Coalition-Proof Mechanisms Under Correlated Information

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December 31, 2019

Motivation

To implement efficient allocation rules with interim coalitional incentive compatible (CIC) and ex-post budget balanced (BB) mechanisms.

Earlier papers

- restrict to independent private value environment (Safronov, 2018) or
- restrict coalitional behaviors: no redistribute within a coalition, no information pooling (e.g., Che and Kim, 2006).

Preview

This paper characterizes the information structures under which efficient allocations are guaranteed to be implementable via an CIC and BB mechanisms.

Simple mechanisms: Every efficient allocation rule is implementable via a CIC and BB simple mechanism, iff the Coalitional Identifiability (CI) condition holds.

- Mixed implication.

Ambiguous mechanisms: Every efficient allocation rule is implementable via a CIC and BB ambiguous mechanism, iff the Coalitional Beliefs Determine Preferences (CBDP) condition holds.

- A generic possibility result.

Setup: Asymmetric Information Environment

The environment is common knowledge between the MD and agents:

- $I = \{1, ..., N \geq 2\}$ is the finite set of **agents**;
- A is the set of **feasible outcomes**;
- $\Theta \equiv \times_{i \in I} \Theta_i$ is a finite **type space**, where each $\theta_i \in \Theta_i$ a type of agent i;
- i has a quasi-linear **utility function** $u_i(a, \theta) + b_i$, where $a \in A$ and $b_i \in \mathbb{R}$;
- $p \in \Delta(\Theta)$ is a fully supported **common prior**.

The pair (Θ, p) is called an **information structure**.

An allocation rule $q : \Theta \to A$ is ex-post **efficient**, if

$$\sum_{i \in I} u_i\big(q(\theta), \theta\big) \geq \sum_{i \in I} u_i\big(a, \theta\big), \forall a \in A, \theta \in \Theta.$$

Setup: Coalitions

A **coalition** S is a non-empty subset of agents in I.

Let the **coalition pattern** $\mathcal{S}$ be the class of all coalitions that can be formed from the MD's view. $\mathcal{S}$ includes all singletons.

We say there exist two **complementary coalitions** in $\mathcal{S}$ when there exist $S^1, S^2 \in \mathcal{S}$ such that $S^1 \cap S^2 = \emptyset$ and $S^1 \cup S^2 = I$.

Let $\mathring{\mathcal{S}}$ be the collection of non-singleton non-grand coalitions in $\mathcal{S}$.

Simple Mechanism: BB and CIC

Focus on direct mechanisms. A **simple** mechanism to implement an efficient allocation rule q is a pair (q, ϕ), where $\phi : \Theta \to \mathbb{R}^N$ is the transfer rule.

It is said to satisfy ex-post **budget balance** (BB) if $\sum_{i \in I} \phi_i(\theta) = 0, \forall \theta \in \Theta$.

It is said to satisfy interim **coalitional incentive compatibility** (CIC) if

$$\sum_{i \in S} \sum_{\theta_{-S} \in \Theta_{-S}} [u_i\big(q(\bar{\theta}_S, \theta_{-S}), (\bar{\theta}_S, \theta_{-S})\big) + \phi_i(\bar{\theta}_S, \theta_{-S})] p(\theta_{-S} | \bar{\theta}_S)$$

$$\geq \sum_{i \in S} \sum_{\theta_{-S} \in \Theta_{-S}} [u_i\big(q(\hat{\theta}_S, \theta_{-S}), (\bar{\theta}_S, \theta_{-S})\big) + \phi_i(\hat{\theta}_S, \theta_{-S})] p(\theta_{-S} | \bar{\theta}_S), \ \forall S \in \mathcal{S}, \bar{\theta}_S, \hat{\theta}_S \in \Theta_S.$$

(focusing on pure strategies w.l.o.g; pool information; side contracts are allowed)

Simple Mechanism: Coalitional Identifiability Condition

A **coalition emerging probability** is a distribution ξ over $\mathring{\mathcal{S}} \cup \{\emptyset\}$, where

1. $\xi(\emptyset)$: the probability that no non-trivial coalition is formed;
2. $\xi(S)$: the probability that coalition S is formed.

Let $\delta_S : \Theta_S \to \Delta(\Theta_S)$ be a strategy of coalition S after members in it sharing private information. This is not a profile of individual strategies for agents in S.

Given S adopting δ_S, other agents' truthfully reporting, and the common prior p, the joint distribution of reports received by MD is π^{δ_S}, where

$$\pi^{\delta_S}(\theta) = \sum_{\bar{\theta}_S \in \Theta_S} p(\bar{\theta}_S, \theta_{-S})\delta_S(\bar{\theta}_S)[\theta_S], \forall \theta \in \Theta.$$

Simple Mechanism: Coalitional Identifiability Condition

Consider agent i's (generalized) unilateral deviation from truthful reporting:

With probability $\xi(S)$, agent i is in $S \in \mathring{\mathcal{S}}$ and follows strategy δ_S to report the type profile of agents in S.

With probability $1 - \sum_{S \in \mathring{\mathcal{S}}, S \ni i} \xi(S)$, agent i abstains from any non-singleton non-grand coalition and follows the strategy δ_i to report his type.

The following distribution on reported information is generated:

$$(1 - \sum_{S \in \mathring{\mathcal{S}}, S \ni i} \xi(S))\pi^{\delta_i} + \sum_{S \in \mathring{\mathcal{S}}, S \ni i} \xi(S)\pi^{\delta_S}.$$

Simple Mechanism: Coalitional Identifiability Condition

Definition

The **Coalitional Identifiability** (CI) condition holds if for any coalition emerging probability $\xi \in \Delta(\mathring{\mathcal{S}} \cup \{\emptyset\})$, any distribution function $\mu : \Theta \to \mathbb{R}$, any profile of strategies $(\delta_S)_{S \in \mathcal{S}}$ that is not always truthful, there exists $i \in I$ such that

$$(1 - \sum_{S \in \mathring{\mathcal{S}}, S \ni i} \xi(S))\pi^{\delta_i} + \sum_{S \in \mathring{\mathcal{S}}, S \ni i} \xi(S)\pi^{\delta_S} \neq \mu.$$

Simple Mechanism: Main Result

Theorem 1

Given any information structure (Θ, p), the following statements are equivalent:

1. The CI condition holds.
2. Any ex-post efficient allocation rule q under any profile of utility functions is implementable via an interim CIC and ex-post BB simple mechanism.

Simple Mechanism: Implication

Mixed implication.

It can be proved that the Coalitional Identifiability condition fails under all information structures when there exist two complementary coalitions in $\mathcal{S}$.

In this case, interim CIC and ex-post BB implementation cannot be guaranteed via simple mechanisms.

Ambiguous Mechanism

Definition

An **ambiguous mechanism** *to implement an efficient allocation rule q is a pair $\mathcal{M} = (q, \Phi)$, where Φ is a compact set of transfer rules with a generic element $\phi : \Theta \to \mathbb{R}^N$.*

The MD

- secretly commits to some $\phi = (\phi_1, ..., \phi_N) \in \Phi$;
- tells agents that Φ is the set of potential transfers, and announces q;
- lets agents report their types;
- reveals ϕ;
- assigns transfers and allocations according to reports, q, and ϕ.

Ambiguous Mechanism: BB

An ambiguous mechanism (q, Φ) satisfies ex-post **budget balance** (BB) if

$$\sum_{i \in I} \phi_i(\theta) = 0, \forall \phi \in \Phi, \theta \in \Theta.$$

Ambiguous Mechanism: Agents' Decision

After pooling information within the coalition S, agents in a coalition face risk (known probabilities) and ambiguity (unknown probabilities).

- Risk: they merely knows the distribution of types of S^c.
- Ambiguity: they do not know the distribution of mechanism rules.

Assume that agents are ambiguity-averse and are maxmin expected utility maximizers (Gilboa & Schmeidler, 1989).

Ambiguous Mechanism: CIC

The ambiguous mechanism (q, Φ) is said to satisfy interim **coalitional incentive compatibility** (CIC) if

$$\min_{\phi\in\Phi}\sum_{i\in S}\sum_{\theta_{-S}}[u_i\big(q(\bar{\theta}_S,\theta_{-S}),(\bar{\theta}_S,\theta_{-S})\big)+\phi_i(\bar{\theta}_S,\theta_{-S})]p(\theta_{-S}|\bar{\theta}_S)$$

$$\geq \min_{\phi\in\Phi}\sum_{i\in S}\sum_{\theta_{-S}}\sum_{\hat{\theta}_S}[u_i\big(q(\hat{\theta}_S,\theta_{-S}),(\bar{\theta}_S,\theta_{-S})\big)+\phi_i(\hat{\theta}_S,\theta_{-S})]\delta_S(\bar{\theta}_S)[\hat{\theta}_S]p(\theta_{-S}|\bar{\theta}_S),$$

$$\forall S\in\mathcal{S},\bar{\theta}_S\in\Theta_S,\text{and (mixed) strategy}\,\delta_S:\Theta_S\to\Delta(\Theta_S).$$

- NOT w.l.o.g to focus on pure strategies;
- Agents in S can adopt budget balanced side contract $(\tau_i^{\bar{\theta}_S,\delta_S}(\phi,\theta_{-S}))_{i\in S,\theta_{-S}\in\Theta_{-S},\phi\in\Phi}$ to redistribute wealth.

Ambiguous Mechanism: CBDP Property

The following condition strengthens Neeman (2004)'s Beliefs Determine Preferences condition.

Definition

Given the information structure (Θ, p)*, the* **Coalitional Beliefs Determine Preferences** *(CBDP) condition holds if for any non-grand coalition* $S \in \mathcal{S}$ *and* $\bar{\theta}_S, \hat{\theta}_S \in \Theta_S$ *with* $\bar{\theta}_S \neq \hat{\theta}_S$

$$(p(\theta_{-S}|\bar{\theta}_S))_{\theta_{-S}\in\Theta_{-S}} \neq (p(\theta_{-S}|\hat{\theta}_S))_{\theta_{-S}\in\Theta_{-S}}.$$

The CBDP property is weaker than Coalitional Identifiability condition.
The CBDP property holds for all almost all information structures.

Ambiguous Mechanism: Main Result

Theorem 2

Given an information structure (Θ, p), the following statements are equivalent:

1. the CBDP condition holds;
2. any ex-post efficient allocation rule q under any profile of utility functions is implementable via an interim CIC and ex-post BB ambiguous mechanism.

A rough intuition why ambiguous mechanism works better.

An Example

Three agents $i = 1, 2, 3$ and $\mathcal{S} = \{\{1, 2\}, \{1\}, \{2\}, \{3\}\}$.

Each agent has two types $\Theta_i = \{\theta_i^1, \theta_i^2\}$. A common prior p is given below.

p	$\theta_1^1, \theta_2^1, \theta_3^1$	$\theta_1^1, \theta_2^1, \theta_3^2$	$\theta_1^1, \theta_2^2, \theta_3^1$	$\theta_1^1, \theta_2^2, \theta_3^2$
	0.2	0.15	0.1	0.05
	$\theta_1^2, \theta_2^1, \theta_3^1$	$\theta_1^2, \theta_2^1, \theta_3^2$	$\theta_1^2, \theta_2^2, \theta_3^1$	$\theta_1^2, \theta_2^2, \theta_3^2$
	0.05	0.1	0.15	0.2

An Example: Non-implementable via a Simple Mechanism

$A = \{x_0, x_1, x_2\}$. The outcome x_0 gives all agents zero payoffs at all type profiles. The payoffs given by x_1 and x_2 are presented below.

	x_1	x_2
$\theta_1 = \theta_1^1$	$(1, 1, 1)$	$(5, -1, -1)$
$\theta_1 = \theta_1^2$	$(0, 1, 1)$	$(1, 1, 1)$

The 1st, 2nd, or 3rd component denotes agent 1, 2, or 3's payoff respectively.

The efficient allocation rule q: $q(\theta_1^1, \cdot) = x_1$ and $q(\theta_1^2, \cdot) = x_2$ is not implementable via a simple mechanism.

Example: Implementable via an Ambiguous Mechanism

An interim CIC and ex-post BB ambiguous mechanism $\Phi = \{\phi^1, \phi^2, \phi^3\}$ can implement q.

Example: Implementable via an Ambiguous Mechanism

First, we define a transfer rule $\phi^1 \in \Phi$ below:

$$\begin{array}{llll}
\phi_1^1(\theta_1^1\theta_2^1\theta_3^1)=120, & \phi_1^1(\theta_1^1\theta_2^1\theta_3^2)=-160, & \phi_1^1(\theta_1^1\theta_2^2\theta_3^1)=-240, & \phi_1^1(\theta_1^1\theta_2^2\theta_3^2)=480, \\
\phi_1^1(\theta_1^2\theta_2^1\theta_3^1)=-480, & \phi_1^1(\theta_1^2\theta_2^1\theta_3^2)=240, & \phi_1^1(\theta_1^2\theta_2^2\theta_3^1)=160, & \phi_1^1(\theta_1^2\theta_2^2\theta_3^2)=-120.
\end{array}$$

Let $\phi_2^1(\theta) = \phi_3^1(\theta) = -0.5\phi_1^1(\theta)$ for all $\theta \in \Theta$.

Example: Implementable via an Ambiguous Mechanism

The second transfer rule is defined by $\phi_i^2(\theta) = -\phi_i^1(\theta)$ for all $\theta \in \Theta$ and $i \in I$.

Example: Implementable via an Ambiguous Mechanism

The third transfer rule $\phi^3 \in \Phi$ is defined below:

$$\phi_1^3(\theta_1^1\theta_2^1\theta_3^1){=}60, \quad \phi_1^3(\theta_1^1\theta_2^1\theta_3^2){=}{-80}, \quad \phi_1^3(\theta_1^1\theta_2^2\theta_3^1){=}0, \quad \phi_1^3(\theta_1^1\theta_2^2\theta_3^2){=}0,$$
$$\phi_1^3(\theta_1^2\theta_2^1\theta_3^1){=}{-240}, \quad \phi_1^3(\theta_1^2\theta_2^1\theta_3^2){=}120, \quad \phi_1^3(\theta_1^2\theta_2^2\theta_3^1){=}0, \quad \phi_1^3(\theta_1^2\theta_2^2\theta_3^2){=}0,$$

$$\phi_2^3(\theta_1^1\theta_2^1\theta_3^1){=}{-60}, \quad \phi_2^3(\theta_1^1\theta_2^1\theta_3^2){=}80, \quad \phi_2^3(\theta_1^1\theta_2^2\theta_3^1){=}120, \quad \phi_2^3(\theta_1^1\theta_2^2\theta_3^2){=}{-240},$$
$$\phi_2^3(\theta_1^2\theta_2^1\theta_3^1){=}0, \quad \phi_2^3(\theta_1^2\theta_2^1\theta_3^2){=}0, \quad \phi_2^3(\theta_1^2\theta_2^2\theta_3^1){=}0, \quad \phi_2^3(\theta_1^2\theta_2^2\theta_3^2){=}0,$$

and $\phi_3^3(\theta) = -\phi_1^3(\theta) - \phi_2^3(\theta)$.

Example: Implementable via an Ambiguous Mechanism

Each of the transfer rule satisfies the ex-post BB condition, and thus the ambiguous mechanism also satisfies the ex-post BB condition.

Example: Implementable via an Ambiguous Mechanism

All interim CIC constraints hold.

e.g., The MEU for type-(θ_1^1, θ_2^1) coalition $\{1, 2\}$ is

$$\min\{2, 2, 2\} = 2.$$

The first 2 is computed from q and $\phi_1^1 + \phi_2^1$, the second from $\phi_1^2 + \phi_2^2$, and the third from $\phi_1^3 + \phi_2^3$.

Example: Implementable via an Ambiguous Mechanism

e.g., $CIC(\theta_1^1, \theta_2^1)$.

- The two transfer rules ϕ^1 and ϕ^2 jointly guarantee pure strategy CIC.
  - When type-(θ_1^1, θ_2^1) misreports (θ_1^1, θ_2^2), his MEU is

$$\min\{2+\frac{240}{7}, 2-\frac{240}{7}, 2-\frac{240}{7}\} = 2-\frac{240}{7} < 2.$$

  - When type-(θ_1^1, θ_2^1) misreports (θ_1^2, θ_2^1), his MEU is

$$\min\{4-\frac{600}{7}, 4+\frac{600}{7}, 4-\frac{600}{7}\} = 4-\frac{600}{7} < 2.$$

  - When type-(θ_1^1, θ_2^1) misreports (θ_1^2, θ_2^2), his MEU is

$$\min\{4+\frac{140}{7}, 4-\frac{140}{7}, 4-\frac{600}{7}\} = 4-\frac{140}{7} < 2.$$

- ϕ^3 can prevent deviation in mixed strategies.
  - e.g., deviating to (θ_1^1, θ_2^2) and (θ_1^2, θ_2^1) w.p. $\frac{5}{7}$ and $\frac{2}{7}$ can hedge against the uncertainty of ϕ^1, ϕ^2.

Conclusion

- This paper studies what information structures can guarantee implementation of efficient allocation rules via CIC and BB mechanisms.
- Under simple mechanisms:
  - CI $\Leftrightarrow$ CIC and BB implementation.
  - Coalition-proof implementation can usually be impossible to guarantee when there are complementary coalitions.
- Under ambiguous mechanisms:
  - CBDP $\Leftrightarrow$ CIC and BB implementation.
  - Coalition-proof implementation can usually be achieved under ambiguous mechanisms.

Literature Review

Coalition-proof mechanisms

- Safronov (2018), Laffont & Martimort (1997, 1998, 2000), Forges et al. (2002), Chen & Micali (2012), Che & Kim (2006) etc.

Mechanism design under correlated beliefs

- Crémer & McLean (1985, 1988), McAfee & Reny (1992), Neeman (2004), etc.
- Kosenok & Severinov (2008), McLean & Postlewaite (2004, 2015), Matsushima (1991, 2007), d'Aspremont et al. (2004), etc.

Mechanism design with ambiguity-averse agents

- Bose et al. (2006), Bose & Daripa (2009), Bodoh-Creed (2012), de Castro et al. (2009, 2017), Wolitzky (2016), Song (2016), etc.
- Bose & Renou (2014), Di Tillio et al. (2017), Guo (2019), Tang & Zhang (2018).