Who benefits the most? Risk pooling in Mortgage Loan Insurance: Evidence from the Canadian Mortgage Market*

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Abstract

This paper evaluates the effect of mortgage loan insurance (MLI), an essential macroprudential tool available to policymakers, on housing affordability, household leverage and the overall welfare of the economy. A dynamic model of the housing market with heterogeneous households and competitive housing and mortgage markets is constructed and is calibrated to Canadian data. The model shows that, through a flat MLI premium, low-risk households are subsidizing high-risk borrowers who have low incomes and low levels of wealth. We find that relaxing the mandatory nature of MLI required for mortgages with a loan-to-value ratio of 80% or more, has significant consequences for homeownership and affordability as well as vulnerabilities and default rates. Removing mandatory MLI dampens demand for housing to purchase and puts downward pressure on house prices. Some of the households with low income and low asset holdings can no longer afford a house; therefore, the aggregate homeownership rate drops. In contrast, demand for rental units increases, and rents go up. Compared to a baseline calibration with mandatory MLI, the Canadian housing finance system would feature lower leverage among households and significantly lower default rates.

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1 Introduction

Mortgage loan insurance (MLI) is a form of credit enhancement that protects mortgage lenders against loss by reason of borrower default when the collateral property value is insufficient to pay off the outstanding debt.[1] By reducing a lender's credit risk, MLI encourages lenders to qualify more prospective borrowers for a mortgage loan. Besides covering borrower defaults, MLI provides support to underserved borrowers through its cross-subsidization of insurance premiums by risk pooling. Risk pooling allows for charging lower-risk borrowers a greater premium than needed to achieve a targeted return and for charging higher-risk borrowers less than needed instead. Most commonly, the availability of MLI can expand lending for a home purchase by reducing the amount of cash that a first-time buyer is required to accumulate for the down payment. By serving as an "equity substitute," MLI can permit more families to own their own home sooner.[2]

As in most countries, the increase in house prices in recent years has made housing affordability a challenge in Canada. Moreover, the escalating house prices has coincided with an increase in the number of homeowners taking on high debt-to-income ratios. This has put a spotlight on the role of MLI in Canadian housing finance and has intensified debate on whether further reforms to MLI are needed to limit the build-up of risk in housing markets.[3] As a result, some of the current policy discussions imply the refinement of the current regulatory framework, such as a significant reduction in the supply of government-backed MLI with a level pricing.[4] This paper develops a dynamic model of the Canadian housing market to study the effect of MLI on housing affordability, household leverage and the overall welfare (a measure of utility or happiness) of the economy. The model is novel because it features agents that are heterogeneous in their idiosyncratic income shocks and that receive idiosyncratic house value shocks. In the model, a housing market with heterogeneous households and competitive housing and mortgage markets is constructed, and

[1] It is not to be confused with mortgage payment protection insurance, which covers the borrower's mortgage payments in the event the borrower loses their job or falls victim to ill health.

[2] Another potential use of MLI— again by reducing mortgage credit risk— is to induce institutional investors (the "secondary market") to purchase mortgage-backed investments, thereby increasing the flow of capital into home mortgage lending. In addition, MLI serves as a macroprudential tool that acts as a shock absorber and insulates the financial system from the large losses that follow a housing crash.

[3] Since mortgage insurers currently cover most of a lender's costs associated with default, such as mortgage principal and foregone interest payments, insurers are exposed to the bulk of potential losses from a housing crash.

[4] For example, see the Department of Finance's most recent proposal on lender risk sharing for government-backed insured mortgages to limit risk build-up in the housing sector: https://www.fin.gc.ca/n16/16-136-eng.asp.

both mortgage spreads and household tenure decisions are endogenously determined.

Key features of current regulations in Canada that are especially applicable to MLI include mandatory MLI for a mortgage with a loan-to-value (LTV) ratio of 80% or more. Low-down-payment mortgages are called high-ratio mortgages. By law, federally regulated depositories can hold a high-ratio mortgage only if there is mortgage insurance to protect against default. This mandatory mortgage insurance can be provided by a government corporation (Canada Mortgage and Housing Corporation—CMHC), or a private company.[5] The federal government currently backstops both mortgages insured by CMHC as well as those insured by private mortgage insurers. Furthermore, CMHC insurance premium pricing for high-ratio mortgages is based solely on LTV ratios and it can range from 2.4% to 4.5%. This method of pricing allows risk pooling across different borrower types and market segments. CMHC currently acts as a price leader, and private insurers follow by setting comparable premiums.

In our baseline model, we have both public and private MLI companies, and they charge an LTV- based flat premium. The flat premium creates a cross subsidy between low-risk and high-risk borrowers. Low-risk households with high incomes and high levels of wealth subsidize high-risk households with low incomes and low levels of wealth. By relaxing the mandatory nature of MLI required for mortgages with an LTV of 80% or more, borrowers are free to choose between insured and uninsured loans. Lenders and insurance companies charge a rate associated with the idiosyncratic characteristics of the borrower.[6] Therefore, lenders' marginal cost is the same for insured and uninsured loans under the optional MLI regime.

As a result, in our counterfactual world, borrowers pay either a risk-based interest rate on their loan (if they get an uninsured loan), or a risk-based MLI premium, plus interest rate (if they get an insured loan). Credit subsidy receivers (i.e., households with low income and low wealth) lose the credit subsidy embedded in the world with a flat premium. As a result, some households can no longer afford to buy a house. This dampens housing demand and puts downward pressure on house prices. Households who cannot afford a risk-based interest rate and/or a risk-based MLI premium switch to the rental market. Therefore, demand for rental units increases, and rents go up. More middle-income and middle-wealth households become homeowners (extensive

[5]Genworth Canada and Canada Guaranty are the two private insurance companies in Canada.

[6]The structural change also implies relaxing the explicit government guarantee; however, we are not directly modeling the guarantee.

margin), and they can afford to purchase bigger houses (intensive margin). Importantly, as a result of this change, the aggregate homeownership rate in the economy drops by 6.7%. House prices, meanwhile, decline by 5.5%, and rents increase by 8.3%.

Overall, compared to a baseline calibration with mandatory MLI, vulnerabilities and risks in the counterfactual world are lower than the status quo with mandatory MLI. The share of highly indebted borrowers with loan-to-income ratios over 450% drops by 78%. This translates to lower aggregate household leverage. The rate of foreclosure also drops by 87%. Lenders are taking on lower risk as the average LTV among mortgage holders drops by 43%.

Our findings have important implications for how the structural changes to the current mortgage market impact debt and housing market outcomes in different segments of the market. First, removing the supply of government-backed MLI can improve the welfare of most Canadians and strengthen the financial stability of the Canadian economy. However, under a no-mandatory-MLI regime, some low-income borrowers with small equity holdings lose their credit subsidy. Some of these potential borrowers would not be able to afford the purchase of a home and would remain renters. As a result, demand for houses to purchase decreases, and demand for rental units increases. Therefore, any significant reduction in the supply of government-backed MLI needs to be accompanied by targeted programs aimed at providing affordable housing options to underserved populations. Many renters face housing affordability challenges, any increase in rent prices would exacerbate affordability problems for a group that is already in core housing need.[7] Therefore, the key to making this policy successful is to make sure that there is enough supply of affordable rental units to dampen the increased demand for rental units.

1.1 Related literature

Our paper builds on and contributes to several strands of literature. First, several studies explore the impacts of structural changes to the housing finance system in the U.S. These studies mainly focus on government bailout guarantees against tail events. For example, Jeske et al. (2013) study the aggregate and distributional impacts of government bailout guarantees. They model a

[7] A household is said to be in "core housing need" if its housing falls below at least one of the adequacies, affordability or suitability standards and it would have to spend 30% or more of its total before-tax income to pay the median rent of alternative local housing that is acceptable. More details on the dimensions of core housing need by housing tenure can be found here: https://www12.statcan.gc.ca/census-recensement/2016/dp-pd/chn-biml/index-eng.cfm.

bailout guarantee as a tax-financed mortgage interest rate subsidy that is passed through to the borrowers. Their model augments an Aiyagari (1994)-type framework with a housing sector and a mortgage market. Jeske et al. (2013) find that the removal of the bailout guarantee benefits low-income and low-asset households. Elenev et al. (2016) study the role of bank bailout guarantee fees in a general equilibrium framework. They focus on the impacts of raising guarantee fees in an environment with underpriced bailout guarantee fees. They find that increasing the price of bank bailout guarantees increases wealth inequality. Gete and Zecchetto (2017) study the distributional impacts of removing government-sponsored enterprises' (GSEs) credit risk guarantees. They show that the removal of GSEs' credit risk guarantees benefits high-income households, hurts low-income households, and decreases aggregate welfare. Our paper develops a general equilibrium framework, extending Gete and Zecchetto (2017), that reflects the Canadian mortgage system. Our key innovation is to capture both public and private mortgage insurers. While we focus on Canada, it is worth noting that such a housing finance structure is common and can be seen, for example, in the U.S. and the Netherlands, where both public and private mortgage insurers coexist. Moreover, we evaluate the effects of mandatory MLI as an essential macroprudential tool available to policy makers and abstract from government bailout guarantee.

A number of other empirical studies have analyzed the effects of changes and discontinuities in the supply of government guarantees on the housing market. Kaufman (2014) uses a regression discontinuity design around GSEs' conforming loan limits (CLLs) to estimate the effects of GSE eligibility on mortgage characteristics. The paper finds no measurable effect of GSE purchase eligibility on loan performance and only a modest impact on loan terms. More recently, Grundl and Kim (2019) studied the marginal effect of government mortgage guarantees on homeownership by applying a difference in difference method to exploit the variation of CLLs, and they found no robust effect on homeownership.

Our paper also follows a strand of literature that uses heterogeneous-agent models with idiosyncratic income and housing shocks to study the impacts of housing policies. Floetotto et al. (2016), Chambers et al. (2009), Gervais (2002) and Chatterjee and Eyigungor (2015) use heterogeneous-agent models to study the distributional impacts of U.S. tax policies on the economy.

2 Mortgage loan insurance in Canada

In Canada, legislation prohibits federally regulated financial institutions from providing residential mortgages without MLI to borrowers with a loan-to-value (LTV) ratio above 80%. This type of MLI is called "high-ratio transactional mortgage insurance."[8] Most provincial regulators impose similar rules. Insured residential mortgages accounted for 38% of total outstanding residential mortgages in 2019 Q4. The mortgage insurance premiums are paid by the borrower, either up front in a lump sum, or added to the mortgage principal and paid along with the regular mortgage payment. Note that in Canada, unlike the U.S., mortgage interest payments are not tax-deductible.[9] Lenders also have the option of purchasing portfolio insurance after the origination of loans. Portfolio insurance premiums are paid by lenders for each insured portfolio. Both transactional and portfolio insurance cover the entire amount of the loan and are for the entire life of the mortgage. The insurance remains in place at term renewal and can be transferred if the borrower decides to change lenders or change homes.[10]

In some countries, like Australia and Denmark, MLI is only available through private insurance companies. In other countries, like the U.S. or Canada, government-controlled insurance agencies play a pivotal role. The main difference is insurance premium pricing and the explicit guarantee offered by the government.

In Canada, Canada Mortgage and Housing Corporation (CMHC), a federal Crown corporation, and two private insurers, Genworth and Canada Guaranty, provide transactional mortgage insurance. Portfolio insurance is only offered by CMHC. In case of mortgage insurers' insolvency, the Government of Canada, under the *Protection of Residential Mortgage or Hypothecary Insurance Act* (PRMHIA), provides an explicit guarantee for all insured mortgages.[11] Insurance premium pricing for CMHC's transactional product is based solely on LTV ratios and it can range from 0.6% to 4.5%.[12] This method of pricing allows risk pooling across different borrower

[8] Borrowers with an LTV ratio below 80% do not require insurance, but can obtain it at their discretion and the discretion of their lender. This insurance is called "low-ratio transactional mortgage insurance."

[9] In the U.S., MLI payments by the borrower were tax-deductible until 2018.

[10] As of October 2016, federal rules prohibit insurance for refinance loans where the balance of the loan has increased upon renewal or refinance of the loan.

[11] The PRMHIA guarantee provides 100% coverage for CMHC-insured loans, and 90% coverage for privately-insured mortgages. The Government of Canada collects a guarantee fee to cover tail exposure to insured mortgages.

[12] For more information on CMHC premium rates, see here: https://www.cmhc-schl.gc.ca/en/finance-and-investing/mortgage-loan-insurance/mortgage-loan-insurance-homeownership-programs/cmhc-mortgage-loan-insurance-cost.

types and market segments. CMHC currently acts as a price leader, and private insurers follow CMHC's LTV-ratio-based pricing. Premium pricing for portfolio insurance is more risk-based and includes LTV ratios, credit scores and other risk factors.

In the U.S., there is no legal obligation, but in the context of most conventional (non-government-backed) mortgage programs, lenders typically impose MLI upon borrowers when borrowers' LTV ratios are above 80%. MLI is typically provided by private mortgage insurers, and the insurance premium structure is more risk-based, and considers percent of the loan insured, LTV ratio, a fixed or variable interest rate structure, and credit score. The premium is much lower than in Canada, and ranges from 0.14% to 2.24% of the principal balance per year.

Unlike in Canada, the coverage provided by private mortgage insurance in the U.S. is "partial," meaning that the insurance covers only a portion of the total loan amount. Mortgage insurance is also offered by the federal government through the Federal Housing Administration (FHA), the Department of Veterans Affairs (VA) and the U.S. Department of Agriculture (USDA). FHA, VA and USDA insurance coverage are similar to that offered by Canadian mortgage insurers, protecting lenders for the entire amount of the loan and the entire life of the mortgage. While FHA coverage is comparable to Canadian coverage, the premium structure is different. An FHA loan requires both an up-front amount (1.75% of the base loan amount), regardless of loan or borrower characteristics, and an annual premium ranging from 0.45% to 1.05% of the base loan amount, depending on three variables: term, loan amount, and LTV ratio.

3 Model environment

The model economy is composed of five components; households, markets, lenders, mortgage insurers and the government. Households receive income, consume, save, make housing tenure and housing financing decisions. Households decide whether to purchase their house with cash or use (insured or uninsured) financing. The model features five markets: owner-occupied housing, rental housing, consumption goods, loan, and deposit. Risk neutral lenders supply mortgages in a perfectly competitive loan market. We have a fixed supply of housing, but rental supply is endogenously determined. Houses are risky assets, therefore mortgage borrowers may default on their mortgages. Borrowers pay a premium and purchase MLI coverage for mortgages with loan-

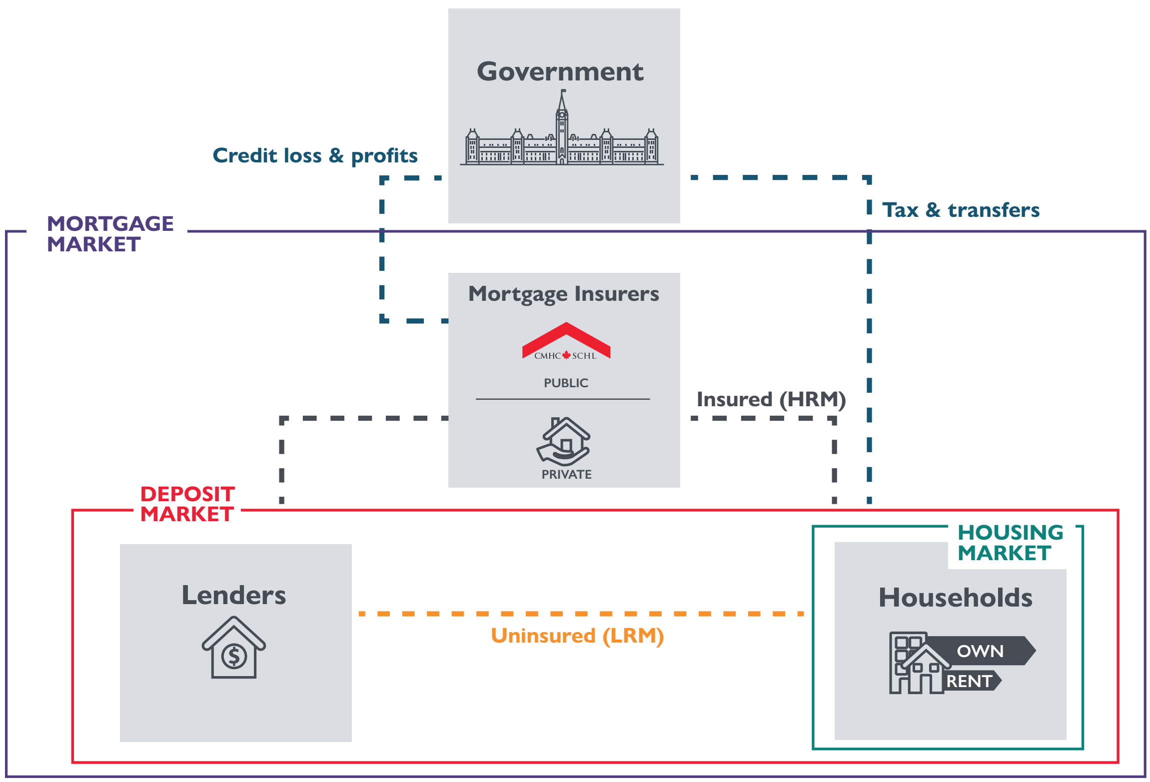


Figure 1: Model players and markets

to-value ratio of at least 80 percent. There are two suppliers of MLI: public and private mortgage default insurance companies. Lenders foreclose defaulted houses and absorb losses caused by uninsured mortgage defaults. In case of Insured mortgage defaults, MLI covers the entire lender losses. Figure 1 shows model players and the structure of markets.

3.1 Households

Households derive utility from consumption c and housing services s. Housing services can be either owned or rented,

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, s_t\right),$$

where $\beta \in (0,1)$ is the discount factor. The tenure status of a household is denoted by

$I_h \in \{0,1\}$, 0 for renter and 1 for homeowner. The utility function $u(c_t, s_t)$ is given by

$$u(c_t, s_t) = \frac{\left[\eta c^{\frac{\epsilon-1}{\epsilon}} + (1-\eta) s^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\sigma)}{\epsilon-1}}}{1-\sigma},$$

where ϵ is the elasticity of intratemporal substitution, σ is the degree of relative risk aversion and η is the share of nonhousing goods in the utility. Household receives an idiosyncratic stochastic disposable income $y \in Y$, which follows a finite state Markov chain with transition probabilities $\pi(y'|y)$ and unique invariant distribution $\Pi(y)$, and mean $\bar{y} = \sum_{y \in Y} y \Pi(y)$. Because of law of large number, π and Π describe the fraction of households receiving a particular income shock, and $\bar{y}$ is the aggregate disposable income.

3.2 Markets

There are five markets: owner-occupied housing, rental housing, consumption goods, loan, and deposit. Households can invest in one-period deposits $P_d d'$which pay d' next period, therefore the gross risk-free rate is $\frac{1}{P_d}$.

Shelter services can be rented at rental price P_s or obtained from owning a house. The price of a house is $P_h h$, where h is the size of the house. The stock of housing (H) is in fixed supply, but rental supply is endogenously determined. A household can be a renter $(h = 0)$, a homeowner $(h = s)$, or a landlord who rents out part of her housing holdings $(h > s)$. To have well-defined renters and owners, there is a minimum house size for ownership, $h \geq \underline{h}$, but no minimum size for rental. Moreover, to match the relative size of owner-occupied and rental housing, there is a minimum housing consumption for landlords, $\underline{s} < \underline{h}$.

The value of a house is uncertain because of a stochastic depreciation shock. The shock δ is idiosyncratic, such that if a house of size h is bought today, then next period, the size of the house is $(1-\delta')h$. The associated CDF is $F(\delta')$ with support $[\underline{\delta}, 1]$, where $\underline{\delta} \leq 0$. Thus, houses are risky assets.

If a household buys a house, she can use it as collateral for one-period mortgage debt. The principle of the loan is denoted by $P_m m'$, and the amount to be repaid next period is m'. The gross rate is $\frac{1}{P_m}$ and the spread is $s_m = \frac{1}{P_m} - \frac{1}{P_d}$.

A borrower can default at the cost of losing her housing stock, a fraction $\phi_y < 1$ of her disposable income, and a fraction ϕ_d of her deposit. ϕ_y and ϕ_d represent garnishment rates of disposable income and deposits, respectively. Thus, default is optimal if and only if

$$y' + d' + P_h \left(1 - \delta'\right) h - m' < (1 - \phi_y)\, y' + (1 - \phi_d)\, d'. \tag{3.1}$$

As a consequence, the probability of default is a function of (m', h, d') and y, which affects the realization of y' through $\pi\,(y'|y)$.

Households can choose between HRM (high-ratio $LTV > 80\% = \theta^L$), and LRM (low-ratio $LTV \leq 80\% = \theta^L$) mortgage loans. Indicators (I_H, I_L) take the value of 1 if the household chooses specific mortgage types respectively. HRM must be associated with a mortgage loan insurance, either from CMHC or from private insurers[13]. Note HRM loans are subject to a maximum loan size $\bar{l}$, and to a maximum loan-to-value ratio $\theta^H = 5\%$[14].

3.3 Lenders

Lenders are risk neutral and participate in a competitive market. Loans are funded using deposits at cost $\frac{1}{P_d}$. In addition, lenders also incure an origination cost r_w per each unit of mortgage issued. The mortgage industry is perfectly competitive in the sense that lenders originate any mortgage that in expectation allows them to cover their cost of funds. Lenders take into account that households may default. If the borrower defaults, then the loss for the lender is

$$L\left(m', h, d', y', \delta'\right) = m' - \phi_y y' - \phi_d d' - \gamma P_h \left(1 - \delta'\right) h. \tag{3.2}$$

Therefore, in case of a default, the lender receives a fraction $\gamma < 1$ of the house value, a share ϕ_y of borrower's after-tax labour income, and a share of ϕ_d of her deposits.

In case of an HRM loan default, the insurer of the mortgage completely assumes the lender's loss[15]. Borrowers pay a premium, proportional to the loan amount, g^H, to receive mortgage

[13] Currently, private insurers, Genworth and Canada Guranty, both follow CMHC's MLI premium structure.

[14] In Canada, the maximum purchase price for Government-backed insured mortgages is limited to \$1 million value. Also, for a residential property with purchase price less than \$1 million, the minimum down payment is at 5% for the first \$500 thousand and 10% for the rest, respectively. Therefore, the dollar value of maximum loan size for CMHC MLI is given by \$925 thousand. And for simplicity, we assume $\theta^H = 5\%$.

[15] In Canada, mortgages insured by CMHC are fully backed by the government. Lender's default loss on privately insured mortgages are covered by the private insurance companies. Since the government spending G is exogenous and free to choose in our model, whether the government or private sector, absorbs default loss on insured mortgages is not relevant for our study.

insurance for HRM loans. In contrast, for LRM loans, the lender absorbs all mortgage default losses. Thus, a borrower owing mortgage repayments m', with house size h, deposits d', and realized labour income y' will default whenever her depreciation shock δ' is larger than the threshold δ^* implicit in the following equation:

$$\delta^* \left(m', h, d', y'\right) = 1 + \frac{\phi_y y' + \phi_d d' - m'}{P_h h}. \tag{3.3}$$

Given the above, lenders price mortgages according to zero-profit conditions as follows.

For HRM loans

$$\frac{\left(1 + r_w + g^H\right) P_m^H m'}{P_d} = m'. \tag{3.4}$$

Here, we assume a flat mortgage insurance premium for all mortgages with $LTV > 80\%$.

For LRM loans,

$$\frac{\left(1 + r_w\right) P_m^L \left(m', h, d', y\right) m'}{P_d} = \sum_{y' \in Y} \pi \left(y'|y\right) \left\{ \begin{array}{c} m' F\left(\delta^*\right) + \\ \int_{\delta^*}^1 \left[\left\{ \phi_y y' + \phi_d d' + \gamma P_h \left(1 - \delta'\right) h \right\} \right] dF\left(\delta'\right) \end{array} \right\}. \tag{3.5}$$

Note, since there is no guarantee on LRM loans, the above expression does not include any guarantee fee.

3.4 Government

The government collects guarantee fees and raises taxes to finance transfers, government spending, and the credit risk guarantees. The credit losses absorbed by the government from HRM loans is CMHC's share of total losses in the mortgage insurance market:

$$\Psi_H = \Theta_{CMHC} \int_X \sum_{y' \in Y} \pi \left(y'|y\right) \int_{\delta^*}^1 I_H\left(x\right) L\left(m'\left(x\right), h\left(x\right), d'\left(x\right), y', \delta'\right) dF\left(\delta'\right) d\mu,$$

where, Θ_{CMHC} is the exogenous market share of CMHC in the mortgage insurance market. Thus, the government budget constraint equals the government's mortgage guarantee-fee income to government's expenditures. These expenditures include mortgage losses and exogenous government spending net of tax receipts and transfers:

$$g^H \int_X I_f(x)\, P_m^H m'(x)\, d\mu = \Psi_H + G. \tag{3.6}$$

3.5 Recursive formulation of household problem

The household decides her consumption, savings in deposits, tenure choice, and whether to take a HRM, or a LRM mortgage loan. Household wealth includes wealth from housing and financial assets. The details of the recursive formulation of household's problem is in appendix A.1

3.6 Equilibrium

A stationary recursive competitive equilibrium is a set of value and policy functions for HRM and LRM mortgagors, and renters: $V_H(x)$, $V_L(x)$, $c_H(x)$, $s_H(x)$, $d'_H(x)$, $h_H(x)$, $m'_H(x)$, $c_L(x)$, $s_L(x)$, $d'_L(x)$, $h_L(x)$, $m'_L(x)$,$c_r(x)$, $s_r(x)$, $d'_r(x)$, household tenure and mortgage choices $I_H(x)$, $I_L(x)$, $I_r(x)$, house price P_h, shelter price P_s, deposit interest rate $\frac{1}{P_d}$, mortgage prices for HRM and LRM loans P_m^H and $P_m^L(m', h, d', y)$, and a probability measure μ over X such that:

1. Given prices, the value and policy functions solve the household problems.
2. Given prices, the HRM and LRM mortgage pricing satisfy (3.4) and (3.5) for any household's choice.
3. The government budget constraint is satisfied.
4. All markets are cleared.
   - Housing market

$$\int_x s(x) = H,$$
$$\int_X h(x) = H.$$

   - Rental market

$$\int_X (1 - I_h(x))\, s(x)\, d\mu = H - \int_X I_h(x)\, s(x)\, d\mu.$$

– Credit marekt

$$\int_X P_d d'(x)\, d\mu = \left\{ \begin{array}{c} \left(1 + r_w + g^H\right) \int_X I_H(x)\, P_m^H m'(x)\, d\mu \\ + (1 + r_w) \int_X I_L(x)\, P_m^L\left(m'(x), h(x), d'(x), y\right) m'(x)\, d\mu \end{array} \right\}.$$

– The goods market

$$\bar{y} = \left\{ \begin{array}{c} G + \int_X c(x)\, d\mu + i_h + r_w \int_X I_H(x)\, P_m^H m'(x)\, d\mu \\ + r_w \int_X I_L(x)\, P_m^L\left(m'(x), h(x), d'(x), y\right) m'(x)\, d\mu \end{array} \right\},$$

where i_h is the investment to cover both the housing net depreciation and the foreclosure costs:

$$i_h = P_h \int_X \sum_{y' \in Y} \pi\left(y'|y\right) \left[\int_{\underline{\delta}}^{\delta^*} \delta' dF\left(\delta'\right) + \int_{\delta^*}^{1} \left(1 - \gamma\left(1 - \delta'\right)\right) dF\left(\delta'\right)\right] h(x)\, d\mu.$$

3.6.1 Welfare

The Consumption Equivalent Variation (CEV) could be used to evaluate the welfare changes after the policy change. The CEV, $\omega(a, y)$, measures the change in per-period composite consumption such that a household is indifferent when moving from a stationary economy with public MLI to another without public MLI. Note here we only compare the steady states before and after the policy change.

Let $\tilde{u}(\tilde{c}) = u(c, s)$ be the utility of a household in terms of composite consumption, i.e. $\tilde{c} = \left[\eta c^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)\, s^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$. For each state (a, y), we solve for $\omega(a, y)$ such that

$$E_o\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left((1 + \omega(a, y))\, \tilde{c}_t\right) | (a, y)\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(\tilde{c}_t^N\right) | (a, y)\right], \tag{3.7}$$

where the superscript N refers to the economy after the policy change. If $\omega(a, y) > 0$, the household has higher utility after the policy changes. In order to make the household indifferent before and after the policy change, she needs to be compensated in the economy before the change. We use the above expression to calculate welfare gains and losses of policies.

4 Calibration and computation

The model is mapped to the Canadian economy. Some parameters are selected endogenously, and the remaining parameters are calibrated jointly in the model. Exogenous household-level parameters are based on existing literature on Canada (Brzozowski et al. (2010) and Bruneau et al. (2016)). Exogenous mortgage market parameters are selected based on CMHC-insured administrative data.

Households' disposable income follows an AR(1) process

$$\log\, y' = \bar{w} + \rho \log\, y + \varepsilon,$$

where ε is a random process with $E\left(\varepsilon\right) = 0$ and $E\left(\varepsilon^2\right) = \sigma_\varepsilon^2$. We set the standard deviation of the innovation σ_ε to 0.1735 similar to Brzozowski et al. (2010)[16]. We set the persistence parameter ρ to match a Gini index of disposable income in Canada at 0.322. The process is approximated with a 7 state Markov chain using Rouwenhorst (1995) procedure.

Following Jeske et al. (2013), we assume a generalized Pareto distribution for the housing depreciation shock δ'. Comparing to a standard log-normal distribution, a generalized Pareto distribution gives us a fat right tail, which helps us better match the foreclosure targets. We truncate the distribution to the interval $[\underline{\delta}, 1]$, where $\underline{\delta} \leq 0$. the CDF is

$$F\left(\delta'\right) = \frac{1 - \left(1 + \xi \frac{(\delta' - \underline{\delta})}{\sigma_\delta}\right)^{-\frac{1}{\xi}}}{1 - \left(1 + \xi \frac{(1 - \underline{\delta})}{\sigma_\delta}\right)^{-\frac{1}{\xi}}},$$

where $\underline{\delta}$, σ_δ and ξ are location, scale and shape parameters of the above generalized Pareto distribution, respectively. The results of the calibration exercises is presented in appendix A.2 (Table 3 show parameter values, which are selected endogenously and table 4 shows parameter values, which are selected endogenously and those which are jointly calibrated.)

In order to compute the model, we solve the simplified household problem using grid-search methods. For a guessed set of prices, we derive the stationary distributions of the state variables

[16]Brzozowski et al. (2010) estimate σ_ε^2 with the assumption of unit root process for the persistent component of disposable income. Our estimation of $\rho = 0.9580$ represents a highly persistent AR(1) process, which is essential to generate a stationary income distribution.

and check if equilibrium conditions hold within our convergence criteria. If the equilibrium conditions do not hold, we update the initial guess for prices and we solve again until we meet our convergence criteria. The details of our computational approach is in appendix A.3.

5 Results

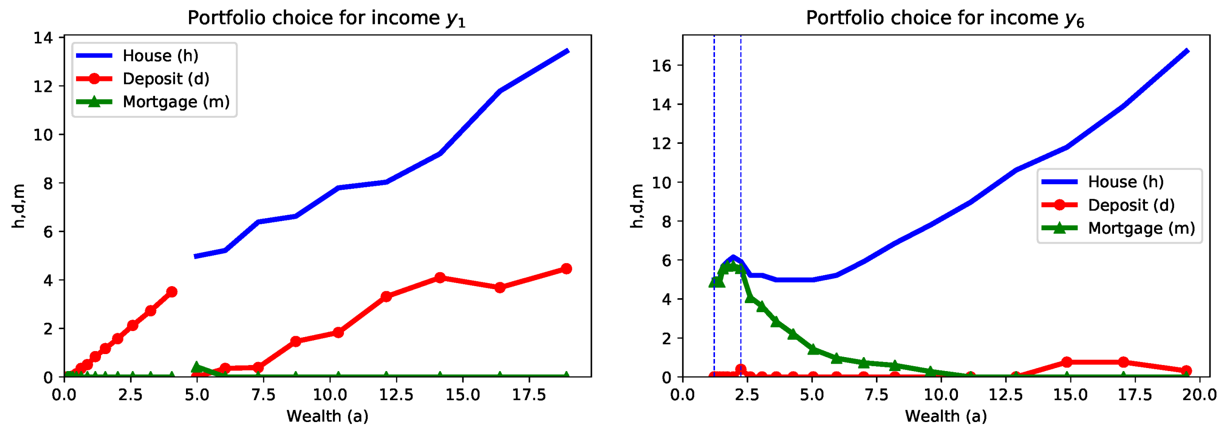
Before analyzing the impacts of mandatory MLI by comparing the equilibria with and without mandatory MLI in place, it is instructive to explain household behaviour and outcomes in the baseline economy. First, we present household behaviour and aggregate outcomes in the baseline economy. Then, we discuss the results of our counterfactual policy exercise.

Household behavior in the baseline economy

In figures 2 to 4, we present household choices for different levels of wealth and select income levels. As explained in the section on calibration and computation, we have seven different levels of income. In this section, we show household choices for only select income levels to present the main behaviours and outcomes in the economy. Where present in graphs, the area between the two dashed vertical lines shows the population of homeowners who purchased with MLI.

Figure 2 shows households' portfolio choices for low-income and high-income households. At the lowest level of income, y_1, households do not qualify for insured mortgages. This is because of the regulatory limit on debt-service ratios and the minimum size of houses for ownership. By law, lenders cannot issue insured mortgages when the borrower has a debt-service ratio above a prescribed ceiling. This regulatory limit forces low-income households to save in deposits in order to purchase a house in the future. Thus, households with low wealth holdings are renters. Low-income households who have enough savings to pass the less stringent debt-service ratio limits for uninsured mortgages purchase houses using uninsured mortgage financing.

Figure 2b shows that most high-income households are homeowners. A small portion of low-wealth homeowners finance through insured mortgages. Since MLI allows borrowers to obtain mortgages with LTV ratios up to 95%, these households can purchase bigger houses. As wealth increases, homebuyers put greater down payments toward the purchase of their houses to get uninsured mortgages and, as a result, household leverage declines.



(a) Portfolio choice for low-income (b) Portfolio choice for high-income

Figure 2: Household portfolio choice for low-income households(y_1) and high-income households (y_6). The area between the two vertical lines shows homeowners with insured mortgages.

Figure 3 shows households' probability of default for low-income and high-income households (income levels $y_1$and y_6, respectively). At the lowest level of income, only high-wealth households are homeowners and hold a mortgage. As wealth increases, the probability of default declines because of households' available equity. The expression for default threshold (δ^* in 3.3) shows as homeowner equity increases, there needs to be a larger depreciation shock to trigger a foreclosure. As such, for homeonwers with high enough equity, we do not observe any foreclosures.

High-income households (y_6) with low equity qualify for insured mortgages. MLI allows these households to borrow more. Figure 3 shows these borrowers' chance of default is much higher, due to the higher leverage allowed by MLI. As equity increases, households are able to put enough downpayment for uninsured mortgagses. Uninsured borrowers' minimum required LTV is 80% and a high income borrower has a very low chance of default. This can be confirmed in expression for δ^* in equation 3.3.

Figure 4 graphs the implicit credit subsidy provided through MLI. Since all of the borrowers at income y_1 hold uninsured mortgages, there is no credit subsidy paid or received at this income level. Insured borrowers with income level y_6 have a high chance of default. As such, they are net receivers of the credit subsidy. Figure 4c shows cross-subsidization between borrowers of insured mortgages at the highest level of income. Low-wealth borrowers of insured mortgages are net credit subsidy receivers, and as their wealth increases, the amount of subsidy received declines.

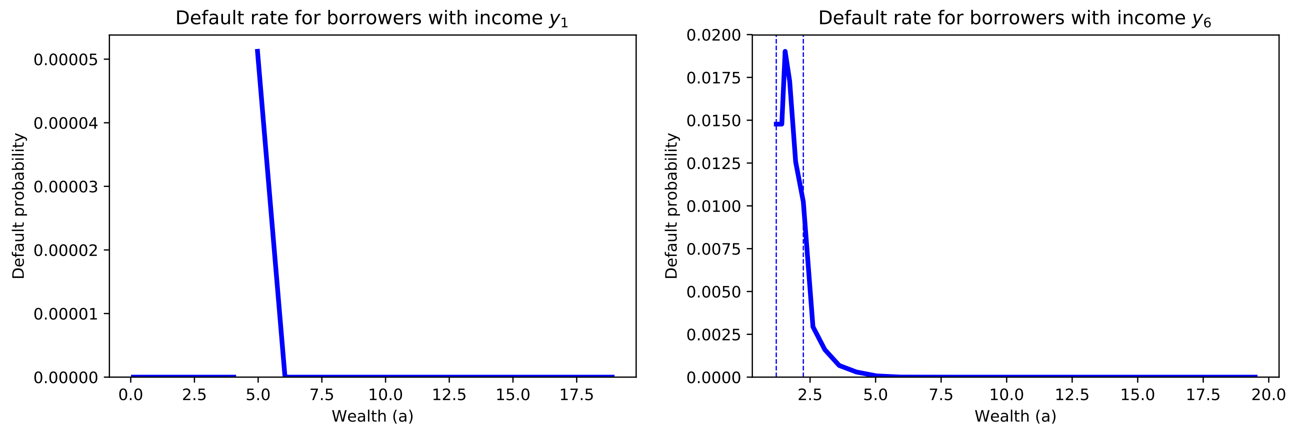


Figure 3: Household default probability for income y_1 and y_6. The area between two vertical lines shows homeowners with insured mortgages.

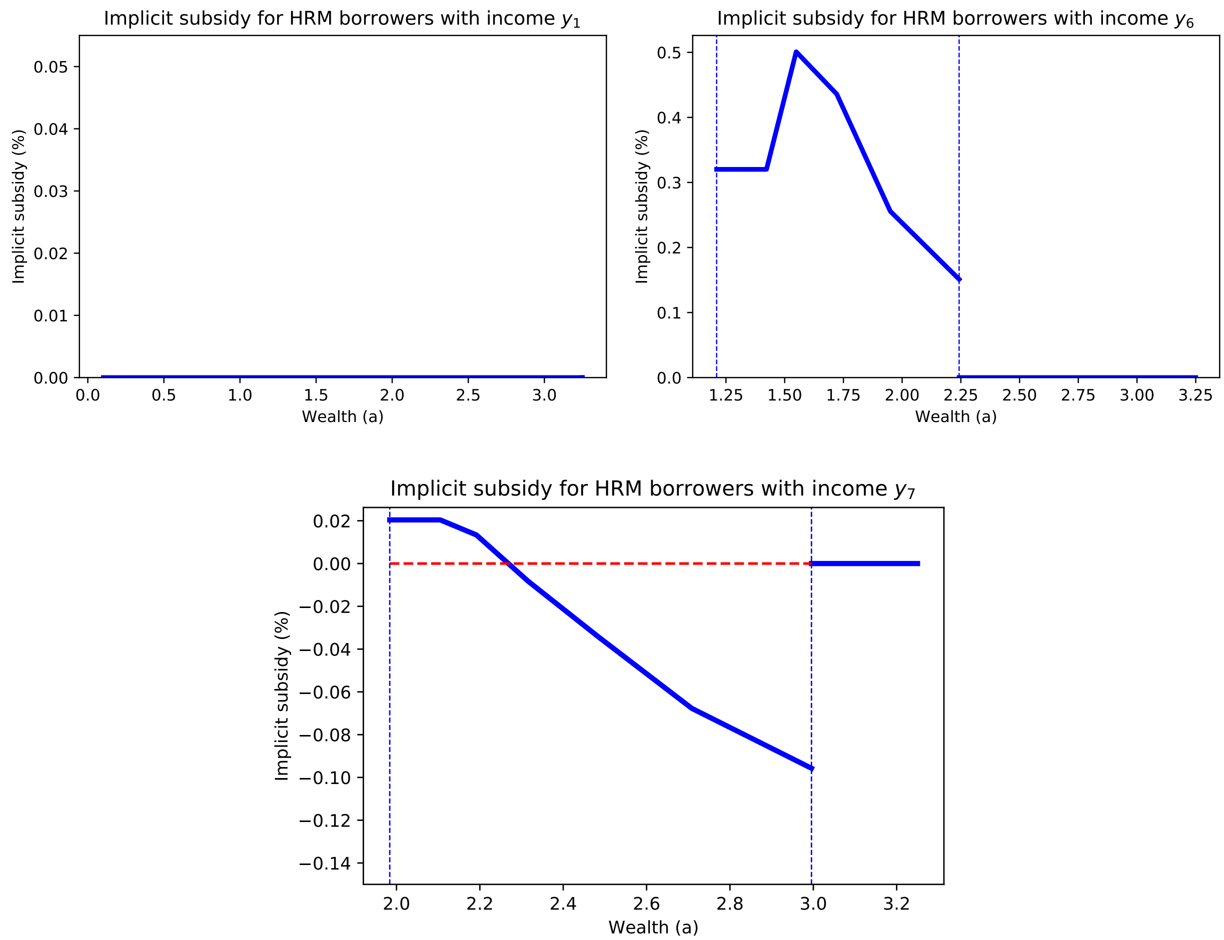
High-wealth borrowers of insured mortgages are net credit subsidy payers.

Aggregate outcomes in the baseline economy

Figure 5a shows the distribution of households across tenure and mortgage types, and figure 5b shows the steady state distribution of wealth. As can be seen in figure 5a, households with the lowest level of income are renters. In general, low-income homeowners do not qualify for MLI due to the tight regulatory debt-service ratio requirement for insured mortgages. Those low income-homeowners who have enough equity to purchase a house and pass the less stringent debt-service ratio limits for uninsured mortgages purchase houses using LRM. Borrowers of HRM mortgages are generally middle-income households (y_4 and y_5). At the right tail of distribution of income, all mortgage holders are uninsured borrowers, and some high-wealth borrowers purchase homes with no financing. Table 1 shows the results of our calibration exercise. Our model closely matches the share of housing services in total consumption and the share of HRM loans as a percentage of total mortgages.

5.1 Counterfactual policy exercise

This section explores how the economy reacts to changes in mandatory MLI requirements, which we can interpret as changes in macroprudential policy. Using our calibrated model, we remove the mandatory MLI requirement for mortgages with LTV ratios above 80%. The counterfactual

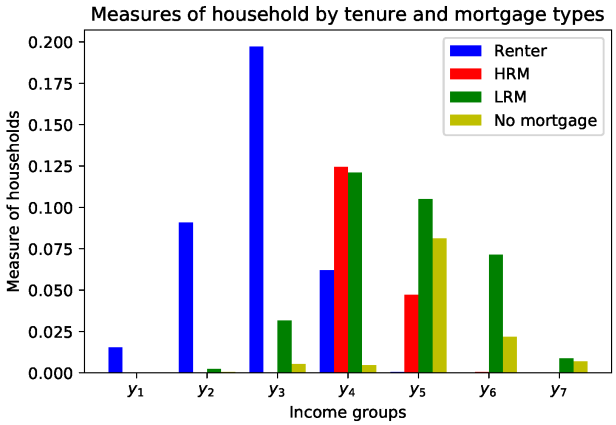


(c) Implicit subsidy for income y_7

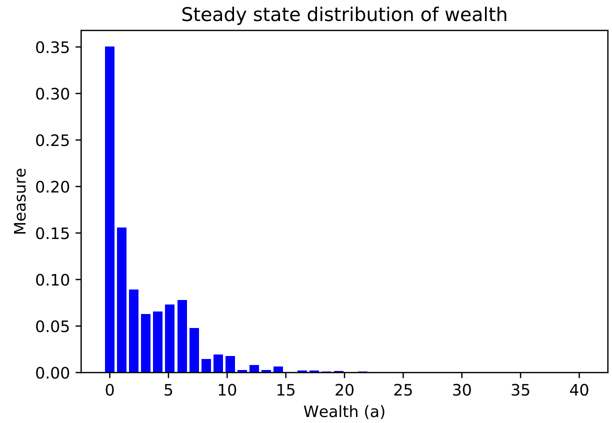
Figure 4: Implicit subsidy for income y_1, y_6 and y_7. The area between two vertical lines shows homeowners with insured mortgages.

	Target	Calibrated model
Housing services in total consumption (%)	14.1	14.26
Homeownership rate (%)	68	63.36
% of homeowners with mortgage debt	61	80.96
HRM loans as % of total mortgage volume	31	33.61
Median size of owner-occupied-to-rental housing	1.58	2.55
Average size of owner-occupied-to-rental housing	1.58	2.57
Foreclosure rate (%)	0.16	0.94

Table 1: Calibration results



(a) Distribution of households across tenure and mortgage types.



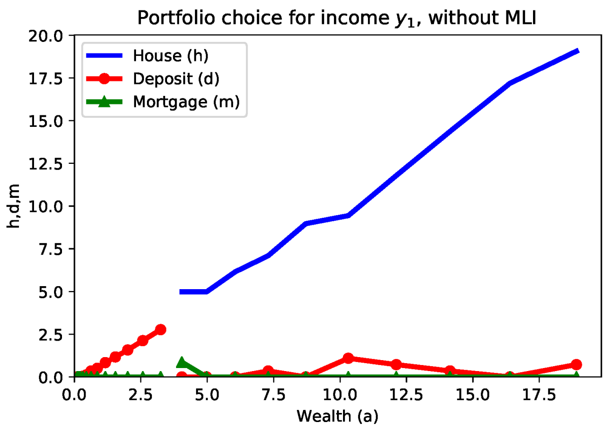
(b) Steady state distribution of wealth

Figure 5: Distribution of households across different states.

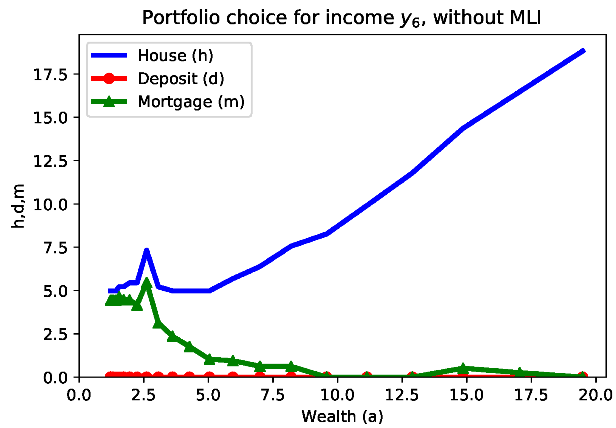
world still features optional MLI. We assume that, after the removal of mandatory MLI, optional MLI is provided by insurers in a perfectly competitive industry. Borrowers of mortgages with LTV ratios of at least 80% are free to choose between uninsured mortgages and insured mortgages with competitive premium prices. Since our lending market is perfectly competitive, uninsured mortgage rates are risk-based. We further assume that the cost structure of insurance companies is the same as the cost structure of lenders. Borrowers are free to choose between uninsured mortgages and insured mortgages, the uninsured mortgage rates are equal to insured mortgage rates plus insurance premiums. As such, competitive insurance premiums are risk-based. Insurers set premiums based on borrowers' observable risk factors (i.e. income and wealth).

Figure 6 shows the calibrated policy functions on portfolio choice for low-income and high-income households. Comparison of figures 2b and 6b indicates that, in a world with no mandatory MLI, high-income borrowers with limited equity would purchase smaller houses. Lower down payment requirements for mortgages with MLI allows these households to borrow more. This is mainly due to the implicit subsidy that MLI provides to these borrowers (figure 4).

Figure 7 shows the probability of default at different wealth levels. The probability of default for high-income borrowers with low wealth is lower than in a world with mandatory MLI. Lower default rates are the result of lower leverage in this space. Low-income borrowers with LTV ratios above 80% pay higher insurance premiums or interest rates, which makes borrowing more



(a) Portfolio choice for low-income, no MLI



(b) Portfolio choice for high-income, no MLI

Figure 6: Household portfolio choice for low-income and high income

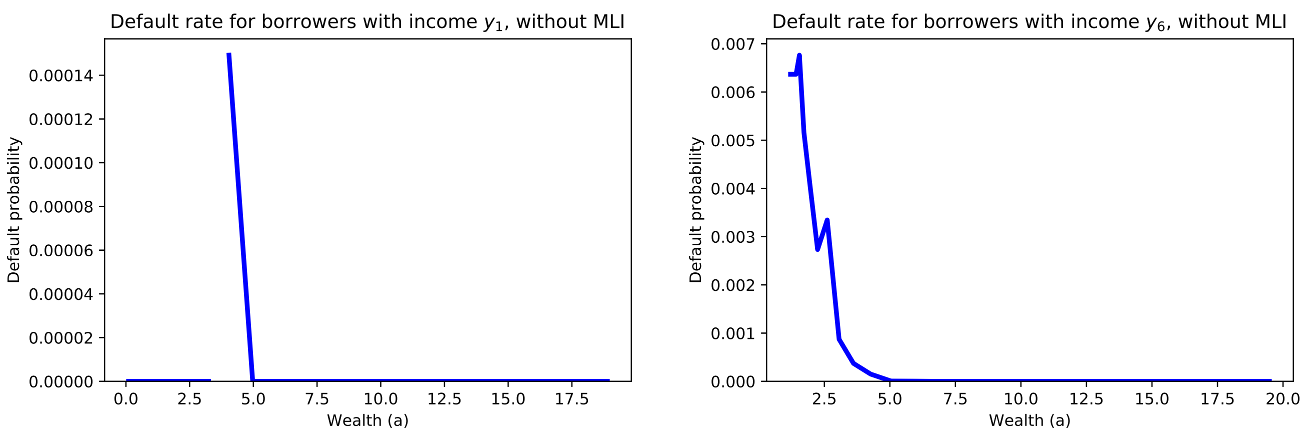
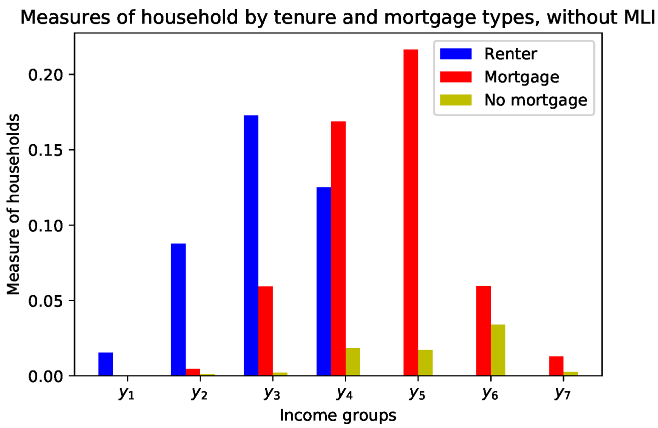


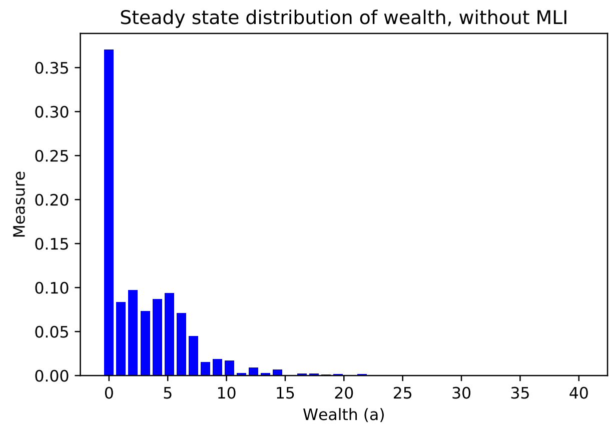
Figure 7: Household default probability for income y_1 and y_6.

expensive for them. Lower leverage decreases their probability of default. Figures 8a and 8b show the distribution of households across tenure types and the distribution of wealth in a steady-state world without mandatory MLI.

Table 2 summarizes the results of our counterfactual policy exercise. The table highlights two channels of impact: 1. House-price channel: as a result of the removal of mandatory MLI, demand for houses to purchase declines, and house prices fall by 5.5%. 2. Rent channel: Households who cannot afford a risk-based interest rate or a risk-based MLI premium switch to the rental market. Demand for rental units increases, and rents go up by 8.3%. In total, the aggregate homeownership rate drops by 6.7%. As a result of this policy, aggregate leverage in the economy decreases, and



(a) Measure of households by tenure and mortgage types.



(b) Distribution of wealth

Figure 8: Distribution of households across states, counterfactual world with no mandatory MLI.

	With MLI	Without MLI	% change
House prices	1	0.95	-5.5
Rent prices	0.0415	0.045	8.31
Home ownership (%)	63.36	59.12	-6.69
Borowers with LTV> 80% (%)	33.62	7.42	-77.93
Average mortgage rate	2.78	2.59	-6.83
Average mortgage size	2.66	1.47	-44.74
Borrowers with LTI>450% (%)	26,19	5.55	-78.81
Average loan-to-income (among mortgagors)	275.14	148.9	-43.21
Average loan-to-value (among mortgagors)	51.66	29.34	-43.21
Foreclosure rate	0.94	0.12	-87.24

Table 2: The impacts of removal of mandatory MLI

lower leverage translates to lower foreclosure rates.

We use equation 3.7 to calculate welfare for our baseline calibration. Then, we calculate welfare gains and losses for agents with different wealth and income levels. Figure 9 shows consumption equivalent variation (CEV). High-income borrowers of insured mortgages in the baseline world are net payers of credit subsidy, and are better off post-removal of mandatory MLI. The rest of borrowers of insured mortgages in the baseline world loose their credit subsidy and are worse off. High-income borrowers of uninsured mortgages and high-income cash purchasers are better off, because of the decline in house prices. As rent prices are higher in the counterfactual world, renters are also worse off. Figure 10 shows Consumption Equivalent Variation (CEV) by income groups in the aggregate economy. Most households are better off either because of lower house

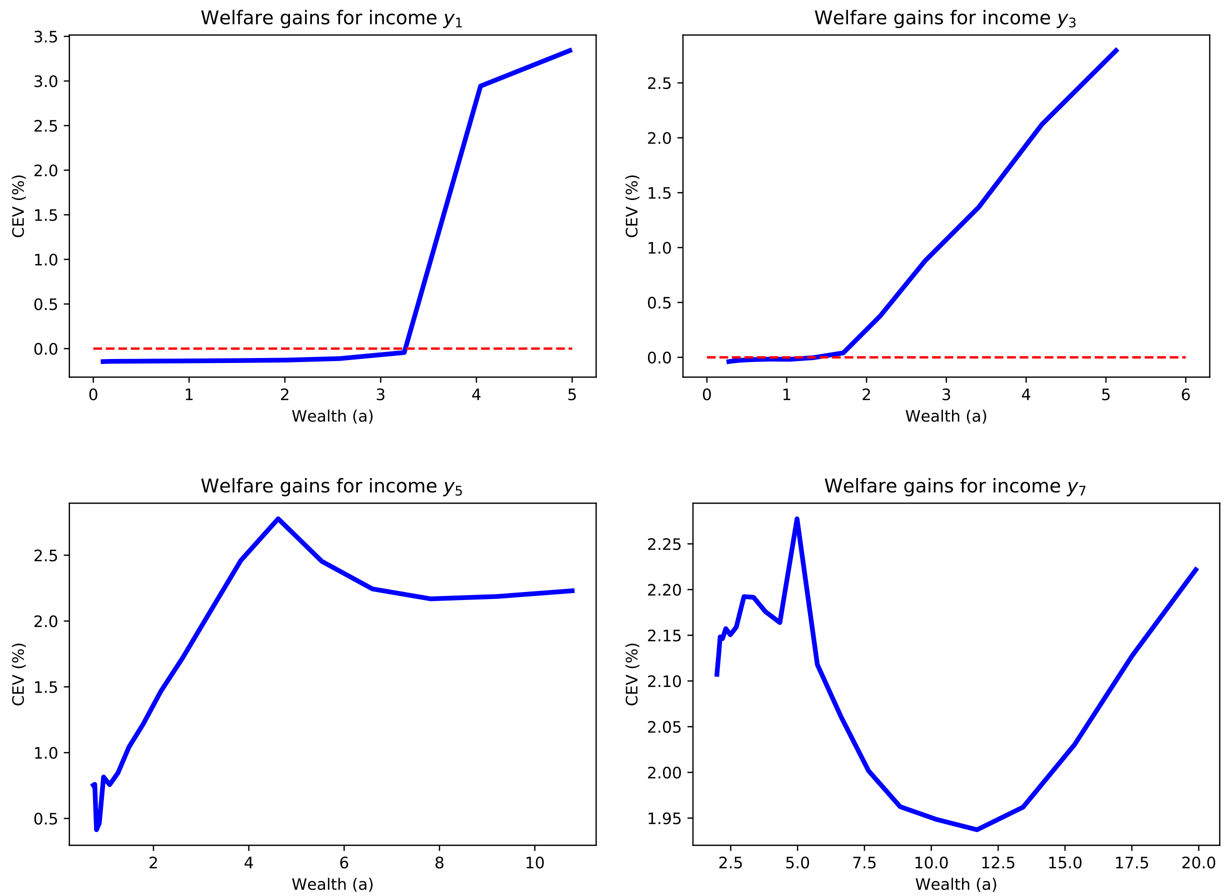


Figure 9: Welfare gains or losses

prices or removal of implicit credit subsidy. Increase in rent prices makes households with lowest income worse off.

6 Conclusion

As it is in most countries, housing in Canada is critically important, not only as a basic need for all people, but as a measure of one's comfort and well-being and an object of one's aspirations. The mandatory MLI was introduced over 60 years ago in Canada with the purpose of promoting the efficient functioning and competitiveness of the housing finance market, while contributing to the stability of the financial system. At the time this requirement was introduced, the objective was to encourage increased mortgage lending, thereby supporting access to credit for borrowers who otherwise might not obtain credit or would receive credit at higher interest rates. With

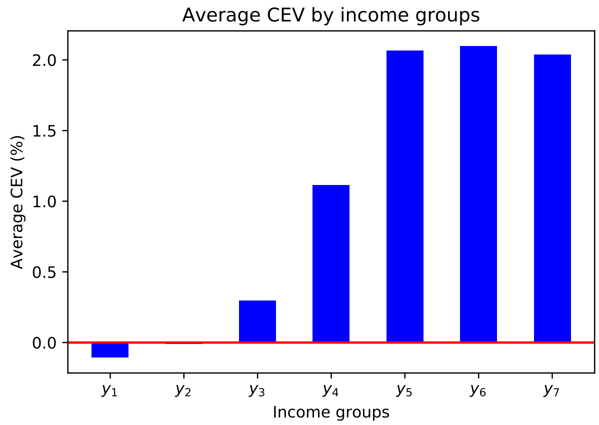


Figure 10: Consumption Equivalent Variation (CEV) by income groups

innovations in the financial industry, low interest rates and the attractiveness of housing finance to domestic and international investors, credit has been more accessible in the housing finance sector. In this environment, some policy discussions are based on the assumption that broad, government-backed MLI has distortionary impacts and has lost its mandate of providing access to housing finance. This has intensified debate on whether further reforms to mortgage insurance are needed to limit the build-up of risk in housing markets.

This paper evaluates the impacts of changing the pricing regime for mortgage insurance by removing the mandatory MLI on the distributions of wealth and homeownership in the economy. More precisely, our results provide evidence to inform policy makers about how the structural changes to the current mortgage market impact housing affordability, household tenure decision, household leverage position, wealth inequality, and the overall default rates in the system.

We construct a dynamic model of the housing market with heterogeneous households and competitive housing and mortgage markets based on the Canadian housing and mortgage market institutions. We calibrate the model to match statistics from the Canadian housing market. The model generates cross-subsidization, which is embedded in a flat-premium insurance scheme. Low-wealth borrowers of insured mortgages are net credit subsidy receivers and, as borrowers' wealth increases, the amount of subsidy received declines. High-wealth borrowers of insured mortgages are net credit subsidy payers.

Our model shows that, after the removal of mandatory MLI, some low-income borrowers with small equity holdings lose their credit subsidy. Some of these potential borrowers would not be able to afford a house, and therefore remain renters. As a result, demand for houses to purchase decreases, and demand for rental units increases. In addition, the counterfactual world with no mandatory MLI features significantly lower leverage.

The key to making this policy successful is to make sure that there is enough supply of affordable rental units to dampen the increased demand for them. Targeted programs, such as the Rental Construction Financing initiative, National Housing Co-Investment Fund, Multi-Unit MLI and Social Housing Securitization Pools, could help alleviate some of the housing affordability challenges in this space. Our framework can be extended to study alternative macroprudential policies, such as changes in LTV limits and increases in MLI premiums, and to investigate the differential impacts of these policies in different segments of the market.

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A Appendix

A.1 Recursive formulation of household problem

The household decides her consumption, savings in deposits, tenure choice (renter or owner), and whether to take a HRM, or a LRM mortgage loan. a is household's wealth after realization of income and housing depreciation shocks. Household wealth is composed of disposable income plus the value from all assets brought into the period plus transfers. The value function $V(a, y)$ is the value of the optimal tenure and mortgage choice. Households take prices$\left(P_h, P_s, P_d, P_m^H, P_m^L(m', h, d', y)\right)$ as given. Next, we characterize the problem of a home buyer who chooses HRM or LRM mortgage, the problem of a renter, and the household's decision between rental, ownership and the type of mortgage.

First, the household facing a HRM-insured mortgage solves:

$$V_H(a, y) = \max_{c,d',m' \geq 0, s \geq \underline{s}, h \geq \underline{h}} \left\{ u(c, s) + \beta \sum_{y' \in Y} \pi(y'|y) \int_{\underline{\delta}}^{1} V(a', y')\, dF(\delta') \right\} s.t. \tag{A.1}$$

$$c + P_d d' + P_h h = a + P_s(h - s) + P_m^H m', \tag{A.2}$$

$$s \leq h, \tag{A.3}$$

$$P_m^H m' \leq \min\left\{\theta^H P_h h, \kappa_h y, \bar{l}\right\}, \tag{A.4}$$

$$P_m^H m' > \theta^L P_h h, \tag{A.5}$$

$$a' = \max \left\{ \begin{array}{c} y' + d' + P_h(1 - \delta') h - m', \\ (1 - \phi_y) y' + (1 - \phi_d) d' \end{array} \right\}, \tag{A.6}$$

where, $P_s(h - s)$ is the rental income received by landlords. (A.2) is the household's budget constraint. (A.3) indicates that a homeowner cannot lease more rental space than her housing space. The maximum loan-to-value and loan size on HRM loans are summarized in (A.4). In addition to constraints on loan-to-value, HRM borrowers face two size limits on their mortgage: 1-limit on debt servicing ratio: here we use κ_h portion of borrowers income to represent existing limits on Insured borrowers' income[17], 2-size limit: Canadian federal legislation prohibits issuance

[17]HRM borrowers face two Debt Service Ratio (DSR) constraints: 1- Gross Debt Service (GDS) ratio, which limits borrower's carrying cost of housing as a percentage of income, 2-Total Debt Service (TDS) ratio, which applies a limit on borrowers's carrying costs of housing plus costs of other debt. We used CMHC's administrative data to translate DSR constraints to a limit on mortgage size.

of Insured mortgages for houses with a market value of above $1 million. We combine this with LTV limit and calculate an imputed limit on mortgage size ($\bar{l}$).

A mortgage is considered as a HRM loan if its LTV is greater than $\theta^L = 80\%$. Equation (A.6) defines the beginning-of-next period wealth following the optimal default rule.

Second, the household borrowing a LRM loan solves:

$$V_L(a,y) = \max_{c,d',m'\geq 0, s\geq \underline{s}, h\geq \underline{h}} \left\{ u(c,s) + \beta \sum_{y'\in Y} \pi(y'|y) \int_{\underline{\delta}}^{1} V(a',y')\, dF(\delta') \right\} \; s.t. \tag{A.7}$$

$$c + P_d d' + P_h h = a + P_s(h-s) + P_m^L(m',h,d',y)\, m', \tag{A.8}$$

$$s \leq h,$$

$$P_m^L m' \leq \min\left\{\theta^L P_h h, \kappa_l y\right\}, \tag{A.9}$$

$$a' = \max \left\{ \begin{array}{c} y' + d' + P_h(1-\delta')h - m', \\ (1-\phi_y)y' + (1-\phi_d)d' \end{array} \right\}. \tag{A.10}$$

In addition to the upper bound limit on LTV, we limit mortgage size by $\kappa_l y$. This constraint represents limits on Debt Service Ratios (DSRs) in the uninsured space. The Office of the Superintendent of Financial Institutions (OSFI)'s B-20 guidelines[18], which applies to the Federally Regulated Financial Intuitions (FRFIs), requires FRFIs to set prudent limits on DSRs. B-20 guidelines are not prescriptive in setting hard limits on DSRs. Provincially Regulated Financial Institutions (PRFIs) are regulated by provincial regulators. Provincial regulators generally follow OSFI in setting soft requirements on DSRs. Therefore, in our baseline model we relax this constraint for all LRM mortgages. The lending rate depends on the mortgage m', house size h, deposits d', and current income y'.

Fourth, households who are renting solve:

$$V_r(a,y) = \max_{c,s,d\geq 0'} \left\{ u(c,s) + \beta \sum_{y'\in Y} \pi(y'|y)\, V(a',y') \right\} \; s.t. \tag{A.11}$$

$$c + P_s s + P_d d' = a,$$

$$a' = y' + d'.$$

[18]See here for more information on B-20 guidelines.

Fifth and finally, the household's value function $V(a,y)$ is the maximum of the above four options:

$$V(a,y) = \max_{I_H,I_L,I_r\in\{0,1\}} \{I_H V_H(a,y) + I_L V_L(a,y) + I_r V_r(a,y)\} \ s.t. \tag{A.12}$$

$$I_H + I_L + I_r = 1, \tag{A.13}$$

$$I_h = 1 - I_r. \tag{A.14}$$

To simplify the notation, we denote the overall optimal choice variables as

$$c(a,y) = I_H(a,y)\, c_H(a,y) + I_L(a,y)\, c_L(a,y) + I_r(a,y)\, c_r(a,y).$$

The Individual state is $x = (a,y)$ and the state space is $X = (A\times Y)$. The probability measure over X is denoted by μ, which is constant across time on stationary equilibrium.

We can simplify the household's maximization problem by analytically solving the static problem of how to allocate resources between consumption and shelter. Let g denote the resources available for current consumption of a households with state (a,y), housing tenure and mortgage choice (I_H, I_L, I_r), and a feasible portfolio choice (d', m', h):

$$g = a - (P_h - P_s)\, h + I_H P_m^H m' + I_L P_m^L\left(m', h, d', y\right) m' - P_d d'.$$

Thus, the static problem is

$$U(g,h,I_h) = \max_{c,s\geq 0} \frac{\left[\eta c^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)\, s^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\sigma)}{\epsilon-1}}}{1-\sigma} \ s.t.$$

$$c + P_s s = g,$$

$$\underline{s} \leq s \leq h \ \ if\, I_h = 1$$

Substitute the budget constraint to $U(\cdot)$, the first-order condition with respect to s is:

$$s \geq \frac{(1-\eta)^{\epsilon}}{\eta^{\epsilon} P_s^{\epsilon} + (1-\eta)^{\epsilon} P_s}.$$

Given $\underline{s} \leq s \leq h$, the closed-form solution for s is

$$s\left(g,h,I_h\right)=\begin{cases}\underline{s} & if\ \underline{s}>\left(1-\theta\right)\frac{g}{P_s}\ and\ I_h=1,\\ h & if\ h<\left(1-\theta\right)\frac{g}{P_s}\ and\ I_h=1,\\ \left(1-\theta\right)\frac{g}{P_s} & else.\end{cases}$$

Note the last line includes two cases: (1) owner with $\underline{s} < s < h$, and (2) renter without any constraint on the housing services she receives. θ is the optimal share allocated to consumption absent the constraints on shelter:

$$\theta=\frac{\eta^{\epsilon}}{\eta^{\epsilon}+\left(1-\eta\right)^{\epsilon}P_s^{1-\epsilon}}.$$

Therefore, the indirect utility function is given by:

$$U\left(g,h,I_h\right)=\begin{cases}\frac{\left[\eta(g-P_s\underline{s})^{\frac{\epsilon-1}{\epsilon}}+(1-\eta)\underline{s}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\sigma)}{\epsilon-1}}}{1-\sigma} & if\ \underline{s}>\left(1-\theta\right)\frac{g}{P_s}\ and\ I_h=1,\\ \frac{\left[\eta(g-P_sh)^{\frac{\epsilon-1}{\epsilon}}+(1-\eta)h^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\sigma)}{\epsilon-1}}}{1-\sigma} & if\ h<\left(1-\theta\right)\frac{g}{P_s}\ and\ I_h=1,\\ \frac{\left[\eta^{\epsilon}+(1-\eta)^{\epsilon}P_s^{1-\epsilon}\right]^{\frac{1-\sigma}{\epsilon-1}}g^{1-\sigma}}{1-\sigma} & else.\end{cases}$$

The closed-form expression for the LRM mortgage pricing function is derived by integrating by parts on the last term of the RHS of (3.5):

$$\int_{\delta^*}^{1}\left(1-\delta'\right)dF\left(\delta'\right)=\begin{cases}\int_{\underline{\delta}}^{1}F\left(\delta'\right)d\delta' & if\ \delta^*<\underline{\delta},\\ -\left(1-\delta^*\right)F\left(\delta^*\right)+\int_{\delta^*}^{1}F\left(\delta'\right)d\delta' & if\ \underline{\delta}\leq\delta^*\leq 1,\\ 0 & else.\end{cases}$$

A.2 Exogenous and jointly calibrated parameters

Tables 3 and 4 show parameter values, which are selected exogenously and those which are jointly calibrated, respectively.

Table 3: Parameters selected exogenously

Parameter	Value	Interpretation	Source
ϵ	0.4	Intratemporal elasticity of substitution	Bruneau et al. (2016)
σ	1.2	Intertemporal elasticity (CES)	Bruneau et al. (2016)
ρ	0.958	Labor income persistence	
σ_ϵ	0.1735	Labor income volatility	Brzozowski et al. (2010)
θ^L	0.8	Down payment requirement LRMs	
θ^H	0.95	Down payment requirement HRMs	
g^H	22 BPS	Guarantee fee	Calculated by discounting MLI premium over the life of the loan
γ	0.67	Foreclosure recovery rate	Internal CMHC calculations
ϕ_y	0.23	Labor income garnishment	Weighted average of provincial exemptions for bankruptcy garnishments
r_w	63 BPS	Mortgage origination cost	Internal CMHC calculations

Table 4: Parameters jointly calibrated

Parameter	Description	Target	Value	Source
η	Non-housing share in consumption	Risk-free rate	2.77%	Bruneau et al. (2016)
β	Discount factor	Homeownership rate	68%	Census 2016
$\underline{h}$	Minimum house size	% of homeowners with mortgage debt	61%	Census 2016
$\bar{l}$	Limit conforming mortgage	HRM loans as % of total volume	31%	FSR[19] Nov. 2017
$\underline{s}$	Min shelter Cons (owners)	Median deposit-to-asset ratio for mortgagors	16.73%	SFS[20] 2016
ϕ_d	Recourse on deposits	Median size of owner-occupied-to-rental housing	1.58	Census 2016
ξ	Pareto shape parameter	Foreclosure rate	0.16%	CMHC calculations
σ_δ	Pareto scale parameter	Average house depreciation	1.5%	Kostenbauer (2001)
$\underline{\delta}$	Pareto location parameter	House price volatility	5.9%	CMHC calculations

[19] Financial Systems Review
[20] Survey of Financial Security

A.3 Computation

A.3.1 Household problem

We solve the simplified household problem using grid-search methods. The detail algorithm is as follows:

Step 1: Initialize the value function $V^{(0)}$ for state space (a, y). To discretize the space, for each income points (y), we create a grid $A(y) = \{a(y)_i\}_{i=1}^n$ of n points for the wealth level a. We set the minimum element of each grid $a_1(y) = (1 - \phi_y) y$, which is the starting wealth next period in case of default in the current period, when $d' = 0$. Polynomial spaced grids are used to generate more density at the lower bound by using a linearly spaced grid z over $[0, 1]$, and then by constructing the grid for $a(y) = a_1(y) + (a_n(y) - a_1(y)) z^{1/\alpha}$, where α controls the curvature.

Step 2: At each grid point (a, y), the ith iteration maximization problem searches for the housing tenure and mortgage type choice that solves

$$
\begin{aligned}
V^{(i)}(a, y) = & \max_{I_g, I_f, I_j, I_r \in \{0,1\}} \left\{ I_g V_g^{(i)}(a, y) + I_f V_f^{(i)}(a, y) + I_j V_j^{(i)}(a, y) + I_r V_r^{(i)}(a, y) \right\} \ s.t. \\
& I_g + I_f + I_j + I_r = 1
\end{aligned}
$$

The simplified value function of a homeowner facing a HRM insured mortgage is

$$
\begin{aligned}
V_g^{(i)}(a, y) = & \max_{d' \geq 0, m' \geq 0, h \geq \underline{h}} \left\{ U(g, h, 1) + \beta \sum_{y' \in Y} \pi(y'|y) \int_{\underline{\delta}}^{1} V^{(i-1)}(a', y') \, dF(\delta') \right\} \ s.t. \\
& g = a - (P_h - P_s) h + P_m^H m' - P_d d', \\
& P_m^H m' \leq \min \left\{ \theta^H P_h h, \bar{l} \right\}, \\
& P_m^H m' > \theta^L P_h h \\
a' = & \max \left\{ \begin{array}{c} y' + d' + P_h (1 - \delta') h - m', \\ (1 - \phi_y) y' + (1 - \phi_d) d' \end{array} \right\}.
\end{aligned}
$$

The value function of a homeowner facing a LRM mortgage is

$$V_j^{(i)}(a,y) = \max_{d'\geq 0, m'\geq 0, h\geq \underline{h}} \left\{ U(g,h,1) + \beta \sum_{y'\in Y} \pi(y'|y) \int_{\underline{\delta}}^{1} V^{(i-1)}(a',y')\, dF(\delta') \right\} \ s.t.$$

$$g = a - (P_h - P_s)h + P_m^L m' - P_d d',$$

$$P_m^L m' \leq \theta^L P_h h$$

$$a' = \max \left\{ \begin{array}{c} y' + d' + P_h(1-\delta')h - m', \\ (1-\phi_y)y' + (1-\phi_d)d' \end{array} \right\}.$$

Moreover, the value function of a renter is

$$V_r^{(i)}(a,y) = \max_{d'\geq 0} \left\{ U(a - P_d d', 0, 0) + \beta \sum_{y'\in Y} \pi(y'|y) V^{(i-1)}(y'+d', y') \right\}.$$

If the constraint set is empty in any problem of homeowners (particularly in cases of LTV ranges), then the corresponding value function takes value minus infinity.

The conditional expectation is computed using the following partial integral

$$\int_{\underline{\delta}}^{1} V^{(i-1)}(a',y')\, dF(\delta') = \int_{\underline{\delta}}^{\delta^*} V^{(i-1)}(a',y')\, dF(\delta') + (1 - F(\delta^*)) V^{(i-1)}\left(\left[(1-\phi_y)y' + (1-\phi_d)d'\right], y'\right).$$

A Gauss-Legendre integration method (with 16 quadrature nodes) is used to calculate the integral over the payment interval $[\underline{\delta}, \delta^*]$.

To ensure a global solution, we perfom maximization in two steps. First, we solve the household's problem using grid search. We use evenly spaced grids of h, m', and d'. Then, the solution is used to start an optimization algorithm and solve the maximization problem at each grid point.

Step 3. Update the value function $V^{(i)}$.

Step 4. Repeat Steps 2 and 3 until the value of the value function at each state space grid point coverges, i.e. $||V^{(i)} - V^{(i-1)}|| \leq \varepsilon$.

A.3.2 Stationary distribution

Next, we derive the stationary distributions of the state variables. The stationary measure $\mu(a,y)$ is approximated using a discrete density function. Define $\delta^*(a,y,y')$ and $a'(a,y,y',\delta')$ as the default shock threshold and next-period wealth implied by the optimal decision rules.

Step 1. Discretize the state space. Let us denote the income specific grid by $A(y) = \{a_i(y)\}_{i=1}^n$. We define a grid $Q = \{\underline{\delta}, ..., 1\}$ fro the depreciation shock δ' and let $p(\delta')$ be a probability mass function defined over the grid Q. Specifically, $p\left(\delta_i' = \frac{f\left(\delta_i'\right)}{\sum_{\delta' \in Q} f\left(\delta'\right)}\right)$, where $f\left(\delta'\right)$ is the PDF of δ'.

Step 2. Initialize the measure $\mu^{(0)}$ at each grid point of the state space, e.g. a uniform distribution.

Step 3. During the ith iteration, update $\mu^{(i)}(a_j(y'),y')$ and $\mu^{(i)}(a_{j+1}(y'),y')$ as following:

$$
\begin{aligned}
\mu^{(i)}\left(a_j\left(y'\right),y'\right) = & \sum_{y\in Y}\sum_{a\in A(y)}\sum_{\delta'\in Q^*} \pi\left(y'|y\right) F\left(\delta^*\left(a,y,y'\right)\right) p^*\left(\delta'\right) \left[\frac{a_{j+1}\left(y'\right) - a'\left(a,y,y',\delta'\right)}{a_{j+1}\left(y'\right) - a_j\left(y'\right)}\right] \\
& \times \mathbb{I}\left(a_j\left(y'\right) \le a'\left(a,y,y',\delta'\right) \le a_{j+1}\left(y'\right)\right)\mu^{(i-1)}(a,y) \\
& + \sum_{y\in Y}\sum_{a\in A(y)} \pi\left(y'|y\right)\left[1 - F\left(\delta^*\left(a,y,y'\right)\right)\right] \left[\frac{a_{j+1}\left(y'\right) - a'\left(a,y,y',\delta^*\left(a,y,y'\right)\right)}{a_{j+1}\left(y'\right) - a_j\left(y'\right)}\right] \\
& \times \mathbb{I}\left(a_j\left(y'\right) \le a'\left(a,y,y',\delta^*\left(a,y,y'\right)\right) \le a_{j+1}\left(y'\right)\right)\mu^{(i-1)}(a,y)
\end{aligned}
$$

where $\mathbb{I}(x) = 1$ if the statement x is true, 0 otherwise, and $Q^*(a,y,y')$ is the set of $\delta' \in Q$ such that $\delta' \le \delta^*(a,y,y')$ with conditional probability mass function $p^*(\delta')$ defined over Q^*. $\mu^{(i)}(a_{j+1}(y'),y')$ is updated using the same equation as above after replacing the first and third terms in square brackets by $\left[\frac{a'(a,y,y',\delta')-a_j(y')}{a_{j+1}(y')-a_j(y')}\right]$ and $\left[\frac{a'(a,y,y',\delta^*(a,y,y')-a_j(y'))}{a_{j+1}(y')-a_j(y')}\right]$. Note the conditional probability mass function is given by $p^*(\delta_i') = \frac{p\left(\delta_i'\right)}{\sum_{\delta'\in Q^*} p(\delta')}$.

Step 4. Repeat Step 3 until the value of the measure at each state space grid point converges, i.e. $||\mu^{(i)} - \mu^{(i-1)}|| \le \varepsilon$.

A.3.3 Equilibrium

With the optimal decision rules and the stationary distribution, we check if equilibrium condtions (market clearing conditions) hold with tolerance. If they do not, we update the initial guess for

P_h, P_s, and P_d. Then we solve again until we meet our convergence criteria.