Uncertainty and Economic Activity: Identification Through Cross-section Correlations

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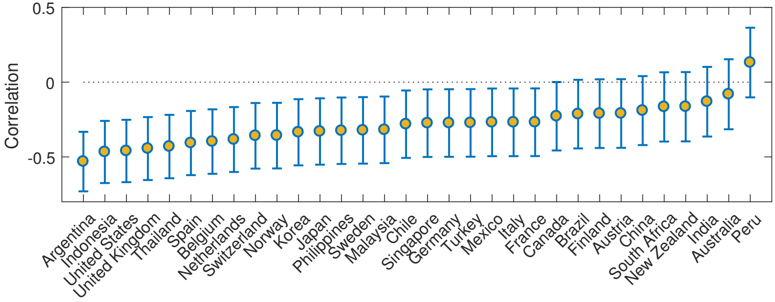
Strong and robust association between measures of uncertainty and economic activity



- Uncertainty proxy: realized volatility equity market volatility
- Economic activity proxy: Quarterly real GDP growth
- Data for 32 countries, covering about 90 percent of world GDP

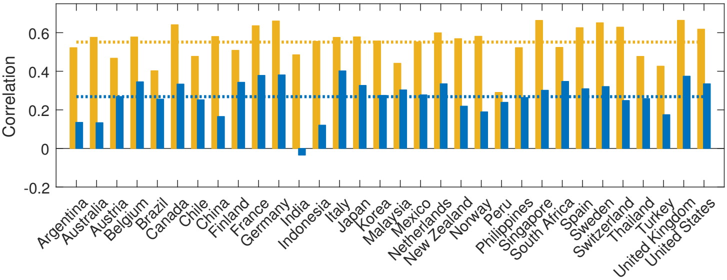
But difficult to interpret

- Uncertainty dampens activity
  - Precautionary savings [Kimball (1990)], irreversible investments [Bernanke (1983), Bloom (2009)], and financial frictions [Christiano et al. (2014), Gilchrist et al. (2014)]
  - Pricing frictions and ZLB can amplify these effects [Basu and Bundick (2017), Fernandez-Villaverde et al. (2011)]

- Recessions can also increase uncertainty
  - Financial and information frictions [Van Nieuwerburgh and Veldkamp (2006), Fostel and Geanakoplos (2012), Decker et al. (2016), Ilut et al. (2017)]

This paper

- Takes a novel multi-country perspective and models the relation between uncertainty and economic activity without restricting the direction of causation
- Identify two factors, a real and financial factor, exploiting different patterns correlation of volatility and growth across countries



NOTE: Average pairwise correlation of volatility (yellow bars) and GDP growth (blue bars) series.

Main findings

1. For most countries, the real common factor accounts for the bulk of the correlation between volatility and growth

2. The ”Endogenous” component of volatility is quite small ($< 5\%$)
   - Innovations to the real factor, to country growth, and growth in other countries explain a very small share of volatility variance
   - The financial common factor and the idiosyncratic components of volatility explain a large share of volatility variance

3. Idiosyncratic components of volatility is small (or well diversified)
   - Only the common components of volatility explain a significant share of growth variance and can have deep imact on country growth when it hits
   - Idysyncratic components of volatility explain very little growth variance

(Large) Related literature

- Volatility does respond to the business and financial cycles [Ludvigson, Ma, and Ng (2015), Carriero, Clark, Marcellino (2016)]
- First vs Second moments factors[e.g., Gorodnichenko and Ng (2017)]: we identify a pure second-moment factor and quantify its importance for and dynamic imact on growth
- International dimension [Carriere-Swallow and Cespedes (2013), Baker and Bloom (2013), Hirata, Kose, Otrok, and Terrones (2012)]
  - Multi-country framework, as opposed to a set of countries considered in isolation.
  - We do not assume volatility is exogenous

Outline

1. Factor model for volatility and growth
2. Data & Empirical Results
3. Conclusions

A static factor model

- For each country i assume that both volatility and GDP growth load on a common factor (f_t) as follows

$$\begin{aligned} v_{it} &= \lambda_i f_t + u_{it} \\ \Delta y_{it} &= \gamma_i f_t + \varepsilon_{it} \end{aligned}$$

- Growth equation easily derived from stochastic RBC/Solow growth model

Consumption-based CAPM interpretation of volatility equation

$$r \approx log(1+r) = \delta + \rho f - \frac{\rho^2 \sigma_f^2}{2}. \tag{1}$$

$$(E_t r_{i,t+1} - r) = \rho Cov\left[\Delta y_{w,t+1}, r_{i,t+1}\right] = \rho Corr\left[\Delta y_{w,t+1}, r_{i,t+1}\right] \sigma_f \sigma_{ir} \tag{2}$$

$$\sigma_i = \left| \frac{(E_t r_{i,t+1} - r)}{\rho \sigma_f} \right| = \left| \frac{\left[E_t r_{i,t+1} - \delta - \rho f + \frac{\rho^2 \sigma_f^2}{2}\right]}{\rho \sigma_f} \right|. \tag{3}$$

A static factor model (Cont.)

- For each country i assume that both volatility and GDP growth load on a common factor (f_t) as follows

$$\begin{aligned} v_{it} &= \lambda_i f_t + u_{it} \\ \Delta y_{it} &= \gamma_i f_t + \varepsilon_{it} \end{aligned}$$

- If we consider only one country in isolation, the model is not identified, even assuming ε_{it} and u_{it} are uncorrelated
  - Four unknown parameters λ_i, γ_i, $\sigma^2_{u,i}$, $\sigma^2_{\varepsilon,i}$ (normalizing $\sigma^2_f = 1$)
  - But covariance matrix of v_{it} and Δy_{it} provides only three independent restrictions
  - Identification usually achieved with an exclusion restriction

- If we take a multi-country approach, we can identify f_t from restrictions implicit in the pattern of correlation the two shocks across countries, even leaving the correlation between ε_{it} and u_{it} unrestricted

Some notation & Identifying assumptions

Notation

- Define global volatility ($\bar{v}_{\omega,t}$) and GDP growth ($\Delta\bar{y}_{\omega,t}$) as weighted (w_i) averages over a large number of countries

$$\bar{v}_{\omega,t} = \sum_{i=1}^{N} w_i v_{it}, \qquad \Delta\bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}$$

Identifying assumptions

1. Loadings: factor f_t is strong (or pervasive) for both volatility and activity
2. Weights: granularity, i.e. weights (w_i) are not dominated by a few cross-section units (can be partially relaxed)
3. Cross-sectional correlations: volatility innovations are strongly correlated across countries (pairwise correlation does not tend to zero), while GDP growth innovations are weakly correlated across countries (pairwise correlation tends to zero)

More details

Identification of the real factor (f_t) by aggregation

Proposition 1 Under these assumptions, for N large enough, f_t can be identified by $\bar{y}_{\omega,t}$ up to a constant

Proof Consider the weighted average of the country systems

$$\begin{aligned} \bar{v}_{\omega,t} &= \lambda f_t + \bar{u}_{\omega,t}, \\ \Delta \bar{y}_{\omega,t} &= \gamma f_t + \bar{\varepsilon}_{\omega,t}, \end{aligned}$$

where $\bar{u}_{\omega,t} = \mathbf{w}'\mathbf{u}_t$ and $\bar{\varepsilon}_{\omega,t} = \mathbf{w}'\boldsymbol{\varepsilon}_t$. For N sufficiently large, we can show that have

$$f_t = \frac{\Delta \bar{y}_{\omega,t}}{\gamma} + \underbrace{\frac{\bar{\varepsilon}_{\omega,t}}{\gamma}}_{O_p\left(N^{-1/2}\right)} .$$

And thus the last term becomes neglible as the sample size increase.

Proof (cont.)

This is becasue:

$$var\left(\bar{\varepsilon}_{\omega,t}\right) = \mathbf{w}'\mathbf{\Sigma}_{\varepsilon}\mathbf{w} \leq \left(\mathbf{w}'\mathbf{w}\right)\varrho_{\max}\left(\mathbf{\Sigma}_{\varepsilon}\right). \tag{4}$$

But under the assumptions made:

$$var\left(\bar{\varepsilon}_{\omega,t}\right) = O\left(\mathbf{w}'\mathbf{w}\right) = O\left(N^{-1}\right), \tag{5}$$

and hence:

$$\bar{\varepsilon}_{\omega,t} = O_p\left(N^{-1/2}\right). \tag{6}$$

QED

Remarks

Remark

(Interpretation of f_t) Because f_t is the same as world growth rescaled, we label it a "real" or "macroeconomic" factor.

Remark

(Estimation of f_t) As f_t is pervasive or strong, we can estimate it with either principal component techniques of cross-section averages of Δy_{it} (for $i = 1, 2, \ldots, N$).

Remark

(Identification of f_t) Nonetheless, f_t cannot be identified from the cross-section average or the principal component of the panel of volatilities series v_{it}.

The finacial factor (g_t)

- By assumption, u_{it} must share at least one more factor than ε_{it}
- Assume for simplicity that, conditional on f_t, u_{it} share only one additional strong factor

$$u_{it} = \theta_i g_t + \eta_{it}$$

- The model becomes

$$\begin{aligned} v_{it} &= \lambda_i f_t + \theta_i g_t + \eta_{it} \\ \Delta y_{it} &= \gamma_i f_t + \varepsilon_{it} \end{aligned}$$

- Different pattern of correlation across countries of volatility and growth innovations implicitly provides a restriction on the factor loadings

$$\begin{bmatrix} v_{it} \\ \Delta y_{it} \end{bmatrix} = \begin{bmatrix} \lambda_i & \theta_i \\ \gamma_i & 0 \end{bmatrix} \begin{bmatrix} f_t \\ g_t \end{bmatrix} + \begin{bmatrix} \eta_{it} \\ \varepsilon_{it} \end{bmatrix}$$

Identification of the financial factor (g_t) by aggregation

- If one factor is enough, volatility innovations η_{it} are cross-sectionally weakly correlated
  - That is, similarly to ε_{it}, we have that $\bar{\eta}_{\omega,t} = O_p\left(N^{-1/2}\right)$
- **Proposition 2** Conditional on f_t, for N large enough, g_t is given by

$$g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta\gamma}\Delta\bar{y}_{\omega,t} + \underbrace{\frac{\bar{\eta}_{\omega,t}}{\theta}}_{O_p\left(N^{-1/2}\right)}$$

- Factors f_t and g_t can then be estimated up to a scalar and a rotation of the coefficients in the expression for gt

More details

Remarks

Remark

We label g_t "financial" factor to stress the idea that g_t ***must be capturing*** *time variation in either systematic risk or time and risk preferences not affecting the growth series contemporaneously.*

Remark

(Relation to structural models) We are agnostic: some models are consistent others are not with the triangular factor structure identified. Approach similar to APT applied to second moments.

Additional results

Proposition

Denote with $\tilde{f}_t$ and $\tilde{g}_t$ a consistent, orthogonalized estimate of estimate of f_t and g_t, respectively. We can obtain $\tilde{f}_t$ by rescaling $\Delta\bar{y}_{\omega,t}$ so that its equal to 1, while $\tilde{g}_t$ can be obtained for $t = 1, 2, ..., T$ as the standardized residual of a regression of $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$.

Remark

(Equivalent models) The derived empirical model is equivalent to a factor augmented VAR (FAVAR) model in which $\tilde{f}_t$ and $\tilde{g}_t$ have been orthogonalized with a Cholesky decomposition of the variance-covariance matrix of the global variables $\bar{v}_{\omega,t}$ and $\Delta\bar{y}_{\omega,t}$, ordering world GDP growth first, but is not consistent with a FAVAR model in which $\tilde{f}_t$ and $\tilde{g}_t$ have been orthogonalized with a Cholesky decomposition and the opposite ordering of the global variables.

Dynamic model (Factor-augmented large VAR)

- Theoretical results carry through a fully heterogeneous dynamic version of the model
  - Country interactions and spillovers through unrestricted variance-covariance matrix and the factors

- Country-specific model with orthonormal factors

$$\begin{bmatrix} v_{it} \\ \Delta y_{it} \end{bmatrix} = \begin{bmatrix} \phi_{i,11} & \phi_{i,12} \\ \phi_{i,21} & \phi_{i,22} \end{bmatrix} \begin{bmatrix} v_{it,-1} \\ \Delta y_{i,t-1} \end{bmatrix} + \begin{bmatrix} \beta_{i,11} & \beta_{i,12} \\ \beta_{i,21} & 0 \end{bmatrix} \begin{bmatrix} \tilde{f}_t \\ \tilde{g}_t \end{bmatrix}$$

$$\ldots + \begin{bmatrix} \psi_{vi,11} & \psi_{vi,12} \\ \psi_{\Delta yi,11} & \psi_{\Delta yi,12} \end{bmatrix} \begin{bmatrix} \bar{v}_{\omega,t-1} \\ \Delta \bar{y}_{\omega,t-1} \end{bmatrix} + \begin{bmatrix} \eta_{it} \\ \varepsilon_{it} \end{bmatrix}$$

- Country-specific models can be combined in a large model of the global economy

Volatility measurement

- We compute the realized volatility for country i in quarter t as:

$$\sigma_{it} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \left(r_{it}(\tau) - \bar{r}_{it}\right)^2} \tag{7}$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter t, and D_t is the number of trading days in quarter t.

- We work with log of σ_{it}

Data & Empirical Results

Data & Empirical Results

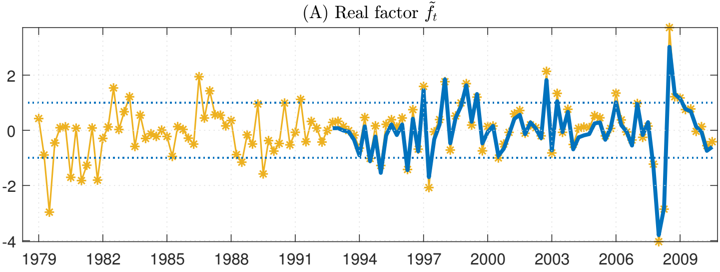
Data

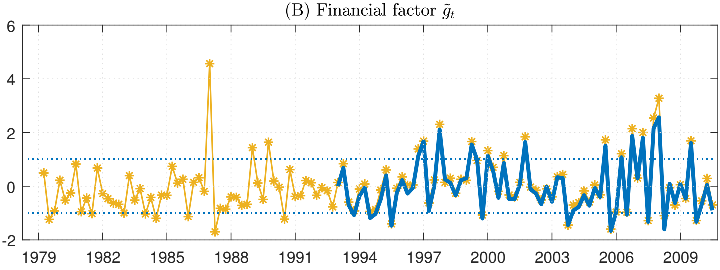
- Balanced panel data for 32 countries from 1993:Q1 to 2011:Q2
- Results robust to
  - Using a slightly longer sample with fewer countries (from 1988:Q1, excluding China, Indonesia, Brazil, and Peru)
  - Using a significantly longer sample in an unbalanced panel data set of the same 32 countries (some empirical results gets hard to compute)

Empirical results

- Factors estimates
- Evidence in support of identifying assumptions
- Within-country identification
- IRFs and FEVDs to factors and country-specific shocks

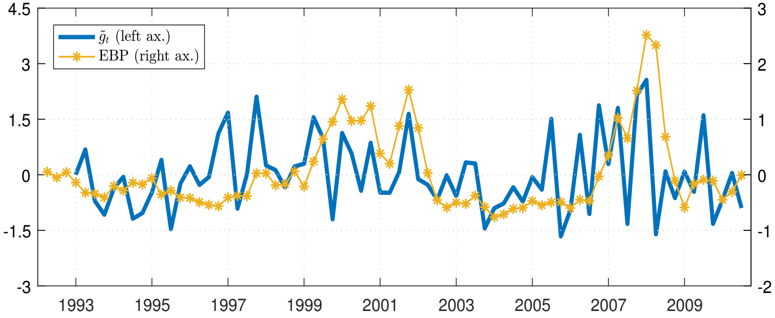
Estimated orthogonal factors ($\tilde{f}$ and $\tilde{g}$)





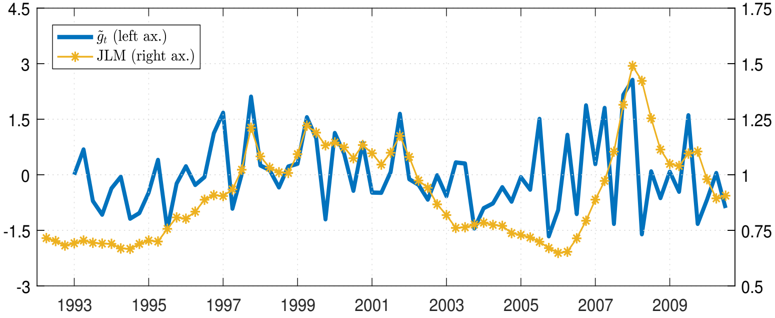
Interpreting the $\tilde{g}$ factor

- Correlation between $\tilde{g}$ and Excess Bond Premium (Gilchrist and Zakrajsek, 2012) is 0.34



Interpreting the $\tilde{g}$ factor

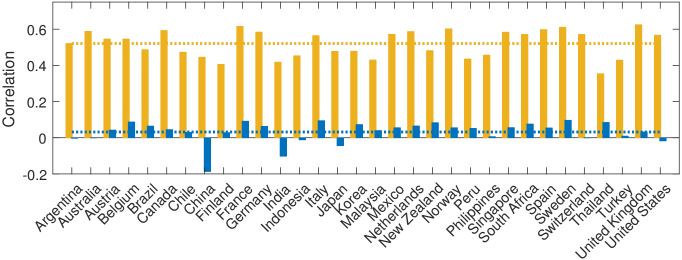
- Correlation between $\tilde{g}$ and Ludvigson, Ma and Ng (2017)'s financial uncertainty measure is 0.4



Is the identifying assumption on cross-sectional dependence consistent with the data?

- Estimate country models with $\tilde{f}_t$ only:

$$\begin{aligned} v_{it} &= \beta_{i,11}\tilde{f}_t + lags + u_{it} \\ \Delta y_{it} &= \beta_{i,21}\tilde{f}_t + lags + \varepsilon_{it} \end{aligned}$$

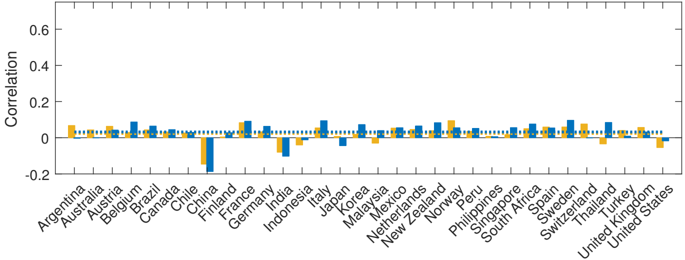


NOTE. Average pairwise correlation of the u_{it} (yellow bars) and the ϵ_{it} (blue bars).

Is one additional strong factor sufficient to model country volatilities?

- Estimate country models with $\tilde{f}_t$ and $\tilde{g}_t$:

$$\begin{aligned} v_{it} &= \beta_{i,11}\tilde{f}_t + \beta_{i,12}\tilde{g}_t + lags + \eta_{it} \\ \Delta y_{it} &= \beta_{i,21}\tilde{f}_t + lags + \varepsilon_{it} \end{aligned}$$



NOTE. Average pairwise correlation of the η_{it} (yellow bars) and the ϵ_{it} (blue bars).

Tests of cross-sectional dependence don't reject identifying assumptions

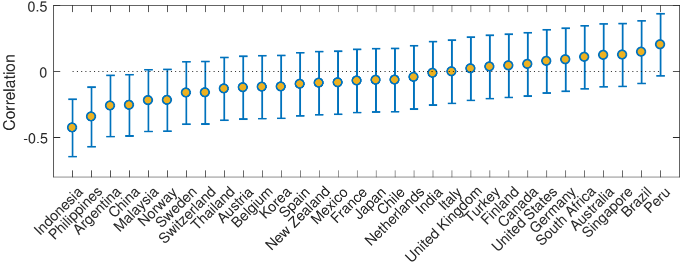
- CD and Exponent of cross-sectional dependence tests [Pesaran, 2015 and Bailey et al, 2016]
- Results in accordance with assumptions of
  - Weak/strong cross-sectional dependence of ε_{it}/u_{it}, respectively
  - Weak cross-sectional dependence of both ε_{it} and η_{it}

	CD	$\alpha_{0.05}$	α	$\alpha_{0.95}$
v_{it}	53.95	0.94	1.00	1.06
Δy_{it}	29.64	0.82	1.00	1.17
u_{it}	49.76	0.92	0.99	1.06
ϵ_{it}	5.40	0.73	0.79	0.85
η_{it}	1.09	0.50	0.59	0.68
ϵ_{it}	5.40	0.73	0.79	0.85

Within-country correlations between volatility and growth

- Estimate country models conditional on $\tilde{f}_t$ and $\tilde{g}_t$ factor

$$\begin{aligned} v_{it} &= \beta_{i,11}\tilde{f}_t + \beta_{i,12}\tilde{g}_t + lags + \eta_{it} \\ \Delta y_{it} &= \beta_{i,21}\tilde{f}_t + lags + \varepsilon_{it} \end{aligned}$$

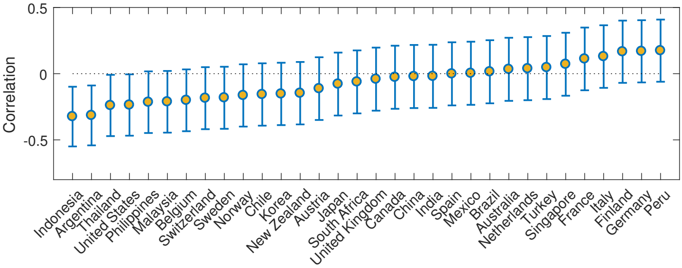


- Remarks
  - Important result: Country VCM approximately diagonal
  - Result robust to conditioning on fundamental factor $\tilde{f}_t$ only

Within-country correlations between volatility and growth conditioning on $\tilde{f}$ only

- Estimate country models conditional on $\tilde{f}_t$ factor only

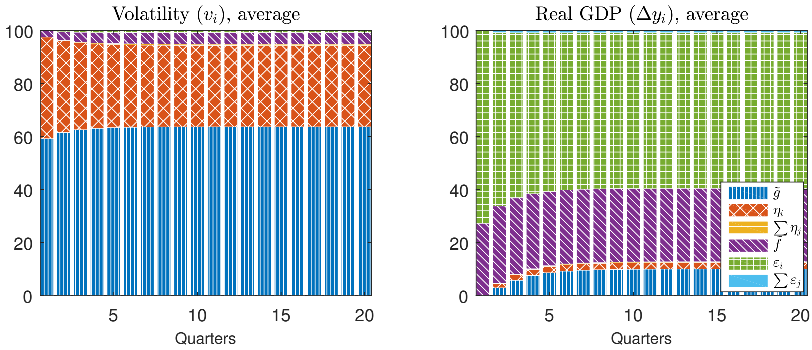
$$
\begin{aligned}
v_{it} &= \beta_{i,11}\tilde{f}_t + lags + u_{it} \\
\Delta y_{it} &= \beta_{i,21}\tilde{f}_t + lags + \varepsilon_{it}
\end{aligned}
$$



Statistically significant pairwise correlations

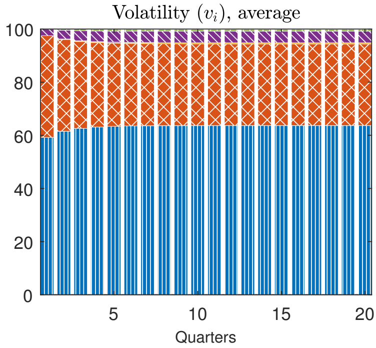
Between-county correlations			Within-county correlations
Groth-Growth Pairs	Volatility-Volatility Pairs	Volatility-Growth Pairs	Volatility-Growth Pairs
AUT GDP PHL GDP -0.43			
	BEL VOL ITA VOL 0.51		
	BEL VOL NLD VOL 0.60		
	BEL VOL CHE VOL 0.51		
	BEL VOL GBR VOL 0.54		
BEL GDP CHN GDP -0.40			
	BRA VOL MEX VOL 0.56		
BRA GDP CHN GDP -0.44			
	CAN VOL NOR VOL 0.40		
	CHN VOL FRA VOL -0.58		
	CHN VOL ITA VOL -0.42		
	CHN VOL NLD VOL -0.46		
	CHN VOL ESP VOL -0.41		
	CHN VOL SWE VOL -0.40		
	CHN VOL CHE VOL -0.45		
	CHN VOL GBR VOL -0.49		
	CHN VOL USA VOL -0.57		
CHN GDP FRA GDP -0.39			
		CHN GDP JPN VOL 0.55	
CHN GDP USA GDP -0.51			
		FIN VOL KOR GDP -0.41	
		FIN VOL TUR GDP 0.41	
	FRA VOL DEU VOL 0.50		
	FRA VOL IND VOL -0.46		
	FRA VOL IDN VOL -0.39		
	FRA VOL ITA VOL 0.46		
	FRA VOL NLD VOL 0.63		

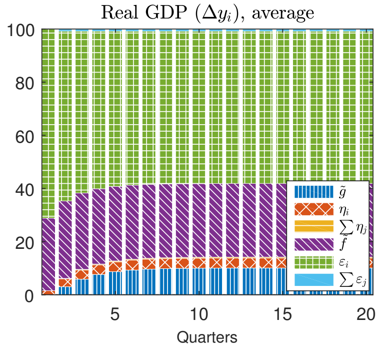
Average FEVD: Diagonal covariance matrix



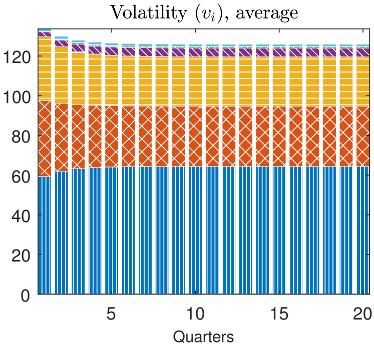
- Real factor (**purple areas**) and country specific growth innovations (**green and light blue areas**) explain less than $< 5\%$ of country volatilities
- Financial factor (**dark blue areas**) explains a significant share of growth variance (about $\sim 10\%$), but country-specific volatility shocks (**orange and yellow areas**) diversified away

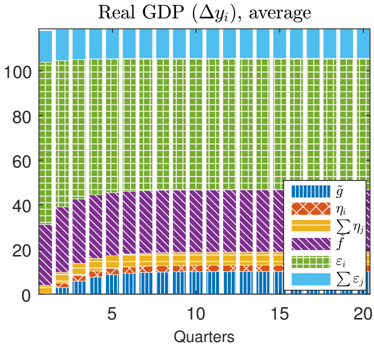
Average FEVD: Block Diagonal covariance matrix



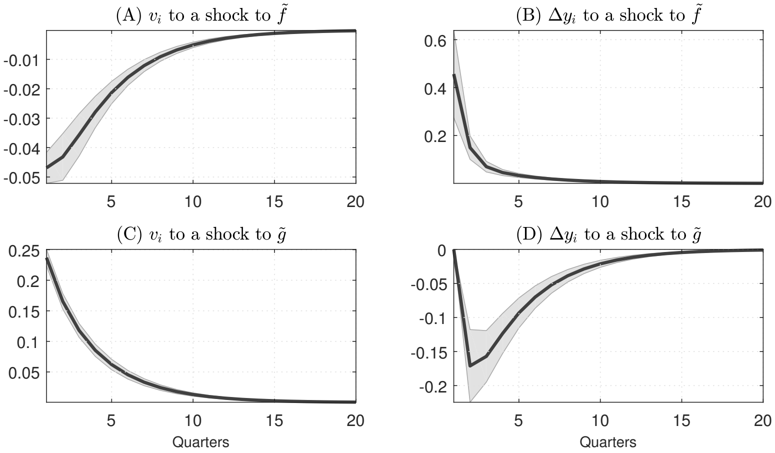


Average Generalized FEVD: Threshold covariance matrix





Shocks to the factors have expected effects



- Countercyclical volatility response to $\tilde{f}_t$ shock and recession induced by $\tilde{g}_t$ shock
- Size of volatility responses to $\tilde{g}_t$ shock larger than responses to $\tilde{f}_t$ shock, but comparable growth responses

Conclusions

- Paper takes a multi-county approach to model the relation between volatility and growth without imposing restrictions on the direction of causation
- Paper exploits the different cross-country correlation structure of volatility and growth innovations to identify a ”real” and a ”financial” factor
- Main take-aways
  - Much of the unconditional correlation between volatility and growth is driven by the real factor
  - Endogenous component of volatility small
  - Country volatility shocks wash away and only shocks to financial factor explains some share of growth variance with impact comparable to real factor when they realize

Appendix

Assumption 1: Loadings

- The factor loadings, λ_i and γ_i, are distributed independently across i and the common factors f_t, for all i and t, with non-zero means λ and γ
- Assume that

$$N^{-1}\sum_{i=1}^{N}\lambda_i^2 = O(1) \quad \text{and} \quad N^{-1}\sum_{i=1}^{N}\gamma_i^2 = O(1),$$

$$\lambda = \sum_{i=1}^{N}\mathring{w}_i\lambda_i \neq 0 \quad \text{and} \quad \gamma = \sum_{i=1}^{N}w_i\gamma_i \neq 0,$$

  for all N, and as $N \to \infty$
- **Interpretation** Factor f_t is strong (or pervasive) for both volatility and growth
  - Standard in the factor literature (see Bai and Ng 2002)
  - Factor can be estimated using principal components or the cross-section averages

Assumption 2: Weights

- Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ and $\mathring{\mathbf{w}} = (\mathring{w}_1, \mathring{w}_2, ..., \mathring{w}_N)'$ be $N \times 1$ vectors of non-stochastic weights with $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N} \mathring{w}_i = 1$

- Weights $\mathbf{w}$ and $\mathring{\mathbf{w}}$ are granular, in the sense that, for instance:

$$||\mathbf{w}|| = O(N^{-1}), \; \frac{w_i}{||\mathbf{w}||} = O(N^{-\frac{1}{2}}), \quad \forall i,$$

- **Interpretation** Granularity rules our dominance of one or more cross-section units
  - Could be problematic for realized volatility
  - We can relax this assumption to derive f, leaving volatility weights $\mathring{\mathbf{w}}$ unrestricted, but cannot make certain statements about the financial factor in the US case.

(Key) Assumption 3: Cross-section correlations

- Let the variance-covariance matrices of the $N \times 1$ error vectors $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and $\mathbf{u}_t = (u_{1t}, u_{2t}, ..., u_{Nt})'$ be $\boldsymbol{\Sigma}_\varepsilon = Var(\boldsymbol{\varepsilon}_t)$ and $\boldsymbol{\Sigma}_u = Var(\mathbf{u}_t)$, respectively

- Assume that

$$
\begin{aligned}
\varrho_{\max}(\boldsymbol{\Sigma}_u) &= O(N), \\
\varrho_{\max}(\boldsymbol{\Sigma}_\varepsilon) &= O(1).
\end{aligned}
$$

  where $\varrho_{\max}(\mathbf{A})$ denotes the largest eigenvalue of matrix $\mathbf{A}$

- **Interpretation** Weak cross-country correlation means that, as N becomes large, the average pairwise correlations of growth innovations tends to zero, since the largest eigenvalue is bounded in N.

Back

Estimating observable and orthogonal factors

- **Issue** Factors f_t and g_t are unobservable, and even if known, would be correlated with each other
- For ease of interpretation it is standard to work with the orthogonalized version of the factors
  - This task is simplified due to the triangular way the factors affect the global variables, $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$
- Proceed recursively
  - Factor f_t can be identified up to a constant

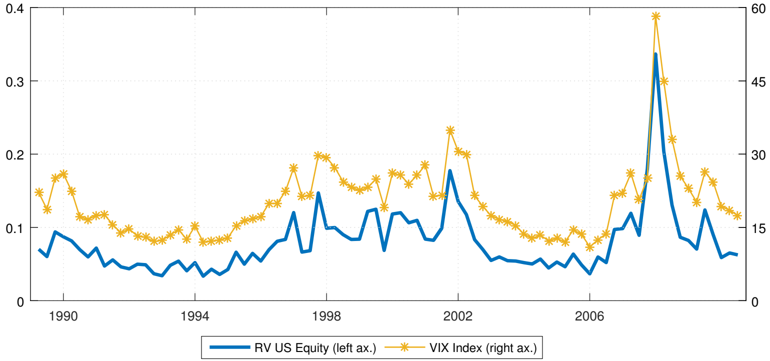
$$f_t = \frac{\Delta\bar{y}_{\omega,t}}{\gamma} \quad \Rightarrow \quad \tilde{f}_t = \Delta\bar{y}_{\omega,t}$$

  - Factor g_t can then be approximated by the residuals of a regression of world volatility $\bar{v}_{\omega,t}$ on world growth

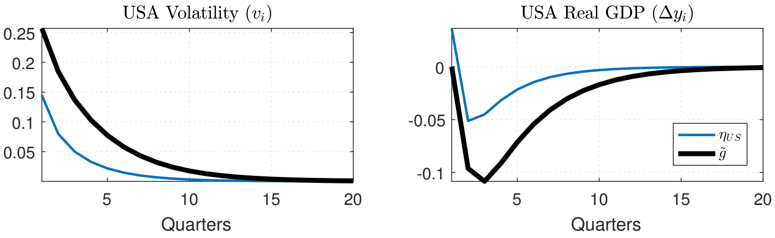
$$g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta\gamma}\Delta\bar{y}_{\omega,t} \quad \Rightarrow \quad \bar{v}_{\omega,t} = \hat{\beta}\Delta\bar{y}_{\omega,t} + \tilde{g}_t$$

Back

Comparison between VIX and US realized volatility

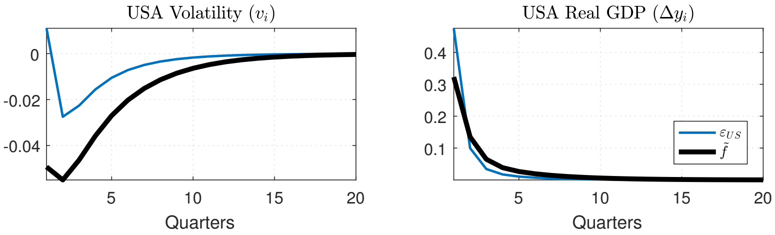


Volatility shocks in the United States have similar recessionary impacts



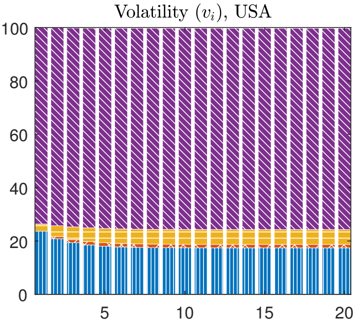
- Shocks to $\tilde{g}_t$ larger impact than shocks to η_{US}
- However, we need to be cautious with interpretation of this split
  - US might not be granular in global financial markets

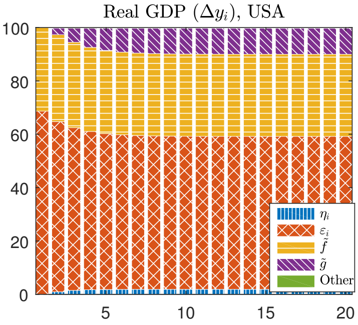
Growth shocks in the United States



- Both shocks to $\tilde{f}_t$ and ε_{US} lead to a fall in volatility, but global component has a larger effect
- Shock to $\tilde{f}_t$ has smaller impact of country specific one

Variance decomposition in the United States is similar





Country-specific response to the factors