Optimal Contracts with Randomly Arriving Tasks

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Motivation and Outline

- Long-term principal-agent relationship where the environment changes over time (random opportunities, demand shocks,...)
- Study the effect of fluctuations in the environment
- This paper: A stylized contacting problem:
  - Unique optimal contract:
    - Promotion based dynamics: Wage increases over time while effort decreases over time
    - Wage stickiness

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Model

- Principal-agent with infinite horizon; discount factor δ
- Every period:
  - Nature draws available task $i \in \mathcal{I} = \{1, \ldots, I\}$ with prob q_i
  - Agent observes i and exerts effort $e \in [0, \infty)$ on task
  - Principal observes i, e and pays wage $w \in [0, \infty)$

- Payoffs for (i, e, w):
  - Principal: $\pi_i(e) - w$
  - Agent: $g(w) - e$
  - $\pi_i'(\cdot), g'(\cdot) > 0 > \pi_i''(\cdot), g''(\cdot)$ and satisfy $\pi_i(0) = g(0) = 0$

- Additional assumptions:
  - Tasks are ordered: $\pi_{i+1}'(e) > \pi_i'(e)$ for all e
  - Interior solutions: $\pi_I'(0) > \frac{1}{g'(0)}$, $\lim_{w \to \infty} \frac{1}{g'(w)} > lim_{e \to \infty} \pi_1'(e)$

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Contracts

- History at beginning of period t: $h_t = \{i_s, e_s, w_s\}_{s<t}$
- A contract specifies:
  1. $work(h_t, i_t) \to [0, \infty)$ – (Job description)
  2. $pay(h_t, i_t, e_t) \to [0, \infty)$ – (Compensation plan)

- Principal's problem:
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Constructing the Auxiliary Problems

- Auxiliary problem $P^{(I)}$
  1. First period: task I is available
  2. Future: Tasks arrive as in the original problem
  3. Principal is restricted to contracts of the form
     - Vector of required efforts $(e_1^{(I)}, \ldots, e_I^{(I)})$
     - Fixed periodic compensation $w^{(I)}$
- Auxiliary problem $P^{(I-1)}$
  1. First period: task $I-1$ is available
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Incentive Compatibility Constraints and Solution

- Let $\lambda_i = 1 - \sum_{j>i} q_j$
- The IC constraint when task j is available in $P^{(i)}$ is

$$e_j \leq g(w) + \sum_{s=1}^{\infty} (\lambda_i \delta)^s \left(g(w) - \frac{1}{\lambda_i} \sum_{k \leq i} q_k e_k \right),$$

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Properties of the Solution(s)

Lemma

The only binding constraint in the solution to $P^{(i)}$ is $IC_i^{(i)}$.

Lemma

In the solution to $P^{(i)}$ $\pi_j'(e_j^{(i)}) \leq \frac{1}{g'(w^{(i)})}$ with equality if $e_j^{(i)} > 0$.

Lemma

The sequence $(w^{(1)}, w^{(2)}, \ldots, w^{(I)})$ is strictly increasing. (proof)

Corollary

Let $j \leq i$.

1. *For $j > 1$, $e_j^{(i)} \geq e_{j-1}^{(i)}$, with a strict inequality if $e_j^{(i)} > 0$, and*
2. *For $i < I$, $e_j^{(i)} \geq e_j^{(i+1)}$, with a strict inequality if $e_j^{(i)} > 0$.*

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Phase mechanism

- Define $\mathscr{I}(h_t; i_t) = max\{i_s : s \leq t\}$
- The Phase Mechanism is defined by:

$$work(h_t, i_t) = e_i^{(\mathscr{I}(h_t; i_t))}$$

$$pay(h_t, i_t, e_t) = \begin{cases} w^{(\mathscr{I}(h_t; i_t))} & \text{if } e_s = work(h_s, i_s) \text{ for all } s \leq t \\ 0 & \text{otherwise.} \end{cases}$$

- In each period, contract is given by solution to $P^{(\mathscr{I}(h_t; i_t))}$
- Contract exhibits downward wage rigidity and upward effort rigidity

Main Result

Proposition

Phase Mechanism is the (essentially) unique optimal contract.

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  - Concavity is what connects between periods
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  - Robustness: companion paper

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General contracting environment with symmetric info

- Dynamic contracting environment:
  - Principal and agent interact over time
  - Time is discrete, common discount factor
  - Periodic game in t is drawn from $f(h_t)$
  - h_t specifies past periodic games and actions
  - Principal can commit to a long term strategy, agent cannot
- The environment accommodates
  - "Incentivizing Randomly Arriving Tasks"
  - Labor Contracts (Harris and Holmstrom 1982, Holmstrom 1983, Postal-Vinay and Robin 2002); Dynamic Risk Sharing (Marcet and Marimon 1992, Kruger and Uhlig 2006); Foreign Investment and Entrepreneur Financing (Thomas and Worrall 1994, Albuquerque and Hopenhayn 2004); Dynamic Project Selection (Forand and Zapal 2018)
  - Other potential models with seasonal demand, R&D investments, long term projects etc.

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Results:

- Define a class of components – "convex separable activities"
- Tight condition guaranteeing that, as time goes by, these components change only in the direction that favors the agent

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Implications:

- Generalize and unify downward wage rigidity results
- Establish a general upward effort rigidity result
  - Monotonicity results in previous model are "detail free"
- New insights on foreign investment/entrepreneur financing

Proofs

- If $w^{(i+1)} \leq w^{(i)}$ then $e_j^{(i)} \leq e_j^{(i+1)}$
- Consider the continuation of $P^{(i+1)}$ when task i is available. Until the arrival of a task $l > i$:
  - The worker exerts weakly more effort than under the solution of $P^{(i)}$
  - None of the IC constraints are binding
- Compensation of strictly less than $w^{(i)}$ can incentivize weakly more effort than $\{e_j^{(i)}\}$ in auxiliary problem i. $\lightning$

(return)