PRICE DESTABILIZING SPECULATION: THE ROLE OF STRATEGIC LIMIT ORDERS*

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We show that under quantity competition with only a few strategic sellers, a large speculator with access to storage facilities can destabilize prices and profit from it. Through clever use of a combination of limit, and market orders, the speculator can lower the price while buying and raise the price while selling. This creates price volatility even though there is no fundamental uncertainty in the economy and all market participants act rationally. When there is free disposal, the speculator uses a combination of limit and stop-loss order, and the resulting market price is more volatile.

Keywords: Cournot competition, Limit orders, Price manipulation, Price volatility, Strategic storage, Stop-loss order.

JEL Codes: G32, G34, G38, K20.

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*We would like to thank Ailsa Roell, and the participants at the Stevens Institute of Technology brown bag seminar and FIU seminar for valuable comments. The usual disclaimer applies.

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I. Introduction

"People who argue that speculation is generally destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on the average sell when the currency is low in price and buy when it is high."

— Milton Friedman, Essays in Positive Economics (1953), p.175

A large literature has followed Milton Friedman's argument that speculators engaging in destabilizing speculation will incur losses.[1] Hart and Kreps (1986) (henceforth, HK) provide an example where speculation is destabilizing but profitable on average. In their example, the speculator buys and stores a commodity in anticipation of a severe shortage, which occurs infrequently. Most of the time the speculator is forced to dump inventories, which depresses prices – speculation is destabilizing and results in losses to the speculator, consistent with Friedman's argument. On the few occasions when the severe shortage is realized, the speculator's supply does not affect the high prices, and the speculator earns huge profits. In HK, on average, speculation is profitable even though it is destabilizing, and the speculator behaves competitively taking prices as given. In this article, we contribute to this literature, by showing that speculative activities can have a large impact on spot/forward prices of a commodity even when all market participants act rationally as in HK. However, in contrast to HK, in our model, the speculator is not a price taker and can be viewed as "manipulating prices," consistent with popular complaints.

We consider a three-periods model economy with a single commodity called "widgets," a large speculator, and two other groups of market participants: a large number of small price-taking "nonstrategic" buyers (henceforth, consumers); and two strategic

[1] See for example, Hart and Kreps (1986), Chari, Jagannathan, and Jones (1990), De Long, Shleifer, Summers, and Waldmann (1990), Stein (1987), Turnovsky (1983), Kemp (1963), Hart (1977), and Johnson (1976).

sellers.[2] Only the speculator has access to storage facilities and storage is costly. The strategic sellers take the consumers' excess demand for widgets in any period as given and choose their supply to maximize profits. The forward market for widgets operates in period 1 and period 2. All deliveries happen in the next period – i.e., delivery happens in period 2 for the forward contracts traded in period 1, and period 3 for the forward contracts traded in period 2. There are no spot markets. All market participants have to plan one period ahead for supplying or taking possession of the single consumption good. There is no information asymmetry or uncertainty in our model. All market participants have the same beliefs about the structure of the economy and the behavior of market participants.[3]

First we consider the case without the speculator – we call this the "benchmark" case. In our benchmark case, periods 1 and 2 look the same. Since there are no fundamental demand or supply uncertainties in our economy, forward prices will be the same in periods 1 and 2.

Next, we consider the case with a speculator. We show that the speculator can lower the market-clearing forward price of widgets in period 1 when he buys to build an inventory of widgets in period 2, and raise the market-clearing forward price of widgets in period 2, when he sells his inventory forward for delivery in period 3. The speculator does this by changing the aggregate demand curves that the duopolistic sellers take as given through clever use of limit and stop-loss orders. The speculator's ability to alter the shape of the demand curve allows the speculator to purchase widgets and at the same time lower the equilibrium price. This introduces price volatility that the speculator profits from. We show that the two strategic sellers are worse off. Overall, the consumers are better off when the two periods are taken together, but will complain

[2]In the appendix, we extend our model with more than two strategic sellers and we find results which are qualitatively similar to the duopoly case.

[3]See, e.g., Hart and Kreps (1986) and Chari et al. (1990) paper for examples of destabilizing speculation in the presence of private information.

about high price due to speculation in period 2.

The rest of the paper is organized as follows. We introduce related literature in section II. We then describe the model economy with duopolistic strategic sellers in section III. In section IV and V, we introduce a speculator who behaves like a Stackelberg leader and show that speculative storage without and with free disposal has a destabilizing effect. We conclude in section VI.

II. Related Literature

The academic literature examining whether speculation stabilizes or destabilizes prices has a long history – for example, it is discussed in Adam Smith (Wealth of Nations 1789). Friedman (1953) essentially argued that speculators profit by buying (selling) when prices are higher (lower) than corresponding fundamental values. Therefore, according to Friedman, profitable speculation would necessarily move prices towards fundamental values thereby stabilizing prices.

Kawai (1983) used a mean-variance rational expectations framework to show that futures markets where speculators participate can make prices more volatile in the presence of production and storage uncertainties. Newbery (1987) makes the intuition behind this clearer. He points out that hedging using the futures market by producers reduce price risk, resulting in larger inventories and a tilt towards riskier production/storage technologies. The effect on price volatility across periods will depend on the magnitude of the shocks to production and storage. When there are no shocks to production or storage, futures trading will stabilize prices, as in Turnovsky (1983). Stein (1987) shows that when market participants infer what others know from prices, introducing speculators can make that inference process difficult and can destabilize prices. Hart and Kreps (1986) show that speculation can be profitable and destabilizing when speculators have access to storage and have superior information relative to other market participants. Chari et al. (1990) construct examples showing that even when there are no production uncertainties, and the commodity cannot be stored, futures markets where speculators participate can increase the variance of spot prices. In their examples, even though price volatility increases, welfare can also increase at the same time.

In the literature above, all market participants behave rationally and competitively. Hart (1977) considers the case where some market participants are sophisticated while others are not. In such a market, he shows that the sophisticated speculator can profit by exploiting the forecasting rule of the naive, destabilizing prices. Attari, Banerjee, and Noe (2006) and Cooper and Donaldson (1998) examine price manipulation using pump-and-dump and corner-and-squeeze strategies respectively. Kyle and Viswanathan (2008) discuss the difference between illegal price manipulation and rational speculation based on information production. Newbery (1984) shows that a producer with market power may engage in profitable destabilizing speculation even though all market participants behave rationally. De Long et al. (1990) construct an economy where there are positive feedback traders who buy when prices go up and sell when prices go down. In such an economy, rational speculators can destabilize prices since their trades trigger positive feedback trades by other investors, and part of rational speculators' trades will be due to anticipating such positive feedback behavior. The findings in Slade (1991) that exchange prices of commodities were less stable than producer prices, while consistent with destabilizing speculation, does not establish causality.

We contribute to this literature by showing that even when there are no demand or supply shocks and no role for information production, and all agents behave rationally, a speculator with exclusive access to storage can profit from destabilizing manipulation of prices.

III. Model Preliminaries

We consider a three-period economy where, one period forward contracts for the delivery of a hypothetical commodity, which we call "widgets" for convenience, are traded on period 1 and period 2. We assume that commodity prices are denoted in "dollars", and the interest rate is zero. Consumers and strategic sellers trade in the one-period forward market, for supply and consumption in the next period. A buyer of one-period ahead forward contract in period t, $t = \{1, 2\}$, is required to exchange p_t dollars per widget delivered in period $t + 1$, $t = \{1, 2\}$. There are three types of participants in the commodity market:

(a) A large number of consumers, each with a different reservation price for widgets. A given consumer will buy one unit of the widget in the forward market if the price is equal or below her reservation price . This gives rise to the consumers' aggregate demand curve.

(b) Two identical strategic sellers (duopoly) who participate in the forward market in period 1 and period 2, and deliver widgets to the buyers. Each seller maximizes her aggregate profits over the two periods.

(c) A large speculator with access to storage. The speculator lacks the ability to produce widgets. He buys widgets in the forward market of period 1 for taking delivery in period 2; intending to sell it in the forward market in period 2 for delivery in period 3, to earn profits. The speculator stores acquired widgets from period 2 to period 3.

We further assume that the demand and supply curves for the widgets are known with no uncertainty. We assume that the speculator's buy-sell decisions are common knowledge. More importantly, we assume a perfect contracting environment – an environment where there is no possibility of natural and/or strategic default at the time of delivery.[4] We

[4]In the assumed environment forward and spot prices are same.

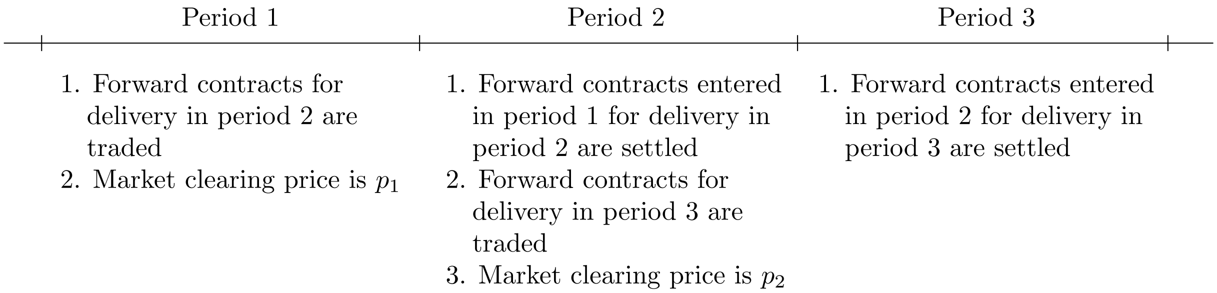

FIGURE I
Timeline of The Three-period Commodity Market Model

focus only on the symmetric equilibrium.[5] The temporal evolution of events is depicted in Figure I.

III.A. Benchmark Equilibrium: No speculator case

First, we consider the benchmark case where there are no speculator. We characterize the Cournot equilibrium following Varian (1992). We denote the two strategic sellers as A and B. In each period $t = \{1, 2\}$, each strategic seller decides how much to produce in the next period and sell in the forward market. We denote the amount produced in period 2, which is sold in the forward market in period 1 by the two strategic sellers, A and B, as q_{A1}, q_{B1} respectively. Similarly, we denote the amount produced in period 3, which is sold forward in period 2 as q_{A2}, q_{B2}. In each period $t = \{1, 2\}$, strategic seller A chooses the supply q_{At} so as to maximize the payoff in period $t + 1$, taking q_{Bt}, and the demand schedule as given. Symmetrically, strategic seller B optimally chooses the quantity q_{Bt}, $t = \{1, 2\}$. We assume the aggregate demand of the price-taking consumers is as given below,

$$\boldsymbol{p}_t(Q_t) = a - b\,Q_t \qquad t = \{1, 2\}, \tag{III.1}$$

[5]Even though we acknowledge the existence of a few asymmetric equilibria. We believe the symmetric equilibrium is full of insights and the asymmetry will not add to many different insights.

where Q_t is the aggregate demand from the price-taking consumers. In equilibrium, the aggregate demand will be equal to the aggregate supply for each period of $t = \{1, 2\}$. We use boldface letters to denote functions and normal letters to denote specific values taken by the variables. For expositional convenience, we assume that the marginal cost of production of all strategic sellers is zero.

Since the benchmark equilibrium is symmetric, we remove the subscript i for notational convenience, and denote the equilibrium supply of each strategic seller as q^* where,

$$q^* = \frac{a}{3\,b}, \tag{III.2}$$

and the market clearing price as p^* where,

$$p^* = \frac{a}{3}. \tag{III.3}$$

Each strategic seller's payoff is given by:

$$\pi^* = \frac{a^2}{9\,b}. \tag{III.4}$$

The equilibrium supply is determined by solving the reaction function of each strategic suppliers simultaneously. The equilibrium prices and supplies are the same in both periods.

Numerical example: Benchmark equilibrium

Let the demand and sensitivity parameters be $a = 90$ and $b = 1$ respectively. The equilibrium supply, $\frac{a}{3\,b} = 30$, which is given by equation (III.2). The equilibrium price and payoff of strategic sellers in each period are 30 and 900 which is obtained using (III.3) and (III.4) respectively.

In the next subsection, we introduce the speculator whose objective is to maximize his profit by buying q_{S1} in period 1 and selling $q_{S2} \leq q_{S1}$ in period 2 accounting for his storage cost, c_s per unit. We formally analyze the strategic behavior of the speculator in two disposal cost scenarios: first, when the disposal cost is infinite (henceforth, without disposal), and second, when the disposal costs is zero (henceforth, free disposal). In the equilibrium without disposal, the speculator behaves strategically only in period 1 and acts as a price taker in period 2 when he dumps his inventory. In the equilibrium with free disposal, the speculator takes the price impact associated with the quantity sold in determining how much to sell in the second period.

IV. Equilibrium with speculator and no disposal

Since the speculator's strategy to buy widgets in period 1 depends on his ability to profitably sell the acquired inventory of widgets in period 2, we first analyze the equilibrium in period 2 for all possible acquired inventory sizes. In this equilibrium the speculator has to sell the entire acquired inventory in period 2. We relax this assumption and give the speculator an option to dispose part of his inventory in subsection V.

Period 3

In period 3, there are no decisions to be made by the agents. The quantity of forward contracts bought and sold in period 2 are settled. The sellers (or agents with short positions) delivers the widgets at the forward prices established in period 2, and the buyers (or agents with long positions) pay and take delivery of the widgets.

Period 2

The widgets bought and sold in the forward market in period 1 are settled and the forward market for delivery in period 3 opens. We assume that the speculator has bought forward q_S units of widgets in period 1 and had it delivered in period 2. In period 2, the speculator sells the entire q_S units of widgets he has taken delivery of, in the forward market.[6] Without loss of generality, we assume that the speculator sells using a market order in period 2.[7] Therefore, the market-clearing price in period 2 of the equilibrium without disposal is stated as,

$$\boldsymbol{p}_2(Q_2) = a - b\,(Q_2 + q_S)\,, \tag{IV.1}$$

where $Q_2 = q_{A2} + q_{B2}$ is the aggregate supply of the strategic sellers in period 2, q_S is the supply of the speculator. Each strategic seller decides how much she should supply, taking the aggregate demand schedule, the supply of the speculator, and the supply of the other strategic seller as given. In equilibrium, the aggregate demand of the consumers is equal to the aggregate supply of the strategic sellers plus the supply of the speculator, $Q_2 + q_S$.

As the speculator's supply, q_S, is fixed, the aggregate demand schedule in period 2 with the speculator equals the aggregate demand schedule in the benchmark case reduced by q_S units.The strategic sellers maximize their period 2 payoff as follows.

$$\begin{aligned} &\max_{q_A} \boldsymbol{p}(q_A + q_B + q_S)\, q_A, \\ &\max_{q_B} \boldsymbol{p}(q_A + q_B + q_S)\, q_B, \end{aligned} \tag{IV.2}$$

where $\boldsymbol{p}(q_A + q_B + q_S) = a - b(q_A + q_B + q_S)$. The best response functions of strategic sellers are:

$$\boldsymbol{q}_A(q_B) = \frac{a}{2\,b} - \frac{q_B}{2} - \frac{q_S}{2}, \quad \text{and} \quad \boldsymbol{q}_B(q_A) = \frac{a}{2\,b} - \frac{q_A}{2} - \frac{q_S}{2}. \tag{IV.3}$$

The equilibrium supply of the strategic sellers as a function of the speculator's supply

[6] Although we assume zero transaction loss but it is straightforward to incorporate inventory loss in our setup.

[7] Later, we show that the speculator profit-wise cannot do any better by using limit orders to sell his inventory in period 2 so long he is forced to liquidate the entire inventory in period 2.

q_S is given below.

$$\text{(IV.4)} \qquad \boldsymbol{q}_2(q_S) = \frac{a - b\,q_S}{3\,b} = q^* - \frac{q_S}{3},$$

where q_S is the speculator's supply in period 2. Compared to the benchmark supply, each strategic seller reduces supply by $\frac{q_S}{3}$ units so that the supply reduction taken together is $\frac{2}{3}\,q_S$ which is less than the supply of the speculator, q_S. Therefore, in equilibrium the aggregate supply is higher than in the benchmark supply, q^*, and the market clearing price is lower than the benchmark price, p^*.

$$\text{(IV.5)} \qquad \boldsymbol{p}_2(q_S) = \frac{a - b\,q_S}{3} = p^* - \frac{b\,q_S}{3}.$$

For non-negative price, $\boldsymbol{p}_2(q_S)$ and positive $\boldsymbol{q}_2(q_S)$ as stated in equation (IV.4) and (IV.5) we need an upper bound of the speculator's inventory $q_S < \frac{a}{b}$. This is a practical condition. It implies although the speculator is quite large relatively to individual consumer, he is still small with respect to the aggregate market size. Then each strategic seller's payoff in period 2 is

$$\text{(IV.6)} \qquad \boldsymbol{\pi}_2(q_S) = \frac{(a - b\,q_S)^2}{9\,b} = \pi^* - \frac{q_S}{9}\,(2\,a - b\,q_S).$$

Compared to the benchmark equilibrium, each strategic seller supplies less as stated in Equation (IV.4), and the price is strictly lower as shown by Equation (IV.5), and hence resulting per firm profit is lower in period 2 as shown by Equation (IV.6).

To see that the speculator cannot do better than supplying using market order when disposal cost is infinite, we consider the strategy where the speculator uses a limit order to supply his entire inventory but he chooses a limit price that is ϵ higher than the equilibrium period 2 price, $p^* - \frac{b\,q_S}{3}$. For such a price to become the equilibrium price, at least one strategic seller has to be better off by reducing her supply by $\frac{\epsilon}{b}$ units. This gives the deviating strategic seller a payoff equal to $(q_2 - \frac{\epsilon}{b})(p_2 + \epsilon) = \frac{1}{b}\left(p_2^2 - \epsilon^2\right)$ which is lower than $\frac{1}{b}p_2^2$ which is the payoff if she does not reduce her supply. Note that the speculator has to supply q_S units in period 2 so that he will comply with the market-clearing price in (IV.5). This argument also applies to the case where the speculator supplies using a combination of market order and limit order since it still requires at least one strategic seller to reduce supply to achieve the speculator's limit price and if she does not have the incentive to reduce supply the equilibrium clearing price will remain unchanged.

Numerical example: The speculator participates without disposal

Let the demand and price sensitivity parameters be $a = 90$ and $b = 1$ respectively, and the speculator supplies 15 units of widgets using a market order in period 2, i.e., $q_S = 15$. In Figure II, the two solid lines represent the best response functions of the two strategic sellers in period 2 in the presence of the speculator but without disposal while the dashed lines are the best response functions of the two strategic sellers in period 2 in benchmark equilibrium. The two solid lines cross each other at the equilibrium supply, $\frac{a - b\,q_S}{3\,b} = 25$, which is given by equation (IV.4). Hence, the equilibrium aggregate supply, the market clearing price and the payoff of each strategic sellers are 65, 25, and 625 respectively.

Period 1

In period 1, the speculator uses a limit order to acquire his inventory. Specifically, we assume that the speculator

$$\boldsymbol{q}_S(p_1) = \begin{cases} 0 & \text{for } p_1 > p_S \\ [0,\, q_S] & \text{for } p_1 = p_S \\ q_S & \text{for } p_1 < p_S \end{cases} \tag{IV.7}$$

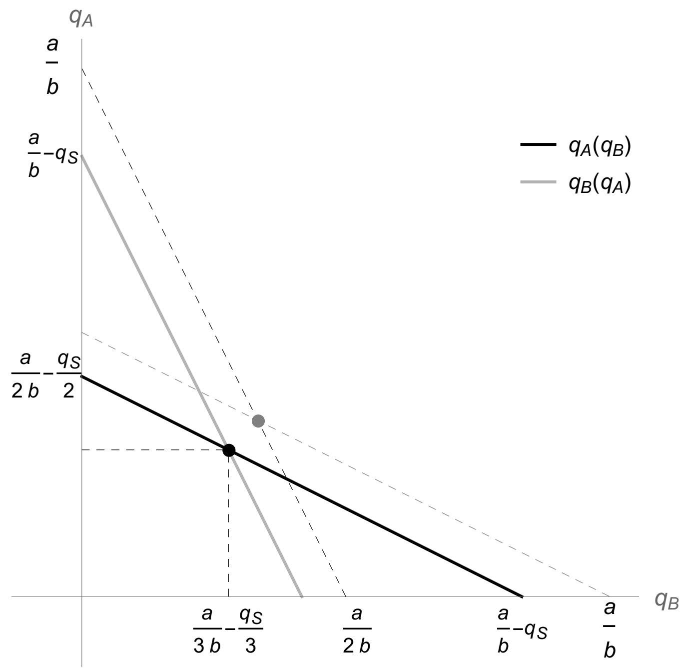

FIGURE II
Best Response Functions of the Strategic Sellers in Period 2

This figure depicts the best response functions of the two strategic sellers in period 2 of equilibrium without the option to dispose. We assume that $a = 90$, $b = 1$, and $q_S = 15$. The black and gray solid lines are the best response functions of strategic seller A and strategic seller B respectively. The black and gray dashed lines represent the best response functions of the strategic seller A and strategic seller B respectively, and the gray dot $(30, 30)$ denotes the aggregate equilibrium supply in period 2 in the benchmark case (without the speculator). The black dot $(25, 25)$ denotes the equilibrium supply in period 2 when the speculator cannot dispose of his inventory.

where p_1 is the market clearing price in period 1, and q_S is the quantity that the speculator buys when the clearing price is below the limit price p_S. When the clearing price is equal to p_S, the speculator accepts any partial execution. Each strategic seller maximizes the sum of her payoffs in the two periods, taking as given the supply of the other strategic sellers, and the limit order of the speculator, and the aggregate demand from the consumers.

The market price in period 1 of the equilibrium without disposal can be represented as a function of aggregate supply of both strategic sellers conditional on both the limit price p_S and limit quantity q_S, in the speculator's limit order, as follows:

$$\text{(IV.8)} \qquad \boldsymbol{p}_1(Q_1; q_S, p_S) = \begin{cases} a - b\,Q_1 & \text{for } Q_1 < \frac{a-p_S}{b} \\ p_S & \text{for } Q_1 \in \left[\frac{a-p_S}{b}, \frac{a-p_S}{b} + q_S\right] \\ a - b(Q_1 - q_S) & \text{for } Q_1 > \frac{a-p_S}{b} + q_S, \end{cases}$$

where $Q_1 = q_{A1} + q_{B1}$ is the aggregate supply of the strategic sellers in period 1. In equilibrium, the aggregate demand of the consumers in period 1 is equal to the aggregate supply of the strategic sellers minus the demand of the speculator. Figure III depicts the clearing price in period 1 of the equilibrium without disposal. If the strategic sellers find it is optimal to produce enough to meet the speculator's demand in full even though, this lowers the price, the speculator's limit order will be executed and the equilibrium price will be p_S.

The objective functions of each strategic sellers are to maximize their payoffs in period 1 while taking into account the other strategic sellers' supply decisions and the speculator's demand in period 1 and the speculator's potential supply in period 2. Again, note that when the disposal cost is infinite, the speculator has to be exactly equal to the acquired inventory – neither he can sell more (no other supply source) nor he can

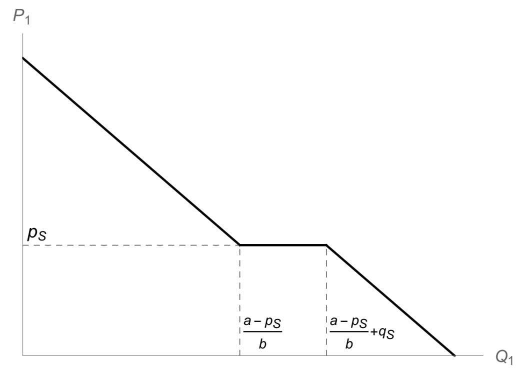

FIGURE III
Aggregate Demand Function and the Speculator's Limit Order in Period 1

This figure depicts the aggregate demand curve in period 1 when the speculator is a buyer. There are two kink points. In each of these kink points, the price is the limit buy-price, p_S chosen by the speculator. The horizontal segment of the demand curve is the limit buy-quantity, q_S chosen by the speculator.

sell less. Specifically,

$$\text{(IV.9)} \qquad \begin{aligned} &\max_{q_{A1}} \boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_2(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))), \\ &\max_{q_{B1}} \boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_2(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))), \end{aligned}$$

where

$$\text{(IV.10)} \qquad \begin{aligned} \boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_S,p_S) &= \boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S)\,q_{A1}, \\ \boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_S,p_S) &= \boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S)q_{B1}, \end{aligned}$$

and $\boldsymbol{\pi}_2(\cdot)$ is given by equation (IV.6).

The objective of the speculator is to maximize his trading profit, i.e., to maximize the difference between the cost of acquiring the inventory in period 1 and the sell revenue in period 2 net of the storage costs. We need the following two constraints to be satisfied: first, the participation constraint needs to hold; i.e., the trading profit of the speculator must be positive. Second, the incentive compatibility constraints of the strategic sellers need to be satisfied, i.e., it is in the interest of the strategic sellers to meet the speculator's demand. However, we do not need to express the speculator's participation constraint since the speculator will always choose $q_S = 0$ when per unit round trip profit is negative. Formally, the speculator's objective can be written as follows:

$$\text{(IV.11)} \qquad \max_{q_S, p_S} \quad \boldsymbol{q}_S\Big(\boldsymbol{p}_2\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))\big) - \boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S) - c_s\Big)$$

$$\text{(IV.12)} \qquad \begin{aligned} \text{s.t.} \quad & \boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_S,p_S) + \boldsymbol{\pi}_{A2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))\big) \\ & \qquad \geq \boldsymbol{\pi}_{A1}(q'_{A1}+q_{B1};q_S,p_S) + \boldsymbol{\pi}_{A2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q'_{A1}+q_{B1};q_S,p_S))\big) \\ & \boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_S,p_S) + \boldsymbol{\pi}_{B2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))\big) \\ & \qquad \geq \boldsymbol{\pi}_{B1}(q_{A1}+q'_{B1};q_S,p_S) + \boldsymbol{\pi}_{B2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q'_{B1};q_S,p_S))\big) \end{aligned}$$

where q'_{A1} and q'_{B2} denote any alternative supply strategies of the seller A and seller B in period 1 respectively (see, e.g., Kyle and Wang (1997)). To derive the equilibrium we first note that the speculator can choose the limit order price and quantity in such a way that the incentive compatibility constraints of the strategic sellers given by (IV.12) hold as equalities.[8] We solve the model following the standard technique: we propose an equilibrium and show that none of the active agents has a unilateral incentive to deviate from the proposed equilibrium. Further, we assume that both the strategic seller on the equilibrium path adopt identical strategies.

Given that the constraints (IV.12) holds with strict equality and the fact that strategic sellers A and B adopt identical strategies on the equilibrium path, the speculator's choice reduces to choosing only one of the two choice variables: p_S or q_S but not both. This is because on the equilibrium path if the incentive compatibility constraint one of the strategic seller is satisfied, all the other strategic sellers' will also be satisfied.

The speculator searches across potential price-quantity combinations corresponding to different limit orders and chooses the one that is the best for him. Without loss of

[8]We assume that the strategic sellers will supply enough to meet the speculator's demand if deviating or not deviating yield the same payoff. This assumption is made for expositional convenience.

generality, we use q_S as the choice variable of the speculator and let p_S be determined by the incentive compatibility constraints. We denote the optimal p_S for a given q_S that satisfies the incentive compatibility constraints as a function $\boldsymbol{p}_S(q_S)$. We then have the following lemma for the $\boldsymbol{p}_S(q_S)$.

Lemma 1. *Let p^* denote the benchmark equilibrium price as defined in equation* (III.3)*; q_S denote the limit buy-quantity in the limit order of the speculator; and a and b are the demand and sensitivity parameters respectively. Then, for any given q_S, there exists a limit buy-price,*

$$\boldsymbol{p}_S(q_S) = p^* + \frac{3\,b\,q_S - \sqrt{b\,q_S\,(4\,a + 13\,b\,q_S)}}{6} \tag{IV.13}$$

such that p_S is also the market clearing price and the speculator's demand q_S is fully satisfied.

Proof: See Appendix B.

To understand the intuition behind Equation (IV.13), first note that the strategic sellers' incentive compatibility constraint has two implications on the speculator's limit buy-price. First, one strategic seller needs incentive to supply more until the speculator's demand is met in full when the other strategic seller supplies the benchmark supply. Second, both strategic sellers do not have incentive to reduce or increase supply when the speculator's demand is met in full and the two strategic sellers supply equally. The first implication leads to equation (IV.13), and this condition is always stricter than what the second implication leads to with our assumption $q_S < \frac{a}{b}$. When the speculator chooses any limit quantity $q_S < \frac{a}{b}$ and a limit price $\boldsymbol{p}_S(q_S)$, the market will be cleared at price $\boldsymbol{p}_S(q_S)$ in period 1. Equation (IV.13), however, is not the necessary condition for the speculator's demand to be met in full. If the speculator sets the limit price lower than (IV.13) while the second implication is satisfied, the speculator's limit price can still be the clearing price even though it is not unique since the benchmark price can also be the clearing price.

Although we allow for partial execution of the speculator's order in the model setup, it is also important to note that in equilibrium the speculator's demand q_S will always be fully supplied. This is because the gain of strategic sellers by supplying a greater portion of the speculator's demand in period 1 outweigh the corresponding loss caused by the speculator's supply in period 2.

Numerical example: Aggregate payoff without disposal

Let the demand and price sensitivity parameter be $a = 90$ and $b = 1$ respectively (same as the previous examples). We suppose that the speculator submits a limit order – the price-quantity combination using Equation (IV.7). Specifically, he chooses the limit-quantity, $q_S = 15$ and sets the limit-buy price using Equation (IV.7) to $p_S = 22.29$. Given the price-quantity combination of the speculator of the speculator, we show that if one of the strategic seller, say B, sticks to the "benchmark" strategy of supplying 30 units of widgets, then it is optimal for the strategic seller A to adopt "supply more" strategy and generate a payoff weakly better than the benchmark payoff of 1800 dollars. We state the payoffs associated with different strategy combinations in Table I.

Since the two strategic sellers are identical and they make supply simultaneously supply decisions, it is optimal for strategic seller B to adopt the "supply more" strategy if strategic seller A sticks to the "benchmark" strategy. Hence, the unique pair of strategies for the two sellers is {supply more, supply more} and the equilibrium supplies of the strategic sellers is (41.35, 41.35) which gives the two strategic sellers a two-period aggregate payoff of 1547 dollars.

The two panels in Figure IV demonstrate how strategic seller A takes optimal supply decision in period 1 taking into account its impact on her period 2 payoff given strategic seller B supplies the benchmark quantity, 30, in period 1. The solid black line in the

TABLE I
The Strategic Sellers' Supply Strategies

		Strategic Seller B	
		Benchmark	Supply More
Strategic Seller A	Benchmark	$\pi^*(= 1800), \pi^*(= 1800)$	$\pi_A(= 1294), \pi_B(\geq 1800)$
	Supply More	$\pi_A(\geq 1800), \pi_B(= 1294)$	$\pi_A^{(*S)}(= 1547), \pi_B^{(*S)}(= 1547)$

left panel is the period 2 demand curve faced by strategic seller A, when B produces 25, and speculator supplies 15 that he bought in period 1. The solid gray line is the corresponding marginal revenue curve of strategic seller A. In this situation, strategic seller A's optimal response is to supply 25 and the corresponding price is 25 denoted by the black dot. The dashed black and gray lines correspond to the case where the speculator's limit order is not filled in period 1 and thus supplies 0 in period 2 and strategic seller B supplies 30.

Strategic seller A's decision to produce 30 to strategic seller B is producing 30 and the resulting market-clearing price is 30 is denoted by the gray dot in the figure. The solid black line in the right panel is the period 1's demand curve faced by strategic seller A, when strategic seller B sticks to benchmark quantity of 30, and speculator puts a limit order to buy 15 units of widgets at a price of 22.29 per unit. The solid gray line is the corresponding marginal revenue curve of strategic seller A. Since we assumed that the marginal costs (MC) is zero, the marginal revenue is equal to the MC when strategic seller A supplies 30 and the resulting market-clearing price is 30. But the speculator's limit order will not be filled. If the gray dot is the equilibrium in period 1, then the equilibrium in period 2 (as shown in the left panel) has to be the gray dot, where both produce 30 units. Strategic seller A's payoff in period 1 (gray dot) and period 2 (gray dot) together is 1800. If strategic seller A supplies 52.71 in period 1, speculator's order will be filled, and the price is 22.29, which corresponds to the black dot.

The supply increase is greater than the speculator's demand since the atomistic consumers will demand more when the the market price is lower. The equilibrium in period 2 corresponds to the black dot in the left panel where both strategic sellers supplying 25 units each. Strategic seller A's optimal response-payoffs in period 1 (black dot) and period 2 (black dot) again sum to ≥ 1800. We assume that when the payoff is such that the strategic seller A is indifferent between supplying $(30, 30)$ and $(52.71, 25)$, she will supply $(52.71, 25)$ such that speculator's order is filled and consumers are better-off. The Figure V further shows the sum of the two period payoffs of strategic seller A as a function of her first period's response, when strategic seller B supplies 30 in period 1, and the Cournot equilibrium prevails in period 2.

Both Figure IV and Figure V take as given strategic seller B supplies 30 in period 1 and strategic seller A's optimal response is 52.71. Then we plot the best response of strategic seller A for any quantity that the strategic seller B supplies in the left panel of Figure VI. The vertical axis is the best response q_A of strategic seller A, when the supply of strategic seller B is q_B (on the x-axis), and Cournot equilibrium prevails in period 2. For $q_B \in (0, 30)$, the best response function of strategic seller A is similar to what she has in the benchmark scenario, i.e., $\boldsymbol{q}_A(q_B) = \frac{a}{2} - \frac{q_B}{2}$, with a slope of $-\frac{1}{2}$. For $q_B \in [30, 52.71)$, it is optimal for strategic seller A to supply enough to meet the speculator's limit demand, i.e., $\boldsymbol{q}_A(q_B) = a + q_S - p_S - q_B$, with a slope of -1. For $q_B \geq 52.71$, the best response function of strategic seller A becomes $\boldsymbol{q}_A(q_B) = \frac{a}{2} + \frac{q_S}{2} - \frac{q_B}{2}$, with a slope of $-\frac{1}{2}$. The horizontal axis in the middle panel of Figure VI gives the best response q_B of strategic seller B, when the supply of strategic seller A is q_A (on the y-axis), and again Cournot equilibrium prevails in period 2. Then in the right panel, we superimpose the two best response curves in the left and middle panel. The two best response functions overlaps on the black interval and the black dot ($q_A = 41.35, q_B = 41.35$) describes the symmetric equilibrium supplies.

An interesting "prisoner dilemma" underlies the strategic sellers' supply strategy

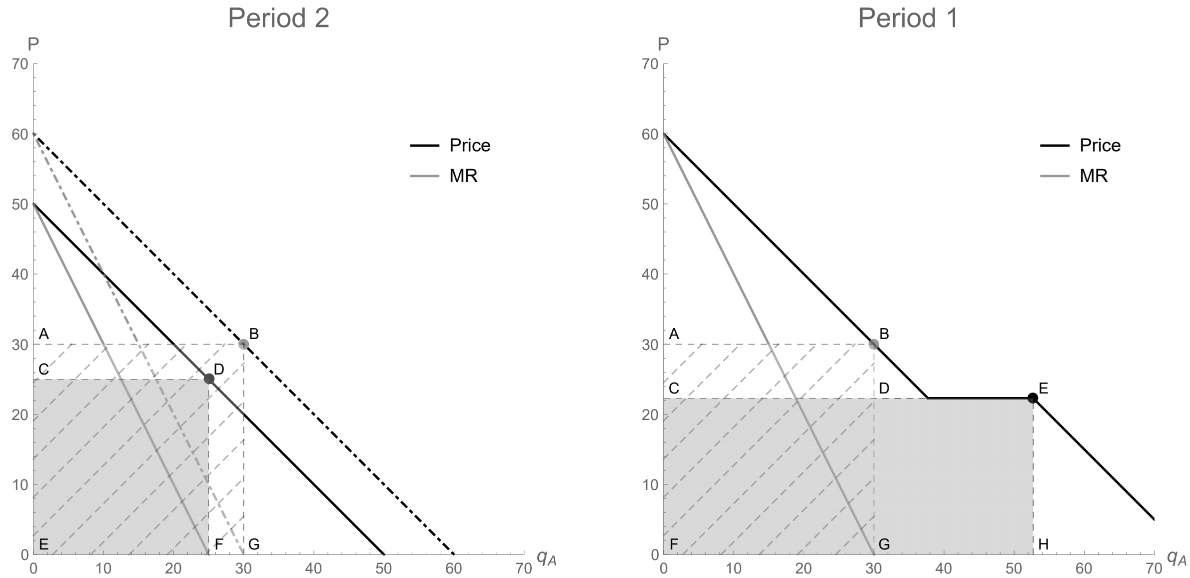

FIGURE IV

Best Response of strategic seller A (or B) in Periods 1 and 2

This figure depicts the best responses of seller A (B) for a given response of seller B (A). *Left panel depicts Period 2*: The solid black line is the demand curve faced by strategic seller A (B), when the strategic seller B (A) produces 25 units, and the speculator dumps 15 units. The solid gray line depicts the marginal revenue (MR) curve of strategic seller A (B). We assume that the marginal cost (MC) is zero. We show that under these assumptions it is optimal for A (B) to produce 25 units. The resulting market-clearing price is 25 (the black dot). The dot-dashed black and gray lines correspond to the case without the speculator, where A and B both produces 30 units and the resulting market price is 30 (the gray dot). The gray color shaded area corresponds to the strategic sellers' payoff when the speculator's limit order is fullfilled in period 1, and the area with tilted dashed lines correspond to the their payoff in the benchmark case.

Right panel depicts Period 1: The gray dot represents the benchmark equilibrium where MR = MC = 0 (note that the gray dot is the equilibrium in period 1 implies that the gray dot has to be the equilibrium in period 2). The solid black line is the demand curve faced by the strategic sellers, when the speculator submits a limit order to buy, 15 units at a price of 22.29 per unit. We show that if strategic seller B (A) sticks to the benchmark equilibrium quantity of 30 units, then it optimal for strategic seller A (B) to supply 52.71 resulting in a market-clearing price of 22.29 (the black dot). The speculator buys 15 units. Although, the black dot in the right panel is not a symmetric equilibrium, but the period 2's symmetric equilibrium corresponds to the black dot in the left panel where both supply 25 units and the speculator sells 15 units. The strategic seller A (B) earns profit weakly greater than the benchmark profit. The gray color shaded area corresponds to the payoff of the strategic seller A (B) if the strategic seller B (A) sticks to the benchmark supply while A (B) increases the supply to meet the speculator's limit buy order in full, and the area with tilted dashed lines correspond to the their payoff in the benchmark case.

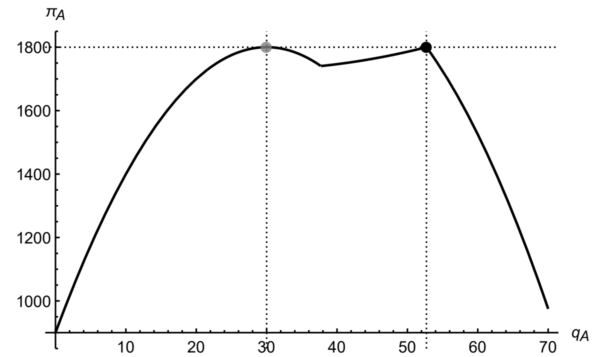

FIGURE V
Strategic Seller A (B)'s Payoffs

The vertical axis gives the sum of the two period payoffs of strategic seller A (B) as a function of her first period response, when strategic seller B (A) supplies 30 in period 1, and the symmetric equilibrium with the speculator supplying his entire inventory of 15 units prevails in period 2.

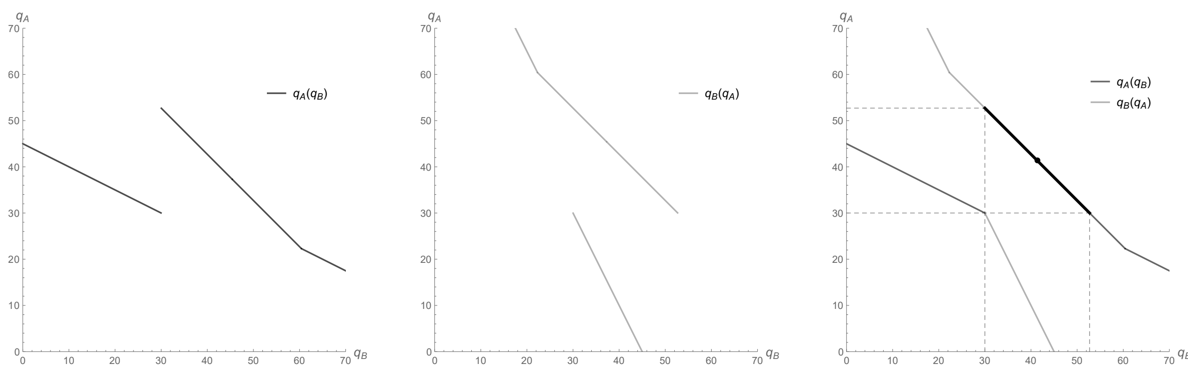

FIGURE VI
Equilibrium in Period 1

Left panel: The vertical axis gives the best response q_A of strategic seller A, when the supply of strategic seller B is q_B (on the x-axis), and the symmetric equilibrium with the speculator supplying his entire inventory of 15 units prevails in period 2. *Middle panel*: The horizontal axis gives the best response q_B of strategic seller B, when the supply of strategic seller A is q_A (on the y-axis), and symmetric equilibrium with the speculator supplying his entire inventory of 15 units prevails in period 2. *Right panel*: The two best response curves in the left and middle panel are superimposed to arrive at the equilibria. The two best response functions overlaps on the black interval. The dark black interval depicts all equilibria, and the black dot ($q_A = 41.35, q_B = 41.35$) depicts the symmetric equilibrium.

which is depicted in Figure VI when the speculator chooses $\boldsymbol{p}_S(q_S)$ as the limit price for a given q_S. If both strategic sellers supply the benchmark quantity $q^* = 30$, then each of them gets 900 dollars each period (1800 for two periods). If strategic seller B supplies the benchmark quantity $q^* = 30$, the strategic seller A can choose either increasing her supply to $a - p_S + q_S - q^* = 52.71$ to meet the speculator's demand, or supplying $q^* = 30$.

Note that if strategic seller A increases her supply to 52.71, the speculator will sell his inventory $q_S = 15$ in period 2 which results in a payoff 625 dollars for both strategic sellers in period 2. However, the supply increase of strategic seller A in period 1 increases her payoff to $22.29 \times 52.71 = 1175$ dollars in period 1. Together with the period 2 payoff, the strategic seller A gets a same two-period payoff when she supplies 52.71 or 30, and hence strategic seller A chooses to supply 52.71 to meet the speculator's demand given our assumption that the strategic sellers will supply enough to meet the speculator's demand if reducing supply or not lead to equal payoff.

As the strategic sellers make supply decision simultaneously and the argument applies to both strategic sellers, the strategic seller B would also increase her supply anticipating that strategic seller A increases supply. In the symmetric-supply equilibrium, both strategic seller A and B supply $\frac{a-p_S+q_S}{2} = 41.35$ which yields a payoff $22.29 \times 41.35 = 922$ in period 1 and 625 in period 2 (1547 for two periods together). As a result, both strategic sellers earn less payoffs by supplying more than the benchmark quantity in period 1, while neither strategic seller has incentive to reduce supply.

According to Lemma 1, if the speculator chooses the limit price as $\boldsymbol{p}_S(q_S)$ for a given q_S, the clearing price in period 1 will be $\boldsymbol{p}_S(q_S)$, i.e., $\boldsymbol{p}_1(q_{A1} + q_{B1}; q_S, \boldsymbol{p}_S(q_S)) = \boldsymbol{p}_S(q_S)$, and the speculator's demand q_S will be fully supplied, i.e., $\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1}; q_S, \boldsymbol{p}_S(q_S))) = q_S$. Then, we can get rid of the incentive compatibility constraints of the strategic sellers in the speculator's optimization problem in equation (IV.11) and represent it as follows:

$$\max_{q_S} \quad q_S\big(\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S) - c_s\big) \tag{IV.14}$$

We solve the simple maximization problem of the speculator and summarize the equilibrium without disposal in Proposition 1.

Proposition 1. *When the storage cost is lower than $\bar{c}_S$, the speculator submits a limit buy order in period 1, then there exists an equilibrium where the market clearing prices are different in period 1 and period 2 and the speculator makes a positive round-trade profit. When the storage cost is greater than $\bar{c}_S$, the speculator does not affect the market outcomes and prices are same in both periods. The threshold of the storage cost is $\bar{c}_S = \frac{5-2\sqrt{3}}{39}\,a \approx 0.04\,a$.*

Proof: See Appendix B.

The prices become volatile compared to the constant price in the benchmark equilibrium. In the period 2 of the equilibrium without disposal, the speculator is a price-taker so that the clearing price is determined by the competition of the two strategic sellers. In period 1, the speculator, however, submits a limit order which gives the two strategic sellers incentive to supply the quantity he demands while lowering the clearing price to a make a profit. In equilibrium, the period 1 price is lower than the period 2 price, and as long as the maximum price spread the speculator can generate is greater than his inventory cost per unit, he is able to make a positive round-trade profit. Note that if the unit net price, $(p_2 - p_1 - c)$ is negative, the speculator will not trade, i.e., $q_S = 0$.

Numerical Example: The speculator's payoff and widget price volatility

Let the demand and sensitivity parameters be $a = 90$ and $b = 1$ respectively, and let the speculator's storage cost be $c_S = 0.48$.[9] The left panel of Figure VII gives

[9]We choose this specific inventory cost parameter because the optimal level of inventory for the

the relationship between the price spread, $p_2 - p_1$, and the limit order quantity of the speculator. The right panel of Figure VII gives the relationship between the speculator's profit, π_S, and the limit order quantity.

Recall that in the benchmark case price of widgets in both periods is equal to 30 implying that the price spread is zero. With the presence and participation of the speculator and everything else remaining the same as the benchmark case, widget prices are volatile and the speculator can profit from it. The speculator can manufacture substantial price volatility in the widget market.

We show that the size of the speculator's capacity plays a critical role in price spread and the speculator's profit. Specifically, the limit order quantity that maximizes the speculator's profit is significantly higher than the one that maximizes the price spread. We depict the relationship between the price spread and the speculator's profit as a function of capacity size in Figure VII. Based on our assumption about the parameters ($a = 90$, $b = 1$, and $c = 0.48$), we find that the speculator's profit is maximized if the speculator storage capacity is greater than or equal to 15 which is about 16.67% of the total market size (a). In that case, the speculator sets 15 as the limit buy-quantity in period 1 at a unit price of 22.29 – his limit order is filled and then he dumps all 15 units for sale in period 2 which results in a market-clearing price of 25.00 in period 2 and a price spread ($p_2 - p_1$) equal to 2.71. The manufactured volatility is about 9.03% of the benchmark price. And the speculator earns 33.38 as round-trip profit.

We find that the price spread is maximized if the speculator's storage capacity is significantly smaller than his profit-maximizing storage capacity. As shown in Figure VII the price spread is maximized if the speculator's storage capacity is around 6 units.[10] In that case, the speculator sets 6 as the limit buy-quantity in period 1 for 24.46 per

speculator is 15 when $c_S = 0.48$, $a = 90$, and $b = 1$ which is consistent with the parameters we provide in the example for period 2.

[10] Although the price spread is maximized at 6.14 units, we assume the nearest integer value of 6 for storage capacity.

unit – his limit order is filled – and then he dumps all 6 units for sale in period 2 which results in the market-clearing price of 28 in period 2 and a dollar price spread $(p_2 - p_1)$ of 3.54. The resulting volatility is about 11.8% of the benchmark price. And the speculator earns 19 as round-trip profit. Note that although the speculator manufactures significant volatility in the widget market – as high as 11.8% relative to the benchmark price, the presence of the speculator without disposal helps to lower the level of market-clearing price in the widget market by approximately 6.67% in period 2 and 18.47% in period 1 relative to the benchmark price.

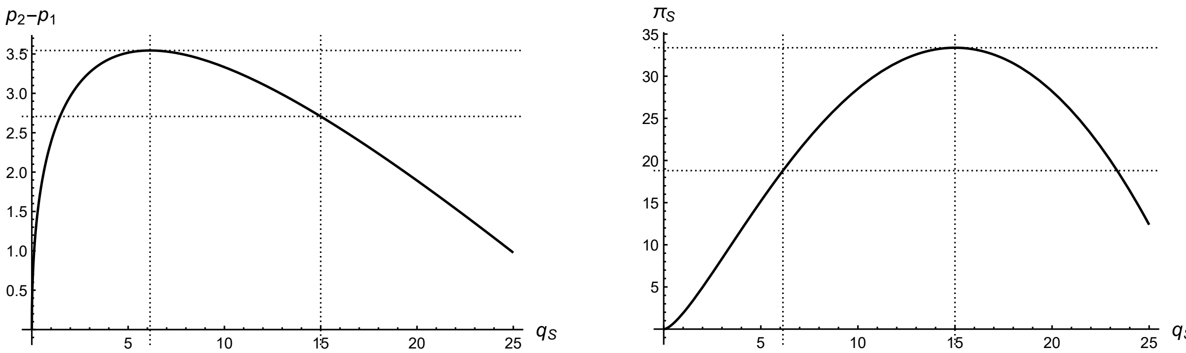

FIGURE VII
Price Volatility and the Speculator's Profit as a Function of Limit Buy-Quantity
The figure shows the price spread between period 2 and period 1 in the equilibrium without disposal (on the left panel) and the round-trade profit π_S of the speculator (on the right panel) given different value of q_S with parameters $a = 90$, $b = 1$, and $c_S = 0.48$. In both figures, we use the limit price that satisfies the incentive compatibility constraints of the strategic sellers as the clearing price in period 1. The price spread reaches its maximum 3.54 when $q_S = 6.14$, while the speculator earns highest profit 33.38 when $q_S = 15$.

V. Equilibrium with speculator and free disposal

In the equilibrium without disposal, we show that the speculator is able to buy at lower price in period 1 and sell at a relatively higher price in period 2, even though the period 2 price does not exceed the benchmark price. In this section, we solve for an equilibrium where the speculator manufacture enough volatility such that the period 2's price is higher than the benchmark price level. We show that free disposal is necessary condition. For expositional simplicity, we assume that the speculator has the exact same limit buy-quantity and limit price in period 1 as he does in equilibrium without disposal. A more general version of the equilibrium where limit quantity and limit price in period 1 are also choice parameters of the speculator is presented in the next section.

Period 3

No decisions are made by the agents in period 3. The forward contracts bought and sold in period 2 are settled by making payments and taking deliveries.

Period 2

The speculator has the inventory $q_{S1} = 15$, he bought in period 1 using forward contract, that is delivered at the beginning of period 2 and submits two orders to supply: a market order to supply $\alpha\, q_{S1}$, $\alpha \in [0, 1]$, and a stop-loss order to supply $(1-\alpha)\, q_{S1}$ when the clearing price in period 2 is below p_{S2} and zero otherwise. We assume that the speculator can dispose $(1-\alpha)q_{S1}$ units of widgets without incurring any costs when the clearing price is above or equal to p_{S2}. Taking the two orders together, we have the speculator's supply schedule given as below.

$$\boldsymbol{q}_{S2}(p_2) = \begin{cases} \alpha\, q_{S1} & \text{for } p_2 \geq p_{S2} \\ q_{S1} & \text{for } p_2 < p_{S2} \end{cases} \tag{V.1}$$

where p_2 is the clearing price in period 2, p_{S2} is the limit price the speculator chooses, and as stated α is between 0 and 1.[11] The supply schedule in (V.1) implies that the speculator

[11] We add subscripts 1 and 2 to q_{S1} to differentiate the speculator's quantity demand in period 1 and quantity supply in period 2. We also add subscripts 1 and 2 to p_S to differentiate the limit prices the speculator chooses in period 1 and period 2.

is willing to “sacrifice” a fraction, $1-\alpha$, of his inventory when the clearing price in period 2 is above the limit price p_{S2}, while if the price is below p_{S2} the speculator supplies his entire inventory q_{S1}. Figure VIII depicts the supply schedule of the speculator in period 2 given by equation (V.1).

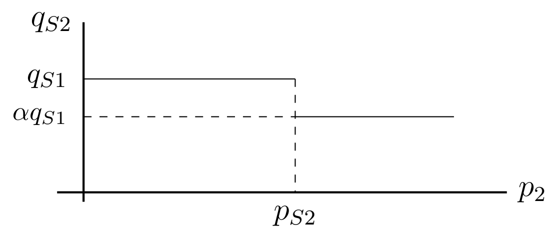

FIGURE VIII
Speculator’s Supply in Period 2 with Free Disposal

The figure depicts the speculator’s period 2 supply function using a combination of market order and stop-loss order. The speculator supplies a fraction α of his acquired inventory in period 1, q_{S1}, using a market order. The remaining fraction $1-\alpha$ of his acquired inventory is supplied using a stop-loss function with a trigger price, p_{S2}.

Taking the speculator’s supply schedule into account, the clearing price in the period 2 of the equilibrium with free disposal is given as

$$\text{(V.2)} \qquad \boldsymbol{p}_2(Q_2;\alpha,p_{S2},q_{S1}) = \begin{cases} a-b(Q_2+\alpha q_{S1}) & \text{for } Q_2 \leq \frac{a-p_{S2}}{b}-\alpha q_{S1} \\ a-b(Q_2+q_{S1}) & \text{for } Q_2 > \frac{a-p_{S2}}{b}-\alpha q_{S1} \end{cases}$$

where $Q_2 = q_{A_2} + q_{B_2}$ is the aggregate supply of the two strategic sellers in period 2. In addition to the aggregate supply of the strategic sellers, there are two parameters affecting the clearing price in period 2:

1. The fraction of his acquired inventory, α that the speculator supplies as a market order; and
2. the stop-loss trigger price, p_{S2} that the speculator chooses.

If the speculator does not give up any portion of his inventory when the clearing price is above p_{S2}, the clearing price in period 2 will be the same as the one of the equilibrium without disposal since the strategic sellers do not have incentive to reduce supplies. The Figure IX depicts the clearing price in period 2 of the equilibrium with free disposal when the two strategic sellers supply Q_2 in total and the speculator supplies α fraction of his inventory q_{S1} via market order and supplies the rest via stop-loss order when the clearing price is above p_{S2}.

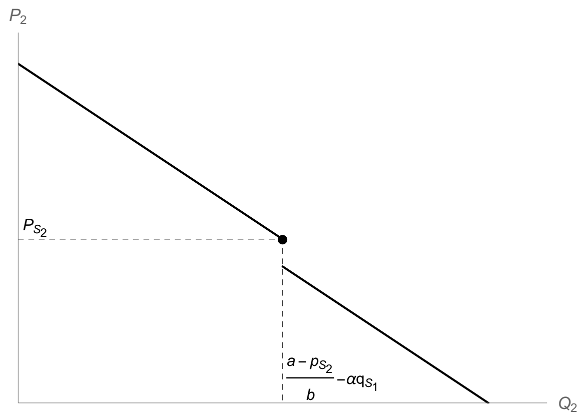

FIGURE IX
Period 2's Price as a Function of Aggregate Supply of only the Strategic Sellers
This figure depicts the clearing price in period 2 equilibrium with free disposal where Q_2 denotes the total supply of the strategic sellers. When the aggregate supply of the strategic sellers is greater than $\frac{a-p_{S2}}{b} - \alpha\, q_{S1}$ price drops below p_{S2}. That kicks an additional supply of $(1-\alpha)\, q_{S1}$ from the speculator causing the price drop even further.

The strategic sellers' objectives in period 2 are to maximize their payoffs in period 2 by taking into account the speculator's stop-loss order, the supply of the other strategic seller, and the demand curve, i.e.,

$$\begin{aligned} &\max_{q_{A2}} q_{A2}\boldsymbol{p}_2(q_{A_2} + q_{B_2}; \alpha, p_{S2}, q_{S1}), \\ &\max_{q_{B2}} q_{B2}\boldsymbol{p}_2(q_{A_2} + q_{B_2}; \alpha, p_{S2}, q_{S1}), \end{aligned} \tag{V.3}$$

The speculator's objective is to maximize the his profit in period 2 when the incentive compatibility constraints of the strategic sellers are satisfied, i.e.,

$$
\text{(V.4)} \qquad
\begin{aligned}
\max_{\alpha, p_{S2}} \quad & \boldsymbol{q}_{S2}(q_{S1})\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1}) \\
\text{s.t.} \quad & q_{A2}\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1}) \geq q_{A2}\boldsymbol{p}_2(q'_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1}) \\
& q_{B2}\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1}) \geq q_{A2}\boldsymbol{p}_2(q_{A_2}+q'_{B_2};\alpha,p_{S2},q_{S1})
\end{aligned}
$$

where q'_{A2} and q'_{B2} denote any alternative supply strategies of the strategic seller A and strategic seller B in period 2 respectively. Meeting the incentive compatibility constraints guarantees that the strategic sellers reduce supply compared to the period 2 of the equilibrium without disposal. We assume that the speculator chooses the limit price p_{S2} at the level which makes the strategic sellers just be indifferent from deviating from the coordination game or not deviating. The following lemma summarizes the speculator's strategy in period 2 of the equilibrium with free disposal.

Lemma 2. *For any given level of inventory $q_{S1} < \frac{a}{b}$, the speculator supplies a fraction*

$$\boldsymbol{\alpha}(q_{S1}) = \frac{3\,a}{2\,a + b\,q_{S1}} - \frac{1}{2}$$

of his inventory using a market order and $1 - \boldsymbol{\alpha}(q_{S1})$ of his inventory using a stop-loss order when the clearing price is above p_{S2}, which is given by:

(V.5)

$$\boldsymbol{p}_{S2}(q_{S1}) = p^* + \frac{b\,q_{S1}(1-3\,\boldsymbol{\alpha}(q_{S1})) + \sqrt{3\,b\,q_{S1}(1-\boldsymbol{\alpha}(q_{S1}))(4\,a - b\,q_{S1}(1+3\,\boldsymbol{\alpha}(q_{S1}))}}{6},$$

where p^* denotes the benchmark price; and a and b are the demand and price sensitivity parameters respectively. The stop-loss order's price-quantity pair, $(\boldsymbol{p}_{S2}, (1-\boldsymbol{\alpha}(q_{S1}))\,q_{S1})$ will be such that the period 2 clearing price p_2 will be equal to p_{S2} and the stop-loss order will not be executed. The speculator gains $\boldsymbol{\alpha}(q_{S1})q_{S1}p_2$ and is better off when compared to the equilibrium without disposal. In the symmetric equilibrium, each strategic seller *supplies* $\boldsymbol{q}_2^*(q_{S1}) = \frac{a-p_2-\boldsymbol{\alpha}(q_{S1})bq_{S1}}{2b}$ *and has a payoff of*

$$\boldsymbol{\pi}_2(q_{S1}) = \frac{a - p_2 - \boldsymbol{\alpha}(q_{S1})bq_{S1}}{2b}p_2 \tag{V.6}$$

and is better off when compared to the equilibrium without disposal. The consumers are worse off when compared to the equilibrium without disposal and the benchmark equilibrium.

Proof: See Appendix B.

Suppose that the speculator has inventory q_{S1} and supplies α fraction of his inventory using a market order and chooses his stop-loss price as p_{S2} which is higher than the period 2 clearing price of equilibrium without disposal. The two strategic sellers have to reduce their supplies to reach the stop-loss price. If one strategic seller increases her supply which induces the execution of the speculator's stop-loss order, it optimal for her to supply more than the period 2 quantity of equilibrium without disposal. This is because the other strategic seller supplies less than the period 2 quantity of the equilibrium without disposal, the supply increasing strategic seller will take the advantage of this by supplying more. This results in a higher payoff of the supply increasing strategic seller when compared to what she gets in the period 2 of the equilibrium without disposal. To prevent the strategic seller A from increasing supply, the speculator has to adjust either α or p_{S2} to accommodate the strategic sellers' deviating incentive.[12] To ensure that the clearing price in period 2 is his stop-loss price, the speculator has to further make sure that both strategic sellers will reduce supply from the period 2 of the equilibrium without disposal. Thus, the two strategic sellers will get a payoff which is higher than or equal to what they would get in the period 2 of the equilibrium without disposal.

[12] We focus on the case that the speculator adjusts his limit sell price for a given α.

Numerical Example: Period 2's payoffs with free disposal

Let the demand and sensitivity parameters be $a = 90$ and $b = 1$ respectively, and assume that the speculator has inventory $q_{S1} = 15$. The left panel of Figure X depicts the period 2 clearing price when the speculator chooses an $\alpha \in [0.5, 1.0]$ and a stop-loss price given by equation (V.5). As the fraction to supply through market order, α, gets smaller, the clearing price in period 2 becomes higher. Recall that the benchmark clearing price is $p^* = 30$, which is lower than $\boldsymbol{p}_{S2}(q_{S1})$ when $\alpha < 0.944$, and the period 2 clearing price in equilibrium without disposal is 25, which is lower than $\boldsymbol{p}_{S2}(q_{S1})$ as long as $\alpha < 1$. The speculator optimally chooses disposed 12% of his acquired inventory; i.e., $1 - \alpha = 0.12$ which results in a market clearing price of $p_2 = 32.5$ and the speculator's revenue is equal to $q_{S2}\, p_2 = 429$ in period 2. We show that incentive compatibility constraints of both the strategic sellers are satisfied. Recall that the benchmark market-clearing price is 30 which is lower than p_2, implying that the consumers are worse off in period 2.

In the equilibrium without disposal, the speculator earned $25 \times 15 = 375$ in period 2 by supplying his entire inventory, while in the equilibrium with free disposal, the speculator earns more than 375 in period 2 as long as he disposes less than 37% of his total acquired inventory; i.e., $(1 - \alpha) \leqslant 0.37$. This is depicted on the right panel of Figure X. Each of the two strategic sellers supplies $q_2 = 22.12$ in period 2 which is less than 25 each of them supplied in the period 2 supply of equilibrium without disposal. Recall that the strategic seller's payoff is 625 dollars which is lower than what they get in period 2 of the equilibrium with free disposal $q_{S2}\, p_2 = 718.9$ dollars.

Period 1

In the last example, we have already shown that the clearing price in period 2 of the equilibrium with free disposal can be greater than the benchmark price. To complete the round-trade game of the speculator, we only need to further show that his participation

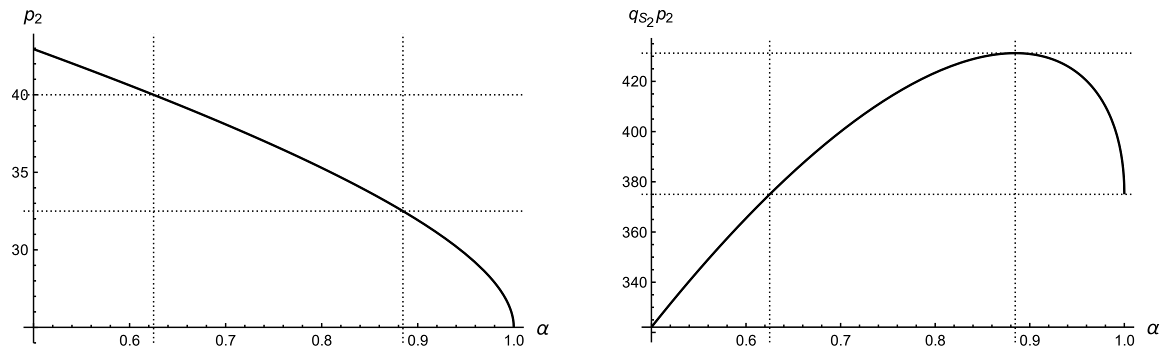

FIGURE X

Period 2 Market Clearing Price and Speculator's Profit vs Fraction Supplied Using Market Order

This figure depicts the period 2 clearing price (on the left panel) and the period 2 profit of the speculator (on the right panel) according to different value of $\alpha \in [0.5, 1.0]$ with parameters $a = 90$, $b = 1$, and $q_{S1} = 15$. Both panels include the highest limit price that satisfies the incentive compatibility constraints and the highest limit price which guarantees that it is the unique clearing price in period 2.

constraint and the strategic sellers' incentive compatibility constraints are satisfied in period 1 when the period 1 clearing price and the limit quantity are at the same levels as in the equilibrium without disposal.

Numerical example: Aggregate payoffs with free disposal

Let the demand and sensitivity parameters be $a = 90$ and $b = 1$, and the speculator's storage cost be $c_S = 0.48$. In period 1, the speculator submits a limit buy order where the limit price $p_{S1} = 24.41$ and the limit quantity $q_{S1} = 15$. Note that these numbers are consistent with the ones in the example of the equilibrium without disposal. In period 2, the speculator submits two orders to sell his inventory: (1) a marketable order which sells $\alpha\, q_{S1} = 0.88 \times 15 = 13.2$ units, and (2) a stop-loss order where the quantity is $(1 - \alpha)\, q_{S1} = 0.12 \times 15 = 1.8$ and the price $p_{S2} = 32.5$. As stated earlier the strategic sellers reduce their supplies to avoid the execution of the speculator's stop-loss order and the resulting market-clearing price $p_2 = 32.5$. The speculator earns $\alpha\, q_{S1}\, p_2 = 429$ and each strategic seller earns $22.12 \times 32.5 = 718.9$.

We then proceed to check whether the strategic sellers have incentive to accept the speculator's limit order. In period 1, when one seller sticks to the benchmark supply $q^* = 30$, then the other strategic seller can increase supply to $\frac{a-p_{S1}}{b} + q_{S1} - q^* = 90 - 24.41 + 15 - 30 = 50.59$ which gives the deviating seller a period 1 payoff equal to $50.59 \times 24.42 = 1234.90$ dollars. Taking the payoffs of the two periods together, the deviating seller earns $1234.90 + 718.90 = 1953.80$ dollars which is strictly greater than the benchmark payoff $\pi^* = 1800$ dollars. Hence, one of the strategic seller has incentive to increase supply when the other strategic seller sticks to the benchmark supply.

In the symmetric equilibrium both the strategic sellers increase their supply up to $\frac{1}{2}\left(\frac{a-p_{S1}}{b} + q_{S1}\right) = 40.30$ units in response to the speculators limit order, which yields a payoff of $40.30 \times 24.41 = 983.72$ dollars in period 1, and hence an aggregate payoff of $983.72 + 718.90 = 1702.62$ dollars. If one of the strategic seller reduces supply implying that the speculator's limit buy order will not be filled and the best supply for the deviating strategic seller is $\frac{a}{2b} - \frac{40.30}{2} = 24.85$ units according to her best response function. The period 1 payoff of the strategic seller who reduces supply is $24.85 \times 24.85 = 617.52$ as the resulting market-clearing price is $90 - (24.85 + 40.30) = 24.85$. Note that the speculator's limit order is not executed so that each strategic seller gets benchmark payoff 900 in period 2. Again, taking the two periods together, the deviating seller gets $617.52 + 900 = 1517.52$ dollars in aggregate, which is strictly less than 1702.62 dollars which she can get if she does not reduce her supply. Thus, the strategic sellers will fully supply the speculator's limit buy order even though both of them are worse off.

The speculator acquire 15 units in period 1 at price 24.41 per unit and sells only 13.2 units in period 2 at price 32.5 per unit. The speculator earns a round-trade profit of 62.85 $(= 13.2 * 32.5 - 15 * 24.41)$ dollars. The speculator participates as long as the unit storage cost is less than $62.85/15 = 4.19$. Note that the clearing price in period 2 is greater than the benchmark price 30.

In Appendix A, we solve a more general version of the equilibrium with free disposal by letting the speculator optimally chooses the limit buy-quantity and limit buy-price in period 1. To measure the consumer's welfare, let $w^* = \frac{1}{b}(a - p^*)^2$ denote the consumer surplus and $w^{*s} = \frac{1}{2b}(a - p_1)^2 + \frac{1}{2b}(a - p_2)^2$ the sum of the consumer surpluses in period 1 and period 2 in the equilibrium with the speculator and free disposal. In proposition 2 below, we show that the consumers are better off when both period 1 and period 2 are taken together, while they can be worse off in period 2. Since consumers are myopic and price takers in our model, they will complain about high prices due to speculation in period 2. The equilibrium is summarized by the following proposition.

Proposition 2. *When the speculator has access to free disposal and there are only two strategic sellers, the speculator and consumers are always better-off in terms of aggregate welfare, and the strategic sellers are worse-off in aggregate profit in comparison to the benchmark equilibrium.*

Proof: See Appendix B.

We will use a numeric example to show that the consumers can be worse off even taking the two periods together when there are more strategic sellers. Suppose that the demand and price sensitivity parameters are $a = 90$ and $b = 1$ and there are 18 strategic sellers. The benchmark price is 4.74 and the benchmark supply of the strategic sellers is 4.74. Assume that speculator has 15 units of widgets at the beginning of period 2 and supplies 0.67 of his inventory through market order and supplies the rest 0.33 through a stop-loss order with a limit price 11.45. It can be verified that the each strategic seller will supply 3.80 in equilibrium which results in a clearing price equal to the limit price 11.45 and the stop-loss order is not executed. In period 1, the speculator demands 15 units of widgets through a limit buy order with a limit price 0.05. In equilibrium each strategic seller supplies 5.83 to meet the speculator's demand in full and the clearing price is equal the speculator's limit buy-price 0.05.

The consumer surplus is 4045 units in period 1 and 3085 units in period 2, 7130 in aggregate, while it is 7270 in the benchmark equilibrium. The fact the consumers are worse off taken the two periods together when there are more strategic sellers is because there is a lower bound 0 for the clearing price in period 1, while the speculator can still raise the price higher in period 2. When there are more strategic sellers the clearing price is low in the benchmark equilibrium so that the speculation will hurt the consumers in aggregate when the gain caused by the price discount in period 1 cannot offset the loss caused by the price increase in period 2.

VI. Summary and Conclusion

We construct a three-period model economy with a large speculator with access to storage facility, two strategic duopolistic sellers and many atomistic consumers. Only the speculator has access to storage. He is able to manipulate the market for widgets in such a way that he obtains a lower price while buying and a higher price while selling thereby profiting from speculation. Contrary to Friedman's (1953) conjecture, speculation is destabilizing because it is the buying (selling) by the speculator that makes the price go down (up).

The two duopolistic strategic sellers are made worse off in both periods. In the case with disposal is not possible, consumers are better off in both periods. When the speculator has access to free disposal, consumers are better off in period 1, worse off in period 2, but better off when both periods are taken together. However, when there are more than two strategic sellers, consumers can be worse off overall. In our model, both strategic sellers and consumers will complain about the destabilizing effect of the speculation in period 2.

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Appendix A. The equilibrium with free disposal

To complete the equilibrium with free disposal, we allow the speculator to optimally choose limit buy price and limit quantity in period 1. Similar to the period 1 of the equilibrium with no disposal, the speculator submits a limit buy order:

$$\boldsymbol{q}_{S1}(p_1) = \begin{cases} 0 & \text{for } p_1 > p_{S1} \\ [0, q_{S1}] & \text{for } p_1 = p_{S1} \\ q_{S1} & \text{for } p_1 < p_{S1} \end{cases} \tag{A.1}$$

where p_1 is the market clearing price in period 1, and q_{S1} is the quantity that the speculator buys when the clearing price is below the limit price p_{S1}. This results in a market price given as follows.

$$\boldsymbol{p}_1(Q_1; q_{S1}, p_{S1}) = \begin{cases} a - bQ_1 & \text{for } Q_1 < \frac{a-p_{S1}}{b} \\ p_{S1} & \text{for } Q_1 \in \left[\frac{a-p_{S1}}{b}, \frac{a-p_{S1}}{b} + q_{S1}\right] \\ a - b(Q_1 - q_{S1}) & \text{for } Q_1 > \frac{a-p_{S1}}{b} + q_{S1}, \end{cases} \tag{A.2}$$

where $Q_1 = q_{A1} + q_{B1}$ is the aggregate supply of the strategic sellers in period 1.

The objective of the speculator is to maximize his round-trade profit, purchasing in period 1 and selling in period 2, subject to two constraints: First, the trading profit of the speculator must cover his cost of inventory. Second, the two strategic sellers have to be incentivized to increase supply to meet the speculator's demand in full. Notice that the speculator will choose $q_S = 0$ if he expects a negative profit by participating, therefore, we have omit his participation constraint. Formally, the speculator's objective can be written as follows.

$$\max_{q_{S1},p_{S1}} \quad \boldsymbol{q}_{S1}\Big(\boldsymbol{\alpha}(q_{S1})\boldsymbol{p}_2\big(\boldsymbol{q}_{S1}(\boldsymbol{p}_1(q_{A1}+q_{B1};q_{S1},p_{S1}))\big) - \boldsymbol{p}_1(q_{A1}+q_{B1};q_{S1},p_{S1}) - c_S\Big) \tag{A.3}$$

$$\begin{aligned} \text{s.t.} \quad & \boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_{S1},p_{S1}) + \boldsymbol{\pi}_{A2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_{S1},p_{S1}))\big) \\ & \qquad \geq \boldsymbol{\pi}_{A1}(q'_{A1}+q_{B1};q_{S1},p_{S1}) + \boldsymbol{\pi}_{A2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q'_{A1}+q_{B1};q_{S1},p_{S1}))\big) \\ & \boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_{S1},p_{S1}) + \boldsymbol{\pi}_{B2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_{S1},p_{S1}))\big) \\ & \qquad \geq \boldsymbol{\pi}_{B1}(q_{A1}+q'_{B1};q_{S1},p_{S1}) + \boldsymbol{\pi}_{B2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q'_{B1};q_{S1},p_{S1}))\big) \end{aligned} \tag{A.4}$$

where q'_{A1} and q'_{B1} denote any alternative quantity that the strategic seller A and B can supply in period 1. The incentive compatibility constraints (A.4) of the speculator's maximization problem can be omitted when the speculator chooses a minimum limit buy price at which the strategic sellers' incentive compatibility constraints are satisfied. This leads us to Lemma 3 given blow.

Lemma 3. *Let p^* denote the benchmark equilibrium price; q_{S1} denote the quantity in the speculator's limit buy order; p_{S2} denote the speculator's stop-loss price in period 2; and q_2^* denote the supply of each strategic seller in period 2; and a and b are the demand and price sensitivity parameters respectively. For any given $q_{S1} < \frac{a}{b}$, it is optimal for the speculator to chooses a limit buy price as follows,*

$$\boldsymbol{p}_{S1}(q_{S1}) = p^* + \frac{1}{6}\left(3q_{S1} + \sqrt{-4a^2 + 12abq_{S1} + 9bq_2^*(4p_{S2} + bq_{S1}^2)}\right) \tag{A.5}$$

which guarantees that it is the clearing price in period 1. The clearing price in the equilibrium with free disposal is lower than the corresponding clearing price in the equilibrium without disposal.

Proof: See Appendix B.

For a given level of demand q_{S1}, the speculator can further lower clearing price in period 1 when compared to the equilibrium without disposal. This is because the period

2 payoff of the strategic sellers are higher than what they would get in the equilibrium without disposal and the speculator's limit buy price makes the strategic sellers have same payoff when supplying the benchmark quantity or supplying more.

In equilibrium, the speculator chooses the optimal q_{S1} which maximize his round-trade profit as in equation (A.3). The gain of the speculator in period 2 is determined by choosing the optimal α and p_{S2} for given q_{S1}, while the cost of buying forward contracts in period 1 is determined by choosing the optimal p_{S1} for given q_{S1}. Together with the inventory cost c_S which is exogenous in our model, the speculator will enter the market as long as the round-trade profit is positive, i.e.,

$$\max_{q_{S1}} \quad q_{S1}\Big(\boldsymbol{\alpha}(q_{S1})\boldsymbol{p}_2(q_{S1}) - \boldsymbol{p}_{S1}(q_{S1}) - c_S\Big) \tag{A.6}$$

Proposition 2 summarizes the outcomes of the equilibrium with free disposal.

Appendix B. Proofs

Proof of Lemma 1.

To make the limit buy price meet the strategic sellers' incentive compatibility constraints, the speculator's limit buy price has to satisfy two conditions. First, the strategic sellers will produce more than the benchmark quantity to supply the demand of the speculator. Second, when the strategic sellers supply more than the benchmark quantity and they supply equally, neither strategic seller has incentive to reduce supply. In addition, we show that the optimal strategy for the strategic sellers is to supply the entire demand of the speculator rather than a fraction of the speculator's demand, and they do not have incentive to supply further more when the speculator's demand is fulfilled.

A. The strategic sellers will produce more than the benchmark quantity.

Suppose that the speculator chooses the limit price as $p_S^{(1)}$ for a given q_S and the two strategic sellers supply the benchmark quantity, q^*. If strategic seller A supplies more to meet the speculator's demand in full and reach the limit price, she has to supply $\frac{a-p_S^{(1)}}{b} - q^* + q_S$. This gives her a payoff of $\left(\frac{a-p_S^{(1)}}{b} - q^* + q_S\right) p_S^{(1)}$ in period 1. If the gain by supplying more in period 1 is higher than or equal to the corresponding loss in period 2 caused by the speculator's supply, the strategic seller will supply more to meet the speculator's demand, i.e.,

$$\text{(B.1)} \qquad \left(\frac{a-p_S^{(1)}}{b} - q^* + q_S\right) p_S^{(1)} + \frac{(a-bq_S)^2}{9b} \geq 2\pi^*.$$

LHS is the unilateral deviating payoff of the deviating strategic seller and RHS is her benchmark payoff. As the speculator wants to lower the price in period 1, he will choose the clearing price that just makes the strategic sellers be indifferent from supplying the benchmark quantity and supplying more to meet the speculator's demand. Thus, we solve the above equation by letting the LHS equal to the RHS, and get

$$\text{(B.2)} \qquad \boldsymbol{p}_S^{(1)}(q_S) = p^* + \frac{3bq_S - \sqrt{bq_S(4a+13bq_S)}}{6}.$$

This deviating incentive applies to both strategic sellers, and hence both strategic sellers have incentive to increase supply to meet the speculator's demand unilaterally when the other strategic sellers supply the benchmark quantity and the speculator sets the limit buy price higher than or equal to $\boldsymbol{p}_S^{(1)}$. This rules out existence of the benchmark equilibrium when speculator submits such a limit buy order.

To show the deviating strategic seller does not supply more than $\left(\frac{a-\boldsymbol{p}_S^{(1)}}{b} - q^* + q_S\right)$ when the speculator chooses a limit buy price $p_S^{(1)}$ for a given q_S. Note that any further increase in supply does not affect the strategic sellers' payoffs in period 2 when the speculator's limit order is fulfilled in period 1. The best response functions of strategic sellers in period 1 are thus given as follows:

$$\text{(B.3)} \qquad \boldsymbol{q}_{A1}(q_{B1}) = \frac{a}{2b} - \frac{q_{B1}}{2} + \frac{q_S}{2}.$$

By substituting q_{B1} with q^* in the above equation, we get the optimal quantity for the deviating strategic seller to supply, however, this quantity is smaller than the unilateral deviating quantity $\left(\frac{a-p_S^{(1)}}{b} - q^* + q_S\right)$. So that the deviating strategic seller does not have incentive to further increase supply.

The deviating strategic seller will supply the speculator's demand in full rather than supplying any portion of the speculator's demand when the speculator's limit buy price is $p_S^{(1)}$ and the demand is q_S. To see this, let's first consider that the speculator demands two different quantities $q_S^{(1)}$ and $q_S^{(2)}$, where $q_S^{(1)} < q_S^{(2)}$, and the speculator chooses two different limit prices $\boldsymbol{p}_S^{(1)}(q_S^{(1)})$ and $\boldsymbol{p}_S^{(1)}(q_S^{(2)})$ for the two quantities. Since the $\boldsymbol{p}_S^{(1)}(q_S)$ in (B.2) is a decreasing function of q_S, we have $\boldsymbol{p}_S^{(1)}(q_S^{(1)}) > \boldsymbol{p}_S^{(1)}(q_S^{(2)})$. Recall that the speculator chooses the limit price which make the strategic sellers indifferent between supplying more and supplying the benchmark quantity. If the speculator chooses to demand $q_S^{(2)}$ at price $\boldsymbol{p}_S^{(1)}(q_S^{(2)})$, the deviating strategic seller will be worse off by only supplying $q_S^{(1)}$ of the speculator's demand. Thus, the deviating strategic seller will always fully supply the speculator's order.

B. When the strategic sellers supply more than the benchmark quantity and they supply equally, neither strategic seller has incentive to reduce supply.

Suppose that the speculator chooses a limit price $p_S^{(2)}$ for a given q_S. In order to supply the speculator's demand in full, both strategic sellers have to supply $\hat{q}_1^* = \frac{a-p_S^{(2)}+bq_S}{2b}$, and the clearing price is $p_S^{(2)}$. When the speculator's limit order is not filled, the best response functions of strategic sellers are as follows.

$$\boldsymbol{q}_{A1}(q_{B1}) = \frac{a}{2b} - \frac{q_{B1}}{2}. \tag{B.4}$$

So that if one strategic seller decides to supply less which causes the speculator's order is not filled at all, it is optimal for her to supply $\boldsymbol{q}_{A1}\left(\hat{q}_1^*\right)$ and the corresponding clearing price is $a - b\big(\hat{q}_1^* + \boldsymbol{q}_{A1}(\hat{q}_1^*)\big)$. In order to prevent the strategic sellers from reducing supplies, the speculator has to choose a limit buy price that guarantees the payoff of the deviating strategic seller is less than or equal to supplying $\hat{q}_1^*$, i.e.,

$$\Big(a - b\big(\hat{q}_1^* + \boldsymbol{q}_{A1}(\hat{q}_1^*)\big)\Big)\boldsymbol{q}_{A1}\left(\hat{q}_1^*\right) + \pi^* \leq p_S^{(2)}\hat{q}_1^* + \frac{(a-bq_S)^2}{9b} \tag{B.5}$$

where we have the unilateral deviating payoff on the LHS and the payoff of supplying $\hat{q}_1^*$ on the RHS. Solve it and we get

$$\boldsymbol{p}_S^{(2)}(q_S) = p^* + \frac{5bq_S - 4\sqrt{bq_S(a+2bq_S)}}{9}. \tag{B.6}$$

We need to further show that neither strategic seller will increase supply when both of them supply $\hat{q}_1^*$. Recall that the period 2 payoffs of the strategic sellers do not change as the speculator's demand in fulfilled. By substituting q_{B1} with $\hat{q}_1^*$ in equation (B.3), we get the optimal supply of the strategic seller A and it is smaller than $\hat{q}_1^*$. The argument symmetrically applies to strategic seller B, and hence both strategic sellers do not have incentive to increase supply when the speculator's demand is met in full.

In addition, strategic sellers will not reduce supplies to make the speculator's limit buy order be supplied partially. Suppose strategic seller A reduces her supply to $\hat{q}_1^* - q_S + \hat{q}_S$, where $\hat{q}_S \in [0, q_S]$, so that the speculator gets only $\hat{q}_S$ units of widgets while the clearing price does not change as the speculator's limit order is executed. Then the strategic seller A's payoff in period 2 is $\frac{(a-b\hat{q}_S)^2}{9b}$, and we can verify that the strategic seller A gets her maximum two-period payoff when $\hat{q}_S = q_S$.

To guarantee that his limit buy price is the clearing price in period 1, the speculator has to choose the higher limit price between $\boldsymbol{p}_S^{(1)}$ and $\boldsymbol{p}_S^{(2)}$ for a given q_S, i.e.,

$$\boldsymbol{p}_S(q_S) = \max\{\boldsymbol{p}_S^{(1)}(q_S), \boldsymbol{p}_S^{(2)}(q_S)\}.$$

Comparing the two prices, we have $\boldsymbol{p}_S^{(1)}(q_S) \geq \boldsymbol{p}_S^{(2)}(q_S)$ when $q_S \geq \frac{49a}{23b}$. Together with the assumption $q_S < \frac{a}{b}$, the lowest limit buy price of the speculator is $\boldsymbol{p}_S^{(1)}(q_S)$ which gives equation (IV.13), and this completes the proof. □

Proof of Proposition 1.

According to lemma 1, the speculator's limit price will be the clearing price in period 1 if the speculator demands q_S and chooses $\boldsymbol{p}_S(q_S)$ as the limit price. The period 2 price p_2, which is given by equation (IV.5), depends on how many units of widgets that the speculator acquired. Hence the spread between the period 2 and period 1 clearing prices is

$$\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S) = \frac{1}{6}\left(\sqrt{bq_S(4a + 13bq_S)} - 5bq_S\right)$$

The price spread reaches its maximum $\frac{(5-2\sqrt{3})a}{39} \approx 0.04a$ when $q_S = \frac{(5\sqrt{3}-6)a}{39b} \approx 0.07\frac{a}{b}$. Since the speculator's participation constraint will be satisfied if his per unit profit $\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S)$ is greater than the per unit storage cost c_s, the speculator will enter the market and affect the clearing prices in both periods when $c_S \leq \frac{(5-2\sqrt{3})a}{39}$ which completes the proof. □

Proof of Lemma 2.

The proof proceeds similar to the proof of lemma 1. Suppose that the speculator has inventory q_{S1} and chooses a limit price $p_{S2}^{(1)}$ and an α for the stop-loss order. To avoid the execution of the stop-loss order, strategic seller A has to reduce her supply to $\frac{a-p_{S2}^{(1)}}{b} - q^* - \alpha q_{S1}$ to raise the clearing price to be equal to $p_{S2}^{(1)}$ when strategic seller B supplies $\frac{a-bq_{S1}}{3b}$ which is the period 2 supply of equilibrium without free disposal. The strategic sellers will reduce supply to avoid the execution of the stop-loss order if unilateral supply reduction brings higher payoff to the strategic sellers in period 2, i.e.,

$$\left(\frac{a-p_{S2}^{(1)}}{b} - q^* - \alpha q_{S1}\right) p_{S2}^{(1)} = \frac{(a-bq_{S1})^2}{9b},$$

where on the LHS is the payoff of strategic seller A when she reduces supply unilaterally to avoid the execution of the stop-loss order and on the RHS is the payoff of strategic seller A when she supplies the period 2 supply of the equilibrium without disposal. This yields

$$\boldsymbol{p}_{S2}^{(1)}(\alpha, q_{S1}) = p^* + \frac{bq_{S1}(1-3\alpha) + \sqrt{3bq_{S1}(1-\alpha)(4a - bq_{S1}(1+3\alpha)}}{6}.$$

If the clearing price is equal to the stop-loss price $p_{S2}^{(1)}(\alpha, q_{S1})$, the speculator will choose an optimal α so as to maximize his profit, $\alpha q_{S1} p_{S2}^{(1)}(\alpha, q_{S1})$. This yields $\alpha = \frac{3a}{2a+bq_{S1}} - \frac{1}{2}$ and $\alpha \in (0,1]$ as long as $q_{S1} \in (0, \frac{4a}{b}]$.

We then proceed to show that the supply reducing strategic seller does not have incentive to further reduce her supply which results in a clearing price lower than $p_{S2}^{(1)}$. The best response function of strategic seller A when the speculator's stop-loss order is not executed is as follows.

(B.7) $$\boldsymbol{q}_{A1}(q_{B1}) = \frac{a}{2b} - \frac{q_{B1}}{2} - \frac{\alpha q_{S1}}{2}.$$

By substituting q_{B1} with $\frac{a-bq_{S1}}{3b}$ in her best response function, we get the optimal supply of strategic seller A, and it is greater than $\left(\frac{a-p_{S2}^{(1)}}{b} - q^* - \alpha q_{S1}\right)$ as long as $q_{S1} < \frac{4a}{b}$. So that the deviating strategic seller will not further reduce supply.

Now suppose that the speculator has inventory q_{S1}, and his stop-loss price is $p_{S2}^{(2)}$ and he supplies α portion of his inventory through a market order. If the clear price is $p_{S2}^{(2)}$ and the strategic sellers supply equally, each of them has to supply $\hat{q}_2^* = \frac{a-p_{S2}^{(2)}-\alpha b q_{S1}}{2b}$. If strategic seller A supplies more, the clearing price will be below $p_{S2}^{(2)}$ which induces the speculator dump all his inventory, and strategic seller A's best response function is

$$\boldsymbol{q}_{A1}(q_{B1}) = \frac{a}{2b} - \frac{q_{B1}}{2} - \frac{q_{S1}}{2}.$$

By substituting q_{B1} with $\hat{q}_2^*$, the strategic seller A's optimal supply is $\boldsymbol{q}_{A1}(\hat{q}_2^*)$, which results in a clearing price $a - b\left(\boldsymbol{q}_{A1}(\hat{q}_2^*) + \hat{q}_2^* + q_{S1}\right)$. To keep the strategic seller A from not supplying more, the following condition has to be satisfied:

$$\boldsymbol{q}_{A1}(\hat{q}_2^*)\big(a - b\left(\boldsymbol{q}_{A1}(\hat{q}_2^*) + \hat{q}_2^* + q_{S1}\right)\big) = \hat{q}_2^* p_{S2}^{(2)}$$

where on the LHS is the payoff of strategic seller A if she supplies more and on the RHS is the payoff of strategic seller A if she does not deviate. Solving this equality, we have

$$\boldsymbol{p}_{S2}^{(2)}(\alpha, q_{S1}) = p^* + \frac{b q_{S1}(2-5\alpha) + 4\sqrt{b q_{S1}(1-\alpha)(3a - b q_{S1}(2+\alpha)}}{9}.$$

Comparing the above stop-loss price with $\boldsymbol{p}_{S2}^{(1)}(\alpha, q_{S1})$, we get $\boldsymbol{p}_{S2}^{(1)}(\alpha, q_{S1}) <= \boldsymbol{p}_{S2}^{(2)}(\alpha, q_{S1})$ as long as $q_S <= \frac{a}{b}$. Hence, choosing a stop-loss price lower than or equal to $\boldsymbol{p}_{S2}^{(1)}(\alpha, q_{S1})$ is a stricter condition than $\boldsymbol{p}_{S2}^{(2)}(\alpha, q_{S1})$ when $q_S <= \frac{a}{b}$.

It is clear that strategic sellers are better off compared to the equilibrium without disposal since choosing a stop-loss price lower than or equal to $\boldsymbol{p}_{S2}^{(1)}(\alpha, q_{S1})$ guarantees that the strategic sellers have incentive to deviate from the period 2 supply of equilibrium without disposal. Moreover, it is always the case that $\boldsymbol{p}_{S2}^{(1)}(\alpha, q_{S1}) > p^*$ so that the consumers are worse off compared to the benchmark equilibrium, and this completes the proof. $\square$

Proof of Lemma 3.

The proof of lemma 3 follows the same steps of the proof of lemma 1. So that we omit some of the analysis here while giving the two conditions that correspond to (B.1) and (B.5) in the proof of lemma 1. First, for any q_{S1} the speculator has to set his limit buy price at a level where the strategic sellers have incentive to supply more than the benchmark supply to meet the speculator's demand in full, i.e.,

$$\left(\frac{a - p_{S1}^{(1)}}{b} - q^* + q_{S1}\right) p_{S1} + q_2^* p_{S2} \geq 2\pi^*$$

where on the LHS is the payoff of the supply increasing strategic seller and on the RHS is the benchmark payoff. This yields

$$\boldsymbol{p}^{(1)}(q_{S1}) = p^* + \frac{1}{6}\left(3q_{S1} + \sqrt{-4a^2 + 12abq_{S1} + 9bq_2^*(4p_{S2} + bq_{S1}^2)}\right). \tag{B.8}$$

Second, when the speculator's order is fulfilled and the strategic sellers supply equally, $\hat{q}_1^* = \frac{a - p_{S1}^{(1)} - bq_{S1}}{2b}$ the speculator has to prevent the strategic sellers from reducing supplies, i.e.,

$$\left(a - b\big(\hat{q}_1^* + \boldsymbol{q}_{A1}(\hat{q}_1^*)\big)\right)\boldsymbol{q}_{A1}\left(\hat{q}_1^*\right) + \pi^* \leq p_S^{(2)}\hat{q}_1^* + \frac{(a - bq_S)^2}{9b}$$

where on the LHS is the maximum payoff of the supply reducing strategic seller according to her best response function given by equation (B.4) and on the RHS is the payoff if she does not reduce supply. This yields

$$\text{(B.9)} \qquad \boldsymbol{p}^{(2)}(q_{S1}) = p^* + \frac{1}{9}\left(5q_{S1} + 4\sqrt{-a^2 + 3abq_{S1} + 9bq_2^* p_{S2} + b^2 q_{S1}^2)}\right).$$

The limit price in (B.8) is greater than the limit price in (B.9) as long as $q_{S1} < \frac{343a}{239b}$. Together with the restriction appears in Lemma 2 that $q_{S1} < \frac{a}{b}$, the proof is complete. □

Proof of Proposition 2.

According to lemma 3 which gives the speculator's limit price when q_{S1} is taken as given, solving the equation (A.6) yields $q_{S1} = \frac{a}{b}$ – a corner solution for the speculator's maximization problem. This is saying the speculator always chooses the maximum storage capacity as the quantity in his limit buy order when his profit can cover the storage cost. With the same trading cost and limit quantity chosen by the speculator, we verify that the trading profit per widget in equilibrium with free disposal, $\boldsymbol{\alpha}(q_{S1})\boldsymbol{p}_2(q_{S1}) - \boldsymbol{p}_{S1}(q_{S1})$ where q_{S1} is the limit quantity chosen by the speculator in period 1 in the equilibrium with free disposal, is always greater than the trading profit per widget in equilibrium without disposal, $\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S)$ where q_S is the limit quantity chosen by the speculator in period 1 in the equilibrium with no disposal. This results guarantees that the speculator is always made better off in the equilibrium with free disposal compared to the equilibrium with no disposal. As the consumer surplus is given by $w^* = \frac{1}{b}(a-p^*)^2$ in the benchmark equilibrium and $w^{*s} = \frac{1}{2b}(a - p_1)^2 + \frac{1}{2b}(a - p_2)^2$ in the equilibrium with free disposal, we also verify that the consumers are always made better off when the two periods are taken together for any given limit quantity chosen by the speculator in period 1 which completes the proof. □