky debt rollover and bank runs (Diamond and Dybvig, 1983; Morris and Shin, 2004; Rochet and Vives, 2004; Goldstein and Pauzner, 2005; He and Xiong, 2012; Vives, 2014), credit market freezes (Bebchuk and Goldstein, 2011), and investment funds (Chen, Goldstein, and Jiang, 2010; Liu and Mello, 2011). As is generally the case in coordination games, multiple equilibria may exist. Establishing a unique equilibrium is important because is resolves the uncertainty around the occurrence of a run, allowing agents to determine the equilibrium contracting terms ex-ante.

Our paper is also related to the theoretical literature that characterizes optimal contracting terms and instability in collateralized short-term funding markets. Fostel and Geanakoplos (2015) derive the optimal haircuts on secured debt. Geanakoplos (2003), Fostel and Geanakoplos (2008)—including a series of subsequent papers—and Simsek (2013) study the interlinkages between asset prices, haircuts and leverage over the cycle as well as the implications for investment and financial stability. Martin, Skeie and Von Thadden (2014) detail the contracting terms that lead to traditional cash-driven repo runs. Ahnert, Anand, Gai and Chapman (2018) study how the over-collateralization of long-term secured debt can affect the incentives of short-term unsecured debt holders to run.[6] Similarly, Donaldson et al. (2019) argue that secured debt creates a "collateral rat race", where creditors optimally choose to secured their claims to avoid dilution at the cost of curtailing future borrowing. We differ from these papers because we examine a distinct source of instability in repo intermediation. In the aforementioned papers, the instability stems from the liability side of the balance sheet; cash lenders may be less willing to provide funding and either require higher margins, leading to borrower deleveraging, or withdraw their funding altogether in a coordinated run episode. In contrast, the instability we study in this paper is borne from the asset side of an intermediaries balance sheet; borrowers may collectively withdraw their

[6] Many other theoretical papers have studied spirals and freezes in short-term funding markets. Some examples are Brunnermeier and Pedersen (2009), Acharya, Gale and Yorulmazer (2011), Diamond and Rajan (2011), and Ahnert (2016). As mentioned, we differ from this literature because we mute the rollover risk of cash lenders positions.

collateral even if cash-lenders' claims are safe with stable haircuts and have no incentive to run. Other papers characterizing instabilities arising from the asset side of a lender's balance sheet is Bond and Rai (2009) in the context of micro finance lending and Huang (2017) in the context of borrowers drawing down credit lines from a distressed institution. We differ from these papers by studying fragility in dealer intermediated markets.

This type of cash and collateral intermediation studied in this paper is consistent with Gottardi, Maurin and Monnet (2017) who show that an optimal rehypothecation chain can arise whenever a dealer is more trustworthy than a hedge fund counterparty. Put differently, dealers are agents with a better technology to source and distribute collateral, making them the natural intermediary, similar in spirit to the warehouse view of banking in Donaldson et al. (2018). We take this intermediation chain as given, and focus on a new type of fragility that can arise from this activity.

The rest of the paper is structured as follows. The following section discusses the institutional setting and a motivating example. Section 3 presents the model setup, detailing the economic environment, the main actors, and their incentives. Section 4 characterizes the coordination problem hedge funds face, their threshold strategies, and the regions where fundamental- or panic-based runs can materialize. Section 5 presents the problem the dealer faces given hedge funds' threshold strategies, characterizes the optimal contracting terms, and shows how the equilibria can change with fundamentals. Section 6 gives policy recommendations aimed at mitigating the risks from a collateral run. Finally, section 7 gives some concluding remarks. All proofs are relegated to the Appendix.

2 Institutional Setting & Motivating Example

The main fragility in our model stems from dealers' ability to use and reuse collateral provided by counterparites. There is ample evidence to suggest that primary dealers, the main counterparties of the Federal Reserve, engage in large amounts of reverse repo and repo with the same underlying collateral, across different segments of the US repo market. These segments can be separated into two distinct markets: the tri-party repo market and the bilateral repo market. The tri-party repo market is where creditworthy dealers, such as the primary dealers, raise funds from cash rich investors, such as money market funds, to finance their inventory and reverse repos. The tri-party statistical release from the Federal Reserve Bank of

New York (FRBNY) shows that during 2015 the total outstanding in tri-party market repo was in the order of \$1.2 Trillion, of which about half used US Treasury collateral.[7] Copeland et al. (2014) provide evidence that the majority of these repos are short-term. The bilateral repo market is where less creditworthy investors, such as hedge funds, borrow from dealers to finance their positions. Although the total size of this market is hard gauge, Baklanova et al. (2019) use survey results from the nine largest dealers and show that in 2015 their total outstanding in bilateral reserve repo was approximately \$ 1.5 trillion, of which about half used US Treasury collateral. Their measures of total outstanding in bilateral repo was approximately \$ 1 trillion, suggesting these dealers intermediate cash and collateral between markets. Baklanova et al. also report that the majority of these repos have short maturity. These numbers highlight the important role primary dealers play in intermediating repo, that much of the activity is between repo markets, and these contracts are typically short term.

Indeed, data from FRBNY's weekly survey of primary dealers (FR 2004) shows that total amount of repos and reverse repos backed by US Treasuries across all repo markets is in the order of \$1.5 Trillion each. Moreover, Infante et al. (2018) show data on dealers' rehypothecation activity and estimate that the one US Treasury can be reused up to 7 times, underscoring the high degree of collateral reuse. Infante (2019) provides a brief discussion of the relevant institutional details surrounding the reuse of collateral in the United States. In particular, in the context of repo, there are no limits to rehypothecation.

An important motivating example of our paper is the demise of Bear Stearns in March 2008. Anecdotally, in the days leading up to its collapse, the firm suffered a large outflow of counterparties that not only pulled their cash but also their collateral from the firm. Using the FR 2004 we can estimate a lower bound on the total amount of cash that Bear Stearns accessed through rehypothecation. Specifically, the FR 2004 asks primary dealers to report the total amount of secured financing extended (Securities In), the total amount of secured financing received (Securities Out), and their outright positions for different asset classes. Importantly, the survey asks dealers to report the total amount of funds received and distributed though secured financing transactions, not the value of the collateral posted. Therefore, these data can be used to estimate the amount of liquidity obtained through

[7]These data also show that approximately \$400 billion is in non-fedwire-eligable collateral, which includes risky collateral classes such as equities and private label mortgage securities.

contracting differences. Figure 2 plots the estimated cash windfall as a fraction of the total repo book for both Bear Stearns and the average of the remaining primary dealers.[8] From the figure, it can be appreciated that before the sharp drop in activity, the estimated cash stemming from different contracting terms reached approximately a third of the firm's entire repo book. A withdrawal of collateral effectively eliminated this additional liquidity windfall. These estimates suggest that, relative to its peers, Bear Stearns relied heavily on differences in contracting terms as a source of liquidity.

3 Model Setup

The model consists of three periods $t \in \{0, 1, 2\}$ and is populated by three types of agents; a broker-dealer (D), a continuum of hedge funds (H), and a continuum of money market funds (M). The dealer is (potentially) risk averse, with a payoff function u that satisfies $u(0) = 0$, hedge funds are risk-neutral, and money market funds are "very" risk averse.[9] All agents discount the future the same way. The timeline is presented in Figure 3.

Hedge funds would like to borrow to invest in a (safe) asset T, which is in perfect elastic supply and is worth 1 in every period. Abusing notation, T will also denote the amount of the asset purchased. Each hedge fund borrows money from the dealer at $t = 0$ (a reverse repo from the dealer's point of view), purchases the asset, and pledges it as collateral. Simultaneously, the dealer enters into a repo contract with money market funds, using the same collateral posted by the hedge fund, that is, the dealer rehypothecates the pledged collateral. Repo contracts are short term, i.e., they mature after one period, and can be rolled over at $t = 1$ as we describe further below.

Apart from intermediating funds and collateral between hedge funds and money funds, the dealer can also invest at $t = 0$ in a risky project $\tilde{R}$ which pays off R^U with probability θ and R^D otherwise in $t = 2$ per unit purchased, where $R^U > 1 > R^D \geq 0$. The state of the

[8] The lower bound depends on an important restriction that securities dealers face: the box constraint. Broadly speaking, the box constraint is a physical restriction that forces dealers to have access to securities, either by owning them outright or by borrowing them, in order to deliver to a counterparty. Huh and Infante (2017) characterize how this constraint is important for bond market intermediation and how to interpret the data in the FR 2004. Details on the lower-bound calculation and some potential caveats are in subsection C of the Appendix.

[9] The assumption that the dealer's payoff function has $u(0) = 0$ is merely for simplicity. "Very" risk averse money funds will be useful to focus on the collateral channel, rather than the traditional repo-run channel.

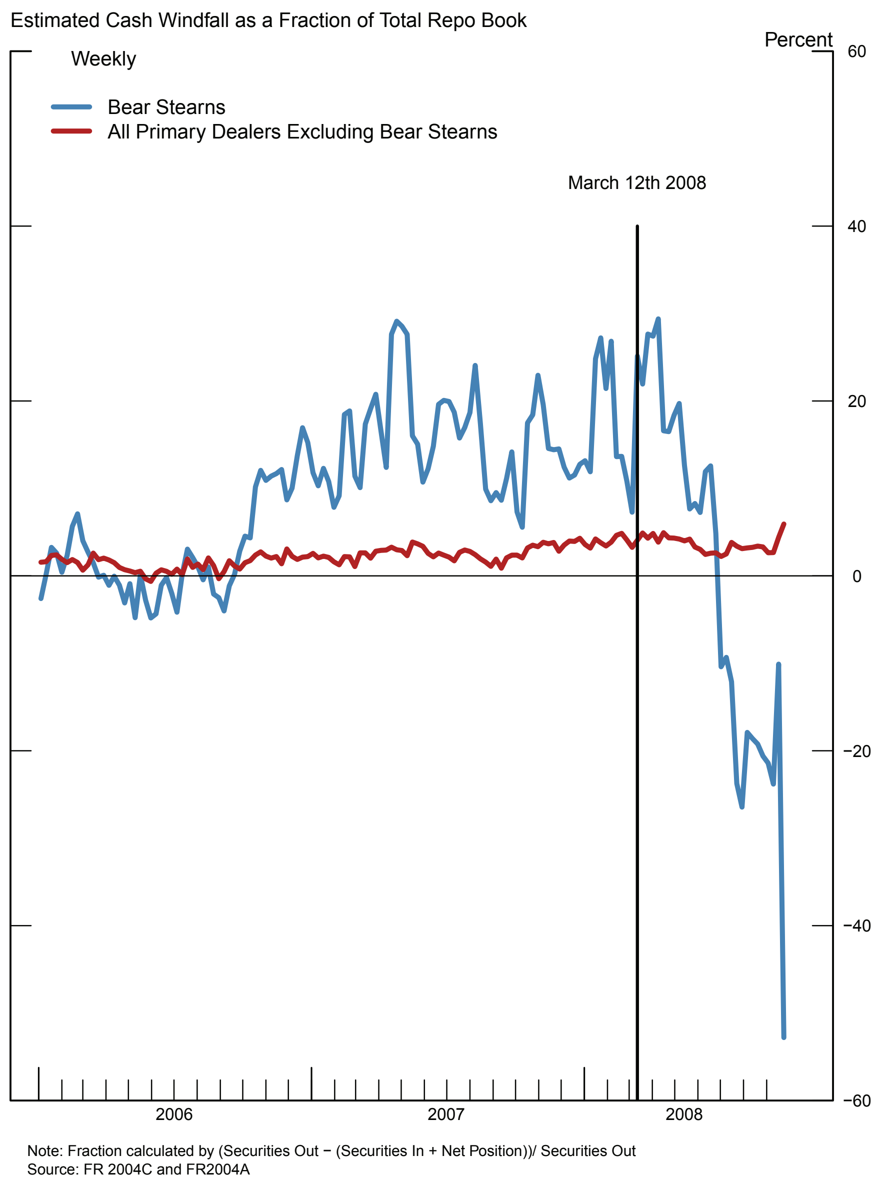


Figure 2: Liquidity from Rehypothecation as a Fraction of Total Repo Activity
Figure shows the lower-bound estimate of the liquidity sourced through dealers' repo activity (securities out minus securities in and net position) as a fraction of their total secured financing (securities out) for both Bear Stearns and average fraction for the rest of the primary dealer community. Source: FR 2004.

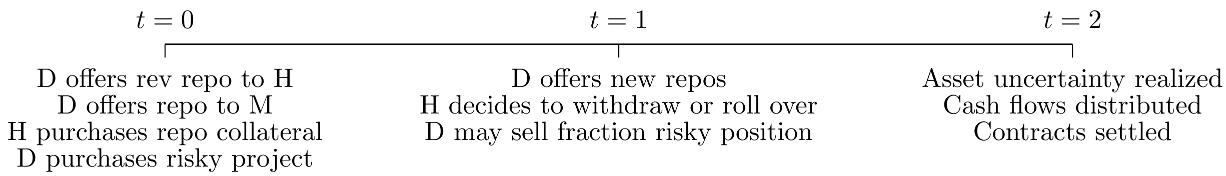


Figure 3: Model Timeline

world θ is realized at $t = 1$ and follows a uniform distribution $\theta \sim U[0, 1]$. The risky project has price of 1, is in perfect elastic supply, and its expected value, conditional on θ, is denoted by $\mathbb{E}_\theta(\tilde{R}) = \overline{R}_\theta$. The risky project can be liquidated in $t = 1$ at a fraction, $\lambda \in (0, 1]$, of $\overline{R}_\theta$.[10] We will assume that the unconditional expected liquidation value is higher than the initial price of the asset, i.e., $\lambda(R^U + R^D)/2 > 1$. Thus, there is liquidity risk because liquidation is generally inefficient, but unconditionally the project has a positive net present value even if liquidated. In particular, we impose a stricter version of this condition—$\lambda R^U > 2$—to also allow R^D to go arbitrarily close to 0 without altering the other parameters. Note that only the unconditional—ex-ante—liquidation value is higher than one; and the liquidation value conditional on the realization of θ can be as low as $\lambda R^D < 1$, which introduces illiquidity and creates incentives to withdraw.

At $t = 1$, the dealer offers new repo contracts to counterparties, and both hedge funds and money funds decide whether to roll over their positions.[11] Given our assumptions, described in detail later, money funds will always roll over their repos as along as the dealer rehypothecates the safe asset.[12] If the repo is rolled over, we assume that the closing leg of

[10] All our results go through even if there is no liquidity discount, i.e., $\lambda = 1$. The reason is that the liquidation value, $\overline{R}_\theta$, varies with the realization of fundamentals and for low enough θ the dealer will not have enough liquid resources to meet all obligations/withdrawals. However, $\lambda < 1$ helps justify our assumption that the dealer will first use liquid resources and then liquidate illiquid assets to meet withdrawals from hedge funds that choose not to roll over their positions.

[11] The dealer can either offer new contracting terms bilaterally to each hedge fund or mutual fund, or can post the new terms to all participants publicly. The distinction is inconsequential for our case. But, importantly, funds have perfect foresight about the terms they will get if they decide to roll over their positions. In other words, the contract terms are not contingent on neither the realization of the state of the world nor the number of hedge or mutual funds that decide to roll-over. This is a natural characterization of the way repo markets operate, as most repo markets clear in the early morning.

[12] We intentionally abstract from the dynamics governing the roll over decision of cash providers (money funds), which have received ample attention in the literature, in order to focus on the dynamics governing the roll-over decision of the providers of collateral (hedge funds).

existing repos (morning) and the opening leg of new repos (evening) happen simultaneously and, thus, we focus on net flows of funds.

However, an individual hedge fund may decide against rolling over its repo and withdraw its collateral at $t = 1$. If enough hedge funds withdraw their collateral, the dealer must sell a fraction of its risky project at its liquidation value in order to collect the collateral from money funds, which are the property of the withdrawing hedge funds. If asset sales are not enough to recuperate withdrawing hedge funds' collateral, the dealer is liquidated. Upon the dealer's liquidation, money funds that were not repaid keep the collateral, and hedge funds that were not served receive nothing.

At $t = 2$, conditional that the dealer survives, the final payoff on the risky project is realized, cash flows are distributed, and contracts are settled. The payoffs accruing to the three agents will not only depend on the repo contract terms, but also on the realization of θ and the portion of hedge funds that withdraw their collateral at $t = 1$, which we denote by $\mu \in [0, 1]$. It should be noted that (as we will show) in equilibrium either all hedge funds roll over ($\mu = 0$) or all hedge funds withdraw their collateral ($\mu = 1$). However, an individual hedge fund will need to form beliefs about the portion of hedge funds withdrawing based on a noisy signal it receives about θ. The signal helps the hedge fund to update its beliefs about the fundamental θ, but also about the strategies of other hedge funds. These noisy signals will allow hedge funds to coordinate their decisions and a unique equilibrium emerges whereby either all or none hedge funds withdraws their collateral.[13]

In order to derive the unique equilibrium, we need to specify all out-of-equilibrium outcomes for conjectured level of fundamentals, θ, and conjectured portion of hedge funds withdrawing, μ. In section 3.1-3.3 we present the payoffs to the dealer, the money funds,

[13]The noisy signals about θ induce coordination. Under complete information about θ multiple equilibria emerge. As Atkeson 2000 has pointed out, market prices can aggregate diverse private information and reveal the true state θ. Hence, if market prices aggregate information perfectly and are observable by agents when they are deciding whether to withdraw, a unique equilibrium does not obtain. As is typical in the literature, we assume that the withdrawal decision is taken before the dealer sells the risky project and, hence, before the true value of θ is observed (see, for example, Rochet and Vives, 2004, for a similar assumption). In our case this assumption is natural given that repo markets clear very early in the day, before asset prices transmit all the relevant information to market participants. Moreover, the risky project we have in mind is not traded in a deep, liquid market and should be thought of more as a subprime mortgage-backed security rather than the S&P index. Hence, the asset's true, realized liquidation value may not be readily observable to hedge funds that need to decide whether to roll over their positions early in the day. Another way to address the Atkeson critique is to consider that prices only imperfectly aggregate dispersed private information (see Angeletos and Werning, 2006).

and the hedge funds as function of the contract terms, as well as the level of fundamentals, θ, and the portion of hedge funds withdrawing, μ.

3.1 Dealer

The dealer offers take-it-or-leave-it repo contracts to hedge funds and money funds.[14] The repo contracts issued at time t, to/from counterparty $j \in \{H, M\}$,[15] have two terms: haircut m_t^j and repurchase price F_t^j.[16] It will be useful to introduce some additional notation. Let $\Delta m_t = (T - m_t^M) - (T - m_t^H) = m_t^H - m_t^M$ be the incremental cash flow the dealer receives from intermediating the initial leg of the repo at t. Moreover, denote by $\Delta F_t = F_t^H - F_t^M$ the incremental cash flow that the dealer receives from the closing leg of the repo at t, paid at $t + 1$.[17]

In the initial leg of the rehypothecation process at $t = 0$, the dealer receives $T - m_0^M$, a portion of which is then distributed to the hedge fund $T - m_0^H$. Simultaneously, the hedge fund delivers the collateral T to the dealer, which then passes it on to the money fund. Hence, the net cash flow to the dealer in $t = 0$ is Δm_0, which is used to purchase the risky project.

At $t = 1$, the dealer needs to unwind the repos for the μ hedge funds withdrawing, which is achieved by repurchasing the collateral from each money fund at a price F_0^M and returning it to each hedge fund at a price F_0^H. For an individual hedge fund withdrawal, the dealer will have to find $-\Delta F_0$ funds in order to return the collateral. Hence, the total net cash flow from these operations is $\mu \Delta F_0 < 0$.

The available resources to meet this negative cash flow can come either from collecting additional cash from hedge funds that roll over their repos or from liquidating (part of) the risky project. The first option to meet the liquidity shortfall yields in total $(1 - \mu)((F_0^H -$

[14]For simplicity we assume that dealers have all the market power. The results are qualitatively similar if hedge funds' have some bargaining power when setting contracting terms. Yet, the dealer needs to have some market power in order to extract some surplus from rehypothecation. Otherwise, collateral runs cannot occur.

[15]Note that repo counterparties are with respect to the dealer.

[16]Hence, $F_t^H/(T - m_t^H) - 1$ and $F_t^M/(T - m_t^M) - 1$ are the implied interest rates promised to D from H and to M from D, when the reverse repo and repo contracts mature, respectively. In addition, the market practice to quote haircuts is 1 minus the loan amount over the collateral value, which in the model translates to m_t^j/T.

[17]Throughout the paper we will assume that in equilibrium $\Delta F_t \leq 0$ and $\Delta m_t \geq 0$. We prove that this is indeed the case in section 5.

$F_0^M) + (T - m_1^M) - (T - m_1^H)) = (1-\mu)(\Delta F_0 + \Delta m_1)$, i.e., the sum of cash owed from repos in $t = 0$ and cash received from repos in $t = 1$, which can be positive or negative. The second option yields $\xi(\mu, \theta)\lambda \overline{R}_\theta \Delta m_0$, where $\xi(\mu, \theta) \in [0, 1]$ is the fraction of the risky project the dealer liquidates as a function of the portion μ of hedge fund withdrawing their collateral.

For the subsequent analysis, we shall consider the case in which the dealer will have a positive net cash flow in the interim period from hedge funds rolling over their position. That is, the positive cash flow from the rehypothecation of collateral at $t = 1$ is higher than the outflow from closing the existing repo contracts. Given that the dealer does not have an initial endowment, this assumption eliminates their incentive to save funds for a liquidity shortfall when all hedge funds rolls over.

$$\text{C0:} \qquad \Delta m_1 + \Delta F_0 \geq 0. \tag{1}$$

Depending on the number of hedge funds withdrawing for a given θ, three outcomes are possible at $t = 1$ which are defined by two cutoff points of hedge fund withdrawals: μ_S, μ_R. First, for $\mu \in [0, \mu_S]$, the dealer can raise additional funds at $t = 1$ to meet the withdrawals and refrain from selling a fraction of the risky project. Second, for $\mu \in (\mu_S, \mu_R]$ the dealer needs to liquidate part of the risky project to meet the withdrawals of collateral, i.e. $\xi \in (0, 1)$. Third, for $\mu \in (\mu_R, 1]$ the dealer cannot meet all the withdrawals even if they liquidate all of the risky project, i.e. $\xi = 1$. We have implicitly considered that the dealer will first use all the excess cash she raises from hedge funds that roll over before liquidating the risky project. Figure 4 illustrates the dealer's balance sheet at the *end* of the refinancing period for these three cases.

The threshold μ_S is the maximum number of withdrawals that can be fulfilled by the additional cash collected from $1 - \mu$ hedge funds, that is,

$$\begin{aligned} &\mu \Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) > 0 \\ \Rightarrow &\mu < \mu_S \equiv 1 + \frac{\Delta F_0}{\Delta m_1}. \end{aligned} \tag{2}$$

Given that $\Delta m_1 > 0$ and $\Delta F_0 < 0$, μ_S is less than one but is strictly positive only if hedge funds that roll over contribute additional cash, i.e., $\Delta F_0 + \Delta m_1 > 0$.

The threshold μ_R is the maximum number of withdrawals that can be fulfilled by the

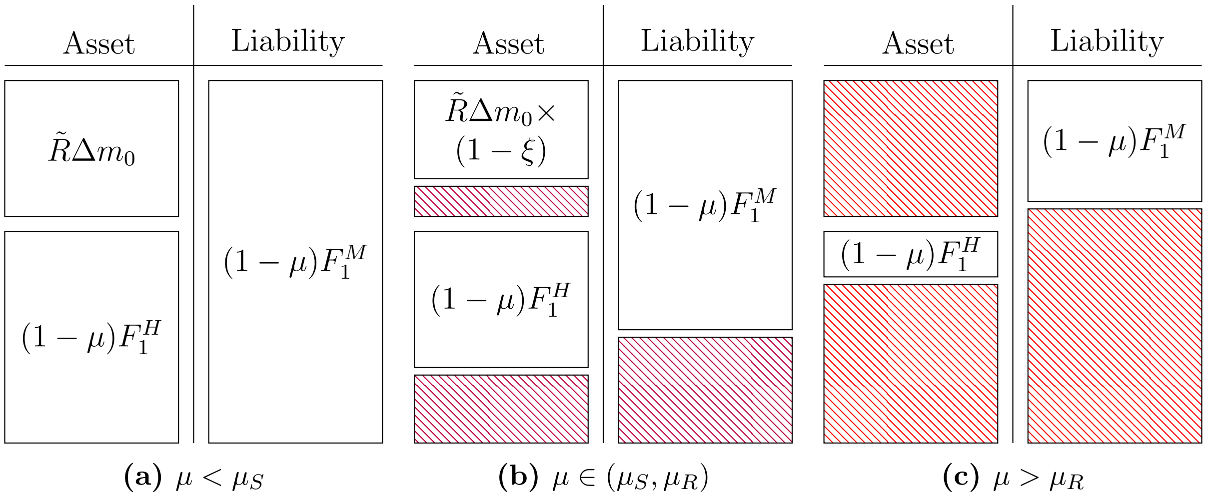


Figure 4: Dealer Balance Sheet at End of $t = 1$
In the left panel, only a small fraction of hedge fund withdraw, swapping $t = 0$'s outstanding repos and reverse repos to $t = 1$ repos and reverse repos. In the middle panel, an intermediate amount of hedge funds withdraw, implying a reduction in the dealer's balance sheet and partial liquidation of the risky project. In the right panel, a large amount of hedge funds withdraw, implying a severe reduction in the dealer's balance sheet and liquidation of the entire risky project. The newly created repos and reverse repos depend on the fraction of hedge funds that rolled over.

additional cash collected plus the liquidation of the entire risky project, that is,

$$\begin{aligned} &\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) + \lambda\overline{R}_\theta \Delta m_0 > 0 \\ \Rightarrow &\mu < \mu_R \equiv 1 + \frac{\Delta F_0 + \lambda\overline{R}_\theta\Delta m_0}{\Delta m_1}. \end{aligned} \tag{3}$$

Intuitively, $\Delta F_0 + \lambda\overline{R}_\theta\Delta m_0 < 0$ implying that $\mu_R < 1$, because the liquidation value of the risky position cannot satisfy all hedge fund withdrawals. As we will show further on, this is source of the coordination problem in the model.

For $\mu \in [\mu_R, 1]$, only a fraction of the hedge funds withdrawing will collect their collateral. That is, a fraction $f(\mu, \theta)\mu$ of money funds get their repayment back and deliver the collateral to the dealer, which is routed back to the hedge funds that decided to withdraw, following a sequential service constraint.[18] The fraction that gets repaid, whenever the entire risky position is sold, is given by

$$
\begin{aligned}
&f(\mu,\theta)\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) + \lambda\overline{R}_\theta\Delta m_0 = 0 \\
\Rightarrow &f(\mu,\theta) = -\frac{\lambda\overline{R}_\theta\Delta m_0 + (1-\mu)(\Delta F_0 + \Delta m_1)}{\mu\Delta F_0}.
\end{aligned} \tag{4}
$$

Comparing (2) and (3) it is clear that $\mu_S < \mu_R$, setting the range for partial liquidation of the dealer's risky project. The fraction that is liquidated is given by

$$
\xi(\mu,\theta) = -\frac{\Delta F_0 + (1-\mu)\Delta m_1}{\lambda\overline{R}_\theta\Delta m_0}. \tag{5}
$$

Having pinned down the relevant cash follows in $t = 1$ for all levels of hedge fund withdrawals, we can calculate the dealer's payoffs in $t = 2$. First, consider the case that the dealer has enough money to serve early withdrawals without liquidating any assets, i.e., $\mu \in [0, \mu_S)$. The cash flow to the dealer in the final period is equal to $\tilde{R}\Delta m_0 + \Delta F_0 + (1 - \mu)(\Delta m_1 + \Delta F_1)$. When there is no selling at $t = 1$, dealer optimization should result in positive cash flow if R^U realizes. However, if R^D realizes, the cash flow may be negative, resulting in a dealer default. In that case, the available resources are distributed pro rata to the $1 - \mu$ hedge funds that rolled over at $t = 1$, and each individual hedge fund receives:

$$
G_S^D(\mu,\theta) = \frac{R^D\Delta m_0 + \Delta F_0 + (1-\mu)\Delta m_1}{1-\mu}. \tag{6}
$$

To keep the model interesting, we ensure that after a bad outcome the amount raised in the interim period is not enough to make all money funds whole in the final one; that is,

$$
\text{C1:} \qquad R^D\Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 \leq 0. \tag{7}
$$

Condition (7) implies that even if all hedge funds roll over in the interim period (i.e., $\mu = 0$), there is not enough wealth to payoff cash lenders' entire claim if R^D realizes. This restriction is important to guarantee the existence of a region for fundamentals where an individual

[18] Sequential servicing is a natural assumption given that repo markets are Over-the-Counter, and the dealer needs to negotiate and settle trades with every hedge fund bilaterally.

hedge fund withdraws its collateral independent of their beliefs about the actions of other hedge funds, i.e., the lower dominance region (see section 4 for details). It is also economically meaningful because it generates a fundamental motive for hedge funds to run.

Similarly, condition (8) below implies that the dealer is solvent at $t = 2$ if R^U realizes and all funds decide to roll over at $t = 1$. This condition will always hold in equilibrium, because the dealer would optimally choose contract terms yielding positive profit in the good state, i.e., $R^U \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 > 0$, even if condition (1) binds.

$$\text{C2:} \qquad R^U \Delta m_0 + \Delta F_1 > 0. \tag{8}$$

Second, consider the case that the dealer has to liquidate some, but not all of the risky project to serve early withdrawals, i.e., $\mu \in [\mu_S, \mu_R)$. The cash flow to the dealer in the final period is equal to $\tilde{R}\Delta m_0(1 - \xi(\mu, \theta)) + (1 - \mu)\Delta F_1$, where $\xi(\mu, \theta)$ is given by (5). For realization $\tilde{R} = R^D$ it is obvious, given condition (7), that the dealer defaults for all $\mu \in [\mu_S, \mu_R)$. Hence, what is left in the dealer's portfolio is distributed pro rata to hedge funds that rolled over at $t = 1$, and each individual hedge fund receives

$$G_I^D(\mu, \theta) = \frac{R^D \Delta m_0 + \frac{R^D}{\lambda \bar{R}_\theta} (\Delta F_0 + (1 - \mu)\Delta m_1)}{1 - \mu}, \tag{9}$$

or, in other words, they collect the payoff on the fraction of the risky project that is not liquidated at $t = 1$, since $\Delta F_0 + (1 - \mu)\Delta m_1 \leq 0$ for $\mu \in [\mu_S, \mu_R)$.

If the amount of liquidation is severe enough the dealer may also default when $\tilde{R} = R^U$. That is, the portfolio payoff may not cover the costs of returning the collateral to hedge funds that rolled over. Denote by μ_I the maximum number of withdrawals after which the dealer default. Then, for $\mu \in [0, \mu_I)$ the dealer is solvent if R^U realizes, while insolvent for $\mu \in [\mu_I, \mu_R)$. In the latter case, what is left in the dealer's portfolio is distributed pro-rate to hedge funds that rolled over at $t = 1$, and each individual hedge fund receives

$$G_I^U(\mu, \theta) = \frac{R^U \Delta m_0 + \frac{R^U}{\lambda \bar{R}_\theta} (\Delta F_0 + (1 - \mu)\Delta m_1)}{1 - \mu}. \tag{10}$$

Therefore, the threshold μ_I is determined at the largest $\mu \in [\mu_S, \mu_R)$, such that the dealer is just solvent at $t = 2$ if R^U realizes, i.e.,

$$R^U \Delta m_0 \left(1 + \frac{\Delta F_0 + (1-\mu)\Delta m_1}{\lambda \overline{R}_\theta \Delta m_0}\right) + (1-\mu)\Delta F_1 \geq 0$$

$$\Rightarrow \mu \leq \mu_I \equiv 1 + \frac{R^U \left(\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0\right)}{\lambda \overline{R}_\theta \Delta F_1 + R^U \Delta m_1}. \tag{11}$$

Lemma 1. *The maximum level of withdrawals that the dealer is solvent in the good state at $t = 2$ is above the level that she starts liquidating assets and below the level that she is fully liquidated at $t = 1$, i.e., $\mu_S < \mu_I \leq \mu_R$.*

Lemma 1 will be useful to show the existence and uniqueness of a run equilibrium in section 4. Figure 5 summarizes the different outcomes for given fundamental θ depending on the number of hedge funds withdrawing at $t = 1$.

$\mu = 0$ to $\mu = \mu_S$	$\mu = \mu_S$ to $\mu = \mu_I$	$\mu = \mu_I$ to $\mu = \mu_R$	$\mu = \mu_R$ to $\mu = 1$
No run	No run	No run	Run
No asset liquidation	Asset liquidation	Asset liquidation	Full liquidation
No default for $\tilde{R} = R^U$	No default for $\tilde{R} = R^U$	Default for $\tilde{R} = R^U$	
Default for $\tilde{R} = R^D$	Default for $\tilde{R} = R^D$	Default for $\tilde{R} = R^D$	

Figure 5: Outcomes as μ Varies for Zero to One for Given Fundamental θ.

3.2 Money Funds

Money funds are the providers of cash and lend funds to dealers at $t = 0$ and $t = 1$ via repo contracts. There is a continuum of identical money funds, each providing the dealer with $T - m_t^M$ at t, where m_t^M is the margin that the dealer has to contribute and T is the value of collateral pledged to money funds. Denote by F_t^M the repurchase price agreed at t.

Given that our focus is on the incentives of collateral providers to withdraw we make assume such that cash lenders do not face a coordination problem which prompts them to run. This will allow us to isolate our mechanism and focus on the run dynamics stemming, instead, from a coordination problem among the providers of collateral. Specifically, we assume that money funds are "very risk averse" (i.e., infinitely risk averse) such that they will not tolerate a loss. In other words, the repo contracts between the dealer and money funds are over-collateralized or, equivalently, $F_t^M \leq T$. All contracts that satisfy this condition are acceptable because they completely eliminate the money fund's exposure to the dealer. In case of a run or insolvency of the dealer, the money fund would have immediate access to the collateral (because repo are exempt from automatic stay), selling it onto the market for a value of T—and possibly returning any surplus above and beyond it was owed. It is in the dealer's interest to maximize the funds they obtains from money funds at $t = 0$ and $t = 1$ rather than receiving some residual cash at $t = 2$. In other words, using the safe asset as collateral, the dealer can borrow (from a competitive money fund market) at zero haircuts, i.e., $m_t^M = 0$, and at repurchase prices $F_t^M = T$, which implies that the recovery value from the sale of Treasuries is zero.

3.3 Hedge Funds

There are a continuum of hedge funds, each of which approaches the dealer to finance the purchase of the riskless asset T. Hedge funds are ex ante identical and value holding T above and beyond its fair value. That is, the hedge fund receives non-pecuniary benefits for holding the asset, which potentially accrue from hedging motives, demand for safe assets or other reasons, which magnify its value by $\eta > 1$. This extra benefit could accrue from insurance motives, since Treasuries are said to be negative-beta assets and may help hedge funds hedge other (not modeled) exposures in the portfolio. Or it could reflect the specialness of specific Treasuries, which hedge funds want to purchase (see Duffie 1996 for a discussion of specialness).

The role of η in the model is to provide incentives for hedge funds to take leverage in order to invest in T. For a given repurchase price, F, a levered position yields $\eta T - F$, i.e., the hedge fund enjoys the full benefit ηT for a price F. On the contrary, the unlevered position yields, $\eta(T - F)$, i.e., the hedge funds can only invests its own funds, $T - F$, in the riskless asset. The levered position yields a higher payoff as long as $\eta > 1$. Without loss of generality, we assume that hedge funds get the extra valuation for assets held at the final period.[19]

[19] Alternatively, hedge funds could also receive these non-pecuniary benefits for holding the asset between $t = 0$ and 1. This assumption unnecessarily complicates the model without providing any meaningful insights.

Hedge funds start out with an initial endowment of W_0 which, along with the repo raised from the dealer, allows them to purchase T. Hedge funds do not initially own T but the extra benefit η provides incentives to enter into a repo contract with the dealer to purchase as much T as possible.[20]

The payoff to an individual hedge fund depends on the realization of θ, the number of hedge funds that withdraw, μ, and the action that it takes in the roll-over stage. Denote by $\alpha = \{0, 1\}$ the strategy set of a hedge fund, where $\alpha = 0$ stands for withdrawing and $\alpha = 1$ for rolling over. The utility that a hedge fund receives can be expressed by $U^H(\mu, \theta; \alpha)$.

First, consider the case that a hedge fund rolls over at $t = 1$, i.e., $\alpha = 1$. The available cash at the end of $t = 1$ after rolling over, which can be invested in additional Treasuries, is equal to $W_0 + (T - m_0^H) - F_0^H + (T - m_1^H) - T$, i.e., what is left of the initial wealth after receiving cash from the starting leg of both repos $(T - m_0^H) + (T - m_1^H)$, paying the closing leg of the initial repo F_0^H, and purchasing the collateral at the onset of the game T.[21] Note that the new repo terms are not contingent on the realization of fundamentals, θ, nor of the portion of hedge funds withdrawing, μ. However, the final payoff at $t = 2$ will depend on θ and μ as they determine whether a run occurs and the probability that the dealer defaults at $t = 2$.

For a given realization of fundamentals θ and $\mu < \mu_S$, a hedge fund that rolls over can repurchase its collateral at price F_1^H and enjoy a utility payoff ηT if the dealer does not default at $t = 2$. This occurs with probability θ. On the other hand, if R^D realizes, the dealer defaults and the hedge fund is repaid its share of the dealer's remaining portfolio: $G_S^D(\mu, \theta)$ in cash which does not yield the utility benefit η. The expected utility of an individual hedge fund that rolls over is, then,

$$U^H(\mu < \mu_S, \theta; 1) = \theta(\eta T - F_1^H) + (1 - \theta)G_S^D(\mu, \theta) + \eta\left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \quad (12)$$

If $\mu \in [\mu_S, \mu_I)$, a hedge fund that rolls over can still repurchase its collateral if R^U realizes at $t = 2$, but it receives a cash payment $G_I^D(\mu, \theta)$ which incorporates the liquidation cost

[20] Alternatively, one could motivate trade by assuming heterogeneous beliefs over risky collateral, which would complicate the analysis without providing additional insights, since we have neutralized potential instability from cash lenders.

[21] Recall that Treasury holdings at $t = 2$ yield a utility payoff $\eta > 1$, thus a hedge fund will invest all available cash at $t = 1$ in Treasuries.

from selling part of the risky project. The expected utility is, then,

$$U^H(\mu_S \leq \mu < \mu_I, \theta; 1) = \theta(\eta T - F_1^H) + (1-\theta)G_I^D(\mu, \theta) + \eta\left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \tag{13}$$

If more hedge funds withdraw, the dealer will default in the good state as well, for $\mu \in [\mu_I, \mu_R)$, receiving its share of the dealer's remaining portfolio in either state,

$$U^H(\mu_I \leq \mu < \mu_R, \theta; 1) = \theta G_I^U(\mu, \theta) + (1-\theta)G_I^D(\mu, \theta) + \eta\left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \tag{14}$$

Comparing (12) or (13) to (14), the role of η in the model becomes more clear. A hedge fund will only enjoy the benefit η if the dealer is solvent and, thus, returns the *physical* collateral T to the hedge fund. If the dealer defaults, the hedge fund receives *cash* payment pro-rata, which does not yield the benefit η. Around the default threshold μ_I, the payoff in the good state to a hedge fund that rolls over is $\eta T - F_1^H$ if the dealer does not default and $T - F_1^H$ if the dealer defaults.

If withdrawals continue, the dealer will eventually run out of money and will be fully liquidated at $t = 1$ for $\mu \in [\mu_R, 1]$. In this case, a hedge fund that rolled over at $t = 1$ will receive utility

$$U^H(\mu_R \leq \mu \leq 1, \theta; 1) = \eta\left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \tag{15}$$

In these last two cases, the hedge fund cannot repurchase back its collateral and the utility benefit η applies *only* to the additional Treasuries purchased with the remaining cash at $t = 1$.

Next, consider the case that a hedge fund that does not roll over at $t = 1$, i.e., $\alpha = 0$. This hedge fund is able to invest $W_0 - m_0^H$ in Treasuries, plus any incremental cash from closing the initial repo position. The latter will depend on whether the dealer is fully liquidated at $t = 1$. If the dealer has enough resources to serve all early withdrawals, a hedge fund that does not roll over can repurchase its collateral at $t = 1$ at price F_0^H, receiving a net cash flow $T - F_0^H$ and final utility equal to

$$U^H(\mu < \mu_R, \theta; 0) = \eta\left(W_0 - m_0^H + T - F_0^H\right). \tag{16}$$

If the dealer cannot serve all of the early withdrawals, then a hedge fund that does not roll over will only be able to repurchase its collateral with probability $f(\mu, \theta)$ given by (4), and the expected utility is equal to

$$U^H(\mu_R \leq \mu \leq 1, \theta; 0) = \eta \left(W_0 - m_0^H + f(\mu, \theta) \cdot (T - F_0^H) \right). \tag{17}$$

As derived later in section 4, hedge funds will follow a strategy such that all roll over at $t = 1$ if the realization of fundamentals, θ, is above a threshold θ^*, and all withdraw their collateral if θ is below θ^*. Moreover, every individual hedge fund should be willing to enter into a repo contract both at $t = 0$ and $t = 1$ independently given that all other hedge funds follow the equilibrium strategy. That is, regardless of whether an hedge fund participates in both periods, their willingness to participate should hold in each period.

Specifically, a hedge fund would not choose to enter the repo contract at $t = 0$ if the following participation constraint is satisfied:

$$PC_0 : \quad \int_{\theta^*}^{1} \cdot (T - F_0^H) d\theta + \int_0^{\theta^*} \cdot f(1, \theta) \cdot (T - F_0^H) d\theta - \cdot m_0^H \geq 0, \tag{18}$$

where $f(1)$ is given by (4) for $\mu = 1$. In other words, an individual hedge fund will participate in $t = 0$ if the expected cash flow at $t = 1$, including the possibility of a run, is higher than the original margin contribution.

In addition, a hedge fund will not deviate from the equilibrium strategy at $t = 1$ if for every $\theta \geq \theta^*$ the following participation constraint is satisfied:

$$PC_1 : \quad \theta(\eta \cdot T - F_1^H) + (1 - \theta) G_S^D(0, \theta) - \eta \cdot m_1^H \geq 0, \tag{19}$$

where $G_S^D(0, \theta)$ is given by (6) for $\mu = 0$. In other words, an individual hedge fund will only roll over at $t = 1$ for $\theta \geq \theta^*$ if the expected benefit is higher than the outside option of investing the margin in Treasuries, which is equal to $\eta \cdot m_1^H$. The former is equal to the utility benefit of repurchasing the collateral, $\eta \cdot T$, minus the repurchase price, F_1^H occurring with probability θ plus the cash flow received when the dealer defaults, $G_S^D(0, \theta)$, occurring with probability $1 - \theta$. Given that the left-hand side in (19) is increasing in θ, it suffices that the participation constraint is satisfied for θ^*. We establish this in Corollary 1 in section 4.

Note that the decision to enter a repo at $t = 0$ is independent of the decision to enter a repo at $t = 1$. If PC_0 is not satisfied, but PC_1 is, then an individual hedge fund will deviate from the equilibrium strategy at $t = 0$, and vice versa. Hence, both (18) and (19) need to hold in equilibrium, which restricts the ability of the dealer to extract all surplus from hedge fund. Moreover, the participation constraint in $t = 1$ does not depend on whether the hedge fund participated in $t = 0$. That is, a hedge fund can decide not to participate in $t = 0$, yet decide to participate in $t = 1$. In this sense, these per period constraints are independent.

As we discuss later, equation (19) will not be binding in equilibrium because hedge funds need to have the proper incentives to roll over in the incomplete information game, while equation (18) will be binding as it restricts the ability of the dealer to set a very high margin, m_0^H, or repurchase price, F_0^H.

Finally, note that integrating (19) over $[\theta^*, 1]$, and adding (18) as well as $\eta \cdot W_0$ on both sides yields $\int_{\theta^*}^1 U^H(0, \theta; 1)d\theta + \int_0^{\theta^*} U^H(1, \theta; 0)d\theta \geq \eta \cdot W_0$. Hence, under the optimal contracting terms the overall utility of a hedge fund playing the equilibrium strategy is higher than the utility in autarky. $U^H(\mu = 0, \theta; 1)$ and $U^H(\mu = 1, \theta; 0)$ which are given by (12) through to (17).

Using the period 0 participation constraint and condition (1), we can prove the following Lemma, which will be useful in later analysis.

Lemma 2. *The contract terms are such that:*

1. *The dealer's liabilities at $t = 0$ are higher than the cash inflow from the rehypothecation of collateral, i.e., $-\Delta F_0 > \Delta m_0$.*

2. *The cash inflow from the rehypothecation of collateral at $t = 1$ is higher than at $t = 0$, i.e., $\Delta m_1 > \Delta m_0$.*

As discussed in section 5, the dealer will choose contract terms that push hedge funds to their period 0 participation constraint *in equilibrium*. Hence, we can rewrite (18) as $-\Delta F_0 \geq g(\theta^*)\Delta m_0$, where $g(\theta^*) > 1$ from Lemma 2 and given by

$$g(\theta^*) = \frac{1 - \lambda \left[\left(R^U - R^D \right) \frac{\theta^{*2}}{2} + \theta^* R^D \right]}{1 - \theta^*}. \tag{20}$$

We should note that equation (20) implies that the dealer needs to offer hedge funds a repurchase price, F_0^H, that is lower than the amount they borrow, $T - m_0^H$, i.e., the interest rate on first period reverse repos is negative.[22] As we will show in Proposition 2, the dealer will be willing to offer a negative rate in order to provide incentives for hedge funds to participate and, thus, use the liquidity windfall Δm_0 to invest in the risky project. In turn, hedge funds require a negative interest rate to participate at $t = 0$ because of the probability of a run on the dealer in which case they could losing everything. We could have dispensed with negative rates if we had assumed that hedge funds also enjoy a non-pecuniary benefit, $\tilde{\eta} > 1$, from holding T from $t = 0$ to $t = 1$ or alternatively, if agents in the model discounted future cash flows. Given that these assumptions would unnecessarily complicated the model, we have opted to proceed without them.

4 Collateral Runs and Coordination Failure

This section examines the decision of an individual hedge fund to withdraw its collateral or roll over its repo contract in the intermediate period with pre-determined contract terms. Knowing the equilibrium outcome of the global game, in Section 5, we can derive the equilibrium contracting terms and show under what conditions they are consistent with the existence of a coordination problem.

At the beginning of $t = 1$, each hedge fund i receives a private noisy signal $x_i = \theta + \epsilon_i$ where the error terms ϵ_i are independently and uniformly distributed over $[-\epsilon, \epsilon]$. The signal provides information about dealer's solvency at $t = 2$, i.e., the probability that R^U realizes, but also about the liquidation value of the dealer's risky investment, $\lambda \overline{R}_\theta \Delta m_0$.[23] The signal also provides information about other hedge funds' signals, which allows an inference regarding their actions. An individual hedge fund may decide to withdraw its collateral not only because it believes that fundamentals are bad, but also because the conjectured portion of hedge funds withdrawing is high enough to push the dealer into illiquidity.

[22] The interest rate is $F_0^H/(T - m_0^H) - 1 = (\Delta F_0 + \Delta m_0)/(T - m_0^H) = (1 - g(\theta^*))\Delta m_0/(T - m_0^H) < 0$, using $T = F_0^M$ and m_0^M from the money funds problem, (20), and condition (8).

[23] Incorporating this feature is the methodological contribution of this paper, contributing to the Goldstein-Pauzner type of bank-runs model where the payoffs of withdrawing agents depend on endogenous equilibrium choices. In turn, such endogenous payoffs are important are important for policy analysis (see section 6. It is true that the liquidation value depends on the state θ in Rochet-Vives as well. However, they assume exogenous payoffs for withdrawing agents, rendering their model less suitable for policies that affect contracting terms. Finally, we show in an online appendix that collateral runs exist also in these two alternative frameworks.

As it common in incomplete information games, in Lemma 3 we derive the upper and lower dominance regions where the actions of an individual hedge fund are independent of the actions of other hedge funds. These regions are essential for the existence of the run threshold.

Lemma 3. *There are two regions of fundamentals defined by thresholds θ^{LD} and θ^{UD} where the decision of an individual hedge fund to withdraw its collateral is independent of the decisions of other hedge funds. A hedge fund will always withdraw its collateral for $\theta \leq \theta^{LD} = ((\eta-1)\Delta m_1 - R^D \Delta m_0 - \Delta F_0)/((\eta-1)T - R^D \Delta m_0 - \Delta F_0 - \Delta m_1 - \Delta F_1)$, and will always roll over for $\theta \geq \theta^{UD} = -(\Delta F_0 + \lambda R^D \Delta m_0)/(\lambda(R^U - R^D)\Delta m_0)$. Moreover, θ^{LD} and θ^{UD} lie in the support of θ, i.e., $\theta^{LD}, \theta^{UD} \in (0,1)$.*

We seek a symmetric equilibrium characterized by two thresholds (x^*, θ^*) such that an individual hedge fund will withdraw its collateral if its private signal realization x_i is lower than a threshold x^* and the dealer will be fully liquidated at $t = 1$ if the fundamentals realization θ is lower than a threshold θ^*.

Under such a threshold strategy, the portion of hedge funds that withdraw their collateral at a given level of fundamentals θ is

$$\mu(\theta, x^*) = \begin{cases} 1 & \text{if } \theta < x^* - \epsilon \\ Prob(x_i \leq x^* | \theta) & \text{if } x^* - \epsilon \leq \theta \leq x^* + \epsilon \\ 0 & \text{if } \theta > x^* + \epsilon. \end{cases} \tag{21}$$

If the fundamental value θ is lower than $x^* - \epsilon$, then all hedge funds receives signals $x_i < x^*$. Hence, all hedge funds, following a threshold strategy, withdraw and $\mu(\theta, x^*) = 1$. The opposite is true for $\theta > x^* + \epsilon$, whereby all hedge funds receive signals $x_i > x^*$ and roll over, thus $\mu(\theta, x^*) = 0$. Finally, if fundamentals are not sufficiently higher or lower than x^*, i.e., $\theta \in [x^* - \epsilon, x^* + \epsilon]$, some hedge funds will receive signals that are lower than x^* and, thus, will withdraw their collateral; and others will receive a signal higher than x^* and, thus, will roll over their repo. Given that private noise, ϵ_i, is independently and identically distributed, from the law of large numbers the portion of hedge funds withdrawing for a given level of θ in the intermediate region is $\mu(\theta, x^*) = Prob(x_i \leq x^* | \theta) = (x^* - \theta + \epsilon)/2\epsilon$.

The signal and fundamentals thresholds are derived in two steps as follows. First, given the threshold strategy x^*, we can derive the threshold for fundamentals, θ^*, which determines whether the dealer is fully liquidated at $t = 1$ or survives to $t = 2$. Because the portion of hedge funds withdrawing is decreasing in θ from (21), the dealer is fully liquidated only if $\theta < \theta^*$. That is, θ^* as a function of x^* is the solution to $f(\mu(\theta^*, x^*), \theta^*) = 1$, which from equation (4) gives:

$$\theta^* = x^* - \epsilon\frac{\Delta m_1 + 2\Delta F_0 + 2\lambda\overline{R}_{\theta^*}\Delta m_0}{\Delta m_1}. \tag{22}$$

In other words, for threshold strategy x^*, if θ is lower than θ^*, then the portion of hedge funds withdrawing is higher than what the dealer can serve by liquidating all of her assets, or $f(\mu(\theta, x^*), \theta) < 1$. On the contrary, if θ is higher than θ^*, fewer hedge funds withdraw, allowing the dealer to decrease asset liquidations and survive to $t = 2$.[24]

Second, given the fundamentals threshold θ^*, an individual hedge fund can compute the signal threshold x^*, below which it is optimal to withdraw conditional on its expectation over the portion of hedge funds withdrawing and the private signal it receives. This signal threshold depends on the utility differential between rolling over and withdrawing for a given level of θ and μ. The difference in expected payoff is given by $U^H(\mu, \theta; a = 1)$-$U^H(\mu, \theta; a = 0)$ derived from (12)-(17). Given that in equilibrium $F_t^M = T$ and $m_t^M = 0$, the utility differential $\nu(\mu, \theta)$ is given by the following piecewise function:

$$\nu(\mu,\theta) = \begin{cases} \theta\left[(\eta-1)T - \Delta F_1\right] + (1-\theta)G_S^D(\mu,\theta) - \eta\Delta m_1 & \mu \in [0, \mu_S) \\ \theta\left[(\eta-1)T - \Delta F_1\right] + (1-\theta)G_I^D(\mu,\theta) - \eta\Delta m_1 & \mu \in [\mu_S, \mu_I) \\ \theta G_I^U(\mu,\theta) + (1-\theta)G_I^D(\mu,\theta) - \eta\Delta m_1 & \mu \in [\mu_I, \mu_R) \\ -\eta\frac{\lambda\overline{R}_\theta\Delta m_0 + \Delta F_0 + \Delta m_1}{\mu} & \mu \in [\mu_R, 1] \end{cases} \tag{23}$$

where $G_S^D(\mu,\theta) = \left(R^D\Delta m_0 + \Delta F_0 + (1-\mu)\Delta m_1\right)/(1-\mu)$, $G_I^D(\mu,\theta) = (R^D\Delta m_0 + R^D/\lambda\overline{R}_\theta \cdot (\Delta F_0 + (1-\mu)\Delta m_1))/(1-\mu)$, and $\theta G_I^U(\mu,\theta) + (1-\theta)G_I^D(\mu,\theta) = (\overline{R}_\theta\Delta m_0 + 1/\lambda \cdot (\Delta F_0 + (1-\mu)\Delta m_1))/(1-\mu)$ from (6), (9) and (10).

Looking at the two first legs, the payment in the bad state for a hedge fund that rolled over is decreasing in μ and $\lim_{\mu\to\mu_S^-} G_S^D(\mu,\theta) = \lim_{\mu\to\mu_S^+} G_I^D(\mu,\theta)$. [25] The third leg is also decreasing in μ for $\lambda\overline{R}_\theta\Delta m_0 + \Delta F_0 < 0$, i.e., for $\theta < \theta^{UD}$. The utility differential is

[24] The fraction of assets distributed is strictly decreasing in μ, i.e., $\partial f(\mu,\theta)/\partial\mu = (\lambda\overline{R}_\theta\Delta m_0 + \Delta F_0 + \Delta m_1)/(\mu^2\Delta F_0) < 0$ given condition in equation (1).

[25] $\partial G_S^D(\mu,\theta)/\partial\mu = (R^D\Delta m_0 + \Delta F_0)/(1-\mu)^2 < 0$, because $R^D < 1$ and $\Delta m_0 + \Delta F_0 < 0$ from Lemma 2, while $\partial G_I^D(\mu,\theta)/\partial\mu = R^D/(\lambda\overline{R}_\theta(\lambda\overline{R}_\theta\Delta m_0 + \Delta F_0)/(1-\mu)^2 < 0$ for $\theta < \theta^{UD}$.

discontinuous at μ_I because the hedge fund fails to receive the non-pecuniary benefit η if the dealer defaults. However, $\lim_{\mu \to \mu_I^-} \nu(\mu, \theta) - \lim_{\mu \to \mu_I^+} \nu(\mu, \theta) = \theta(\eta - 1)T > 0$, and, thus, $\nu(\mu, \theta)$ is strictly decreasing in $\mu \in [0, \mu_R)$ for $\theta < \theta^{UD}$ which is the relevant region where a coordination problem may occur. The final leg in (23) is increasing in μ given that $\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 + \Delta m_1 > 0$ from the condition in equation (1). Thus, the model features one-sided, rather than global, strategic complementarities as in Goldstein and Pauzner (2005). That is, once the dealer is fully liquidated, a hedge fund has fewer incentives to withdraw as withdrawals increase. Note that ν "crosses" zero as μ increases from above. That is, depending on the contract terms, this can happen within any of the first two legs or at the jump, but not within the third and fourth legs where $\nu(\mu, \theta)$ always takes negative values.[26]

Consider an individual hedge fund that receives signal x_i. The hedge fund will use the signal to update its beliefs about the realization of θ. Given that both θ and ϵ_i are uniformly distributed, the posterior distribution of θ given x_i is $\theta | x_i \sim U[x_i - \epsilon, x_i + \epsilon]$. This implies that the utility differential between rolling over and withdrawing for a hedge fund that receives signal x_i as a function of the cutoff value is

$$\Delta(x_i, x^*) = \frac{1}{2\epsilon} \int_{x_i - \epsilon}^{x_i + \epsilon} \nu(\mu(\theta, x^*), \theta) d\theta. \tag{24}$$

In a threshold equilibrium, a hedge fund prefers to withdraw, i.e., $\Delta(x_i, x^*) < 0$, for all $x_i < x*$, and prefers to roll over, i.e., $\Delta(x_i, x^*) > 0$, for all $x_i > x*$. $\Delta(x_i, x^*)$ is continuous in x_i because a change in the signal only changes the limits of integration $[x_i - \epsilon, x_i + \epsilon]$ and the integrand is bounded. Hence, a hedge fund that receives signal $x_i = x^*$ is indifferent between rolling over and withdrawing if

$$\Delta(x^*, x^*) = \frac{1}{2\epsilon} \int_{x^* - \epsilon}^{x^* + \epsilon} \nu(\mu(\theta, x^*), \theta) d\theta = 0. \tag{25}$$

Equations (22) and (25) jointly determine the threshold for fundamentals θ^* and the threshold strategy x^*. As in Goldstein-Pauzner, the model features one-sided strategic complementarities. Hence, we follow the steps in Goldstein-Pauzner and make similar assumptions, most importantly that noise is uniformly distributed, to show the uniqueness of a

[26] $\nu(\mu, \theta) < 0$ for $\mu \in [\mu_I, \mu_R)$ —third leg— requires $\mu > 1 + (\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0)/\Delta m_1 > \mu_I$, which is always true.

threshold equilibrium. However, our framework features two additional complications akin to Kashyap et al. (2017).[27] First, due to limited liability, the dealer's default threshold is endogenous. Second, and most importantly, the liquidation value matters for the payoff in a run and, thus, state monotonicity of $\nu(\mu, \theta)$ which typically used to find unique equilibria, is not straightforward.

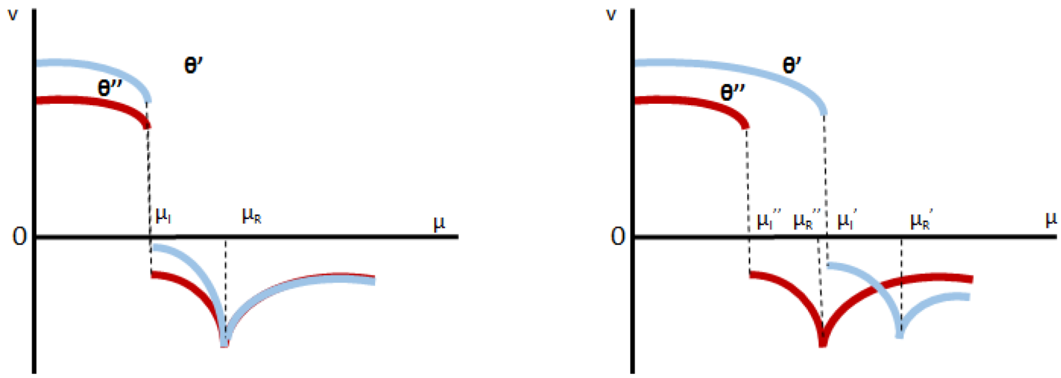
Figure 6 illustrates how the liquidation value in our framework affects the utility differential ν for different values of fundamentals, θ' and θ'', as μ varies. The left panel corresponds to the case of a fixed liquidation value, while the right panel corresponds to the case of an uncertain liquidation value, modeled in this paper. Both cases are characterized by one-sided strategic complementarities as ν increases in the region where a run occurs. Namely, once a run has materialized, the incentive to withdraw is lower for a higher μ. This is akin to Goldstein-Pauzner and is typical in bank-run models. The two cases differ in how liquidation values change with fundamentals. Consider that $\theta' > \theta''$. When the liquidation value is fixed (left panel), the utility differential unambiguously increases, because θ only affect the probability of getting a high payoff conditional on a run not occurring. Once we allow the liquidation value to vary with θ (right panel), the utility differential changes in a non-monotone way: it is increasing in θ in the region where a run does not occur, but it is decreasing in the region where a run materializes. This latter effect is because the relative payoff from withdrawing is higher conditional on a run materializing. We employ the same strategy as in Kashyap et al. (2017) to address this issue of non-state monotonicity. Proposition 1 establishes the existence and uniqueness of a threshold equilibrium.[28]

Proposition 1. *Given contract terms satisfying $\theta^{LD} < \theta^{UD}$ in Lemma 3, there exist a threshold, x^*, such that a hedge fund rolls over if $x_i > x^*$ and withdraws if $x_i < x^*$, and a threshold θ^*, such that the dealer does not experience any withdraws if $\theta \geq \theta^*$ and is fully liquidated if $\theta < \theta^*$. Moreover, the thresholds are unique if noise is not too large.*

Proposition 1 establishes the existence of a *unique* threshold strategy conditional that

[27] Our frameworks differ because we consider a different source of stochastic uncertainty. In Kashyap et al. (2017) the liquidation value of the risky assets can vary independently of their expected long-term payoff. In our model, the variation in the liquidation value stems from the variation in the long-term expected payoff. In technical terms, they allow λ to vary keeping θ constant, while we allow θ to vary keeping λ constant.

[28] To simplify the analysis we have restricted attention to a uniformly distributed probability of a good realization. The difficulty arises from the uncertain liquidation value. Goldstein-Pauzner allow for more general distributions of the probability of a good realization, $p(\theta)$, where θ is uniformly distributed and $p'(\theta) > 0$. In our case, $p(\theta) = \theta$ and $p'(\theta) = 1$.



(a) Fixed liquidation value

(b) Uncertain liquidation value

Figure 6: Utility differential for different fundamental values under certain and uncertain liquidation values

there exist equilibrium contract terms such that the lower dominance threshold, θ^{LD}, is strictly smaller than the upper dominance threshold, θ^{UD}. In the proof of Proposition 2 we actually show that the equilibrium contracting terms are such that $0 < \theta^{LD} < \theta^{UD} < 1$. As a result, *collateral runs* accrue from the dealer's optimal behavior in equilibrium.

Hereafter, we focus on the case that the noise goes arbitrarily close to zero. Note that taking the limit $\epsilon \to 0$ implies that $x^* \to \theta^*$ from (22). A hedge fund that receives signal x^*, the posterior distribution of θ is uniform over the interval $[x^* - \epsilon, x^* + \epsilon]$. Thus, that hedge fund's belief of the portion of hedge funds withdrawing as a function of θ, $\mu(\theta, x^*)$, is uniform over $[0, 1]$.[29] In other words, as θ decreases from $x^* + \epsilon$ to $x^* - \epsilon$, μ increases from 0 to 1. Changing variables in $\Delta(x^*, x^*) = 0$ provides the indifference condition that determines the unique value θ^*:

$$V(\theta^*) = \int_0^1 \nu(\mu, \theta^*) d\mu = 0. \tag{26}$$

The detailed expression for $V(\theta^*)$, with its derivatives with respect to θ^* and the contract

[29]This is true because $\lim_{x^* \to \theta^*} Prob(\mu(\theta, x^*) \leq N) = Prob(\mu(\theta, \theta^*) \leq N) = 1 - Prob(\theta \leq \theta^* + \epsilon - 2\epsilon N) = 1 - (\theta^* + \epsilon - 2\epsilon N - \theta^* + \epsilon)/(2\epsilon) = N$. Hence, $\mu(\theta, \theta^*) \sim U[0, 1]$.

terms, are shown in equation (B.31) in Appendix B. Moreover, (26) implies that $\nu(0, \theta^*) > 0$ given that ν is decreasing in μ when positive. Because in equilibrium μ is zero or one we can establish the following Corollary.

Corollary 1. *The period 1 participation constraint* (19) *is always slack for all* $\theta \geq \theta^*$.

5 Threshold Equilibrium

Having characterized hedge funds' threshold strategy under incomplete information, we turn to see the take-it-or-leave-it contracting terms the dealer chooses, anticipating hedge funds' optimal strategy. Because a hedge fund's problem is scalable, we normalize $T = 1$ for simplicity. This implies that feasible contract terms should satisfy $\Delta m_t \in [0, 1]$ and $\Delta F_t \in [-1, 0]$.

Given a threshold for fundamentals θ^* defined in (26), all hedge funds withdraw their collateral for $\theta < \theta^*$, inducing the dealer to default and receive zero profits. Conversely, if the realization of θ is above θ^* all hedge funds roll over their repos to period $t = 2$ and the dealer is exposed to the risky projects's payoff. With probability θ, the good state realizes and the dealer enjoys positive profits. Otherwise, the bad state realizes and the dealer defaults receiving nothing. Her expected utility is, then, given by:

$$\begin{aligned} U^D &= \int_0^{\theta^*} u(0) d\theta + \int_{\theta^*}^1 \left[\theta u(R^U \Delta m_0 + \Delta m_1 + \Delta F_0 + \Delta F_1) + (1 - \theta) u(0)\right] d\theta \\ &= \frac{(1 - \theta^{*2})}{2} u(R^U \Delta m_0 + \Delta m_0 + \Delta F_0 + \Delta F_1), \end{aligned} \tag{27}$$

where $u(\cdot)$ is a concave utility function–not excluding linear utility–with $u(0) = 0$.[30]

The dealer will internalize that changing the contracting terms directly affects hedge funds' threshold strategy θ^* through the global game condition (26), and, thus, the probabil-

[30] Note that a coordination problem can exist only if $\Delta m_0 > 0$. If hedge funds do not have an unsecured claim on the dealer, i.e., $\Delta m_0 = 0$, there cannot be an advantage to withdraw early. This situation can be appreciated graphically through Figure 4: if there is nothing hedge funds' can claim, beyond their collateral, there is no reason to withdraw. In this case the dealer would not invest in the risky project, and the only feasible contracting terms that give dealer non-negative profits are $\Delta m_0 = \Delta m_1 = \Delta F_0 = \Delta F_1 = 0$, i.e., $u(0) = 0$. This implies that the dealer is better off setting contracting terms that expose them to a run as long as the profits in the good state are positive and $\theta^* < 1$

ity of a collateral run. In many global games applications, the run threshold can be derived in closed-form using a condition similar to (26). Given the complexity herein, we are not able to solve for the threshold in closed-form to substitute intp the dealer's problem.[31] Instead, we will explicitly impose (26) as a constraint that the dealer faces and have her optimize also over θ^*, respecting its relationship with the other contract terms.

Hence, the dealer chooses $\{\Delta m_0, \Delta m_1, \Delta F_0, \Delta F_1, \theta^*\}$ to maximize (27) subject to hedge funds' period-0 participation constraint (18), the global game constraint (26), the positive liquidity injection constraint (1), and the bad-state default constraint (7) (see the proof of Proposition 2 for the Lagrangian of the dealer's problem and the first-order optimality conditions, which determine the equilibrium of the model).[32] We have imposed the last constraint in the dealer's problem to guarantee the existence of a lower dominance region, which is essential for the existence of a threshold equilibrium in the incomplete information game. We will elaborate further on the presence of these four constraints below, after we have established the existence of contract terms that give rise to a coordination problem and, hence, the possibility of a collateral run.[33]

Proposition 2. *For $\lambda R^U > 2$, $R^D < \eta R^U/(\eta + R^U)$, and dealer's risk-aversion sufficiently high enough, there exist optimal contracting terms $\Delta m_t(\theta^*)$ and $\Delta F_t(\theta^*)$ under which hedge funds adopt a threshold strategy θ^*.*

Note that the existence result in Proposition 2 requires a high degree of dealer risk aversion so that that the marginal utility of the dealer is low enough to push θ^* below its upper bound θ^{UD}. In other words, conditional on survival at θ^{UD} the dealer would prefer a lower run probability over higher profits in the good state. As we will see later on, this can be true when the dealer is risk-neutral, but under stricter parameter conditions.

The optimal contracting terms of Proposition 2 are only a function of the threshold θ^* and given by $\Delta m_0(\theta^*) = -\theta^*(\eta - 1)\mu_I/h(\theta^*)$, $\Delta m_1(\theta^*) = g(\theta^*)\Delta m_0(\theta^*)$, $\Delta F_0(\theta^*) =$

[31] A closed-form solution is attainable in Goldstein and Pauzner (2005) because the liquidation value does not depend on fundamentals. In Rochet and Vives (2004), the liquidation value depends on fundamentals, but the payoff structure does not, allowing for a closed-form solution.

[32] Alternatively, the problem can be stated with θ^* determined implicitly, and the dealer internalizing how contracting terms change the threshold. These approaches are mathematically equivalent.

[33] The participation constraint in period one is always slack from Corollary 1, and the contract terms will be interior. Thus, for the sake of conciseness, we do not include (19), $0 \leq \Delta m_t \leq 1$ and $-1 \leq \Delta F_t \leq 0$ in the dealer's optimization problem. The Lagrangian and the first-order optimization conditions are reported in (A.18)-(A.23) in Appendix A.

$-g(\theta^*)\Delta m_0(\theta^*)$, and $\Delta F_1(\theta^*) = -R^D \Delta m_0(\theta^*)$, where $g(\theta^*)$ and $h(\theta^*)$ are given by (20) and (A.25), respectively. Finally, the run threshold is the solution to the following optimization condition:

$$\frac{1}{2}(1-\theta^{*2})u'((R^U - R^D)\Delta m_0(\theta^*))(R^U - R^D) + \frac{\theta^* u\left((R^U - R^D)\Delta m_0(\theta^*)\right) h(\theta^*)}{\frac{\partial V}{\partial \theta^*} - \Delta m_0(\theta^*) g'(\theta^*)\left(\frac{\partial V}{\partial \Delta F_0} - \frac{\partial V}{\partial \Delta m_1}\right)} = 0, \tag{28}$$

which can be easily interpreted. The first term captures the incremental utility to the dealer from an increase in the initial risky investment while keeping the run probability unchanged. The second term captures the effect on the probability that the dealer suffers a run, and, hence, forfeits any profits in the final period (note that the term is negative). So, the dealer balances the higher profits conditional on a run not occurring, with the associated increase in the probability of a run.

The optimal contracting terms of Proposition 2 are characterized by four binding constraints: the global game constraint, the initial participation constraint, the positive liquidity injection constraint, and the dealer default constraint.[34] The last three constraints allow us to write the contract terms ΔF_0, Δm_1, and ΔF_1 as function of Δm_0 and θ^*. The global game constraint gives Δm_0 as a function of θ^*. The binding initial participation constraint relates the initial margin with the initial repurchase price, and implies that an individual hedge fund is indifferent between participating initially or waiting to participate in $t = 1$. We, now, discuss in more detail the intuition behind the other two binding constraints.

The binding liquidity injection constraint implies that the dealer does not collect any net funds in $t = 1$. The intuition behind the tightness of this restriction stems from inspecting θ^*'s sensitivity to both Δm_1 and ΔF_0. To see this, consider contracting terms which do not have a binding PC_0 constraint. In that case, the only difference between Δm_1 and ΔF_0 is how those variables affect θ^*. In the proposed equilibrium we have, $\partial\theta^*/\partial\Delta m_1 - \partial\theta^*/\partial\Delta F_0 > 0$.[35]

[34] In a model extension where the dealer can default for reasons outside of the rehypothecation process, a threshold equilibrium exists even when the default condition is slack. In that case, we can show that the contracting term ΔF_1 is an interior solution to the dealer's optimization problem rather than being pinned down by the binding default constraint. In this version of the model all other contracting terms are determined as in the original model. Given that this extension does not add additional intuition for our mechanism we have decided to omit its exposition.

[35] From the implicit function theorem $\partial\theta^*/\partial\Delta m_1 - \partial\theta^*/\partial\Delta F_0 = (\partial V/\partial\Delta F_0 - \partial V/\partial\Delta m_1)(\partial V/\partial\theta^*)^{-1}$. $\partial V/\partial\Delta F_0 - \partial V/\partial\Delta m_1$ pins down the value of the multiplier on the participation constraint and thus is

Intuitively this is because a hedge fund loss due to an increase in Δm_1 only affects hedge funds that roll over. A hedge fund loss due to an increase in ΔF_0 is mutualized between those that roll over and those that do not. That is, the dealer gets more "bang for its buck" by altering the roll over haircut. Thus, to increase the possibility for hedge funds to rollover (i.e., lower θ^*), it is optimal to reduce hedge funds' loss via a reduction in Δm_1 rather than ΔF_0, which in equilibrium is pinned down by the initial participation constraint.

The final active constraint is the dealer's default condition. The intuition behind this active constraint is that an increase in ΔF_1 affects the dealer's payoff directly with a minimal impact on the sensitivity of θ^*: It has a small impact on μ_I, and only affects the final repayment of hedge funds' that roll over directly, with no effect on the incentives *between* rolling over and withdrawing. This is why in equilibrium the dealer decides to set contracting terms such that they default in the bad state.

Finally, it is important to note that not extracting any liquidity in $t = 0$ is a feasibly strategy, yet the optimal contracting terms characterized in Proposition 2 call for $\Delta m_0 > 0$. This implies that the dealer is willing to expose itself to the possibility of a run in order to gain exposure to the risky project. One could consider more sophisticated strategies, like for example investing a fraction of the windfall in the risky project and holding the remaining in the safe asset to safeguard for possible withdrawals. But as long as the value of the risky project is sufficiently low in the bad state (small R^D), to ensure the existence of a lower dominance region, any small positive allocation to the risky project would generate a coordination problem amongst hedge funds, regardless of the additional liquidity the dealer may have to meet withdrawals. That is, the main mechanism behind the model still holds.[36]

Having a general characterization of the equilibrium in Proposition 2 we focus on a specific case that gives a more precise characterization of the equilibrium outcome, and also allows us to do comparative statics. Specifically, we shall assume that the risky project's payoff is zero in the down state $R^D = 0$ and the dealer is risk neutral. In this case, we have the following result,

Corollary 2. *For $R^D = 0$, $\lambda R^U \in \left(2, \frac{4+8\sqrt{2}}{7}\right)$, and risk neutral dealer, there exist optimal*

positive. Given that $\partial V/\partial\theta^* > 0$ from the proof of Proposition 1, $\partial\theta^*/\partial\Delta m_1 - \partial\theta^*/\partial\Delta F_0$ is positive as well.

[36]We discuss a policy prescription that would force a dealer to safely store the windfall in section 6.2.

contracting terms

$$\Delta m_0(\theta^*) = \frac{(\eta-1)\theta^*}{\eta g(\theta^*)\left(1-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right)}, \quad \Delta m_1(\theta^*) = g(\theta^*)\Delta m_0(\theta^*)$$
$$\Delta F_0(\theta^*) = -g(\theta^*)\Delta m_0(\theta^*), \qquad \Delta F_1(\theta^*) = 0$$

under which hedge funds adopt a threshold strategy θ^* *that solves,*

$$(1+\theta^*)\left[(1-\theta^*) - \theta^*\left(1-\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right]\left(2-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right) - 2\theta^{*2}\left(1-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right) = 0$$

with $\frac{\partial\theta^*}{\partial\lambda R^U} > 0$.

The optimal contracting terms have the same functional form as in Proposition 2 with many of the expressions simplified because in this case $\mu_I = \mu_R$. Because $R^D = 0$, the no default condition can be replaced by the reasonable restriction of having the $\Delta F_1 \leq 0$. If this were not the case, the dealer would never default.

Note that the partial derivative is with respect to λR^U, that is the comparative statics with respect to R^U, but these are identical to the ones with respect to λ.[37] As the asset becomes more valuable at liquidation for all realizations of θ, either due to a higher payment on the good state or a lower liquidity discount, the possibility of a run becomes higher. This result may seem counterintuitive, but it is not that surprising within bank-run models featuring one-sided strategic complementarities. For *given* contract terms, a higher liquidation value increases the payoff from running conditional on a collateral run occurring, while at the same time it decreases the region of withdrawals that the run occurs. The former effects dominates the later in this special case of our model resulting in higher θ^* all else equal.[38] The dealer facing a more unfavorable run risk/liquidity windfall tradeoff can respond by reducing the liquidity windfall and risky project holdings in order to mitigate the increase in run risk. But, as we show, the first order effect on θ^* is not undone and θ^* is higher under the new equilibrium contract terms.

Finally, changes in η do not affect the equilibrium threshold, but matter for the optimal

[37] With dealer risk aversion this would not be the case, because the dealer's marginal utility is affected by the final payoff (see equation (28)).

[38] This can be seen by totally differentiating V for this special case and evaluating $d\theta^*/d\lambda R^U = -[(\partial V/\partial\lambda R^U)/(\partial V/\partial\theta)]|_{\theta=\theta^*} > 0$, because $(\partial V/\partial\lambda R^U)|_{\theta=\theta^*} < 0$ and $(\partial V/\partial\theta)_{\theta=\theta^*} > 0$.

contracting terms. Hedge funds are willing to accept a higher haircut (or, equivalently, Δm_0) for given θ^* if the benefits from leveraging (η) are higher. Because the payoff in the bad state is zero, Δm_0 depends proportionally on the relative marginal no-run and run expected gains with the leverage gain, $(\eta - 1)/\eta$ being the degree of proportionality. A risk-neutral dealer also benefits from a higher η, but the tradeoff she faces is between the marginal no-run and run expected gains which is independent of the leverage gain. Thus, the equilbrium θ^* is independent of η.

6 Policy Analysis

The evidence provided in the Section 1 is suggestive that during the 2007–09 financial crisis some firms relied on repo intermediation as a source of liquidity. In addition, Gorton and Metrick (2012) provide evidence that haircuts in the bilateral repo market increased, while Krishnamurthy et al. (2014) and Copeland et al. (2014) show stable haircuts in the tri-party market. This is consistent with the idea that dealers sourced liquidity by lending less than what they received when intermediating collateral.[39] Infante (2019) also provides evidence suggestive of dealers' ability to extract a liquidity windfalls from repo intermediation, and shows that this extraction can be sizable. In this paper, we characterize how this activity can generate a coordination failures amongst cash borrowers and introduce a new source of fragility. These observations raise two natural policy questions: how can regulators monitor and identify the risks of a collateral run, and what regulatory framework might reduce the risk of these runs to materialize? To address these questions it is useful to remember three necessary ingredients for a collateral run: to rehypothecate collateral (i.e., to create a collateral liability), set different contracting terms between borrowers and lenders, and reinvest cash windfalls into risky investments.

6.1 Monitoring and Identifying Collateral Runs

The three ingredients for a collateral run suggest that the risk is higher for firms with some degree of market power that rehypothecate large amounts of securities between different

[39] Although Gorton and Metrick focus on repos backed by riskier collateral classes, the increase in haircuts they document would correspond to a windfall for dealers providing funds through repo intermediation.

counterparties. To monitor these risks it would be important to measure the degree of overcollateralization of their secured borrowing and lending for each collateral class. If differences in these contracting terms correspond to a significant part of their total secured funding—much like the calculation in Figure 2—then the exposure of collateral providers would be relatively high, increasing the risk of a run.

Nevertheless, in the context of rehypothecation, it would be challenging to distinguish a collateral run from a more traditional repo run, as the equilibrium outcome is an abrupt reduction of both borrowing and lending. Haircut dynamics leading up to the run event may be informative. The theoretical framework in Infante (2019) suggests that haircuts to cash lenders are collateral specific while haircuts to collateral providers depend on the dealer–borrower relationship. Moreover, in this paper collateral runs involves safe collateral, which are likely to hold have more stable cash lending haircuts, as uncertainty over the collateral's value is relatively low.[40] Thus, a dealer engaged in repo intermediation is more likely suffer from a collateral run we observe an increase in reverse repo haircuts for relatively safe collateral across all counterparty types, along with relatively stable repo haircuts.

6.2 Regulatory Framework

Since the 2007–09 financial crisis there have been numerous proposals to make the repo market more resilient. The vast majority have focused on the liability side of the balance sheet, and thus, their impact on the risk of a collateral run are unclear. For example, the Financial Stability Board (FSB) has proposed minimum haircuts requirements, i.e., "haircut floors", to limit the amount of leverage a cash borrower can take in a single repo transactions.[41] In this section we analyse the impact of this specific policy proposal and show that in our model haircut floors make collateral runs more likely. We then discuss alternative policy prescriptions that may directly address the risks of a collateral run.

[40]Copeland et al. (2014) show that during the crisis tri-party haircuts for government collateral were less volatile than for private collateral, and of similar magnitude when compared to stable times.

[41]See the FSB, "Strengthening Oversight and Regulation of Shadow Banking: Regulatory framework for haircuts on non-centrally cleared securities financing transactions," October 14, 2014, http://www.fsb.org/wp-content/uploads/r_141013a.pdf. Note, however, that the overcollateralization of secured positions can increase run risk from unsecured claimants, which has been pointed out by Ahnert et al. (2018).

6.2.1 Haircut Floors

The introduction of haircut floors is directly aimed at limiting the risks borne by cash providers. We show that even under assumptions that should alleviate the risks borne by collateral providers, haircut floors increase the risk of a collateral run. Specifically, for the analysis we have to take a stand as to what happens with the money funds overcollateralization in case of a dealer default. Current repo contracting conventions stipulate that upon a default event, if the underlying collateral is valuable enough to make the lender's claim whole, then the lender must return the excess funds to the defaulted borrower. In the context of rehypothecation, it is unclear as to whether these excess funds would be pooled with the rest of the borrowers defaulted assets, immediately returned to the collateral provider, or someway split between these two extreme alternatives. In this section we assume that in case of a dealer default the hedge fund immediately recieves its corresponding money fund's overcollateralization. This resolution assumption would be the safest option for collateral providers and gives rise to the following proposition.

Proposition 3. *For $R^D = 0$, $\lambda R^U \in \left(2, \frac{4+8\sqrt{2}}{7}\right)$, and a risk neutral dealer, if haircuts have to satisfy $m_t^M, m_t^H \geq \underline{m} > 0$ and money funds' overcollateralization are directly returned to hedge funds after a dealer default, then, for $\underline{m}$ sufficiently small, there exist optimal contracting terms*

$$\begin{aligned} \Delta m_0(\theta^*) &= \frac{(\eta-1)(\theta^*-\underline{m})}{\eta\lambda g(\theta)\left(1-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right)}, \quad & \Delta m_1(\theta^*) &= g(\theta^*)\Delta m_0(\theta^*) \\ \Delta F_0(\theta^*) &= -g(\theta^*)\Delta m_0(\theta^*), & \Delta F_1(\theta^*) &= 0 \end{aligned}$$

under which hedge funds adopt a threshold strategy θ^* that solves,

$$\begin{aligned} & (1+\theta^*)\left[(1-\theta^*) - \theta^*\left(1 - \frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right]\left(2 - \ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right) - 2\theta^{*2}\left(1 - \ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right) \\ = \quad & -\left(\frac{\underline{m}}{\theta - \underline{m}}\right)(1-\theta^2)\left(1 - \ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right) \end{aligned}$$

with $\frac{\partial\theta^*}{\partial\underline{m}} > 0$.

Proposition 3 shows that even if the money fund's overcollateralization is directly returned to each individual hedge fund, an increase in minimum haircuts makes collateral runs more likely. The intuition is that haircut floors increase hedge funds' incentives to run because they receive the money fund's haircut earlier, allowing them to invest in Treasuries which they value more than cash. On the contrary, overcollaterlization constitutes an opportunity cost absent a run because it is remains idle. Thus, haircut floors would increase hedge funds' incentives to withdraw. It is easy to imagine that any other bankruptcy regimes that pools a fraction of money funds overcollateralization would increase hedge funds incentives to run even more, making collateral runs more likely.

6.2.2 Restrictions to Rehypothecation

Regulations that limit rehypothecation activity are more suitable to reduce the risk of a collateral run. The three necessary ingredients we identify for a collateral run are instructive to consider what type of regulation may be more effective. The first is to limit rehypothecation in the altogether. By doing so dealers would not be able to reap any windfall from differences in contracting terms, and pledged collateral would be easier to seize in bankruptcy. The second is to restrict dealers' use of said windfall. By safely storing differences in contracting terms, collateral providers exposures would still be pooled, but there would be enough funds to the dealer to recuperate all rehypothecatd collateral, regardless of what others do.

Naturally, taken to an extreme, these policy prescription would eliminate a collateral run altogether. In fact, versions of these prescriptions have been adopted in other contexts. For example, the Securities Exchange Act Rule 15c3-3 allows brokers to rehypothecate up to 140% of a customer's total loan balance, and prohibits brokers from financing their own activity with clients assets, that is, use the windfall for their own purposes. Currently, this rule do not apply to repo because repo counterparties are not considered to be "clients", but rather as investors who entrust their securities to a dealer.[42]

However, this type of regulation would have important repercussions in repo and other markets. In effect, the ability to use and reuse securities using repo is said to be crucial for market functioning.[43] Although introduction limits to rehypothecation would alleviate

[42]See Mitchell and Pulvino (2012) for a description of broker limits on client asset use and Infante (2019) for how these rules apply to repo.

[43]See the FSB. "Re-hypothecation and collateral re-use: Potential financial stability issues, market evolution and regulatory approaches" January 25, 2017, `http://www.fsb.org/wp-content/uploads/Re-hypothecation-and-collateral-re-use.pdf`. See Infante et al. (2018) for the role of repo in collateral reuse.

the financial stability concerns that come with it, the overall welfare implications due to a decrease in aggregate secured funding and/or market liquidity for the underlying collateral are unclear. Future work in needed to address the overall costs and benefits of limiting rehypothecation.

7 Conclusion

This paper presents a model which highlights fragility that can arise from the re-use of collateral in a short-term collateralized lending context. Specifically, this paper formalizes the idea of a coordination failure that can arise amongst cash borrowers, inducing a panic-based default on an intermediary. In contrast to traditional wholesale funding runs, the model shows that when intermediating secured financing fragility can materialize on the asset side of a dealer's balance sheet. The model delivers a unique threshold equilibria in which panic-based runs can ensue. In addition, the paper shows how different repo contracting terms, specifically the haircut and repurchase price, can have differential effects on collateral providers' incentives to run. Namely, when rolling over short-term contracts new haircuts affect those choosing to roll over, whereas existing repurchasase prices affect all those that participated initially. This provides another mechanism in which to disentangle different repo contracting terms.

The results in this paper also pose a challenge for regulators concerned with the fragility of large broker dealers. Much of the regulation introduced since the 2007–09 is designed to monitor and restrict the repo contracting terms to avoid runs from the liability side. This paper cautions that this focus is too narrow. Given that the total amount of collateral received is an important source of liquidity, fragility can present itself on the asset side of the balance sheet, as well.

Policy prescriptions that could address the source of fragility studied in this paper might be to restrict the amount of over-collateralization in dealers' rehypothecation activity, which effectively limits the cash windfall dealers are able to extract, or to restrict dealers' reinvestment of said cash windfall. This type of policy intervention can be implemented using existing rules that limit rehypothecation in other contexts, but the overall impact must also balance the effect these rules may have on market functioning. These are important areas of future research.

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Appendices

A Proofs

Proof of Lemma 1

For values of θ such that a run of the dealer is possible, i.e., $\mu_R < 1$, or equivalently $\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0 < 0$, the default threshold, μ_I, is always lower than the run threshold, μ_R, if $\lambda \overline{R}_\theta \Delta F_1 + R^U \Delta m_1 > 0$. The latter expression can be written as $\lambda \overline{R}_\theta \Delta F_1 + R^U \Delta m_1 > (R^U \Delta m_0 \Delta m_1 - \Delta F_0 \Delta F_1)/\Delta m_0$ using the fact that $\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0 < 0$ and $\Delta F_0, \Delta F_1 < 0$. Given that $\Delta m_1 \geq -\Delta F_0$ from condition $C0$ in (1), $R^U \Delta m_0 \Delta m_1 - \Delta F_0 \Delta F_1 > -\Delta F_0 (R^U \Delta m_0 + \Delta F_1) > 0$, using condition $C2$ in (8). For $\Delta F_1 = 0$, $\mu_I = \mu_R$.

Finally, to ensure that the dealer will begin to sell before it comes insolvent, i.e., $\mu_S < \mu_I$, we also need $R^U \Delta m_1 \Delta m_0 - \Delta F_1 \Delta F_0 > 0$, which we proved above.

Proof of Lemma 2

From (18), and using $T = F_0^M$ and m_0^M from the money funds problem, we get that:

$$-\Delta F_0 \left[\int_{\theta^*}^{1} d\theta + \int_0^{\theta^*} f(1,\theta) d\theta \right] \geq \Delta m_0 \quad \Rightarrow \quad -\Delta F_0 > \Delta m_0, \tag{A.1}$$

because $\int_{\theta^*}^{1} d\theta + \int_0^{\theta^*} f(1,\theta) d\theta < 1$. This proves claim 1.

Combining (A.1) and (1), we get that $\Delta m_1 > \Delta m_0$. This proves claim 2.

Proof of Lemma 3

The lower dominance region is defined by the values of θ for which an individual hedge fund chooses to withdraw even if other hedge funds do not. The utility differential between rolling over and withdrawing when $\mu = 0$ from (12) and (16) is $U^H(0,\theta;1) - U^H(0,\theta;0) = \theta[(\eta - 1)T - \Delta F_1] + (1-\theta)[R^D \Delta m_0 + \Delta F_0 + \Delta m_1] - \eta \Delta m_1$, where we have substituted the equilibrium conditions $F_t^M = T$ and $m_t^M = 0$ derived in section 3.2. Given that the differential is increasing in θ the lower dominance region comprises of values for $\theta \leq \theta^{LD}$, where θ^{LD} is the solution to $U^H(0,\theta^{LD};1) - U^H(0,\theta^{LD};0) = 0$, i.e.,

$$\theta^{LD} = \frac{(\eta - 1)\Delta m_1 - R^D \Delta m_0 - \Delta F_0}{(\eta - 1)T - R^D \Delta m_0 - \Delta F_0 - \Delta m_1 - \Delta F_1}.$$

The lower dominance threshold θ^{LD} is greater than zero because, from Lemma 2 and $R^D < 1$, $R^D \Delta m_0 + \Delta F_0 < (R^D - 1)\Delta m_0 < 0$, and from condition (7) $R^D \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 < 0$. It is also lower than one because $(\eta - 1)T - \Delta F_1 > \eta \Delta m_1$ from the period 1 participation constraint (19).

The upper dominance region is defined by the values of θ for which an individual hedge fund rolls over even if all other hedge funds withdraw. First, we need to guarantee that for $\theta \geq \theta^{UD}$, the dealer has enough liquidity to serve all early withdraws, i.e., $\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 \geq 0$. Given that the last expression is increasing in θ, the upper dominance threshold is the root, i.e.,

$$\theta^{UD} = -\frac{\Delta F_0 + \lambda R^D \Delta m_0}{\lambda(R^U - R^D)\Delta m_0}.$$

The upper dominance threshold θ^{UD} is greater than zero because, from Lemma 2 and $\lambda R^D < 1$, $\Delta F_0 + \lambda R^D \Delta m_0 < (\lambda R^D - 1)\Delta m_0 < 0$. To show that θ^{UD} is lower than one, we impose a binding participation constraint (18), which is always the case *under the equilibrium contract terms.* Then, $\theta^{UD} = (g(\theta^*) - \lambda R^D)/(\lambda R^U - \lambda R^D)$, where $g(\theta^*)$ is given by (20), and is smaller than one if $g(\theta^*) < \lambda R^U$. Substituting the expression for $g(\theta^*)$, the latter condition can be written as $\mathcal{X}(\theta^*) \equiv \lambda(R^U - R^D)\theta^{*2} - 2\lambda(R^U - R^D)\theta^* + 2(\lambda R^U - 1) > 0$. $\mathcal{X}$ does not have real roots because the discriminant $4\lambda(R^U - R^D)\left[\lambda(R^U - R^D) - 2(\lambda R^U - 1)\right] < 0$ given the restriction $\lambda(R^U + R^D)/2 > 1$.

We have shown that for $\theta \geq \theta UD$, the hedge fund believes the liquidation price is high enough to avoid a liquidity default. Now we must ensure that the hedge fund in fact wants to roll over. To conclude the proof, we need to show that for $\theta \geq \theta^{UD}$ and $\mu \to 1$, the utility differential between rolling and withdrawing for an individual hedge fund is positive. Technically, we need to check whether an infinitesimal hedge fund with mass ε that deviates from the strategy of other fund that withdraw can repurchase its collateral at $t = 2$. In other words, we need to check whether the dealer default on the remaining $-\varepsilon\Delta F_1$ obligations when $\varepsilon \to 0$. The dealer will not default if the value of her remaining asset is higher that her remaining obligations, i.e., $\lim_{\mu\to 1} G_I^U(\mu,\theta)/(-\Delta F_1) > 1$ with $G_I^U(\mu,\theta)$ given by (10). Changing variables such that $\mu = 1 - \varepsilon$ and substituting $\lambda\overline{R}_\theta \Delta m_0 = \lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Omega(\theta)\Delta m_0$, where $\Omega(\theta) \geq 0$ because $\theta \geq \theta^{UD}$, we get that:

$$\begin{aligned}
\lim_{\varepsilon\to 0} \frac{G_I^U(1-\varepsilon,\theta)}{-\Delta F_1} &= -\frac{R^U}{\lambda\overline{R}_\theta} \lim_{\varepsilon\to 0} \frac{\lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Omega(\theta)\Delta m_0 + \Delta F_0 + \varepsilon\Delta m_1}{\varepsilon\Delta F_1} \\
&= -\frac{R^U}{\lambda\overline{R}_\theta}\frac{\Delta m_1}{\Delta F_1} + \frac{\Omega(\theta)\Delta m_0}{\Delta F_1} \lim_{\varepsilon\to 0}\frac{1}{\varepsilon},
\end{aligned}$$

where we used the fact that $\lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Delta F_0 = 0$ from the definition of θ^{UD}. Given that $lim_{\varepsilon\to 0}\frac{\Omega(\theta)\Delta m_0}{\varepsilon\Delta F_1} \to \infty$ for $\theta > \theta^{UD}$, it suffices to show that for $\theta = \theta^{UD}$, i.e., $\Omega(\theta) = 0$ irrespective of the value of ε, the limit converges to a value higher than 1. Using $\lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Delta F_0 = 0$, we get that the limit converges to $-(R^U\Delta m_1/(\lambda\overline{R}_{\theta^{UD}}\Delta F_1) = R^U\Delta m_0\Delta m_1/(\Delta F_0\Delta F_1)$, which is greater than 1 because, as proved in Lemma 1, $R^U\Delta m_0\Delta m_1 - \Delta F_0\Delta F_1 > 0$. Hence, the dealer will not default at $t = 2$ if R^U realizes and the hedge fund will be able to repurchase its collateral.

If, instead R^D realizes at $t = 2$, the limit is

$$\lim_{\varepsilon\to 0} \frac{G_I^U(1-\varepsilon,\theta)}{-\Delta F_1} = -\frac{R^D}{\lambda\overline{R}_\theta}\frac{\Delta m_1}{\Delta F_1} + \frac{\Omega(\theta)\Delta m_0}{\Delta F_1} \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}.$$

Again, for the $\theta > \theta^{UD}$, the second term goes to infinity and the dealer does not default. But, for $\theta = \theta^{UD}$, the limit goes to $-(R^D\Delta m_1/(\lambda\overline{R}_{\theta^{UD}}\Delta F_1) = R^D\Delta m_0\Delta m_1/(\Delta F_0\Delta F_1)$. Using conditions (1) and (7) we get that $R^D\Delta m_0 + \Delta F_1 < 0$ and, hence, the limit is higher than 1 for $\Delta m_1 > \Delta F_0\Delta F_1/(R^D\Delta m_0) > -\Delta F_0$

and less than one otherwise. If the former case, the dealer does not default and the hedge fund is able to repurchase its collateral. In the latter case, the dealer defaults and the hedge fund gets $-R^D \Delta m_1 \Delta m_0 / \Delta F_0$.

In most cases described above the dealer does not default either in the good or in the bad state and it is straightforward that the hedge fund's period 1 participation constraint (19) is satisfied. Thus, the hedge fund rolls over its collateral. However, for the special cases that $\theta = \theta^{UD}$ and $\Delta m_1 \in [-\Delta F_0, \Delta F_0 \Delta F_1/(R^D \Delta m_0)]$, the dealer defaults in the bad state. A sufficient condition such that the period 1 participation constraint is satisfied is $-R^D \Delta m_1 \Delta m_0/\Delta F_0 \geq G_S^D(0, \theta) = R^D \Delta m_0 + \Delta F_0 + \Delta m_1$. For the lower bound on $\Delta m_1 = -\Delta F_0$ the latter relationship becomes $R^D \Delta m_0 \geq R^D \Delta m_0$, while for the upper bound on $\Delta m_1 = \Delta F_0 \Delta F_1/(R^D \Delta m_0)$ it becomes $\Delta F_1 < R^D \Delta m_0 + \Delta F_0 + \Delta F_1$, which is always true because of condition (7). Given that both sides of the inequality are monotonically increasing in Δm_1, we conclude that the repayment is the case of default is higher than $G_S^D(0, \theta)$ and, thus, the participation constraint is satisfied.

Note that the participation constraint (19) holds for $\theta \geq \theta^*$, while in all the aforementioned cases the utility differential is computed for $\theta^{UD} > \theta^*$. Hence, the participation constraint is easily satisfied for $\theta \geq \theta^{UD}$ and an individual hedge fund will always roll over even if all other hedge fund withdraw.

Proof of Proposition 1

The proof follows Goldstein and Pauzner (2005), but introduces additional steps and derivations due to the complexity accruing from the limited liability of the dealer and the fact that the liquidation value of the risky project depends on θ.

An equilibrium with threshold x^* exists only if $\Delta(x^*, x^*) = 0$ given by (25). Consider a potential threshold x'. We will show that x' exists and it satisfies (25) at exactly one point, $\xi' = \xi^*$.

By the existence of θ^{LD} and θ^{UD} in Lemma 3, $\Delta(x', x')$ is negative for $x' < \theta^{LD} - \epsilon$ and positive for $x' > \theta^{UD} + \epsilon$. Thus, it suffices to show that $\Delta(x', x')$ is continuous in x' to establish that a threshold equilibrium exists. It is convenient to write the utility differential $\Delta(x', x')$ as $\Delta(\hat{x} + \Delta x, \hat{x} + \Delta x)$ for some $\hat{x}$ such that Δx is the change in both the signal that the marginal hedge fund receives and the threshold strategy. Then,

$$
\begin{aligned}
\Delta(\hat{x} + \Delta x, \hat{x} + \Delta x) &= \frac{1}{2\epsilon} \int_{\hat{x}+\Delta x-\epsilon}^{\hat{x}+\Delta x+\epsilon} \nu(\mu(\theta, \hat{x} + \Delta x), \theta) d\theta \\
&= \frac{1}{2\epsilon} \int_{\hat{x}-\epsilon}^{\hat{x}+\epsilon} \nu(\mu(\theta + \Delta x, \hat{x} + \Delta x), \theta + \Delta x) d\theta \\
&= \frac{1}{2\epsilon} \int_{\hat{x}-\epsilon}^{\hat{x}+\epsilon} \nu(\mu(\theta, \hat{x}), \theta + \Delta x) d\theta, \qquad \text{(A.2)}
\end{aligned}
$$

because $\mu(\theta + \Delta x, \hat{x} + \Delta x) = \mu(\theta, \hat{x})$ from (21). In other words, the marginal hedge fund's belief about how many other hedge funds withdraw is unchanged when its private signal and the threshold strategy change by the same amount. Yet, it expects the θ to be higher for $\Delta x > 0$ and lower for $\Delta x < 0$ which is reflected in the calculation of $\nu(\mu(\theta, \hat{x}), x + \Delta x)$. Thus, we need to show that for a given distribution of $\mu' s$ the integral in (A.2) is continuous in Δx.

The integrand $\nu(\mu(\theta, \hat{x}), \theta+\Delta x)$ in (A.2) is a piecewise function such that each sub-function is computed over a distribution of μ unaffected by Δx, but the interval for each sub-function depends on Δx apart from $\mu \in [0, \mu_S)$ in (2). In other words, μ_I and μ_R given by (11) and (3), respectively, move with $\theta+\Delta x$. Note that θ always lies between $\hat{x}-\epsilon$ and $\hat{x}+\epsilon$, hence only Δx will matter. Given that the distribution of μ is unchanged, we can compute the levels of threshold $\theta's$ as functions of Δx, such that $\mu(\theta_{\mu_S}(\Delta x), \hat{x}) = \mu_S(\theta_{\mu_S}(\Delta x)+\Delta x)$, $\mu(\theta_{\mu_I}(\Delta x), \hat{x}) = \mu_I(\theta_{\mu_I}(\Delta x) + \Delta x)$ and $\mu(\theta_{\mu_R}(\Delta x), \hat{x}) = \mu_R(\theta_{\mu_R}(\Delta x) + \Delta x)$ as follows:[44]

$$\begin{aligned} &\mu(\theta_{\mu_S}(\Delta x), \hat{x}) = \mu_S(\theta_{\mu_S}(\Delta x) + \Delta x) \\ \Rightarrow &\frac{\hat{x} - \theta_{\mu_S}(\Delta x) + \epsilon}{2\epsilon} = 1 + \frac{\Delta F_0}{\Delta m_1}, \end{aligned} \tag{A.3}$$

$$\begin{aligned} &\mu(\theta_{\mu_I}(\Delta x), \hat{x}) = \mu_I(\theta_{\mu_I}(\Delta x) + \Delta x) \\ \Rightarrow &\frac{\hat{x} - \theta_{\mu_I}(\Delta x) + \epsilon}{2\epsilon} = 1 + \frac{R^U(\Delta F_0 + \lambda \overline{R}_{\theta_{\mu_I}(\Delta x)+\Delta x} \Delta m_0)}{\lambda \overline{R}_{\theta_{\mu_I}(\Delta x)+\Delta x} \Delta F_1 + R^U \Delta m_1}, \end{aligned} \tag{A.4}$$

$$\begin{aligned} &\mu(\theta_{\mu_R}(\Delta x), \hat{x}) = \mu_R(\theta_{\mu_R}(\Delta x) + \Delta x) \\ \Rightarrow &\frac{\hat{x} - \theta_{\mu_R}(\Delta x) + \epsilon}{2\epsilon} = 1 + \frac{\Delta F_0 + \lambda \overline{R}_{\theta_{\mu_R}(\Delta x)+\Delta x} \Delta m_0}{\Delta m_1}. \end{aligned} \tag{A.5}$$

Because the number of hedge funds withdrawing decreases as fundamentals improve for given strategy threshold (see equation (21)), we have $\theta_{\mu_R}(\Delta x) < \theta_{\mu_I}(\Delta x) < \theta_{\mu_S}(\Delta x)$, which is the reverse ordering of μ_S, μ_I and μ_R from Lemma 1. Thus, using (23), (A.2) can be written as:

$$\begin{aligned} &\Delta(\hat{x} + \Delta x, \hat{x} + \Delta x) = \\ &\quad \frac{1}{2\epsilon} \int_{\hat{\mathrm{x}}-\epsilon}^{\theta_{\mu_\mathrm{R}}(\Delta \mathrm{x})} -\eta \frac{\lambda \overline{\mathrm{R}}_{\theta+\Delta \mathrm{x}} \Delta \mathrm{m}_0 + \Delta \mathrm{F}_0 + \Delta \mathrm{m}_1}{\mu(\theta, \hat{\mathrm{x}})} \mathrm{d}\theta \\ &\quad + \frac{1}{2\epsilon} \int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)} \left[(\theta + \Delta x) G_I^U(\mu(\theta, \hat{x}), \theta + \Delta x) + (1 - (\theta + \Delta x)) G_I^D(\mu(\theta, \hat{x}), \theta + \Delta x) - \eta \Delta m_1 \right] d\theta \\ &\quad + \frac{1}{2\epsilon} \int_{\theta_{\mu_I}(\Delta x)}^{\theta_{\mu_S}(\Delta x)} \left[(\theta + \Delta x)[(\eta - 1)T - \Delta F_1] + (1 - (\theta + \Delta x)) G_I^D(\mu(\theta, \hat{x}), \theta + \Delta x) - \eta \Delta m_1 \right] d\theta \\ &\quad + \frac{1}{2\epsilon} \int_{\theta_{\mu_S}(\Delta x)}^{\hat{x}+\epsilon} \left[(\theta + \Delta x)[(\eta - 1)T - \Delta F_1] + (1 - (\theta + \Delta x)) G_S^D(\mu(\theta, \hat{x}), \theta + \Delta x) - \eta \Delta m_1 \right] d\theta. \end{aligned} \tag{A.6}$$

[44]Note that for θ^* and x^*, then $\mu(\theta^*, x^*) = \mu_R(\theta^*)$, which yields (22).

Then, $\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)$ in (A.6) is continuous in Δx, because all the integrands are bounded and continuous, θ_{μ_I} and θ_{μ_R} change continuously with Δx from (A.4) and (A.5) (θ_{μ_S} doesn't move from (A.3)), and the discontinuity in ν occurs only at one discrete point, θ_{μ_I}. Hence, a threshold equilibrium exists.

We will now establish that the threshold equilibrium is unique. By implicitly differentiating (A.4) and (A.5) we get:

$$\frac{d\theta_{\mu_I}(\Delta x)}{d\Delta x} = -\frac{2\epsilon\Gamma_{\theta_{\mu_I}}(R^U - R^D)}{1 + 2\epsilon\Gamma_{\theta_{\mu_I}}(R^U - R^D)} < 0 \tag{A.7}$$

because $\Gamma_{\theta_{\mu_I}} \equiv \lambda R^U(R^U\Delta m_0\Delta m_1 - \Delta F_0\Delta F_1)/(\lambda\overline{R}_{\theta_{\mu_I}(\Delta x)+\Delta x}\Delta F_1 + R^U\Delta m_1)^2 > 0$ from Lemma 1, and

$$\frac{d\theta_{\mu_R}(\Delta x)}{d\Delta x} = -\frac{2\epsilon\Gamma_{\theta_{\mu_R}}(R^U - R^D)}{1 + 2\epsilon\Gamma_{\theta_{\mu_R}}(R^U - R^D)} < 0, \tag{A.8}$$

because $\Gamma_{\theta_{\mu_R}} \equiv \lambda\Delta m_0/\Delta m_1 > 0$

The derivative of (A.6) with respect to Δx is:

$$\begin{aligned}
&d\frac{\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)}{d\Delta x} = \\
&-\frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\theta_{\mu_R}(\Delta x)} \eta\frac{\lambda(R^U - R^D)}{\mu(\theta,\hat{x})}d\theta - \frac{1}{2\epsilon}\frac{d\theta_{\mu_I}(\Delta x)}{d\Delta x}(\theta_{\mu_I}(\Delta x) + \Delta x)(\eta - 1)T + \frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)} \frac{R^U - R^D}{1-\mu(\theta,\hat{x})}\Delta m_0 d\theta \\
&+ \frac{1}{2\epsilon}\int_{\theta_{\mu_I}(\Delta x)}^{\theta_{\mu_S}(\Delta x)} \left[(\eta-1)T - \Delta F_1 - G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x) + (1-\theta-\Delta x)\frac{dG_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)}{d\Delta x}\right]d\theta \\
&+ \frac{1}{2\epsilon}\int_{\theta_{\mu_S}(\Delta x)}^{\hat{x}+\epsilon} \left[(\eta-1)T - \Delta F_1 - G_S^D(\mu(\theta,\hat{x}),\theta+\Delta x)\right]d\theta.
\end{aligned} \tag{A.9}$$

The third, fourth and fifth terms in (A.9) are positive – $dG_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)/d\Delta x = -(R^D(R^U - R^D)(\Delta F_0 + (1-\mu)\Delta m_1)/(\lambda(1-\mu)\overline{R}^2_{\theta+\Delta x}) > 0$ in the respective region of fundamentals. The second term in (A.9) represents the utility change from changing the threshold $\theta_{\mu_I}(\Delta x)$ where default occurs and hedge funds forfeit the extra benefit $\eta - 1$ and is positive due to (A.7). However, the first term, which correspond to the change in the range that the run occurs, is negative. In order to establish that the positive terms outweigh the negative term we will evaluate (A.9) at a candidate threshold $\hat{x}$, which we know that exists. If the derivative is positive at candidate threshold, we can conclude that (A.6) does not cross zero from above and, given continuity, the threshold is unique. Using (A.6), we can derive the following lower bound for

(A.9):

$$
\begin{aligned}
&d\frac{\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)}{d\Delta x} > \\
&\frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\theta_{\mu_R}(\Delta x)} \eta\frac{\lambda\overline{R}_{\theta+\Delta x}\Delta m_0+\Delta F_0+\Delta m_1-\lambda(R^U-R^D)\Delta m_0}{\mu(\theta,\hat{x})}d\theta \\
&-\frac{1}{2\epsilon}\frac{d\theta_{\mu_I}(\Delta x)}{d\Delta x}(\theta_{\mu_I}(\Delta x)+\Delta x)(\eta-1)T \\
&+\frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)}\left[\eta\Delta m_1+\frac{R^U-R^D}{1-\mu(\theta,\hat{x})}\Delta m_0-\frac{\overline{R}_{\theta+\Delta x}\Delta m_0+1/\lambda[\Delta F_0+(1-\mu(\theta,\hat{x}))\Delta m_1]}{1-\mu(\theta,\hat{x})}\right]d\theta \\
&+\frac{1}{2\epsilon}\int_{\theta_{\mu_I}(\Delta x)}^{\theta_{\mu_S}(\Delta x)}\left[\eta\Delta m_1-G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)+(1-\theta-\Delta x)\frac{G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)}{d\Delta x}\right]d\theta \\
&+\frac{1}{2\epsilon}\int_{\theta_{\mu_S}(\Delta x)}^{\hat{x}+\epsilon}\left[\eta\Delta m_1-G_S^D(\mu(\theta,\hat{x}),\theta+\Delta x)\right]d\theta. \qquad (A.10)
\end{aligned}
$$

From the lower dominance region, the last three terms on the right-hand side are positive. The second term in also positive as mentioned above. The first term is always positive if $(\hat{x}-\epsilon-1)(R^U-R^D)+R^D>0$, which in general would not be true for low values of R^D and certainly not true for $R^D=0$. Thus, we consider that $(\hat{x}-\epsilon-1)(R^U-R^D)+R^D<0$ and show that the negative part is outweighed by the other positive terms given that noise is not too big. Taking the first and third terms in (A.10) in isolation we obtain:

$$
\begin{aligned}
&\frac{1}{2\epsilon}\left[\left(\theta_{\mu_I(\Delta x)}-\theta_{\mu_R(\Delta x)}\right)\eta\Delta m_1-2\epsilon(\theta_{\mu_R(\Delta x)}-\hat{x}+\epsilon)\eta\lambda\Delta m_0(R^U-R^D)\right]+\Omega \\
=&\eta\left[\left(\mu(\theta_{\mu_R}(\Delta x),\hat{x})-\mu(\theta_{\mu_I}(\Delta x),\hat{x})\right)\Delta m_1-2\epsilon\lambda\Delta m_0(R^U-R^D)\mu(\theta_{\mu_R}(\Delta x),\hat{x})\right]+\Omega, \qquad (A.11)
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega =& \frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)}\left[\frac{(R^U-R^D)\Delta m_0-\overline{R}_{\theta+\Delta x}\Delta m_0-1/\lambda[\Delta F_0+(1-\mu(\theta,\hat{x}))\Delta m_1]}{1-\mu(\theta,\hat{x})}\right]d\theta \\
&+\frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\theta_{\mu_R}(\Delta x)}\eta\frac{\lambda\Delta m_0 R^D+\Delta F_0+\Delta m_1}{\mu(\theta,\hat{x})}d\theta \\
&-\eta\lambda\Delta m_0(R^U-R^D)(1-\hat{x}-\epsilon)\left[1-ln\left(\mu(\theta_{\mu_R}(\Delta x),\hat{x})\right)\right]ln(2\epsilon)>0. \qquad (A.12)
\end{aligned}
$$

(A.11) is positive for small enough noise, satisfying

$$
\epsilon<\epsilon^A\equiv\frac{\left(\mu_R(\theta_{\mu_R}(\Delta x)+\Delta x)-\mu_I(\theta_{\mu_I}(\Delta x)+\Delta x)\right)\Delta m_1}{2\lambda\Delta m_0(R^U-R^D)\mu_R(\theta_{\mu_R}(\Delta x)+\Delta x)}, \qquad (A.13)
$$

using (A.4) and (A.5).

The second and third terms in (A.12) are positive given that $\epsilon < 1/2$. The first term is unambiguously positive if $(1-\theta)(R^U - R^D) - R^D > 0$. Thus, θ should be lower than $(R^U - 2R^D)/(R^U - R^D)$. Given that a threshold equilibrium exists, the threshold for fundamentals θ^* is between the upper and lower dominance regions and, thus, bounded above by a hypothetical threshold $\bar{\theta}$, which solves $\lambda \overline{R}_{\bar{\theta}} - g(\bar{\theta}) = 0$, where $g(\cdot)$ is given by (20). This threshold is given by:

$$\bar{\theta} = 1 - \sqrt{1 - \frac{2(1 - \lambda R^D)}{\lambda(R^U - R^D)}}. \tag{A.14}$$

From (22) the upper bound for the signal threshold is $\bar{\theta} + \epsilon$, and a hedge fund that receives threshold signals believes that θ^* can be at most $\bar{\theta}$. Hence, the first term in (A.12) is positive if

$$\epsilon < \epsilon^B \equiv \frac{1}{2}\left(\frac{R^U - 2R^D}{R^U - R^D} - \bar{\theta}\right), \tag{A.15}$$

where $\epsilon^B > 0$ because $\lambda R^U > 2$. In sum, the threshold equilibrium is uniqueness for $\epsilon < \min(\epsilon^A, \epsilon^B)$ given by (A.13) and (A.13).

To conclude the proof we need to show that the threshold equilibrium is indeed an equilibrium, i.e., $\Delta(x_i, x^*)$ in (24) is positive for all $x_i > x^*$, and negative for all $x_i < x^*$. The steps (and notation) below are the same as in Goldstein and Pauzner (2005).

First, consider that $x_i < x^*$. Then we can decompose the intervals $[x_i - \epsilon, x_i + \epsilon]$ and $[x^* - \epsilon, x^* + \epsilon]$ into a common part $c = [x_i - \epsilon, x_i + \epsilon] \cap [x^* - \epsilon, x^* + \epsilon]$, and two disjoint parts $d^i = [x_i - \epsilon, x_i + \epsilon]\backslash c$ and $d^* = [x^* - \epsilon, x^* + \epsilon]\backslash c$. Thus, (24) and (25) can be written as:

$$\Delta(x_i, x^*) = \frac{1}{2\epsilon}\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta + \frac{1}{2\epsilon}\int_{\theta \in d^i} \nu(\mu(\theta, x^*), \theta) d\theta, \tag{A.16}$$

$$\Delta(x^*, x^*) = \frac{1}{2\epsilon}\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta + \frac{1}{2\epsilon}\int_{\theta \in d^*} \nu(\mu(\theta, x^*), \theta) d\theta. \tag{A.17}$$

From (23), ν is always one over d^i, thus $\int_{\theta \in d^i} \nu(\mu(\theta, x^*), \theta) d\theta < 0$. As a result, it suffices to show that $\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta < 0$. ν only crosses zero once and it is positive for higher values of θ and negative for lower values of θ in the interval $[x^* - \epsilon, x^* + \epsilon]$. Hence, given that (A.17) is zero, we get that $\int_{\theta \in d^*} \nu(\mu(\theta, x^*) > 0$ and $\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta < 0$, since the fundamentals are higher over d^* than c. Essentially, observing a signal x_i below x^* shifts probability from positive values of ν to negative values of ν because noise is uniformly distributed. Note that the argument goes through even if the interval c is empty. The proof for $x_i > x^*$ is similar.

Proof of Proposition 2

In a threshold strategy equilibrium, the dealer's optimization problem has the following Lagrangian,

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(1-\theta^2)u(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)\\ &\quad -\xi_0(\Delta F_0+g(\theta)\Delta m_0)+\xi_{PL}(\Delta m_1+\Delta F_0)-\xi_{DD}(R^D\Delta m_0+\Delta F_0+\Delta m_1+\Delta F_1)+\xi_V V(\theta)\end{aligned} \tag{A.18}$$

Taking the first order conditions with respect to the contract terms and the run threshold gives:

$$\frac{\partial\mathcal{L}}{\partial\Delta m_0}=\frac{1}{2}(1-\theta^2)u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)R^U-\xi_0 g(\theta^*)-\xi_{DD}R^D+\xi_V\frac{\partial V}{\partial\Delta m_0}=0, \tag{A.19}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta m_1}=\frac{1}{2}(1-\theta^2)u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)+\xi_{PL}-\xi_{DD}+\xi_V\frac{\partial V}{\partial\Delta m_1}=0, \tag{A.20}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta F_0}=\frac{1}{2}(1-\theta^2)u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)-\xi_0+\xi_{PL}-\xi_{DD}+\xi_V\frac{\partial V}{\partial\Delta F_0}=0, \tag{A.21}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta F_1}=\frac{1}{2}(1-\theta^2)u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)-\xi_{DD}+\xi_V\frac{\partial V}{\partial\Delta F_1}=0, \tag{A.22}$$

$$\frac{\partial\mathcal{L}}{\partial\theta}=-\theta u(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)-\xi_0 g'(\theta)\Delta m_0+\xi_V\frac{\partial V}{\partial\theta}=0, \tag{A.23}$$

where $g(\theta)$ is given by (20) and $g'(\theta)=(g(\theta)-\lambda\overline{R}_\theta)/(1-\theta)$.

The global games expression holds always will equality in equilibrium, i.e., $\xi_V\neq 0$. Moreover, we conjecture that the participation constraint in $t=0$, the positive liquidity injection constraint, and the dealer default constraint are all binding in equilibrium, i.e., $\xi_0,\xi_{PL},\xi_{DD}>0$. These constraints pin down the optimal ΔF_0, Δm_1 and ΔF_1 as functions of θ and Δm_0 such that $\Delta F_0=-g(\theta)\Delta m_0$, $\Delta m_1=-\Delta F_0=g(\theta)\Delta m_0$ and $\Delta F_1=-R^D\Delta m_0$. Substituting in the conjectured contract terms, the global game expression $V(\theta)=0$ (the detailed expression is reported in (B.31) in Appendix B) becomes:

$$\theta(\eta-1)\mu_I+h(\theta)\Delta m_0=0, \tag{A.24}$$

where $\mu_I=\lambda\overline{R}_\theta(R^U-R^D)/(g(\theta)R^U-\lambda\overline{R}_\theta R^D)$, $\mu_R=\lambda\overline{R}_\theta/g(\theta)$, and

$$\begin{aligned}h(\theta)&=\theta R^D\mu_I-\eta g(\theta)\mu_R+\frac{g(\theta)}{\lambda}(\mu_R-\mu_I)+\frac{(\lambda\overline{R}_\theta-g(\theta))}{\lambda}\ln\left(\frac{1-\mu_I}{1-\mu_R}\right)+\eta\lambda\overline{R}_\theta\ln(\mu_R)\\ &\quad+(1-\theta)\frac{R^D}{\lambda\overline{R}_\theta}\left[g(\theta)\mu_I-(\lambda\overline{R}_\theta-g(\theta))\ln(1-\mu_I)\right].\end{aligned} \tag{A.25}$$

Subtracting A.20 from A.21 gives an expression for ξ_0, which substituted in A.23 gives us the following expression

$$\xi_V=\frac{\theta u(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)}{\frac{\partial V}{\partial\theta}-g'(\theta)\Delta m_0\left(\frac{\partial V}{\partial\Delta F_0}-\frac{\partial V}{\partial\Delta m_1}\right)}.$$

Using the expression for ξ_V, and equation (A.22) to solve for ξ_D, equation (A.19) gives

$$\frac{1}{2}(1-\theta^2)u'((R^U-R^D)\Delta m_0(\theta))(R^U-R^D)+\frac{\theta u\left((R^U-R^D)\Delta m_0(\theta)\right)h(\theta)}{\frac{\partial V}{\partial\theta}-\Delta m_0(\theta)g'(\theta)\left(\frac{\partial V}{\partial\Delta F_0}-\frac{\partial V}{\partial\Delta m_1}\right)}=0, \tag{A.26}$$

where we used the fact the $h(\theta)=\frac{\partial V}{\partial\Delta m_0}-g(\theta)\left(\frac{\partial V}{\partial\Delta F_0}-\frac{\partial V}{\partial\Delta m_1}\right)-\frac{\partial V}{\partial\Delta F_1}R^D$ when the partial derivatives of $V(\theta)$ are evaluated at the conjectured contract terms (the detailed expressions for these derivatives are reported in (B.32)-(B.36) in Appendix B).

Using the above characterization, and the expression for $\Delta m_0(\theta^*)$ in equation (A.24), reduces the problem down to one equation (A.29) for the equilibrium θ^*. We proceed to show that a solution $\theta^*\in(\theta^{LD},\theta^{UD})$ exists. Recall that $0<\theta^{LD},\theta^{UD}<1$ from Lemma 3, which will also verify in equilibrium (see last part of the proof).

As a first step, consider the hypothetical upper bound $\bar{\theta}=1-\sqrt{(\lambda R^U+\lambda R^D-2)/(\lambda(R^U-R^D))}$. This upper bound was used to show the uniqueness of threshold strategy, derived it in (A.14). Then, $\Delta m_0(\bar{\theta})=\bar{\theta}(\eta-1)/(\eta\lambda\overline{R}_{\bar{\theta}}-R^D)$, which is strictly positive if $R^D<\eta R^U/(\eta+R^U)$.

Next, consider a θ' relatively close to $\bar{\theta}$. In that case, because $\Delta m_0(\theta)$ is continuous and $\Delta m_0(\bar{\theta})=\bar{\theta}(\eta-1)/(\eta\lambda\overline{R}_{\bar{\theta}}-R^D)$ is strictly greater than zero, $\Delta m_0(\theta')$ is strictly positive. In addition, $(\lambda\overline{R}_{\theta'}-g(\theta'))$ is very close to zero, $\partial V/\partial\theta$ is finite and positive, and $h(\theta')$ is negative. Thus for a dealer sufficiently risk averse, i.e., u' small enough, the left-hand side of equation (A.29) is negative. Moreover, for $\theta''=0$, $f(0)$ is finite, because $\lambda R^D<1$ implying that $\mu_I,\mu_R\in(0,1)$, and therefore $\Delta m_0(0)=0$. Hence, for $\theta''=0$, the left-hand side of (A.29) becomes $1/2u'(0)(R^U-R^D)>0$. Because of continuity there exists θ^* between zero and $\bar{\theta}$ such that the equation holds.

Finally, given that $\lambda\bar{R}_\theta-g(\theta)$ is an increasing function in θ, $\lambda\bar{R}_{\theta^*}-g(\theta^*)<0$, and hence lower than θ^{UD}. In addition, for any $\Delta m_0(\theta^*)$ solving (A.24), we know from Corollary 1 that the participation constraint (19) in period 1 is not binding for θ^*. Rearranging (19), we get that $\eta-1-\Delta F_1>(\eta\Delta m_1-(1-\theta^*)(R^D\Delta m_0+\Delta F_0+\Delta F_1))/\theta^*$, which implies that $\theta^{LD}<\theta^*$ by substituting the latter expression in the definition of θ^{LD} in Lemma 3. These observations ensures that $\theta^*\in(\theta^{LD},\theta^{UD})$, completing the proof.

Proof of Corollary 2

With $R^D=0$ the proposed equilibrium has $\Delta F_1=0$, therefore $\mu_I=\mu_R=\frac{\lambda\overline{R}_\theta}{g(\theta)}:=f_{\mu_R}(\theta,\lambda R^U)$, allowing us to solve for Δm_0 as a function of θ (from $V(\theta)=0$) and the relevant partial derivatives of V

$$\Delta m_0=\frac{(\eta-1)\theta}{\eta g(\theta)\left(1-\ln\left(f_{\mu_R}\right)\right)},\ \frac{\partial V}{\partial\theta}=(\eta-1)\left(\frac{2-\ln\left(f_{\mu_R}\right)}{1-\ln\left(f_{\mu_R}\right)}\right)f_{\mu_R},\ \text{and}\ \frac{\partial V}{\partial\Delta F_0}-\frac{\partial V}{\partial\Delta m_1}=\eta\left(2-\ln\left(f_{\mu_R}\right)\right)f_{\mu_R}$$

This gives the following expression for $\frac{\partial V}{\partial\theta}-g'(\theta)\Delta m_0\left(\frac{\partial V}{\partial\Delta F_0}-\frac{\partial V}{\partial\Delta m_1}\right)$ and $h(\theta)$:

$$\frac{(\eta-1)}{(1-\theta)}\left(\frac{2-\ln\left(f_{\mu_R}\right)}{1-\ln\left(f_{\mu_R}\right)}\right)f_{\mu_R}\left(1-\theta-(1-f_{\mu_R})\theta\right)\ \text{and}\ -\eta\lambda\overline{R}_\theta(1-\ln(f_{\mu_R})),$$

respectively. Replacing this expression in (28) results in the expression in the Corollary, which we can rewrite as,

$$T(\theta) := (1+\theta)\left[(1-\theta) - \theta\left(1 - f_{\mu_R}\right)\right]\left(2 - \ln\left(f_{\mu_R}\right)\right) - 2\theta^2\left(1 - \ln\left(f_{\mu_R}\right)\right) = 0$$

To ensure the existence of an equilibrium we have to determine under what conditions $T(\theta) = 0$ has a solution. Because of continuity it suffices to show under what conditions T is positive and negative for the possible limits of θ^*. Recall that from the proof of Proposition 28 we know that θ^* must be below $\bar{\theta}$, defined by $\lambda\overline{R}_{\overline{\theta}} = g(\overline{\theta})$. [45]
Note that,

$$\lim_{\theta \longrightarrow 0} T(\theta) = \infty \quad \text{and} \quad \lim_{\theta \longrightarrow \bar{\theta}} T(\theta) = 2(1 - 2\bar{\theta}^2)$$

therefore we have to ensure that $1 < 2\bar{\theta}^2$. Using the expression for $\bar{\theta} = 1 - \sqrt{1 - 2/\lambda R^U}$ the inequality is reduced to $7(\lambda R^U)^2 - 8\lambda R^U - 16 < 0$ which holds for $\lambda R^U < \frac{4+8\sqrt{2}}{7}$. From Proposition 28 we also know that λR^U needs to be greater than 2. Therefore, a solution to the equation exists if $\lambda R^U \in \left(2, \frac{4+8\sqrt{2}}{7}\right)$ (see proof of proposition 2 for why $\theta^* \in (\theta^{LD}, \theta^{UD}$; same argument applies).

For the comparatives statics we rewrite the equilibrium condition as,

$$T(\theta) = 2\left(1 - \theta - 3\theta^2 + \theta(1+\theta)f_{\mu_R}\right) - \ln\left(f_{\mu_R}\right)\left(1 - \theta - 4\theta^2 + \theta(1+\theta)f_{\mu_R}\right) = 0 \tag{A.27}$$

where $f_{\mu_R}(\theta, \lambda R^U) = \frac{\lambda\overline{R}_\theta}{g(\theta)}$. It will be useful to note that from equation (A.27)

$$\left(1 - \theta^* - 4\theta^{*2}\right) = -\frac{2\theta^{*2}}{2 - \ln\left(f_{\mu_R}\right)} - \theta(1+\theta^*)f_{\mu_R} < 0.$$

Taking the partial derivative of T with respect to λR^U gives,

$$\frac{\partial T}{\partial \lambda R^U} = \frac{\partial f_{\mu_R}}{\partial \lambda R^U}\left((1 - \ln\left(f_{\mu_R}\right))\theta(1+\theta) - (1 - \theta - 4\theta^2)\frac{1}{f_{\mu_R}}\right)$$

which is positive in θ^* because $\theta^* > \frac{-1+\sqrt{17}}{8}$ and $\frac{\partial f_{\mu_R}}{\partial \lambda R^U} > 0$. Because $T(0) > 0$ and $T(\bar{\theta}) < 0$, we know that (at least one) θ^* that solves $T(\theta^*) = 0$ also satisfies $\frac{\partial T}{\partial \theta}|_{\theta=\theta^*} < 0$. Focusing on that equilibria, and invoking the implicit function theorem, gives $\frac{\partial \theta^*}{\partial \lambda R^U} > 0$.
Proof of Proposition 3

Because of money funds' risk aversion we know that $1 - F_t^M = m_t^M$, that is, the degree of money funds' overcollateralization is equal to their haircut. Because hedge funds receive money funds' overcollateralization if a dealer defaults, hedge funds' payoff after default—either through a bad asset outcome in $t = 2$ or a collateral run in $t = 1$—increases by the degree of money funds overcollateralization, that is, m_t^M. Specifically, using equations (6) – (10) the payoff of hedge funds' that roll over after a dealer default in $t = 2$ can be expressed as $\hat{G}_S^D = G_S^D + m_1^M, \hat{G}_I^U = G_I^U + m_1^M$ and $\hat{G}_I^D = G_I^D + m_1^M$. Moreover, in case of a collateral run,

[45] θ^* must be below $\bar{\theta}$ because $\lambda\overline{R}_\theta - g(\theta)$ is increasing and any feasible threshold equilibria requires that the liquidation value $\lambda\overline{R}_\theta\Delta m_0$ be below ΔF_0.

hedge funds that do not get their collateral in $t = 1$ receive m_0^M, which they will value by η. As we will show below, this will give hedge funds even more of an incentive to run.

Because the money funds haircut is always returned to the hedge fund, the expression of the $t = 0$ participation constraint in equation (18) does not change. The $t = 1$ participation constraint in equation (19) can be rewritten as $\theta((\eta - 1) - \Delta F_1) + (1 - \theta)(\Delta F_0 + \Delta m_1) \geq \eta \Delta m_1 + (\eta - 1)m_1^M$, but, as before, it remains slack due to the global game constraint. From these observations it is direct to see that Lemmas 1 and 2 still hold.

In this case, upper dominance takes the same functional form as in the original model (solves $\lambda R^U \theta^{UD} \Delta m_0 + \Delta F_0 = 0$) and lower dominance is,

$$\hat{\theta}^{LD} = \theta^{LD} + \frac{(\eta - 1)m_1^M}{(\eta - 1) - (\Delta F_1 + \Delta m_1 + \Delta F_1)}$$

where θ^{LD} is the lower dominance threshold in the original model. Because upper dominance takes the same expression as before, and because lower dominace is larger than the original expression and the participation constraint in $t = 1$, we have that $\theta^{\hat{U}D}, \hat{\theta}^{LD} \in (0, 1)$.

We must ensure that for $\theta \geq \theta^{UD}$ and $\mu \longrightarrow 1$, the dealer would not default on those hedge funds that decide to roll over. That is, it suffices to show that for $\theta = \theta^{UD}$,

$$\lim_{\epsilon \longrightarrow 0} \frac{\hat{G}_I^U(1 - \epsilon, \theta^{UD})}{-\Delta F_1} = \frac{\frac{R^U \Delta m_1}{\lambda R^U \theta^{UD}} + m_1^M}{-\Delta F_1} = \frac{R^U \Delta m_0 \Delta m_1}{\Delta F_0 \Delta F_1} + \frac{m_1^M}{-\Delta F_1} \geq 1$$

which holds because of condition 1 and 8. Thus, in this case, a hedge fund that deviates gets $\theta^{UD}((\eta - 1) - \Delta F_1) + (1 - \theta^{UD})(\Delta F_0 + \Delta m_1) + m_1^M$ if it rolls over which is greater than $\eta(\Delta m_1 + m_1^M)$ if it does not because of $t = 1$ the participation constraint.

Putting all these observations together implies that in this case hedge funds differential $\hat{\nu}$, and its integral $\hat{V}$ are expressed as

$$\hat{\nu}(\mu, \theta) = \begin{cases} \nu - (\eta - 1)m_1^M & \mu \in [0, \mu_S) \\ \nu - (\eta - 1)m_1^M & \mu \in [\mu_S, \mu_I) \\ \nu - (\eta - 1)m_1^M & \mu \in [\mu_I, \mu_R) \\ \nu & \mu \in [\mu_R, 1] \end{cases} \quad \text{and} \quad \hat{V} = V - \mu_R(\eta - 1)m_1^M \tag{A.28}$$

where ν and V are the expressions of the utility differential in the original model. Intuitively, because hedge funds receive the money funds haircut, regardless of what happens to the dealer, the only difference in the utility differential is when hedge funds can access those funds for investing in Treasuries to capture η. In particular, for those agents that decide to roll over, if there is no run they access those funds in $t = 2$ while if there is a run they access those funds in $t = 1$.

Given that the $\hat{V}$ is different that in the original model, we show briefly that the run threshold is unique under minimum haircuts as well. For ease of exposition, we focus on the limiting case that noise goes to zero. At the threshold, θ^*, we have that $\hat{V} = 0$ or $(\eta-1)(\theta^* - \underline{m}) - \eta g(\theta^*)(1 - log(\lambda R_{\theta^*}/g(\theta^*)))\Delta m_0$. Then, taking the derivative with respect to θ and evaluating it at $\theta = \theta^*$—employing $\hat{V} = 0$—we get $dV^*/d\theta = {\theta^*}^{-1}[(\eta-1)\underline{m} + (2 - log(\lambda R_{\theta^*}/g(\theta^*)))\eta\Delta m_0(\lambda R^U - g(\theta^*)\theta^*/(1-\theta^*)] > 0$, because $\lambda R^U > g(\theta^*)$ from upper dominance. Hence, using the same argument as in the original model, θ^*.

Therefore to the new dealer's optimization problem is exactly the same before except now $\hat{V} = V - \mu_R(\eta-1)m_1^M = 0$ is the global game condition and $m_t^M \geq \underline{m}$.[46]. The Lagrangian in this case, $\hat{\mathcal{L}}$, has two additional multipliers corresponding to the lower bound on m_0^M and m_1^M. Taking the derivative for m_t^H and m_t^M separately, we have that $\frac{\partial\hat{\mathcal{L}}}{\partial m_0^H} = -\frac{\partial\hat{\mathcal{L}}}{\partial m_0^M} + \underline{\xi}_0$ and $\frac{\partial\hat{\mathcal{L}}}{\partial m_1^H} = -\frac{\partial\hat{\mathcal{L}}}{\partial m_1^M} - \xi_V\mu_R(\eta-1) + \underline{\xi}_1$. Conjecturing that $\xi_0 = 0$ and $\xi_1 = \xi_V\mu_R(\eta-1)$ give the following set of first order conditions:

$$
\begin{aligned}
&\frac{\partial\hat{\mathcal{L}}}{\partial m_0^H} = \frac{\partial\mathcal{L}}{\partial\Delta m_0} - \xi_V(\eta-1)\frac{m_1^M}{\Delta m_1}\lambda\overline{R}_\theta = 0, \quad && \frac{\partial\hat{\mathcal{L}}}{\partial m_1^H} = \frac{\partial\mathcal{L}}{\partial\Delta m_1} - \xi_V(\eta-1)\frac{m_1^M}{\Delta m_1}(1-\mu_R) = 0,\\
&\frac{\partial\hat{\mathcal{L}}}{\partial\Delta F_0} = \frac{\partial\mathcal{L}}{\partial\Delta F_0} - \xi_V(\eta-1)\frac{m_1^M}{\Delta m_1}, && \frac{\partial\hat{\mathcal{L}}}{\partial\Delta F_1} = \frac{\partial\hat{L}}{\partial\Delta F_1} = 0,\\
&\frac{\partial\hat{\mathcal{L}}}{\partial\theta} = \frac{\partial\mathcal{L}}{\partial\theta} - \xi_V(\eta-1)\frac{m_1^M}{\Delta m_1}\lambda R^U.
\end{aligned}
$$

With $R^D = 0$, the proposed equilibrium has $\Delta F_1 = 0$, therefore $\mu_I = \mu_R = \frac{\lambda\overline{R}_\theta}{g(\theta)} := f_{\mu_R}(\theta, \lambda R^U)$. Evaluating in the conjectured equilibrium and subtracting $\frac{\partial\hat{\mathcal{L}}}{\partial m_1^H}$ from $\frac{\partial\mathcal{L}}{\partial\Delta F_0}$ gives an expression for ξ_0, which substituted in equation $\frac{\partial\hat{\mathcal{L}}}{\partial\theta}$ gives

$$
\xi_V = \frac{\theta u(R^U\Delta m_0)}{\frac{\partial V}{\partial\theta} - g'(\theta)\Delta m_0\left(\frac{\partial V}{\partial\Delta F_0} - \frac{\partial V}{\partial\Delta m_1}\right) + (\eta-1)\frac{m}{g}(f_{\mu_R}g' - \lambda R^U)}
$$

Using the expression for ξ_V, under risk neutrality, $\frac{\partial\hat{\mathcal{L}}}{\partial m_0^H}$ gives

$$
\frac{1}{2}(1-\theta^2)\Delta m_0(\theta) + \frac{\theta\Delta m_0(\theta)\eta\lambda\overline{R}_\theta(1-\ln(f_{\mu_R}))}{\frac{\partial V}{\partial\theta} - \Delta m_0(\theta)g'(\theta)\left(\frac{\partial V}{\partial\Delta F_0} - \frac{\partial V}{\partial\Delta m_1}\right) + (\eta-1)\frac{m}{g}(f_{\mu_R}g' - \lambda R^U)} = 0, \tag{A.29}
$$

We solve for Δm_0 as a function of θ (from $\hat{V}(\theta) = 0$) and the relevant partial derivatives of $\hat{V}$

$$
\Delta m_0 = \frac{(\eta-1)(\theta-\underline{m})}{\eta g(\theta)\left(1-\ln\left(f_{\mu_R}\right)\right)}, \qquad \frac{\partial\hat{V}}{\partial\theta} = (\eta-1)\left(\frac{2-\left(\frac{\theta+m}{\theta}\right)\ln\left(f_{\mu_R}\right)}{1-\ln\left(f_{\mu_R}\right)}\right)f_{\mu_R},
$$

$$
\text{and} \quad \frac{\partial\hat{V}}{\partial\Delta F_0} - \frac{\partial\hat{V}}{\partial\Delta m_1} = \left(\frac{1}{\theta-\underline{m}}\right)\eta\left(2\theta - \ln\left(f_{\mu_R}-\underline{m}\right)\right)f_{\mu_R}
$$

Placing these ingredients together allows us to solve for the expression in the Proposition, and assuming

[46]In equilibrium, $m_t^H \geq \underline{m}$ will not bind and any optimal solution will have $\Delta m_0 > 0$.

$\theta > \underline{m}$, we have the following equilibrium expression,

$$\begin{aligned}\hat{T}(\theta) &= (1+\theta)\left[(1-\theta)-\theta\left(1-f_{\mu_R}\right)\right]\left(2-\ln\left(f_{\mu_R}\right)\right)-2\theta^2\left(1-\ln\left(f_{\mu_R}\right)\right)\\ &\quad +\frac{\underline{m}}{\theta-\underline{m}}\left(1-\theta^2\right)\left(1-\ln\left(f_{\mu_R}\right)\right)=0\end{aligned}$$

Note that the first term of $\hat{T}$ coincides with the equilibrium condition in Corollary 2 when $\underline{m} = 0$. As before, to ensure the existence of an equilibrium we have to determine under what conditions $\hat{T}(\theta) = 0$ has a solution. Because of continuity it suffices to show under what conditions $\hat{T}$ is positive and negative for the possible limits of θ^*. Evaluating in $\theta = 0$ and $\theta = \bar{\theta}$ (defined by $f_{\mu_R}(\bar{\theta}, \lambda R^U) = 1$, gives
Note that,

$$\lim_{\theta \longrightarrow 0} \hat{T}(\theta) = \infty \quad \text{and} \quad \lim_{\theta \longrightarrow \bar{\theta}} \hat{T}(\theta) = 2(1-2\bar{\theta}^2) + \frac{\underline{m}}{\bar{\theta}-\underline{m}}(1-\bar{\theta}^2)$$

Therefore, we have to ensure that

$$2(1-2\bar{\theta}^2)\bar{\theta} - \underline{m}(1-3\bar{\theta}^2) < 0 \tag{A.30}$$

Note that from the proof of Corollary 2 we know that $1 - 2\bar{\theta}^2$ is negative for the admisible λR^U of the propsition. Thus, there exists an $\underline{m}$ sufficiently small enough for the condition in equation (A.30) to hold. Thus, there exists a θ^* such that $\hat{T}(\theta) = 0$.
For the comparatives statics, note that the derivative of $\hat{T}$ with respect to $\underline{m}$ is

$$\frac{\partial \hat{T}}{\partial \underline{m}} = \frac{\theta}{(\theta-\underline{m})^2}\left(1-\theta^2\right)\left(1-\ln\left(f_{\mu_R}\right)\right) > 0.$$

Because $\hat{T}(0) > 0$ and $\hat{T}(\bar{\theta}) < 0$, we know that (at least one) θ^* that solves $\hat{T}(\theta^*) = 0$ also satisfies $\frac{\partial \hat{T}}{\partial \theta}|_{\theta=\theta^*} < 0$. Focusing on that equilibria, and invoking the implicit function theorem, gives $\frac{\partial \theta^*}{\partial \underline{m}} > 0$.

B Detailed expression for $V(\theta^*)$ and its derivatives

Expanding (26), the threshold θ^* is the solution to the $V(\theta^*) = 0$ shown in (B.31) below.

$$\begin{aligned}V(\theta^*) =&\theta^*\left[(\eta-1)-\Delta F_1\right]\mu_I - \eta\Delta m_1\mu_R + \frac{\Delta m_1}{\lambda}(\mu_R-\mu_I) + (1-\theta^*)\Delta m_1\mu_S\\ &- (1-\theta^*)\Delta F_0\ln(1-\mu_S) + \frac{\Delta F_0 + \lambda\overline{R}_{\theta^*}\Delta m_0}{\lambda}\ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta(\lambda\overline{R}_{\theta^*}\Delta m_0 + \Delta F_0 + \Delta m_1)\ln(\mu_R)\\ &+ (1-\theta^*)\frac{R^D}{\lambda\overline{R}_{\theta^*}}\left[-\lambda\overline{R}_{\theta^*}\Delta m_0\ln(1-\mu_S) + (\Delta F_0 + \lambda\overline{R}_{\theta^*}\Delta m_0)\ln\left(\frac{1-\mu_S}{1-\mu_I}\right) + \Delta m_1(\mu_I-\mu_S)\right].\end{aligned} \tag{B.31}$$

The derivative of $V(\theta^*)$ with respect to the contract terms and θ^* are shown in (B.32)–(B.36) below.

$$\frac{\partial V}{\partial \Delta m_0} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta m_0} + \overline{R}_{\theta^*}\ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta\lambda\overline{R}_{\theta^*}\ln(\mu_R) + (1-\theta^*)R^D\left[-\ln(1-\mu_S) + \ln\left(\frac{1-\mu_S}{1-\mu_I}\right)\right], \tag{B.32}$$

$$\frac{\partial V}{\partial \Delta m_1} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta m_1} - \eta\mu_R + \frac{(\mu_R-\mu_I)}{\lambda} + (1-\theta^*)\mu_S + \eta\ln(\mu_R) + (1-\theta^*)\frac{R^D}{\lambda\overline{R}_{\theta^*}}(\mu_I-\mu_S), \tag{B.33}$$

$$\frac{\partial V}{\partial \Delta F_0} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta F_0} - (1-\theta^*)\ln(1-\mu_S) + \frac{1}{\lambda}\ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta\ln(\mu_R) + (1-\theta^*)\frac{R^D}{\lambda\overline{R}_{\theta^*}}\ln\left(\frac{1-\mu_S}{1-\mu_I}\right), \tag{B.34}$$

$$\frac{\partial V}{\partial \Delta F_1} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta F_1} - \theta^*\mu_I, \tag{B.35}$$

$$\begin{aligned}
&\frac{\partial V}{\partial \theta^*} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \theta^*} + \left[(\eta-1) - \Delta F_1\right]\mu_I - \Delta m_1\mu_S \\
&+ \Delta F_0\ln(1-\mu_S) + (R^U-R^D)\Delta m_0\ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta\lambda(R^U-R^D)\Delta m_0\ln(\mu_R) \\
&- \frac{R^D}{\lambda\overline{R}_{\theta^*}}\left[-\lambda\overline{R}_{\theta^*}\Delta m_0\ln(1-\mu_S) + (\Delta F_0+\lambda\overline{R}_{\theta^*}\Delta m_0)\ln\left(\frac{1-\mu_S}{1-\mu_I}\right) + \Delta m_1(\mu_I-\mu_S)\right] \\
&+ (1-\theta^*)R^D\left[-\Delta F_0\frac{(R^U-R^D)}{\lambda\overline{R}^2_{\theta^*}}\ln\left(\frac{1-\mu_S}{1-\mu_I}\right) - \Delta m_1\frac{(R^U-R^D)}{\lambda\overline{R}^2_{\theta^*}}(\mu_I-\mu_S)\right],
\end{aligned} \tag{B.36}$$

with

$$\frac{\partial \mu_I}{\partial \theta^*} = \frac{\lambda(R^U-R^D)R^U\left(R^U\Delta m_0\Delta m_1 - \Delta F_0\Delta F_1\right)}{(\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1)^2}, \quad \frac{\partial \mu_I}{\partial \Delta m_0} = \frac{R^U\lambda\overline{R}_{\theta^*}}{\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1},$$

$$\frac{\partial \mu_I}{\partial \Delta m_1} = -\frac{R^U(\Delta F_0+\lambda\overline{R}_{\theta^*}\Delta m_0)R^U}{(\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1)^2} = \frac{R^U(1-\mu_I)}{\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1}, \quad \frac{\partial \mu_I}{\partial \Delta F_0} = \frac{R^U}{\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1},$$

$$\frac{\partial \mu_I}{\partial \Delta F_1} = -\frac{R^U(\Delta F_0+\lambda\overline{R}_{\theta^*}\Delta m_0)\lambda\overline{R}_{\theta^*}}{(\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1)^2} = \frac{\lambda\overline{R}_{\theta^*}(1-\mu_I)}{\lambda\overline{R}_{\theta^*}\Delta F_1 + R^U\Delta m_1}.$$

C Interpretation of FRBNY's Primary Dealer Survey

The model's notation is useful to interpret the data from FRBNY's primary dealer survey. The total amount of funds distributed and collected (i.e., Securities In and Securities Out) can be interpreted as loans made to hedge funds $T_I - m^H$ and loans received from money funds $T_O - m^M$, respectively. In this case, T_I is the total amount of collateral received from hedge funds and T_O is the total amount of collateral posted with money funds. It is important to note that the total amount of collateral posted with money funds T_O may not necessarily come from hedge fund counterparties. That is, a fraction of collateral posted in Securities Out can be part of the dealer's own asset position. But when dealing in cash and secured financing markets, dealers have a natural collateral restriction to follow, known as the *box constraint*. This constraint forces dealers to have a non-negative stock of collateral. That is, denoting L and S the dealer's long and short position, respectively, the box constraint can be translated into

$$(L - S) + (T_I - T_O) \geq 0.$$

That is, the amount of collateral owned and sourced must be larger than the amount of collateral sold and posted.

In figure 2 we argue that the difference between Securities Out and Securities In plus net position is a lower bound for the amount of liquidity coming from different haircuts. In effect,

$$\underbrace{(T_O - m^M)}_{sec-out} - (\underbrace{(T_I - m^H)}_{sec-in} + (L - S)) \leq (T_O - T_I) + m^H - m^M + (T_I - T_O) = m^H - m^M$$

where the inequality comes from imposing the box constraint.

An important caveat to this lower bound is that survey asks respondents to also report the total amount of long and short positions in forward contracts.[47] Because forward contracts are derivatives, they do not enter into the box constraint, which is strictly a cash market restriction. Regrettably, we cannot tease out how much the lower bound is attributable to haircut differences and how much is due to large forward positions.

From Figure 2, we can see that in the last year of Bear Stearn's activity, the estimated amount of liquidity the firm captured through rehypothecation was at least between \$10 and \$50 billion, equivalent to 1/10 or 1/3 of its entire repo activity.

[47]Forwards are the only derivative contracts that are reported in the FR 2004.