Corporate governance in the presence of active and passive delegated investment*

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Abstract

We examine the governance role of delegated portfolio managers. In our model, investors decide how to allocate their wealth between passive funds, active funds, and private savings, and fund fees are endogenously determined. Funds' ownership stakes and fees determine their incentives to engage in governance. Whether passive fund growth improves governance depends on whether it crowds out private savings or active funds. In the former case, it improves governance even though it is accompanied by lower fund fees, whereas in the latter case it can be detrimental to governance. Overall, passive fund growth improves governance only if it does not increase fund investors' returns too much. Regulations that decrease funds' costs of engagement can be opposed by both fund investors and fund managers even though they are value-increasing.

Keywords: corporate governance, delegated asset management, passive funds, index funds, competition, investment stewardship, engagement

JEL classifications: G11, G23, G34, K22

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1 Introduction

Institutional ownership has grown tremendously over the last decades, rising to more than 70% of US public firms. The composition of institutional ownership has also changed, with a remarkable growth in index fund ownership. The fraction of equity mutual fund assets held by passive funds is now greater than 30%, and the Big Three index fund managers (BlackRock, Vanguard, and State Street) alone cast around 25% of votes in S&P 500 firms (Appel et al., 2016; Bebchuk and Hirst, 2019a). How active and passive asset managers monitor and engage with their portfolio companies has thus become of utmost importance for the governance and performance of public firms. In 2018, the SEC chairman Jay Clayton encouraged the SEC Investor Advisory Committee to examine "how passive funds should approach engagement with companies," and during the 2018 SEC Roundtable on the Proxy Process, Senator Gramm noted that "what desperately needs to be discussed [in the context of index fund growth] ... is corporate governance."[1]

There is considerable debate in the literature about the governance role of asset managers and the different incentives faced by active vs. passive funds. Some argue that index funds "have incentives to underinvest in stewardship" (Bebchuk and Hirst, 2019b) and even propose that "lawmakers consider restricting passive funds from voting at shareholder meetings" (Lund, 2018). Others disagree and counter that passive investors have "significant incentives ... to play their current roles in corporate governance responsibly" (Kahan and Rock, 2020) and that "existing critiques of passive investors are unfounded" (Fisch et al., 2019). The existing empirical evidence is also mixed: on the one hand, Appel, Gormley, and Keim (2016, 2019) find that passive ownership is associated with more independent directors, fewer antitakeover defenses, and greater success of activist investors. On the other hand, Brav et al. (2020) and Heath et al. (2020) conclude that index funds vote against management more rarely than active funds, and Schmidt and Fahlenbrach (2017) and Heath et al. (2020) find that passive ownership is associated with more CEO power, less board independence, and worse pay-performance sensitivity.

Motivated by these ongoing academic and policy discussions, the goal of our paper is to provide a theoretical framework to analyze the governance role of active and passive asset managers. We are particularly interested in the following questions. How does competition

[1] See the SEC chairman's statement at https://www.sec.gov/news/public-statement/statement-clayton-iac-091318 and the 2018 SEC roundtable transcript at https://www.sec.gov/files/proxy-round-table-transcript-111518.pdf.

between funds affect their assets under management and fees and, in turn, fund managers' incentives to engage in governance? What are the effects of passive fund growth? What is the relation between asset management fees and governance? And what are the expected effects of policy proposals that have been put forward to improve the governance role of asset managers?

In our model, fund investors decide how to allocate their capital by choosing between three options: they can either save privately or invest with either an active or a passive (index) fund manager by incurring a search cost. If an investor decides to delegate his capital to a fund manager, they negotiate an asset management fee, which is a certain fraction of the realized value of the fund's assets under management (AUM) at the end of the game. Next, trading takes place. Passive funds invest all their AUM in the value-weighted market portfolio. Active funds invest strategically, exploiting trading opportunities due to liquidity investors' demand: they buy stocks with low liquidity demand, i.e., those that are "undervalued," and do not invest in "overvalued" stocks with high liquidity demand. After investments are made, fund managers decide how much costly effort to exert in order to increase the value of their portfolio companies. Effort captures multiple actions that a shareholder can take to increase firm value: interacting and engaging with the firm's management and board, investing resources to make informed voting decisions, ongoing monitoring activities, and more confrontational tactics such as submitting shareholder proposals, nominating directors, and aggressively questioning management at annual meetings and on conference calls. All of these tactics are regularly employed by institutional investors, as evidenced by the survey of McCahery, Sautner, and Starks (2016). We refer to these actions as engaging in governance or monitoring.

The key determinants of a fund manager's incentives to engage in governance are the fund's stake in the firm and the fees it charges to its investors: The higher is the fund's stake, the more its AUM increase in value due to monitoring; and the higher are the fees, the more is captured by the fund manager from this increase in value.[2] (See Lewellen and Lewellen (2020) for an empirical estimate of funds' incentives to engage based on the analysis of their portfolios and fees.) The equilibrium ownership stake and fees, in turn, depend on

[2] These properties are consistent with the observed empirical evidence. For example, Iliev and Lowry (2015) and Iliev, Kalodimos, and Lowry (2020) show that funds with higher equity stakes are more likely to conduct governance research and to vote "actively" instead of relying on proxy advisors' recommendations, while Heath et al. (2020) document that index funds with high expense ratios are more likely to vote against management than those with low expense ratios.

the fund's AUM and the fees of other funds in the market. All of these characteristics are determined endogenously; they are affected by the returns fund managers realize by trading in financial markets and by the competition between funds.

Jointly analyzing these aspects and their combined effect on governance is critical, because focusing only on one aspect (e.g., fund fees) can miss other important effects. For example, it is frequently argued that the growth in passive funds is detrimental to governance because of the low fees they charge to investors which, in turn, can lead to lower incentives to be engaged shareholders. However, this argument does not take into account that fees do not change in isolation, and a decrease in fees is accompanied by other changes relevant for governance, such as the reallocation of investor funds from private savings to asset managers and across different types of asset managers, and changes in funds' ownership stakes. While our model captures all of these general equilibrium effects, it is very tractable, allowing us to analyze the combined effects on governance, firm valuations, and investors' payoffs.

In particular, one implication of our analysis is that the relation between funds' fees and governance is far from obvious: easier access to passive funds could simultaneously decrease passive funds' fees but increase their engagement efforts and improve overall governance. Intuitively, when passive funds are more easily available and charge lower fees, their aggregate AUM increase, which, in turn, increases their ownership stakes and strengthens their incentives to engage. Moreover, if passive funds primarily crowd out fund investors' private savings, rather than their allocation to active funds, then passive fund growth does not significantly affect active funds' fees. Hence, active funds continue to engage, and the dominant effect of passive fund growth is to replace retail shareholders (who have neither ability nor incentives to monitor) in firms' ownership structures. As a result, the overall level of investor engagement increases, so passive fund growth improves aggregate governance despite the decrease in fund fees.

However, if passive fund growth crowds out investors' allocations to active funds, rather than their private savings, then it can be detrimental to governance.[3] In this case, passive funds primarily replace active funds, rather than retail shareholders, in firms' ownership structures. Since passive funds charge lower fees than active funds, they have lower incentives to engage, so the overall level of investor engagement can decrease. The accompanying decline

[3] Passive funds seem to be crowding out active funds in recent years: according to Morningstar (2019), actively managed U.S. stock funds have posted net outflows in 11 out of the last 12 years, while passive funds have posted net inflows in all these years. See https://www.morningstar.com/insights/2019/06/12/asset-parity.

in both active and passive fund fees further reduces funds' combined incentives to engage.

An implication of these results is that there can be a trade-off between governance and fund investors' well-being: if passive fund growth substantially increases fund investors' returns on their investment, then it is detrimental to governance, and vice versa. Intuitively, passive fund growth is especially beneficial to fund investors if it creates strong competition between funds and substantially decreases fund fees. But competition implies that funds' combined AUM and ownership stakes do not increase too much, which, combined with lower fees, decreases funds' combined level of engagement. Put differently, effective fund manager engagement requires that funds earn sufficient rents from managing investors' assets, which comes at the expense of fund investors.

Our model also has implications for policy proposals to reduce asset managers' costs of engaging in governance. A common criticism, especially regarding passive funds, is that they have small stewardship teams and thus do not have sufficient resources to monitor their portfolio firms. Based on this criticism, it is natural to suggest regulations inducing passive funds to increase investments in their stewardship teams, which in the context of the model can be interpreted as reducing funds' costs of engagement. However, our analysis shows that the effects of such regulations are generally subtle. On the one hand, decreasing fund managers' costs of engagement induces them to engage more, which increases the value of their portfolio firms and improves governance. On the other hand, this increase in engagement can come at the expense of fund investors' well-being. Intuitively, traders in financial markets rationally anticipate the benefits of increased engagement and bid up the prices, so the funds do not make trading profits on their engagement efforts. Moreover, increased prices imply a lower ability of the funds to realize gains from trade, which can harm fund investors.

Likewise, fund managers themselves do not always benefit from decreasing their costs of engagement. Since this induces the fund to engage more, and more engagement can, in turn, be detrimental to fund investors, the fund would attract less investor capital and thereby collect lower asset management fees. Interestingly, this implies that regulations inducing funds to increase their stewardship teams can be value-increasing but nevertheless be strongly opposed both by fund managers and fund investors. More generally, our analysis suggests that to understand the effects of governance regulations, it is important to consider the potential effects of regulations on funds' assets under management.

To summarize, our results have three main implications. First, whether passive fund growth improves governance crucially depends on whether passive funds grow at the expense of active funds' AUM and primarily replace active funds as firms' shareholders, or whether passive funds bring new investor capital (from investors' private savings) and primarily replace retail shareholders in firms' ownership structures. While passive fund growth can be detrimental in the former case, it is beneficial in the latter case. Second, there is often a trade-off between governance and fund investors' well-being: factors that increase fund engagement decrease fund investors' returns, and vice versa. Finally, the link between fund fees and governance is not immediate, and fee reductions can be accompanied by improvements in governance.

Related literature

Our paper is related to the literature on shareholder activism and the interaction between shareholders' trading and monitoring decisions.[4] Our key contribution to this literature is to study the activism by delegated asset managers and examine how the simultaneous presence of active and passive funds and the competition between them affect funds' fees, AUM, portfolio allocation decisions, and the effect of these factors on funds' engagement in governance. Given our interest in these questions, we abstract from more specific details of the activism process, such as negotiations with management (Corum, 2020), the role of the board (Cohn and Rajan, 2013), communication (Levit, 2019), pushing for the sale of the firm (Burkart and Lee, 2020; Corum and Levit, 2019), and the interaction between multiple shareholders (e.g., Edmans and Manso, 2011; Brav, Dasgupta, and Mathews, 2019). Dasgupta and Piacentino (2015) and Cvijanovic, Dasgupta, and Zachariadis (2019) also study the governance role of asset managers, but differently from our paper, which focuses on governance through voice, these papers focus on how governance via exit is affected by funds' flow-based incentives. Edmans, Levit, and Reilly (2019) and Levit, Malenko, and Maug (2020) analyze index funds in extensions of their models but focus, respectively, on the interaction between voice and exit, and on index funds' role in voting. Baker, Chapman, and Gallmeyer (2020) study the interaction between passive funds and activists in general equilibrium, assuming that passive funds do not engage in governance, while activists engage but do not have mispricing skills to strategically select securities. In contrast, in our paper, both active and passive funds engage in governance, and active funds are also skilled stock pickers, which can make passive fund growth beneficial for aggregate governance and also

[4] E.g., Admati, Pfleiderer, and Zechner (1994), Kahn and Winton (1998), and Maug (1998), among many others. See Edmans and Holderness (2016) for a comprehensive survey.

lead to heterogeneous effects of passive fund growth across firms.

Our paper is also related to empirical studies of index reconstitutions, which examine how the resulting changes in firms' ownership structures affect corporate governance.[5] In the context of our model, if institutional investors replace retail shareholders in a firm's ownership structure, the firm's governance is expected to improve. In contrast, if index inclusion primarily affects the distribution of ownership between active and passive funds (as, e.g., in Bennett, Stulz, and Wang, 2020, and Heath et al., 2020), the effects on governance are subtle and depend on active and passive funds' ownership stakes, fees, and costs of engagement. This can potentially reconcile the conflicting findings in the literature on the effects of index inclusion. Differently from the index reconstitution papers, our focus is on the time-series effects of passive fund growth on aggregate governance. The time-series implications of passive fund growth could be quite different from the cross-sectional effects of index reconstitutions. First, the types of investors that passive funds replace in the time-series could differ from those they replace upon index reconstitutions. Second, as our results emphasize, the time-series analysis needs to take into account not only the changes in firms' ownership structures, but also the simultaneous changes in fund fees and AUM – factors that stay constant in the index reconstitution setting.

Finally, our paper contributes to the literature on delegated asset management and the role of passive investing. This literature examines investor learning about fund manager skills (e.g., Berk and Green, 2004; Pastor and Stambaugh, 2012), endogenous formation of mutual funds by informed agents (e.g., Admati and Pfleiderer, 1990; Garcia and Vanden, 2009), and the asset pricing implications of benchmarking and asset management contracts in general (e.g., Cuoco and Kaniel, 2011; Basak and Pavlova, 2013; Buffa, Vayanos, and Woolley, 2019). Within this literature, our paper is most related to studies that examine the equilibrium levels of active and passive investing and their implications for price efficiency and welfare (Stambaugh, 2014; Brown and Davies, 2017; Bond and Garcia, 2020; Garleanu and Pedersen, 2020; Malikov, 2020). Among these papers, the closest is Garleanu and Pedersen (2020), as we build on Garleanu and Pedersen (2018, 2020) in modeling the asset management industry with endogenously determined fees. But differently from all the above papers, our focus is on the corporate governance role of delegated asset management. In particular, while the asset payoffs in the above papers are exogenous, the asset payoffs in our

[5]They include Appel, Gormley, and Keim (2016, 2019), Bennett, Stulz, and Wang (2020), Crane, Michenaud, and Weston (2016), Heath et al. (2020), Schmidt and Fahlenbrach (2017), and others.

paper are determined endogenously by fund managers' decisions on monitoring. Like our paper, Buss and Sundaresan (2020) and Kashyap et al. (2020) also study the effects of asset managers on corporate outcomes, but through very different channels: Buss and Sundaresan (2020) show that passive ownership reduces firms' cost of capital and induces them to take more risk, while Kashyap et al. (2020) show that due to benchmarking in asset management contracts, firms inside the benchmark are more prone to invest and engage in mergers.

The paper proceeds as follows. Section 2 describes the setup. Section 3 derives the equilibrium and discusses its properties. Section 4 analyzes the implications for governance, fund investor returns, and fund managers' profits. Finally, Section 5 concludes.

2 Model setup

Our model is motivated by Garleanu and Pedersen (2018, 2020): we follow their approach in modeling investors' search for fund managers and their bargaining over asset management fees. Our trading and governance stages are broadly based on Admati, Pfleiderer, and Zechner (1994). We extend their model to a continuum of firms (rather than one firm in Admati et al.), multiple shareholders that can take actions (rather than one shareholder in Admati et al.), and we introduce active and passive delegated asset management. In addition, differently from Admati et al., in which agents are risk-averse, we assume that all agents are risk-neutral, and trading occurs not due to risk-sharing motives but because of heterogeneous private valuations.

There are three types of agents: (1) fund investors, who decide how to allocate their capital; (2) fund managers, who make investment and governance decisions; and (3) liquidity investors. All agents are risk-neutral.

Timeline

The timeline of the model is illustrated in Figure 1. At $t = 1$, fund investors decide whether to search for a fund manager or invest their capital outside the financial market, which we refer to as private savings. At $t = 2$, investors who meet a fund manager negotiate with the fund manager over the asset management fees. At $t = 3$, fund managers decide how to invest their assets under management and trading takes place. At $t = 4$, each fund manager decides on effort to exert for each firm in his portfolio. Finally, at $t = 5$, all firms pay off, and the payoffs are split between fund managers and their investors according to the asset management fees decided upon at $t = 2$.

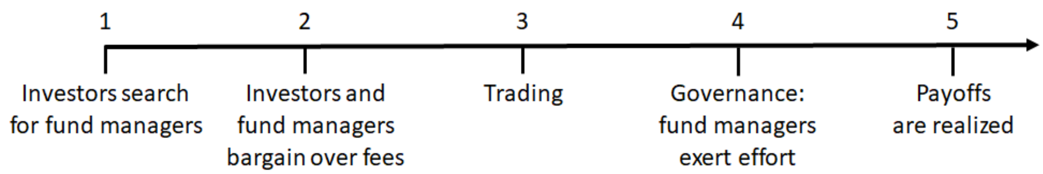

Figure 1. Timeline of the model.

We now describe the three types of agents and each of these stages in more detail.

Fund managers and fund investors

There are two types of risk-neutral fund managers, active and passive (index). The number of active managers is N_A; the number of passive managers is N_P. For now, we focus on the case of $N_A = N_P = 1$. While an active fund manager optimally chooses his investment portfolio, a passive fund manager is restricted to hold a value-weighted index of stocks. Assets in financial markets can be accessed by fund investors only through fund managers. Each fund manager offers to invest the capital of fund investors in exchange for an asset management fee. To focus on the effects of contractual arrangements that are observed in the mutual fund industry, we ignore the issues of optimal contracting and, following Pastor and Stambaugh (2012), assume that the fee charged to fund investors is a fraction of the fund's realized value of AUM at date 5. In particular, let f_A and f_P denote the fees as the percent of AUM charged by the active and passive fund manager, respectively. These fees are determined by bargaining between investors and fund managers, as described below. Then, if the realized value of fund manager i's portfolio at date 5 is $\tilde{Y}_i$, he keeps $f_i\tilde{Y}_i$ to himself and distributes $(1 - f_i)\,\tilde{Y}_i$ among fund investors in proportion to their original investments to the fund.

There is a mass of risk-neutral investors with capital, who have combined capital (wealth) W. Each investor has an infinitesimal amount of capital. At $t = 1$, each investor decides whether to invest his capital in the financial market by delegating his capital to one of the fund managers, or whether to invest outside the financial market (private savings). We normalize the return of the outside asset to zero. It can be interpreted as immediate consumption, saving at a bank deposit, or simply keeping the funds under the mattress.

If the investor decides to invest his capital with a fund manager, he needs to incur a search cost. Specifically, to find a passive (active) fund manager, an investor with wealth ε needs to incur a cost $\psi_P \varepsilon$ ($\psi_A \varepsilon$).[6] These costs can be interpreted as the costs of searching for relevant information, such as the fund's portfolio characteristics, investment process, and fee structure, and spending the time to understand it (see, e.g., Appendix B in Garleanu and Pedersen (2018) for a detailed description of investors' search process and the associated costs). For index fund investors, the key component of these costs would be finding out the fund's fee structure; these costs are likely to be especially large for less financially sophisticated investors. Consistent with this, Hortaçsu and Syverson (2004) conclude that investors' search frictions contribute to explaining the sizable dispersion in fund fees across different S&P 500 index funds despite their financial homogeneity, and Choi, Laibson, and Madrian (2010) show, in an experimental setting, that search costs for fees play an important role in subjects' decisions to invest across similar S&P 500 index funds. Some sources of growth in index funds over time have been the move of 401(k) plans into passive funds, as well as improved information about them: increased investor awareness about what index funds do and how their after-fee returns compare to those of active funds; the increased ability to find fund information on the internet; improved disclosures; and the increased availability of financial advisors – these trends could be interpreted as a decrease in ψ_P.

We assume that $\psi_A \geq \psi_P$: intuitively, it takes more time and effort to understand the investment strategy and fee structure of an active fund, compared to an index fund. Since active fund managers in our model exploit trading opportunities and thus outperform passive fund managers, who simply invest in the market portfolio, fund investors face a trade-off between earning a higher rate of return on their portfolio but at a higher search cost vs. a lower rate of return at a lower cost. In a richer model with heterogeneity of skill among active fund managers, ψ_A could be interpreted as the cost of searching for skill.

If an investor incurs the search cost $\psi_i \varepsilon$, he finds fund manager of type $i \in \{A, P\}$, and they negotiate the asset management fee $\tilde{f}_i$ through Nash bargaining, as in Garleanu and Pedersen (2018, 2020). We assume that fund managers have bargaining power η, and fund investors have bargaining power $1 - \eta$. Modeling the fee setting through bargaining leads to a very tractable setup, which allows us to derive the equilibrium in closed form. This assumption is natural if we think of fund investors as institutional investors, but may be less

[6] The assumption that search costs are proportional to wealth ε is just a normalization, which substantially simplifies the exposition.

natural in the context of individual investors. However, the qualitative effects that arise in our model would also arise in other models of imperfect competition among fund managers. The feature that is needed for our effects is that easier access to passive funds, by improving fund investors' outside options, lowers the fees and AUM of the active fund, and the extent of this effect depends on how much the active fund competes with the passive fund vs. with investors' private savings.

Let W_A and W_P denote the assets under management of the active and passive fund after the investors make their capital allocation decisions.

Assets and trading

There is a continuum of measure one of firms (stocks), indexed by $j \in [0,1]$. Each stock is in unit supply. The date-5 payoff of stock j is

$$R_j = R_0 + \sum_{i=1}^{M_j} e_{ij}, \tag{1}$$

where R_0 is publicly known, M_j is the number of shareholders of firm j, and e_{ij} is the amount of "effort" exerted by shareholder i in firm j at date 4, as described below.

The initial owners of each firm are assumed to have low enough valuations to be willing to sell their shares regardless of the price (for example, we can think of these initial owners as venture capitalists, who would like to exit the firm, and normalize their valuations to zero), so that the supply of shares in the market is always one. In addition to the initial owners, there are three types of traders who initially do not hold any stocks: active fund managers, passive fund managers, and competitive liquidity investors.

The trading model is broadly based on Admati, Pfleiderer, and Zechner (1994), augmented by passive fund managers: The active fund is strategic in that it takes into account the impact of its trading on the price, the passive fund buys the index portfolio, and the price is set to clear the market (i.e., a Walrasian trading mechanism). It can be microfounded by the following game: first, the active and passive fund each submits a market order, then competitive liquidity investors submit their demand schedules as a function of the price, and the equilibrium price is the one that clears the market. Short sales are not allowed.

More specifically, for each stock, there is a large mass of competitive risk-neutral liquidity investors (or noise traders), who can each submit any demand of up to one unit. Liquidity investors value an asset at its common valuation, given by (1), perturbed by an additional private value component. In particular, liquidity investors' valuation of stock j is $R_j - Z_j$, where Z_j captures the amount of liquidity demand driven by hedging needs or investor sentiment: Stocks with large Z_j have relatively low demand from liquidity investors, while stocks with small Z_j have relatively high demand. We assume that Z_j are i.i.d. (across stocks) draws from a binary distribution: $\Pr(Z_j = Z_L) = \Pr(Z_j = Z_H) = \frac{1}{2}$, where $Z_L > Z_H$. We will refer to these two types of stocks as L-stocks and H-stocks, i.e., stocks with low and high liquidity demand, respectively. The realizations of Z_j are publicly observed for all j. We assume that $\frac{Z_L+Z_H}{2} > 0$, i.e., the liquidity investors' private valuations of the market portfolio are negative, which automatically also implies $Z_L > 0$. In other words, the market portfolio and, even more so, the L-stocks, are undervalued by liquidity investors, which enables fund managers to realize gains from trade by buying these stocks. The role of different realizations of Z_j for different stocks ($Z_L > Z_H$) is to create potential gains from active portfolio management.

When trading: (1) liquidity investors have rational expectations in their assessment of asset payoffs and trade anticipating the equilibrium level of effort exerted by fund managers; (2) fund managers of active funds are not price takers: they are strategic in that they take into account the price impact of their trades; and (3) fund managers of passive funds follow the mechanical rule of investing all assets under management in a value-weighted portfolio of all stocks. We denote x_{ij} the number of shares held by investor i in firm j.

Governance stage

After establishing a position in firm j, each fund manager decides on the amount of effort to exert in the firm. If he exerts effort e and is of type $i \in \{A, P\}$, he bears a private cost of effort $c_i(e)$. This cost is not shared with fund investors, capturing what happens in practice (although the equilibrium fees charged to fund investors will be indirectly affected by these costs). We impose the standard assumptions that $c_i(0) = 0$, $c_i'(e) > 0$, $c_i''(e) > 0$, $c_i'(0) = 0$, and $c_i'(\infty) = \infty$, which guarantee an interior solution to fund managers' decisions on governance.

As discussed in the introduction, we think of the fund's effort as any action that a shareholder can take to increase value: informed voting, monitoring, engagement with management, submission of shareholder proposals, as well as more confrontational activism tactics. We refer to these actions broadly as engagement in governance or monitoring. We allow for different cost functions for active and passive funds: for example, active funds' trading in the firm's stock could give them access to firm-specific information, which could be helpful for their engagement efforts and reduce their costs of monitoring. Alternatively, a fund's cost of monitoring may depend on whether it has incentives to side with management to preserve their business ties, such as managing the firm's 401(k) plans (Davis and Kim, 2007; Cvijanovic, Dasgupta, and Zachariadis, 2016; Bebchuk and Hirst, 2019b). More generally, passive funds tend to use different engagement strategies compared to active funds because they have a different comparative advantage. For example, as Kahan and Rock (2020) and Fisch et al. (2019) point out, while active funds are better positioned to identify firm-specific problems, passive funds have the advantage of setting and implementing broad, market-wide standards in areas such as governance, sustainability, and risk management.

3 Analysis

We solve the model by backward induction, starting with the fund managers' decisions about monitoring.

3.1 Governance stage

If fund manager $i \in \{A, P\}$ with fee f_i and x_{ij} shares in firm j exerts effort e_{ij}, his payoff, up to a constant that does not depend on e_{ij}, is $f_i x_{ij} e_{ij} - c_i(e_{ij})$. The first-order condition implies that the fund manager's optimal effort level satisfies

$$e_{ij} = c_i'^{-1}(f_i x_{ij}). \tag{2}$$

The fund manager exerts more effort if his fund owns a higher fraction of the firm (higher x_{ij}) or if he keeps a higher fraction of the payoff to himself rather than giving it out to fund investors (higher f_i). This implies that there are two layers of the free-rider problem. First, the fund manager does not get all the benefits from his engagement efforts because his fund does not own the entire firm. Second, even if the fund owned the entire firm, the fund manager would have to share the benefits of his engagement efforts with the fund's investors.

Note also that at this stage, fund investors benefit from the fund manager's engagement and would like him to exert as much effort as possible. As we discuss below, however, engagement does not benefit fund investors from the ex-ante perspective.

3.2 Trading stage

During the trading stage, all players rationally anticipate that the effort decisions will be made according to (2).

Liquidity investors. Each liquidity investor has rational expectations about the effort that the active and passive fund managers will undertake. Specifically, if he expects the active fund to hold x_{Aj} shares and the passive fund to hold x_{Pj} shares of stock j, then his assessment of the payoff (1) of the stock is

$$R_j\left(x_{Aj}, x_{Pj}\right) = R_0 + c_A^{\prime -1}\left(f_A x_{Aj}\right) + c_P^{\prime -1}\left(f_P x_{Pj}\right). \tag{3}$$

Thus, each liquidity investor finds it optimal to buy stock j if and only if $R_j\left(x_{Aj}, x_{Pj}\right) - Z_j \geq P_j$, i.e., his valuation of this stock exceeds its price. We focus on the parameter range such that liquidity investors are the marginal traders in each type of stock, L and H. This holds when the combined AUM of active and passive funds, $W_A + W_P$, are not too high, so that their combined demand for the stock is lower than its supply (a sufficient condition for this to hold is specified in Proposition 1 below). Thus, the price of stock j is given by:

$$P_j = R_j - Z_j. \tag{4}$$

Equation (4) has intuitive properties. First, the price is decreasing in Z_j: all else equal, the price is lower if demand from liquidity investors is lower, for example, if there is lower hedging demand or lower investor sentiment (i.e., higher Z_j). Second, the price is higher if $R_j = R_j\left(x_{Aj}, x_{Pj}\right)$ is higher, i.e., if either the active fund or the passive fund holds more shares. This is because higher ownership by a fund manager implies higher expected engagement and firm payoff given (2), and consequently, a higher valuation of the stock by liquidity investors, leading to a higher price. We assume that $R_0 > Z_L$, which ensures that the price of each stock is always positive.

The fact that market participants incorporate the expected governance improvements into the price implies that the fund cannot make profits on its engagement. This is similar to the corresponding results of Admati, Pfleiderer, and Zechner (1994) and Grossman and Hart (1980), where the price incorporates the benefits of an activist's (raider's) future value improvement. As a result, as investor engagement increases, the return $\frac{R_j}{P_j}$ decreases and funds can realize lower gains from trade. This, as we show below, can lead to a trade-off between governance and fund investor well-being.

Passive fund manager. The passive fund manager is restricted to investing his assets under management W_P into the value-weighted portfolio of stocks. Denote this market portfolio by index M, and note that its price, i.e., the total market capitalization, is $P_M \equiv \int_0^1 P_j dj = \frac{P_L+P_H}{2}$. The passive fund manager would like to buy x_{Pj} units of stock j such that the proportion of his AUM invested in this stock, $\frac{x_{Pj}P_j}{W_P}$, equals the weight of this stock in the market portfolio, i.e., $\frac{P_j}{P_M}$. It follows that x_{Pj} is the same for all stocks and equals

$$x_P = \frac{W_P}{P_M}. \tag{5}$$

Note that the passive fund manager's demand for each stock does not depend on the stock's individual price and only depends on the price of the market portfolio.

Active fund manager. The active fund manager strategically chooses which assets to invest in, choosing between stocks of type L, stocks of type H, and the outside asset with return zero. We focus on the case when the active fund manager finds it optimal to only buy L-stocks, but not H-stocks or the outside asset, and to diversify across all L-stocks (a sufficient condition for this to hold is specified in Proposition 1). Intuitively, stocks with higher liquidity demand are "overpriced" relative to stocks with lower liquidity demand, and the active fund manager only finds it optimal to buy the relatively cheaper stocks. Since the total AUM of the active fund manager are W_A and are allocated evenly among mass $\frac{1}{2}$ of L-stocks, the fund manager's investment in each L-stock is

$$x_{AL} = \frac{2W_A}{P_L}. \tag{6}$$

Summary of the equilibrium at the trading and governance stage. Combining the above arguments, we can characterize the equilibrium in the financial market and the payoffs of all stocks as functions of funds' assets under management W_A and W_P and the fees f_A and f_P that are determined at stages 1 and 2. Denote the aggregate liquidity demand for the market portfolio by $Z_M \equiv \frac{Z_L+Z_H}{2}$. Since active fund managers only invest in L-stocks, which constitute half of all stocks, the equilibrium prices and payoffs of L-stocks and of the market portfolio are given by the following equations:

$$P_L = R_L - Z_L, \tag{7}$$

$$P_M = R_M - Z_M, \tag{8}$$

$$R_L = R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P), \tag{9}$$

$$R_M = R_0 + \frac{1}{2} c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P), \tag{10}$$

where x_P and x_{AL} are given by (5) and (6), respectively. Note that there is a one-to-one mapping between W_A and x_{AL}, and between W_P and x_P. Therefore, we can treat x_{AL} and x_P as state variables at date 3, which will simplify the exposition.

We next consider fund investors' capital allocation decisions and the determination of fees.

3.3 Capital allocation by investors

Infinitesimal investors decide whether to invest their capital into an outside asset and get a return of zero, or whether to search for an active or passive fund manager and invest with them. Our baseline analysis focuses on the case where the equilibrium AUM of each fund are positive (a sufficient condition for this to hold is specified in Proposition 1).[7] Then, there are two possible cases, depending on, as we show below, the cost ψ_P, which captures how easy it is to invest with the passive fund. First, if ψ_P is sufficiently large, then in equilibrium, investors earn a low rate of return and are indifferent between all the three options: investing in the outside asset (private savings), investing with the active fund, and investing with the passive fund. Second, if ψ_P is small, then investors are indifferent between investing with the active fund and the passive fund, and both options dominate investing in the outside asset, i.e., they earn a sufficiently high rate of return. Consider each of these cases separately.

3.3.1 Case 1: Low investor returns

Suppose first that private savings occur in equilibrium, i.e., investors earn a low rate of return from investing in the financial market through the funds.

[7] Lemma 1 in the appendix analyzes equilibria where only one of the funds raises positive AUM, and we examine these equilibria in some of the implications.

Negotiations over fees. We start by finding the active fund manager's fees. Consider an investor with wealth ε, and suppose this investor has already incurred the cost to find an active fund manager. To determine the Nash bargaining solution, we find each party's payoff upon agreeing and upon negotiations failing.

First, consider the fund investor. The investor's payoff from agreeing on fee $\tilde{f}_A$ is

$$(1-\tilde{f}_A)\frac{\varepsilon}{P_L}\left(R_0+c_A'^{-1}\left(f_A x_{AL}\right)+c_P'^{-1}\left(f_P x_P\right)\right). \tag{11}$$

This is because the fund manager will invest all the investor's wealth into L-stocks, which have price P_L, and the payoff of each of these stocks is given by (9). The investor's payoff if negotiations fail is ε because the net return of private savings is zero. The investor also has an option to search for the passive fund manager, but given the assumption that private savings occur in equilibrium, the investors are indifferent between all three options, so it is sufficient to consider her private savings as the outside option.

Consider the active fund manager. Note that by the envelope theorem, the effect of a marginal additional investment on the fund manager's utility via a change in effort is second-order.[8] Hence, the fund manager's additional utility from agreeing on fee $\tilde{f}_A$ and getting additional assets under management ε is $\tilde{f}_A R_L \frac{\varepsilon}{P_L}$, where R_L is given by (9). Given the fund manager's bargaining power η, fee $\tilde{f}_A$ is determined via the Nash bargaining solution:

$$\max_{\tilde{f}_A}\left((1-\tilde{f}_A)R_L\frac{\varepsilon}{P_L}-\varepsilon\right)^{1-\eta}\left(\tilde{f}_A R_L\frac{\varepsilon}{P_L}\right)^{\eta}. \tag{12}$$

Since the total surplus created from bargaining is $R_L\frac{\varepsilon}{P_L}-\varepsilon$, the fee must be such that the fund manager gets fraction η of this surplus:

$$\tilde{f}_A R_L\frac{\varepsilon}{P_L}=\eta\left(R_L\frac{\varepsilon}{P_L}-\varepsilon\right), \tag{13}$$

or $\tilde{f}_A=\eta\left(1-\frac{P_L}{R_L}\right)$. This implies that the active management fees for all investors are the

[8]To see this, note that the active manager's payoff is $\frac{1}{2}[f_A x_{AL}\left(R_0+e+c_P'^{-1}\left(f_P x_P\right)\right)+\tilde{f}_A\left(R_0+e+c_P'^{-1}\left(f_P x_P\right)\right)\frac{2\varepsilon}{P_L}-c_A(e)]$, and by the envelope theorem, the derivative with respect to ε is $\tilde{f}_A\left(R_0+e+c_P'^{-1}\left(f_P x_P\right)\right)\frac{1}{P_L}$, where $e=c_A'^{-1}\left(f_A x_{AL}\right)$.

same, $\tilde{f}_A = f_A$, and satisfy the following fixed point equation:

$$f_A = \eta \left(1 - \frac{P_L}{R_L}\right). \tag{14}$$

Second, consider the passive fund manager. By exactly the same arguments, the Nash bargaining solution $\tilde{f}_P$ satisfies:

$$\tilde{f}_P R_M \frac{\varepsilon}{P_M} = \eta \left(R_M \frac{\varepsilon}{P_M} - \varepsilon\right), \tag{15}$$

or $\tilde{f}_P = \eta \left(1 - \frac{P_M}{R_M}\right)$. This implies that the passive management fees for all investors are the same, $\tilde{f}_P = f_P$, and satisfy the following fixed point equation:

$$f_P = \eta \left(1 - \frac{P_M}{R_M}\right). \tag{16}$$

Asset allocation. Finally, we need to determine the assets under management. Capital flows into funds until, in equilibrium, investors are indifferent between investing with the active fund, investing with the passive fund, and investing in the outside asset. This gives

$$(1 - f_A)\, R_L \frac{\varepsilon}{P_L} - \psi_A \varepsilon = (1 - f_P)\, R_M \frac{\varepsilon}{P_M} - \psi_P \varepsilon = \varepsilon. \tag{17}$$

Dividing by ε, we get the following conditions for investor indifference

$$1 + \psi_A = (1 - f_A) \frac{R_L}{P_L}, \tag{18}$$

$$1 + \psi_P = (1 - f_P) \frac{R_M}{P_M}. \tag{19}$$

Combining these arguments, the equilibrium $(f_A, f_P, x_{AL}, x_P, P_L, P_M, R_L, R_M)$ is given by the solution to the following system of equations: market clearing and optimal monitoring decisions (7)-(10); fee negotiation conditions (14) and (16); and investor capital allocation conditions (18) and (19). We characterize this equilibrium in Proposition 1 below.

3.3.2 Case 2: High investor returns

Next, suppose that investors earn a high rate of return from investing in the financial market and thus private savings do not occur in equilibrium. The solution follows the same steps as those in Section 3.3.1, but with two differences. First, the investor indifference conditions at the capital allocation stage, (17)-(19), are replaced by: (a) the indifference condition between investing with active and passive funds,

$$(1 - f_A) \frac{R_L}{P_L} - \psi_A = (1 - f_P) \frac{R_M}{P_M} - \psi_P, \tag{20}$$

and (b) the condition that the combined AUM of the funds are equal to W:

$$W_A + W_P = W. \tag{21}$$

The second difference is that during bargaining, the fund investor's outside option is now to invest with the other fund manager, which is no longer equivalent to using private savings. The fund managers' outside options remain unchanged. First, consider negotiations with the active fund manager. Since the investor's outside option is to search for the passive fund manager and get $(1 - f_P) R_M \frac{\varepsilon}{P_M} - \psi_P \varepsilon$, the total surplus created from bargaining is now $R_L \frac{\varepsilon}{P_L} - (1 - f_P) R_M \frac{\varepsilon}{P_M} + \psi_P \varepsilon$. Hence, the fee must be such that the fund manager gets fraction η of this surplus:

$$\tilde{f}_A R_L \frac{\varepsilon}{P_L} = \eta \left(R_L \frac{\varepsilon}{P_L} - (1 - f_P) R_M \frac{\varepsilon}{P_M} + \psi_P \varepsilon \right), \tag{22}$$

which yields $\tilde{f}_A = f_A$ that satisfies the following fixed point equation:

$$f_A = \frac{P_L}{R_L} \eta \left(\frac{R_L}{P_L} - (1 - f_P) \frac{R_M}{P_M} + \psi_P \right). \tag{23}$$

Similarly, in negotiations with the passive fund manager, the investor's outside option is to search for the active fund manager and get $(1 - f_A) R_L \frac{\varepsilon}{P_L} - \psi_A \varepsilon$. Therefore, fee $\tilde{f}_P$ is determined from:

$$\tilde{f}_P R_M \frac{\varepsilon}{P_M} = \eta \left(R_M \frac{\varepsilon}{P_M} - (1 - f_A) R_L \frac{\varepsilon}{P_L} + \psi_A \varepsilon \right), \tag{24}$$

which yields $\tilde{f}_P = f_P$ that satisfies the following fixed point equation:

$$f_P = \frac{P_M}{R_M} \eta \left(\frac{R_M}{P_M} - (1 - f_P) \frac{R_L}{P_L} + \psi_A \right). \tag{25}$$

Combining these arguments, the equilibrium $(f_A, f_P, x_{AL}, x_P, P_L, P_M, R_L, R_M)$ is given by the solution to the following system of equations: market clearing and optimal monitoring decisions (7)-(10); fee negotiation conditions (23) and (25); and investor capital allocation conditions (20) and (21). We characterize this equilibrium in Proposition 1 below.

3.4 Equilibrium

We derive the equilibrium in each of the above cases by combining the market clearing and optimal monitoring conditions, fee negotiation conditions, and investor capital allocation conditions derived above. From this point on, we assume that fund managers' costs of effort are quadratic, i.e.,

$$c_i(e) = \frac{c_i}{2} e^2.$$

While the assumption of quadratic costs is not necessary to characterize the equilibrium and is not important for many equilibrium properties discussed after Proposition 1 and in Section 4,[9] assuming quadratic costs allows us to formulate in closed form the sufficient conditions for the existence of this equilibrium and simplifies the exposition. In particular, funds' equilibrium effort levels are then given by $e_P = \frac{f_P x_P}{c_P}$ and $e_{AL} = \frac{f_A x_{AL}}{c_A}$.

Denote by

$$\lambda \equiv (1 - f_A) \frac{R_L}{P_L} - \psi_A \tag{26}$$

the equilibrium gross rate of return that fund investors earn on their investment. In Case 1 above, $\lambda = 1$ since investors are indifferent between investing in the outside asset (that earns a gross return of one) and investing with the fund managers, while in Case 2, $\lambda > 1$.

Proposition 1 (equilibrium). *Suppose* $c_P \geq \frac{\psi_P}{\psi_A} c_A$, $r_1 < \frac{Z_M}{Z_L} < r_2$ *and* $W_1 < W < W_2$, *where* r_i, W_i *are given by (38)-(39) in the appendix. Then the equilibrium is as follows.*

(*i*) *The asset management fees are* $f_A = \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}$ *and* $f_P = \frac{\eta\psi_P}{\psi_P + \lambda(1-\eta)}$, *and* $f_A \geq f_P$.

(*ii*) *The payoffs of the L-stocks and the market portfolio are* $R_L = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}) Z_L$ *and* $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}) Z_M$.

[9] For example, for general costs of effort, the equilibrium characterized by Proposition 1 takes exactly the same form, except that equation (27) becomes $W = \frac{P_L}{2 f_A} c'_A \left(2\left(R_L - R_M\right)\right) + \frac{P_M}{f_P} c'_P \left(2R_M - R_L - R_0\right)$. The proof of Proposition 1 in the appendix is presented for this more general case.

(*iii*) *The prices of the L-stocks and the market portfolio are* $P_L = \frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)} Z_L$ *and* $P_M = \frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)} Z_M$.

(*iv*) *There exists* $\bar{W}$ *such that if* $W \geq \bar{W}$, *the investors' gross rate of return satisfies* $\lambda = 1$, *whereas if* $W < \bar{W}$, λ *strictly decreases in* W *and satisfies the fixed point equation*

$$W = \frac{c_A}{f_A} \left(R_L - R_M\right) P_L + \frac{c_P}{f_P} \left(2R_M - R_L - R_0\right) P_M. \tag{27}$$

The restrictions on parameters in the statement of the proposition ensure that we consider the interesting case, i.e., one in which both the active and the passive fund raise positive AUM, liquidity investors are marginal in both types of stocks, and the active fund finds it optimal to invest in L-stocks, and not in H-stocks or the outside asset. As a result, the active fund holds a less diversified portfolio than the passive fund, which is consistent with the observed evidence. For the remainder of the paper, we assume that these assumptions hold, with a few exceptions that we explicitly point out.

The assumption $c_P \geq \frac{\psi_P}{\psi_A} c_A$ is intuitive: if passive and active funds have relatively similar monitoring technologies ($c_P \approx c_A$), it automatically follows from the assumption that active funds are harder to search for, $\psi_A \geq \psi_P$. Moreover, Lund (2018) notes that "governance interventions are especially costly for passive funds, which do not generate firm-specific information as a byproduct of investing," and Bebchuk and Hirst (2019b) point out that "index fund managers ... have a web of financially significant business ties with corporate managers." Both effects could potentially make passive funds' costs of monitoring higher, i.e., increase c_P relative to c_A.

The properties of the equilibrium are as follows. If aggregate investor wealth is large, investors' outside options in negotiations are limited, which makes the fees charged by asset managers relatively high and investors' rate of return equal to the rate of investing in the outside asset, $\lambda = 1$. If, in contrast, aggregate investor wealth is limited, asset managers compete for investor funds and have to offer relative low asset management fees, allowing investors to earn a rate of return $\lambda > 1$.

Comparing the active and the passive fund, we note that the active fund outperforms the passive fund before fees. Indeed, the active fund earns a return of $\frac{R_L}{P_L} = \frac{\psi_A}{1-\eta} + \lambda$ on its investments, which is greater than $\frac{\psi_P}{1-\eta} + \lambda = \frac{R_M}{P_M}$, the return of the passive fund. Accordingly, and consistent with practice, the fee charged by the active fund is higher than the fee charged by the passive fund: $f_A = \frac{\eta\psi_A}{\psi_A+\lambda(1-\eta)} \geq \frac{\eta\psi_P}{\psi_P+\lambda(1-\eta)} = f_P$.

Because we are interested in the role of passive funds for corporate governance, it is useful to understand how the search cost ψ_P affects the equilibrium. As discussed in Section 2, the easier access to passive funds over time can be interpreted as a decrease in ψ_P, coming from increased investor awareness about index funds, their growing inclusion in 401(k) plans, and improved disclosures about their fee structures.

Proposition 2. *As access to passive funds becomes easier (ψ_P decreases): (1) funds' fees, f_A and f_P, decrease; (2) funds' AUM, $W_A + W_P$, increase; and (3) fund investors' rate of return, λ, increases. In particular, there exists a cutoff $\bar{\psi}_P$ such that $\lambda = 1$ for $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ for $\psi_P < \bar{\psi}_P$.*

Intuitively, easier access to passive funds is beneficial for fund investors: it decreases both active and passive fund fees and increases investors' returns on their investment. As a result, investors allocate more funds from private savings to fund managers, so funds' combined AUM grow (all the monotonicity statements in the proposition apply in a weak sense). The cutoff $\bar{\psi}_P$ separates the region $\psi_P > \bar{\psi}_P$, where investors are indifferent between investing through the funds and saving privately, and the region $\psi_P < \bar{\psi}_P$, where they strictly prefer to invest through the funds ($\lambda > 1$) and allocate all their wealth between active and passive fund managers.

Proposition 2 is broadly consistent with the observed empirical evidence if we think of the recent trends in the asset management industry as stemming from easier access to passive funds over time, i.e., a decrease in ψ_P. The assets held by passive funds have increased substantially over the last decades, both in absolute value and as a fraction of all fund assets. For example, the total AUM of passive funds have grown from less than \$1 trillion in the early 2000s to more than \$5 trillion in recent years. These trends have been accompanied by a decrease in both active and passive funds' expense ratios (captured by f_A and f_P in the model), from around 1% (0.23%) for active (passive) funds in the 2000s, to less than 0.7% (0.15%) in recent years.[10]

[10] These stylized facts are based on the data on funds' AUM and expense ratios from the CRSP Mutual Fund database. We thank Davidson Heath, Daniele Macciocchi, Roni Michaely, and Matthew Ringgenberg for generously sharing these data with us.

4 Implications for governance

In this section, we examine the properties of the equilibrium and derive the implications of delegated asset management for corporate governance, investor returns, and total welfare.

4.1 The governance role of passive funds

It is often argued that passive fund growth is detrimental to governance due to lower fees that passive fund managers charge and, thereby, their lower incentives to stay engaged. This argument implicitly assumes that as passive funds grow, fund fees decrease, while other factors that affect fund managers' monitoring efforts do not change. However, in reality, fees do not change exogenously and in isolation: changes in fees are likely to be accompanied by other changes, such as changes in funds' AUM, changes in funds' ownership stakes, and the substitution between delegated asset management and private savings. In this section, we use our model to analyze the governance role of passive funds while formally accounting for a combination of all these effects. Among other things, we show that passive fund growth can be beneficial for governance even if it results in lower fund fees.

As in Proposition 2, to study the implications of passive fund growth, we consider the comparative statics of parameter ψ_P. To understand its effect on aggregate governance, we examine the payoff of the market portfolio R_M, since R_M reflects the level of investor monitoring in an average firm.

Proposition 3. *Easier access to passive funds (lower ψ_P) improves aggregate governance R_M if $\psi_P > \bar{\psi}_P$. If, in addition, $c_P \geq c_A$ and $e_{AL} < \frac{Z_L - Z_H}{2}$, then lower ψ_P hurts governance if $\psi_P \leq \bar{\psi}_P$.*

The intuition is the following. As ψ_P declines, both active and passive fund fees weakly decrease (see Proposition 2), which, other things equal, decreases funds' incentives to engage. However, in addition, investing with the passive fund becomes increasingly attractive to investors. Hence, capital flows to the passive fund, allowing it to take increasingly large stakes in its portfolio companies and increasing its incentives to engage. Whether these higher passive fund stakes are beneficial for governance and outweigh the effect of lower fees depends on whether the passive fund primarily replaces liquidity investors or the active fund in firms' ownership structures. Liquidity investors do not engage in governance (we can think of them as retail shareholders, who have neither ability nor incentives to monitor), so replacing liquidity investors increases the overall level of investor monitoring. In contrast, active funds have higher incentives to monitor than passive funds given their higher fees, $f_A \geq f_P$. If, in addition, active funds have a higher ability to monitor (as captured by the assumption $c_P \geq c_A$ in Proposition 3), then replacing active funds in firms' ownership structures is detrimental to governance.

The cutoff $\bar{\psi}_P$ separates the region where the passive fund primarily replaces liquidity investors and active fund fees do not decrease too much ($\psi_P > \bar{\psi}_P$), from the region where it primarily replaces the active fund and active fund fees decrease substantially ($\psi_P < \bar{\psi}_P$). In particular, recall from Proposition 2 that when $\psi_P > \bar{\psi}_P$, investors are indifferent between saving privately and investing in the stock market through the funds. Thus, passive fund growth primarily crowds out investors' private savings and is not accompanied by a decrease in active funds' fees. Hence, active funds continue to engage, and the dominant effect of lower ψ_P is that the passive fund replaces liquidity investors in firms' ownership structures, which improves governance (the discussion after Corollary 2 below provides a more detailed intuition for this case).

In contrast, when $\psi_P < \bar{\psi}_P$, all investor wealth is invested with fund managers, so passive fund growth comes entirely at the expense of investors' allocations to the active fund. As a result, the passive fund primarily replaces the active fund in firms' ownership structures, which harms governance because the passive fund monitors less given the same ownership stake. In addition, both active and passive fund fees decrease substantially, reducing funds' incentives to monitor on the stakes they continue to own.[11]

Figure 2 presents a numerical example illustrating this result.[12] When $\psi_P > \bar{\psi}_P$, investors are indifferent between investing with the funds and in the outside asset, so their rate of return is $\lambda = 1$ (see panel a). In this region, passive fund growth does not change active

[11] There are two additional nuanced effects in this case, one negative and one positive. The negative effect is that since the passive fund invests in more expensive stocks than the active fund ($P_H > P_L$), the combined ownership by the active and passive fund declines, while liquidity investors' ownership increases, which further reduces overall investor monitoring. The positive effect is that the reduction in R_L means that the active fund can buy L-stocks at a lower price, and hence the ownership stakes of the active fund do not decrease as much. Condition $e_{AL} < \frac{Z_L - Z_H}{2}$ in Proposition 3 ensures that this positive effect is relatively minor. In the proof of Proposition 3, we show that there exists a cutoff $\underline{\psi}_P$ such that this condition is satisfied for $\psi_P < \underline{\psi}_P$.

[12] The parameters in this example satisfy the sufficient conditions for the equilibrium in Proposition 1, and we do not impose the additional condition $e_{AL} < \frac{Z_L - Z_H}{2}$. Thus, the result of Proposition 3 is even more general, i.e., the nuanced positive effect discussed in footnote 11 for the case $\psi_P < \bar{\psi}_P$ is relatively minor under more general conditions than those in Proposition 3.

fund fees (panel c), and its dominant governance effect is to replace liquidity investors (i.e., retail shareholders) in firms' ownership structures. Hence, as ψ_P decreases, overall investor monitoring increases, increasing the market payoff R_M (panel b). However, once ψ_P falls below $\bar{\psi}_P$, passive funds start primarily replacing active funds and active fund fees start declining, so a further decrease in ψ_P reduces the overall level of investor monitoring and R_M.

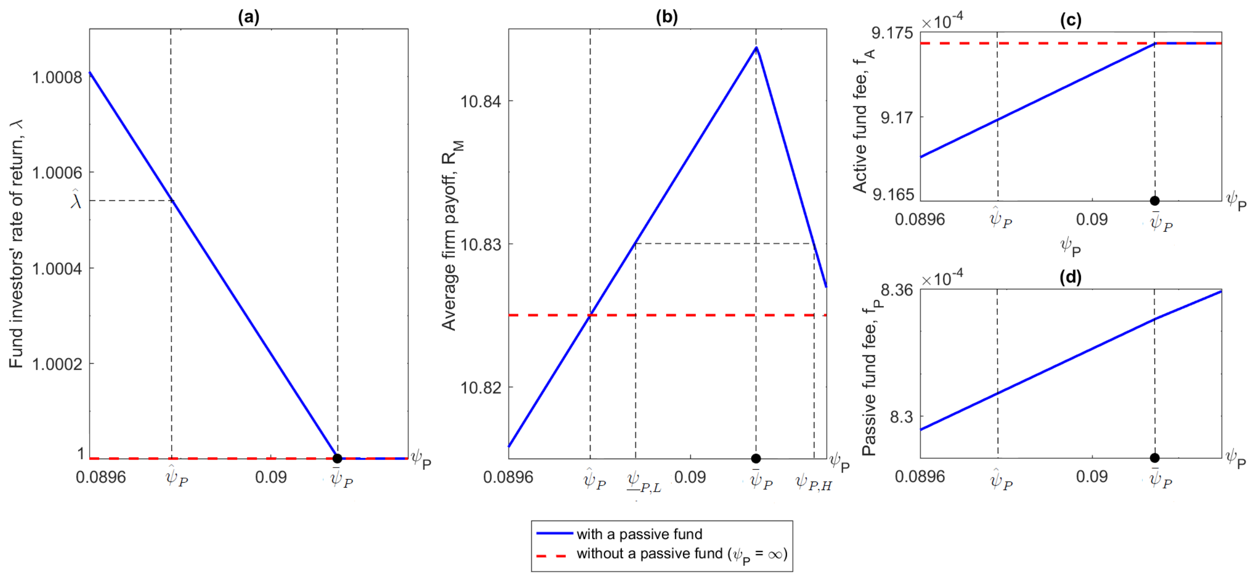

Figure 2. The figure plots fund investors' rate of return, average firm payoff, and fund fees as a function of search costs ψ_P. The blue solid line corresponds to the parameters $\eta = 0.01$, $c_A = 0.001$, $c_P = 0.002$, $\psi_A = 0.1$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$, and $W = 1.5$. The red dashed line uses the same parameters but for the benchmark case $\psi_P = \infty$, where the passive fund does not exist.

An interesting implication of Proposition 3 is that there can be a trade-off between fund investors' well-being and governance. To see this, consider any $\psi_{P,H} > \bar{\psi}_P$ (e.g., see panel b in Figure 2) and compare it with some $\psi_{P,L} < \bar{\psi}_P$. If $\psi_{P,L}$ is not too low, e.g., close to $\bar{\psi}_P$, then both λ (which captures investors' well-being) and the market payoff R_M are higher than for $\psi_{P,H}$, i.e., passive fund growth is beneficial both for investors and for governance. However, once $\psi_{P,L}$ falls below a certain level (e.g., below $\underline{\psi}_{P,L}$ in panel b), a further decrease in ψ_P increases λ even further, but now becomes detrimental to governance. The same dynamics is observed if we compare the baseline case (where both the active and passive fund are present) to a benchmark case with $\psi_P = \infty$, where there is no passive fund at all and investors allocate their wealth between the active fund and private savings. The red dashed line in Figure 2 corresponds to this benchmark case.[13] It shows that while the introduction of a passive fund always weakly increases λ and hence is beneficial for fund investors, it only improves governance if it does not decrease ψ_P too much (below $\hat{\psi}_P$ in Figure 2) and, accordingly, does not increase λ too much (above $\hat{\lambda}$ in panel a). The following corollary summarizes these observations:

Corollary 1. *Easier access to passive funds (lower ψ_P) improves aggregate governance if and only if it does not increase fund investors' returns too much.*

Intuitively, passive fund growth is especially beneficial for fund investors (i.e., increases λ substantially) when it results in strong competition between funds and significantly decreases fund fees. However, this competition implies that funds primarily replace each other, rather than liquidity investors, in firms' ownership structures. Moreover, a reduction in fees implies lower incentives to monitor: to have incentives to stay engaged, fund managers need to earn enough rents from managing investors' portfolios and not leave too much money to fund investors. Both of these effects create a trade-off between governance and fund investor well-being.

Importantly, this trade-off does not arise in the region $\psi_P > \bar{\psi}_P$, where aggregate governance strictly improves even though fund fees decline. Hence, the link between asset management fees and incentives to engage in governance is not immediate:

Corollary 2. *If $\psi_P > \bar{\psi}_P$, then easier access to passive funds (lower ψ_P) improves aggregate governance, even though it decreases fund fees.*

Intuitively, in this region, the funds primarily compete with investors' private savings and not with each other, which limits the negative effects of passive fund growth. In particular, the negative effect of lower fees is dominated by the positive effect of the passive fund's increased ownership stakes. This is because as ψ_P decreases, capital starts flowing to the passive fund, increasing its AUM and holdings x_P in its portfolio firms, so that in equilibrium, investors remain indifferent between investing with the passive fund and in the outside asset, i.e., $(1 - f_P)\,\frac{R_M}{R_M - Z_M} - \psi_P = 1$ (see condition (19)). Hence, the decrease in f_P and ψ_P must be accompanied by a decrease in $\frac{R_M}{R_M - Z_M}$, i.e., an increase in R_M. In other words, the positive

[13] Lemma 1 in the appendix presents sufficient conditions for such a "corner" equilibrium to exist and for investors' rate of return in this equilibrium to be $\lambda = 1$.

effect of higher passive fund's ownership stakes must outweigh the negative effect of lower fees, leading to a higher payoff R_M. This argument emphasizes that fee-related criticisms of passive funds need to take into account that lower fees are frequently accompanied by higher AUM and funds' ownership.

Another implication is that passive fund growth can have heterogeneous effects on the governance of different types of firms, depending on whether they are primarily held by retail shareholders or active fund managers. For example, the positive governance effect of passive fund growth in the case $\psi_P > \bar{\psi}_P$ comes entirely from improvements in H-firms. Because the active fund does not hold these relatively more expensive firms, the passive fund is only replacing liquidity investors in these firms' ownership structures, which increases shareholder engagement in these firms. In contrast, the value R_L of the cheaper L-firms remains unaffected. Intuitively, the passive fund replaces not only liquidity investors but also the active fund in these firms' ownership structures (x_P increases, while x_{AL} declines), and the combined effect is neutral.[14]

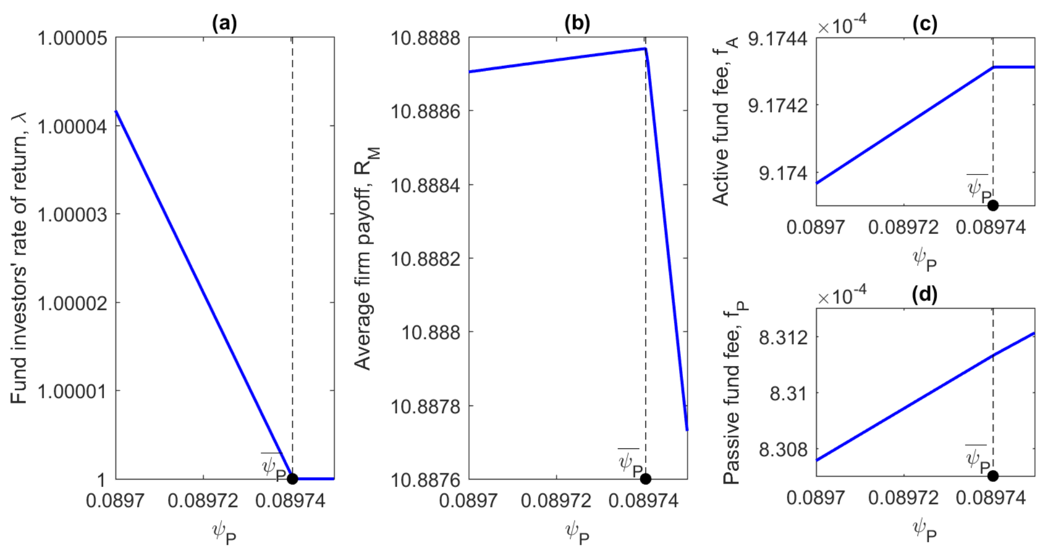

Figure 3. The figure plots fund investors' rate of return, average firm payoff, and fund fees as a function of search costs ψ_P. The parameters are the same as in Figure 2 ($\eta = 0.01$, $c_A = 0.001$, $\psi_A = 0.1$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$, and $W = 1.5$) except for c_P, which now equals $c_P = 0.0009 < c_A$.

Finally, it is useful to discuss the assumption $c_P \geq c_A$ in Proposition 3. In some cases,

[14] Formally, because investors are indifferent between investing with the active fund and the outside asset, the active fund's after-fee return $(1 - f_A)\frac{R_L}{R_L - Z_L}$ must remain the same (see (18)), which together with (14), implies that both the active fund fee f_A and the return $\frac{R_L}{R_L - Z_L}$ must remain unaffected.

passive funds could be more effective in their engagement efforts than active funds, i.e., $c_P < c_A$ (see, e.g., the arguments in Kahan and Rock, 2020). Then, passive funds replacing active funds in firms' ownership structures could have an ambiguous effect: passive funds would have lower incentives to engage due to lower fees ($f_P \leq f_A$), but a greater ability to do so. As a result, there could be an additional positive effect of passive fund growth. All the other effects would remain the same, and hence the trade-offs described above would arise in this setting as well. To see this, consider Figure 3, which uses the same parameters as Figure 2 except c_P, which is now lower than c_A. It shows that while the negative effects of passive fund growth are not as strongly pronounced in the region $\psi_P \leq \bar{\psi}_P$ as in Figure 2, the results remain qualitatively unchanged.

4.2 Who benefits from investments in governance?

It is frequently noted that asset managers may not have sufficient resources to engage in effective monitoring of their portfolio companies. For example, Bebchuk and Hirst (2019b) point out that for each of the Big Three passive fund families, the size of its stewardship team is between 12 and 45 people, even though it manages more than 11,000 portfolio firms, and that its stewardship budget is less than 0.2% of the fees it charges for managing equity assets. Based on this criticism, some observers propose regulations inducing asset managers, and especially passive funds, to invest more resources into their stewardship teams. In the context of our model, we can think of these regulations as reducing the ex-post costs of engaging in governance (c_A and c_P) at the expense of some unmodeled ex-ante cost. In this section, we study the effects of such proposals on governance, fund investors' and fund managers' payoffs, and total welfare. The next result shows that while they generally have a positive effect on governance and firm valuations, they can be detrimental to fund investors and fund managers themselves.

Proposition 4. *Suppose fund manager i's cost of monitoring c_i decreases. Then:*

(i) firms' payoffs and prices always weakly increase, and strictly increase if $\psi_P < \bar{\psi}_P$;

(ii) fund investors' rate of return always weakly decreases, and strictly decreases if $\psi_P < \bar{\psi}_P$;

(iii) fund manager i's payoff strictly decreases if $\psi_P \geq \bar{\psi}_P$.

This result emphasizes that policy proposals that decrease investors' costs of engagement – for example, by inducing funds to invest more resources into their stewardship teams – are not universally beneficial. While a decrease in c_i increases the fund's engagement and thus firms' payoffs (R_L and R_M), it can make fund investors and, potentially, fund managers worse off. Intuitively, because investors in financial markets have rational expectations about the effect of c_i on the fund's equilibrium effort and firms' payoffs, a decrease in c_i translates into higher prices. In particular, even though R_j increases as c_i decreases, the price $P_j = R_j + Z_j$ increases by the same amount, so the fund can only make money on gains from trade, Z_j, and neither fund investors nor fund managers can benefit from the fund's monitoring.[15] In fact, they can be made worse off: higher prices imply that funds can buy a lower number of shares and hence realize lower gains from trade. More precisely, as part (ii) of Proposition 4 shows, fund investors do not benefit from increased monitoring when $\psi_P \geq \bar{\psi}_P$ (when their rate of return is $\lambda = 1$) and are harmed by the fund's increased monitoring when $\psi_P < \bar{\psi}_P$. Thus, while initial owners of the firm (e.g., venture capitalists) are better off as they can now sell their shares for a higher price, the new owners of the firm, i.e., fund investors, are weakly worse off.

The fact that *all* fund investors are worse off when monitoring becomes cheaper is a property of our static model. In a richer dynamic model, lower costs of monitoring would be harmful for some fund investors but beneficial for others. Specifically, suppose that a given point in time, the fund already has some existing investors and has acquired ownership stakes using their capital. If, at this point, the fund's cost of monitoring unexpectedly declines, this benefits existing investors on the positions that the fund already holds (as discussed in Section 3.1, once trading has already taken place, fund investors always benefit from more monitoring). However, and for the same reason as in our setting, this decrease in c_i hurts all future investors of the fund, as well as its existing investors on any of their future contributions to the fund.

Whether decreasing the costs of monitoring is beneficial for the fund itself depends on the interaction of several forces. The positive effect is that for a given level of effort, the fund's costs of engagement decrease. However, there can also be a negative effect: given that greater monitoring decreases fund investors' return, the fund may experience outflows, leading to lower management fees. This is exactly what happens when $\psi_P \geq \bar{\psi}_P$ (and $\lambda = 1$):

[15] As discussed in Section 3.2, this inability to profit from ex-post monitoring is similar to Admati, Pfleiderer, and Zechner (1994) and the free-rider problem in Grossman and Hart (1980).

because fund investors can invest in the outside asset that earns a gross return of one, the fund ends up with lower AUM when c_i decreases, and as part (*iii*) of Proposition 4 shows, this effect dominates the decrease in the costs of effort.

In contrast, when $\psi_P < \bar{\psi}_P$, so that investors only choose between the active and passive fund (and earn a return higher than that of the outside asset, $\lambda > 1$), the fund manager may find it optimal to decrease its costs of monitoring. Moreover, the passive fund manager's incentives to decrease c_P are generally stronger than the active fund manager's incentives to decrease c_A. The reason is that the passive fund can actually experience inflows as a result of such a policy change. Intuitively, more monitoring by the funds increases stock prices and decreases funds' ability to realize gains from trade. Since the ability to realize gains from trade is relatively more important for the active fund, this hurts the active fund more than the passive fund, resulting in outflows from the active fund and inflows into the passive fund. Note that this effect arises due to the interaction between the active and passive fund and would not arise with a single fund.[16] Hence, while the active fund manager is often hurt when funds' costs of monitoring decrease, the passive fund manager can benefit from such a change.

Figure 4 illustrates this logic. It considers the same set of parameters as in Figure 2, but varies parameters c_A and c_P. Panels (a) and (b) show the trade-off between the positive effect of lower monitoring costs on firm valuations and its potential negative effect on fund investors (parts (*i*) and (*ii*) of Proposition 4). Panels (d) and (e) show the difference between active and passive funds: while the active fund would prefer to keep its costs of monitoring high, the passive fund benefits from decreasing its costs of monitoring.

4.2.1 Implications for total welfare

In this section, we examine the effects of the above regulations on the combined welfare of all the players. To analyze welfare, we interpret Z_i as liquidity investors' private valuations coming from motives such as hedging or liquidity needs, rather than investor sentiment. Whether decreasing funds' costs of monitoring is beneficial for total welfare depends on its combined effect on firms' initial owners, fund investors, fund managers, and liquidity investors. Since liquidity investors are marginal traders and Z_i are their private valuations,

[16] To show this formally, we analyze the setting in which ψ_A (ψ_P) is so large that only the passive (only the active) fund manager raises positive AUM, as in Lemma 1 in the appendix. In this equilibrium, as Lemma 9 in the online appendix demonstrates, both the active and the passive fund manager are always worse off if their cost of monitoring decreases, similar to result (*iii*) of Proposition 4.

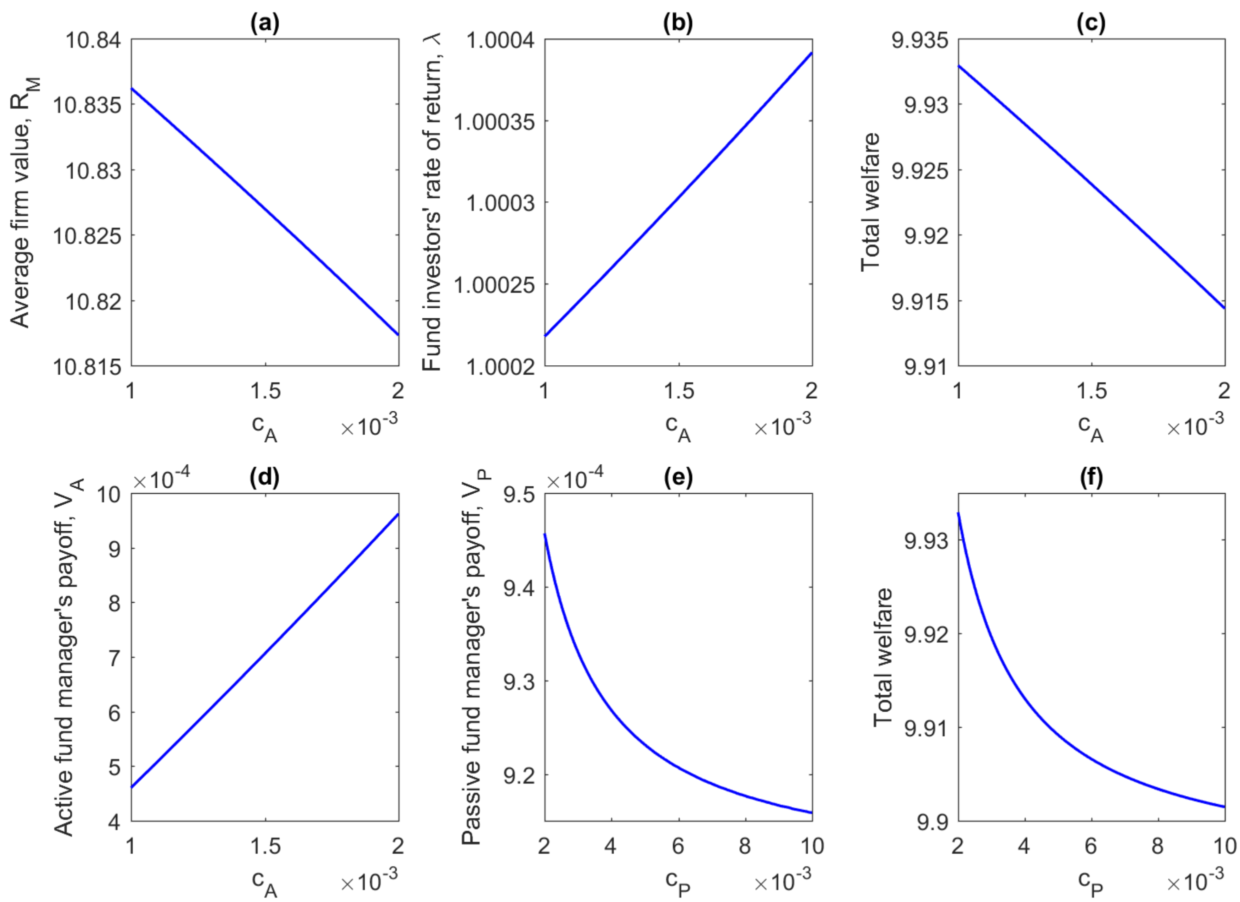

Figure 4. The figure plots average firm value, investors' rate of return, active and passive fund managers' payoffs, and total welfare as a function of fund managers' costs of monitoring c_A and c_P. The parameters are $c_A = 0.001$ (when c_P varies), $c_P = 0.002$ (when c_A varies), $\eta = 0.01$, $\psi_A = 0.1$, $\psi_P = 0.09$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$, $W = 2$.

their payoff is zero. Hence, the effect of such policies on total welfare depends on the trade-off between their positive effect on governance and initial owners' payoff on the one hand, and their potential negative effect on fund investors and fund managers on the other hand.

In the example above, total welfare increases when either of the fund's costs of monitoring decrease (panels (c) and (f) of Figure 4), i.e., regulations that induce funds to increase the size of their governance teams are welfare improving. Interestingly, however, both the active fund manager and fund investors would push against such welfare improving regulations because it would make them worse off.

However, as we point out next, such regulations are not always welfare improving. In particular, decreasing funds' costs of engagement beyond a certain threshold is always detrimental to total welfare:

Proposition 5 (welfare effects of decreasing the costs of monitoring). *Define $\bar{c}_i$ as the infimum of c_i for which $\lambda > 1$. If $c_i < \bar{c}_i$, then decreasing c_i harms total welfare.*[17]

The logic is the following. According to Proposition 4, as a fund's cost of engagement decreases, fund investors' rate of return decreases as well, until it reaches the point (at $c_i = \bar{c}_i$) where investors are indifferent between investing with the fund managers and saving privately, i.e., $\lambda = 1$. At this point, a further decrease in the fund's cost of engagement has no additional marginal benefit because, as follows from Proposition 1, the fund's monitoring levels and hence firm valuations stay constant in c_i when $\lambda = 1$. Therefore, the only welfare effect of further decreasing c_i is the decline in fund managers' profits (condition $\psi_P \geq \bar{\psi}_P$ in part (*iii*) of Proposition 4 corresponds to the case $\lambda = 1$).

The reason why funds' monitoring and thus firm value do not change with c_i when $\lambda = 1$ is as follows. Suppose, for example, that the passive fund's effort increased as c_P decreased (assuming for a moment that the fund's ownership stakes x_P would not change). Higher effort would raise firms' payoffs (R_M) and hence market prices (P_M). Since, as discussed above, the fund does not gain from increased monitoring, the only effect of higher valuations would be the fund's lower ability to realize gains from trade. This would make investing in the fund less attractive to investors relative to investing in the outside asset, leading to outflows into private savings and decreasing the fund's AUM. These outflows, in turn, would lead the fund to take smaller positions in the underlying stocks, and these smaller

[17] If this infimum does not exist, i.e., $\lambda = 1$ for all c_i satisfying the conditions of Proposition 1, then decreasing c_i harms total welfare for all c_i satisfying these conditions.

positions would have a counteracting effect of decreasing the fund's incentives to monitor. In equilibrium, the fund's AUM and, accordingly, its ownership stakes x_P decrease in a way that the combined effects of lower c_P and lower x_P on the fund's effort cancel out, so that the equilibrium effort and hence firm valuations remain unchanged.

Overall and more generally, this logic emphasizes that to understand the effects of governance regulations, it is important to consider their potential effects on funds' assets under management, since those effects can potentially counteract the desired effects of regulations.

Note also that as passive funds become easier to access (ψ_P declines), funds' AUM grow and investors are likely to strictly prefer investing with the funds over their private savings (Proposition 2), which makes the counteracting effect described above less likely. Accordingly, as we show in the proof of Proposition 5, the threshold $\bar{c}_i$ increases with ψ_P, which leads to the following implication: Regulations that reduce funds' costs of engagement are more likely to be welfare improving if (1) passive funds are easier to access, and (2) funds' AUM are sufficiently large.

Proposals that restrict passive funds from voting. Lund (2018) suggests that lawmakers consider restricting passive funds from voting at shareholder meetings. In the context of our model, this would be equivalent to substantially increasing passive funds' costs of monitoring c_P, and Proposition 5 implies that such a proposal could indeed be potentially beneficial for total welfare. However, the reasoning emphasized in our paper is very different from the reasoning put forward by Lund (2018). In particular, Lund (2018) points out that if a passive fund chooses to intervene, "it will rationally adhere to a low-cost, one-size-fits-all approach to governance that is unlikely to be in the company's best interest," or, in other words, that passive fund monitoring decreases firm value. In contrast, we emphasize that even if passive fund monitoring has the potential to increase firm value, restricting it could be welfare-improving because too much monitoring may have a negative effect on fund investors and, potentially, fund managers.

5 Conclusion

The governance role of delegated portfolio managers, and passive funds in particular, is the subject of an ongoing debate among academics and policymakers. This paper develops a theoretical framework to study the governance effects of active and passive funds in a general equilibrium setting. Analyzing market equilibrium is critical for understanding the implications of passive fund growth because their greater availability changes not only firms' ownership structure, but also the fees and AUM of both active and passive funds, which all affect investor engagement.

We show that whether passive fund growth is beneficial for governance depends on whether it primarily crowds out investors' private savings or their allocation to active funds. In the former case, passive fund growth improves governance because retail shareholders (who play no governance role) are replaced by passive funds in firms' ownership structures, and passive funds have incentives to engage given their large ownership stakes. Moreover, passive fund growth improves governance even though it is accompanied by a decrease in fund fees. However, if passive fund growth crowds out investors' allocation to active funds, it is more likely to have a negative effect. The increased competition between funds decreases active funds' fees, which weakens their incentives to monitor. In addition, passive funds replace active funds in firms' ownership structures, which can further reduce overall investor monitoring since passive funds' fees, and hence their monitoring incentives, are lower than those of active funds. Overall, passive fund growth improves governance only if it does not substantially increase the returns of fund investors, i.e., there can be a trade-off between governance and fund investor well-being.

We also study the effect of regulations that decrease funds' costs of engaging in governance, e.g., by mandating larger stewardship teams. While such regulations increase funds' monitoring and thus firm valuations, they can be detrimental to fund investors and, potentially, fund managers themselves. As a result, fund managers and fund investors may oppose such regulations even when they are value-increasing. Moreover, if such regulations reduce funds' costs of engagement beyond a certain threshold, they can harm total welfare.

To focus on the role of funds' assets under management, fees, and the competition between funds, we abstract from several important features of the engagement process, such as the interaction between different shareholders in their engagement efforts, the role of fund managers' private information about firms, or dynamic considerations due to differences in investors' horizons. An in-depth look at these questions and their interaction with the mechanisms we study in the paper provides interesting avenues for future research.

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Appendix

Proof of Proposition 1. We consider each case separately.
(1) Equilibrium in Case 1: low investor returns, $\lambda = 1$.

Consider the three equations for active fund managers and L-assets, i.e., (7), (14), and (18), which we can rewrite as:

$$\begin{aligned} f_A &= \eta \frac{Z_L}{R_L} \quad \text{(fee bargaining)} & (28) \\ (1-f_A)\frac{R_L}{P_L} &= 1+\psi_A \quad \text{(investor indifference)} & (29) \\ R_L - P_L &= Z_L \quad \text{(market clearing)} & (30) \end{aligned}$$

Plugging f_A from (28) and P_L from (30) into (29) gives:

$$\left(1 - \frac{\eta Z_L}{R_L}\right)\frac{R_L}{R_L - Z_L} = 1 + \psi_A \Leftrightarrow (1+\psi_A - \eta) Z_L = \psi_A R_L.$$

Hence, $R_L = \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L$. Then, (30) implies $P_L = R_L - Z_L = \frac{1-\eta}{\psi_A} Z_L$, and (28) implies

$$f_A = \eta \frac{Z_L}{\frac{1+\psi_A-\eta}{\psi_A} Z_L} = \frac{\eta\psi_A}{1+\psi_A-\eta}.$$

Similarly, we can rewrite the three equations for passive fund managers and the market asset, i.e., (8), (16), and (19), as

$$\begin{aligned} f_P &= \eta \frac{Z_M}{R_M} \quad \text{(fee bargaining)} \\ (1-f_P)\frac{R_M}{P_M} &= 1+\psi_P \quad \text{(investor indifference)} \\ R_M - P_M &= Z_M \quad \text{(market clearing)} \end{aligned}$$

Since this system looks exactly the same as the corresponding system for active fund managers and the L-asset, the solution looks the same: $R_M = \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M$, $P_M = \frac{1-\eta}{\psi_P} Z_M$, and $f_P = \frac{\eta\psi_P}{1+\psi_P-\eta}$, which completes the derivation of Case 1.

(2) Equilibrium in Case 2: high investor returns, $\lambda > 1$.

We start by deriving (27). Using (5) and (6) and plugging them into (21), we get

$$W = \frac{1}{2} x_{AL} P_L + x_P P_M. \tag{31}$$

Next, using (9) and (10),

$$R_L - R_M = \frac{1}{2} c_A'^{-1} (f_A x_{AL}) \Leftrightarrow c_A'(2(R_L - R_M)) = f_A x_{AL}, \tag{32}$$

$$2R_M - R_L = R_0 + c_P'^{-1} (f_P x_P) \Leftrightarrow c_P'(2R_M - R_L - R_0) = f_P x_P. \tag{33}$$

Plugging these into (31) gives (27).

We next characterize the equilibrium as a function of λ, using (7)-(10); (23), (25); and (20), (27).

First, consider asset L and the active fund manager and use (23), (20), and (7):

$$f_A \frac{R_L}{P_L} = \eta \left(\frac{R_L}{P_L} - \lambda \right) \quad \text{(fee bargaining)} \tag{34}$$

$$(1 - f_A) \frac{R_L}{P_L} = \psi_A + \lambda \quad \text{(investor indifference)} \tag{35}$$

$$P_L = R_L - Z_L \quad \text{(market clearing)} \tag{36}$$

From (34), $\frac{R_L}{P_L} = \frac{\eta\lambda}{\eta - f_A}$, and plugging this into (35) gives

$$(1 - f_A) \frac{\eta\lambda}{\eta - f_A} = \psi_A + \lambda \Leftrightarrow f_A = \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}.$$

Plugging this into (34) gives

$$\frac{R_L}{P_L} \eta \left(1 - \frac{\psi_A}{\psi_A + \lambda(1-\eta)} \right) = \eta\lambda \Leftrightarrow (\psi_A + \lambda(1-\eta)) P_L = (1-\eta) R_L,$$

and using (36) gives

$$(\psi_A + \lambda(1-\eta)) Z_L = (\psi_A + \lambda(1-\eta)) R_L - (1-\eta) R_L \Leftrightarrow$$

$$R_L = \left(1 + \frac{1-\eta}{\psi_A + (\lambda - 1)(1-\eta)} \right) Z_L. \tag{37}$$

Finally, using (36) and (37),

$$P_L = R_L - Z_L = \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L.$$

Second, consider asset M (the market portfolio) and the passive fund manager. Since the system of equations (8), (25), and (20) looks exactly the same as the corresponding system for active fund managers and the L-asset (34)-(36), the solution looks the same as well, which gives the expressions for f_P, R_M, and P_M in the statement of the proposition.

Thus, all equilibrium outcomes – f_A, f_P, R_L, R_M, P_L, P_M – are expressed as a function of λ and the exogenous parameters of the model. The equilibrium λ is then determined from the equilibrium condition that investors invest all of their capital either with the active or with the passive fund manager, i.e., the fixed point solution to (27). This completes the derivation of Case 2.

(3) Combining the two cases together.

According to Lemma 2 in the online appendix, if $c_P \geq \frac{\psi_P}{\psi_A} c_A$, then λ is decreasing in W. Hence, there exists $\bar{W}$ such that $\lambda > 1$ for $W < \bar{W}$ and $\lambda = 1$ for $W \geq \bar{W}$. Therefore, again due to Lemma 2, λ is strictly decreasing in W if $W < \bar{W}$ and $c_P \geq \frac{\psi_P}{\psi_A} c_A$. It remains to verify that in the conjectured equilibrium: (1) the active fund indeed finds it optimal to only invest in L-stocks (and not H-stocks or the outside asset) and to diversify across all L-stocks; (2) both the active and passive fund raise positive AUM; and (3) liquidity investors are marginal in each stock. Lemma 3 in the online appendix shows that under the quadratic cost function, the active fund will indeed diversify across L-stocks. Part (ii) of Lemma 4 and Part (ii) of Lemma 5 in the online appendix impose conditions that are sufficient for the active fund to not deviate to investing in either H-stocks or the outside asset. Lemma 6 in the online appendix imposes sufficient conditions for both funds' AUM to be positive, and Lemma 7 in the online appendix imposes sufficient conditions for liquidity investors to be marginal. Combining these conditions together yields the following two conditions:

$$\max\left\{0.64, \frac{\frac{R_0}{Z_L} + \left[1 + \frac{1-\eta}{\psi_A}\right]}{2\left[1 + \frac{1-\eta}{\psi_P}\right]}, \frac{\xi_A \xi_P + \xi_A - \xi_P}{\xi_P^2}, \frac{\frac{1}{2} + \frac{1-\eta}{\psi_A}}{1 + \frac{1-\eta}{\psi_P}}\right\} < \frac{Z_M}{Z_L} < \frac{1 + \frac{1-\eta}{\psi_A}}{1 + \frac{1-\eta}{\psi_P}}, \tag{38}$$

$$\hat{W} \leq W < \frac{R_0 - Z_L}{2}, \tag{39}$$

where ξ_A and ξ_P are given by (86)-(87) and $\hat{W} < \bar{W}$ is defined in Lemma 6 in the online appendix. The numerical example in Figure 3 satisfies this set of parameters for all values of ψ_P displayed, i.e., it is a non-empty set. Note also that even though the numerical example in Figure 2 satisfies this set of parameters for only $\psi_P \in [0.08965, 0.09018]$, nevertheless all values of ψ_P displayed in the figure satisfy the sufficient conditions for the equilibrium in Proposition 1. This is because for $\psi_P < 0.08965$ that are displayed in Figure 2, the only violated parameter restriction is $\frac{\xi_A \xi_P + \xi_A - \xi_P}{\xi_P^2} < \frac{Z_M}{Z_L}$, which is imposed to ensure that $W_A > 0$ and $W_P > 0$ (by Lemma 6 in the online appendix), and $W_A > 0$ and $W_P > 0$ are still satisfied. Similarly, for $\psi_P > 0.09018$ that are displayed in Figure 2, the only violated parameter restriction is $\frac{Z_M}{Z_L} < \frac{1+\frac{1-\eta}{\psi_A}}{1+\frac{1-\eta}{\psi_P}}$, which is imposed (by part (ii) of Lemma 4 in the online appendix) to ensure that the active fund manager does not deviate to investing in H-stocks, and this behavior of the active fund manager is still satisfied since part (i) of Lemma 4 holds for all such ψ_P. ■

Proof of Proposition 2. (1) We start by deriving the expressions for active and passive funds' AUM. Using Proposition 1 and (75),

$$\begin{aligned} W_P &= x_P P_M = \frac{c_P e_P}{f_P} \frac{R_M}{\frac{\psi_P}{1-\eta} + \lambda} = c_P \left(2R_M - R_L - R_0\right) \frac{\psi_P + \lambda \left(1-\eta\right)}{\eta \psi_P} \frac{R_M \left(1-\eta\right)}{\psi_P + \lambda \left(1-\eta\right)} \\ &= \frac{1-\eta}{\eta} \frac{c_P}{\psi_P} R_M \left(2R_M - R_L - R_0\right). \end{aligned} \tag{40}$$

Similarly, using Proposition 1 and (74),

$$\begin{aligned} W_A &= \frac{1}{2} x_{AL} P_L = \frac{1}{2} \frac{c_A e_{AL}}{f_A} \frac{R_L}{\frac{\psi_A}{1-\eta} + \lambda} = \frac{1}{2} 2c_A \left(R_L - R_M\right) \frac{\psi_A + \lambda \left(1-\eta\right)}{\eta \psi_A} \frac{R_L \left(1-\eta\right)}{\psi_A + \lambda \left(1-\eta\right)} \\ &= \frac{1-\eta}{\eta} \frac{c_A}{\psi_A} R_L \left(R_L - R_M\right). \end{aligned} \tag{41}$$

Note, as an auxiliary result, that these expressions imply that in Case 1, AUM of fund i are decreasing in ψ_i. Indeed, if $\lambda = 1$, then R_L does not depend on ψ_P, and W_P strictly decreases in ψ_P if and only if

$$-\frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \frac{dR_M}{d\psi_P} < 0,$$

which holds since $2R_M - R_L - R_0 > 0$ and $\frac{dR_M}{d\psi_P} < 0$. Similarly, if $\lambda = 1$, then R_M does not depend on ψ_A, and W_A strictly decreases in ψ_A if and only if

$$-\frac{c_A}{\psi_A^2} R_L \left(R_L - R_M\right) + \frac{c_A}{\psi_A} \left(2R_L - R_M\right) \frac{dR_L}{d\psi_A} < 0,$$

which holds since $R_L - R_M > 0$ and $\frac{dR_L}{d\psi_A} < 0$. Note also that the same arguments hold for the equilibria of Lemma 1, in which only one fund raises AUM – this is because the above expressions for W_A (W_P) are still valid in the equilibrium where only the active (passive) fund raises AUM.

(2) Next, we show that the combined AUM of active and passive fund managers, W_A+W_P, strictly decrease in ψ_P in Case $\lambda = 1$. This automatically implies that $W_A + W_P$ always weakly decrease in ψ_P (because when $\lambda > 1$, $W_A + W_P = W$). To show that total AUM decrease in ψ_P, note, using (41)-(40), that

$$W_A + W_P = \frac{1-\eta}{\eta} \left(\frac{c_A}{\psi_A} R_L \left(R_L - R_M\right) + \frac{c_P}{\psi_P} R_M \left(2R_M - R_L - R_0\right) \right). \tag{42}$$

Since, in Case 1, R_L does not depend on ψ_P, total AUM strictly decrease in ψ_P if and only if

$$
\begin{aligned}
-\frac{c_A}{\psi_A} R_L \frac{dR_M}{d\psi_P} - \frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \frac{dR_M}{d\psi_P} &< 0 \Leftrightarrow \\
\left[-\frac{c_A}{\psi_A} R_L + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right)\right] \frac{dR_M}{d\psi_P} - \frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) &< 0.
\end{aligned}
$$

Since $2R_M - R_L - R_0 > 0$ and $\frac{\partial R_M}{\partial \psi_P} < 0$, it is sufficient to show that

$$
-\frac{c_A}{\psi_A} R_L + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \geq 0. \tag{43}
$$

Note that $e_P = 2R_M - R_L - R_0 \geq 0$ and hence $2R_M - R_L > 0$, and summing up these two inequalities gives $4R_M - R_L - R_0 > R_L$. This, together with the assumption of Proposition 1 that $\frac{c_P}{\psi_P} \geq \frac{c_A}{\psi_A}$, implies (43), as required. The same result with respect to ψ_P also applies in the equilibrium of Lemma 1, in which only the passive fund raises AUM.

The fact that $W_A + W_P$ decrease in ψ_P implies the last statement of the lemma, i.e., that Case 1 of low investor returns ($\lambda = 1$) only applies when ψ_P is large enough. Indeed, in Case 1, fund investors invest their funds both with the fund managers and in the outside asset, and hence $W_A + W_P < W$, while in Case 2, all investor funds are allocated to the fund managers, i.e., $W_A + W_P = W$. Hence, Case 1 applies if and only if $W_A + W_P < W$, or if and only if ψ_P is large enough.

(3) Next, we prove that λ decreases in ψ_P under the conditions of Proposition 1. This is weakly satisfied for Case 1 because $\lambda = 1$. To see this for Case 2, note that the combined AUM of the two funds, $W_A + W_P$, satisfy (42). In addition, for a fixed λ, R_L does not depend on ψ_P and R_M decreases in ψ_P, so repeating the steps subsequent to (42), implies that for a fixed λ, $W_A + W_P$ decreases in ψ_P. Moreover, for Case 2, $W_A + W_P = W$. On the other hand, as follows from the proof of Lemma 2 in the online appendix, equality (51) holds, where the right-hand side decreases in λ. Combined, we have

$$
W_A\left(\lambda, \psi_P\right) + W_P\left(\lambda, \psi_P\right) = W,
$$

and hence,

$$
\frac{\partial\left(W_A + W_P\right)}{\partial \lambda} \frac{d\lambda}{d\psi_P} + \frac{\partial\left(W_A + W_P\right)}{\partial \psi_P} = 0,
$$

where $\frac{\partial(W_A + W_P)}{\partial \lambda} < 0$ and $\frac{\partial(W_A + W_P)}{\partial \psi_P} < 0$. Thus, $\frac{d\lambda}{d\psi_P} < 0$, as required.

(4) Finally, we prove the result for fund fees, i.e., that both f_A and f_P increase in ψ_P. Since $f_A = \frac{\eta \psi_A}{\psi_A + \lambda(1-\eta)}$, it weakly increases in ψ_P (it does not depend on ψ_P in Case 1 and strictly increases in Case 2 given $\frac{d\lambda}{d\psi_P} < 0$). And, since $f_P = \frac{\eta \psi_P}{\psi_P + \lambda(1-\eta)}$, it always strictly increases in ψ_P: In Case 1, this is because $f_P = \frac{\eta \psi_P}{\psi_P + 1 - \eta}$, while in Case 2, this is because $\frac{df_P}{d\psi_P} = \frac{\partial f_P}{\partial \lambda} \frac{d\lambda}{d\psi_P} + \frac{\partial f_P}{\partial \psi_P} > 0$, which follows from $\frac{\partial f_P}{\partial \lambda} < 0$, $\frac{d\lambda}{d\psi_P} < 0$, and $\frac{\partial f_P}{\partial \psi_P} > 0$. This completes the proof. ■

Proof of Proposition 3. Note that $c_P \geq c_A$ and $\psi_P \leq \psi_A$ together imply that $c_P \geq \frac{\psi_P}{\psi_A} c_A$. Recall that by Proposition 2, $\lambda = 1$ if $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ if $\psi_P < \bar{\psi}_P$. Therefore, if $\psi_P > \bar{\psi}_P$, Proposition 1 implies that R_M strictly increases as ψ_P decreases.

Second, to establish that the continuity of equilibrium also applies at $\psi_P = \bar{\psi}_P$, we prove that $\lim_{\psi_P \uparrow \bar{\psi}_P} \lambda = 1$, and that $\psi_P = \bar{\psi}_P$ satisfies the fixed point equation (27) with $\lambda = 1$. To see this, note that Propositions 1 and 2 imply that for all $\psi_P < \bar{\psi}_P$, (27) is satisfied for the equilibrium λ. Denote the right hand side of (27) by $RHS(\lambda, \psi_P)$, and recall that by the proof of Proposition 1, $RHS(\lambda, \psi_P)$ represents the total AUM of active and passive funds (that is, $W_A + W_P$). Also note that $RHS(\lambda, \psi_P)$ is continuous w.r.t. λ and ψ_P, is strictly decreasing with ψ_P (by step (3) of the proof of Proposition 2), and is strictly decreasing in λ (by Proposition 1). Therefore, it is sufficient to show that $\psi_P = \bar{\psi}_P$ satisfies (27) with $\lambda = 1$ (since it would also imply that $\lim_{\psi_P \uparrow \bar{\psi}_P} \lambda = 1$). Suppose this is not the case. Then, since $\lambda = 1$ has to hold by Proposition 2, it must be that $W \neq RHS(1, \bar{\psi}_P)$. Since $RHS(\lambda, \psi_P)$ represents the total AUM, it cannot be $W < RHS(1, \bar{\psi}_P)$, and hence it must be $W > RHS(1, \bar{\psi}_P)$. However, then by continuity of $RHS(\lambda, \psi_P)$ in ψ_P, there exists $\delta > 0$ such that $W > RHS(1, \psi'_P)$ for any $\psi'_P \in (\bar{\psi}_P - \delta, \bar{\psi}_P)$. Therefore, for any such $\psi_P = \psi'_P$, $\lambda = 1$ should be an equilibrium according to step (1) in the proof of Proposition 1, which yields a contradiction with Proposition 2 since $\psi'_P < \psi_P$.

Third, we prove that if W_A weakly increases as ψ_P decreases and $\psi_P \leq \bar{\psi}_P$, then R_M strictly decreases as ψ_P decreases. Note that as ψ_P decreases, Proposition 2 implies that λ strictly increases, where "strictly" follows step (3) in the proof of Proposition 2. Therefore, Proposition 1 implies that R_L strictly decreases as ψ_P decreases. Therefore, since W_A is given by (41) in the proof of Proposition 2, for W_A to weakly increase it must be that R_M strictly decreases.

Fourth, we re-formulate R_H and R_L. Denote the total capital invested by the passive fund in L-firms and H-firms by W_{PL} and W_{PH}, respectively. Then, using this notation, we can re-formulate R_H and R_L as follows.

(a) Re-formulation of R_H: By (2), (3), and $x_{AH} = 0$, we have $R_H = R_0 + e_P$, where $e_P = \frac{f_P x_P}{c_P}$. Plugging in $x_P = \frac{W_{PH}}{\frac{1}{2} P_H}$ (since there is $\frac{1}{2}$ measure of H-firms) and $P_H = R_H - Z_H$ (due to (4)) yields

$$
\begin{aligned}
R_H &= R_0 + \frac{f_P}{c_P} \frac{2W_{PH}}{R_H - Z_H} \Leftrightarrow R_H (R_H - Z_H) = R_0 (R_H - Z_H) + \frac{f_P}{c_P} 2W_{PH} \\
&\Leftrightarrow R_H^2 - (R_0 + Z_H) R_H - \left(\frac{f_P}{c_P} 2W_{PH} - R_0 Z_H \right) = 0
\end{aligned}
$$

The discriminant of this quadratic equation is given by

$$
\Delta = (R_0 + Z_H)^2 + 4 \left(\frac{f_P}{c_P} 2W_{PH} - R_0 Z_H \right) = (R_0 - Z_H)^2 + 8 \frac{f_P}{c_P} W_{PH}
$$

Since $\sqrt{\Delta} > R_0 - Z_H$, the smaller root for R_H is smaller then Z_H, contradicting with

$P_H = R_H - Z_H > 0$. Therefore, R_H is given by the larger root:

$$R_H = \frac{1}{2}(R_0 + Z_H) + \sqrt{\frac{1}{4}(R_0 - Z_H)^2 + 2\frac{f_P}{c_P}W_{PH}} \tag{44}$$

Note that

$$\frac{dR_H}{d\psi_P} = \frac{2}{2R_H - Z_H - R_0}\left(\frac{f_P}{c_P}\frac{dW_{PH}}{d\psi_P} + \frac{1}{c_P}W_{PH}\frac{df_P}{d\psi_P}\right), \tag{45}$$

where the last equality follows from (44).

(b) Re-formulation of R_L: By (2) and (3), we have $R_L = R_0 + e_P + e_{AL}$, where $e_P = \frac{f_P x_P}{c_P}$ and $e_{AL} = \frac{f_P x_{AL}}{c_A}$. Plugging in $x_P = \frac{W_{PL}}{\frac{1}{2}P_L}$ and $x_{AL} = \frac{W_A}{\frac{1}{2}P_L}$ (since $x_{AH} = 0$ and there is $\frac{1}{2}$ measure of H-firms) and using derivations analogous to part (a) yields

$$R_L = \frac{1}{2}(R_0 + Z_L) + \sqrt{\frac{1}{4}(R_0 - Z_L)^2 + \frac{f_P}{c_P}2W_{PL} + \frac{f_A}{c_A}2W_A} \tag{46}$$

Note that

$$\frac{dR_L}{d\psi_P} = \frac{2}{2R_L - Z_L - R_0}\left(\frac{f_P}{c_P}\frac{dW_{PL}}{d\psi_P} + \frac{f_A}{c_A}\frac{dW_A}{d\psi_P} + \frac{1}{c_P}W_{PL}\frac{df_P}{d\psi_P} + \frac{1}{c_A}W_A\frac{df_A}{d\psi_P}\right), \tag{47}$$

where the last equality follows from (46).

Fifth, we prove that if W_A strictly decreases as ψ_P decreases, $\psi_P \le \bar{\psi}_P$, and $Z_L - Z_H > 2e_{AL}$, then $\frac{dR_M}{d\psi_P} > 0$. Note that as noted in the third step above, as ψ_P decreases, λ strictly increases and R_L strictly decreases. Denote the total capital invested by the passive fund in L-firms and H-firms by W_{PL} and W_{PH}, respectively. Then, combining $W_A + W_P = W_A + W_{PL} + W_{PH}$ with $W = W_A + W_P$ (where the latter follows by the arguments in the second step above) yields

$$\frac{dW_A}{d\psi_P} + \frac{dW_{PL}}{d\psi_P} = -\frac{dW_{PH}}{d\psi_P}. \tag{48}$$

(Note that when $\psi_P = \bar{\psi}_P$, we replace all derivatives with left-hand derivatives, i.e., derivatives as $\psi_P \uparrow \bar{\psi}_P$.) Note that $\frac{dW_A}{d\psi_P} > 0$ since we are focusing on the case where W_A strictly decreases as ψ_P decreases. Also note that $\frac{d\lambda}{d\psi_P} < 0$ together with Propositions 1 and 2 imply that $\frac{df_P}{d\psi_P} > 0$ and $\frac{df_A}{d\psi_P} > 0$. There are two scenarios to consider:

(1) Suppose that $\frac{dW_A}{d\psi_P} + \frac{dW_{PL}}{d\psi_P} \le 0$. Then, (48) implies that $\frac{dW_{PH}}{d\psi_P} \ge 0$. Therefore, $\frac{df_P}{d\psi_P} > 0$ and (45) imply that $\frac{dR_H}{d\psi_P} > 0$, i.e., R_H strictly decreases as ψ_P decreases. Since we have previously established that $\frac{dR_L}{d\psi_P} > 0$, this implies that $\frac{dR_M}{d\psi_P} = \frac{1}{2}\left(\frac{dR_L}{d\psi_P} + \frac{dR_H}{d\psi_P}\right) > 0$.

(2) Suppose that $\frac{dW_A}{d\psi_P} + \frac{dW_{PL}}{d\psi_P} > 0$. Note that due to (48), this implies that $\frac{dW_{PH}}{d\psi_P} < 0$. Since $\frac{df_P}{d\psi_P} > 0$ and $\frac{df_A}{d\psi_P} > 0$, (45) and (47) imply that to show $\frac{dR_M}{d\psi_P} = \frac{1}{2}\left(\frac{dR_L}{d\psi_P} + \frac{dR_H}{d\psi_P}\right) > 0$ it

is sufficient to prove that

$$0 < \frac{1}{2R_H - Z_H - R_0}\frac{f_P}{c_P}\frac{dW_{PH}}{d\psi_P} + \frac{1}{2R_L - Z_L - R_0}\left(\frac{f_P}{c_P}\frac{dW_{PL}}{d\psi_P} + \frac{f_A}{c_A}\frac{dW_A}{d\psi_P}\right). \tag{49}$$

Recall that $\frac{dW_A}{d\psi_P} > 0$. Combining with $c_P \geq c_A$ and $f_P \leq f_A$ (where the latter is implied by Proposition 1 since $\psi_P \leq \psi_A$), this implies that to show that (49) holds, it is sufficient to show

$$0 < \frac{1}{2R_H - Z_H - R_0}\frac{dW_{PH}}{d\psi_P} + \frac{1}{2R_L - Z_L - R_0}\left(\frac{dW_{PL}}{d\psi_P} + \frac{dW_A}{d\psi_P}\right). \tag{50}$$

In turn, (48) and $\frac{dW_{PH}}{d\psi_P} < 0$ imply that (50) is equivalent to

$$0 < -\frac{1}{2R_H - Z_H - R_0} + \frac{1}{2R_L - Z_L - R_0},$$

or, equivalently,

$$2R_L - Z_L < 2R_H - Z_H \Leftrightarrow 2e_{AL} < Z_L - Z_H$$

where the equivalence follows from $R_H = R_0 + e_P$ and $R_L = R_0 + e_P + e_{AL}$, which in turn follow from (2), (3), and $x_{AH} = 0$. Since $Z_L - Z_H > 2e_{AL}$ holds by assumption, this concludes the proof of the proposition.

We now show that there exists a cutoff $\underline{\psi}_P$ such that condition $e_{AL} < \frac{1}{2}(Z_L - Z_H)$ is satisfed if $\psi_P < \underline{\psi}_P$. Since $e_{AL} = 2(R_L - R_M)$, $e_{AL} < \frac{1}{2}(Z_L - Z_H)$ reduces to $\frac{1}{2}(Z_L - Z_H) > 2(R_L - R_M)$. Plugging in $Z_H = 2Z_M - Z_L$ and R_L and R_M from Proposition 1, this inequality becomes

$$\begin{aligned} Z_L - Z_M &> 2\left(1 + \frac{1-\eta}{\psi_A + (\lambda - 1)(1-\eta)}\right) Z_L - 2\left(1 + \frac{1-\eta}{\psi_P + (\lambda - 1)(1-\eta)}\right) Z_M \\ \Leftrightarrow & \quad \frac{1 + 2\frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}}{1 + 2\frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}} > \frac{Z_L}{Z_M}. \end{aligned}$$

Since $\psi_P \leq \psi_A$, the left-hand side is decreasing in λ. Therefore, since $\lambda_{\max} = \frac{R_0}{R_0 - Z_L} - \psi_A$ by Lemma 8, it is sufficient to show that the inequality above holds for $\lambda = \lambda_{\max}$, i.e.,

$$\psi_P < 2\frac{1-\eta}{\frac{Z_L}{Z_M}\left(1 + 2\frac{1-\eta}{\psi_A + (\lambda_{\max} - 1)(1-\eta)}\right) - 1} - (\lambda_{\max} - 1)(1-\eta) \Leftrightarrow \psi_P < \underline{\psi}_P,$$

where

$$\underline{\psi}_P \equiv 2\frac{1-\eta}{\frac{Z_L}{Z_M}\left(1 + 2\frac{1-\eta}{\psi_A + \left(\frac{R_0}{R_0 - Z_L} - \psi_A - 1\right)(1-\eta)}\right) - 1} - \left(\frac{R_0}{R_0 - Z_L} - \psi_A - 1\right)(1-\eta).$$

■

Proof of Proposition 4.

Note that by Proposition 2, $\lambda = 1$ if $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ if $\psi < \bar{\psi}_P$. By Proposition 1, $\lambda = 1$ if $W \geq \bar{W}$ and $\lambda > 1$ if $W < \bar{W}$. Therefore, it must be that if $\psi_P \geq \bar{\psi}_P$, then $W \geq \bar{W}$, and if $\psi_P < \bar{\psi}_P$, then $W < \bar{W}$.

We start by proving (ii). Fund investors' payoff is characterized by their equilibrium rate of return λ. When $W \geq \bar{W}$, their rate of return is $\lambda = 1$ and is unaffected by c_i. When $W \leq \bar{W}$, λ increases with c_i. To see this, recall that λ is the solution to

$$W = \frac{c_A}{f_A(\lambda)} \left(R_L(\lambda) - R_M(\lambda)\right) P_L(\lambda) + \frac{c_P}{f_P(\lambda)} \left(2R_M(\lambda) - R_L(\lambda) - R_0\right) P_M(\lambda), \tag{51}$$

where $f_A(\lambda)$, $f_P(\lambda)$, $R_L(\lambda)$, $R_M(\lambda)$, $P_L(\lambda)$, and $P_M(\lambda)$ are given by the expressions in Proposition 1. According to Lemma 2 in the online appendix, the right-hand side decreases with λ whenever $\psi_A > \psi_P$ and $c_P \geq \frac{\psi_P}{\psi_A} c_A$. Since the right-hand side increases in c_i, it follows that λ increases in c_i (otherwise, if c_i increased, the right-hand side would increase both through the effect of c_i and through the effect of λ, while the left-hand side would not).

We next prove (i). Consider R_L and R_M. If $W \geq \bar{W}$, they do not depend on c_i. If $W \leq \bar{W}$, then $R_L = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}) Z_L$ and $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}) Z_M$. Since λ increases with c_i as shown above, then both R_L and R_M decrease with c_i, and thus P_L and P_M decrease with c_i as well.

Finally, we prove (iii). Let e_P (e_{AL}) denote the passive (active) fund manager's equilibrium effort. Then, the passive fund manager's payoff is given by

$$\begin{aligned} V_P = f_P x_P R_M - \tfrac{c_P}{2} e_P^2 = c_P e_P \left(R_M - \tfrac{1}{2} e_P\right) = c_P \left(2R_M - R_L - R_0\right) \left(R_M - \tfrac{1}{2}\left(2R_M - R_L - R_0\right)\right) \\ = \tfrac{c_P}{2} \left(2R_M - R_L - R_0\right)\left(R_L + R_0\right), \end{aligned} \tag{52}$$

and the active fund manager's payoff is given by

$$\begin{aligned} V_A = \tfrac{1}{2}\left(f_A x_{AL} R_L - \tfrac{c_A}{2} e_{AL}^2\right) = \tfrac{1}{2} c_A e_{AL} \left(R_L - \tfrac{1}{2} e_{AL}\right) \\ = c_A \left(R_L - R_M\right)\left(R_L - \tfrac{1}{2} 2 \left(R_L - R_M\right)\right) = c_A \left(R_L - R_M\right) R_M. \end{aligned} \tag{53}$$

If $W \geq \bar{W}$, then by Proposition 1, R_L and R_M do not change with c_P and c_A, which implies that V_P strictly increases with c_P and V_A strictly increases with c_A. ∎

Proof of Proposition 5. Welfare equals the sum of the payoffs of the initial shareholders, the payoffs of liquidity investors, the payoffs of fund managers, and the payoffs of fund investors:

$$Welfare = P_M + 0 + \left[\frac{1}{2} f_A x_{AL} R_L + f_P x_P R_M - \frac{1}{2}\frac{c_A}{2} e_{AL}^2 - \frac{c_P}{2} e_P^2\right] + (\lambda - 1) W \tag{54}$$

The first term is the payoff of the initial owners of the firms, which is $\frac{P_L + P_H}{2}$ up to a constant (initial owners' valuations). The second term equals zero because liquidity investors are marginal traders. The third term, in the square brackets, captures the combined payoff of the active and passive fund manager, which is their share of the fund's payoff minus their costs of engaging in governance. The last term captures the payoff of the fund investors: since their initial wealth is W and they earn equilibrium rate of return λ on it, their final payoff is λW. Note that in the expression above, W has a multiplier of $(\lambda - 1)$, rather than just λ. This has an effect on the comparative statics of welfare only with respect to W, and not any other parameters. The rationale behind this choice is that if W increases, the increase in W must be financed from another source in the economy that is not explicitly modeled in our framework. For example, if W increases by ΔW, it must be that ΔW less is invested in the rest of the overall economy, and to capture that, we subtract ΔW from our welfare calculation, resulting in the term $(\lambda - 1) W$.

Using $f_A x_{AL} = c_A e_{AL}$, $f_P x_P = c_P e_P$, $e_{AL} = 2(R_L - R_M) \geq 0$, and $e_P = 2R_M - R_L - R_0 \geq 0$, we can rewrite (54) as

$$\begin{aligned} Welfare &= P_M + \frac{1}{2} c_A e_{AL} R_L + c_P e_P R_M - \frac{1}{2}\frac{c_A}{2} e_{AL}^2 - \frac{c_P}{2} e_P^2 + (\lambda - 1) W \\ &= P_M + \frac{1}{2} c_A e_{AL} \left(R_L - \frac{1}{2} e_{AL} \right) + c_P e_P \left(R_M - \frac{1}{2} e_P \right) + (\lambda - 1) W \\ &= P_M + c_A (R_L - R_M) R_M + \frac{c_P}{2} (2R_M - R_L - R_0)(R_L + R_0) + (\lambda - 1) W. \quad (55) \end{aligned}$$

Below, we show that $\bar{c}_i$ is given by (56)-(57) and prove that $\lambda > 1$ for $c_i > \bar{c}_i$ and $\lambda = 1$ for $c_i \leq \bar{c}_i$. Now, consider any $c_i < \bar{c}_i$, so that $\lambda = 1$. Then, according to Proposition 1, P_M, R_M, and R_L do not change with c_P and c_A. Note that $R_L - R_M = \frac{1}{2} e_{AL} = \frac{1}{2} \frac{f_A x_{AL}}{c_A} > 0$ and $2R_M - R_L - R_0 = e_P = \frac{f_P x_P}{c_P} > 0$, because f_A and f_P are positive by Proposition 1, and both x_{AL} and x_P are positive by the proof of Proposition 1. Hence, (55) implies that welfare strictly increases with c_P and c_A, as required.

We next show that $\bar{c}_P$ and $\bar{c}_A$ are given by

$$W = \frac{1-\eta}{\eta} \left(\begin{array}{c} \frac{c_A}{\psi_A} \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \right) \\ + \frac{\bar{c}_P}{\psi_P} \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \left(2\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M - \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0 \right) \end{array} \right), \quad (56)$$

$$W = \frac{1-\eta}{\eta} \left(\begin{array}{c} \frac{\bar{c}_A}{\psi_A} \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \right) \\ + \frac{c_P}{\psi_P} \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \left(2\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M - \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0 \right) \end{array} \right), \quad (57)$$

respectively. Indeed, recall that in equilibrium described by Proposition 1, $W_A + W_P$ is given by the right-hand side of (27). Consider any $i \in \{A, P\}$. We show that $\lambda > 1$ for $c_i > \bar{c}_i$ and $\lambda = 1$ for $c_i \leq \bar{c}_i$. First, consider $c_i \leq \bar{c}_i$. Then, it must be $\lambda = 1$. This is because then, (56), (57), and Proposition 1 imply $W \geq W_A + W_P$, which is consistent with $\lambda = 1$. This also implies that it cannot be $\lambda > 1$, because if we had $\lambda > 1$, then (56), (57), Proposition 1, and Lemma 2 in the online appendix would imply that $W > W_A + W_P$, yielding a contradiction since no investor would invest in the outside asset given $\lambda > 1$. Second, consider $c_i > \bar{c}_i$.

Then it must be $\lambda > 1$. Indeed, if we had $\lambda = 1$, then (56), (57), and Proposition 1 would imply $W < W_A + W_P$, yielding a contradiction since then the total investor endowment would not be sufficient for funds to raise total AUM of $W_A + W_P$.

As an auxiliary result, we next also show that $\bar{c}_P$ and $\bar{c}_A$ strictly increase with ψ_P. This follows from (56) and (57), because the right-hand side in both of them is strictly decreasing in ψ_P. To see this, take any $i \in \{A, P\}$, and let $c_i = \bar{c}_i$. If $i = P$, consider (56), and if $i = A$, consider (57). Then $\lambda = 1$, and using the expressions for R_L and R_M from Proposition 1, the partial derivative of the right-hand side w.r.t. ψ_P is negative if and only if

$$0 > \left(-\frac{c_A}{\psi_A} R_L + \frac{c_P}{\psi_P}(4R_M - R_L - R_0)\right)\frac{\partial R_M}{\partial \psi_P},$$

which always holds since $\frac{\partial R_M}{\partial \psi_P} < 0$, $c_P \geq \frac{\psi_P}{\psi_A} c_A$, and $4R_M - R_L - R_0 > 2R_M > R_L$, where the last set of inequalities follow from $2R_M - R_L - R_0 = e_P > 0$ as argued after expression (55) above.

While this completes the proof, in what follows, we also provide the sufficient conditions that ensure that (1) $c_P > \frac{\psi_P}{\psi_A}\bar{c}_A > 0$ and (2) $\bar{c}_P > \frac{\psi_P}{\psi_A} c_A$. Together, these two inequalities in turn ensure that the set of values of c_i that satisfy both the conditions of Proposition 1 ($c_P \geq \frac{\psi_P}{\psi_A} c_A$) and the condition $c_i < \bar{c}_i$, is non-empty for each $i \in \{A, P\}$. We show that these sufficient conditions are given by $W_L < W < W_H$, where

$$\begin{aligned}
W_L &\equiv \frac{1-\eta}{\eta}\max\left\{\begin{array}{c}
\frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L - \left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\
+\frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M - \left(1+\frac{1-\eta}{\psi_A}\right)Z_L - R_0\right), \\
\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M - \left(1+\frac{1-\eta}{\psi_A}\right)Z_L - R_0\right)
\end{array}\right\} \\
W_H &\equiv \frac{1-\eta}{\eta}\left(\begin{array}{c}
\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L - \left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\
+\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M - \left(1+\frac{1-\eta}{\psi_A}\right)Z_L - R_0\right)
\end{array}\right).
\end{aligned}$$

Note that $W_L \leq W_H$ is satisfied since $\frac{c_P}{\psi_P} \geq \frac{c_A}{\psi_A}$ as assumed in Proposition 1, and that $W_L < W_H$ whenever $\frac{c_P}{\psi_P} > \frac{c_A}{\psi_A}$. The reason why $W_L < W < W_H$ is a sufficient condition is that from (56)-(57), it follows that $W_L < W$ implies that $\bar{c}_P > \frac{\psi_P}{\psi_A} c_A$ and $c_P > \frac{\psi_P}{\psi_A}\bar{c}_A > 0$, and $W < W_H$ implies $c_P > \frac{\psi_P}{\psi_A}\bar{c}_A$, as required.

Finally, we point out that the set $\{W : W_L < W < W_H\}$ overlaps with the other parameter assumptions made in Proposition 1. In other words, it does not result in an empty set of parameters. To see this, consider the allowed range of W in the example provided in Figure 2 with $\psi_P = 0.09$ (that is, $\eta = 0.01$, $c_A = 0.001$, $c_P = 0.002$, $\psi_A = 0.1$, $\psi_P = 0.09$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$). Then, $W_L = \max\{1.184\,2, 1.672\,4\} < 2.631\,6 = W_H$, and (W_L, W_H) is a subset of (W_1, W_2) imposed by Proposition 1, since $W_1 = 0.503$ and $W_2 = 4.875$. ■

Lemma 1 (equilibria with one type of fund) *Suppose*

$$Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}} \tag{58}$$

and

$$\frac{R_0 - Z_L}{2} > W. \tag{59}$$

(*i*) *Suppose*

$$W > W_P(1) \equiv \frac{1-\eta}{\eta}\frac{c_P}{\psi_P}\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \left(\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M - R_0\right). \tag{60}$$

Then, the equilibrium where $\lambda = 1$ and only the passive fund raises AUM exists if and only if

$$\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \geq \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \tag{61}$$

and

$$\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M > R_0. \tag{62}$$

If this equilibrium exists, then $W_P = W_P(1)$, f_P, R_M, and P_M are as described in Proposition 1, $R_L = R_M$, and $P_L = R_L - Z_L$. Moreover, if $\psi_A > \frac{Z_L}{R_0 - Z_L}$, then this equilibrium is unique.

(*ii*) *Suppose*

$$W > W_A(1) \equiv \frac{1}{2}\frac{1-\eta}{\eta}\frac{c_A}{\psi_A}\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0\right). \tag{63}$$

Then, the equilibrium where $\lambda = 1$ and only the active fund raises AUM exists if and only if

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L + R_0 \geq 2\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \tag{64}$$

and

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L > R_0. \tag{65}$$

If this equilibrium exists, then $W_A = W_A(1)$, f_A, R_L, and P_L are as described in Proposition 1, $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$, and $P_M = R_M - Z_M$. Moreover, if $\psi_P > \frac{Z_M}{R_0 - Z_M}$, then this equilibrium is unique.

Proof of Lemma 1. Note that $Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}}$ automatically implies $Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A}}$, and hence condition (58) implies that the conditions of part (*iii*) of Lemma 4 and part (*iii*) of Lemma 5 in the online appendix are all satisfied. This, together with Lemma 3 in the online appendix, implies that under the conjectured equilibrium, the active fund does not find it optimal to deviate from its strategy of only investing in L-stocks (and not H-stocks or outside asset) and equally diversifying across them.

Proof of part (i)**.** Consider the equilibrium in part (i), i.e., where only the passive fund raises positive AUM and the gross rate of return λ that fund investors earn on their investment satisfies $\lambda = 1$. Then, following the same steps in the proof of Proposition 1 yields the same expressions for f_P, R_M, and P_M as described in that proposition. Note that

$$f_P = \frac{\eta\psi_P}{\psi_P + 1 - \eta} = \eta\frac{Z_M}{R_M}, \tag{66}$$

Since the active fund does not raise any AUM, we have $R_L = R_M$, and $P_L = R_L - Z_L$. Moreover, the AUM of the passive fund are given by

$$\begin{aligned} W_P = x_P P_M = \tfrac{c_P e_P}{f_P} P_M = \tfrac{c_P}{f_P}(2R_M - R_L - R_0)P_M = \tfrac{c_P}{f_P}(R_M - R_0)P_M \\ = c_P \tfrac{\psi_P + 1 - \eta}{\eta\psi_P}\left(\left(1 + \tfrac{1-\eta}{\psi_P}\right) Z_M - R_0\right)\tfrac{1-\eta}{\psi_P} Z_M \equiv W_P(1), \end{aligned} \tag{67}$$

where the second equality follows from (2) and the third equality follows from (9)-(10). Therefore, $W_P = W_P(1)$. Note that $W > W_P(1)$ by assumption, which implies that $W > W_P$, which is consistent with $\lambda = 1$.

Let us now derive the necessary and sufficient conditions for this equilibrium to exist. Note that a fund investor gets a return of $1 + (1-\eta)\left(\frac{R_L}{P_L} - 1\right)$ from his bargaining with the active fund, and therefore the fund investor does not prefer to deviate to search for the active fund if and only if

$$\begin{aligned} 1 \;&\geq\; 1 + (1-\eta)\left(\frac{R_L}{P_L} - 1\right) - \psi_A \Leftrightarrow 1 \geq \frac{R_L}{R_L - Z_L} - \frac{\psi_A}{1-\eta} \\ &\Leftrightarrow\; R_L \geq \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L, \end{aligned}$$

which is equivalent to (61) due to $R_L = R_M$. Positive AUM for the passive fund require $x_P > 0$, i.e., $2R_M - R_L - R_0 > 0$, which is equivalent to (62). Finally, liquidity investors are marginal in this equilibrium, i.e., $x_P < 1$ is satisfied, because

$$x_P = \frac{W_P}{P_M} = \frac{W_P}{R_M - Z_M} < \frac{W}{R_0 - Z_L} < \frac{1}{2},$$

where the last inequality holds by assumption (59).

Next, we show that if $\psi_A > \frac{Z_L}{R_0 - Z_L}$, then the equilibrium described in part (i) is unique. Proving this result consists of two substeps. First, we show that the investors' return from searching for and investing in the active fund is always strictly smaller than one. This holds because this return is bounded from above by $\frac{R_L}{P_L} - \psi_A$, which satisfies

$$\frac{R_L}{P_L} - \psi_A = \frac{R_L}{R_L - Z_L} - \psi_A < \frac{R_0}{R_0 - Z_L} - \psi_A < 1,$$

where the first inequality follows from $R_L = R_M > R_0$ and the last inequality follows from $\psi_A > \frac{Z_L}{R_0 - Z_L}$. Second, we prove that there is no equilibrium where only the passive fund raises positive AUM and $\lambda > 1$. To see this, consider any equilibrium where $W_A = 0$ and $W_P > 0$, but without restricting λ to be equal to one (that is, allowing for $\lambda > 1$). Then, the derivation of the equilibrium is slightly different than in Proposition 1, because the outside option of the fund investor in his bargaining with the passive fund is not equal to λ, but is equal to one. This is because the only other option of the investor is to invest in the outside asset, which has a gross return of one. Therefore, following the same steps as those used in deriving (25), but plugging in ε for the outside option of the investor in the fee bargaining, yields the following fixed point equation:

$$f_P = \eta \left(1 - \frac{P_M}{R_M}\right). \tag{68}$$

Since $R_L = R_M$ and $P_M = R_M - Z_M$ still hold, we have

$$W_P = x_P P_M = \frac{c_P e_P}{f_P} P_M = c_P \frac{R_M}{\eta Z_M}(R_M - R_0)(R_M - Z_M), \tag{69}$$

where the second equality follows from (2) and the third equality utilizes (9)-(10). Note that λ is given by $\lambda = (1 - f_P)\frac{R_M}{P_M} - \psi_P$, and plugging in (68) and $P_M = R_M - Z_M$, λ can be expressed as

$$\lambda = (1 - f_P)\frac{R_M}{P_M} - \psi_P = (1 - \eta)\,\frac{R_M}{R_M - Z_M} + \eta - \psi_P,$$

which strictly decreases in R_M. Since the right-hand side in (69) strictly increases in R_M, this implies that W_P strictly decreases in λ. Moreover, if $\lambda = 1$, then (69) is equal to (67), since (66) and (68) are equal. Combining this with the continuity of (69) in R_M, as λ converges to 1 from above, (69) converges to (67). Thus, $W_P < W_P(1)$ for all $\lambda > 1$. Since $W_P(1) < W$, this implies that $W_P < W$ for all $\lambda > 1$, and hence it cannot be $\lambda > 1$ in equilibrium, because if it were, then no investor would invest in the outside asset, resulting in a contradiction.

Proof of part *(ii)***.** Consider the equilibrium in part *(ii)*, i.e., where only the active fund raises positive AUM and $\lambda = 1$. Then, following the same steps in the proof of Proposition 1 yields the same expressions for f_A, R_L, and P_L as described in that proposition. Note that

$$f_A = \frac{\eta \psi_A}{\psi_A + 1 - \eta} = \eta \frac{Z_L}{R_L}, \tag{70}$$

where the last equality follows from (28). Since the passive fund does not raise any AUM, we have $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$ and $P_M = R_M - Z_M$. Moreover, the AUM of the active fund are given by

$$\begin{aligned} W_A = \tfrac{1}{2}x_{AL}P_L = \tfrac{1}{2}\tfrac{c_A e_{AL}}{f_A}P_L = \tfrac{1}{2}\tfrac{c_A}{f_A}2(R_L - R_M)P_L = \tfrac{1}{2}\tfrac{c_A}{f_A}(R_L - R_0)P_L \\ = \tfrac{1}{2}c_A\tfrac{\psi_A+1-\eta}{\eta\psi_A}\left(\left(1+\tfrac{1-\eta}{\psi_A}\right)Z_L - R_0\right)\tfrac{1-\eta}{\psi_A}Z_L \equiv W_A(1), \end{aligned} \tag{71}$$

where the second equality follows from (2) and the third equality follows from (9)-(10). Therefore, $W_A = W_A(1)$. Note that $W > W_A(1)$ by assumption, which implies that $W > W_A$, which is consistent with $\lambda = 1$.

Let us now derive the necessary and sufficient conditions for this equilibrium to exist. Note that a fund investor gets a return of $1 + (1-\eta)\left(\frac{R_M}{P_M} - 1\right)$ from his bargaining with the passive fund, and therefore the fund investor does not prefer to deviate to search for the passive fund if and only if

$$\begin{aligned} 1 &\geq 1 + (1-\eta)\left(\frac{R_M}{P_M} - 1\right) - \psi_P \Leftrightarrow 1 \geq \frac{R_M}{R_M - Z_M} - \frac{\psi_P}{1-\eta} \\ &\Leftrightarrow R_M \geq \left(1 + \frac{1-\eta}{\psi_P}\right)Z_M, \end{aligned}$$

which is equivalent to (64) due to $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0$. Positive AUM for the active fund require $x_{AL} > 0$, i.e., $R_L - R_0 > 0$, which is equivalent to (65). Finally, liquidity investors are marginal in this equilibrium, i.e., $x_{AL} < 1$ is satisfied, because

$$x_{AL} = \frac{W_A}{\frac{1}{2}P_L} = 2\frac{W_A}{R_L - Z_L} < 2\frac{W}{R_0 - Z_L} < 1,$$

where the last inequality holds by assumption (59).

Next, we show that if $\psi_P > \frac{Z_M}{R_0 - Z_M}$, then the equilibrium described in part (ii) is unique. Proving this result consists of two substeps. First, we show that the investors' return from searching for and investing in the passive fund is always strictly smaller than one. This holds because this return is bounded from above by $\frac{R_M}{P_M} - \psi_P$, which satisfies

$$\frac{R_M}{P_M} - \psi_P = \frac{R_M}{R_M - Z_M} - \psi_P < \frac{R_0}{R_0 - Z_M} - \psi_P < 1,$$

where the first inequality follows from $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0 > R_0$ and the last inequality follows from $\psi_P > \frac{Z_M}{R_0 - Z_M}$. Second, we prove that there is no equilibrium where only the active fund raises positive AUM and $\lambda > 1$. To see this, consider any equilibrium where $W_P = 0$ and $W_A > 0$, but without restricting λ to be equal to one (that is, allowing for $\lambda > 1$). Then, the derivation of the equilibrium is again slightly different from that in Proposition 1, because the outside option of the fund investor in his bargaining with the active fund is not equal to λ, but is equal to one. Therefore, following the same steps as those used in deriving (23), but plugging in ε for the outside option of the investor in the fee bargaining, yields the following fixed point equation:

$$f_A = \eta \left(1 - \frac{P_L}{R_L}\right). \tag{72}$$

Since $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$ and $P_L = R_L - Z_L$ still hold, we have

$$W_A = \frac{1}{2}x_{AL}P_L = \frac{1}{2}\frac{c_A e_{AL}}{f_A}P_L = \frac{1}{2}c_A\frac{R_L}{\eta Z_L}2(R_L - R_M)(R_L - Z_L), \tag{73}$$

where the second equality follows from (2) and the third equality utilizes (9)-(10). Note that λ is still given by (26), and plugging (72) and $P_L = R_L - Z_L$ in (26), λ can be expressed as

$$\lambda = (1 - f_A)\frac{R_L}{P_L} - \psi_A = (1 - \eta)\,\frac{R_L}{R_L - Z_L} + \eta - \psi_A,$$

which strictly decreases in R_L. Since the right-hand side in (73) strictly increases in R_L, this implies that W_A strictly decreases in λ. Moreover, if $\lambda = 1$, then (73) is equal to (71), since (70) and (72) are equal. Combining this with the continuity of (73) in R_L, as λ converges to 1 from above, (73) converges to (71). Thus, $W_A < W_A(1)$ for all $\lambda > 1$. Since $W_A(1) < W$, this implies that $W_A < W$ for all $\lambda > 1$, and hence it cannot be $\lambda > 1$ in equilibrium, since if it were, then no investor would invest in the outside asset, resulting in a contradiction. ■