Information Choice and Shock Transmission *

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Abstract

During the 2007-2008 financial crisis, countries that were relatively more exposed to the crisis epicenter, the United States, were among the least affected. This counters the intuition that the impact of a shock increases with exposure to it, and raises the question of the mechanism through which the impact of a shock can decrease with exposure. I develop a model in which decision-makers learn about the risk factors they are exposed to, but have limited capacity to process information. I find that decision-makers optimally choose to learn more about the risk factors they are more exposed to, and this informational advantage mitigates the impact of shocks by improving the optimality of their decisions. Relative to an exogenous information benchmark, the impact of shocks to risk factors that decision-makers are relatively more exposed to is attenuated, while shocks to risk factors that are relatively less important in terms of exposure are amplified through decision-markers' poorly informed actions.

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1 Introduction

During the 2007-2008 financial crisis, countries that were relatively more exposed to the crisis epicenter, the United States, were among the least affected in real terms (Rose and Spiegel, 2010, 2011). In financial markets, equity portfolios that were relatively more exposed to a United States-specific factor experienced relatively lower declines in returns (Bekaert, Ehrmann, Fratzscher, and Mehl, 2014).[1] This evidence points to the existence of a negative relation between the degree of exposure to a risk factors and the impact of a shock to that risk factor, which cannot be rationalised using existing theories of shock transmission. The evidence counters the intuition that the impact of a shock increases with exposure to it, and raises the question of the mechanism through which the impact of a shock can decrease with exposure.

I develop an information-based theory of shock transmission and characterize the conditions under which the impact of a shock can decrease with exposure to it. The theory builds on the intuition that the consequences of events depend not only on decision-makers' direct exposure to events, but also on their reactions to these events. Reactions can thus represent an indirect shock transmission mechanism, which can serve to amplify or mitigate the direct impact of shocks. Risk perception research suggests that an important factor shaping reactions to events is the degree of understanding of these events, which depends in turn on the information set of decision-makers.[2] Hence, understanding how decision-makers learn is central to understanding how their conditional actions contribute to the transmission of shocks. This paper studies the role that endogenous information choice plays in the transmission of shocks, through the actions of decision-makers.

I propose a framework in which decision-makers learn about the risk factors that they are exogenously exposed to, but have limited capacity to process information. I focus on studying the real consequences of shocks to these risk factors by embedding my framework in a simple model of corporate investment. The baseline model features a representative firm which undertakes investment to maximize profits. The return on investment depends on the fundamentals of the

[1] During the 1998 Russian financial crisis, Peruvian subsidiaries of international banks that had been directly exposed to the Russian government default shock were less affected relative to domestic Peruvian banks that had no such direct exposure (Schnabl, 2012). Evidence from the literature on natural disasters indicates that more hurricane casualties occur inland relative to the coast, despite the higher direct exposure of coastal areas (Moser, Davidson, Kirshen, Mulvaney, Murley, Neumann, Petes, and Reed, 2014). Evidence from sociology indicates that relatively minor risks that people have little exposure to are amplified and produce massive public reactions accompanied by substantial social and economic impacts, while ubiquitous and well-documented hazards that people are significantly exposed to are attenuated and elicit little public concern and responses (Kasperson, Renn, Slovic, Brown, Emel, Goble, Kasperson, and Ratick, 1988).

[2] See Slovic (1987); Kasperson et al. (1988); Renn, Burns, Kasperson, Kasperson, and Slovic (1992).

economy in which the firm operates. Economic fundamentals, in turn, are modelled as a sum of exogenous exposures to risk factors. Before investing, the firm chooses how much information to observe about the risk factors that fundamentals are exposed to, but has limited resources to acquire and process information.

This simple framework is intended to serve as a paradigm of a more general phenomenon, namely that of decision-makers which are exogenously exposed to a number of risk factors, and which dispose of limited resources to hedge against shocks to these risk factors. I study learning as a form of hedging, insofar as acquiring information about a risk factor represents an action that reduces the future uncertainty surrounding that risk factor and so it is conceptually similar to hedging. The assumption of exogenous exposures to risk factors is particularly suited to the corporate investment setup that I study, since corporate decision-makers are often exposed to risk factors that are beyond their immediate control. Firms operating in certain industries or countries cannot readily disentangle themselves from their respective industry or country specific risks, and are thus faced with relatively fixed or sticky risk exposures. The framework I propose is less suitable for studying financial investment setups, such as portfolio allocation, since in that case the fundamental problem faced by decision-makers is that of choosing their exposure to risk factors.

I find that learning about a risk factor optimally increases with exposure to it, such that the firm chooses to learn more about the risk factors that it is relatively more exposed to. The reduction in uncertainty achieved through learning mitigates the impact of shocks to risk factors that the firm is relatively more exposed to, by enabling it to take a better informed investment decision. On the other hand, the impact of shocks to risk factors that the firm is relatively less exposed to is amplified through the poorly informed investment decision of the firm. The interpretation of these predictions in terms of the motivating evidence is that countries which were relatively less exposed to the United States shock were more affected because decision-makers operating in these countries had a poorer understanding of the shock and, as a consequence, took actions that aggravated their circumstances.[3]

My model highlights a trade-off between the cost of being highly exposed to a risk factor and the benefit of having a better understanding of it. This trade-off would not obtain in a standard, fully rational Bayesian learning model in which information is abundant and decision-makers are not limited in their ability to process information. In this setup, learning would not be guided

[3]Ongena, Tümer-Alkan, and Von Westernhagen (2018) provide micro evidence from banking consistent with this interpretation, by documenting that in the lead-up to the crisis, German banks which had a higher exposure to United States exhibited stronger flight to quality responses to potential losses.

by aspects such as the degree of exposure, so there would be no relation between exposure to risk factors and conditional actions. On the other hand, in the boundedly rational setup I study, decision-makers devote their limited information processing resources to learning more about the risk factors that are more important in terms of exposure, and as a consequence the deviation of conditional actions from optimality decreases with exposure.

The information-based shock transmission mechanism I propose in this paper operates through decision-makers' uncertainty. Learning reduces the uncertainty that the firm faces when deciding on the optimal level of investment. Relative to a benchmark model in which the firm is endowed with an exogenous and equal amount of information about all its risk factor exposures, overall uncertainty is lower when the firm optimally chooses how much information to observe about its risk factor exposures. Furthermore, uncertainty is convex in exposure to a risk factor when the firm is exogenously endowed with information about its risk factor exposures, but it is concave in exposure when the firm endogenously chooses its information. Uncertainty dynamics in the benchmark model illustrate a classic diversification effect whereby high exposure to a risk factor implies high overall uncertainty, and the lowest level of uncertainty is achieved when exposure to the risk factors is equal. As exposure to a risk factor become relatively higher, uncertainty increases with exposure and results in a convex relationship that operates though a so-called *exposure channel*. However, in the endogenous learning model I propose, a change in exposure has two competing effects. On the one hand, higher exposure to a risk factor increases uncertainty mechanically, through the exposure channel. On the other hand, higher exposure to a risk factor entails a reduction in uncertainty via learning, which operates through a so-called *information channel*. As exposure increases, the exposure channel which is operative in the benchmark model is overturned by the information channel, and as a consequence uncertainty decreases with exposure, resulting in a concave relationship between uncertainty and exposure.

The reduction in uncertainty that is achieved through learning enables the firm to accurately incorporate the shocks affecting fundamentals into investment decisions. To the extent that the firm's investment deviates from the first-best optimum obtained under perfect information, such deviation is suboptimal. In the case of positive shocks the deviation from optimality manifests as under-investment, while in the case of negative shocks it manifests as over-investment.[4] It is thus convenient to define the loss due to suboptimal investment as the squared deviation from the perfect information optimum. The loss due to suboptimal investment essentially measures

[4]The deviation from optimality can be more generally conceptualized as a decision error.

the contribution of actions to the transmission of shocks. I find that the loss due to suboptimal investment decreases with exposure when information is endogenously chosen, and it increases with exposure when information is exogenously given. Consequently, the endogenous information model I propose predicts that the transmission of shocks is intensified as exposure to shocks decreases. Relative to the exogenous information benchmark, the impact of shocks to risk factors that are relatively less important in terms of exposure is amplified, while the impact of shocks to risk factors that are relatively more important in terms of exposure is attenuated.

I consider a number of extensions to the baseline model. First, I extend the baseline model to account for the degree of anticipation of shocks, and find that it is optimal to learn less about shocks that are expected to occur with low probability. The model thus predicts that the impact of unanticipated shocks is amplified, in line with the empirical evidence on the transmission of unanticipated crises (Kaminsky, Reinhart, and Vegh, 2003; Rigobon and Wei, 2003; Mondria and Quintana-Domeque, 2013). Second, I allow for strategic interactions and find that the amplification of shocks increases with the degree of strategic complementary in investment. Third, I relax the assumption that the risk factors affecting fundamentals are independent, and find that the amplification of shocks increases with the degree of correlation between the risk factors. Finally, I relax the assumption that exposures to the risk factors are exogenous, and find that it is optimal to specialize in learning about one risk factor and to be relatively more exposed to that factor.

The paper proceeds as follows. The rest of this introduction considers related literature and clarifies the contribution of my model. Section 2 formally introduces the mechanism through which the impact of shocks can decrease with exposure. Section 3 discusses the baseline results, Section 4 considers a number of extensions to the baseline model, and Section 5 concludes. All proofs and derivations can be found in the Appendix.

1.1 Related Literature

The model proposed in this paper incorporates endogenous learning and rational inattention (Sims, 2003) in a classic model of contagion that takes a risk factor view of the world (Forbes and Rigobon, 2002; Kodres and Pritsker, 2002; Pericoli and Sbracia, 2003). Thus, my paper mainly contributes to the literature on rational inattention, and the literature on financial contagion.

The rational inattention literature popularized by Sims (2003) builds on the idea that attention, rather than information is a scarce resource. A large number of rational inattention applications focus on attention allocation across multiple risk factors.[5] The paper most related to mine is Mondria and Quintana-Domeque (2013), who use fluctuations in attention allocation to explain financial contagion. They study the implications of endogenous information choices for the transmission of idiosyncratic shocks via a portfolio rebalancing channel by analyzing how a shock to one risk factor affects the attention allocated to other risk factors, and what this implies for subsequent portfolio allocation decisions. On the other hand, I study how attention allocation is affected by exposure to risk factors, rather than the occurrence of shocks to risk factors, and what this implies for the optimality of real investment decisions. Worth noting is that my baseline model differs markedly from a portfolio allocation problem in which decision-makers choose both their information about and exposure to risk factors, because in my framework decision-makers only choose how much information to acquire about the risks they are exogenously exposed to.

Another, comparatively smaller but growing, strand of the rational inattention literature focuses on attention allocation across a number of possible states of the world.[6] The paper most related to mine in this literature strand is Maćkowiak and Wiederholt (2018), who study how the degree to which decision-makers are prepared for events can exacerbate their consequences. While their model focuses on degree of anticipation of shocks to study how decision-makers make state-contingent plans, my baseline model focuses on the degree of exposure to shocks to study how the transmission of a systemic shock varies in the cross-section of decision-makers exposed to it. My baseline model is extended to account for the degree of anticipation of shocks as well, so my work complements theirs in that I study attention allocation across risk factors with vary in terms of their degree of exposure and which are also subject to shocks that vary in terms of their degree of anticipation.

My paper can be framed in the context of the contagion literature that studies the transmission of shocks in the presence of linkages between the crisis epicenter and the entities subsequently affected, but which cannot be unexplained by or is disproportionate to the observable size of these linkages.[7] The basic idea underlying this literature is that decision-makers transmit shocks by

[5]Rational inattention applications to setups in which decision-makers learn about many risks include asset pricing and portfolio choice (Mondria, 2010; Van Nieuwerburgh and Veldkamp, 2010; Kacperczyk, Van Nieuwerburgh, and Veldkamp, 2016), monetary policy (Woodford, 2001, 2009; Paciello and Wiederholt, 2013; Alvarez, Lippi, and Paciello, 2015), consumption dynamics (Luo, 2008; Tutino, 2013), price setting (Mackowiak and Wiederholt, 2009; Matějka and McKay, 2012; Matějka, 2015; Stevens, 2020).

[6]Rational inattention applications to setups in which decision-makers learn about future states of the world has been explored in a static setting by Maćkowiak and Wiederholt (2018) and in a dynamic setting by Ilut and Valchev (2017), Sundaresan (2018), and Nimark and Sundaresan (2019).

[7]This literature strand is to be contrasted with the theories of "pure contagion" which study the transmission of shocks in the absence of direct linkages between the crisis epicentre and the entities subsequently affected (Forbes, 2012). The types of linkages that have been studied in the literature have been both real linkages such as trade exposures (Eichengreen, Rose, and Wyplosz, 1996; Glick and Rose, 1999; Forbes, 2004), as well as financial linkages

directly altering the linkages or, alternatively stated, by changing their exposure to risks. My paper offers an alternative explanation by showing that information choices and decisions about learning can effectively alter risk exposures, and hence the transmission of shocks, even when decision-makers do not directly change their exposures. The shock transmission channel I propose is distinct from and complementary to the direct channel of changing exposures to shocks, and can be best understood by conceptualizing learning as a form of hedging which reduces the magnitude of, rather than the exposure to shocks.[8]

In my model, the transmission of shocks is disproportionate to the observable measure of exposure to these shocks. Thus, my paper contributes to the financial contagion literature which defines contagion as a change in shock transmission mechanism that cannot be explained by "fundamentals".[9] Specifically, my model explains contagion manifested as shock amplification, or how small shocks can have disproportionately large effects (Allen and Gale, 2004; Krishnamurthy, 2010; Benoit, Colliard, Hurlin, and Pérignon, 2017). Importantly, it sheds light on two important dimensions of shock amplification: (i) which of the shocks that an entity is exposed to are likely to amplify, and (ii) which of the entities exposed to a shock are likely to be more affected.[10] The model predicts that the transmission of shocks is intensified as exposure to shocks decreases. This prediction implies that: (i) the shocks that an entity is less exposed to are amplified, and (ii) the entities that are less exposed to a shock are relatively more affected.

Within the contagion literature, my paper also contributes to the literature strand on information-based models of contagion.[11] Most of these models take information as exogenous and focus on studying how idiosyncratic shocks, which should not be transmitted across entities if observable, are in fact transmitted because of imperfect information. In contrast, I focus on studying how endogenous information choices results in the differential transmission of a common shock in the cross-section of entities that are exposed to it.

such as interbank exposures (Allen and Gale, 2000; Dasgupta, 2004; Freixas, Parigi, and Rochet, 2000; Iyer and Peydro, 2011) and portfolio exposures (Yuan, 2005; Pavlova and Rigobon, 2008; Jotikasthira, Lundblad, and Ramadorai, 2012; Manconi, Massa, and Yasuda, 2012).

[8] To the extent that we can conceptualize the total size of a shock as being given by the magnitude of the shock and the exposure to it, learning essentially reduces its magnitude. In other words, the exposure to risk factors is fixed and only the magnitude of these risks is reduced through learning.

[9] This notion of contagion is used in King and Wadhwani (1990); Forbes and Rigobon (2002); Karolyi (2003); Pericoli and Sbracia (2003); Jotikasthira et al. (2012); Bekaert et al. (2014).

[10] Although my baseline model is a representative agent model, comparative statics with respect to exposure are informative about a cross-section of entities which are exposed to the same set of risk factors, but which vary in their degree of exposure to these factors. Note that the terms entities is used generically to encompass firms, countries, financial markets or financial institutions.

[11] Such as King and Wadhwani (1990); Calvo and Mendoza (2000); Kodres and Pritsker (2002); Calvo (2004); Acharya and Yorulmazer (2008); Allen, Babus, and Carletti (2012).

2 Model

This section formally introduces the mechanism through which the impact of a shock can decrease with exposure to it. It outlines a baseline model of learning about exogenous risk factor exposures when the capacity to process information is limited. The basic result is that it is optimal to learn more about higher risk factor exposures, and this information advantage can mitigate the consequences of shocks.

2.1 Structure of the Economy

I illustrate the mechanism in the context of a simple canonical model of investment. There is a risk-neutral, representative firm in the economy. The firm undertakes investment with an aim to maximize expected profits. Realized profits are given by

$$\pi = \lambda\theta - C(\lambda), \tag{1}$$

where λ is the chosen level of investment, $C(\lambda)$ is the cost of investment and θ is the exogenous gross return on investment. The random variable θ parametrizes some unknown exogenous state variable that affects the return on investment. Following Angeletos and Pavan (2004) and in line with the motivation, I interpret the random variable θ as the underlying fundamentals in the economy, but it can also be thought of as exogenous productivity or production technology. My preferred interpretation captures the intuition that real investment returns are affected by the state of the economy in which firms operate. The cost function $C(\lambda)$ is increasing and convex in investment, and assumed to take the quadratic form $\frac{\lambda^2}{2}$.

Economic fundamentals θ are a function of exogenous exposures to independent risk factors. More specifically, economic fundamentals are modelled as a sum of independent risks

$$\theta = \alpha f_1 + (1-\alpha) f_2, \tag{2}$$

where f_1 and f_2 are exogenous risk factors which affect fundamentals in proportion to exogenous exposures or factor loadings, α and $1-\alpha$, respectively.[12] The risk factors, f_i, are given by

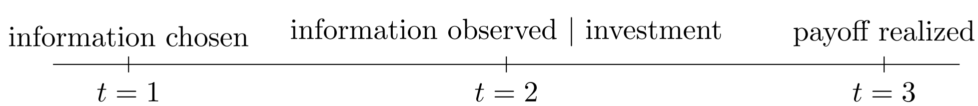
$$f_i = \mu_i + \epsilon_i, \; i = 1, 2,$$

[12] The model can be extended without loss of generality to include more than two factors.

where μ_i are constants and ϵ_i are independently distributed normal random variables, $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$, which will be further referred to as shocks.

This simple factor structure accommodates a number of interpretations. The interpretation preferred here is that they are country-specific risks. This captures the intuition that aggregate economic outcomes in a country are affected both by events occurring within the country, and to the extent that it engages in economic relations with other countries, by events occurring within those other countries. Alternatively stated, fundamentals in a country are determined by risks that are specific to that country, i.e. domestic risks, and risks that are specific to other countries that the country has links with, i.e. foreign risks. The extent to which these risks affect economic fundamentals is captured by the exposure parameters α and $1-\alpha$. For instance, the first factor f_1 can be thought of as capturing domestic risks and the second factor f_2 can be thought of as capturing foreign risks; a relatively high exposure parameter $\alpha > 0.5$ would thus describe a relatively closed economy for which domestic risks are more important, while a relatively low parameter $\alpha < 0.5$ would describe a relatively open economy that is more exposed to foreign risks.

Fundamentals are realized but unknown when the firm chooses its investment and this introduces uncertainty about the optimal level of investment. To reduce this uncertainty, the firm chooses how much information to observe about the risk factors affecting fundamentals, before investing. Reducing the uncertainty about the unobserved fundamentals will enable the firm to reduce the loss due to suboptimal investment and thus achieve a higher payoff and utility. The sequence of events is illustrated below.



The model can thus be broken down into three periods 1, 2 and 3. In the first period the representative firm chooses its information. In the second period, the firm observes the chosen information and optimally decides on a level of investment. In the third period the payoff of the investment is realized and utility is consumed. The firm's objective function is to maximize date-1 utility given by

$$U_1 \equiv E_1[E_2[\pi]], \tag{3}$$

where $U_t[\cdot]$ and $E_t[\cdot]$ denote the expected utility and expected value, respectively, conditional on the information available at time t.

2.2 Information Structure

The firm takes the structure of risk factor exposures as given and decides how much to reduce uncertainty about each risk factor through learning. However, the firm has limited capacity to process information, meaning that its choice of how much information to observe about each risk factor is subject to a constraint on the total amount of information that it can observe.

The firm devotes limited information processing resources to learn about the factor-specific shocks affecting fundamentals. It is endowed with the prior beliefs that the shocks follow a normal distribution, $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$, and acquires noisy signals about each shock

$$s_i = \epsilon_i + \epsilon_{s_i}, \quad i = 1, 2 \tag{4}$$

where the signal noise is normally distributed, $\epsilon_{s_i} \sim \mathcal{N}(0, \sigma_{s_i}^2)$, and uncorrelated with the other random variables. The firm combines the prior beliefs with the acquired signals and forms posterior beliefs according to Bayes' law. Let $\hat{\theta}$ and $\hat{\sigma}^2$ denote the posterior mean and variance of fundamentals, respectively, conditional on the information available at time 2

$$\hat{\theta} \equiv E[\theta|s_1, s_2] = \alpha \left(\mu_1 + \frac{\hat{\sigma}_1^2}{\sigma_{s_1}^2} s_1\right) + (1-\alpha)\left(\mu_2 + \frac{\hat{\sigma}_2^2}{\sigma_{s_2}^2} s_2\right), \tag{5}$$

$$\hat{\sigma}^2 \equiv V[\theta|s_1, s_2] = \alpha^2 \hat{\sigma}_1^2 + (1-\alpha)^2 \hat{\sigma}_2^2, \tag{6}$$

where $\hat{\sigma}_i^2 \equiv \left(\sigma_i^{-2} + \sigma_{s_i}^{-2}\right)^{-1}, \; i = 1, 2$ denotes the factor-specific posterior variance.

The firm has limited resources or capacity to process information about the risk factors that fundamentals are exposed to. Let K denote the total capacity to process information and let k_i denote the amount of capacity devoted to learning about risk factor $i = 1, 2$. The information processing constraint can be generically formulated as

$$k_1 + k_2 \leq K. \tag{7}$$

Essentially, this condition tells us that the firm's choice of how much information to observe about each risk factor is subject to a constraint on the total amount of information it can observe. It also implies that for a given total capacity, learning more about one risk factor reduces the resources that can be devoted to learning about the other risk factor.

The capacity devoted to learning about a risk factor, k_i, henceforth referred to as the factor-specific information processing capacity, is essentially a measure of the reduction in uncertainty that can be achieved through learning. I employ the rational inattention framework proposed by Sims (2003), and model this reduction in uncertainty using tools from information theory, namely entropy and mutual information. Entropy is the standard measure of information in information theory, and it measures the amount of uncertainty in a distribution. Mutual information is the difference between the entropy of an unconditional and a conditional distribution, and it measures the amount of uncertainty resolved by conditioning on information. The factor-specific information processing capacity, k_i, is a mutual information measure defined as the difference between the entropy of prior and posterior beliefs. The higher the factor-specific information processing capacity, k_i, the higher the uncertainty resolved by an information signal and the more informative or precise the signal is said to be. Given the assumption of normally distributed priors and signals, the factor-specific information processing capacity is given by

$$k_i \equiv \frac{1}{2} \ln \frac{\sigma_i^2}{\hat{\sigma}_i^2}, \quad i = 1, 2, \tag{8}$$

where $\hat{\sigma}_i^2$ is the factor-specific posterior variance, and $\hat{\sigma}_i^{-2} \equiv \left(\sigma_i^{-2} + \sigma_{s_i}^{-2}\right)$ is the factor-specific posterior precision.

The entropy-based learning technology essentially imposes a bound on the product of factor-specific posterior precisions.[13] This has two implications that make this information processing technology suitable for the setup considered here. First, it has a form of increasing returns to learning built into it, which means that it is less costly to learn about risks that are already well understood.[14] This captures the intuition that learning about risk factor exposures that are fixed or sticky, and which take a long time to build or terminate, such as the cross-country real or financial links considered in the motivating example, is a process of refined learning whereby the interpretation of new information builds on existing information. Second, the entropy-based learning technology implies that the marginal cost of increasing precision about one risk factor is proportional to the precision of information about the other risk factors, which means that it is more costly to learn about all risk factor exposures rather than specialize in learning about one. This captures the intuition that learning about one risk factor will affect the ability to learn about the other risk factors, and it is costly to learn about all relevant risk factor exposures.

[13] This follows from combining (7) and (8) and re-writing the capacity constraint as $\prod_{i=1}^{2} \sigma_i^2 \hat{\sigma}_i^{-2} \leq e^{2K}$. Thus, more information capacity allows for a higher product of posterior precisions, $\hat{\sigma}_i^{-2}$, weighted by prior variances, σ_i^2.

[14] Given that posterior precision is the sum of prior precision and signal precision, an increase in signal precision when prior precision is high (prior variance is low) increases the product capacity constraint by less, since $\prod_{i=1}^{2} \sigma_i^2 \hat{\sigma}_i^{-2} \leq e^{2K}$.

In addition to the capacity constraint, the agent also faces a no-forgetting constraint which rules out the possibility of forgetting information about one risk factor in order to obtain more information about another one, without violating the capacity constraint

$$k_i \geq 0, \quad i = 1, 2. \tag{9}$$

This is a condition that posterior variance should not exceed prior variance, which effectively restricts the precision of each signal from being negative. It captures the intuition that learning about a risk factor should not increase uncertainty.

2.3 Solving the model

Given a level of information processing capacity K, a solution to the model is: a choice of factor-specific information processing capacity k_i to maximize date-1 utility (3), subject to the capacity constraint (7), the no-forgetting constraint (9) and rational expectations about the date-2 conditional investment; posterior beliefs which are formed according to Bayes' law (5) and (6), given a signal about the risk factors; a choice of investment that maximizes expected utility, given the signal realization.

The model is solved using backward induction. First, given an arbitrary information choice, the firm decides the optimal investment. Then, given the optimal investment for each information choice, the firm decides the optimal information choice.

3 Results

In this section, I derive the equilibrium allocation of information processing capacity across risk factors. Then I discuss the implications of these information choices in terms of the uncertainty faced by the firm when investing. Finally, I discuss the implications in terms of the chosen level of investment and the loss due to suboptimal investment.

3.1 Optimal Information Choice

The date-2 problem consists of the firm choosing an investment level to maximize expected profits (1), while taking information choice as given

$$\max_{\lambda} U_2 \equiv E_2[\pi] = \lambda E_2[\theta] - \frac{\lambda^2}{2} \tag{10}$$

where E_2 denotes the expected value conditional on the information available at date 2.

The first order condition with respect to λ yields best investment response $\lambda = E_2[\theta] = \hat{\theta}$. Thus, for any given information choice, the optimal investment level is the expected level of economic fundamentals. Substituting this optimal investment choice into the objective (10) delivers the indirect date-2 utility of having any posterior beliefs and investing optimally

$$U_2 = \frac{(E_2[\theta])^2}{2} = \frac{\hat{\theta}^2}{2}. \tag{11}$$

The date-1 problem consists of the firm choosing the optimal level of information processing capacity devoted to learning about each risk factor to maximize the expected value of (11), and subject to the capacity constraint (7) and the no-forgetting constraint (9). The posterior mean, $\hat{\theta}$, is unknown at date 1 when the investment decision is made. It is a normally distributed random variable, $\hat{\theta} \sim \mathcal{N}(\theta, \sigma^2 - \hat{\sigma}^2)$, so date-1 utility is given by

$$U_1 = E_1[U_2] = \frac{E_1[\hat{\theta}^2]}{2} = \frac{E_1[\hat{\theta}]^2 + V_1[\hat{\theta}]}{2} = \frac{\theta^2 + \sigma^2 - \hat{\sigma}^2}{2}. \tag{12}$$

Since date-1 utility is decreasing in posterior uncertainty, $\hat{\sigma}^2$, and all other terms are exogenous variables, maximizing equation (12) subject to (7) and (9), is equivalent to

$$\max_{k_1, k_2} \; - \hat{\sigma}^2 = -\alpha^2 \hat{\sigma}_1^2 - (1-\alpha)^2 \hat{\sigma}_2^2$$

$$\text{s.t. } \hat{\sigma}_i^2 = \sigma_i^2 e^{-2k_i} \quad \text{and} \quad \sum_{i=1}^{2} k_i \leq K \quad \text{and} \quad 0 \leq k_i, \quad i = 1, 2.$$

The unique solution to this problem delivers the following optimal factor-specific capacity allocation

$$k_1 = \begin{cases} 0 & \text{if} \quad \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} < e^{-K} \\ \frac{1}{2}\left(K + \ln\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) & \text{if} \quad e^{-K} \leq \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} \leq e^{K} \\ K & \text{if} \quad \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} > e^{K} \end{cases} \tag{13}$$

and $k_2 = K - k_1$.

At the interior optimum, the optimal level of capacity devoted to learning about a risk factor increases with factor-specific exposure, factor-specific prior uncertainty and total information processing capacity. For a given level of information processing capacity, K, the firm will optimally choose to learn more about the risk factors that are relatively more important in terms of exposure and which are ex-ante more uncertain. If exposure to and prior uncertainty regarding a risk factor are sufficiently low (high) relative to the other factor, the firm will optimally choose to allocate no (all) capacity to learning about it. The higher the capacity to process information, K, the smaller the range of parameter values for which corner solutions are obtained, but corner solutions will be obtained for any limited information processing capacity. Alternatively stated, for any finite level of capacity, $K < \infty$, there exists a level of exposure, $0 < \alpha < 1$, for which corner solutions are obtained, and the firm will stop learning about one of the risk factors.

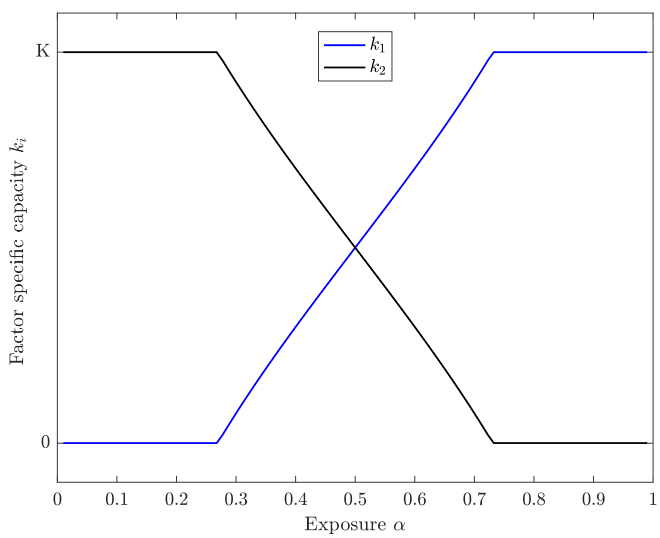


Figure 1. The parameter values are $\sigma_1 = \sigma_2 = 1$ and $K = 1$.

Figure 1 plots the optimal level of information processing capacity allocated to factor 1, k_1, and factor 2, k_2, against exposure to factor 1, α. If exposure to factor 1 is sufficiently low, then the firm optimally chooses not to learn about it and instead devotes all information processing resources to learning about factor 2. The economic intuition is that when exposure to factor 1 is very low, the marginal benefit of learning about factor 1 is lower than the benefit of learning about factor 2, whose exposure is relatively higher. Consequently, the firm would like to forget information about factor 1 in order to obtain more information about factor 2 but the no-forgetting constraint prevents it from doing so and the zero capacity corner solution is obtained. A similar reasoning is applied when exposure to factor 1 is very high, leading to the full capacity corner solution for factor 1. At the interior optimum, factor-specific capacity increases with factor-specific exposure and the firm optimally learns more about the risk factor fundamentals are relatively more exposed to.

3.2 Implications for Uncertainty

Information choice is a tool to reduce the uncertainty that the firm faces when deciding on the level of investment. Relative to an equal capacity, exogenous information benchmark, uncertainty is lower when the firm can optimally choose what risk factors to learn about. Importantly, uncertainty is convex in exposure when the firm is exogenously endowed with information, but it is concave in exposure when the firm optimally learns about risk factor exposures.

Given the optimal information processing capacity allocated to each factor in (13), uncertainty can be backed out using results (8) and (6), and it is given by

$$\hat{\sigma}^2 = \begin{cases} \alpha^2\sigma_1^2 + (1-\alpha)^2\sigma_2^2 e^{-2K} & \text{if} \quad \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} < e^{-K} \\ 2\alpha(1-\alpha)\sigma_1\sigma_2 e^{-K} & \text{if} \quad e^{-K} \leq \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} \leq e^{K} \\ \alpha^2\sigma_1^2 e^{-2K} + (1-\alpha)^2\sigma_2^2 & \text{if} \quad \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} > e^{K} \end{cases} . \tag{14}$$

At the interior optimum, uncertainty increases with exposure to and prior uncertainty regarding the risk factors, and decreases with the total capacity to process information. The uncertainty expressions for the corner solutions reflect the intuition that when there is no learning about a risk factor its factor-specific posterior uncertainty is equal to its prior uncertainty, while the posterior uncertainty surrounding the other factor is reduced in proportion to the total capacity i.e. if $k_1 = 0$ then $\hat{\sigma}_1^2 = \sigma_1^2$ and $\hat{\sigma}_2^2 = \sigma_2^2 e^{-2K}$.

In order to assess the implications of learning for uncertainty, and thus conditional investment, a suitable benchmark is needed for comparison. I consider as benchmark a model in which the firm is exogenously endowed with an equal amount of information about each risk factor. This model will be further referred to as the exogenous information benchmark, while my model in which the firm can optimally choose its information about risk factor exposures will be referred to as the endogenous information model. The representative firms in the two models share the same prior beliefs regarding the distribution of shocks affecting fundamentals, $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$, but the firm in the exogenous information benchmark is endowed with signals with exogenous noise, $\tilde{\epsilon}_{s_i} \sim \mathcal{N}(0, \tilde{\sigma}_{s_i}^2),\ i = 1, 2$. Posterior beliefs are formed according to Bayes' rule, such that benchmark posterior uncertainty is

$$\tilde{\sigma}_B^2 = \alpha^2 \tilde{\sigma}_1^2 + (1 - \alpha)^2 \tilde{\sigma}_2^2, \tag{15}$$

where $\tilde{\sigma}_i^2 \equiv \left(\sigma_i^{-2} + \tilde{\sigma}_{s_i}^{-2}\right)^{-1},\ i = 1, 2$ denotes the factor-specific posterior variance when information is exogenous.

Figure 2 depicts the relationship between the degree of exposure to a risk factor and the uncertainty implied by the endogenous information model $\hat{\sigma}^2$ (blue line) and the exogenous information benchmark model $\tilde{\sigma}^2$ (red line). This exercise is informative about the uncertainty faced by firms operating in economies whose fundamentals share the same factor structure but vary in the degree of exposure to a risk factor, which in this case is the exposure to factor 1, measured by α. To enable meaningful comparisons, the total capacity in the endogeous learning model is set equal to the capacity implied by exogenous signal precisions in the benchmark model when the capacity constraint is binding i.e. $\frac{1}{2} \ln \frac{\sigma_1^2}{\tilde{\sigma}_1^2} + \frac{1}{2} \ln \frac{\sigma_2^2}{\tilde{\sigma}_2^2} = K$. This ensures that the two models are otherwise identical except for the ability to optimally reallocate information processing resources. Note that Figure 2 illustrates a symmetric equilibrium whereby the factors are ex-ante equally risky and the exogenous signals are equally informative, hence the symmetry around and intersection of the two lines at the exposure midpoint, $\alpha = 0.5$.

In terms of levels, note that relative to the equal capacity, exogenous information benchmark, uncertainty is lower when the firm can optimally allocate information processing resources across risk factors. This is because learning effectively reduces the uncertainty about individual risk factors. The reduction in uncertainty achieved through learning operates through what will be further referred to as the *information channel.* Given that learning optimally increases with exposure, the regions at the left and right of the exposure midpoint, $\alpha = 0.5$, depict situations of relatively higher exposure to a risk factor whose uncertainty is reduced more through learning, and as a consequence overall uncertainty will be lower under the endogenous information model than under the exogenous information benchmark. Appendix A.1 provides an analytical treatment of this intuition.

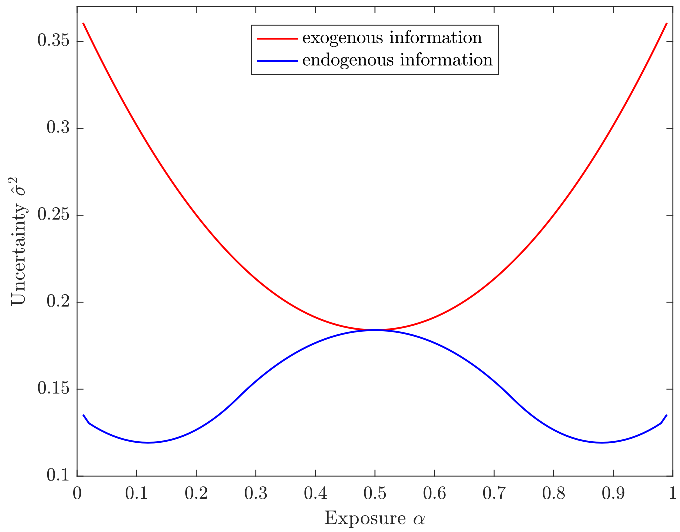


Figure 2. The parameter values are $\sigma_1 = \sigma_2 = 1$, $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 0.75$ and the total capacity implied by these parameters is $K = \frac{1}{2}\ln\sigma_1^2\left(\sigma_1^{-2} + \tilde{\sigma}_{s_1}^{-2}\right) + \frac{1}{2}\ln\sigma_1^2\left(\sigma_2^{-2} + \tilde{\sigma}_{s_2}^{-2}\right) = 1$.

In terms of dynamics, note that both models predict a non-monotonic relationship between overall uncertainty and the degree of exposure to a risk factor. While the benchmark model predicts a convex relationship between uncertainty and exposure, in the endogenous learning model uncertainty is concave in exposure at the interior optimum, and it is convex in exposure when corner solutions are obtained. The exogenous information benchmark model illustrates a classic diversification effect whereby high exposure to a risk factor implies high overall uncertainty, and the lowest level of uncertainty is achieved when exposure to the two factors is equal. Consequently, uncertainty initially decreases with exposure, but once the factor becomes important in terms of exposure, uncertainty starts to increase. This works through what will be further referred to as the *exposure channel*. The endogenous learning model, on the other hand, is convex in exposure when the firm optimally learns about one risk factor only, and it is concave in exposure when the firm learns about both risk factors.[15] In the corners, when the firm devotes all capacity to learning about one factor, information is essentially exogenous, so uncertainty dynamics will be the same as in the benchmark and will operate through the exposure channel. At the interior optimum though, uncertainty initially increases with exposure, but once the factor becomes important in terms of exposure, uncertainty starts to decrease. To understand the factors that are at play, start from the situation in which exposure to the two risk factors is equal i.e. $\alpha = 0.5$. As exposure to factor 1

[15] Note that uncertainty in the endogenous information model is convex on the same interval of exposure parameters over which the factor-specific capacity allocation illustrated in Figure 1 is extreme i.e. either zero or maximum.

increases beyond this exposure midpoint, uncertainty about factor 1 is effectively reduced through learning. The reduction in uncertainty achieved through learning is stronger than the increase in uncertainty due to higher exposure, and as a result overall uncertainty decreases with exposure to factor 1. In other words, the information channel dominates the exposure channel. However, if exposure to factor 1 is sufficiently high that the firm optimally devotes all information processing capacity to learning about factor 1, then the exposure channel will overturn the information channel. The result is an eventual increase in uncertainty, as reflected by the turning point in the uncertainty, which occurs when the learning-adjusted risk exposures of the two factors are equal.[16]

It is important to note the markedly different predictions of the exogenous information benchmark and of the endogenous information model at the interior optimum, where the firm optimally learns about both risk factors rather than specializing in learning about one risk factor, i.e. $k \in (0, K)$. To ease understanding, it is again useful to focus on the right side of Figure 2, where the exposure parameter $\alpha > 0.5$ indicates a relatively higher exposure to factor 1. Provided that risk factor 1 is relatively important in terms of exposure, uncertainty increases with exposure when information is exogenously given, but it decreases with exposure when information about both risks is endogenously chosen. Thus, an implication of the endogenous learning model is that in the cross-section of entities exposed to a relatively important risk factor, an entity that is more exposed to that risk will face a lower overall uncertainty, due to learning, relative to an entity that is less exposed to it.

PROPOSITION 1. *Provided that a risk is relatively important in terms of exposure, uncertainty decreases with exposure to it when the firm optimally learns about all risk factors i.e.* $\frac{\partial \hat{\sigma}^2}{\partial \alpha} < 0$ *if* $\alpha > 0.5$ *and* $k \in (0, K)$.

Proof. See Appendix A.1

Proposition 1 contains the key result of the paper and explains why the shock transmission patterns observed during the financial crisis of 2007-2008 were different relative to those observed during past contagious crises, such as the Mexican crisis of 1994 and the Asian crisis of 1997.[17] The 2007-2008 financial crisis was different because the epicentre of the crisis was a country playing a

[16] At the first corner (when learning about factor 2) uncertainty decreases if $\alpha\sigma_1^2 < (1-\alpha)\sigma_2^2 e^{-2K}$, which essentially reads that increasing exposure to a factor whose learning-adjusted risk is relatively lower decreases overall uncertainty. At the second corner (when learning about factor 1), uncertainty starts to increase when $\alpha\sigma_1^2 e^{-2K} > (1-\alpha)\sigma_2^2$ i.e. when learning-adjusted risk is higher.

[17] During these past crises, the epicentre country in which the shock originated was more severely affected than the other countries that were subsequently affected by the shock. In other words, the impact of the original shock increased with exposure to it during the Mexican and Asian crises, but it decreased with exposure during the last financial crisis of 2007-2008.

central role in the global economy: the United States is a country that is that is relatively important in terms of exposure and one that foreign decision-makers are likely to learn about, i.e. $\alpha > 0.5$ and $k \in (0, K)$. Thus, variation in the extent to which other countries were exposed to the United States translates into variation in uncertainty which follows the dynamics illustrated at the right of the exposure midpoint illustrated in Figure 2. The implication is that countries which were relatively more exposed to the United States shock faced a relatively lower level of uncertainty regarding the implications of the shock. Insofar as uncertainty affects the optimality of actions, and thus contributes to the transmission of shocks, this result can rationalize why during the financial crisis of 2007-2008, unlike during previous crises, the transmission of the shock decreased with exposure to the shock.

In sum, the benchmark illustrates a classic diversification effect which operates through the exposure channel. In the endogenous learning model, learning substitutes for diversification in reducing risk, resulting in a specialization effect. In a portfolio allocation context, Van Nieuwerburgh and Veldkamp (2010) also make the point that diversification is not optimal if portfolio choice is preceded by information choice. While they focus on the implications of information choice for portfolio holdings in a setup in which agents choose both their information about and exposure to risks, I focus on the role of information for the transmission of shocks to the real economy in a a setup in which agents choose their information about risks they are exogenously exposed to. Section 4.4 relaxes the assumption of exogenous exposures to risk factors and provides a characterization of the optimal exposure points.

3.3 Implications for Investment

Reducing uncertainty will enable the firm to take an investment decision that is more aligned with underlying fundamentals, and thus reduce the loss due to suboptimal investment.[18] Insofar as the firm' investment deviates from the first-best optimum obtained under perfect information, it indirectly contributes to the transmission of shocks. Whereas the transmission of shocks increases with exposure when information is exogenous given, I find that the transmission of shocks decreases with exposure when information is endogenously chosen. Consequently, relative to the exogenous information benchmark, the endogenous information model I propose predicts that the impact of shocks that fundamentals are relatively less exposed to is amplified, while the impact of shocks that

[18]Mathematically a lower level of uncertainty means higher response to signals that are informative about the underlying shocks affecting fundamentals.

fundamentals are relatively more exposed to is attenuated.

Recall that the firm optimally chooses a level of investment that is given by the expected level of fundamentals conditional on the information available at the intermediate date 2, i.e. $\lambda = E_2[\theta] = \hat{\theta}$. Thus, the investment decision is essentially a response to information about the realization of shocks affecting fundamentals. Figure 3 depicts the relationship between the degree of exposure to risk factor 1 and the response or sensitivity of investment to a shock to factor 1, which is implied by the exogenous information benchmark (red line), the endogenous information model (blue line) as well as a full information model (black line). Note that the shock sensitivity increases with exposure under all the three models considered but the rate of increase is different across models.

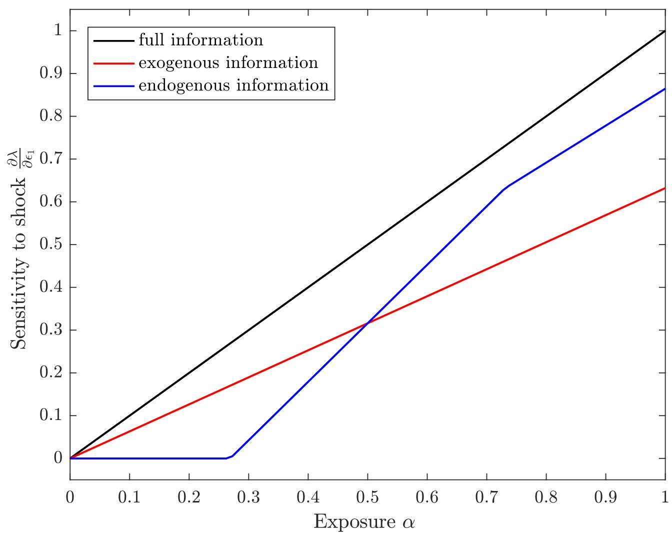


Figure 3. The parameter values are $\sigma_1 = \sigma_2 = 1$, $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 0.75$ and $K = 1$.

Under the full information model shocks can be perfectly observed, so there is a one-to-one mapping between exposure to a shock, and the degree to which the shock is incorporated into investment decisions. This represents the first-best optimal response. To the extent that investment responses obtained under the other models deviate from this first-best optimum, they are said to be suboptimal and to act as a shock transmission mechanism. Under the exogenous information benchmark, the investment response deviates increasingly more from the optimal one as exposure increases, because this amounts to increasing exposure to a shock that is observed with a constant precision. Under the endogenous information model three regimes can be observed. First, when exposure to factor 1 is sufficiently low that the firm does not acquire any information about it, shock sensitivity is zero. Investment does not respond to the shock to factor 1 because the firm is essentially unaware of the underlying shock realization. Second, as exposure increases and the firm starts learning about factor 1, its investment response to the shock will become increasingly more aligned with the optimal one because the precision of information about the underlying shock optimally increases with exposure. Finally, when exposure is sufficiently high that the firm only learns about factor 1, the investment response starts to deviate again from the first-best optimum as exposure increases, because information precision, albeit set at the maximum, is essentially exogenous.

Thus, information frictions introduce a loss or inefficiency relative to the perfect information model. When shocks can be perfectly observed, investment decisions are fully responsive to the shocks affecting economic fundamentals and, as a consequence, investment returns. However, when shocks cannot be perfectly observed, investment decisions are relatively less responsive to shocks, and the firm fails to fully incorporate underlying shocks into investment decision-making. In case of positive shocks, the deviation from the perfect information optimum is negative and the firm increases investment by less than it optimally should. This represents a situation of under-investment whereby business opportunities are lost. In case of negative shocks the deviation is positive and the firm reduces investment by less than it optimally should. This is a situation of over-investment, resulting in excess capacity and wasted resources. In order to ease analysis and abstract from the nature of shocks, I define the loss due to suboptimal investment as $L \equiv (\hat{\theta} - \theta)^2$. This symmetric loss function captures a more general situation in which shocks are changes in circumstances that the firm needs to adapt to, such as the introduction of standards, technological disruption or terms of trade changes, rather than positive or negative events.

Figure 4 plots the loss due to suboptimal investment against exposure to factor 1. The example considers a one standard deviation shock to factor 1, abstracts from factor 2 shocks as well as from information shocks. Under the exogenous information benchmark, loss increases monotonically with exposure to the shock. Given that signal precision is fixed, this result works through the exposure channel and is due to increasing exposure to a constant amount of uncertainty. Under the endogenous information model, three regimes can be observed as exposure increases. First, when exposure is sufficiently low such that the firm does not learn about factor 1, the loss due to suboptimal investment is increasing in exposure. The rate of increase is higher relative to the exogenous information benchmark because the firm chooses to observe no information, as opposed to fixed precision information, about factor 1. Second, as exposure increases and the firm starts to learn about factor 1, the loss due to suboptimal action decreases with exposure because more information enables a more accurate incorporation of shocks into investment decisions. Third, when the firm learns only about factor 1, the loss due to suboptimal investment starts to increase again because information precision is essentially exogenous and as a consequence dynamics resemble the benchmark.

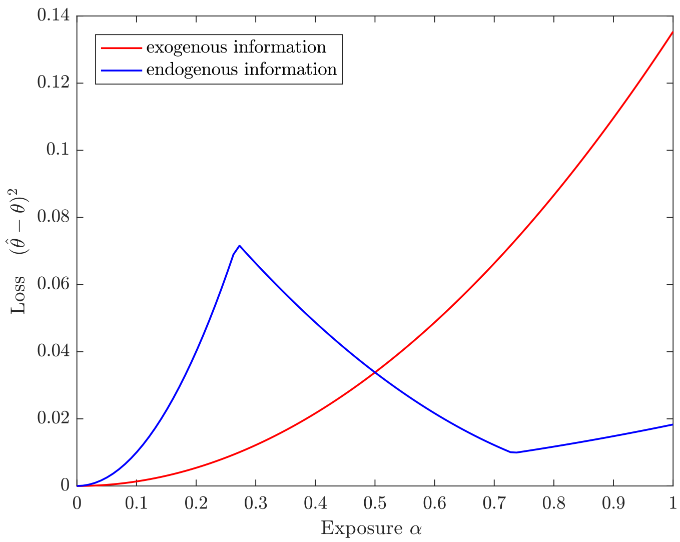


Figure 4. This example considers a one standard deviation shock to factor 1 i.e. $\epsilon_1 = \sigma_1$, abstracts from factor 2 shocks i.e. $\epsilon_2 = 0$ and information shocks i.e. $\epsilon_{s_1} = \epsilon_{s_2} = 0$. The parameter values are $\mu_1 = \mu_2 = 1$, $\sigma_1 = \sigma_2 = 1$ and $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 0.75$ and the total capacity implied by these parameters is $K = \frac{1}{2}\ln\sigma_1^2\left(\sigma_1^{-2} + \tilde{\sigma}_{s_1}^{-2}\right) + \frac{1}{2}\ln\sigma_1^2\left(\sigma_2^{-2} + \tilde{\sigma}_{s_2}^{-2}\right) = 1$.

Worth emphasizing is the fact that the predictions of the two models are starkly different when an interior solution is obtained under the endogenous information model i.e. when the firm learns about both risks. Whereas the loss due to suboptimal action is increasing with exposure when the firm is exogenously endowed with information about the risk factors, it is decreasing with exposure when the firm optimally chooses what risk factors to learn about. Consequently, relative to the exogenous learning benchmark, shocks that the firm is relatively more exposed to are attenuated, while shocks that the firm is relatively less exposed to are amplified.[19] If the shock to factor 1 is interpreted as a shock to United States and the representative firm's investment is interpreted as an aggregate economy quantity, Figure 4 implies that a country which is less exposed to the United States (and which is situated towards the left end of the x-axis) will incur a higher loss due to suboptimal investment compared to a country which is more exposed to the United States (and which is situated towards the right end of the x-axis). It also implies that a country which is less exposed to the United States will incur a higher loss due to suboptimal investment compared to

[19]The magnitude of amplification decreases with the firm's capacity to learn.

the United States itself (which is likely to be situated towards the right end of the x-axis). These predictions are in line with evidence from the financial crisis of 2007-2009 suggesting that countries other than the epicentre have been more severely affected than United States itself, and, more generally, countries relatively less exposed to the United States have been relatively more affected.

The results illustrated in Figure 4 can be understood by examining analytically how investment decisions act as a shock transmission mechanism. To that end, I define the shock transmission mechanism as the change in the loss due to suboptimal investment that is induced by a shock i.e. $\frac{\partial L}{\partial \epsilon_i}$. The shock transmission mechanism is a measure of the extent to which a shock translates into losses due to suboptimal investment, which effectively captures the negative consequences caused by decision-makers' suboptimal responses. The interaction between the strength of the shock transmission mechanism and the magnitude of the shock determines the impact of a shock. Consequently, statement about the shock transmission mechanism are informative about the impact of a shock. The stronger the shock transmission mechanism, the higher the loss due to suboptimal investment, and the higher the impact of the shock is said to be.

PROPOSITION 2. *The transmission of shocks decreases with exposure when the firm optimally learns about all risk factors i.e.* $\frac{\partial^2 L}{\partial \epsilon_1 \partial \alpha} < 0$ *if* $k \in (0, K)$.

Proof. See Appendix A.2.

Proposition 2 pins down the mechanism through which the impact of a shock can decrease with exposure to it. When the firm optimally learns about the risk factors affecting fundamentals, the loss due to suboptimal investment that is induced by a shock to a risk factor decreases with exposure to it. In other words, the contribution of investment actions to the transmission of shocks decreases with exposure. This is because the reduction in uncertainty that is achieved through learning increases with exposure, and as a consequence the deviation of the firm's investment from the perfect information optimum decreases with exposure. Thus, the informational benefit mitigates the direct impact of shocks to risk factors that fundamentals are relatively more exposed to, as it enables the firm to take better informed decisions and minimize the loss due to suboptimal investment. By the same token, the impact of shocks to risk factors that fundamentals are relatively less exposed to is amplified through the firm's poorly informed investment decision, which induce a higher loss due to suboptimal investment.

4 Extensions

In this section, I consider the following extensions to the baseline model studied so far. First, I account for the degree of anticipation of shocks to risk factors. Second, I extend the model to a multi-firm setting and allow for strategic interactions between their investment actions. Third, I relax the assumption that the risk factors affecting fundamentals are independent. Finally, I relax the assumption that exposures to the risk factors are exogenous.

4.1 Extension: Shock Anticipation

The baseline model considers the case in which the firm allocates an exogenously given information processing capacity, K, across risk factor exposures. This section endogenizes the capacity available to the firm in a certain state of nature, by linking it to the degree of anticipation of that state. The basic result is that the impact of unanticipated shocks is amplified because the firm optimally devotes less information processing capacity to learning about states of nature that are expected to occur with a low probability.

One of the risk factors affecting fundamentals, say factor 1, is assumed to be in one of two possible states of nature: a low-probability, low-mean state of nature interpreted as rare times, and a high-probability, high-mean state of nature interpreted as normal times. Let $p_r > 0$ denote the probability that factor 1 is in the rare state of nature, and let $p_n = 1 - p_r > 0.5$ denote the probability that factor 1 is in the normal state of nature. The notion that it is ex-ante unlikely for the rare state to occur is captured by the condition $p_r < p_n$. A crisis is said to occur if factor 1 is revealed to be in the rare state of nature. This setup can thus be graphed as

$$f_1 \begin{cases} \xrightarrow{\;p_r\;} & \underline{\mu_1} + \epsilon_1, \quad \epsilon_1 \sim \mathcal{N}(0, \sigma_1^2) \\ \xrightarrow{\;p_n = 1 - p_r\;} & \overline{\mu_1} + \epsilon_1, \quad \epsilon_1 \sim \mathcal{N}(0, \sigma_1^2) \end{cases}$$

where $\underline{\mu_1}$ and $\overline{\mu_1}$ denote the mean levels of the risk factor in the rare and normal states, respectively, and where it holds that $\underline{\mu_1} < \overline{\mu_1}$. To isolate the effect of the degree of anticipation alone, I assume that only the mean level of the risk factor is expected to be different in the two states, while the priors associated with the shocks affecting the risk factor are the same.[20]

[20] In any state of nature, the capacity allocation across risk factors is mean independent but increases with the

In this setup, there are two dimensions of information choice: how much information processing capacity to devote to a state of nature, and how to allocate that information across risk factors in each state of nature.[21] Let $\mathcal{K}$ denote the *total capacity* or total amount of information processing capacity available to the firm, K_s denote the *state capacity* or amount of information processing capacity dedicated to state of nature $s \in \{n, r\}$, and k_{is} denote the amount of information processing capacity dedicated to factor i in state $s \in \{n, r\}$. The capacity constraint (7) governing the allocation of capacity across risks in any state of nature $s \in \{n, r\}$ can now be formulated as

$$k_{1s} + k_{2s} \leq K_s,$$

and the capacity constraint governing the allocation of total capacity across states of nature is

$$K_n + K_r \leq \mathcal{K}.$$

The model is solved following the same approach as that used in the baseline model in Section 3, except that now the date-1 problem consists of two steps. As before, the first step is to allocate capacity across risk factors, given the optimal investment level and an arbitrary state capacity, K_s. The second step is to allocate total capacity across states of nature, given the optimal investment and the optimal capacity allocation across risk factors in any state, k_{is}. Let $U_1(K_s)$ denote the date-1 utility of investing optimally at the second date, and optimally allocating the available state capacity, K_s, across risk factors at the first date

$$U_1(K_s) = \begin{cases} -\alpha^2\sigma_1^2 - (1-\alpha)^2\sigma_2^2 e^{-2K_s} & \text{if } \ k_1 = 0 \\ -2\alpha(1-\alpha)\sigma_1\sigma_2 e^{-K_s} & \text{if } \ k_1 = \frac{1}{2}\left(K_s + \ln\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \\ -\alpha^2\sigma_1^2 e^{-2K_s} - (1-\alpha)^2\sigma_2^2 & \text{if } \ k_1 = K_s. \end{cases} \tag{16}$$

The date-1 problem for the allocation of total capacity, $\mathcal{K}$, across states is

$$\max_{K_n, K_r} \ p_n U_1(K_n) + p_r U_1(K_r) \tag{17}$$

$$\text{s.t. } \ K_n + K_r \leq \mathcal{K} \ \text{ and } \ K_s \geq 0, \ s \in \{n, r\} \tag{18}$$

The basic result is that the optimal level of information processing capacity dedicated to a state of nature increases with the probability of occurrence of the state. At the interior optimum, when

prior uncertainty surrounding a risk factor i.e. the volatility of the shock affecting the risk factor. Accounting for the intuition that crises episodes are characterized by high volatility, reduces the magnitude of the effect but leaves the main result qualitatively unchanged. This case is dealt with in Appendix A.3.

[21]This can be thought of as capturing situations in which decision-makers prepare for different contingencies.

the firm learns about both risk factors, the optimal level of capacity dedicated to the rare state is given by

$$K_r = \begin{cases} 0 & \text{if } \frac{p_r}{p_n} < e^{-\mathcal{K}} \\ \frac{1}{2}\left(\mathcal{K} + \ln\frac{p_r}{p_n}\right) & \text{if } \frac{p_r}{p_n} \geq e^{-\mathcal{K}} \end{cases} \tag{19}$$

Note that a corner solution is possible. More specifically, if the probability of the state of nature is sufficiently low, no capacity is allocated to the state. Otherwise, the information processing capacity allocated to a state of nature increases with its degree of anticipation, as well as with the total capacity, $\mathcal{K}$, available. Appendix A.3 provides a full characterization of the equilibria.

The implication of this capacity allocation in terms of state contingent investment schedules is that the impact of shocks decreases with their degree of anticipation because the loss due to suboptimal investment is lower the more anticipated the shock. Given that the firm optimally acquires less information about low-probability events, the deviation of the firm's investment from the perfect information optimum is higher the lower the probability of occurrence of a shock. Thus, the contribution of investment decisions to the transmission of shocks is higher the lower their ex-ante probability of occurrence and their transmission is said to be intensified, resulting in the amplification of unanticipated shocks.

PROPOSITION 3. *The transmission of shocks decreases with their degree of anticipation when the firm optimally learns about all risk factors i.e.* $\frac{\partial^2 L}{\partial \epsilon_1 \partial p_s} < 0$ *if* $k_{1s} \in (0, K_s)$, $s \in \{n, r\}$.

Proof. See Appendix A.3.

Proposition 3 is in line with empirical evidence suggesting that the degree of anticipation of shocks plays an important role in generating contagion. The financial contagion literature has documented a negative relation between the degree of anticipation of crises and the occurrence of contagion (Kaminsky et al., 2003; Rigobon and Wei, 2003; Didier, Mauro, and Schmukler, 2008; Mondria and Quintana-Domeque, 2013), and has even argued that a necessary condition for the contagious transmission of shocks is that they are unexpected. The academic and regulatory discussions around the 2007-2009 financial crisis have also noted that the highly unexpected nature of the Lehman shock has likely amplified its transmission and consequences. My model predicts that contagion is more likely to occur following unexpected crises because decision-makers optimally prepare less for unexpected events.

Figure 5 plots the loss due to suboptimal investment against exposure to a one standard deviation shock to factor 1, for varying degrees of shock anticipation. The loss due to suboptimal investment is larger when the shock occurs with a small probability (solid lines), relative to the case in which the shock occurs with a higher probability (dashed lines). The loss due to suboptimal investment that is induced by a shock decreases with the degree of anticipation of that shock because the firm optimally devotes less information processing resources to learning about less anticipated shocks. Thus, endogenizing the information processing capacity available in a state of nature has the effect of amplifying the impact of shocks occurring in unexpected states; decision-makers are unprepared for unexpected shocks and this amplifies their consequences.

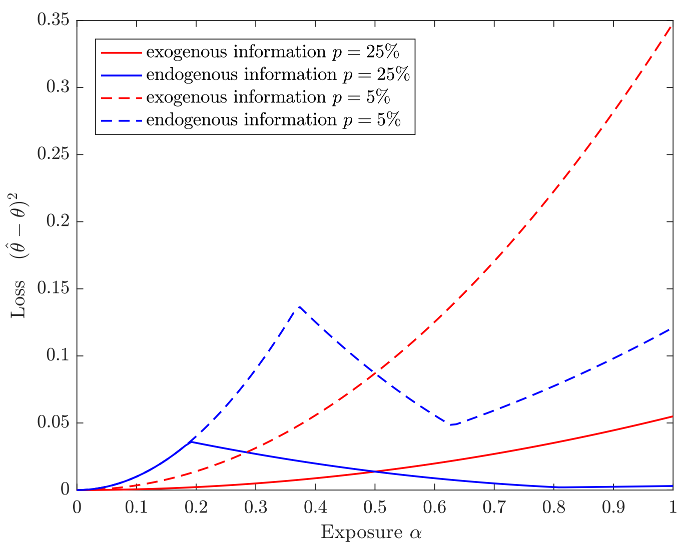


Figure 5. This example considers a one standard deviation shock to factor 1 i.e. $\epsilon_1 = \sigma_1$, abstracts from factor 2 shocks i.e. $\epsilon_2 = 0$ and information shocks i.e. $\epsilon_{s_1} = \epsilon_{s_2} = 0$. The parameter values are $\sigma_1 = \sigma_2 = 1$; when $p = 25\%$ $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 0.55$ and $K = 1.45$, and when $p = 5\%$ $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 1.20$ and $K = 0.53$.

4.2 Extension: Strategic Interactions

This subsection extends the baseline model to a multi-firm setup and explores the implications of strategic interactions for the equilibrium allocation of information processing capacity and the loss due to suboptimal investment. Relative to the baseline model, strategic complementarity in investment reduces the incentive to learn about the risk factors that fundamentals have relatively low exposure to, and results in a higher loss due to suboptimal investment. Consequently, when firms seek to coordinate their investment decisions the impact of shocks that fundamentals are relatively less exposed to is amplified relative to the baseline.

There is a continuum of firms indexed by j. Each firm chooses its level of investment, λ_j, to maximize expected profits. The profit function for firm j is

$$\pi_j = R\lambda_j - \frac{1}{2}\lambda_j^2. \tag{20}$$

The return on investment, R, is a function of the unknown fundamentals in the economy, θ, and the average investment in the population, $\bar{\lambda} = \int_j \lambda_j$, and is given by

$$R = (1-r)\theta + r\bar{\lambda}, \tag{21}$$

where r is a constant governing the type of strategic interactions between firms. Real investment environments have typically been treated as being characterized by strategic complementarity in actions, and the subsequent analysis will focus on this case but the model results can easily accommodate a strategic substitutability interpretation. In environments characterized by strategic complementarities agents want to do what others do. This is captured by a positive interactions coefficient, $r > 0$, which implies that optimal individual responses, λ_j, increase in the average population response in the population, $\bar{\lambda}$. If $r = 0$ individual actions are independent of the average action in the population and the baseline model is obtained.

As in the baseline model, the solution strategy is to work backwards. At date 2, each firm undertakes a level of investment to maximize the expected profit function (20), while taking information choice as given. Thus, the firm problem is

$$\max_{\lambda_j} U_{2j} \equiv E_{2j}[R\lambda_j - \frac{1}{2}\lambda_j^2], \tag{22}$$

where E_{2j} denotes the expected value conditional of the information available to firm j at date 2. The first order condition yields optimal investment response

$$\lambda_j = E_{2j}[R] = (1-r)E_{2j}[\theta] + rE_{2j}[\bar{\lambda}]. \tag{23}$$

I consider equilibria in which the mean investment in the population is a linear function of the shocks affecting fundamentals

$$\bar{\lambda} = \psi + \phi_1\epsilon_1 + \phi_2\epsilon_2 \tag{24}$$

where ψ, ϕ_1 and ϕ_2 are constants determined in equilibrium. Using the fact that $E_{2j}[\theta] = \alpha(\mu_1 + E_{2j}[\epsilon_1]) + (1-\alpha)(\mu_2 + E_{2j}[\epsilon_2])$, and substituting conjecture (24) into the first order condition (23) yields the following investment response for firm j

$$\lambda_j = r\psi + (1-r)(\alpha\mu_1 + (1-\alpha)\mu_2) + [\alpha(1-r) + r\phi_1]E_{2j}[\epsilon_1] + [(1-\alpha)(1-r) + r\phi_2]E_{2j}[\epsilon_2].$$

Calculating first the conditional expectation of the shocks $E_{ij}[\epsilon_i] = (1-\gamma_i)s_{ij}$, and integrating over the investment response of all firms j, the average investment in the population is

$$\bar{\lambda} = r\psi + (1-r)(\alpha\mu_1 + (1-\alpha)\mu_2) + [\alpha(1-r) + r\phi_1](1-\gamma_1)\epsilon_1 + [(1-\alpha)(1-r) + r\phi_2](1-\gamma_2)\epsilon_2.$$

Matching coefficients verifies the conjecture (24) that the average investment level is linear in the shocks and that the linear weights are $\psi = r\psi + (1-r)(\alpha\mu_1 + (1-\alpha)\mu_2)$, $\phi_1 = [\alpha(1-r) + r\phi_1](1-\gamma_1)$ and $\phi_2 = [(1-\alpha)(1-r) + r\phi_2](1-\gamma_2)$. Collecting the unknown coefficients yields

$$\psi = \alpha\mu_1 + (1-\alpha)\mu_2 \tag{25}$$

$$\phi_1 = \frac{\alpha(1-r)(1-\gamma_1)}{1-r(1-\gamma_1)} \tag{26}$$

$$\phi_2 = \frac{(1-\alpha)(1-r)(1-\gamma_2)}{1-r(1-\gamma_2)}. \tag{27}$$

The date-1 problem consists of choosing the optimal level of capacity devoted to learning about each risk factor to maximize the expected utility implied by the investment response (24) and the equilibrium coefficients (25)-(27), subject to the capacity constraint (7) and the no-forgetting constraint (9)

$$\max_{k_1,k_2} \; U_{1j} = E_{1j}[U_{2j}] = \frac{1}{2}\Big[\psi^2 + \frac{\alpha^2(1-r)^2\sigma_1^2(1-\gamma_1)}{(1-r(1-\gamma_1))^2} + \frac{(1-\alpha)^2(1-r)^2\sigma_2^2(1-\gamma_2)}{(1-r(1-\gamma_2))^2}\Big] \tag{28}$$

$$\text{s.t.} \quad \gamma_i = e^{-2k_i}, \quad \sum k_i \leq K, \quad 0 \leq k_i, \quad i = 1, 2. \tag{29}$$

Numerical results indicate that relative to the baseline model with no strategic interactions, i.e. $r = 0$, the firm is more likely to learn about a single risk factor rather than both risks when investment actions are strategic complements, i.e. $r > 0$. Alternatively stated, as the degree of strategic complementarities increases, corner solutions occur more easily. In fact, if the degree of strategic complementarity is sufficiently high, the parameter region for which an interior solution is obtained collapses to a single point.[22] The implication is that for high levels of strategic complementarity, a small change in the exposure parameter can have a large effect on the

[22]For strategic complementary parameters beyond this point multiple equilibria exist.

equilibrium allocation of information processing capacity.

At the interior optimum, when the firm optimally learns about both risk factors, the optimum level of capacity allocated to factor 1 decreases with the degree of complementarity if exposure to factor 1 is relatively low, i.e. $\frac{\partial k_1}{\partial r} < 0$ if $\alpha < 0.5$, but it increases if exposure to factor 1 is relatively high, i.e. $\frac{\partial k_1}{\partial r} > 0$ if $\alpha > 0.5$. Relative to the baseline, strategic complementarity in investment reduces the incentive to learn about risk factors that fundamentals have a relatively low exposure to, but increases the incentive to learn about risk factors that are relatively important in terms of exposure. This is because the firm anticipates that learning behaviour in the population is such that the capacity devoted to learning about a risk factor increases with exposure to it. Given that the firm wants to do what others do, it will also want to learn what other firms learn, and as a consequence it will choose to learn less about low-exposure risk factors and more about high-exposure risk factors. Consequently, the transmission of shocks to risk factors that fundamentals are relatively less exposed to is amplified as the degree of strategic complementarity increases.

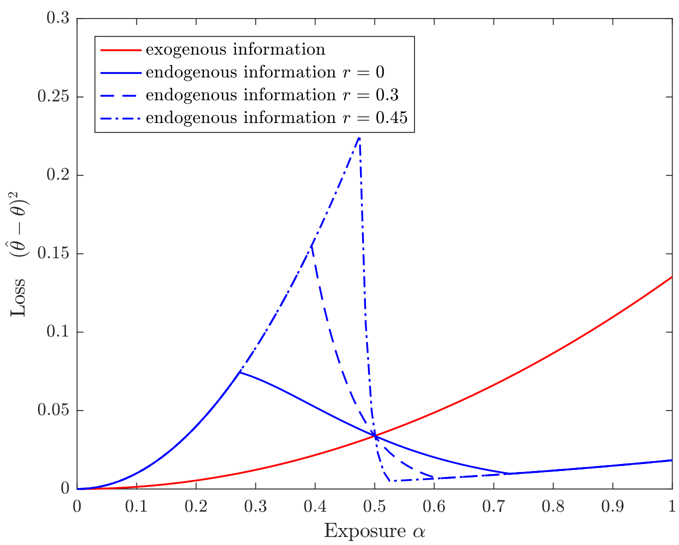


Figure 6. This example considers a one standard deviation shock to factor 1 i.e. $\epsilon_1 = \sigma_1$, abstracts from factor 2 shocks i.e. $\epsilon_2 = 0$ and information shocks i.e. $\epsilon_{s_1} = \epsilon_{s_2} = 0$. The other parameter values are $\sigma_1 = \sigma_2 = 1$, $K = 1$ and $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 0.75$.

The implications in terms of shock impact are illustrated in Figure 6, which plots the loss due to suboptimal investment against exposure to a one standard deviation shock to factor 1, for varying levels of strategic complementarity. The loss due to suboptimal investment is larger in environments characterized by a higher level of strategic complementarity (dashed lines), relative to the baseline case in which there are no strategic interactions (solid lines). This is due to the fact that the firm's incentive to hedge against shocks through learning is weakened by the desire to coordinate its investment decision with the average investment in the population. Since the firm anticipates other firms will optimally choose to learn less about the risk factors that fundamentals have relatively little exposure to, its incentive to learn about these low-exposure risk factors decreases as the degree of strategic complementarity increases. As a consequence, the loss due to suboptimal action that is induced by a shock is amplified in the presence of strategic complementarities.

4.3 Extension: Correlated Risks

The baseline model considers the case in which the risk factors affecting fundamentals are independent. In this section, I allow for the risk factors to be correlated, and find that the loss due to suboptimal investment increases with the degree of correlation between the two risk factors.

To deal with the case of correlated risks it is useful to use matrix notation. As in the baseline model, economic fundamentals are modelled as a sum of risk factors. Let f be a $N \times 1$ vector of risk factors and A be a $N \times 1$ vector of exposures to these factors. Fundamentals can be expressed as

$$\theta = A'f$$

where the factors are ex-ante known to be correlated i.e. the prior variance-covariance matrix of the risk factors f is non-diagonal. Assuming that the prior variance-covariance matrix of the factors f is non-diagonal is equivalent to assuming the following linear structure for the risk factors[23]

$$f = \mu + \Gamma\epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma) \tag{30}$$

where μ is a $N \times 1$ vector of constants measuring the mean level of each risk factor, ϵ is a $N \times 1$ vector of independent random variables or shocks with diagonal variance-covariance matrix Σ, and Γ is an $N \times N$ matrix of loadings which measures the extent to which the independent shocks in ϵ affect the risk factors f.[24] The i^{th} row of the matrix Γ, denoted Γ_i, gives the loadings of the i^{th} risk factor, f_i, on the independent shocks in the vector ϵ. Thus, each risk factor f_i is expressed as

[23] The variance-covariance matrix of the risk factors that is implied by (30) is $\Gamma\Sigma\Gamma'$. Note that an alternative solution method is to assume that the prior variance-covariance matrix of the shocks is non-diagonal, say Ω, and then use eigen-decomposition to re-write is as $\Omega = \Gamma\Sigma\Gamma'$; learning would then be about the principal components with diagonal variance-covariance matrix Σ.

[24] The variance-covariance matrix of the risk factors that is implied by (30) is $\Gamma\Sigma\Gamma'$. An alternative and equivalent approach is to assume that the prior variance-covariance matrix of the shocks is non-diagonal, say Ω, and then use eigen-decomposition to re-write it as $\Omega = \Gamma\Sigma\Gamma'$.

the sum of a factor-specific mean, μ_i, and the independent shocks contained in the vector ϵ, which affect it in proportion to the loadings Γ_i.

This factor structure essentially allows for correlations between the risk factors through shared exposure to underlying independent shocks. It accounts for the existence of underlying forces that might be driving more than one of the risk factors affecting fundamentals. I conceptualize and further refer to these independent shocks as factor-specific shocks. For instance, I interpret shock 1, ϵ_1, as being specific to factor 1, f_1, while the interpretation given to shock 2, ϵ_2, is that of a shock that is specific to factor 2, f_2. Correlation is introduced by allowing factor 1, f_1, to load on the shock specific to factor 2, ϵ_2, and vice versa. Interpreted in the context of the motivating example, this setup accounts for the reality that domestic and foreign risks are likely to be correlated. The linear factor modelling approach adopted above is essentially equivalent to principal component analysis, which provides a way to decompose correlated risks into independent risks. In the portfolio literature it is common to use principal components analysis to decompose sets of correlated asset returns into independent underlying risk factors such as business-cycle risk, industry-specific risk, and firm-specific risk (Ross, 1976). Similarly, correlated domestic and foreign risks can be decomposed into a set of independent underlying risk factors, which can be interpreted as pure country-specific risks.

The firm aims to reduce uncertainty about the underlying shocks, ϵ_i, through learning. Signals will thus be about the independent shocks contained in the vector ϵ. I assume that learning about independent shocks is done independently. In other words, the firm acquires independent noisy signals about each of the independent shocks contained in the vector ϵ, and thus receives a $N \times 1$ vector of independent signals

$$s = \epsilon + \epsilon_s, \quad \epsilon_s \sim \mathcal{N}(0, \Sigma_s)$$

where the variance-covariance matrix of the $N \times 1$ vector of signal noise Σ_s is diagonal.[25]

Applying Bayes's rule on the transformed variable $\Gamma^{-1} f$, and then pre-multiplying this solution by Γ, I obtain that posterior beliefs about the correlated risk factors have mean $E[f|s] = \mu + \Gamma(I - \hat{\Sigma}\Sigma^{-1})s$ and variance $V[f|s] = \Gamma\hat{\Sigma}\Gamma'$, where $\hat{\Sigma} \equiv (\Sigma^{-1} + \Sigma_s^{-1})^{-1}$ denotes the posterior variance-covariance matrix of the independent shocks ϵ.[26] Consequently, the posterior mean and variance

[25] Note that an alternative to assuming that shocks affecting the risk factors are independent and learning is about these independent shocks, is to assume that the shocks are correlated, with non-diagonal variance-covariance matrix Ω. In this case, eigen-decomposition can be used to re-write it as $\Omega = \Gamma\Sigma\Gamma'$ and learning would then be about the principal components with diagonal variance-covariance matrix Σ. These two approaches are equivalent.

[26] Transforming the variable $f^* = \Gamma^{-1} f = \Gamma^{-1}\mu + \epsilon$, allows applying standard Bayesian rules for updating normally

of fundamentals, respectively, conditional on the information available at time 2 are

$$E[\theta|s] = A'E[f|s] = A'(\mu + \Gamma(I - \hat{\Sigma}\Sigma^{-1})s) \tag{31}$$

$$V[\theta|s] = A'V[f|s]A = A'\Gamma\hat{\Sigma}\Gamma'A. \tag{32}$$

The solution strategy follows the same steps as in the baseline model. The date-2 problem is unchanged and yields solution $\lambda = E[\theta|s]$. Just as it was the case in the baseline model, date-1 utility decreases with uncertainty regarding fundamentals, $V[\theta|s]$, so the date-1 problem is to minimize

$$V[\theta|s] = A'\Gamma\hat{\Sigma}\Gamma'A$$

subject to the information processing constraint

$$\frac{1}{2}\ln\frac{|\Sigma|}{|\hat{\Sigma}|} \leq K \tag{33}$$

and the restriction that the matrix Σ_s is positive semi-definite i.e. the no-forgetting constraint. Note that since the variance-covariance matrices that enter the determinants in (33) are diagonal, the information processing constraint can be re-written as a sum. Furthermore, define the information-processing capacity devoted to learning about each of the underlying independent shocks as $k_i \equiv \frac{1}{2}\ln\frac{\Sigma_{ii}}{\hat{\Sigma}_{ii}}$. Thus, the information-processing constraint (33) can be re-written as $\sum_i k_i \leq K$.

The matrix of loadings Γ is essentially a measure of the correlation structure between the risk factors. The rows of the matrix contain the loadings of each risk factor on all the shocks and the columns contain the loadings of all the risk factors on each shock. The degree to which risk factor i loads on the independent shocks in the vector ϵ is captured by the i^{th} row of the loadings matrix Γ, denoted Γ_i. On the other hand, the j^{th} column of the matrix Γ, denoted Γ_j, gives the loadings of all the risk factors on the j^{th} shock. Define exposure to shock j as

$$E_j \equiv A'\Gamma_j = \sum_{i=1}^{N} \alpha_i \Gamma_{ij}.$$

This measure of exposure captures the intuition that when the risk factors are correlated, the degree to which an underlying shock, j, affects fundamentals will depend on the interaction between the observable exposure to the risk factors, captured by A, as well as on the loading of these risk

distributed variables and yields posterior mean $E[f^*|s] = \Gamma^{-1}\mu + V[f^*|s]\Sigma_s^{-1}s$ and posterior variance $V[f^*|s] = V[\epsilon|s] = (\Sigma^{-1} + \Sigma_s^{-1})^{-1} \equiv \hat{\Sigma}$.

factors on the underlying common shock, captured by Γ_j. Thus, the effective exposure to a shock depends on the interaction between the observed exposure and the underlying correlation structure.

In keeping up with the baseline model and to ease exposition, in what follows I consider and solve for the case in which there are two correlated risk factors, i.e. $N = 2$. In this case the relevant effective exposure parameters to the two factor-specific shocks, $j = 1, 2$, are $E_j = \alpha_1\Gamma_{1j} + \alpha_2\Gamma_{2j}$. The date-1 problem can be expressed as

$$\min_{k_1,k_2} \; V[\theta|s] = A'\Gamma\hat{\Sigma}\Gamma' A = \hat{\Sigma}_{11}E_1^2 + \hat{\Sigma}_{22}E_2^2$$

$$\text{s.t.} \quad \hat{\Sigma}_{ii} = \Sigma_{ii}e^{-2k_i}, \quad \sum k_i \leq K, \quad 0 \leq k_i, \quad i = 1, 2.$$

The optimal information processing capacity allocated to the factor-specific shocks is

$$k_1 = \begin{cases} 0 & \text{if } \sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}} < e^{-K} \\ \frac{1}{2}\left(K + \ln\sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}}\right) & \text{if } e^{-K} \leq \sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}} \leq e^{K} \\ K & \text{if } \sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}} > e^{K} \end{cases} \tag{34}$$

and $k_2 = K - k_1$. Note that the optimal capacity allocation for the case in which the risk factors are correlated is similar in spirit to that obtained in the baseline model in which the risk factors are independent, except that now learning is about the underlying independent shocks that affect simultaneously more than one risk factor. However, the implications in terms of magnitude of losses are different relative to baseline model.

Figure 7 plots the loss due to suboptimal investment against exposure to factor 1, for varying degrees of correlation between the two risk factors. The main result on the non-monotonic relationship between exposure and the loss due to suboptimal investment remains unchanged. However, the figure shows that relative to the zero correlation baseline, the loss due to suboptimal investment increases with the degree of correlation between the two factors (Appendix A.4 provides an algebraic derivation). Notably, the effect is stronger for shocks that are specific to risk factors that fundamentals have a relatively low exposure to.

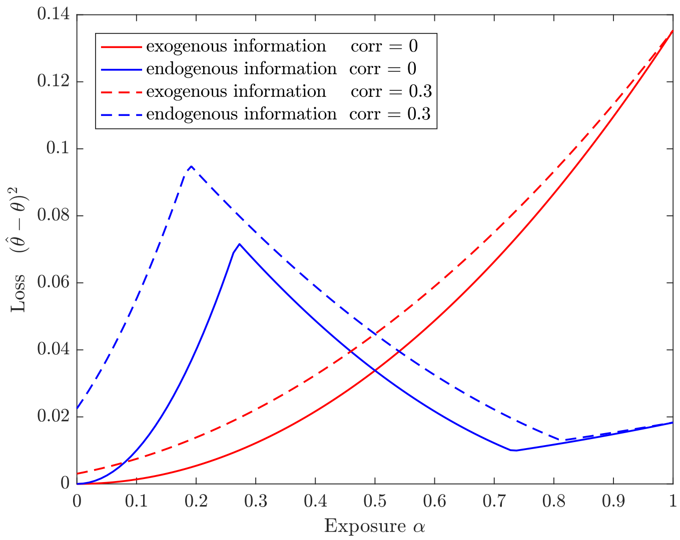


Figure 7. This example considers a one standard deviation factor 1 specific shock i.e. $\epsilon_1 = \Sigma_{11}$, abstracts from factor 2 shocks i.e. $\epsilon_2 = 0$ and information shocks i.e. $\epsilon_{s_1} = \epsilon_{s_2} = 0$. The parameter values are $K = 1$, $\tilde{\sigma}_{s_1} = \tilde{\sigma}_{s_2} = 0.75$, $\Sigma_{11} = \Sigma_{22} = 1$. Both risk factors are assumed to load equally on the underlying independent shocks: $\Gamma_{11} = \Gamma_{22} = 1$ and $\Gamma_{12} = \Gamma_{21} = 0.15$. The implied correlation between the two risk factors is 0.3.

When the risk factors are positively correlated, shocks to risk factors that fundamentals have a seemingly low exposure to are further amplified relative to independent risks baseline. This is because an increasing degree of correlation between the two risk factors has two implications. On the one hand, correlation introduces learning complementarity benefits, as the firm can use information about one factor-specific shock to reduce uncertainty about both risk factors. On the other hand correlation also increases the effective exposure to shocks because now a shock specific to factor 1 will affects fundamentals not only through exposure to factor 1, but also through exposure to factor 2. The effect of complementary in learning is to shift the loss turning points along the x-axis. In the specific case illustrated in Figure 7, the loss function is shifted to the left relative to the zero-correlations baseline because the firm starts learning about the shock specific to factor 1 at a lower level of observable exposure i.e. the effective exposure which drives the learning choices in (34) relies on a lower level of observable exposure when the risk factors are correlated. The effect of increased effective exposure to shocks is to shift the loss function upwards along the y-axis. The loss is higher relative to the zero-correlations baseline because effective exposure is higher than the observable exposure that is plotted on the x-axis. This result highlights that the apparently unexplained transmission of shocks, whereby the transmission of shocks is disproportionately high relative to the observable measure of exposure, is further amplified when the risk factors are positively correlated.

4.4 Extension: Endogenous Exposure

The baseline model considered the case in which exposures to risk factors are exogenous. In this section, I allow for exposure to be endogenous quantities determined in equilibrium. The basic result is that it is optimal for the firm to specialize in learning about one risk factor and to be relatively more exposed to that factor.

The solution strategy follows the same steps as in the baseline model in Section 3, except that at the first date, in addition to choosing the amount of information processing resources devoted to each risk factor, the firm also chooses the exposure to these risk factors. In particular, given the optimal investment and the optimal factor-specific information processing capacity for any given exposure, the firm then chooses the optimal level of exposure by solving

$$\max_{\alpha} U_1 = \begin{cases} -\alpha^2\sigma_1^2 - (1-\alpha)^2\sigma_2^2 e^{-2K} & \text{if } \ k_1 = 0 \\ -2\alpha(1-\alpha)\sigma_1\sigma_2 e^{-K} & \text{if } \ k_1 = \frac{1}{2}\left(K + \ln\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \\ -\alpha^2\sigma_1^2 e^{-2K} - (1-\alpha)^2\sigma_2^2 & \text{if } \ k_1 = K. \end{cases} \tag{35}$$

The solution method and full characterization of the equilibria is detailed in Appendix A.5. The main result is that it is optimal to be relatively more exposed to and learn about one risk factor. In the case of symmetric equilibria whereby the two factors are ex-ante equally volatile, the firm can either learn about factor 1 only, i.e. $k_1 = K$ and $k_2 = 0$, and be relatively more exposed to factor 1, or it can learn about factor 2 only, i.e. $k_1 = 0$ and $k_2 = K$, and be relatively more exposed to factor 2. The optimal level of factor 1 exposure corresponding to these two cases is

$$\alpha^* = \begin{cases} \frac{\sigma_2^2}{\sigma_2^2+\sigma_1^2 e^{2K}} & \text{if } \quad k_1 = 0 \\ \frac{\sigma_2^2}{\sigma_2^2+\sigma_1^2 e^{-2K}} & \text{if } \quad k_1 = K. \end{cases} \tag{36}$$

Thus, the firm prefers to be relatively more exposed to one risk factor, the factor that it learns about. Indifference between exposure allocations arises if the risk factors are ex-ante equally volatile. Otherwise, it is optimal to learn about and to be relatively more exposed to the risk factor that is ex-ante less volatile. In other words, it is ex-ante optimal for the firm to specialize in learning about one risk factor and to be relatively more exposed to it. These ex-ante optimal exposure allocations will expose the firm to a higher loss due to suboptimal investment in the event that the risk factor that it is relatively less exposed to, and about which no information is acquired, is hit by a shock.

The analytical expression (36) reveals that the optimal level of exposure to factor 1 decreases with factor 1 uncertainty and increases with factor 2 uncertainty. Furthermore, the optimal factor 1 exposure increases with total capacity K if the firm chooses to learn about factor 1, and it decreases with capacity K if the firm chooses to learn about factor 2.[27] Note that for any limited capacity and ex-ante uncertain risk factor, full exposure to a risk factor is never optimal and the optimal exposure is an interior solution i.e. for any $K < \infty$ and $\sigma_i^2 > 0,\ i = 1, 2 \Rightarrow 0 < \alpha^* < 1$.

5 Concluding Remarks

The financial crisis of 2007-2008 highlighted the existence of a remarkable and poorly understood type of contagion whereby countries that were relatively less exposed to the crisis epicentre, the United States, were among the most severely affected. In other words, this crisis showed that the impact of a shock can decrease with exposure to it. In this paper, I study how endogenous information choices affect decision-makers' reactions to shocks, and as a consequence the impact of these shocks. By linking information choices and learning behavior to exposure, the model I propose explains the puzzling observation that the impact of a shocks can decrease with exposure to it. The key mechanism in my model is that learning increases with exposure, such that the cost of being highly exposed to a shock is mitigated by the benefit of having a better understanding of it. My model contributes to understanding observed cross-sectional and time-series patterns of contagion by explaining how entities that are more exposed to a crisis can be less affected, and why contagion is more likely to occur following unexpected crises.

[27] This is because less capacity constrained agents are able to learn more and the levels of exposure for which specialized learning occurs are more extreme.

References

Acharya, Viral V, and Tanju Yorulmazer, 2008, Information contagion and bank herding, *Journal of Money, Credit and Banking* 40, 215–231.

Allen, Franklin, Ana Babus, and Elena Carletti, 2012, Asset commonality, debt maturity and systemic risk, *Journal of Financial Economics* 104, 519–534.

Allen, Franklin, and Douglas Gale, 2000, Financial contagion, *Journal of Political Economy* 108, 1–33.

Allen, Franklin, and Douglas Gale, 2004, Financial fragility, liquidity, and asset prices, *Journal of the European Economic Association* 2, 1015–1048.

Alvarez, Fernando E, Francesco Lippi, and Luigi Paciello, 2015, Monetary shocks in models with inattentive producers, *The Review of Economic Studies* 83, 421–459.

Angeletos, George-Marios, and Alessandro Pavan, 2004, Transparency of information and coordination in economies with investment complementarities, *American Economic Review* 94, 91–98.

Bekaert, Geert, Michael Ehrmann, Marcel Fratzscher, and Arnaud Mehl, 2014, The global crisis and equity market contagion, *The Journal of Finance* 69, 2597–2649.

Benoit, Sylvain, Jean-Edouard Colliard, Christophe Hurlin, and Christophe Pérignon, 2017, Where the risks lie: A survey on systemic risk, *Review of Finance* 21, 109–152.

Calvo, Guillermo A, 2004, Contagion in emerging markets: when wall street is a carrier, in *Latin American Economic Crises*, 81–91 (Springer).

Calvo, Guillermo A, and Enrique G Mendoza, 2000, Rational contagion and the globalization of securities markets, *Journal of International Economics* 51, 79–113.

Dasgupta, Amil, 2004, Financial contagion through capital connections: A model of the origin and spread of bank panics, *Journal of the European Economic Association* 2, 1049–1084.

Didier, Tatiana, Paolo Mauro, and Sergio L Schmukler, 2008, Vanishing financial contagion?, *Journal of Policy Modeling* 30, 775–791.

Eichengreen, Barry, Andrew K Rose, and Charles Wyplosz, 1996, Contagious currency crises, Technical report, National Bureau of Economic Research.

Forbes, Kristin, 2012, The big C: identifying and mitigating contagion, in *Proceedings-Economic Policy Symposium-Jackson Hole*, 23–87, Federal Reserve Bank of Kansas City.

Forbes, Kristin J, 2004, The asian flu and russian virus: the international transmission of crises in firm-level data, *Journal of International Economics* 63, 59–92.

Forbes, Kristin J, and Roberto Rigobon, 2002, No contagion, only interdependence: measuring stock market comovements, *The Journal of Finance* 57, 2223–2261.

Freixas, Xavier, Bruno M Parigi, and Jean-Charles Rochet, 2000, Systemic risk, interbank relations, and liquidity provision by the central bank, *Journal of Money, Credit and Banking* 611–638.

Glick, Reuven, and Andrew K Rose, 1999, Contagion and trade: Why are currency crises regional?, *Journal of International Money and Finance* 18, 603–617.

Ilut, Cosmin, and Rosen Valchev, 2017, Economic agents as imperfect problem solvers, *Working Paper* .

Iyer, Rajkamal, and Jose-Luis Peydro, 2011, Interbank contagion at work: Evidence from a natural experiment, *The Review of Financial Studies* 24, 1337–1377.

Jotikasthira, Chotibhak, Christian Lundblad, and Tarun Ramadorai, 2012, Asset fire sales and purchases and the international transmission of funding shocks, *The Journal of Finance* 67, 2015–2050.

Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp, 2016, A rational theory of mutual funds' attention allocation, *Econometrica* 84, 571–626.

Kaminsky, Graciela L, Carmen M Reinhart, and Carlos A Vegh, 2003, The unholy trinity of financial contagion, *Journal of Economic Perspectives* 17, 51–74.

Karolyi, G Andrew, 2003, Does international financial contagion really exist?, *International Finance* 6, 179–199.

Kasperson, Roger E, Ortwin Renn, Paul Slovic, Halina S Brown, Jacque Emel, Robert Goble, Jeanne X Kasperson, and Samuel Ratick, 1988, The social amplification of risk: A conceptual framework, *Risk Analysis* 8, 177–187.

King, Mervyn A, and Sushil Wadhwani, 1990, Transmission of volatility between stock markets, *The Review of Financial Studies* 3, 5–33.

Kodres, Laura E, and Matthew Pritsker, 2002, A rational expectations model of financial contagion, *The Journal of Finance* 57, 769–799.

Krishnamurthy, Arvind, 2010, Amplification mechanisms in liquidity crises, *American Economic Journal: Macroeconomics* 2, 1–30.

Luo, Yulei, 2008, Consumption dynamics under information processing constraints, *Review of Economic Dynamics* 11, 366–385.

Mackowiak, Bartosz, and Mirko Wiederholt, 2009, Optimal sticky prices under rational inattention, *American Economic Review* 99, 769–803.

Maćkowiak, Bartosz, and Mirko Wiederholt, 2018, Lack of preparation for rare events, *Journal of Monetary Economics* 100, 35–47.

Manconi, Alberto, Massimo Massa, and Ayako Yasuda, 2012, The role of institutional investors in propagating the crisis of 2007–2008, *Journal of Financial Economics* 104, 491–518.

Matějka, Filip, 2015, Rationally inattentive seller: Sales and discrete pricing, *The Review of Economic Studies* 83, 1125–1155.

Matějka, Filip, and Alisdair McKay, 2012, Simple market equilibria with rationally inattentive consumers, *American Economic Review* 102, 24–29.

Mondria, Jordi, 2010, Portfolio choice, attention allocation, and price comovement, *Journal of Economic Theory* 145, 1837.

Mondria, Jordi, and Climent Quintana-Domeque, 2013, Financial contagion and attention allocation, *The Economic Journal* 123, 429–454.

Moser, S.C., M. A. Davidson, P. Kirshen, P. Mulvaney, J. F. Murley, J. E. Neumann, L. Petes, and D. Reed, 2014, Coastal zone development and ecosystems., in J. M. Melillo, Terese (T.C.) Richmond, and G. W. Yohe, eds., *Climate Change Impacts in the United States: The Third National Climate Assessment.*, chapter 25, 579–618 (U.S. Global Change Research Program).

Nimark, Kristoffer P, and Savitar Sundaresan, 2019, Inattention and belief polarization, *Journal of Economic Theory* 180, 203–228.

Ongena, Steven, Günseli Tümer-Alkan, and Natalja Von Westernhagen, 2018, Do exposures to sagging real estate, subprime, or conduits abroad lead to contraction and flight to quality in bank lending at home?, *Review of Finance* 22, 1335–1373.

Paciello, Luigi, and Mirko Wiederholt, 2013, Exogenous information, endogenous information, and optimal monetary policy, *Review of Economic Studies* 81, 356–388.

Pavlova, Anna, and Roberto Rigobon, 2008, The role of portfolio constraints in the international propagation of shocks, *The Review of Economic Studies* 75, 1215–1256.

Pericoli, Marcello, and Massimo Sbracia, 2003, A primer on financial contagion, *Journal of Economic Surveys* 17, 571–608.

Renn, Ortwin, William J Burns, Jeanne X Kasperson, Roger E Kasperson, and Paul Slovic, 1992, The social amplification of risk: Theoretical foundations and empirical applications, *Journal of Social Issues* 48, 137–160.

Rigobon, Roberto, and Shang-Jin Wei, 2003, News, contagion, and anticipation understanding the differences between the Asian and the Latin American crises, *LAEBAWorking Paper Series* .

Rose, Andrew K, and Mark M Spiegel, 2010, Cross-country causes and consequences of the 2008 crisis: International Linkages and American Exposure, *Pacific Economic Review* 15, 340–363.

Rose, Andrew K, and Mark M Spiegel, 2011, Cross-country causes and consequences of the crisis: An update, *European Economic Review* 55, 309–324.

Ross, Stephen, 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

Schnabl, Philipp, 2012, The international transmission of bank liquidity shocks: Evidence from an emerging market, *The Journal of Finance* 67, 897–932.

Sims, Christopher A, 2003, Implications of rational inattention, *Journal of Monetary Economics* 50, 665–690.

Slovic, Paul, 1987, Perception of risk, *Science* 236, 280–285.

Stevens, Luminita, 2020, Coarse pricing policies, *The Review of Economic Studies* 87, 420–453.

Sundaresan, Savitar, 2018, Emergency preparedness: Rare events and the persistence of uncertainty, *Working Paper* .

Tutino, Antonella, 2013, Rationally inattentive consumption choices, *Review of Economic Dynamics* 16, 421–439.

Van Nieuwerburgh, Stijn, and Laura Veldkamp, 2010, Information acquisition and under-diversification, *The Review of Economic Studies* 77, 779–805.

Woodford, Michael, 2001, Imperfect common knowledge and the effects of monetary policy, Technical report, National Bureau of Economic Research.

Woodford, Michael, 2009, Information-constrained state-dependent pricing, *Journal of Monetary Economics* 56, S100–S124.

Yuan, Kathy, 2005, Asymmetric price movements and borrowing constraints: A rational expectations equilibrium model of crises, contagion, and confusion, *The Journal of Finance* 60, 379–411.

A Appendix

A.1 Appendix: Uncertainty

This section of the appendix provides an analytical treatment of the factor-specific posterior uncertainty, or the learning-adjusted risk of a factor, and its contribution to overall uncertainty. To contrast the predictions of the two models, it is useful to recall that under the exogenous information benchmark model uncertainty is given by

$$\tilde{\sigma}_B^2 = \alpha^2 \tilde{\sigma}_1^2 + (1-\alpha)^2 \tilde{\sigma}_2^2, \tag{37}$$

while under the endogenous information model uncertainty, $\hat{\sigma}^2 = \alpha^2 \hat{\sigma}_1^2 + (1-\alpha)^2 \hat{\sigma}_2^2$, is given by

$$\hat{\sigma}^2 = \begin{cases} \alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 e^{-2K} & \text{if } \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} < e^{-K} \\ 2\alpha(1-\alpha)\sigma_1\sigma_2 e^{-K} & \text{if } e^{-K} \leq \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} \leq e^{K} \\ \alpha^2 \sigma_1^2 e^{-2K} + (1-\alpha)^2 \sigma_2^2 & \text{if } \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} > e^{K} \end{cases}. \tag{38}$$

Meaningful comparisons require that the total capacity to process information in the endogeous learning model, K, is set equal to the capacity implied by exogenous signals in the exogenous learning benchmark model, $\frac{1}{2}\ln\frac{\sigma_1^2}{\tilde{\sigma}_1^2} + \frac{1}{2}\ln\frac{\sigma_2^2}{\tilde{\sigma}_2^2}$. This implies that $\frac{1}{2}\ln\frac{\sigma_1^2\sigma_2^2}{\tilde{\sigma}_1^2\tilde{\sigma}_2^2} = K$ and so result $e^K = \frac{\sigma_1\sigma_2}{\tilde{\sigma}_1\tilde{\sigma}_2}$ can be plugged into expression (38) to obtain

$$\hat{\sigma}^2 = \begin{cases} \alpha^2 \sigma_1^2 + (1-\alpha)^2 \tilde{\sigma}_2^2 \frac{\tilde{\sigma}_1^2}{\sigma_1^2} & \text{if } \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} < \frac{\tilde{\sigma}_1\tilde{\sigma}_2}{\sigma_1\sigma_2} \\ \alpha^2 \tilde{\sigma}_1^2 \frac{1-\alpha}{\alpha}\frac{\tilde{\sigma}_2}{\tilde{\sigma}_1} + (1-\alpha)^2 \tilde{\sigma}_2^2 \frac{\alpha}{1-\alpha}\frac{\tilde{\sigma}_1}{\tilde{\sigma}_2} & \text{if } \frac{\tilde{\sigma}_1\tilde{\sigma}_2}{\sigma_1\sigma_2} \leq \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} \leq \frac{\sigma_1\sigma_2}{\tilde{\sigma}_1\tilde{\sigma}_2} \\ \alpha^2 \tilde{\sigma}_1^2 \frac{\tilde{\sigma}_2^2}{\sigma_2^2} + (1-\alpha)^2 \sigma_2^2 & \text{if } \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} > \frac{\sigma_1\sigma_2}{\tilde{\sigma}_1\tilde{\sigma}_2} \end{cases} \tag{39}$$

These analytical expressions confirm the intuition conveyed in Figure 2 that relative to the equal capacity, exogenous information benchmark, uncertainty is lower when information processing capacity can be optimally allocated across risk factor exposures. It can be shown that the corner solutions are always smaller than the benchmark solutions. In case of the first corner solution, the inequality $\alpha^2\sigma_1^2 + (1-\alpha)^2\tilde{\sigma}_2^2\frac{\tilde{\sigma}_1^2}{\sigma_1^2} < \alpha^2\tilde{\sigma}_1^2 + (1-\alpha)^2\tilde{\sigma}_2^2$ is equivalent to $\tilde{\sigma}_2^2 > \frac{\alpha^2}{(1-\alpha)^2}\sigma_1^2$. This last inequality holds true in light of the condition for obtaining the corner solution, which can be written as $\frac{\tilde{\sigma}_1}{\sigma_1}\tilde{\sigma}_2 > \frac{\alpha}{1-\alpha}\sigma_1$. Since $\tilde{\sigma}_2 \geq \frac{\tilde{\sigma}_1}{\sigma_1}\tilde{\sigma}_2$, it is verified that $\tilde{\sigma}_2^2 > \frac{\alpha^2}{(1-\alpha)^2}\sigma_1^2$ and overall uncertain is lower under the endogenous information model than under the benchmark. Similarly, for the second corner solution we have that $\alpha^2\tilde{\sigma}_1^2\frac{\tilde{\sigma}_2^2}{\sigma_2^2} + \alpha_2^2\sigma_2^2 < \alpha_1^2\tilde{\sigma}_1^2 + \alpha_2^2\tilde{\sigma}_2^2$, which is equivalent to $\frac{\alpha}{1-\alpha}\frac{1}{\sigma_2} > \frac{1}{\tilde{\sigma}_1}$. Given that the corner solution condition can be re-written as $\frac{\alpha}{1-\alpha}\frac{1}{\sigma_2} \geq \frac{\sigma_2}{\tilde{\sigma}_1\tilde{\sigma}_2}$ and $\frac{1}{\tilde{\sigma}_1}\frac{\sigma_2}{\tilde{\sigma}_2} \geq \frac{1}{\tilde{\sigma}_1}$, if follows that the inequality $\frac{\alpha}{1-\alpha}\frac{1}{\sigma_2} > \frac{1}{\tilde{\sigma}_1}$ holds and uncertainty is lower under the endogenous information model than under the benchmark. For the interior solution, the inequality $2\alpha_1\alpha_2\tilde{\sigma}_1\tilde{\sigma}_2 \leq \alpha^2\tilde{\sigma}_1^2 + (1-\alpha)^2\tilde{\sigma}_2^2$ holds true because $(\alpha_1\tilde{\sigma}_1 - \alpha_2\tilde{\sigma}_2)^2 \geq 0$.

Importantly, these analytical expressions shed further light into the mechanism behind the observed result. When exposure to factor 1 is sufficiently low such that the first corner solution in which there is no learning about factor 1 is obtained, the posterior uncertainty or learning-adjusted risk of factor 1 is higher under the endogenous information model that under the benchmark, $\sigma_1^2 \geq \tilde{\sigma}_1^2$, while the factor 2 posterior uncertainty is lower relative to the benchmark, $\tilde{\sigma}_1^2\frac{\tilde{\sigma}_1^2}{\sigma_1^2} \leq \tilde{\sigma}_1^2$. A similar reasoning applies when exposure to factor 2 is sufficiently low such that the second corner solution in which there is no learning about factor 2 is obtained: relative to the exogenous information benchmark, factor 1 posterior uncertainty is lower, $\tilde{\sigma}_1^2\frac{\tilde{\sigma}_2^2}{\sigma_2^2} < \tilde{\sigma}_1^2$, while factor 2 posterior uncertainty is higher, $\sigma_2^2 \geq \tilde{\sigma}_2^2$, when information choice is endogenously chosen. At the interior optimum, factor 1 posterior uncertainty is higher under the endogenous information model than under the benchmark if exposure to that factor 1 is relatively low, i.e. all else equal $\frac{1-\alpha}{\alpha}\frac{\tilde{\sigma}_2^2}{\tilde{\sigma}_1^2} > 1$ if $\alpha < 1-\alpha$, and it is lower if exposure to factor 1 is relatively high, i.e. all else equal $\frac{1-\alpha}{\alpha}\frac{\tilde{\sigma}_2^2}{\tilde{\sigma}_1^2} < 1$ if $\alpha > 1-\alpha$.

Proof. Proposition (1)

Contrast comparative statics with respect to exposure under the exogenous information model

$$\frac{\partial\tilde{\sigma}_B^2}{\partial\alpha} = 2\alpha\tilde{\sigma}_1^2 - 2(1-\alpha)\tilde{\sigma}_2^2 > 0 \;\text{ if }\; \alpha > 0.5 \tag{40}$$

and when an interior solution is obtained under the endogenous information model

$$\frac{\partial\hat{\sigma}^2}{\partial\alpha} = 2(1-2\alpha)\sigma_1\sigma_2 e^{-K} < 0 \;\text{ if }\; \alpha > 0.5 \tag{41}$$

□

A.2 Appendix: Shock Transmission Mechanism

Proof. Proposition (2)

Recall that the optimal level of investment is given by the expected level of economic fundamentals. Define $\gamma_i \equiv \frac{\hat{\sigma}_i^2}{\sigma_i^2}$ and re-write the conditional mean value of fundamentals obtained under the endogenous information model as

$$\hat{\theta} = E[\theta|s_1, s_2] \quad = \quad \alpha\left[\mu_1 + (1-\gamma_1)s_1\right] + (1-\alpha)\left[\mu_2 + (1-\gamma_2)s_2\right], \tag{42}$$

where $s_i = \epsilon_i + \epsilon_{s_i},\ i = 1, 2,\ \gamma_2 = \gamma_1^{-1} e^{-2K}$ and

$$\gamma_1 = \begin{cases} 1 & \text{if} \quad \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} < e^{-K} \\ \frac{1-\alpha}{\alpha}\frac{\sigma_2}{\sigma_1}e^{-K} & \text{if} \quad e^{-K} \leq \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} \leq e^{K} \\ e^{-2K} & \text{if} \quad \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} > e^{K} \end{cases}. \tag{43}$$

The first-best optimum obtained under the perfect information model is

$$\theta = \alpha(\mu_1 + \epsilon_1) + (1-\alpha)(\mu_2 + \epsilon_2). \tag{44}$$

Define the loss due to suboptimal investment as

$$L \equiv (\hat{\theta} - \theta)^2 = \Big(\alpha\left[(1-\gamma_1)\epsilon_{s_1} - \gamma_1\epsilon_1\right] + (1-\alpha)\left[(1-\gamma_2)\epsilon_{s_2} - \gamma_2\epsilon_2\right]\Big)^2, \tag{45}$$

such that the interpretation of the parameters $\gamma_i,\ i = 1, 2$, is that of the weight given to the shocks affecting fundamentals or, in other words, the extent to which shocks affecting fundamentals are incorporated into investment decisions. The loss function obtained under the exogenous information benchmark is defined as

$$L_B \equiv (\hat{\theta}_B - \theta)^2 = \left(\alpha\left[(1-\tilde{\gamma}_1)\epsilon_{s_1} - \tilde{\gamma}_1\epsilon_1\right] + (1-\alpha)\left[(1-\tilde{\gamma}_2)\epsilon_{s_2} - \tilde{\gamma}_2\epsilon_2\right]\right)^2, \tag{46}$$

where the weight coefficients are $\tilde{\gamma}_i \equiv \frac{\tilde{\sigma}_i^2}{\sigma_i^2},\ i = 1, 2$.

Define the shock transmission mechanism as the loss induced by a shock

$$\frac{\partial L}{\partial \epsilon_1} \quad \equiv \quad -2\alpha\gamma_1\Big(\alpha\left[(1-\gamma_1)\epsilon_{s_1} - \gamma_1\epsilon_1\right] + (1-\alpha)\left[(1-\gamma_2)\epsilon_{s_2} - \gamma_2\epsilon_2\right]\Big) \tag{47}$$

$$\frac{\partial L_B}{\partial \epsilon_1} \quad \equiv \quad -2\alpha\tilde{\gamma}_1\Big(\alpha\left[(1-\tilde{\gamma}_1)\epsilon_{s_1} - \tilde{\gamma}_1\epsilon_1\right] + (1-\alpha)\left[(1-\tilde{\gamma}_2)\epsilon_{s_2} - \tilde{\gamma}_2\epsilon_2\right]\Big). \tag{48}$$

The interest lies in examining how the transmission mechanism varies with exposure, captured by the parameter α. Recall that under the benchmark model, the factor-specific posterior uncertainty parameters, $\tilde{\sigma}_1^2$ and $\tilde{\sigma}_2^2$, that enter the shock weights coefficients $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ are exogenous. On the other hand, under the endogenous information model the factor-specific posterior uncertainty parameters that influence the weight coefficients are endogenous functions of exposure, $\hat{\sigma}_1^2 = \frac{1-\alpha}{\alpha}\sigma_1\sigma_2 e^{-K}$ and $\hat{\sigma}_2^2 = \frac{\alpha}{1-\alpha}\sigma_1\sigma_2 e^{-K}$. Abstracting from factor 2 effects by setting $\epsilon_2 = \epsilon_{s_2} = 0$, and using the result that $\frac{\partial\gamma_1}{\partial\alpha} = -\frac{1}{\alpha(1-\alpha)}\gamma_1$, we have that

$$\frac{\partial^2 L}{\partial\epsilon_1\partial\alpha} = 2\alpha\gamma_1\left[\left(\frac{2\alpha-1}{1-\alpha} - \frac{2\alpha}{1-\alpha}\gamma_1\right)\epsilon_{s_1} - \frac{2\alpha}{1-\alpha}\gamma_1\epsilon_1\right] \tag{49}$$

$$\frac{\partial^2 L_B}{\partial\epsilon_1\partial\alpha} = -4\alpha\tilde{\gamma}_1\left[(1-\tilde{\gamma}_1)\epsilon_{s_1} - \tilde{\gamma}_1\epsilon_1\right]. \tag{50}$$

In the limiting case in which the signal is perfectly informative, i.e. the signal noise is zero $\epsilon_{s_1} = 0$, it is clear that the shock transmission mechanism decreases with exposure under the endogenous information model, i.e. $\frac{\partial^2 L}{\partial\epsilon_1\partial\alpha} < 0$, but it increases with exposure under the exogenous information benchmark, i.e. $\frac{\partial^2 L_B}{\partial\epsilon_1\partial\alpha} > 0$. These results hold more generally if the signal is sufficiently informative i.e. the signal noise is sufficiently small $\epsilon_{s_1} < \frac{\gamma_1}{1-\gamma_1}\epsilon_1$.

□

A.3 Appendix: Shock Anticipation

Proof. Proposition (3)

The date-1 utility function (16) is a continuous piecewise function that is increasing in state capacity. It is useful to distinguish between three types of equilibria: (i) the equilibrium capacity allocation across factors has the property $k_1 = 0$, (ii) the equilibrium capacity allocation across factors has the property $k_1 = K$, and (iii) the equilibrium capacity allocation across factors has the property $k_1 = \frac{1}{2}\left(K + \ln\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right)$. In all three cases, $k_2 = K - k_1$. Substituting the constraint (18) into the objective function (17) and solving for K_r yields

$$K_r = \begin{cases} \frac{1}{4}\left(2\mathcal{K} + \ln\frac{p_r}{p_n}\right) & \text{if } k_1 = 0 \text{ or } k_1 = K \\ \frac{1}{2}\left(\mathcal{K} + \ln\frac{p_r}{p_n}\right) & \text{if } k_1 = \frac{1}{2}\left(K + \ln\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \end{cases}. \tag{51}$$

Imposing the no-forgetting constraint $0 \leq K_r$ and noting that $K_r \leq \mathcal{K}$ is always satisfied because $\ln\frac{p_r}{p_n} < 0$ when $p_r < p_n$, the optimal capacity allocation across states when the firm learns only about one risk factor (when a corner solution is obtained for capacity allocation across factors) is given by

$$K_r = \begin{cases} 0 & \text{if } \frac{p_r}{p_n} < e^{-2\mathcal{K}} \\ \frac{1}{4}\left(2\mathcal{K} + \ln\frac{p_r}{p_n}\right) & \text{if } \frac{p_r}{p_n} \geq e^{-2\mathcal{K}} \end{cases} \tag{52}$$

and $K_n = \mathcal{K} - K_r$. On the other hand, the optimal capacity allocation across states when the firm learns about both risk factors (when an interior solution is obtained for capacity allocation across factors) is given by

$$K_r = \begin{cases} 0 & \text{if } \frac{p_r}{p_n} < e^{-\mathcal{K}} \\ \frac{1}{2}\left(\mathcal{K} + \ln\frac{p_r}{p_n}\right) & \text{if } \frac{p_r}{p_n} \geq e^{-\mathcal{K}} \end{cases} \tag{53}$$

and $K_n = \mathcal{K} - K_r$. Thus, if the firm only learns about one as opposed to both risk factors, it is more likely that it will dedicate information processing resources to the rare state, since $e^{-2\mathcal{K}} < e^{-\mathcal{K}}$, but the overall capacity allocated to the state is smaller, since $\frac{1}{4}\left(2\mathcal{K} + \ln\frac{p_r}{p_n}\right) < \frac{1}{2}\left(\mathcal{K} + \ln\frac{p_r}{p_n}\right)$.

Proving Proposition 3 follows from noting that the loss induced by a shock under the exogenous information model in (47) increases with the weight coefficient $\gamma_{is} \equiv \frac{\hat{\sigma}^2_{is}}{\sigma^2_{is}},\ i = 1, 2,\ s \in \{r, n\}$. Since the factor-specific uncertainty $\hat{\sigma}^2_{is}$ in state of nature s decreases with the information processing capacity available in that state, K_s, which in turn increases with the probability of occurrence of the state, $p_s,\ s \in \{r, n\}$ it follows that the loss decreases with the degree of anticipation of the state.

□

Note that if the volatility of the shock affecting the risk factor is different in the two states, such that the shock occurring in the rare state has associated prior $\epsilon_{1r} \sim \mathcal{N}(0, \sigma^2_{1r})$, while the prior associated with the shock occurring in the normal state is $\epsilon_{1n} \sim \mathcal{N}(0, \sigma^2_{1n})$, then the capacity allocation across states is

$$K_r = \begin{cases} 0 & \text{if } \frac{p_r\sigma_{1r}}{p_n\sigma_{1n}} < e^{-\mathcal{K}} \\ \frac{1}{2}\left(\mathcal{K} + \ln\frac{p_r\sigma_{1r}}{p_n\sigma_{1n}}\right) & \text{if } e^{-\mathcal{K}} \leq \frac{p_r\sigma_{1r}}{p_n\sigma_{1n}} \leq e^{\mathcal{K}} \\ \mathcal{K} & \text{if } \frac{p_r\sigma_{1r}}{p_n\sigma_{1n}} > e^{\mathcal{K}} \end{cases} .$$

and $K_n = \mathcal{K} - K_r$. Accounting for the intuition that crises episodes are characterized by high volatility amount to assuming that $\sigma^2_{1r} > \sigma^2_{1n}$. The information processing capacity allocated to the rare state of nature increases with the ex-ante volatility of the shock associated with the rare state, thus reducing the loss due to suboptimal action and the impact of the shock. Magnitudes are reduced relative to the symmetric case but dynamics remain unchanged.

A.4 Appendix: Correlated Risks

The loss due to suboptimal investment is

$$\begin{aligned} L \equiv (E[\theta|s]-\theta)^2 &= (A'E[f|s]-A'f)^2 = [A'(\mu+\Gamma(I-\hat{\Sigma}\Sigma^{-1})s - A'(\mu+\Gamma\epsilon)]^2 \\ &= [A'(\mu+\Gamma(I-\hat{\Sigma}\Sigma^{-1})(\epsilon+\epsilon_s) - A'(\mu+\Gamma\epsilon)]^2 \\ &= [A'\Gamma(I-\hat{\Sigma}\Sigma^{-1})\epsilon_s + A'\Gamma(I-\hat{\Sigma}\Sigma^{-1})\epsilon - A'\Gamma\epsilon]^2 \\ &= [A'\Gamma(I-\hat{\Sigma}\Sigma^{-1})\epsilon_s - A'\Gamma\hat{\Sigma}\Sigma^{-1}\epsilon]^2 \end{aligned}$$

where the elements of the posterior variance-covariance matrix $\hat{\Sigma}$ are given by the shock-specific posterior uncertainty implied by the capacity allocation (34)

$$\hat{\Sigma}_{11} = \begin{cases} \Sigma_{11} & \text{if} \quad \sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}} < e^{-K} \\ \sqrt{\frac{\Sigma_{11}\Sigma_{22}E_2^2}{E_1^2}}e^{-K} & \text{if} \quad e^{-K} \leq \sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}} \leq e^K \\ \Sigma_{11}e^{-2K} & \text{if} \quad \sqrt{\frac{\Sigma_{11}E_1^2}{\Sigma_{22}E_2^2}} > e^K \end{cases} \tag{54}$$

and $\hat{\Sigma}_{22} = \Sigma_{11}\Sigma_{22}e^{-2K}\hat{\Sigma}_{11}^{-1}$.

Abstracting from information shocks i.e. $\epsilon_s = 0$ we have that

$$L = (A'\Gamma\hat{\Sigma}\Sigma^{-1}\epsilon)^2.$$

A.5 Appendix: Endogenous Exposure

The objective function (35) is a continuous piecewise function, which is concave in exposure when corner solutions are obtained for capacity allocation, and convex in exposure when an interior solution is obtained. Hence, interior solutions is obtained for optimal exposure if corner solutions are obtained for information choice, and corner solutions are obtained for optimal exposure if an interior solution is obtained for information choice. The first-order conditions related to the problem in (35) are

$$\frac{\partial U_1}{\partial \alpha} = \begin{cases} -2\alpha\sigma_1^2 + 2(1-\alpha)\sigma_2^2 e^{-2K} & \text{if} \quad k_1 = 0 \\ -2(1-2\alpha)\sigma_1\sigma_2 e^{-K} & \text{if} \quad k_1 = \frac{1}{2}\left(K + \ln\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \\ -2\alpha\sigma_1^2 e^{-2K} + 2(1-\alpha)\sigma_2^2 & \text{if} \quad k_1 = K \end{cases} \tag{55}$$

It is useful to distinguish between three types of equilibria: (i) the equilibrium capacity allocation has the property $k_1 = 0$, (ii) the equilibrium capacity allocation has the property $k_1 = K$, and (iii) the equilibrium capacity allocation has the property $k_1 = \frac{1}{2}\left(K + \ln \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right)$. In all three cases, $k_2 = K - k_1$.

Equilibria (i) and (ii) represents situations in which the firm is only able to learn about one risk factor. In these situations, it is optimal to be relatively more exposed to the factor the firm learns about, and the optimal point of exposure is the one at which the learning-adjusted risk exposures are equal. More specifically, if $k_1 = 0$ optimal exposure is implied by $\alpha\sigma_1^2 = (1-\alpha)\sigma_2^2 e^{-2K}$. If $k_1 = K$ optimal exposure is implied by $\alpha\sigma_1^2 e^{-2K} = (1-\alpha)\sigma_2^2$.

$$\alpha^* = \begin{cases} \frac{\sigma_2^2}{\sigma_2^2 + \sigma_1^2 e^{2K}} & \text{if} \quad k_1 = 0 \\ \frac{\sigma_2^2}{\sigma_2^2 + \sigma_1^2 e^{-2K}} & \text{if} \quad k_1 = K \end{cases} \tag{56}$$

Indifference between the exposure allocations in (56) arises if the risk factors are ex-ante equally volatile, but it is otherwise optimal to be relatively more exposed to the risk factor that is ex-ante less volatile.

Equilibrium (iii) represents a situation in which the parameter values are such that the firm can learn about both risk factors. In this case, it is optimal to be as exposed as possible to one factor, where the maximum level of exposure to a factor is implied by the condition for learning about both risks $e^{-K} \leq \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} \leq e^K$. To determine which risk factor it is optimal to relatively more exposed to, I compare the expected utility of each corner solution for exposure. The maximum exposure to factor 1 is obtained when $\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} = e^K$, which implies $\overline{\alpha} = \frac{\sigma_2}{\sigma_2 + \sigma_1 e^{-K}}$ and the utility of being relatively more exposed to factor 1 is $U_1(\overline{\alpha}) = -2\left(\frac{\sigma_1\sigma_2}{\sigma_1 + \sigma_2 e^K}\right)^2$. The maximum exposure to factor 2 is obtained when $\frac{\alpha\sigma_1}{(1-\alpha)\sigma_2} = e^{-K}$, which implies minimum factor 1 exposure $\underline{\alpha} = \frac{\sigma_2}{\sigma_2 + \sigma_1 e^K}$ and the utility of being relatively more exposed to factor 2 is $U_1(\underline{\alpha}) = -2\left(\frac{\sigma_1\sigma_2}{\sigma_1 e^K + \sigma_2}\right)^2$. In the case of symmetric equilibria whereby the two factors are ex-ante equally volatile, the firm will be indifferent between the two exposure allocations i.e. $U_1(\overline{\alpha}) = U_1(\underline{\alpha})$ if $\sigma_1 = \sigma_2$. However, in the case of non-symmetric equilibria, it is optimal to be more exposed to the less volatile risk factor i.e. $U_1(\overline{\alpha}) > U_1(\underline{\alpha})$ if $\sigma_1 < \sigma_2$ hence $\alpha^* = \overline{\alpha}$, and $U_1(\overline{\alpha}) < U_1(\underline{\alpha})$ if $\sigma_1 > \sigma_2$ hence $\alpha^* = \underline{\alpha}$.

$$\alpha^* = \begin{cases} \frac{\sigma_2}{\sigma_2 + \sigma_1 e^K} & \text{if} \quad k_1 = \frac{1}{2}\left(K + \ln \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \text{ and } \sigma_1 > \sigma_2 \\ \frac{\sigma_2}{\sigma_2 + \sigma_1 e^{-K}} & \text{if} \quad k_1 = \frac{1}{2}\left(K + \ln \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \text{ and } \sigma_1 < \sigma_2 \\ \frac{\sigma_2}{\sigma_2 + \sigma_1 e^K} \text{ or } \frac{\sigma_2}{\sigma_2 + \sigma_1 e^{-K}} & \text{if} \quad k_1 = \frac{1}{2}\left(K + \ln \frac{\alpha\sigma_1}{(1-\alpha)\sigma_2}\right) \text{ and } \sigma_1 = \sigma_2 \end{cases} \tag{57}$$

However, since the firm is not constrained to learn about both risk factors, i.e. to be in equilibria of the type (iii), it will optimally choose to learn about one factor only and to be relatively more exposed to the factor it learns about. This follows from the fact that the expected utility associated with the optimal levels of exposure (57) obtained in equilibrium (iii) is lower than the utility associated with the optimal levels of exposure (56) that are obtained in equilibria (i) and (ii).