Land and the Rise in the Dispersion of House Prices and Rents across U.S. Cities

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Abstract

Shelter cost is the largest component of household expenditures and there have been large swings in the two shelter costs across cities since 1980, which affect homeownership as well as living standards across cities. In this paper, I first document three stylized facts about the distributions of prices and rents across cities in the U.S.: (i) while both prices and rents vary across cities, prices are more dispersed compared to rents; (ii) the dispersion of house prices

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has increased more than the dispersion of rents from 1980 to 2010; (iii) despite the discrepancy between the two distributions, prices and rents are highly correlated both in terms of levels and growth rates. Cities with higher rents usually have even higher prices. Motivated by the fact that most owners live in detached houses while most renters live in apartments, this paper examines the implication of the land use difference between houses and apartments on these stylized facts. I develop a city-level housing tenure choice model where owner-occupied houses take more land to build compared to rental apartments. When land values vary across cities, house prices vary more compared to rents due to the intensive use of land in the construction of houses. I calibrate the model to house prices, rents, and the fraction of households living in detached/attached homes for each of the largest 181 cities in the U.S. in 1980. Feeding in the model population, income, down payment requirement, and residential land supply in 2010, I show the model can account for 82% of the large increase in house price dispersion and 56% of the moderate increase in rent dispersion. In addition, the model generates an increase in the dispersion of price-rent ratios that matches the data. It suggests the increase in the price-rent ratios in big cities can be explained by changes in economic fundamentals, which challenges the view that the price-rent ratio can be used as an indicator of housing bubbles.

1 Introduction

Shelter cost is a major part of households expenditures. Therefore, understanding why it differs across cities is important for evaluating the quality of life in different places (see, e.g., Moretti, 2013 and Albouy, 2008). As owning and renting are the most prevalent options to obtain housing services, both prices and rents have been commonly used to measure shelter costs in the literature. In this paper, I find that although prices and rents are highly correlated, the distribution of house prices across cities differs from rents. Moreover, house prices have become more dispersed across cities during the past 30-50 years compared to rents.[1] These findings suggest that using prices and rents interchangeably to measure shelter costs could be problematic. Moreover, understanding the joint distribution between the two shelter costs has important implications for housing tenure choice and household welfare in different places. In this paper, I first establish three stylized facts about the distributions of prices and rents across cities in the U.S. and then propose a mechanism that can account for these observations.

Distribution of Prices and Rents across Cities: First, while both prices and rents differ substantially across cities, price dispersion is 80% larger than rent dispersion. For instance, as Table 1 shows, the Coefficient of Variation (CV) of house prices is 0.33 while the CV of rents across cities is only 0.18 in 1980. Second, the price dispersion has increased more than rent dispersion between 1980 to 2010. The dispersion of house prices has almost doubled while the dispersion of rents has only increased by 50% (Table 1). Third, the correlation between prices and rents is high, both in levels and in growth rates. Cities that have higher (growth rates of) prices normally have higher (growth rates of) rents (Section 2).[2]

The main mechanism that I propose to account for the joint distribution of prices and rents across cities is motivated by the fact that most owners live in detached houses while most renters live in apartments. Houses and apartments differ in the use of land, which is the scarce resource in big cities. To quantitatively evaluate the implication of land use

[1] Gyourko, Mayer, and Sinai (2013) and Van Nieuwerburgh and Weill (2010) among others document the significant increase in house price dispersion during the past 30-50 years.

[2] Appendix A.1 provides the density plot of prices and rents.

difference between houses and apartments on the joint distribution between prices and rents, I develop a city-level housing tenure choice model where owner-occupied houses take more land to build compared to rental apartments. The model is calibrated to the distribution of house prices and rents across the cities in the year 1980. Feeding into the model factors that affect the housing demand and residential land supply in 2010, I show that the model can account for the majority of changes in the dispersion of prices and rents from 1980 to 2010.[3]

Table 1: First and Second Moments of Prices and Rents in the data

Variable [a]	Obs.	1980		2010	
		Mean	CV	Mean	CV
Price	181	120900	0.33	146270	0.62
Rent	181	6038	0.18	6264	0.27

[a]Prices are measured by the trimmed mean value of detached houses. Rents are measured by the trimmed mean rent of 2 bedroom apartment. More details about the data can be found in Section 2.1.

According to the American Housing Survey (2009), around 93% of owner-occupied homes are detached with a median lot size of 14,000 square feet (sqf), while the majority of renters (around 63%) live in multifamily apartments with a median land usage of 367 sqf.[4] In other words, the "land footprint" of owner-occupied homes is much larger than that of rental apartments. Moreover, in cities where land is expensive, apartments economize on land use by building up. However, detached houses cannot have less land than the size of the first floor and can be constrained by the minimum lot size requirement imposed by zoning regulations. As a result, the cost of building houses is disproportionally higher compared to apartments in these cities due to the heavy reliance on land as an input.

To quantitatively evaluate the implication of different land use between houses and apartments on the dispersion of prices and rents as well as their changes over time, I develop a city-level housing tenure choice model with competitive housing supplies.

[3]I choose the year 2010 in the simulation for two reasons. First, the dataset I am using adopts a consistent definition of metropolitan areas from 1980 to 2010. Second, during the housing boom period, house prices may contain bubbles, i.e. part of the house price growth cannot be rationalized by economic fundamentals. As this paper focuses on the impact of economic fundamental in the steady states, I choose the year 2010 in the simulation instead of the years around 2007.

[4]Author's calculation. Land use per unit is defined as the ratio between unit size and the number of floors in the multifamily building.

The key feature is that while both owner-occupied houses and rental apartments are constructed using land and material, owner-occupied houses are more land-intensive.

The model allows me to examine how the land values are determined by economic fundamentals (i.e., income and population) and land supply, through calibration in the baseline year 1980. Then I test the performance of the model by feeding in the economic fundamentals and land supply in 2010 to see whether it can account for changes in the distributions of prices and rents from 1980 to 2010.

In the model, each city is an isolated island inhabited by finitely-lived households. Households choose between purchasing a house and renting an apartment, as well as the size to buy/rent in each period. A down payment is required for households who purchase a house. Competitive construction firms build houses and apartments using land and material. A key model feature is that the production function for houses has a higher land share compared to apartments. In addition, houses are subject to a city-specific minimum lot size requirement, which specifies a lower bound on the land input per house. The total amount of land available for residential construction in each city is assumed to be exogenous.

The higher land intensity in building houses allows the model to endogenously generate the feature that prices increase more in land values than rents do, consistent with the empirical evidence. In cities where the price of land is high, both house and apartment developers substitute expensive land with relatively cheap material. However, the reduction in land used for houses is smaller than apartments for two reasons. First, the higher land share in the production function of houses implies that land and material are less substitutable, i.e., the ratio between the marginal productivity of land and material is larger in producing houses compared to apartments for any land-material input ratio. Second, the minimum lot size requirement puts a floor on the land use per house. As a result of the intensive use of land, the total cost of building houses, which equals the house price in a competitive market, grows more than that of apartments, which equals the net present value of rents, when land value rises. From the cross city perspective, when the distribution of land values across cities changes, the dispersion of prices should change more than rents.

The model is first calibrated to house prices, rents, and the fraction of households living in detached or attached houses for each of the largest 181 cities in the U.S. in 1980.[5] My main numerical exercise is to change population, income, residential land supply, and the down payment requirement to the 2010 level to examine whether the model can account for the large increase in price dispersion and the moderate increase in rent dispersion at the same time.

I find that changes in income, population, residential land supply, and the down payment requirement between 1980 and 2010 can account for 82% of the increase in price dispersion and 56% of the increase in rent dispersion. The model captures the changing relationship between prices and rents as well. It can also account for 90% of the increase in the dispersion of price-rent ratios across cities from 1980 to 2010, which implies that most of the rise in the price-rent ratios in large cities can be attributed to these fundamentals. This challenges the conventional view that a rise in the ratio of house prices to rents signals a housing bubble.

This paper provides additional insights on the interaction between owner-occupied and rental markets, as well as the dynamics of prices, rents, and homeownership rates.[6] As households' owning/renting decision depends on both prices and rents, understanding how prices and rents interact is important for estimating the response of aggregate housing demand to policy changes. The previous literature largely adopts one of the two frameworks. In models where rental units are provided by risk-neutral real estate firms (Gervais, 2002; Yang, 2009), rents change at the same rate as prices. In models where the supply of rental units is endogenously provided by the owners (Chambers, Garriga, and Schlagenhauf, 2009; Sommer, Sullivan, and Verbrugge, 2013; Favilukis and Van Nieuwerburgh, 2017), price and rent may change in different directions. For instance, an increase in the credit supply may lead to higher prices and more rental supply, which contributes to lower rents. This paper examines another channel, the role of different land intensities between houses and apartments, which delivers the positive correlation between prices

[5] Appendix A.3 provides a full list of the 181 cities.

[6] For instance, Greenwald and Guren (2019) argue that perfect segmentation of these two markets implies a large impact of credit supply while complete integration implies no impact of credit supply on price.

and rents while allowing the growth rates of prices and rents to differ.

The mechanism proposed in this paper helps reconcile the documented difference in price and rent elasticity with respect to either demand shocks or supply changes. For instance, Saiz (2007) finds that the impact of changing income or immigration on median prices is 40% to 80% larger than that on median rents. Focusing on the impact of relaxing credit constraints, Greenwald and Guren (2019) find that the elasticity of prices to credit is between 0.30 to 0.38 while the elasticity of rents is between 0.21 to 0.26. In addition, Parkhomenko (2018) finds that the elasticity of prices with respect to land scarcity is 0.051, which is twice as big as the elasticity of rents with respect to land scarcity, 0.024. Aladangady, Albouy, and Zabek (2017) document that inequality in house prices has risen more compared to inequality in rents since 1980. The mechanism proposed in this paper suggests that in the long-run equilibrium, demand for houses and apartments are aggregated into demand for land through different production functions. The higher land share in producing houses implies that price responds more than rent when the underlying land value changes due to either changes in total land demand and/or supply.

This paper builds on a growing literature which examines alternative explanations for the rising dispersion of house prices across U.S. cities. Gyourko, Mayer, and Sinai (2013) discuss the impact of sorting on preference towards specific locations on housing price dispersion across cities. Van Nieuwerburgh and Weill (2010) explore how sorting on ability, i.e., people with high ability moves to more productive places, drives up price dispersion across cities. Noticeably, Van Nieuwerburgh and Weill (2010) over-predict the increase in the rent dispersion as they do not distinguish owner-occupied from rental units. In this paper, I consider another type of dwelling which is less land-intensive and only available for rent. Such an extension allows me to complement the previous literature by accounting for both the substantial rise in the price dispersion and the simultaneous moderate increase in the rent dispersion.

Closest in spirit to this paper are Davis and Heathcote (2007), Davis and Palumbo (2008), and Larson et al. (2019), who estimate residential land values using data on home values and costs of housing structures on county-, city-, and country-level.[7] They find

[7]The county-level average price of land used in singe-family housing is available from 2012 to 2018.

that land prices have become more important in determining house prices over the past three decades. This paper differs in modeling the demand and supply of the land market explicitly. The housing tenure choice model developed in this paper maps economic fundamentals to the demand of houses and apartments, which is then translated into the demand of land via production functions. The upside of working with this equilibrium model is that it allows me to evaluate the impact of changes in economic fundamentals on the land values which determine house prices and rents.

The mechanism proposed in this paper allows for a flexible relationship between prices and rents, which is important for explaining the cross-city variation in the price-rent ratios. Previous studies that treat rents as the dividends of housing assets find that the standard asset-pricing approach does not work well to account for the variation in price-rent ratios across cities. Therefore, Glaeser and Gyourko (2007) argue that owner-occupied houses and rental units differ, such that prices and rents are not directly comparable. However, I find that prices and rents are highly correlated both in levels and in growth rates. The mechanism that I propose allows a non-linear relationship between prices and rents. It rationalizes the high correlation (> 0.7) between prices and rents. As both houses and apartments are constructed using the same inputs, land and material, the equilibrium house prices and apartments rents both depend on land values and material values. Meanwhile, it allows price and rent to diverge when land value changes. Due to the higher land share in building houses, the equilibrium house prices change more with land values compared to rents.

My study also complements the literature on the estimation of housing production functions. Previous work focuses on estimating the production function of single-family detached homes (Albouy, Ehrlich, and Shin, 2018; Epple, Gordon, and Sieg, 2010). In addition to single-family detached homes, I estimate the production function of multi-family apartments and find that the land share in constructing houses is almost twice as big as the land share in the construction of multi-family buildings.

The remainder of the paper is organized as follows. Section 2 provides empirical

The city-level price of residential land is available from 1984 to 2014. The country-level price of residential land is available at ten year intervals between 1930 and 2000, and annually from 1975 to 2006.

evidence. Section 3 lays out the model. Section 4 describes the calibration strategy. Section 5 discusses the calibration results. Section 6 presents and discusses the quantitative exercise. Section 7 concludes.

2 Empirical Evidence

This section documents empirical evidence supporting the main mechanism explored in the paper. First, I show the two shelter costs, prices and rents, are highly correlated both in levels and in growth rates. Cities that have higher (growth in) prices tend to have higher (growth in) rents. Second, I examine the implications of the main mechanism. Specifically, the higher land share in the production function of owner-occupied houses implies that (1) the ratio between price and rent is increasing in land value and (2) the cost of building houses grows more than the cost of building apartments when land value changes. The data confirms these two implications. I find that the price-rent ratio is increasing in land values. Moreover, for around 30 cities with a sufficient number of condo owners, the price growth for single family detached homes is higher compared to condos from 1980 to 2010. In addition, as land plays an important role in my mechanism, I examine the use of land between houses and apartments across cities. I find that as population density, i.e. an approximation for land scarcity, increases, both houses and apartments use less land: houses have smaller lots while apartments increase the number of stories. On average, apartments reduce land use more compared to houses.

2.1 Data

Apartment rents: I construct an estimate of the rent of a standard two-bedroom apartment using data from the 1980 Census and the American Community Survey in 2010. To eliminate the influence of rents of luxury and low-quality apartments, I use the trimmed mean and discard the top and bottom 5 percent.

House prices: To construct an estimate of house prices, I combine the mean value of single-family homes from the 1980 Census with the Freddie Mac Conventional Mortgage Home Price Index (CMHPI), an index based on repeated sales to approximate the house price of constant quality, following Van Nieuwerburgh and Weill (2010).[8] To be consistent with the rent data, I use the trimmed mean house value in 1980 with the top and bottom 5 percent discarded.[9]

House price and apartment rents are deflated by the national Consumer Price Index (CPI).[10]

Characteristics of houses and apartments: Land use and the unit size for houses and apartments are computed using data from the American Housing Survey (AHS), 2009 National Sample.[11] The AHS provides detailed information on the year of construction, type of the dwelling (e.g., detached, attached, or multi-family buildings), lot size, unit size, number of stories, and housing tenure status.

2.2 Correlation between Prices and Rents

The correlation between prices and rents is high both in levels and in growth rates. Figure 1 (a) displays the correlation between prices of detached houses and rents of apartments across MSAs in the year 1980. The correlation between these two series is 0.71. This positive correlation is persistent over time and rose to 0.90 in 2010. Figure 1 (b) plots the growth rates of apartment rents against the growth rates of house prices from 1980 to 2010. The correlation is 0.7. Specifically, a 1% increase in house price is associated with a 0.48% increase in apartment rent.

2.3 Price-rent ratios and Land Values

One key model prediction is that price grows more than rent when the cost of land increases as owner-occupied houses are more land-intensive than rental apartments. Figure 2 plots the ratios between prices and rents against the transaction-based land values provided by Albouy, Ehrlich, and Shin (2018) in 2010. Each point corresponds to one city.

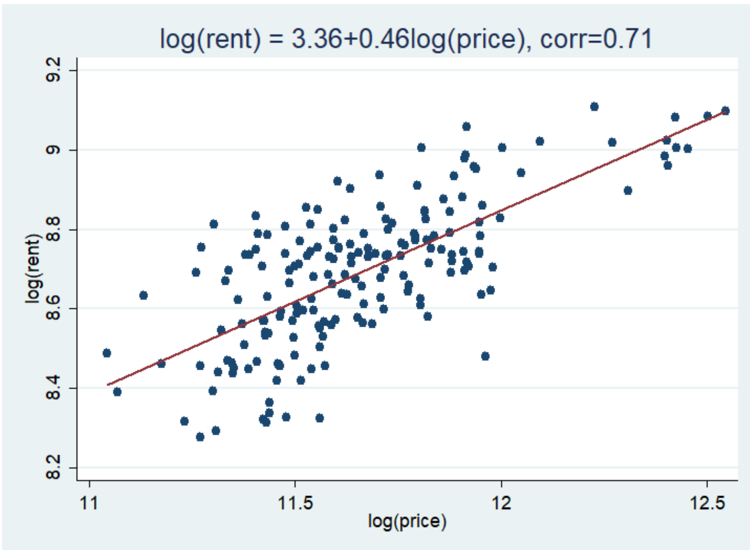
[8]The CMHPI is a quarterly index. I get the annual index by taking the average of the four quarters.

[9]I use trimmed mean house values and apartment rents instead of the commonly used median as house values and contract rents reported in Census were categorical in 1980.

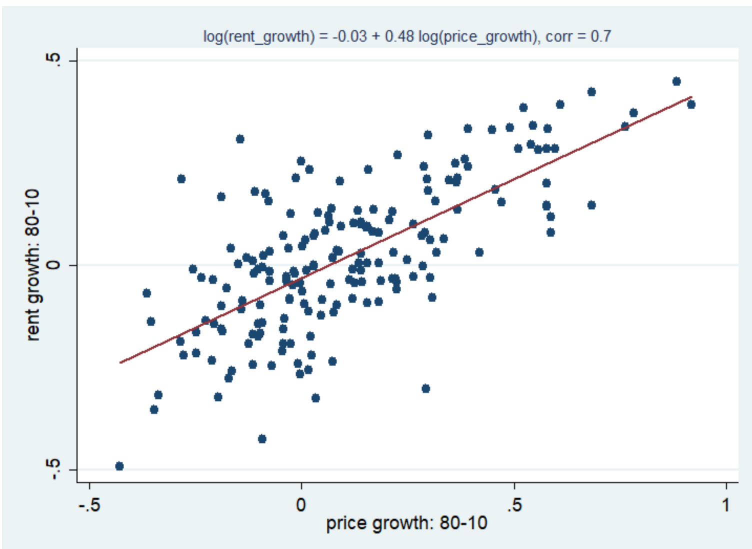
[10]As I focus on the dispersion of prices and rents, which is measured by the Coefficient of Variation, using GDP deflator to adjust would not affect the results.

[11]AHS 2009 is subtracted from National Microdata from the Inter-university Consortium for Political and Social Research (ICPSR).

Figure 1: Correlation between House Prices and Apartment Rents



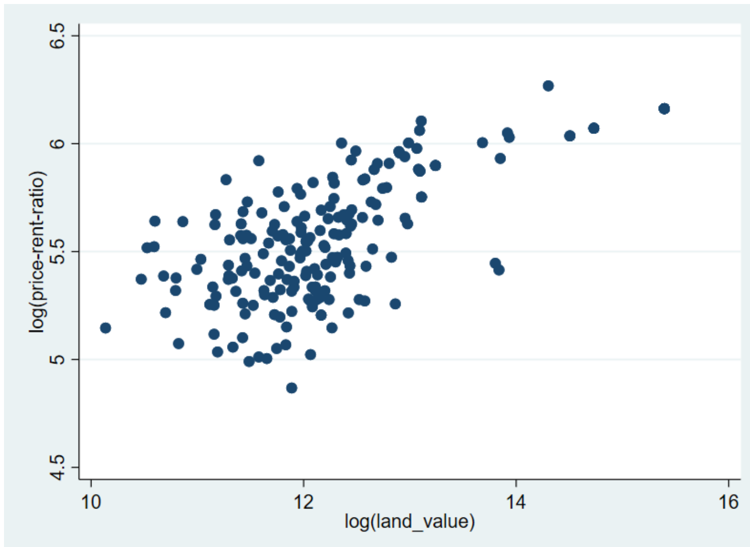
(a) Level in 1980



(b) Growth Rates: 1980-2010

The correlation is positive (around 0.6) and significant, indicating that price grows more than rent with land values, which is consistent with the implication of the mechanism.

Figure 2: Price-rent Ratio and Land Values



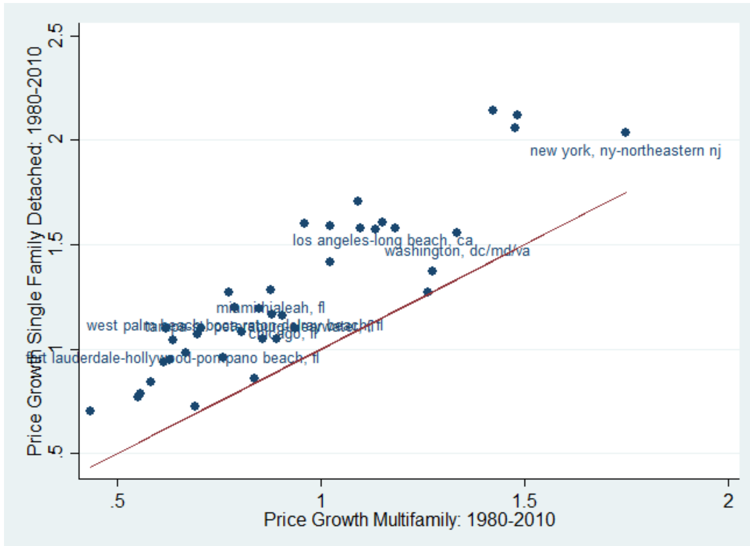
2.4 Price Growth: Houses and Apartments

I have been comparing the price of houses with the rent of apartments due to the fact that most owners live houses while most renters live in apartments and we want to understand the relationship between these two shelter costs.[12] A more direct comparison in prices between houses and apartments would allow us to examine the importance of the difference in land intensities in determining the values of these two types of dwelling. Figure 3 displays the mean price growth for houses and apartments that were built between 1950 to 1970 for cities where we observe owner-occupied houses and owner-occupied apartments at the same time.[13] A 45-degree line is included to facilitate the comparison. Consistent with the implications of the mechanism, price growth for the land-intensive single-family detached houses is larger than that for multi-family apartments for all cities in the sample from 1980 to 2010.

[12]For instance, even in the biggest city, New York City, 55% of the owners live in houses, and 94% of the renters live in apartments.

[13]Due to the limited stock of owner-occupied condos in the 1980, a small numbers of high-quality expensive apartments built in recent years will significantly affect the mean/median value of owned condos. Therefore, I exclude apartments and houses that are built in recent years.

Figure 3: Price Growth: Houses and Apartments



2.5 Land Use in Houses and Apartments

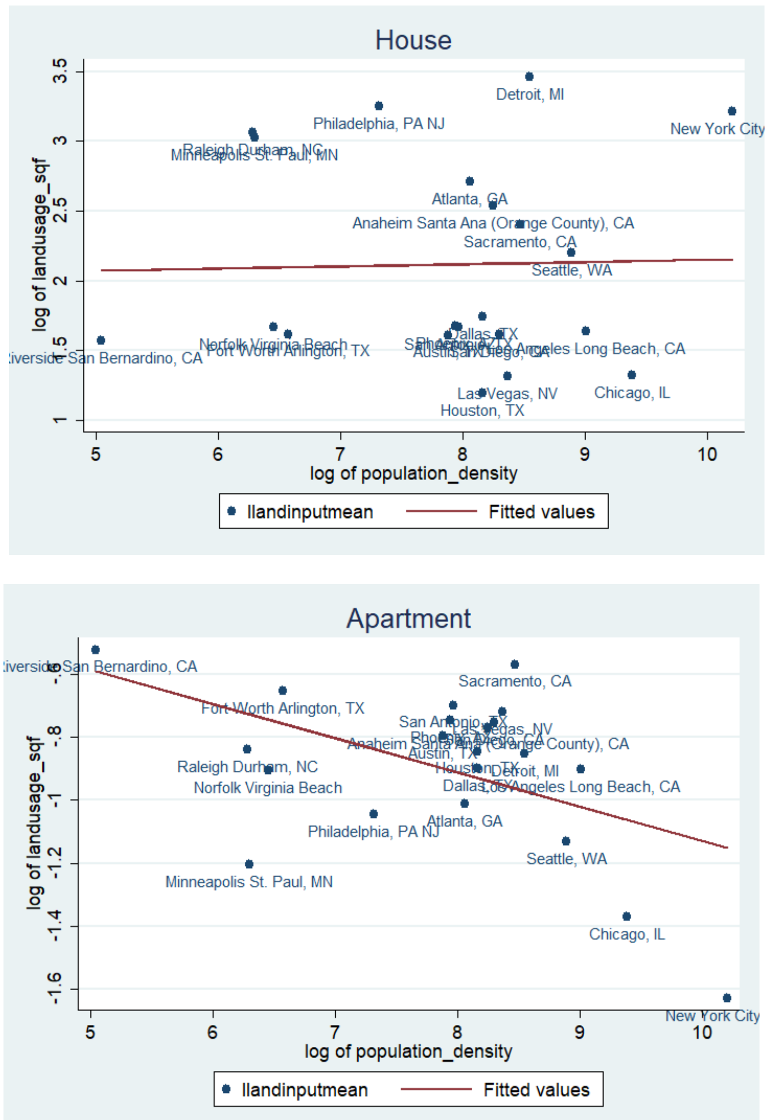
According to the 2009 American Housing Survey, 95% of owners live in a detached or semi-detached house, while 63% of renters live in a multi-family apartment. As land is the key component in my mechanism, I compare the land use of houses and apartments across cities.[14] I find that land input in building apartments is more responsive to land scarcity, which is approximated by the population density, compared to houses.

Figure 4 plots the land input in building one square foot of living space against local population density for houses in panel (a) and apartments in panel (b) in 2010.[15] A 1% increase in local population density is associated with a 10% decline in the land use for building one square foot living space for apartments. Meanwhile, the land input response against population density is not very significant in house constructions.

[14] Appendix A.2 presents more evidence about the land use in different cities and its change over time for houses and apartments.

[15] The land input is measured by the one divided by the number of stories in the building for apartments and lot size divided by the unit size for houses.

Figure 4: Land Input against Population Density: Houses vs Apartments



3 Model

The mechanism suggests that prices and rents differ across cities as underlying land values vary. In addition, the distributions of prices and rents across cities differ due to the differences in land intensities. To quantify the difference in land intensities between houses and apartments and to investigate how land values are determined, I develop a city level housing tenure choice model. I use this model to answer the following questions. First, what factors determine the land values? Second, can change in these factors across cities predict the change in land values that can account for the rise in the dispersion of prices and rents at the same time?

3.1 Model Overview

Each city is an isolated island. In other words, households do not move across islands. Cities differ in total population, income, land supply, material cost, the ownership premium of the local residents, and the minimum lot zoning regulation.

On each island, households live up to J periods. Households are ex-ante identical. They maximize their expected discounted lifetime utility from the consumption of a non-durable good and housing services. At the beginning of each period, households receive income that depends on their age, the city they live in, and an idiosyncratic shock.

To obtain housing services, households choose between buying a house and renting an apartment. Owners get additional utility from living in owned houses compared to renters. Only owners can borrow against (a certain fraction of) the value of their housing asset.

Houses and apartments are produced using land and material but via different production technologies. Houses are more land-intensive. In addition, the construction of houses is subject to a city-specific minimum lot size requirement, which places a lower bound on the amount of land input.[16] Following Albouy and Ehrlich (2018) and Epple, Gordon, and Sieg (2010), I assume that the markets of houses and apartments exhibit perfect competition. As a result, the equilibrium price of a house equals to the total con-

[16] This captures zoning laws, see e.g., Isakson, 2004; Bucovetsky, 1984.

struction cost. Similarly, the net present value of rents equals to the total construction cost of the apartment.

3.2 Households

3.2.1 Preference

Each household has preferences defined over a non-durable good and housing service represented by:

$$\sum_{j=1}^{J} \beta^{j-1} (\prod_{l=1}^{j-1} \psi_l) u(c_j, s_j) \tag{1}$$

where β is the discount factor, ψ_l is the probability that a household survives from age j to age $j+1$, c_j is consumption, and s_j is housing service received at age j. The $u(.)$ is a C^2, increasing, and concave function.

A household can obtain housing service by buying a house or renting an apartment.

$$s = \begin{cases} h & \text{if Rent} \\ \theta_k h \mathbb{1}_{j \geq j_o} \zeta & \text{if Own} \end{cases}$$

Owners on island k derive additional utility θ_k compared to renters. θ_k is supposed to capture the quality difference between a standard owner-occupied house and a standard rental apartment and how residents in that city evaluate this quality difference. Similar to Fisher and Gervais (2011), I assume that old households, i.e., over age j_o, discount ownership premium by a factor ζ. Senior households may not enjoy big houses as much as young households due to smaller family size, and the fact that houses with yards involve heavy housework such as cleaning the snow or maintaining the lawn.

3.2.2 Household Income

Households from island k receive exogenous household income that depends on their age, location, and an idiosyncratic shock. The household income of a household i of age j on island k is represented by

$$lny^i_{j,k} = ln(w_j) + ln(\bar{w_k}) + log(\epsilon^i_j) \quad (2)$$

where w_j is the average household income of an age j household, $\bar{w_k}$ is the average income of households on island k relative to the average income of the whole economy, and ϵ^i_j is the idiosyncratic shock that follows an AR(1) process.

$$ln(\epsilon^i_j) = \nu ln(\epsilon^i_{j-1}) + \xi_{i,j}, \xi_{i,j} \sim N(0, \Sigma^2_y) \quad (3)$$

The AR(1) process is the same for households of all ages and on all islands.

3.2.3 Asset Arrangement

Following the literature, I assume that only collateralized credit is available.[17] The borrowing interest (i.e., the interest on mortgages) equals the deposit interest r plus a spread r_m. The net asset position is denoted by a. To buy a house, a household must satisfy a minimum down payment requirement of γ. That is, a household's financial asset always satisfies

$$a \geq -(1-\gamma)P(h) \quad (4)$$

where $P(h)$ represents the price of a size h house.[18]

A newborn household has no housing asset and draws his/her initial wealth from a probability distribution Π_w defined on $\mathbb{R}_+$.

3.2.4 Costs Related to Owner-Occupied Houses

It is costly to buy or sell a house. These costs include the opportunity cost of time associated with the market search, brokerage, and moving, as well as legal fees. Following the literature (see e.g., Sommer, Sullivan, and Verbrugge, 2013), I assume that the transaction costs are proportional to the value of the house. Specifically, a buyer incurs

[17] See e.g., Yang (2009) and Sommer, Sullivan, and Verbrugge (2013).

[18] Equation 13 shows that due to the minimum lot size requirement, the price of a house is not linear in its size for the whole support.

a total transaction cost of $k_b P(h)$, and a seller incurs a total transaction cost of $k_s P(h)$. In addition, houses depreciate at a rate δ, each year.

3.2.5 Household's Recursive Problem

At the beginning of each period, the state variable of a household in one city is given by (h, a, ϵ, j), which correspond the current housing stock, financial stock, income shock, and age, respectively.

Owner's Problem An owner enjoys his/her current housing stock h, and chooses consumption c, future asset a', and future housing stock h' to maximize his/her expected value

$$V(h, a, \epsilon, j) = max_{c,a',h'} u(c, h) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(h', a', \epsilon', j+1) \tag{5}$$

subject to

$$\begin{aligned}
&(k_b P(h') + k_s P(h)) \mathbb{1}_{h \neq h'} + c + P(h') + a' + \tau P(h) = (1 - \tau_w) w_j \bar{w}_k \epsilon + \\
&\mathbb{1}_{a \geq 0}(1 + r)a + \mathbb{1}_{a \leq 0}(1 + r + r_m)a + (1 - \delta)P(h) \\
&a' \geq -(1 - \gamma)P(h') \\
&c \geq 0
\end{aligned} \tag{6}$$

where k_b and k_s are the transaction cost on buyers and sellers, respectively. τ is the property tax on owners, r is the real interest rate, and $r + r_m$ is the mortgage interest. τ_w is the income tax rate, and δ is the depreciation rate on houses.

Renter's Problem A renter with current housing stock $h = 0$, chooses consumption c, future housing stock h', future asset a', and the size of apartment to rent in the current period h^r to maximize his/her value

$$V(0, a, \epsilon, j) = max_{c,a',h',h^r} u(c, h^r) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(h', a', \epsilon', j+1) \tag{7}$$

subject to

$$\begin{aligned} & c + (1 + k_b)P(h') + R(h^r) + a' = (1 - \tau_w)w_j\bar{w_k}\epsilon + (1 + r)a \\ & a' \geq -(1 - \gamma)P(h') \\ & c \geq 0 \end{aligned} \tag{8}$$

where $R(h^r)$ is the total rent which is a function of the apartment size.

3.3 Housing Supply

Owner-occupied houses and rental apartments are produced through Cobb-Douglas production functions that differ in land shares.[19] Moreover, the construction of houses is subject to a city-specific minimum lot size requirement, which may prevent developers from reducing land inputs when land prices are high. The land is owned by absentee landlords who consume the profit from selling land.

Figure 5 displays the market structure. Each island has a continuum of competitive developers that purchase material and land to construct houses and apartments. Developers are price-takers. I assume that developers on island k can purchase material at an island-specific constant price ϕ_k. The supply of land is given exogenously with the price of land q_k adjust to clear the land market.

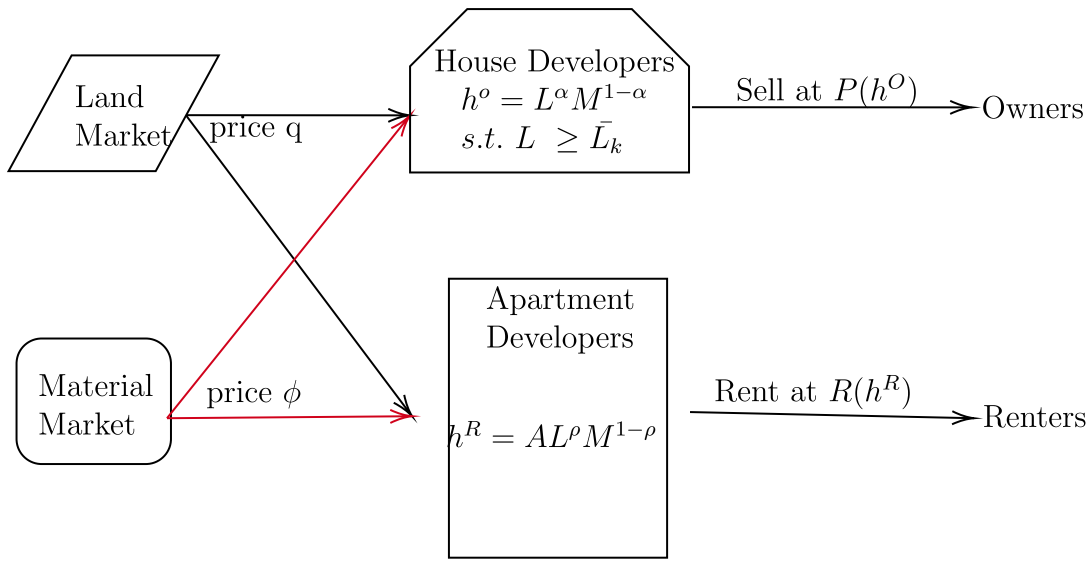
Houses are constructed through a production function with a land share α.

$$h^O = L^\alpha M^{1-\alpha} s.t. \quad L \geq \bar{L_k} \tag{9}$$

To produce a house of size h^O, a developer needs L units of land input and M units of material input. The land input L (lot size of a house) cannot fall below the minimum lot size $\bar{L_k}$ in city k.

[19]The substitutability between land and other inputs (material) is a common assumption in the literature, see e.g., Albouy and Ehrlich (2018), and Epple, Gordon, and Sieg (2010). Moreover, Larson et al. (2019) find that land prices tend to rise faster than house prices, which supports the functional form used in this paper.

Figure 5: Housing Market Structure



Apartments are built using a production function with land share, ρ.

$$h^R = AL^\rho M^{1-\rho} \tag{10}$$

where A is the relative productivity of apartments compared to houses. A captures the fact that a standard rental apartment uses less material and less land compared to a standard house due to the physical difference between these two types of dwellings.[20]

To build a house of size h^O, developers take the price function $P_k(.)$, land price q_k, and material price ϕ_k as given, and choose land input and material input to minimize the cost of each house they build. Upon finishing, developers sell the house to owners.

$$\begin{aligned} &\min_{L^o, M^o} q_k L^o + \phi_k M^o \\ \text{s.t. } & h^o = (L^o)^\alpha (M^o)^{1-\alpha} \\ & L^o \geq \bar{L}_k \end{aligned} \tag{11}$$

Similarly, apartment developers choose land and material input to minimize the cost

[20] $A > 1$ implies that using the same amount of land and material, developers can produce more apartments than houses due to the fact that apartments are smaller. A captures the structural difference between a standard owner-occupied house and a rental apartment, while θ_k in the utility function captures how people treat this difference.

of each apartment they produce. Upon finishing, developers rent the apartments to renters and collect rents every period.

$$
\begin{aligned}
&\min_{L^r, M^r} q_k L^r + \phi_k M^r \\
&\text{s.t. } h^r = A(L^r)^{\rho}(M^r)^{1-\rho}
\end{aligned}
\tag{12}
$$

3.4 Characterization of Stationary Competitive Equilibrium

As cities are isolated (i.e. households cannot move across cities), each city is a closed economy that is described by a unique competitive equilibrium. Appendix A.4 provides the definition for the stationary competitive equilibrium. This section focuses on characterizing the equilibrium.

The competitive market assumption implies zero profit for developers. The price of a house equals the total cost of building it. Solving the house developers' maximization problem (Equation 11) delivers the following price function of house size.

$$
\begin{aligned}
P_k(h^o) &= f(\alpha) q_k^{\alpha} \phi_k^{1-\alpha} h^o \quad if \quad \left(\frac{\alpha \phi_k}{(1-\alpha) q_k}\right)^{1-\alpha} h^O \geq \bar{L}_k \\
&= q_k \bar{L}_k + \left(\frac{h^o}{\bar{L}_k^{\alpha}}\right)^{\frac{1}{1-\alpha}} \phi_k \quad otherwise
\end{aligned}
\tag{13}
$$

where $f(\alpha) = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha}$. The lot size of a house or the land input depends on the relative price of material and land, as well as the house size. When the value of land is high compared to material, developers substitute land with materials, until the minimum lot size binds. The minimum lot size may also bind if the house size is too small such that not much land is required. When the minimum lot size does not bind, as the production function is constant return to scale, the price is linearly increasing in house size. When the minimum lot size binds, the price function is convex in house size. The minimum lot size puts a lower bound on the price function, which equals the cost of buying the minimum lot.

The corresponding optimal land use for a house of size h^o on island k is

$$
\begin{aligned}
L_j^o(h^o) &= {\frac{\alpha\phi_k}{(1-\alpha)q_k}}^{1-\alpha} h^o \;\text{ if }\; {\frac{\alpha\phi_k}{(1-\alpha)q_k}}^{1-\alpha} h^o \geq \bar{L_k} \\
&= \bar{L_k} \;\text{ otherwise}
\end{aligned}
\tag{14}
$$

Assuming apartment developers are risk neutral, the zero profit condition for rental units implies that the net present value of future rent flows equals the construction cost of that unit.

$$
\frac{(1-\delta_r-\tau)R(h^R)}{r} = \frac{f(\rho)q_k^{\rho}\phi_k^{1-\rho}h^r}{A} \tag{15}
$$

where δ_r is the management cost of rental units and τ is the property tax. The management cost includes salaries, insurance, utilities, management fee, administrative, marketing, contract services, and repair/maintenance (Goodman, 2004).

The corresponding optimal land use of an apartment of size h^r is

$$
L_k^r(h^r) = {\frac{\rho\phi_k}{(1-\rho)q_k}}^{1-\rho} h^r \tag{16}
$$

The literature on housing tenure choice typically assumes that both price and rent increase linearly in house/apartment size. To prevent households from buying a very small home and to match homeownership rates, a common assumption is that there is a minimum house size for owner-occupied dwellings (see e.g., Chambers, Garriga, and Schlagenhauf, 2009; Sommer, Sullivan, and Verbrugge, 2013). This paper assumes a minimum lot size constraint, which is qualitatively equivalent to a minimum house size constraint.[21] Such a modification allows prices to grow at an increasing rate with land values when land values are high.

A 1% increase in land price leads to an increase in price bounded from below by α%, and a ρ% growth in rent.

Proof According to Equation 15, $\frac{\partial lnP_k(h^o)}{\partial lnq_k} = \alpha$ if ${\frac{\alpha\phi_k}{(1-\alpha)q_k}}^{1-\alpha} h^o \geq \bar{L_k}$, that is, if

[21] Equation 14 presents the linear relationship between land usage and house size. Conditional on land price and material price, minimum house size binding is equivalent to the minimum lot size binding.

minimum lot size does not bind, a 1% increase in the land price q_k leads to an $\alpha\%$ increase in the price of a fixed-quality house.

When it binds, $\frac{\partial lnP_k(h^o)}{\partial lnq_k} = \frac{q_k L_k^{1/(1-\alpha)}}{q_k L_k^{1/(1-\alpha)} + \phi_k (h^o)^{1/(1-\alpha)}} > \alpha$ as $\frac{\alpha\phi_k}{(1-\alpha)q_k}^{1-\alpha} h^o < \bar{L}_k$, the price grows at a rate greater than α.

On the other hand, if $\frac{\partial lnR_k(h^r)}{\partial lnq_k} = \rho$, a 1% increase in land price leads to a $\rho\%$ increase in the rent of a fixed-quality apartment.

Proposition 3.4 implies that as long as $\alpha \geq \rho$, when land price changes, the price of a fixed-quality house changes more than the rent of a fixed-quality apartment. Proposition 3.4 formalizes the idea that prices may diverge from rents when land value changes due to the intensive use of land in building houses.

The total land used for constructing houses and apartments equals to the exogenous supply of land, LS_k.

$$\int L^o(h^o)H^o d(h^o) + \int L^r(h^r)H^r d(h^r) = LS_k \tag{17}$$

where $L^o(h^o)$ is the land used for constructing a house of size h^o. H^o is the number of households who desire owning a house of size h^o given the equilibrium price function $P^*(h^o)$. Similarly, $L^r(h^r)$ is the land used for constructing an apartment of size h^r. H^r is the number of renters who desire renting an apartment of size h^r given the equilibrium rent function $R^*(h^r)$. The land market clearing condition pins down the equilibrium land price q_k^*.

4 Calibration Strategy

The model is calibrated in three stages. In the first stage, values are assigned to parameters that can be determined directly from the data or from the literature. In the second stage, parameters in the production functions are estimated through an Instrumental Variable (IV) approach. In the last stage, city-specific parameters are calibrated to homeownership rates of different age groups. Due to the model's assumption that owners live in houses and renters live in apartments, the homeownership rates in the model are calibrated to the fraction of households living in detached/attached homes in the data.[22] Given this paper has a special focus on the land market and the high correlation between homeownership rates and the fractions of households living in detached/attached homes, this assumption highlights the land use channel and simplifies the analysis.

4.1 Pre-determined Parameters

Due to the large number of cities with each being described by a general equilibrium model and the city-specific parameters to estimate, it is computationally burdensome to estimate the common parameters, i.e., parameters that apply to all cities, through solving the model. Therefore, I take these common parameters directly from the literature. Table 2 summarizes these parameters and their sources.

4.1.1 Preference

Following Chambers, Garriga, and Schlagenhauf (2009) and Fisher and Gervais (2011), the per-period utility takes the following form

$$u(c,s) = ln(c) + \frac{s^{1-\sigma}}{1-\sigma} \tag{18}$$

This utility function treats housing as a necessity so that the expenditure share on housing rises with the costs of housing services.

Following the literature on housing tenure choice (see, for example, Sommer, Sullivan, and Verbrugge, 2013; and Yang, 2009), the risk aversion parameter, σ is 2.5, and the discount factor β is 0.95 per year. The discount factor on the ownership premium for old households, $\zeta = 0.9116$, is taken from Fisher and Gervais (2011).

[22] As the majority of owners live in detached/attached houses while most of the renters live in apartments, the correlation between homeownership rate and the fraction of households living in detached/attached houses is around 0.80 across cities in 1980. The fraction of households who are owners and living in apartments is 6% and the fraction of households who are renters and living in detached/attached homes is 9.6% for the 181 cities in my sample.

4.1.2 Market Arrangements

Transaction costs for buyers and sellers are set to, $k_b = 0.07$ and $k_s = 0.025$, based on Gruber et al., 2004. Following Sommer, Sullivan, and Verbrugge (2013), the depreciation rate of owner-occupied houses $\delta = 0.025$. The management cost of rental unit $\delta_r = 0.33$, comes from Goodman (2004). Consistent with the literature (see e.g. Sommer, Sullivan, and Verbrugge (2013); Chambers, Garriga, and Schlagenhauf (2009)), down payment requirement, $\gamma = 20\%$, in the baseline calibration. It is lowered to 10% in the simulation of 2010 to capture the mortgage credit expansion from 1980 to 2010.

The risk-free interest rate is, $r = 4\%$, per year and the spread on mortgage, $r_m = 1.5\%$, which are consistent with the literature (see e.g. Amior and Halket, 2014).

4.1.3 Demography and Income

Each period in the model is set to 5 years. At age 20, households enter the model. At the beginning of each period, households receive exogenous income, which depends on location, age, and an idiosyncratic shock. The persistence of income residuals and the standard deviation of error of income residuals, ν and Σ, are set to be 0.75 and 0.45, respectively, following Fernández and Wong (2014) and Chang and Kim (2006). Specifically, I follow Tauchen (1986) to approximate the continuous process with a discrete number of seven states.

4.1.4 Initial Wealth of Young Households

At the beginning of their lives, households receive initial non-housing wealth. I calibrate the wealth distribution of newborns using the distribution of wealth among 21-25-year-olds in the 2016 Survey of Consumer Finances (SCF). Households with negative wealth or no income are dropped from the sample. I parameterize the initial wealth distributions with a log-normal distribution with the mean and standard deviation calibrated to the data.

I translate the initial wealth distribution in the data to that in the model by scaling by the ratio of average household income among 21-25-year-olds in the model to the average household income of the same age group in different cities. In other words, I assume that the initial wealth distribution in city k follows a log-normal distribution with a city-specific mean $\mu_w \bar{w}_k w_0$ and a city-specific standard deviation $\sigma_w \bar{w}_k w_0$, where μ_w and σ_w are the mean and standard deviation of the wealth distribution of 21-25-year-old households adjusted by, $\bar{w}_k w_0$, the average household income of 21-25-year-old households in city k.

4.1.5 Taxes

The income tax is set to be $\tau_w = 0.2$, following Piketty and Saez (2007) and Amior and Halket (2014). The property tax is chosen to be $\tau = 0.01$, which is standard in the literature.

Table 2: Predetermined Parameters

Parameter	Meaning	Value	Target or Source
σ	Risk aversion	2.5	Sommer, Sullivan, and Verbrugge (2013)
β	Discount factor	0.96	Sommer, Sullivan, and Verbrugge (2013)
ζ	Discount factor on ownership premium for seniors	0.9116	Fisher and Gervais (2011)
δ	Depreciation rate of owner-occupied units	2.5%	Sommer, Sullivan, and Verbrugge (2013)
δ_R	Management cost of rental units	33%	Goodman, 2004
k_b	Buying cost	2.5%	Yang (2009)
k_s	Selling cost	7%	Yang (2009)
r	Risk-free interest	0.04	Sommer, Sullivan, and Verbrugge (2013)
r_m	Mortgage interest	0.015	Amior and Halket (2014)
τ_w	Income Tax	0.2	Piketty and Saez (2007)
τ	Tax on residential properties	0.01	Sommer, Sullivan, and Verbrugge (2013)
ν	AR(1) Coefficient of income	0.75	Fernández and Wong (2014)
Σ	Innovation of income process	0.45	Chang and Kim (2006)
μ_w	Mean of initial wealth distribution adj by income	3.4	Survey of Consumer Finance 2016
σ_w	std of initial wealth distribution adj by income	28.76	Survey of Consumer Finance 2016

4.2 Land Shares in Production Functions

The two land shares and the relative productivity $\{\alpha, \rho, A\}$ in the production functions are estimated through regressions. Assuming that the minimum lot size does not bind for the standard owner-occupied houses, I regress the house prices of a standard single-family detached house and the rents of a standard two-bedroom apartment on the cross-sectional transaction-based land values provided by Albouy, Ehrlich, and Shin (2018) for 2010. As

I allow for the variation in unobserved material prices, it is possible that material prices are correlated with land prices. Therefore, I use the fraction of undevelopable land provided by Saiz (2010) as an instrumental variable for the land value. After backing out the land shares, the relative productivity A is estimated by combining Equation 13 and Equation 15.

Table 3 presents the regression results for the two land shares using data in 2010.[23] The first two columns present the results for OLS estimates, and the last two columns present the results using the fraction of undevelopable land as Instrumental Variable. The land share in the house production function is almost twice as high as the land share in the apartment production function. Consistent with the prediction of Proposition 3.4, when the underlying land price changes, house price grows more than apartment rent due to the intensive use of land.

Table 3: Estimates of Land Shares

	OLS	OLS	IV	IV
VARIABLES	$log(P)$	$log(R)$	$log(P)$	$log(R)$
$log(Land\_Value\_Albouy)$	0.377***	0.202***	0.539***	0.280***
	(0.0265)	(0.0158)	(0.0503)	(0.0309)
Constant	7.436***	3.759***	5.479***	2.813***
	(0.319)	(0.192)	(0.608)	(0.373)
Observations	182	182	182	182
R-squared	0.625	0.532	0.510	0.452

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

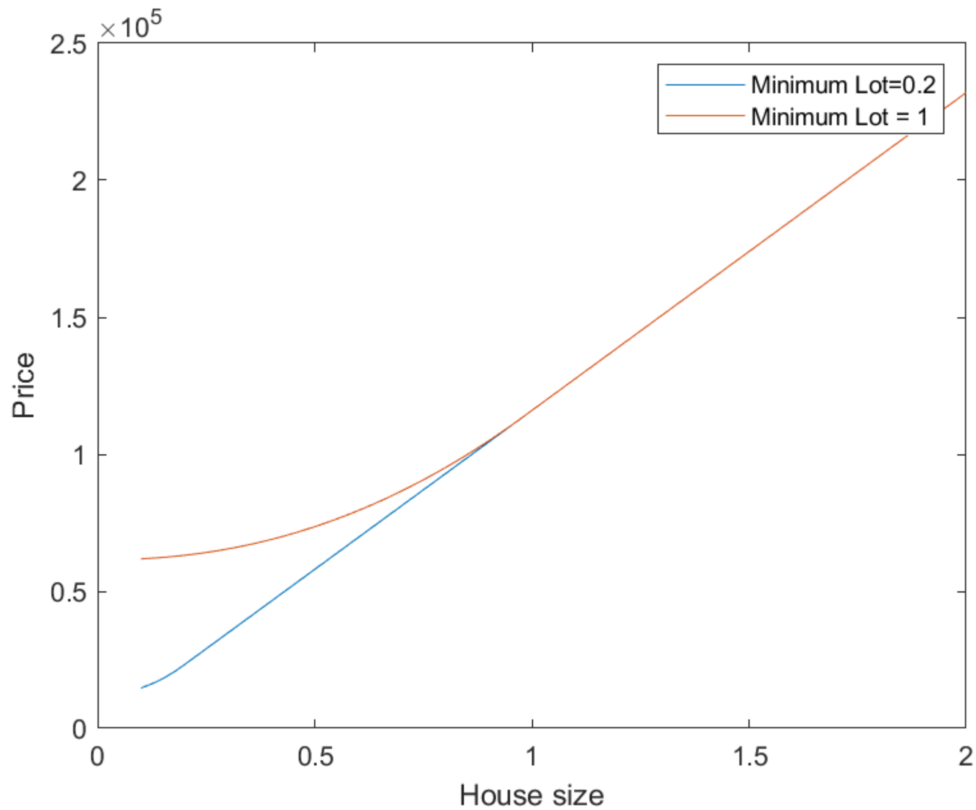
4.3 Local Specific Parameters

For each metropolitan area k, the minimum lot size $\bar{L_k}$ and ownership premium θ_k are calibrated to the fraction of households living in houses of three age groups: young (20-40-year-olds), middle-aged (41-65-year-olds), and old (65-90-year-olds).

[23]The cross-sectional indices of transaction-based land values provided by Albouy, Ehrlich, and Shin (2018) are only available between 2005 and 2010.

The basic idea of the identification is that when land and material price are fixed, an increase in the ownership premium θ_k lifts the homeownership rates of all age groups as households derive more utility from owned houses. On the contrary, an increase in minimum lot size disproportionally affects households with low income. Figure 6 illustrates the impact of an increase in minimum lot size on the price function of house size. A rise in the minimum lot size increases the house price for small homes while having no impact on the large ones, as smaller homes do not need that much land. As a result, an increase in minimum lot size will crowd out low-income households who prefer buying a small home to renting. Given the hump-shaped income profile, young households are more likely to quit owning when minimum lot size goes up. In other words, a rise in minimum lot size has an unbalanced impact on homeownership rates across age groups as it lowers the homeownership rate for young more than the middle-aged households.

Figure 6: Price Function



More details about estimation can be found in Appendix A.5.

4.4 Data

This paper uses the 5 percent sample of the 1980 U.S. Census and the American Community Survey (ACS) 2010 from the Integrated Public Use Microdata Series (IPUMS). These two datasets provide household-level data on household income, age of the household head, geographic location of residence, housing tenure choice (own or rent), self-valued house price for owner-occupied units, contract rent for rental units, and types of the dwelling (single-family detached house, attached house and multi-family buildings). The geographical unit of analysis is the metropolitan statistical areas (MSA) of residence. The dataset adopts a consistent definition of metropolitan areas from 1980 to 2010.

Young households are defined as households headed by 20-40-year-olds. Middle-age households are households headed by 40-60-year-olds, while old households are households with a 60-90-year-old head.

The age-profile of household income is constructed using the national-level data. Specifically, I group households according to the age of the household head and then put them into five-year bins from 20 to 85. Then for each bin, I calculate the average household income. Finally, for each metropolitan area, I calculate the number of households and average household income.[24] Data on survival probabilities comes from the life tables provided by the National Center for Health Statistics. For each age group, I calculate the probability of surviving for another 5 years.

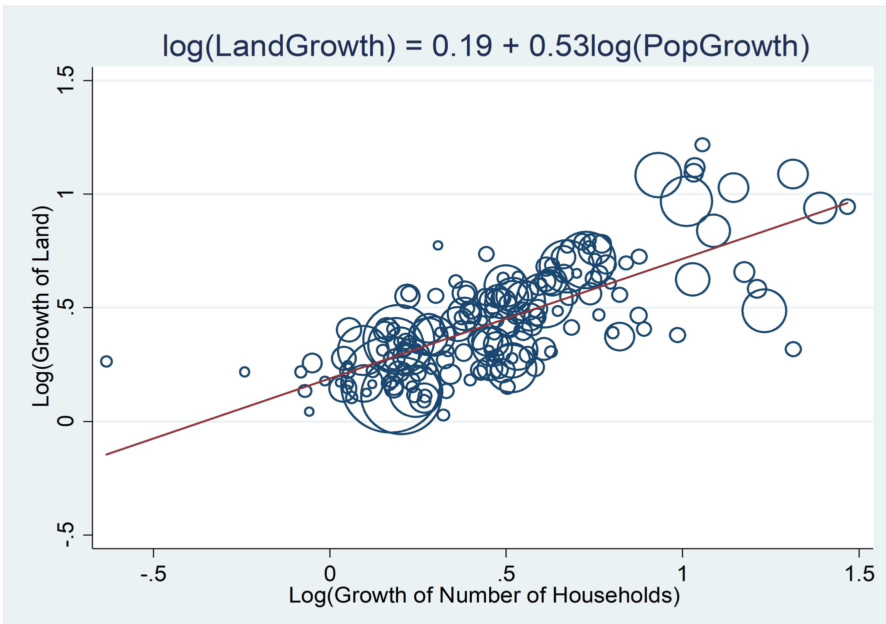
Residential land supply is computed from the Land-Use and Land-Cover Data Sets of U.S. Geological Survey for 1982 and 2012.[25] The dataset classifies the conterminous land area into 19 categories. For each metropolitan area, I calculate the land used for residential constructions for 1982 and 2012. Then I use the growth rate of observed residential land from 1982 to 2012 to approximate the growth of residential land supply from 1980 to 2010.[26]

I start by comparing the growth of the number of households with the growth of residential land for the 181 cities from 1980 to 2010. For most of the cities, land growth

[24]Data on house prices and apartment rents has been discussed in Section 2.1.
[25]https://pubs.er.usgs.gov/publication/ds240
[26]The Geological Survey is not available for 1980 or 2010.

does not catch up with population growth, as most of the points fall below the 45-degree line in Figure 7. Specifically, a 1% increase in population is assoicated with 0.5% increase in rent. In other words, land has become scarcer for many of the cities. As a result, land values grow which leads to an increase in prices and rents in most of the cities. This is consistent with the observation that the average price and rent for the cities in my sample have both increased.

Figure 7: Population Growth and Residential Land Growth



5 Calibration Results

Table 4 shows the estimates of the relative productively A, ownership premium θ_k, and minimum lot size $\bar{L}_k$. The model closely matches the average and the standard deviation of the fraction of households living in houses among young, middle-aged, and old households across cities.

As an additional check of the calibration, I plot the model generated fractions of households living in houses (owners) against their data counterparts for the three age groups in Figure 8 - Figure 10. Each bubble represents one city and the bubble size corresponds to the city population in 1980. A 45-degree line is included to facilitate the

Table 4: Parameters and Moments

Estimated Parameters	Mean	Std		
Relative Productivity A	1.3			
Ownership Premium θ_k	1.7	0.43		
Minimum Lot Size $\bar{L}_k$	0.96	0.1		
Moments	Mean (data)	Std (data)	Mean (model)	Std (model)
Frac in Houses young	0.65	0.097	0.68	0.093
Frac in Houses middle-aged	0.83	0.075	0.86	0.085
Frac in Houses old	0.72	0.113	0.69	0.114

comparison. The model does a great job in matching these moments for each individual city. The correlation between model-generated moments and the data exceeds 0.9 for all the age groups.

6 Quantitative Exercises

6.1 Baseline Results

In this section, I use the calibrated model to quantitatively investigate the impact of changing population, income, land supply, and credit constraint from 1980 to 2010 on the dispersion of house prices and rents. Note that by construction, the model can perfectly match the distribution of prices and rents in 1980. Therefore, I examine the performance of the model with its prediction of prices and rents for each individual city in my sample in 2010.

Specifically, I change population, income, and residential land supply to the 2010 level for each city in my sample. The survival probabilities and the age income profiles are also changed to the 2010 level. In addition, I lower the down payment requirement from 20% to 10% to capture the impact of mortgage innovation on the housing demand during the past several decades.[27] I simulate the model to calculate the equilibrium land price in 2010 for each individual city. Then I combine the land price with material price to calculate prices and rents for all cities. Note that in the simulation, material prices are

[27]Duca, Muellbauer, and Murphy (2016) show that the loan-to-value ratio for 1st-time home-buyers has increased to 90% in 2010.

Figure 8: Fraction of Young Households living in Detached/Attached Houses 1980: Model and Data

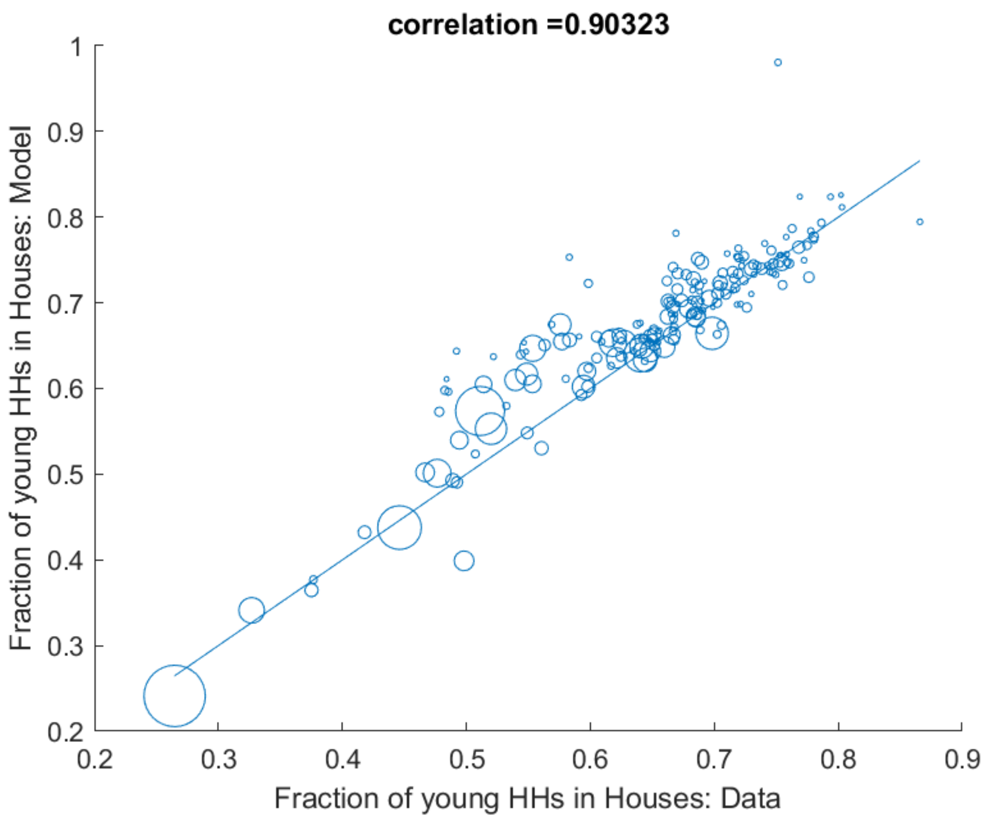


Figure 9: Fraction of Middle-aged Households living in Detached/Attached Houses 1980: Model and Data

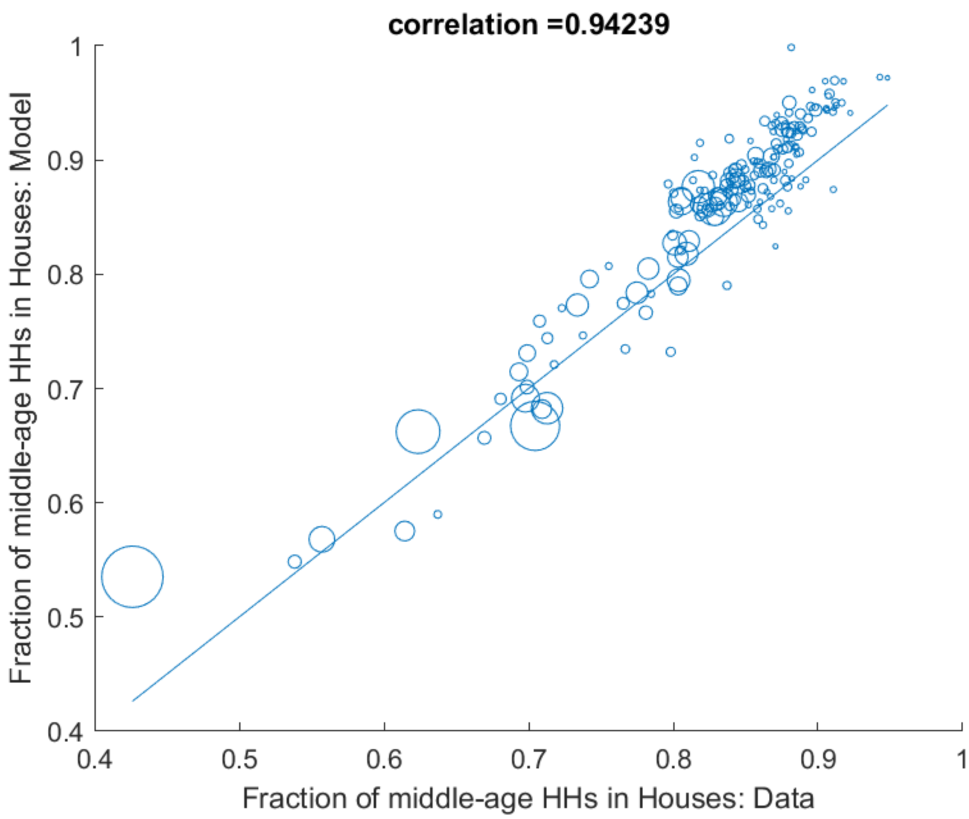
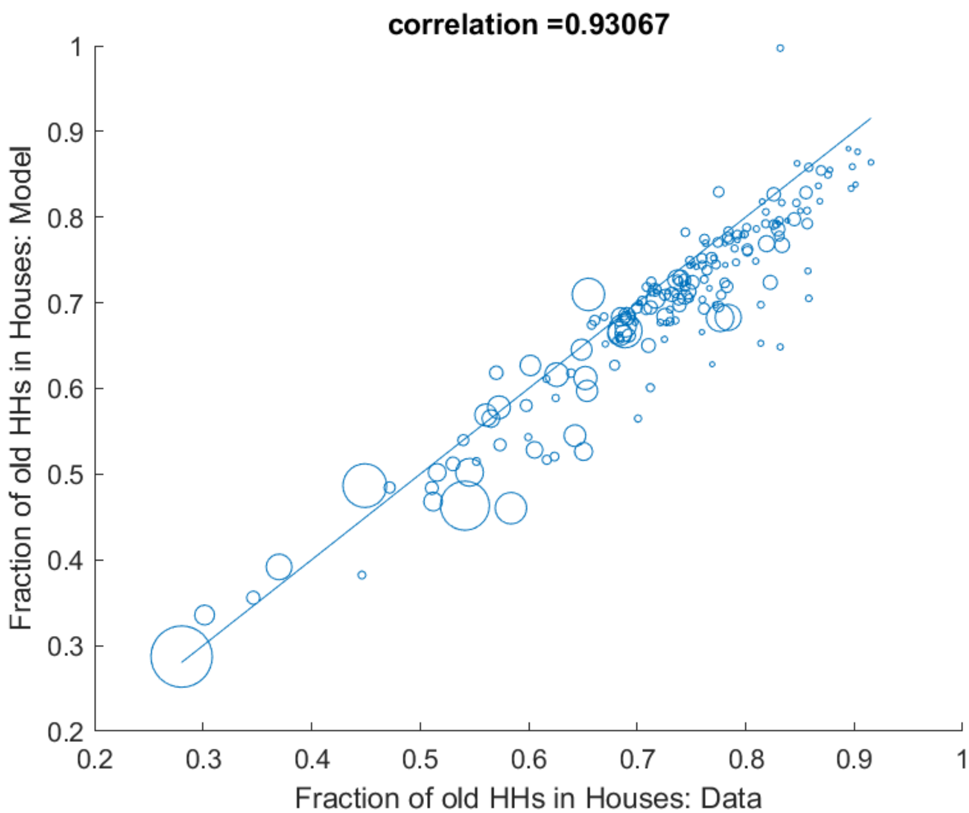


Figure 10: Fraction of Old Households living in Detached/Attached Houses 1980: Model and Data



fixed at the 1980 level.[28]

The results are summarized in Table 5. The model successfully generates a large increase in the price dispersion and a relatively moderate increase in the rent dispersion that match the data. Changes in population, income, land supply, and the down payment requirement can account for 82% of the increase in CV of house prices and 56% of the increase in CV of rents in the data. In addition, the model successfully generates an increase in the dispersion of price-rent ratios. As the last two columns in Table 5 show, the model can account for 90% of the increase in CV of price-rent ratios. These results suggest that most of the rise in the price-rent ratios in large cities can be attributed to fundamentals. This challenges the conventional view that the rise in the ratio of house prices to rents signals a housing bubble. In addition, it shows that the land use difference is important for understanding the relationship between these two shelter costs.

Table 5: Baseline Results

	mean(P)	CV(P)	mean(R)	CV(R)	mean(P/R)	CV(P/R)
1980	120900	0.33	6038	0.18	19.89	0.21
2010	146270	0.62	6264	0.27	22.15	0.31
Simulated 2010 10% down payment	129266	0.57	6085	0.23	20.47	0.30
Simulated 2010: 20% down payment	126095	0.52	6046	0.22	20.26	0.28

In addition to the aggregate moments, the model does a good job in predicting prices and rents for each individual city. Figure 11 and 12 plot the simulated prices and rents against the observed prices and rents in 2010, respectively. Each bubble represents a city, with the size of the bubble determined by the population size in 2010. A 45-degree line is included to facilitate the comparison. The model-generated and observed house prices line up well, with a correlation of 0.9. Similarly, the correlation between the model generated rent and the data is 0.78. The model slightly under-predicts the average level of prices and rents in 2010. Potential explanations are provided in the discussion.

As a robustness check, I also compare the model generated ratios between prices and rents with the data. Figure 13 demonstrates the comparison. The correlation between model-generated price-rent ratios and data is as high as 0.81. In other words, the model

[28] Material prices in 1980 are calculated in Section 4.3.

Figure 11: House Prices in 2010: Model vs Data

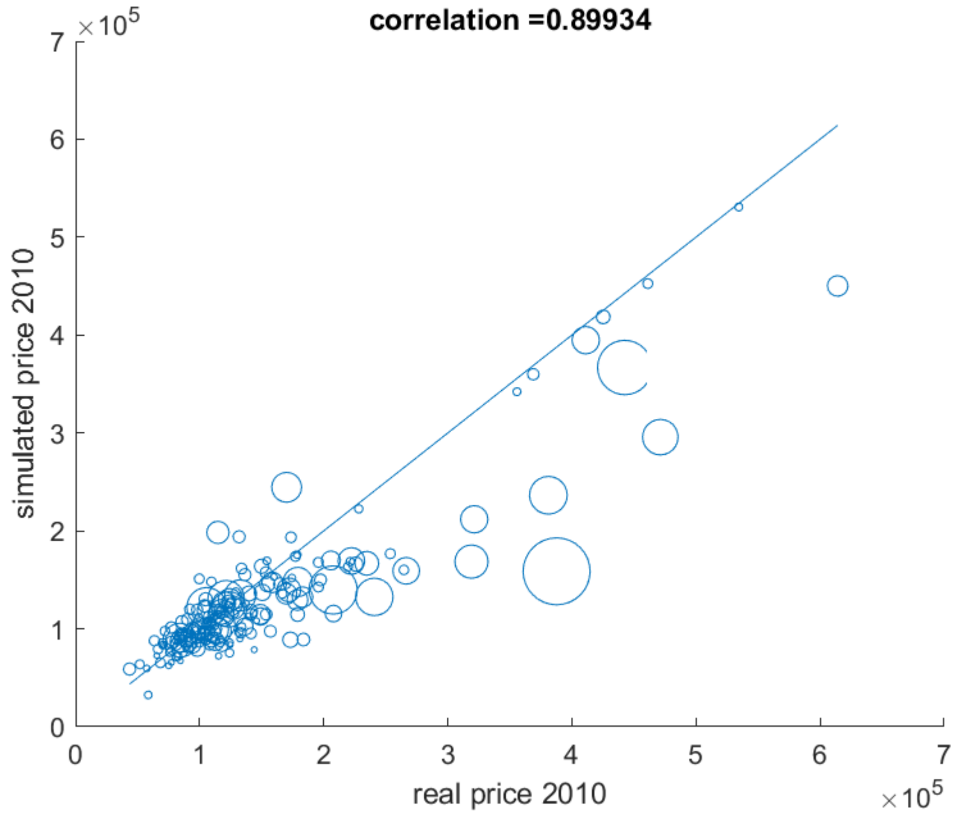
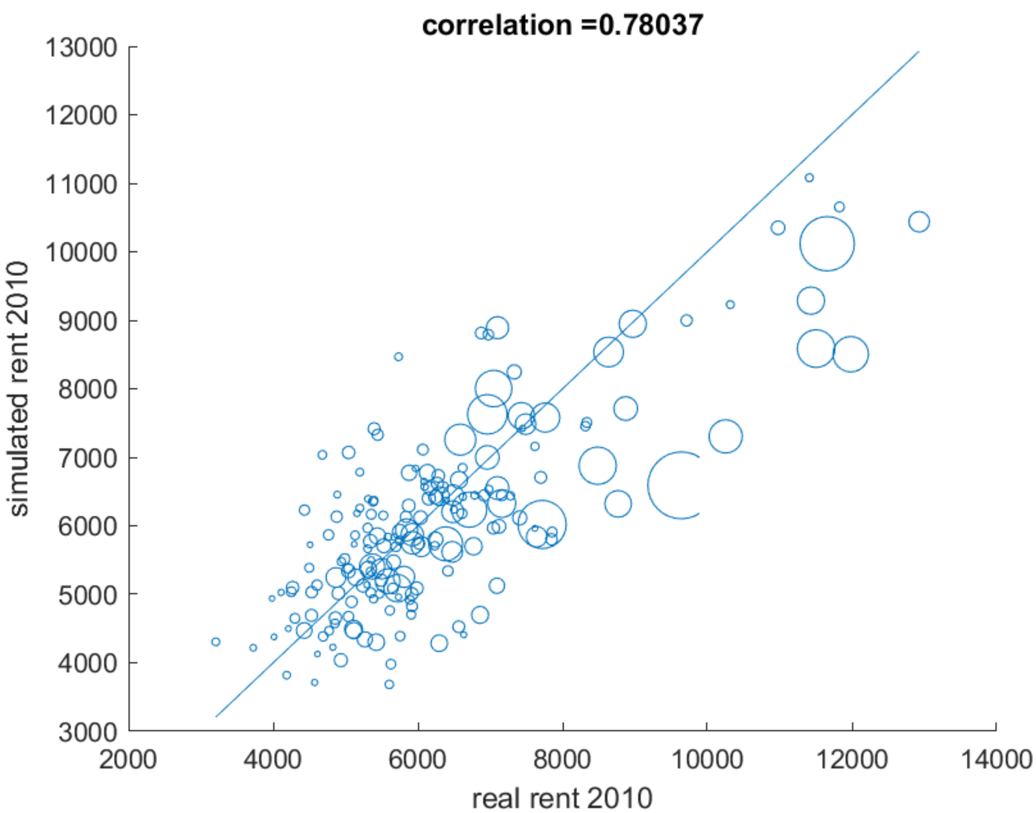
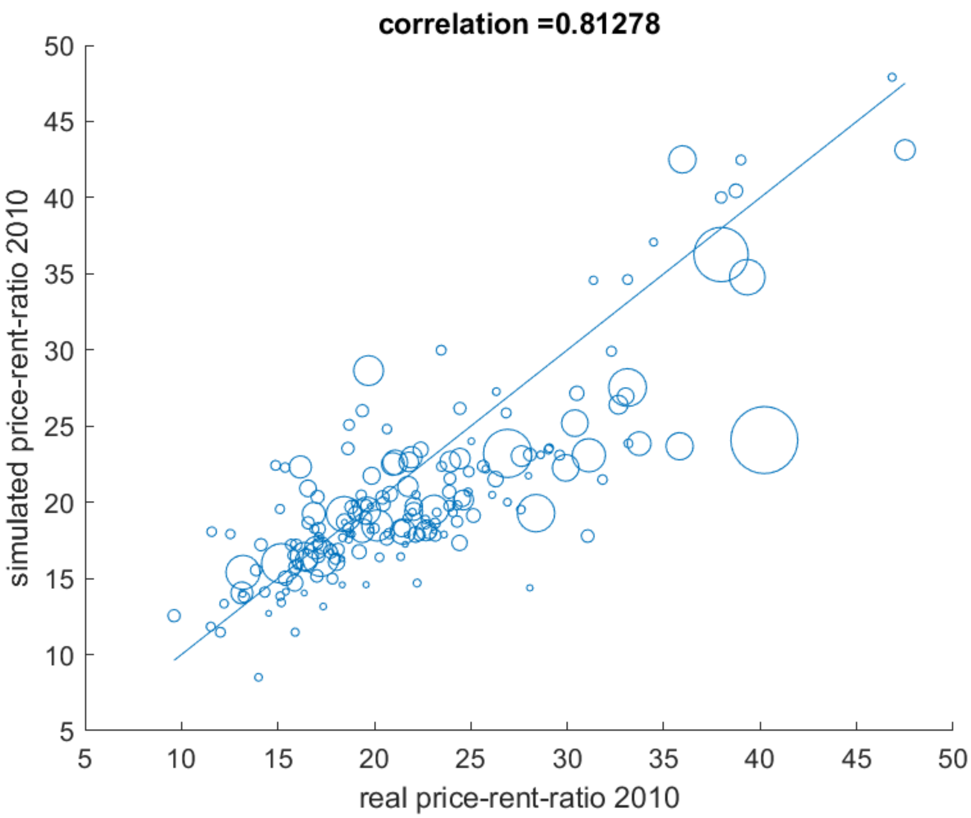


Figure 12: Rents in 2010: Model vs Data



successfully captures the relationship between these two shelter costs across cities.

Figure 13: Price-rent ratios in 2010: Model vs Data



To sum up, the quantitative exercise shows that while changes in land values lead to changes in prices and rents, the difference in land intensities is key to understanding the divergence between these two shelter costs.

6.2 The Contribution of Lowering Down Payment Requirement

Motivated by the recent literature on the heterogeneous effect of mortgage-related policies, such as interest rate cuts across regions (See e.g. Hurst et al., 2016), I investigate the impact of lowering down payment requirement on house prices, rents, and housing price-rent ratios in different cities by changing population, income, and residential land supply to 2010 while holding the down payment requirement at its initial level, 20%. The results are presented at the bottom line in Table 5. Consistent with the previous literature, I find that the impact of lowering the down payment requirement on the underlying land prices is not identical across cities, as evidenced by the changes in the dispersion of prices, rents, and price-rent ratios. The difference between the last two rows in Table 5 reveals that the lowering down payment ratio accounts for 17% of the increase in house price dispersion, 11% of the increase in the rent dispersion, and 22% of the increase in the dispersion of housing price-rent ratios.

6.3 Discussion

Unchanged material price: In the quantitative exercise, I use material prices in 1980, which are estimated using data on prices and rents in the calibration. One concern is that increasing housing demand may push up wages of construction workers, which leads to an increase in the material cost, i.e., total construction cost net of land cost. In the Appendix A.6, I exploit cross-states variation and show that the relative wages of construction workers to other workers are not affected by the number of building permits, which is an approximation for housing demand. This is consistent with Gyourko and Saiz (2006), who document the significant differences in construction costs across U.S. housing markets and find that the construction costs do not change much in regards to housing permits.[29]

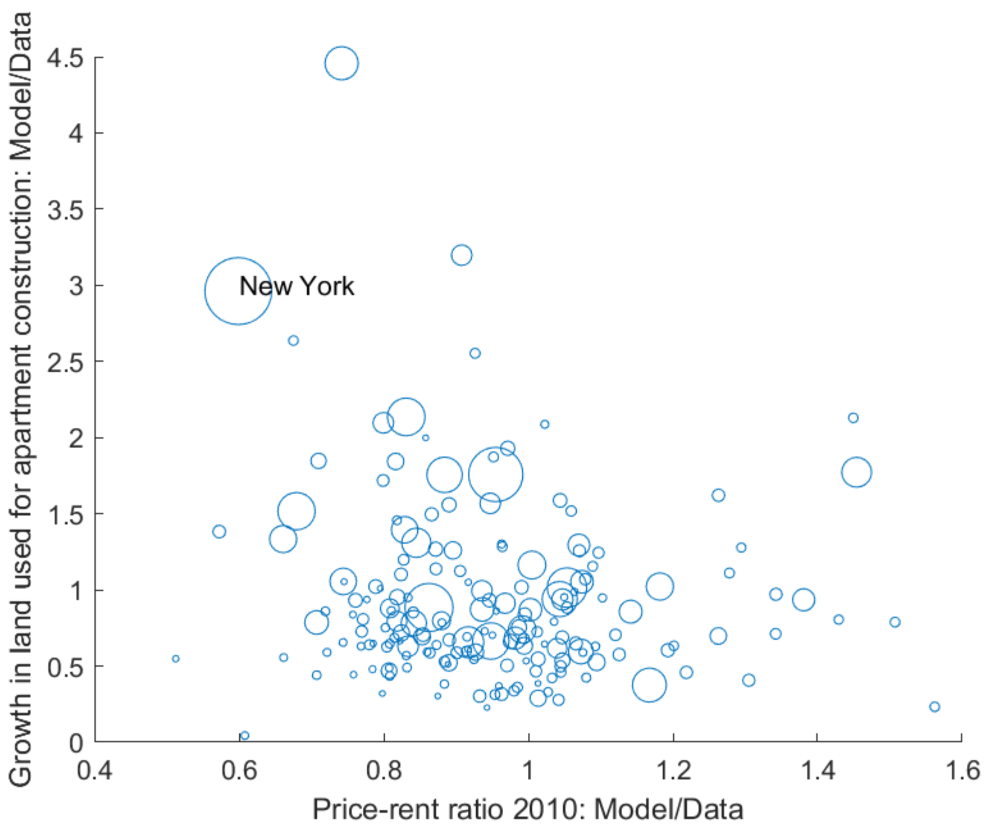
Underpredicted land price for big cities: The model noticeably under-predicts house prices, rents, and housing price-rent ratios in some big cities, which suggests the land prices in these cities are under-predicted. One potential explanation is the lack of redevelopment costs in my model, i.e., costs associated with changing the use of existing land. In reality, it is costly both in terms of time and money to tear down a house and build apartments on the same piece of land, and vice versa. Moreover, there are regulations, such as single-family zoning, that prevent the construction of multifamily buildings in certain areas. In other words, some land in expanding cities is not efficiently used, which may put upward pressure on the land price through the overuse of land.[30]

[29] I use cross-state variation rather than cross-city variation due the data availability. For most of the cities, I do not enough observations for construction workers.

[30] New York Times: Cities Start to Question an American Ideal: A House With a Yard on Every lot https://www.nytimes.com/interactive/2019/06/18/upshot/cities-across-america-question-single-family-zoning.html?mtrref=www.google.com&assetType=REGIWALL.

Figure 14 plots the ratio between the growth of land used for apartment constructions generated by the model and the growth of the amount of land used for high-density construction in the data from 1980 to 2010 against the ratio between model-generated price-rent ratio and price-rent ratio in data in 2010.[31] The negative slope suggests that the model underestimates price-rent-ratios in 2010, i.e., the underlying land prices, in some cities as it over-projects the growth of land used for high-rise buildings. For example, in the case of New York, the model under-predicts the housing price-rent ratio by 40% as it over-predicts the land growth for high-density construction by 200%. The living costs, prices and rents, in these big cities could thus be much lower if land use were flexibly adjusted.[32]

Figure 14: Price-Rent Ratio and Land Growth for Apartments: Model V.S. Data



Allowing for migration across cities For simplicity, in the quantitative exercise,

[31] I use price-rent ratio to approximate land values as it is strictly increasing in land values.

[32] I approximate the land use growth for apartments in the data by the land used for high-density construction.

I assume the city specific ownership premium is fixed at the 1980 level. One concern is that ownership premium in a city may change if internal migration is allowed. Intuitively, allowing people to move across cities ("sorting") will affect the population composition. Specifically, people who value owning more will move to cities with relatively low house prices, resulting in a decline in ownership premium in cities where house price is high and an increase in cities where house price is low from 1980 to 2010. In other words, when I apply the ownership premium estimated using data in 1980 to the 2010 simulation, I should over-predict the fraction of people living in houses in big cities with high house prices and under-predict it in small cities with low house prices, as people with high ownership premium move to these small cities. In other words, the prediction for demand of houses could be biased.

I apply the estimated ownership premium θ_k to simulate the demand of houses in 2010 and compare it to the data. Figure 15 - Figure 16 display the comparison for young and middle-aged households. Overall, the model's prediction of demand for houses lines up well with the data. Noticeably, instead of over-predicting demand for houses, the model slightly under-estimates the fraction of households living in houses for young and middle-age groups especially among the big cities.[33] In other words, allowing for migration across cities would not have a significant impact on my results.

One potential explanation is that the estimated ownership premium of a specific city combines two pieces of information: the structure difference between a standard owner-occupied house and a standard rental apartment in that city as well as how local residents treat this difference, i.e., the taste of local residents.[34] Changing population composition only affects the taste of local residents on the difference between owned-

[33]The comparison for old households (65-90-year-olds) is ruled out from the comparison due to another dynamic aspect that is missing from the model. Assuming stationary equilibrium in 1980 and 2010 alleviates the burden from computing the transitional dynamics. Due to the high transaction cost of housing assets, households may not adjust the size of their houses frequently. Specifically, a large fraction of house owners aged from 65 to 90 bought their house in 1980 when the house prices were lower than today.

[34]The average structure difference is captured by the relative productivity A, while the local residents' taste is captured by the ownership premium θ. As these two terms are not separable in the model, they are identified using different sets of moments. The average relative productivity A is estimated using the relationship between prices and rents while the ownership premium is calibrated to the ownership rates.

Figure 15: Fraction of Young Households living in Detached/Attached Houses 2010: Model and Data

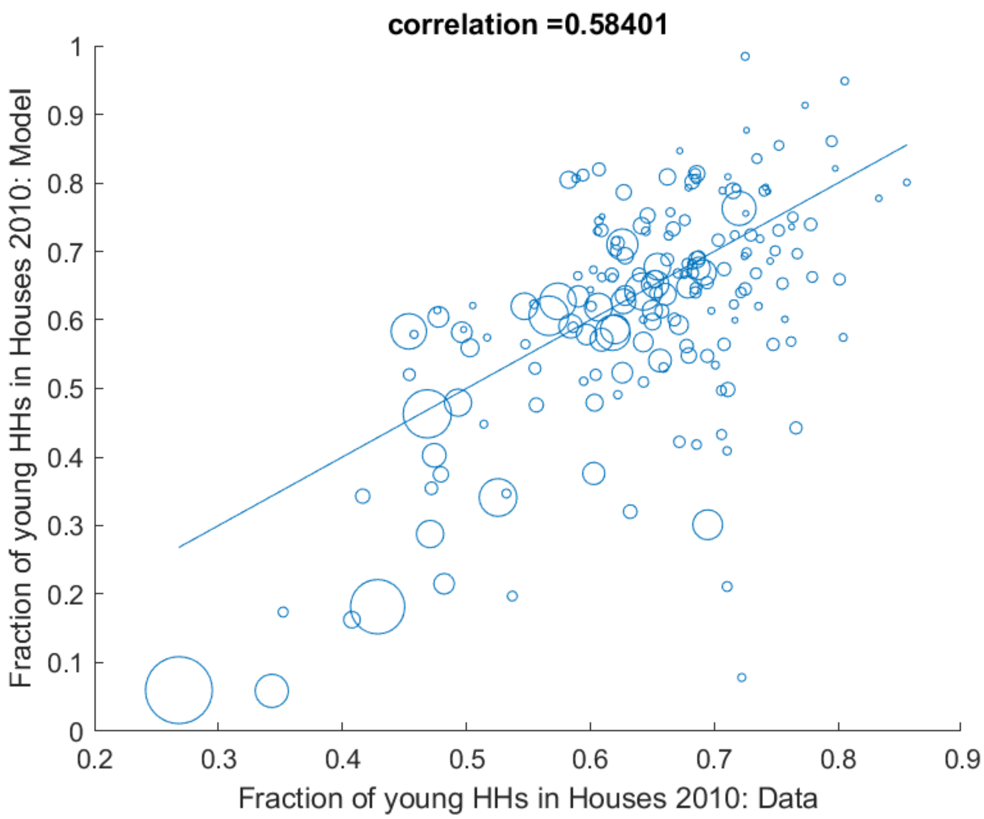
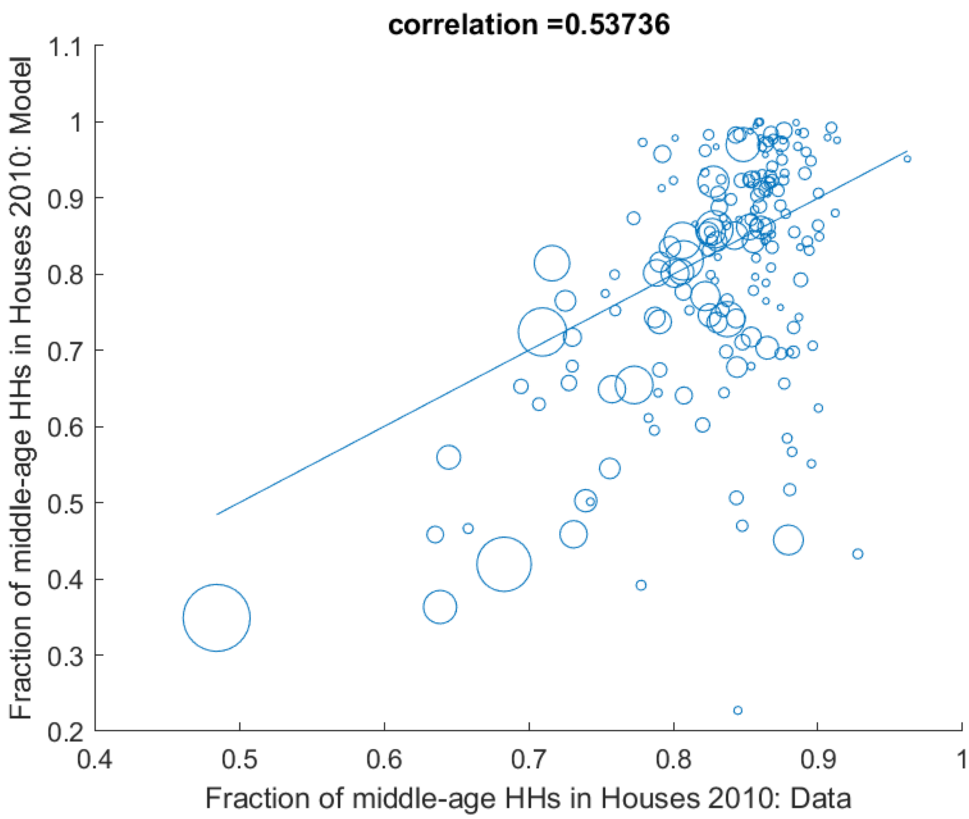


Figure 16: Fraction of Middle-aged Households living in Detached/Attached Houses 2010: Model and Data



houses and rental apartments. If the variation of the estimated ownership premium θ_k mainly comes from the structure differences across cities, i.e., the relative quality of houses compared to apartments differ substantially across cities instead of local residents taste, allowing people to move across cities based on their taste towards owning would not have a significant impact on the results.

7 Conclusion

This paper provides new insights into the joint distributions of the two shelter costs, prices and rents, across metropolitan areas. It extends the housing tenure choice model to account for the difference in land use between owner-occupied houses and rental apartments. Houses are more land-intensive compared to apartments. When increasing land demand drives up land prices in expanding cities, the cost of building houses grows faster than that of apartments, leading to a larger increase in price than rent. The quantitative analysis shows the difference in land intensity is crucial in understanding the divergence between the distribution of these two shelter costs across cities.

Throughout my analysis, I maintain the assumption that both the apartment market and the house market are competitive such that the price (net present value of rents) equals the construction cost of houses (apartments), which simplifies the problem and allows me to highlight the mechanism where the house prices and apartment rents grow at different rates when land price changes due to different land intensities. Admittedly, perfect competition is a strong assumption and developers may have monopolistic powers. Nevertheless, even for monopolistic developers, the marginal cost of constructing houses and apartments is one of the most important factors in pricing their products.

This paper makes the first attempt to study the relationship between house prices and apartment rents by exploring the difference in land intensities. The framework developed in this paper rationalizes the different supply elasticity between owner-occupied and rental units especially in the long-run when land is reusable.

This paper provides new insights on measuring the user cost of owner-occupied dwellings. The commonly-used measurements for shelter costs in the Consumer Price

Index (CPI) are the Rent of Primary Residence and the Owners Equivalent Rent of primary residence (OER). This paper illustrates that when land price changes, the construction costs of houses may deviate from that of apartments. As construction cost is one of the crucial components that determine user costs, using the change in rent of apartments, i.e. primary residence, to approximate the change in the user cost of owner-occupied houses could under-estimate (over-estimate) the growth of shelter cost for residents in areas where land price increases (declines).

The Owner Equivalent Rent is based on a hypothetical question that asks the owners about the rent they will receive if their dwellings were rented out. If the owners' answer is based on the rent of regular rental units, the OER could also be biased. Understandably, house prices are not perfect measurements of owners' user costs due to the high volatility and vulnerability to changes in macroeconomic conditions. Nevertheless, policymakers should be aware of the divergence between housing prices and rents that is sustained and can be accounted for by changes in land values and land use difference when evaluating living expenses for owners.

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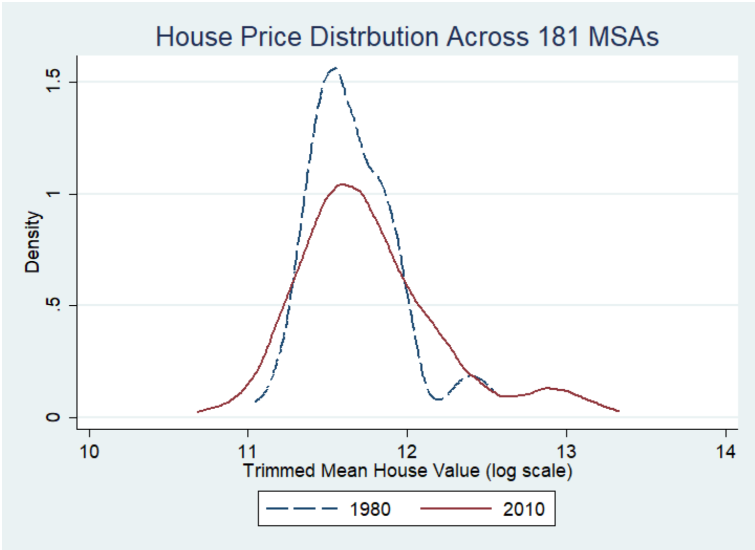
A Appendix

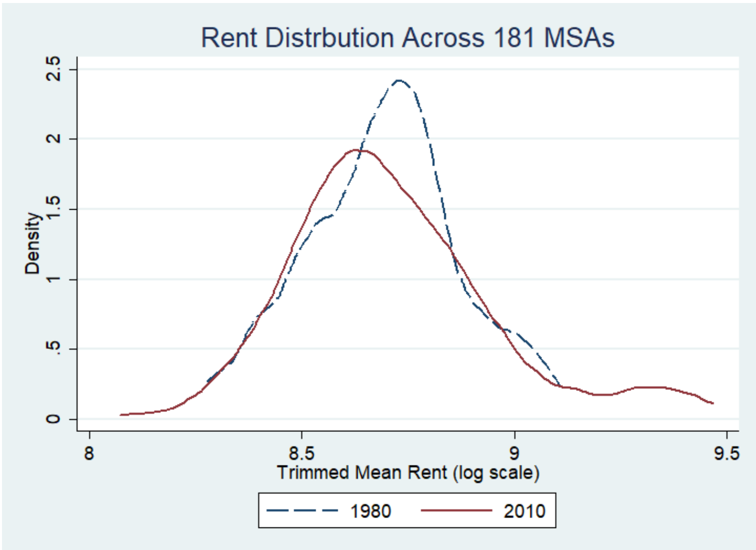
A.1 Distribution of Prices and Rents

Figure 17 plots the density distribution of prices and rents across the largest 181 cities for 1980 and 2010. Both prices and rents become more dispersed with fatter tails on both sides over time. Moreover, the dispersion of prices has increased more than rents.

Figure 17: Distribution of Prices of Rents



(a) Prices



(b) Rents

A.2 Land Use

In this section, I present more evidence about the land use in different cities and its changes over time for houses and apartments separately. Houses use more land on average compared to apartments. Table 2 summarizes the population density, the homeownership rate, the fraction of owners that live in houses with a lot, i.e. detached or attached house, the median lot size of owned houses (i.e. land use per house), and the median land use per rental unit for three cities: New York City, Houston, and Memphis. New York City is a representative large city with strict land regulations, while Houston is a large city known for its permissive zoning regulations. Memphis is an example of a mid-sized city. The land use of houses is represented by the lot size, while the land use of apartments is measured by the unit size divided by the number of stories in the building. Population density from the Census 2010 is included as an approximation of the intensity of land demand.

Table A1: Characteristics of Owner Occupied Dwellings and Rental Units

		New York City	Houston	Memphis
Owners	Population Density 2000 (per sq miles)	8158.7	705	377.7
	Ownership Rate	37	54	58
	Fraction in Detached/ Attached House	55	92	100
	Median Unit (sqf)	1900	1800	1500
	Median Lot (sqf)	5500	5500	9000
	Lot Size Distributions for Owner-Occupied Houses			
	<=1/2 acre	41.4	60.4	62.8
	1/2-1 acre	4.2	4.4	13.5
	>=1 acre	54.4	35.2	23.7
Renters	Fraction in Detached/ Attached House	6	25	29
	Median Unit (sqf)	700	800	900
	Median Land Use (unit size / stories)	129	462	450
	Fraction of High Rise Building (>=4 stories)	88	16	4

Comparing New York City with Memphis suggests that as land becomes scarcer, i.e., population density increases, land use for both houses and apartment declines. However, apartments are more efficient in economizing land use by building up. For instance, the median lot size in Memphis is only 60% higher than New York, while the median land use for apartments in Memphis is three times that of New York. In New York City, 88%

of rental units are in high-rise buildings (more than four stories), while the fraction of high-rise rental units in Memphis is only 4%.

The New York City and Houston comparison points to the impact of regulation on land use and housing demand. According to Gyourko, Saiz, and Summers (2008), fifteen of the sixteen jurisdictions within New York Primary Metropolitan Statistical Areas (PMSA) have a minimum lot size requirement, and seven have a minimum lot size requirement that is greater than one acre. Meanwhile, among the twelve jurisdictions in the Houston PMSA, nine have a minimum lot size requirement, and only four have a minimum lot size requirement that is greater than one acre.[35] The median lot size for owner-occupied houses in Houston is the same as that in New York City, despite the fact that the population density in New York is eight times higher than that of Houston. Moreover, 60% of owner-occupied houses in Houston have a lot smaller than 1/2 acre compared to 41% in New York City. Only 35% of owner-occupied houses in Houston have a lot larger than 1 acre, which is 20 percentage points lower than that in New York City. These observations suggest that the minimum lot size zoning is important in determining the land input in the construction of houses, and therefore the total land demand.

Change Land Use: 1980-2010 Consistent with a rise in the national-wide land price over the past 50 years (Knoll, Schularick, and Steger, 2017), recently built houses and apartments tend to use less land. Table A2 summarizes the characteristics of single-family detached houses built in metropolitan areas by the year of construction.

Table A2: characteristics of single-family detached houses by the construction year in Metropolitan areas

Year Built	1980-1990	1990-2000	2000-2009
Median Unit Size (sqf)	1900	2200	2300
Median Lot Size (sqf)	11000	10000	9000
Lot Size Distribution			
Smaller than 1/2 acre	40%	52%	57%
1/2-1 acre	7%	7%	6%
Larger than 1 acre	53%	41%	37%

Note: Author's calculation using data from AHS

While the median unit size of detached houses has grown steadily, rising from 1900

[35] Memphis has three jurisdictions in the sample, and all of them have a minimum lot size requirement.

sqft in the 1980s to 2300 sqft in the 2000s, more recently built houses tend to have smaller lots. The fraction of houses with a small lot (less than 1/2 acre) increases from 40% to 57% while the share of houses with large lots (bigger than 1 acre) declines from 53% to 37%.

A similar pattern holds for rental apartments as well. Table A3 summarizes the characteristics of multifamily buildings by the year of construction. Newly built apartments tend to be larger compared to the old ones. The median unit size for apartments built after 2000 is 10% larger compared to apartments built in the 1980s. However, newly built apartments building are taller, resulting in less land use per apartment compared to old ones. The fraction of high-rise buildings has more than doubled from the 1980s to the 2000s, increasing from 14% to 36%.

Table A3: Characteristics of Multi-family apartments by the construction year in Metropolitan areas

Year Built	1980-1990	1990-2000	2000-2009
Median Unit Size	845	900	940
Median Stories	2	3	3
Fraction of High-rise (4+ story)	14%	19%	36%
Land usage per unit	350	333	285

Note: Author's calculation using data from AHS
Land usage for apartments is defined by unit size divided by the number of stories.

A.3 List of Sample Cities

This section lists the 181 Metropolian areas used in this paper.

A.4 Definition of Stationary Competitive Equilibrium

As cities are isolated (i.e. households cannot move across cities), each city is a closed economy that is described by a unique competitive equilibrium.

The competitive equilibrium on an island k is defined by the price function and the rent function, $(P_k^*(h^o), R_k^*(h^r))$; land price q_k^*; value functions $V(x)$; the allocation of housing service $h^{o'*}(x), h^{r'*}(x)$ to owners and renters respectively; asset $a'(x)$; consumption $c(x)$; the distribution of people over the state variable x, $g(x)$; the allocation of land

MSA	MSA
Akron, OH	Charleston-N. Charleston, SC
Albany-Schenectady-Troy, NY	Charlotte-Gastonia-Rock Hill, NC/SC
Albuquerque, NM	Chattanooga, TN/GA
Alexandria, LA	Chicago, IL
Allentown-Bethlehem-Easton, PA/NJ	Chico, CA
Altoona, PA	Cincinnati-Hamilton, OH/KY/IN
Amarillo, TX	Cleveland, OH
Ann Arbor, MI	Colorado Springs, CO
Appleton-Oshkosh-Neenah, WI	Columbia, MO
Atlanta, GA	Columbia, SC
Atlantic City, NJ	Columbus, OH
Augusta-Aiken, GA/SC	Corpus Christi, TX
Austin, TX	Dallas-Fort Worth, TX
Bakersfield, CA	Davenport, IA - Rock Island-Moline, IL
Baltimore, MD	Dayton-Springfield, OH
Baton Rouge, LA	Daytona Beach, FL
Beaumont-Port Arthur-Orange, TX	Decatur, IL
Benton Harbor, MI	Denver-Boulder, CO
Billings, MT	Des Moines, IA
Biloxi-Gulfport, MS	Detroit, MI
Binghamton, NY	Duluth-Superior, MN/WI
Birmingham, AL	El Paso, TX
Bloomington-Normal, IL	Erie, PA
Boise City, ID	Eugene-Springfield, OR
Boston, MA/NH	Fayetteville, NC
Bremerton, WA	Fayetteville-Springdale, AR
Brownsville-Harlingen-San Benito, TX	Flint, MI
Buffalo-Niagara Falls, NY	Fort Collins-Loveland, CO
Canton, OH	Fort Myers-Cape Coral, FL
Cedar Rapids, IA	Fort Wayne, IN
Champaign-Urbana-Rantoul, IL	Fresno, CA

MSA	MSA
Gainesville, FL	Little Rock-N. Little Rock, AR
Grand Rapids, MI	Longview-Marshall, TX
Greeley, CO	Los Angeles-Long Beach, CA
Green Bay, WI	Louisville, KY/IN
Greensboro-Winston Salem-High Point, NC	Lubbock, TX
Greenville-Spartenburg-Anderson, SC	Macon-Warner Robins, GA
Hagerstown, MD	Madison, WI
Harrisburg-Lebanon–Carlisle, PA	Mansfield, OH
Hartford-Bristol-Middleton- New Britain, CT	McAllen-Edinburg-Pharr-Mission, TX
Hickory-Morganton, NC	Medford, OR
Houston-Brazoria, TX	Melbourne-Titusville-Cocoa-Palm Bay, FL
Indianapolis, IN	Memphis, TN/AR/MS
Jackson, MS	Miami-Hialeah, FL
Jacksonville, NC	Milwaukee, WI
Janesville-Beloit, WI	Minneapolis-St. Paul, MN
Johnson City-Kingsport–Bristol, TN/VA	Mobile, AL
Johnstown, PA	Modesto, CA
Joplin, MO	Monroe, LA
Kalamazoo-Portage, MI	Montgomery, AL
Kansas City, MO/KS	Nashville, TN
Kileen-Temple, TX	New Haven-Meriden, CT
Knoxville, TN	New Orleans, LA
Lafayette, LA	New York, NY-Northeastern NJ
Lafayette-W. Lafayette, IN	Norfolk-VA Beach–Newport News, VA
Lakeland-Winterhaven, FL	Ocala, FL
Lancaster, PA	Oklahoma City, OK
Lansing-E. Lansing, MI	Olympia, WA
Las Vegas, NV	Omaha, NE/IA
Lexington-Fayette, KY	Orlando, FL
Lima, OH	Pensacola, FL
Lincoln, NE	Peoria, IL

MSA
Philadelphia, PA/NJ
Phoenix, AZ
Pittsburgh, PA
Portland, OR/WA
Providence-Fall River-Pawtucket, MA/RI
Provo-Orem, UT
Racine, WI
Raleigh-Durham, NC
Reading, PA
Redding, CA
Reno, NV
Richland-Kennewick-Pasco, WA
Richmond-Petersburg, VA
Riverside-San Bernardino, CA
Roanoke, VA
Rochester, NY
Rockford, IL
Saginaw-Bay City-Midland, MI
St. Cloud, MN
St. Louis, MO/IL
Salem, OR
Salinas-Sea Side-Monterey, CA
Salt Lake City-Ogden, UT
San Antonio, TX
San Diego, CA
San Francisco-Oakland-Vallejo, CA
San Jose, CA
Santa Barbara-Santa Maria-Lompoc, CA
Santa Cruz, CA

MSA
Santa Rosa-Petaluma, CA
Sarasota, FL
Savannah, GA
Scranton-Wilkes-Barre, PA
Seattle-Everett, WA
South Bend-Mishawaka, IN
Spokane, WA
Springfield, MO
Springfield-Holyoke-Chicopee, MA
State College, PA
Stockton, CA
Syracuse, NY
Tampa-St. Petersburg-Clearwater, FL
Toledo, OH/MI
Trenton, NJ
Tucson, AZ
Tulsa, OK
Tyler, TX
Utica-Rome, NY
Ventura-Oxnard-Simi Valley, CA
Vineland-Milville-Bridgetown, NJ
Visalia-Tulare-Porterville, CA
Washington, DC/MD/VA
Waterloo-Cedar Falls, IA
Wichita, KS
Wichita Falls, TX
York, PA
Youngstown-Warren, OH/PA

$L^{o*}(h^o)$ and material $M^{o*}(h^o)$ to house developers; and the allocation of land $L^{r*}(h^r)$ and material $M^{r*}(h^r)$ to apartment developers; such that on each island k

- Given house price as a function of house size $P_k^*(h^o)$ and rent as a function of apartment size $R_k^*(h^r)$, the functions $V(x), c(x), a'(x), h^o(x), h^r(x)$ solve the maximization problem for households with state variables x specified in Equation 5 and Equation 7.

- Given the land price q_k^*, material price ϕ_k, and the house price function $P_k^*(h)$, land input $L^{o*}(h^o)$ and material input $M^{o*}(h^o)$ minimize the cost specified in Equation 11 for house developers.

- Given the land price q_k^*, material price ϕ_k, and the apartment rent function $R_k^*(h^r)$, land input $L^{r*}(h^r)$ and material input $M^{r*}(h^r)$ minimize the cost specified in Equation 12 for apartment developers.

- House market clears. For all house size h^o, the demand for houses of size h^o equals the supply for houses of the same size.

$$H^o = \int \mathbb{1}_{h^o(x)=h^o} g(x)dx \quad \forall h^o \tag{19}$$

- Apartment market clears. For all sizes h^r, the demand for apartments of size h^r equals the supply for apartments of the same size. As the technology of apartment construction exhibits constant return to scale, this is equivalent to total demand of apartments equaling total supply of apartments.

$$H^r = \int \mathbb{1}_{h^r(x)=h^r} g(x)dx \quad \forall h^r \tag{20}$$

- q_k clears the land market. The total land used for constructing houses and apartments equals the exogenous supply of land.

$$\int L^o(h^o)H^o d(h^o) + \int L^r(h^r)H^r d(h^r) = LS_k \tag{21}$$

where $L^o(h^o)$ is the land used for constructing a house of size h^o. H^o is the number of households who desire owning a house of size h^o given the equilibrium price function $P^*(h^o)$. $L^o(h^o)$ is the land used for constructing a house of size h^o. H^r is the number of renters who desire renting an apartment of size h^r given the equilibrium rent function $R^*(h^r)$. The first integral represents total land used for house constructions, the second integral shows total land used for apartment constructions, and LS_k is the total residential land supply in city k, which is assumed to be exogenous.

A.5 Computation Detail

The solution is computed numerically for each individual city. The algorithm solves the households' problem backward from the last period of their life by plugging the price function, Equation 13, and the rent function, Equation 15, into households' problem in Equation 5 and Equation 7. After solving for the demand for houses and apartments, I aggregate the land use in Equation 14 and Equation 16 to calculate the total land demand as a function of land price. For each individual city, the minimum lot size $\bar{L_k}$ and the ownership premium θ_k are estimated through the following algorithm.

- Given minimum lot size $\bar{L_k}$

  1. Initialize the model so that prices and rents in 1980 is the same in the model as in the data. Specifically, I solve for land price and material price by inverting the price Equation 13 for a standard owner-occupied house of which size is normalized to be 1, i.e. $h^o = 1$, and the rent Equation 15 for a standard two-bedroom apartment, of which size is normalized to be 1, i.e. $h^r = 1$.

  2. Solve for prices and rents for all sizes of owner-occupied and rental units, i.e. $P(h^o)$ and $R(h^r)$ for all h^o and h^r, by plugging the land price and material price provided by step 1 back into Equation 13 and Equation 15.

  3. Search over ownership premium θ_k to minimize distance between model gen-

erated homeownership rates g_j and data g_j^0.

$$\Lambda(\bar{L_k}) = \min_{\theta_k} \sum_j (\frac{g_j(\theta_k; \bar{L_k}) - g_j^0}{g^0})^2 \tag{22}$$

- Loop over minimum lot size $\bar{L_k}$ to minimize $\Lambda(\bar{L_k})$

A.6 Wage of Construction Workers and Building Permits

In this section, I exploit the cross states variation in the relative wages of construction workers compared to workers in other industries and the number of building permits to test whether increasing housing demand affects the labor cost of construction. I use cross states variation instead of cross cities variation due to the concern of sample size. Construction is not a very large sector in the U.S.. The employment share of construction industry is around 4.5% in 2016.[36] As a result, the 1% ACS sample may not have many cities that contain observations for construction workers.

I run the following regression.

$$log(RelativeWage_{j,2017}) = \alpha + \beta log(Permits_{j,2017}) + \epsilon_j \tag{23}$$

where $RelativeWage_{j,2017} = \frac{\text{Mean Wage of Construction Workers}}{\text{Mean Wage of other Workers}}$ is the relative wage of construction workers compared to workers in other industries in state j. The independent variables $Permits_{j,2017}$ is the number of building permits issued in 2017. I use the number of permits for all buildings, 1-unit buildings, 2-unit buildings, buildings with 3 and 4 units, and buildings with 5 units and more as independent variables. The results are presented in the top panel of Table A4. The data on building permits comes from the Building Permits Survey conducted by the Census. The coefficients on building permits are negative and insignificant, suggesting that relative wages of construction workers do not respond much to increasing housing demand. It is possible that the building permits issued in current year do not represent labor demand for construction as it may take

[36]Employment by major industry sector provided by BLS: https://www.bls.gov/emp/tables/employment-by-major-industry-sector.htm.

some time to get housing projects started after getting the permits. Therefore, I also use the building permits issued in the previous year as independent variables. The results are presented in the bottom panel of Table A4. The relative wage of construction workers does not change much towards housing permits issued in the previous year either. These findings are consistent with Gyourko and Saiz (2006), who document large variation in construction costs across housing markets and find that construction costs do not respond significantly to building permits.[37]

Table A4: Regression of relative wage of construction workers

	Total	1 Unit	2 Units	3 and 4 Units	5 Units or More
	$log(RelativeWage)$	$log(RelativeWage)$	$log(RelativeWage)$	$log(RelativeWage)$	$log(RelativeWage)$
$log(Permits_{2017})$	-0.0225	-0.00255	-0.0101	-0.00107	-0.0235**
	(0.0145)	(0.0265)	(0.0192)	(0.0187)	(0.00919)
Constant	0.133	-0.0590	-0.0277	-0.0767	0.111
	(0.150)	(0.252)	(0.117)	(0.108)	(0.0811)
Observations	51	51	51	51	51
R-squared	0.041	0.001	0.007	0.000	0.070
			Lag of Building Permits		
	Total	1 Unit	2 Units	3 and 4 Units	5 Units or More
	$log(RelativeWage)$	$log(RelativeWage)$	$log(RelativeWage)$	$log(RelativeWage)$	$log(RelativeWage)$
$log(Permits_{2016})$	-0.0224	-0.00297	-0.0140	0.00812	-0.0228**
	(0.0156)	(0.0272)	(0.0225)	(0.0271)	(0.00923)
Constant	0.130	-0.0554	-0.00619	-0.124	0.106
	(0.160)	(0.257)	(0.134)	(0.152)	(0.0831)
Observations	51	51	51	51	51
R-squared	0.039	0.001	0.014	0.005	0.065

[37] Gyourko and Saiz (2006) use data from the R.S. Means Company. The construction costs include those for materials, labor, and equipment for four different qualities of single unit residences.