

# International Spillovers of New Monetary Policy

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## Abstract

We study spillovers of conventional and new monetary policies of a large economy to a small open economy (SOE). Building on Sims and Wu (“Evaluating Central Banks’ Tool Kit: Past, Present and Future,” 2020, forthcoming in the *Journal of Monetary Economics*), we employ a medium-scale New Keynesian model that features all the major types of new monetary policies and the conventional monetary policy in a unified framework. We extend their model to an open economy setting. We use our model as a measurement device to quantify the spillovers and study the economic mechanisms behind them. In our quantitative application, Canada is the SOE and the US is the large economy. Our results show similar spillover effects of conventional and new monetary policies on GDP of the SOE. However, the effects on various components of GDP (consumption, investment and net exports) differ by policy. We also simulate counterfactual monetary policy scenarios for the US and Canada around the Great Recession of 2008. Three main conclusions emerge from these simulations: (1) if the Fed had not engaged in quantitative easing (QE), the US recession in the wake of the 2008 financial crisis would have been deeper but Canada would have had better economic outcomes; (2) there are diminishing returns to QE in terms of its effects on both the US and Canadian real variables; and (3) had the Bank of Canada followed the Fed and engaged in QE of its own during the Great Recession, the real economic outcomes would have been better for Canada.

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The new tools of monetary policy “are often referred to as *unconventional* or *nonstandard* policies. Since I will argue that these tools should become part of the standard toolkit, I will refer to them here as *new* or *alternative* monetary tools.” [Bernanke (2020), p. 944, footnote 1, emphasis added]

## I. Introduction

During the 2008 financial crisis, the Federal Reserve decreased its policy rate to the zero lower bound (ZLB). At the ZLB, the Fed used new monetary policy tools like quantitative easing (QE) and forward guidance (FG) to strengthen the US economy. The use of these new monetary policy tools sparked great interest in both academic and policy circles about the possible international spillover effects of these policies [Bernanke (2018)]. In this paper, we study the spillovers of conventional and major new monetary policy tools (QE, FG and negative interest rate policy (NIRP)) from a large economy to a small open economy (SOE).

The debate on international spillovers of monetary policy has a long history but the unconventional nature of the new monetary policy tools prompted calls for new research on the subject. Since then, a new and fast growing literature on international spillovers of new monetary policy has emerged. There are two main branches of this literature. The papers in the first branch are empirical in nature and focus on identifying the new monetary policy shocks originating in the US and exploring their effects on various macroeconomic variables in other countries [see Gilchrist et al. (2019) and the literature cited therein]. In the second and more recent branch, researchers have employed medium size dynamic stochastic general equilibrium (DSGE) models to build quantitative theories of international spillovers of new monetary policy. These models allow researchers to think explicitly about the transmission mechanisms of the policies and run counterfactual experiments to examine the effects of alternative policy paths. This literature is new and evolving.

We contribute to the latter literature by extending the model in Sims and Wu (2020) (from here on, SW) to an open economy setting. A distinguishing feature of SW is that they model conventional and major new monetary policy tools (QE, FG and negative interest rate

policy (NIRP)) in a unified framework. Another feature of their model, following Gertler and Karadi (2011, 2013), is a separate financial sector that buys both government and private bonds, and holds reserves issued by the monetary authority. Our open economy extension retains these attractive features of their framework. This allows us to not only compare the international spillovers of conventional monetary policy and QE, but also to compare the international spillovers caused by FG and NIRP, something that is new to the literature.

There are two economies in our model – a home and a foreign economy. The home economy is a small open economy (SOE). It engages in both goods and asset trades with a foreign economy, which is so large that its transactions with the SOE form a negligible fraction of its total economic activity. So much so, that we model this foreign economy as a closed economy. The foreign economy in our model is identical to the closed economy in SW.

In the SOE, the representative household consumes both home and foreign goods. There is a representative exporting firm that channels part of the final output to the foreign country. There is a representative wholesale firm that must finance a certain fraction of its investment by borrowing. We allow the wholesale firm to issue bonds in both home and foreign currencies. There are financial intermediaries that buy private and government bonds. We allow the financial intermediaries to buy both type of bonds (private and government) in both home and foreign currencies. By construction, there are no spillovers from the SOE to the foreign economy.<sup>1</sup>

Our model features most of the conventional channels of international monetary policy spillovers. One such channel is the exchange rate channel. When the large economy engages in expansionary monetary policy, it is likely to result in an appreciation of the SOE’s currency and may have negative effects on its net exports. However, the higher output in the large economy may partially offset this effect by leading to a higher demand for the SOE exports. Another channel is the portfolio balance channel. The expansionary monetary policy in the large economy may lead to relatively higher bond yields in the SOE and cause investors to move funds from the large economy to the SOE in search of better returns. In our general

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<sup>1</sup>Obstfeld (2019) talks about “mechanisms through which global factors influence the tradeoffs that US monetary policy faces.” By construction, our model abstracts from these reverse spillovers.

equilibrium model, both exchange rate and bond yields are endogenously determined in equilibrium.

We calibrate the SOE in our model to the Canadian economy and the large economy to the US. We then use our model as a measurement device to quantify the size of various spillovers from the US to Canada. Our main findings are the following: (1) The international spillovers of new monetary policy are qualitatively similar but quantitatively different from those of the conventional monetary policy. (2) The expansionary monetary policy shocks in the US, have contractionary effects on most Canadian real variables. These contractionary effects are strongest in the case of forward guidance (FG) and the negative interest rate policy (NIRP), intermediate in the case of conventional monetary policy and the weakest in the case of quantitative easing (QE). In other words, from a Canadian perspective, US QE is the best policy, followed by the US conventional monetary policy. The US FG and NIRP are the least desirable. (3) An increase in the size of US QE increases the spillovers but only at a decreasing rate and, after a certain threshold, further increases in US QE have very small spillover effects on the Canadian economy. (4) If Canada engages in its own expansionary new monetary policy on top of the one by the US, the positive effects on Canadian economy would be much stronger.

### *A. Related Literature*

The literature on international spillovers of monetary policy is large and has a long history. Gali (2015, pp. 252–254) provides a brief and informative overview of this literature.

We use a calibrated general equilibrium model to quantify the international spillovers of new monetary policy and understand the economic mechanisms behind them. The literature using this approach is still in its infancy. To the best of our knowledge, ours is the first paper to develop a quantitative DSGE model that builds the traditional interest rate policy and the three popular new monetary policy tools (namely QE, FG and NIRP) together in an open economy framework. The papers closest to ours in terms of general methodology are Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020) but there are important differences in details.

Alpanda and Kabaca (2020), similar to our paper, evaluate the international spillovers

of new monetary policy using a dynamic stochastic general equilibrium (DSGE) model. But they focus on QE and study two symmetric economies. We, following SW, model all major tools of new monetary policy in a unified framework and focus on spillovers from a large economy to an SOE.

Kolasa and Wesolowski (2020) develop a two-country model with asset market segmentation to investigate the effects of QE by a large country central bank on a small open economy. Similar to us, they also restrict international trade in short-term bonds. This assumption not only reflects the experiences of SOEs but is also crucial in explaining the difference between spillover effects of QE and conventional monetary policy on domestic long term yields. Again, our paper is different from theirs because we also model spillovers due to FG and NIRP. Another difference is in the way we model bonds. We comment more on it below.

Another important difference between our paper and the two papers above is that we incorporate the idea of an endogenous QE policy. This is an important assumption to match the Fed's QE policy (especially its second and third rounds of QE) as it unfolded in the wake of 2008 financial crisis. Nonetheless, Kolasa and Wesolowski (2020) and Kolasa and Wesolowski (2020) are two important contributions to this literature and in this paper, we frequently relate our findings to the insights from their work.

## II. Model

We build on the works of Gali and Monacelli (2016) and Sims and Wu (2020). Gali and Monacelli (2016) build a small open economy model with staggered prices and wages to study gains from wage flexibility in a currency union. Sims and Wu (2020), building on Gertler and Karadi (2011), Gertler and Karadi (2013) and Carlstrom et al. (2017), develop a closed-economy New Keynesian model that features a financial sector and a central bank that engages in both conventional and new (unconventional) monetary policy.<sup>2</sup> Our model is a small-open-economy (SOE) version of the model in Sims and Wu (2020). We make the following additions to their model to make it suitable to address questions about international monetary policy spillovers.<sup>3</sup> We allow consumers to import. We introduce an exporter who buys the final good and sells it to the foreign country. This is the real side of the SOE. We allow the wholesale firm to issue both domestic and foreign bonds, and financial intermediaries to hold both domestic and foreign bonds (both private and government). This is the financial side of the SOE.

The focus of our model is an SOE, which we call the home country ( $H$ ). The rest of the world is represented by a large country, which we call the foreign country ( $F$ ). Later, in the empirical application, Canada is the home country and the US is the foreign country. The foreign country in our model is not affected by economic events in the home country and hence we model it as a closed economy. However, the economic events in the foreign country affect the home country through both real and financial channels. In this section, we describe the SOE (i.e. the economy of the home country) in detail. We describe the foreign economy briefly towards the end of this section.

We have divided this section into seven subsections: (1) household; (2) labor market; (3) non-financial firms; (4) financial intermediaries; (5) fiscal authority; (6) monetary authority; and (7) foreign exchange market.

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<sup>2</sup>As suggested by Bernanke (2020), what the literature calls ‘Unconventional Monetary Policy’ (which mainly consists of quantitative easing, forward guidance and negative interest rate policy), is not unconventional any more. In the last decade, so many central banks around the world have used these new tools that it is more appropriate to call them ‘New Monetary Policy’. This is the label that we use in this paper.

<sup>3</sup>The following short description of our additions to the model in SW assumes that the reader is familiar with their model. For the new readers, we provide details below.

## A. Household

The first part of the household's problem in our model is identical to that in SW except that we have modified some of their notation, changed the utility function from log to CRRA and introduced a shock  $Z_t$  to utility. The problem of the representative household is

$$(1) \quad \max_{\{C_{t+\tau}, L_{1,t+\tau}, D_{t+\tau}\}} \mathbb{E}_t \left( \sum_{\tau=0}^{\infty} \beta^\tau Z_{t+\tau} \left( \frac{(C_{t+\tau} - hC_{t+\tau-1})^{1-\sigma}}{1-\sigma} - \omega \frac{L_{1,t+\tau}^{1+\varphi}}{1+\varphi} \right) \right)$$

subject to the following *nominal* period budget constraint:

$$(2) \quad P_{C,t}C_t + D_t - D_{t-1} \leq W_{1,t}L_{1,t} + \Pi_t - P_{C,t}T_t - P_{C,t}\chi + (R_{t-1}^D - 1)D_{t-1}.$$

$C_t$  is a composite consumption good which is a CES (constant elasticity of substitution) aggregate of consumption on home and foreign goods,  $L_{1,t}$  is the labor supply (work hours) of the household and  $Z_t$  is a shock to the discount rate. We assume that  $\log Z_t$  follows an AR(1) process.  $P_{C,t}$  is the composite price index, which is a CES aggregate of home and foreign prices.  $W_{1,t}$  is the nominal wage a household receives from the labor unions.  $\Pi_t$  is the net nominal dividend from all financial and non-financial firms.  $T_t$  is the real lump sum tax paid to the fiscal authority.  $\chi$  is the real transfer from the household to the new financial intermediaries.  $D_t$  is the household's deposits with the financial intermediaries. These deposits pay a nominal gross interest rate of  $R_t^D$ . The household chooses sequences of consumption ( $C_t$ ), work hours ( $L_t$ ) and deposits ( $D_t$ ) to maximize the expected value of utility over the infinite horizon.

We follow Gali and Monacelli (2016) to model the second part of the household's problem. The household combines home-produced goods,  $C_{H,t}$ , and foreign-produced goods (imports),  $C_{F,t}$  to produce the composite consumption good,  $C_t$  according to the following CES production function:

$$(3) \quad C_t = \left( (1 - \nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1-1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} C_{F,t}^{\frac{\eta_1-1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1-1}}.$$



The problem of the household is to choose  $C_{H,t}$  and  $C_{F,t}$  that minimize the cost of composite consumption  $C_t$  subject to the CES production function above.

The solution to the household's problem is standard and satisfies the nine equations (from eqn. 01/72 to eqn. 09/72) in the appendix.<sup>4</sup>

## **B. Labor Market**

We closely follow SW in our modelling of the labor market. The following flow chart illustrates the flow of labor in our model economy:

Household  $\longrightarrow$  Labor unions  $\longrightarrow$  Labor packer  $\longrightarrow$  Wholesale firm

The household supplies labor ( $L_{1,t}$ ) to labor unions at wage ( $W_{1,t}$ ) that is equal to the marginal rate of substitution of leisure for consumption.

There is a unit mass of labor unions indexed by  $i \in [0, 1]$ . Each labor union takes labor services from the household and repackages them into *specialized labor*  $L_{2,t}(i)$ , which is specific to union  $i$ , and sells it to a labor packer at wage  $W_{2,t}(i)$ . The labor unions face Calvo-type wage rigidity. Each period,  $\theta_w$  fraction of unions do not choose optimal wage. Instead, they update their last-period wage by using an indexation formula. The remaining fraction  $1 - \theta_w$  of unions reoptimize and choose a new optimal wage  $W_{2,t}^*(i)$ .

The labor packer combines specialized union labor  $L_{2,t}(i)$  into a final labor bundle  $L_{2,t}$  according to a CES aggregator. The packer sells  $L_{2,t}$  to the wholesale firm—the only user of labor for production—at the economy-wide wage  $W_{2,t}$ .

In this set up, the wage-choosing labor unions are constrained by Calvo pricing. This allows for a transparent modelling of wage rigidity. This standard formulation of labor market gives the familiar equilibrium conditions in eqns. 10/72 to 15/72 in the appendix. We derive these conditions in the online appendix.

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<sup>4</sup>We provide the basic description of the model in the main text and collect all equilibrium conditions in the appendix. The appendix also contains a full list of variables. The online appendix accompanying this paper contains the steady-state solution to the model and the derivations of equilibrium conditions.

### *C. Non-Financial Firms*

There are five types of non-financial firms in our model: (1) A representative wholesale firm produces output using its own capital, accumulated through purchases of new capital from the capital-good firm, and labor hired from the labor packer. (2) A continuum of retail firms repackage wholesale output for resale to the final good firm. Retail firms behave as monopolistic competitors and are subject to Calvo-type price stickiness. (3) A competitive final good producer aggregates retail output into a final good that is meant for consumption (by both the household and government), investment and export. (4) A representative capital-good firm purchases final output and transforms it into new physical capital subject to an adjustment cost. (5) A representative exporter buys the final good and sells it to the foreign country.

The firms of first four types behave in exactly the same way as in SW except that we allow the wholesale firm to issue foreign currency bonds in addition to the home currency bonds. We add an export firm to the model along the lines of Gali and Monacelli (2016). We now describe the problem of each type.

**Wholesale Firm.**—A representative wholesale firm produces according to a Cobb-Douglas technology:<sup>5</sup>

$$(eqn. 16/72) \quad Y_{2,t} = A_t(u_t K_t)^\alpha L_{2,t}^{1-\alpha}.$$

$Y_{2,t}$  is the flow of output and  $L_{2,t}$  is the labor input. Parameter  $\alpha \in (0, 1)$  is share of capital in production.  $A_t$  is an exogenous productivity variable that follows a stochastic process.  $u_t$  is capital utilization.  $K_t$  is the stock of physical capital owned by the firm. The cost of utilization is faster depreciation and a function  $\delta(u_t)$  maps utilization into depreciation. Physical capital evolves according to:

$$(eqn. 18/72) \quad K_{t+1} = \hat{I}_t + (1 - \delta(u_t))K_t,$$

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<sup>5</sup>There are 72 non-linear equilibrium conditions in our full model of the SOE. These equations are in Appendix B and we have numbered them from eqn. 01/72 to eqn. 72/72. Whenever one of these equations appears in the text, we give it the same number as in the appendix.

where  $\hat{I}_t$  is new gross investment. The wholesale firm is constrained to finance a constant fraction,  $\psi \in [0, 1]$ , of investment by issuing private debt. This constraint, which Sims and Wu (2020) and Carlstrom et al. (2017) call the “loan-in-advance constraint,” is

$$(4) \quad \psi P_{k,t} \hat{I}_t \leq \bar{Q}_{H,t}^{Pvt} (\bar{B}_{H,t}^{Pvt} - \kappa \bar{B}_{H,t-1}^{Pvt}).$$

$P_{k,t}$  is the price at which the wholesale firm purchases new physical capital. The left-hand side of the constraint is fraction  $\psi$  of the wholesale firm’s investment expenditure  $P_{k,t} \hat{I}_t$ . The right-hand side is net addition to the wholesale firm’s debt (i.e. outstanding bonds).<sup>6</sup> The firm has  $\bar{B}_{H,t-1}^{Pvt}$  bonds outstanding from the previous period that are equal to  $\kappa \bar{B}_{H,t-1}^{Pvt}$  bonds this period because of drop in return from one to  $\kappa$ .  $\bar{B}_{H,t}^{Pvt}$  is the firm’s new bond issue. The difference  $\bar{B}_{H,t}^{Pvt} - \kappa \bar{B}_{H,t-1}^{Pvt}$  is the net addition to the wholesale firm’s debt. This debt is priced at  $\bar{Q}_{H,t}^{Pvt}$ . In simple words, the loan-in-advance constraint forces the firm to issue new debt to finance at least  $\psi$  fraction of its current-period investment expenditure.

The loan-in-advance constraint in (4), looks very similar to the one in SW (see their equation 2.21). However, it is not the same. We allow the wholesale firm to issue bonds in home and foreign currencies. The values of the two types of bonds aggregate according to

$$\bar{Q}_{H,t}^{Pvt} \bar{B}_{H,t}^{Pvt} = Q_{H,t}^{Pvt} B_{H,t}^{Pvt} + Q_{H,FC,t}^{Pvt} B_{H,FC,t}^{Pvt},$$

where  $B_{H,t}^{Pvt}$  is the total quantity of outstanding bonds issued in home currency by the wholesale firm,  $B_{H,FC,t}^{Pvt}$  is the total quantity of outstanding bonds issued in foreign currency and

$$(5) \quad \bar{B}_{H,t}^{Pvt} = \left[ \left( \frac{1}{1 - v_3} \right)^{1/\eta_3} (B_{H,t}^{Pvt})^{1+1/\eta_3} + \left( \frac{1}{v_3} \right)^{1/\eta_3} (B_{H,FC,t}^{Pvt})^{1+1/\eta_3} \right]^{\frac{\eta_3}{\eta_3+1}}$$

is a CES-type aggregate. The firm takes the bond prices  $Q_{H,t}^{Pvt}$  and  $Q_{H,FC,t}^{Pvt}$  as given ( $\bar{Q}_{H,t}^{Pvt}$ , which we define below, is a CES-type aggregate of  $Q_{H,t}^{Pvt}$  and  $Q_{H,FC,t}^{Pvt}$ ) and chooses  $B_{H,t}^{Pvt}$  and

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<sup>6</sup>All private and government bonds in this model are perpetuities that cost  $Q_t$  in period  $t$  and pay one unit of the currency in which they are issued in period  $t+1$ ,  $\kappa$  units in period  $t+2$ ,  $\kappa^2$  units in period  $t+3$ , and so on.

$B_{H,FC,t}^{Pvt}$  to maximize its bond-sale proceeds

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt} + Q_{H,FC,t}^{Pvt} B_{H,FC,t}^{Pvt}$$

subject to (5). The solution to this problem gives the wholesale firm's bond supply functions in eqns. 26/72 and 27/72 in the appendix. The shadow price of  $\bar{B}_{H,t}^{Pvt}$  is  $\bar{Q}_{H,t}^{Pvt}$  which is in eqn. 28/72 in the appendix. To see the intuition behind the bond supply functions, take the ratio of eqns. 26/72 and 27/72:

$$\frac{B_{H,FC,t}^{Pvt}}{B_{H,t}^{Pvt}} = \frac{v_3}{1 - v_3} \left( \frac{Q_{F,t}^{Pvt}}{Q_{H,t}^{Pvt}} \right)^{\eta_3}.$$

According to this equation, the relative supply of foreign-currency bonds depend on their relative price. If a unit of foreign bond can bring in more proceeds (in home currency), i.e.  $Q_{F,t}^{Pvt}/Q_{H,t}^{Pvt}$  increases, the wholesale firm would like to issue more foreign bonds, i.e.  $B_{H,FC,t}^{Pvt}/B_{H,t}^{Pvt}$  will increase. The elasticity of relative supply is given by  $\eta_3 > 0$ . Parameter  $v_3$  determines the openness of the wholesale firm to foreign borrowing. For example, if the home and foreign bond prices are equal,  $Q_{F,t}^{Pvt} = Q_{H,t}^{Pvt}$ , the ratio of foreign to home bonds will be equal to  $v_3/(1 - v_3)$ . If  $v_3 = 0$ , the wholesale firm does not issue any foreign currency bonds.

Just like in SW, bonds issued in home currency are held by the home financial intermediaries (indexed by  $j \in [0, 1]$ ) and the home central bank:

$$B_{H,t}^{Pvt} = B_{H,t}^{Pvt}(fi) + B_{H,t}^{Pvt}(ma),$$

where  $B_{H,t}^{Pvt}(fi) \equiv \int B_{H,t}^{Pvt}(j) dj$  is the sum of private bond holdings of the financial intermediaries and  $B_{H,t}^{Pvt}(ma)$  are the private bond holdings of the monetary authority. We assume for simplicity that the foreign currency bonds are held only by foreign financial intermediaries. Because of the SOE assumption, the wholesale firm takes the price of foreign private bonds,  $Q_{H,FC,t}^{Pvt}$ , as given.

$Q_{H,FC,t}^{Pvt}$  is the price of foreign private bonds in home currency. The corresponding price in foreign currency is  $\text{€}Q_{H,FC,t}^{Pvt} = Q_{H,FC,t}^{Pvt}/E_t$ , where the sign  $\text{€}$  represents foreign currency

and  $E_t$  is the nominal exchange rate defined as the price of one unit of foreign currency in terms of home currency. The realized return on foreign private bond in foreign currency is

$$\epsilon R_{H,FC,t}^{Pvt} = \frac{\epsilon(1 + \kappa) \epsilon Q_{H,FC,t}^{Pvt}}{\epsilon Q_{H,FC,t-1}^{Pvt}},$$

which we can convert to a return in home currency of

$$R_{H,FC,t}^{Pvt} = \frac{(1 + \kappa Q_{H,FC,t}^{Pvt}) E_t}{Q_{H,FC,t-1}^{Pvt} E_{t-1}}.$$

Apart from the choice of the wholesale firm between home and foreign bonds, the rest of the problem of the wholesale firm in our model is identical to that in SW. The objective of the wholesale firm is to choose labor, capital, capital utilization and bonds to maximize its real profit

$$\frac{\Pi_{2,t}}{P_{C,t}} = p_{2,t} Y_{2,t} - w_{2,t} L_{2,t} - p_{K,t} \hat{I}_t + \bar{Q}_{H,t}^{Pvt} (\bar{b}_{H,t}^{Pvt} - \kappa \bar{b}_{H,t-1}^{Pvt} \pi_{C,t}^{-1}) - \bar{b}_{H,t-1}^{Pvt} \pi_{C,t}^{-1},$$

subject eqn. 16/72, eqn. 18/72 and (4), where  $p_{2,t} \equiv P_{2,t}/P_{C,t}$  is the relative price of wholesale firm's output and  $p_{k,t} \equiv P_{K,t}/P_{C,t}$  is the relative price of new capital. The first term on the right-hand side is the firm's revenue from sale of output. The second term is its labor cost. The third term is the cost of new investment. The fourth term is the real value of new bond issue and the last term is the real coupon payment on outstanding bonds. The first-order conditions for this problem together with other equilibrium conditions are in eqns. 16/72 to 25/72 in the appendix.

**Retailers.**—Retail firms are indexed by  $f \in [0, 1]$ . They repackage wholesale output  $Y_t(f) = Y_{2,t}(f)$  and sell it to a competitive final-good firm. The final output,  $Y_t$ , is a CES aggregate of retail outputs with elasticity of substitution  $\epsilon_p > 1$ . Hence, retailers face the demand curve:

$$(6) \quad Y_t(f) = \left( \frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\epsilon_p} Y_t$$

Where  $P_{H,t}(f)$  is the price of retail output. The price of the final output good,  $P_{H,t}$ , is given

by

$$(7) \quad P_{H,t}^{1-\epsilon_p} = \int_0^1 P_{H,t}(f)^{1-\epsilon_p} df.$$

The nominal profit of a retail firm is given by

$$(8) \quad \Pi_{3,t}(f) = P_{H,t}(f)Y_t(f) - P_{2,t}Y_{2,t}(f).$$

By using  $Y_{2,t}(f) = Y_t(f)$  and the demand function, we obtain:

$$(9) \quad \Pi_{3,t}(f) = P_{H,t}(f)^{1-\epsilon_p} P_{H,t}^{\epsilon_p} Y_t - P_{2,t} P_{H,t}(f)^{-\epsilon_p} P_{H,t}^{\epsilon_p} Y_t.$$

The retail firms set prices in a Calvo fashion. In each period, a retailer faces a constant probability  $1 - \theta_p$  of being able to adjust its price, with  $\theta_p \in [0, 1]$ . Non-updated prices are indexed to lagged CPI inflation.

The relevant equilibrium conditions from retailers' problem are in eqns. 29/72 to 31/72 in the appendix.

**Final-Good Producer.**—The final-good producer buys  $Y_t(f)$  at price  $P_{H,t}(f)$  from retailers and combines them into a composite final good  $Y_t$ . The equilibrium conditions from the final-good producer's problem are in eqns. 32/72 to 34/72 in the appendix.

**Capital Producers.**—A representative capital producer generates new physical capital according to

$$(10) \quad \widehat{I}_t = \left(1 - O\left(\frac{I_t}{I_{t-1}}\right)\right) I_t.$$

$I_t$  is the final output allocated to investment.  $O(\cdot)$  is an adjustment cost function. The capital producer chooses  $I_t$  to maximize

$$(11) \quad P_{k,t} \left[1 - O\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_{H,t} I_t.$$

We assume the following functional form for  $O(\cdot)$ :

$$O\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa_I}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

The equilibrium conditions for the capital producer's problem are in eqns. 35/72 and 36/72.

**Exporters.**—We follow Gali and Monacelli (2016) to model exports. We assume that the exporting firm is a price taker and has no market power. Moreover, we assume that the demand for exports from the foreign country is given by

$$(12) \quad X_t = \nu_2 \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_2} Y_{F,t}$$

where  $P_{F,t}$  is the price of good produced in the foreign country and  $Y_{F,t}$  is the total output in the foreign country. Also note that

$$(13) \quad P_{F,t} \equiv \text{€}P_{F,t}E_t,$$

where  $\text{€}P_{F,t}$  is the price of foreign good in foreign currency and  $E_t$  is the nominal exchange rate. The equilibrium condition for the exporter's problem is in eqn. 37/72 in the appendix.

#### ***D. Financial Intermediaries***

There is a unit mass of financial intermediaries (FI's) indexed by  $j \in [0, 1]$ . Each period,  $1 - \vartheta$  fraction of financial intermediaries exit and are replaced by an equal mass of new entrants such that the total mass remains the same. The exiting FI's return their networth to the household. The new entrants receive an initial endowment of networth from the household. Each FI holds private bonds,  $B_t^{Pvt}(j)$ , government bonds,  $B_t^{Gov}(j)$ , and interest bearing reserves,  $S_t(j)$ , with the central bank. These assets are matched by the FI's liabilities in the form of household deposits,  $D_t(j)$  and net worth,  $N_t(j)$ . The balance sheet of financial intermediary  $j$  is:

$$Q_t^{Pvt} B_t^{Pvt}(j) + Q_t^{Gov} B_t^{Gov}(j) + S_t(j) = D_t(j) + N_t(j).$$

The value of the FI at  $t$  is:

$$V_t(j) = (1 - \vartheta) \mathbb{E}_t \Lambda_{t,t+1} n_{t+1}(j) + \vartheta \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}(j).$$

The two constraints facing the FI are:

$$V_t(j) \geq \theta_t (Q_t^{Pvt} b_t^{Pvt}(j) + \Delta Q_t^{Gov} b_t^{Gov}(j)),$$

where  $b_t^{Pvt}(j) \equiv B_t^{Pvt}(j) / P_{C,t}$  and  $b_t^{Gov}(j) \equiv B_t^{Gov}(j) / P_{C,t}$ , and

$$s_t(j) \geq \varsigma_t d_t(j),$$

where  $s_t(j) \equiv S_t(j) / P_{C,t}$  and  $d_t(j) \equiv D_t(j) / P_{C,t}$ . The first constraint above is central to the problem of the FI. If this constraint is not binding, the FI's will buy unrestricted quantities of bonds, bid up the bond prices and drive down bond returns to the level of return on deposits. In that case, there would be no excess returns from holding bonds and the FI would be indifferent among the three assets that it can hold. In our model, this constraint is always binding:

$$V_t(j) = \theta_t (Q_t^{Pvt} b_t^{Pvt}(j) + \Delta Q_t^{Gov} b_t^{Gov}(j)).$$

The second constraint is only binding when the monetary authority allows the rate on reserves to be negative. So far, the problem of the FI is identical to that in SW and the relevant equilibrium conditions are appendix eqns. 38/72 to 46/72.

What is new in our model is that we allow the FI's to also hold foreign private and government bonds. Hence the first two terms on the left-hand side of the above equation can be further decomposed as:

$$Q_t^{Pvt} B_t^{Pvt}(j) = Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt} B_{F,t}^{Pvt}(j)$$

and

$$Q_t^{Gov} B_t^{Gov}(j) = Q_{H,t}^{Gov} B_{H,t}^{Gov}(j) + Q_{F,t}^{Gov} B_{F,t}^{Gov}(j).$$



This formulation provides two more channels through which the effects of foreign monetary policy spillover to the small open economy. For example, when the interest rate on foreign private bonds increases, the FI's at home may divert some of their funds to buy foreign private bonds.

The FI's take the prices of foreign private ( $Q_{F,t}^{Pvt}$ ) and government ( $Q_{F,t}^{Gov}$ ) bonds, as given. We assume the foreign private bond market to be frictionless and hence

$$Q_{F,t}^{Pvt} = Q_{H,FC,t}^{Pvt}.$$

The price at which the home FI's buy foreign private bonds is the same as the price at which home wholesale firms sell foreign private bonds. And because of the SOE assumption, home firms, both financial and non-financial, take this price as exogenously given.

For the private bonds, the FI's problem is to choose  $B_{H,t}^{Pvt}(j)$  and  $B_{F,t}^{Pvt}(j)$  to minimize the total cost of private bonds

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt}(j) + Q_{F,t}^{Pvt} B_{F,t}^{Pvt}(j)$$

subject to

$$B_t^{Pvt}(j) = \left[ \left( \frac{1}{1 - \nu_4} \right)^{-1/\eta_4} [B_{H,t}^{Pvt}(j)]^{\frac{\eta_4 - 1}{\eta_4}} + \left( \frac{1}{\nu_4} \right)^{-1/\eta_4} [B_{F,t}^{Pvt}(j)]^{\frac{\eta_4 - 1}{\eta_4}} \right]^{\frac{\eta_4}{\eta_4 - 1}}.$$

The solution to this problem gives the demand function in eqns. 47/72 and 48/72 in the appendix. A similar problem for government bonds gives the demand function in eqns. 49/72 and 50/72 in the appendix. The appendix eqns. 51/72 to 60/72 contain other equilibrium conditions that mainly consist of the definitions of some bond prices and returns.

### ***E. Monetary Authority***

We model the monetary authority in the same way as SW do. The monetary authority conducts monetary policy in two ways: (1) by adjusting the interest rate on reserves and

(2) by adjusting the quantities of home-issued private and government bonds.<sup>7</sup> The choice of the interest rate on reserves is guided by the following Taylor rule:

$$\begin{aligned} \ln R_t^{Pol} &= (1 - \rho_r) \ln (R^{Pol})^{SS} + \rho_r \ln R_{t-1}^{Pol} \\ &+ (1 - \rho_r) [\phi_\pi (\ln \pi_t - \ln \pi^{SS}) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_r \varepsilon_{r,t} \end{aligned}$$

where  $(R^{Pol})^{SS}$  and  $\pi^{SS}$  are steady state values of the policy rate and the CPI inflation target,  $0 < \rho_r < 1$ , and  $\phi_\pi$  and  $\phi_y$  are non-negative parameters. There is no restriction on  $R_t^{Pol}$  and its realized value depends on economic conditions (i.e. on inflation relative to its steady-state value and output relative to its previous-period value) and the policy-rate shock ( $\varepsilon_{r,t}$ ). The realized value of  $R_t^{Pol}$  leads to one of the following three scenarios regarding the monetary authority's choice of the interest rate on reserves.

**Scenario 1:**  $R_t^{Pol} > 1$ .—When  $R_t^{Pol} > 1$ , we call it the normal times, the monetary authority sets the interest rate on reserves equals to the policy rate implied by the Taylor rule:

$$R_t^S = R_t^{Pol}.$$

**Scenario 2:**  $R_t^{Pol} \leq 1$  *and the monetary authority does not want negative interest rate on reserves.*—When  $R_t^{Pol} \leq 1$ , the monetary authority may decide that it will not reduce the rate on reserves beyond zero. In this scenario,

$$R_t^S = \max \{1, R_t^{Pol}\},$$

which implies  $R_t^S = 1$  because  $R_t^{Pol} \leq 1$ .

**Scenario 3:**  $R_t^{Pol} \leq 1$  *and the monetary authority is open to negative interest rate on reserves.*—When  $R_t^{Pol} \leq 1$ , the monetary authority may decide to allow negative interest rate on reserves. Suppose  $\underline{R} < 1$  is the lowest interest rate on reserves that the

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<sup>7</sup>We could allow the monetary authority to hold foreign bonds but it would be an unnecessary complication.

monetary authority is willing to allow. Then

$$R_t^S = \max \{ \underline{R}, R_t^{Pol} \}.$$

In this scenario, because  $R_t^{Pol} \leq 1$ ,  $R_t^S \in [\underline{R}, 1]$ .

Under Scenario 1, the FI's set  $R_t^D = R_t^S$ . Under Scenarios 2 and 3, the FI's set  $R_t^D = 1$ . So the FI's choice of the interest rate on deposits can be summarized as

$$R_t^D = \max \{ 1, R_t^S \}.$$

We summarize the choices of  $R_t^S$  and  $R_t^D$  in the three scenarios in Table 1.

Table 1: Summary of interest rates scenarios

Scenario	Taylor rule	Rate on reserves	Rate on deposits ( $R_t^D = \max \{ 1, R_t^S \}$ )
1	$R_t^{Pol} > 1$	$R_t^S = R_t^{Pol}$	$R_t^D = R_t^S$
2	$R_t^{Pol} \leq 1$	$R_t^S = \max \{ 1, R_t^{Pol} \}$ (which implies $R_t^S = 1$ )	$R_t^D = 1$
3	$R_t^{Pol} \leq 1$	$R_t^S = \max \{ \underline{R}, R_t^{Pol} \}$ (which implies $R_t^S \in [\underline{R}, 1]$ )	$R_t^D = 1$

In addition to the choice of  $R_t^S$ , the monetary authority can also buy or sell private and government bonds. This is what is commonly known as quantitative easing (QE). Note that the monetary authority can potentially engage in QE in any of the above three scenarios. However, in practice, monetary authorities have resorted to QE only when  $R_t^{Pol} \leq 1$  (i.e. Scenarios 2 and 3). To see how QE works in this model, note that the balance sheet of the monetary authority is

$$Q_{H,t}^{Pvt} B_{H,t}^{Pvt} (ma) + Q_{H,t}^{Gov} B_{H,t}^{Gov} (ma) = S_t,$$

where the assets, which consist of bond holdings, are on the left-hand side and the liabilities, which consist of reserves, are on the right-hand side. We can express this balance

sheet in real terms as

$$\text{(eqn. 61/72)} \quad Q_{H,t}^{Pvt} b_{H,t}^{Pvt}(ma) + Q_{H,t}^{Gov} b_{H,t}^{Gov}(ma) = s_t.$$

The monetary authority's choices of  $b_{H,t}^{Pvt}(ma)$  and  $b_{H,t}^{Gov}(ma)$  follow the following AR(1) processes:

$$\text{(eqn. 66/72, Exog.)} \quad b_{H,t}^{Pvt}(ma) = (1 - \rho_1) (b_H^{Pvt}(ma))^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) + s_1 \varepsilon_{1,t}$$

and

$$\text{(eqn. 67/72, Exog.)} \quad b_{H,t}^{Gov}(ma) = (1 - \rho_2) (b_H^{Gov}(ma))^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) + s_2 \varepsilon_{2,t},$$

in the case of exogenous QE policies. We will call  $\varepsilon_{1,t}$  and  $\varepsilon_{2,t}$  the QE shocks. The MA's choices of  $b_{H,t}^{Pvt}(ma)$  and  $b_{H,t}^{Gov}(ma)$  could alternatively follow a Taylor type reaction function:

(eqn. 66/72, Endo.)

$$\begin{aligned} b_{H,t}^{Pvt}(ma) &= (1 - \rho_1) (b_H^{Pvt}(ma))^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) \\ &\quad + (1 - \rho_1) \Psi_1 [\phi_\pi (\ln \pi_{C,t} - \ln \pi_C^{SS}) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_1 \varepsilon_{1,t} \end{aligned}$$

and

(eqn. 67/72, Endo.)

$$\begin{aligned} b_{H,t}^{Gov}(ma) &= (1 - \rho_2) (b_H^{Gov}(ma))^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) \\ &\quad + (1 - \rho_2) \Psi_2 [\phi_\pi (\ln \pi_{C,t} - \ln \pi_C^{SS}) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_2 \varepsilon_{2,t} \end{aligned}$$

in the case of endogenous QE policies. The appendix eqns. 61/72 to 67/72 summarize the equilibrium conditions related to the monetary authority.

## **F. Fiscal Authority**

We model the fiscal authority in the same way as do SW. The nominal period budget constraint of the fiscal authority is:

$$P_{C,t} G_t + B_{H,t-1}^{Gov} = P_{C,t} T_t + \Pi_{ma,t} + Q_{H,t}^{Gov} B_{H,t}^{Gov} - \kappa Q_{H,t}^{Gov} B_{H,t-1}^{Gov}.$$

The fiscal authority consumes an exogenous and stochastic amount of final good,  $G_t$ . The money to pay for these expenses comes from lump sum taxes on the household ( $T_t$ ), monetary-authority profits ( $\Pi_{ma,t}$ ) and borrowing ( $B_{H,t}^{Gov}$ ). Following SW, we assume that the real outstanding government debt is constant and equal to

$$\frac{B_{H,t}^{Gov}}{P_{C,t}} = \frac{B_{H,t-1}^{Gov}}{P_{C,t-1}} = \bar{b}^{Gov}$$

and the lump sum taxes adjust every period to satisfy the government's budget constraint. The market clearing condition for home government bonds is

$$B_{H,t}^{Gov} = B_{H,t}^{Gov}(fi) + B_{H,t}^{Gov}(ma),$$

where  $B_{H,t}^{Gov}(fi) \equiv \int B_{H,t}^{Gov}(j) dj$ . The equilibrium conditions related to the fiscal authority are in eqns. 68/72 to 70/72.

### **G. Foreign Exchange Market**

There are two important equations related to the foreign exchange market. The first is the evolution of the real exchange rate (RER). By definition, the RER is

$$\text{RER}_t = \frac{\text{€}P_{F,t}E_t}{P_{C,t}} = \frac{P_{F,t}}{P_{C,t}} = p_{F,t}.$$

Similarly,

$$p_{F,t-1} \equiv \frac{P_{F,t-1}}{P_{C,t-1}} = \frac{E_{t-1}\text{€}P_{F,t-1}}{P_{C,t-1}}.$$

By taking the ratio of the two we get

$$\frac{p_{F,t}}{p_{F,t-1}} = \frac{E_t}{E_{t-1}} \frac{\text{€}\pi_{F,t}}{\pi_{C,t}},$$

which, we can rearrange to get eqn. 71/72 in the appendix.

The second equation is the balance-of-payment equilibrium condition. To derive this condition, note that the total demand for home currency (in home currency units) in the

forex market in period  $t$  is

$$P_{H,t}X_t + (B_{F,t-1}^{Pvt} + B_{F,t-1}^{Gov}) E_t + Q_{F,t}^{Pvt} (B_{H,FC,t}^{Pvt} - \kappa B_{H,FC,t-1}^{Pvt}).$$

The total supply of home currency in units of home currency in period  $t$  is

$$P_{F,t}C_{F,t} + E_t B_{H,FC,t-1}^{Pvt} + Q_{F,t}^{Pvt} (B_{F,t}^{Pvt} - \kappa B_{F,t-1}^{Pvt}) + Q_{F,t}^{Gov} (B_{F,t}^{Gov} - \kappa B_{F,t-1}^{Gov}).$$

The balance of payment equilibrium condition is

$$\text{Total Demand for Home Currency} = \text{Total Supply of Home Currency},$$

which gives the balance of payment equilibrium condition in eqn. 72/72 of the appendix.

This completes our description of the model. There are 72 equilibrium equations and 72 endogenous variables. We provide a complete list of endogenous variables in Appendix A. The list also includes a short description for each variable. The full set of equilibrium equations is in Appendix B. In an online appendix that accompanies this paper, we solve the model for its non-stochastic steady state, provide other details about the model and derive the equilibrium conditions.

As we said in the introduction, our model is an small-open-economy version of the model in SW. On the real side, we add imports and exports closely following Gali and Monacelli (2016). On the financial side, we allow the wholesale firm to issue bonds in both home and foreign currency. We also allow home financial intermediaries to hold both home and foreign bonds. We summarize the flow of bonds in our model in Figure 1.

The wholesale firm issues  $B_{H,t}^{Pvt}$  bonds in home currency. These bonds are held by the monetary authority,  $B_{H,t}^{Pvt}(ma)$ , and the financial intermediaries,  $B_{H,t}^{Pvt}(fi)$ :

$$B_{H,t}^{Pvt} = B_{H,t}^{Pvt}(ma) + B_{H,t}^{Pvt}(fi).$$

The wholesale firm also issues  $B_{H,FC,t}^{Pvt}$  bonds in foreign currency. The choice between  $B_{H,t}^{Pvt}$  and  $B_{H,FC,t}^{Pvt}$  depends on the home bias of the wholesale firm, the relative price of the two

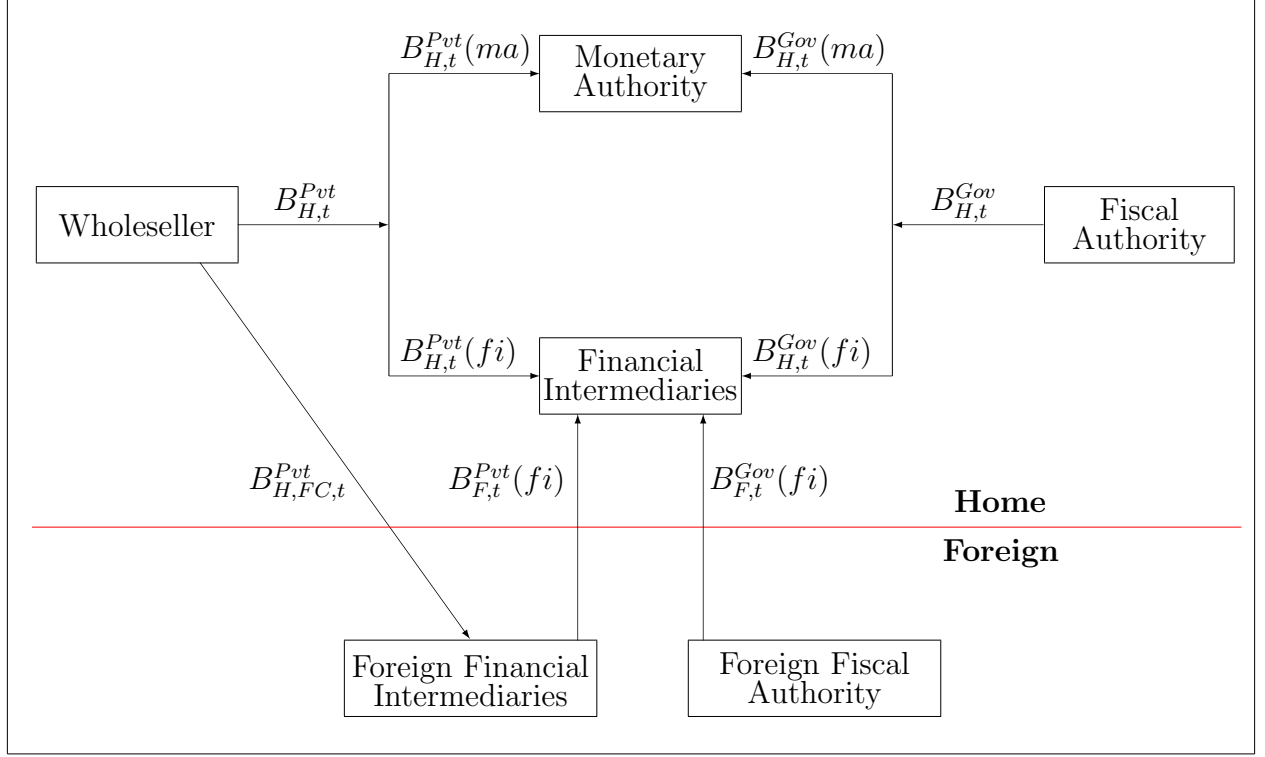


Figure 1: Bond flow chart

types of bonds and an elasticity parameter. The CES aggregate of  $B_{H,t}^{Pvt}$  and  $B_{H,FC,t}^{Pvt}$  is  $\bar{B}_{H,t}^{Pvt}$  (not in the figure), that also satisfies

$$\bar{B}_{H,t}^{Pvt} = \frac{Q_{H,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} B_{H,t}^{Pvt} + \frac{Q_{F,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} B_{H,FC,t}^{Pvt}.$$

The home financial intermediaries hold both private,  $B_t^{Pvt}(fi)$ , and government,  $B_t^{Gov}(fi)$ , bonds (both not in the figure). Within each category, they hold both home and foreign bonds that satisfy

$$B_t^{Pvt}(fi) = \frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}} B_{H,t}^{Pvt}(fi) + \frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}} B_{F,t}^{Pvt}(fi)$$

and

$$B_t^{Gov}(fi) = \frac{Q_{H,t}^{Gov}}{Q_t^{Gov}} B_{H,t}^{Gov}(fi) + \frac{Q_{F,t}^{Gov}}{Q_t^{Gov}} B_{F,t}^{Gov}(fi).$$

The financial intermediaries' choices between home and government bonds of each type depend on the degree of their home bias, the relative price of the two types of bonds and elasticity parameters.

The fiscal authority issues  $B_{H,t}^{Gov}$  bonds, all in home currency. Of these,  $B_{H,t}^{Gov}(ma)$  are held by the monetary authority and  $B_{H,t}^{Gov}(fi)$  are held by the financial intermediaries:

$$B_{H,t}^{Gov} = B_{H,t}^{Gov}(ma) + B_{H,t}^{Gov}(fi).$$

The monetary authority, holds home currency bonds issued by the wholesale firm,  $B_{H,t}^{Pvt}(ma)$ , and by the fiscal authority,  $B_{H,t}^{Gov}(ma)$ .



### III. Quantitative Features of the Model

In this section, we describe our calibration strategy and solution method. We then present results of a simulation experiment that is similar to what SW did for their closed economy model. Their motivation was to compare the relative strength of conventional and new monetary policy tools. Our motivation is to compare the international spillovers of conventional and new monetary policy tools.

#### A. Calibration

We model the foreign country as a closed economy.<sup>8</sup> This closed economy is almost identical to the economy in SW and, to acknowledge their contribution, we call it *Sims and Wu economy*. The only changes that we make to Sims and Wu economy are to generalize their log utility to CRRA and add a preference shock. These changes add three parameters to their model. Our modified Sims and Wu economy has 45 parameters in total.<sup>9</sup> In Tables 5 and 6 in the Appendix, we list these parameters together with their value or target for the home economy. Table 6 has 31 parameters. Table 5 has 14 parameters, all related to the dispersion or persistence of shocks in the model. We take most of these parameter values and targets from SW. The only differences are the following: (1) The ratio of the value of the MA's bond holdings to GDP is 6% in SW. We keep that target for the foreign economy but change it to 3.19% for the home economy to match the Bank of Canada's government bond holdings in 2006. (2) We need to adjust parameters  $\Psi_1$  and  $\Psi_2$  to achieve some quantitative QE targets. We set  $\Psi_1 = \Psi_2 = -20$  for the home economy and  $\Psi_1 = \Psi_2 = -35$  for the foreign economy. Sims and Wu had  $\Psi_1 = \Psi_2 = -7$ . (3) The target for  $L_1^{SS}$  is 1 for home country and 7.44 for foreign country.<sup>10</sup>

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<sup>8</sup>This is consistent with our assumption that the home economy is a small open economy. Although, the home economy engages in both real and financial transaction with the foreign economy, these transactions are so small relative to the size of the foreign economy that they have no effect on it. An alternative would be to have a two-country model with home as a small country and foreign as a large country. However, due to the asymmetry in the size of the two countries, we would also need to add a third player (rest of the world) to close the model. Modelling home as a small open economy obviates the need for a third player and makes the analysis simpler and more transparent.

<sup>9</sup>The model period is a quarter and we calibrate the parameters accordingly.

<sup>10</sup>By the end of 2006, the number of employed people in the US was 119.1 million and in Canada it was 16.0 million. The ratio of the two is 7.44. Given the SOE structure of our model, we could normalize  $L_1^{SS} = 1$  in the foreign country as well and recalibrate  $\omega$  accordingly.

In addition to the 45 parameters that we list in Tables 5 and 6, the home economy has 10 SOE-specific parameters. There are five elasticity parameters:  $\eta_1, \eta_2, \eta_3, \eta_4$  and  $\eta_5$ , and five home/foreign bias (or home/foreign share) parameters:  $\nu_1, \nu_2, \nu_3, \nu_4$  and  $\nu_5$ . There are no readily available counterparts in the literature to our elasticity parameters. We follow Gali and Monacelli (2016) and set all these elasticity parameters to 1. Later, we perform sensitivity analysis on each of these parameters. We calibrate the five home/foreign share parameters to match certain targets in the Canadian data. We list these parameters and their targets in Table 2.

Table 2: SOE-specific Parameters

Parameter Name/Description	Symbol	Target
Home bias in consumption <i>Target: Imports to GDP ratio</i>	$\nu_1$	28.0%
Degree of openness <i>Target: Exports to GDP ratio</i>	$\nu_2$	31.7%
Share of foreign bonds in wholesale firms total bond issue <i>Target: <math>(b_F^{Gov}(fi))^{SS} / (b_F^{Pvt}(fi))^{SS}</math></i>	$\nu_3$	4.7
Share of foreign pvt. bonds in FI's pvt. bond holdings <i>Target: Balance of payments equilibrium</i>	$\nu_4$	0
Share of foreign govt. bonds in FI's govt. bond holdings <i>Target: <math>(b_F^{Gov}(fi))^{SS} / [(b_F^{Gov}(fi))^{SS} + (b_H^{Gov}(fi))^{SS}]</math></i>	$\nu_5$	43.0%

Parameter  $\nu_1$  represents the steady-state share of imports in total consumption in the model. And consumption is a stable function of GDP. We pick  $\nu_1$  such that the steady-state import to GDP ratio in the model is 28%, which is the same as the Canadian import to GDP ratio in 2006, the year before the 2007 crisis. Similarly, we pick  $\nu_2$  such that the steady-state export to GDP ratio is 31.7%. We pick  $\nu_5$  to match the share of foreign government bonds in Canada's five largest banks' government bond holdings. We then pick  $\nu_3$  and  $\nu_4$  together such that the ratio of foreign government and foreign private bonds held by the FI's is 4.7 and the balance of payment condition is satisfied in the steady state. An alternative target to pin down  $\nu_3$  is the share of foreign currency bonds in the total bonds issued by Canadian corporations. We try this alternative calibration in the sensitivity section and show that it does not make any significant difference to our results.

## *B. Solution and Computation*

The solution to our model is based on 51 non-linear equilibrium equations for the foreign economy and 72 non-linear equilibrium equations for the home economy.<sup>11</sup> We list the 72 equations for the home economy in Appendix B. The equations for the foreign economy are a 51-equation subset of the 72 equations. In our description of the solution and in the online appendix accompanying this paper, we focus on the 72-equation equilibrium system that represents the home economy. In the online appendix, we derive the equilibrium conditions and provide the non-stochastic steady-state solution to the model. Following Sims and Wu (2020), we simulate the model by using a linear approximation around the non-stochastic steady state. In the non-stochastic steady state, the zero lower bound (ZLB) constraint does not bind. When the ZLB binds, we follow Guerrieri and Iacoviello (2015) to solve a piecewise linear version of the model. We use Dynare, Adjemian et al. (2011), for simulations.<sup>12</sup>

## *C. International Spillovers of Conventional versus New Monetary Policy*

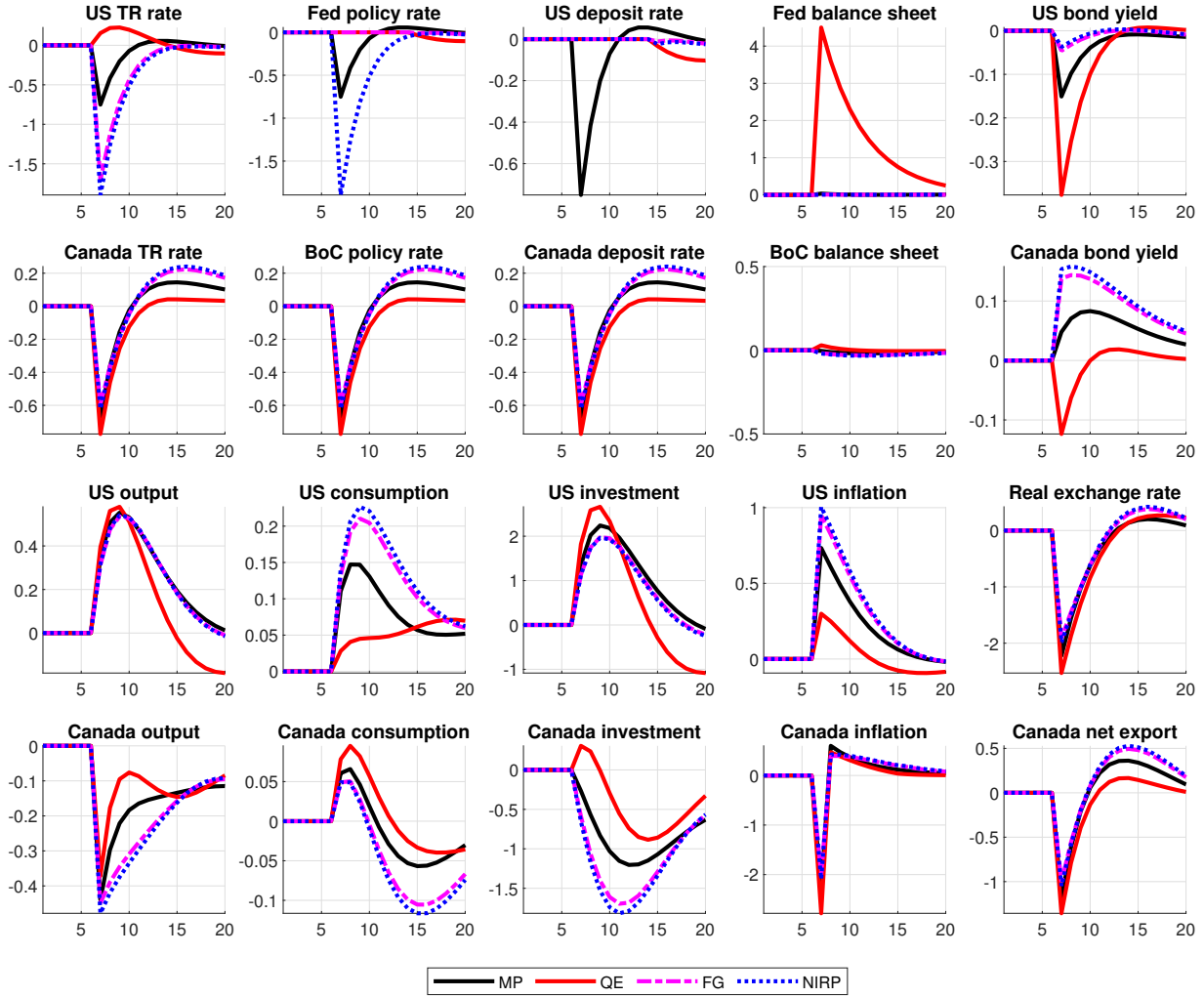
There is an emerging literature that compares the international spillovers of conventional monetary policy with those of QE [Alpanda and Kabaca (2020), Curcuru et al. (2018) and Kolasa and Wesolowski (2020)]. In our first experiment, we do the same but go one step further. In addition to the quantitative easing (QE), our model allows us to talk about the spillover effects of forward guidance (FG) and the negative interest rate policy (NIRP).

We show the first set of results in Figure 2. Here we repeat the experiment that SW did for Figure 1 in their paper. Their goal was to quantify the conventional and new monetary policy steps that would generate a similar effect on GDP. In order to do so, they came up with the following monetary policy interventions: (1) They hit the economy with a  $-1\%$  shock to its annualized policy rate. This is the conventional monetary-policy stimulus. (2) For QE, they allow the central bank to increase its balance sheet by about 4% of GDP. (3) For FG, they shock the economy by  $-2.2\%$  change in the annualized policy rate. (4) For

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<sup>11</sup>Our foreign economy is the same as the Sims and Wu economy except for the two changes noted above. These changes add one more equation for the evolution of preference shock to Sims and Wu's system of 50 equations.

<sup>12</sup>For now, the Dynare codes to replicate our results are available from authors upon request. Eventually, we will make the codes publicly available together with the online appendix.



Notes: We define bond yield,  $RL_t^{Pvt}$ , as the gross yield to maturity on the private investment bond. Specifically,  $RL_t^{Pvt} = \frac{1}{Q_t^{Pvt}} + \kappa$ .

Figure 2: Exogenous monetary policy shocks to US economy only

NIRP, they hit the economy with a  $-2.4\%$  shock to its annualized policy rate. All shocks hit the economy in period 7. We generate a binding ZLB with a sequence of liquidity shocks of 1.5 standard deviations in each of the period 1 through to 6 to the US economy.

The US simulations in Figure 2 are identical to those in Figure 1 in SW. The focus of our discussion will be on the simulations of the Canadian economy. It is important to note that we do not hit the Canadian economy with any exogenous shock. The shocks to the US economy spillover to the Canadian economy through both real and financial channels. The Canadian monetary authority (The Bank of Canada, BoC), continues to follow the Taylor

rule to conduct its monetary policy.

We start with the conventional monetary policy (solid black lines). When the US policy rate drops, the bond yield in the US falls. This increases the foreign bond prices. On the other hand, a decrease in the Fed policy rate causes an appreciation of the Canadian dollar. By definition, bond price in Canada is an aggregate of domestic and foreign bond prices in Canadian dollars. The net equilibrium effect is that the Canadian bond yield increases. This is because the small increase in foreign bond prices as a result of the expansionary conventional monetary policy is dominated by the much large appreciation of the domestic currency. This has a negative effect on investment in Canada. Although the US output increase has a positive effect on Canadian exports, the appreciating Canadian currency has a negative effect. Also, due to the higher inflation in the US and lower inflation in Canada, Canadians import less. The net effect of these changes on Canada's net exports is negative. Despite a small increase in consumption, lower investment and net exports cause Canadian GDP to fall. This is consistent with the finding in Alpanda and Kabaca (2020) that an expansionary monetary policy in the foreign economy has a negative effect on home economy. Rey (2016) and Blanchard et al. (2016) also find a similar contractionary effect of foreign expansionary monetary policy on home GDP.

The effect of QE (solid red lines) is different on the Canadian bond yield. Unlike the increase in the Canadian bond yield caused by an expansionary conventional monetary policy in the US, the yield decreases as a result of the expansionary QE policy in the US.<sup>13</sup> This is because the decrease in US bond yield is significantly larger in the case of QE than in the case of conventional monetary policy. As a result, the positive effect on foreign bond prices dominates the negative effect caused by the appreciating local currency. The net equilibrium effect is a decrease in Canadian yields. This causes a small increase in investment in Canada. However, the increase is not large enough to offset the decline in net exports. The final effect on Canadian GDP is similar in this case except that the GDP recovers faster in the case of QE than it does in the case of conventional monetary policy.

The effects of FG (purple dashed lines) and NIRP (blue dotted lines) on Canadian

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<sup>13</sup>This is consistent with the main result in Gilchrist et al. (2019) "that yields on dollar-denominated sovereign debt are highly responsive to unanticipated changes in the stance of US monetary policy during both the conventional and unconventional policy regimes."

bond yield are qualitatively similar to those of the conventional monetary policy (the yield increases) but quantitatively stronger. This translates into a bigger drop in investment and a slower recovery in the GDP.

To sum up, an expansionary conventional monetary policy in the US leads to small increase in the Canadian bond yield. An expansionary QE leads to a small decrease in the yield. The FG and NIRP lead to even bigger increases in the yield. The immediate net effect on Canadian GDP is negative and similar for all four policies. However, the GDP recovery is fastest in the case of QE and slowest in the cases of FG and NIRP.

A key assumption behind the simulations in Figure 2 is that there is no monetary policy shock in the Canadian economy. However, for highly integrated economies like the US and Canada, it is far more likely that they are simultaneously hit by similar exogenous shocks. To explore the implications of this assumption, we modify the experiment in Figure 2 and allow, in addition to the same monetary policy shocks to the US economy, a  $-1\%$  shock to Canada's policy rate. The results of this experiment are in Figure 3.

The US simulations in Figure 3 are identical to those in Figure 2 because any exogenous shocks to the Canadian economy do not affect the US economy because of the SOE assumption. However, the effects on Canadian economy are qualitatively different. The Canadian bond yield, which increased in the case of three out of four policies in Figure 2, decrease in all four cases in Figure 3. Lower long yields stimulate investment because it is cheaper for the wholesale firm to borrow. Another significant difference is the immediate response of Canadian inflation. In Figure 2, the Canadian inflation decreased but in Figure 3 it increased. The third significant difference is that Canada's real exchange rate depreciates and leads to an increase in Canadian net exports. Increase in investment and net exports, accompanied by a larger increase in consumption, add up to a significant increase in Canadian GDP. This effect on GDP in Figure 3, which is consistent with the finding in Kolasa and Wesolowski (2020) about the spillovers of conventional monetary policy, is opposite of what we saw in Figure 2.

Two overarching conclusions emerge from Figures 2 and 3. First, if the monetary policy shocks originate only in the foreign economy and we choose the shocks such that the immediate effect on foreign GDP of conventional and new monetary policies is roughly the

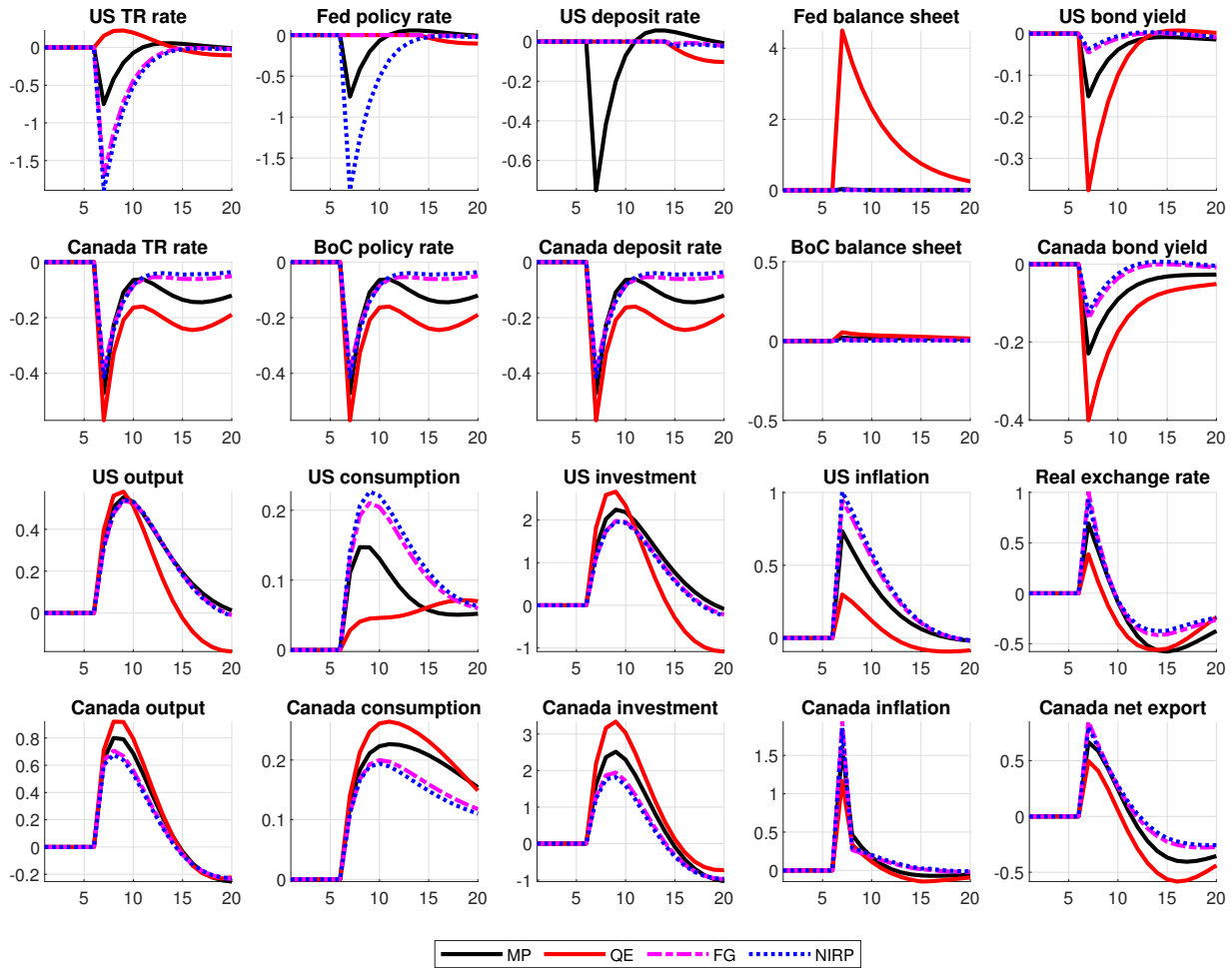


Figure 3: Exogenous monetary policy shocks to US and Canadian economies

same, the immediate effects on home GDP are also the same. However, the recovery rate of home GDP is different across conventional and new monetary policies and also within the various new monetary policy measures. Moreover, the effects on various components of GDP, like consumption, investment and net exports, are also different by policy. In these respects, QE is generally more expansionary than the conventional monetary policy whereas the FG and NIRP are less expansionary.

The second overarching conclusion is that if there are domestic monetary policy shocks, in addition to the foreign monetary policy shocks, the effects on GDP and its components are different. The key drivers of these differences are the differences in equilibrium prices or returns. Specifically, the home bond yield, home inflation and the real exchange rate all behave quite differently, and more favorably to the home economy, when the home monetary

policy shocks are added to the foreign ones. However, even in this case the spillovers are strongest in the case of foreign QE and the weakest in the cases of foreign FG and NIRP.



## IV. Counterfactual Experiments

Before the 2008 financial crisis, the size of the Fed’s balance sheet was around 6% of US GDP. After the third round of QE in 2014, the size of the Fed’s balance sheet had increased to 25% of US GDP. The Bank of Canada, on the other hand, did not engage in any QE around the 2008 crisis. What would have happened to the Canadian economy had the Fed engaged in a more or less aggressive QE? How would the outcomes be different had the Bank of Canada engaged in QE on top of what the Fed did? In this section, we use our model to run some counterfactual experiments to answer these questions. We do so in three steps. First, we construct a benchmark scenario in which we choose credit shocks in such a way that when combined with the actual QE policies of the Fed and the Bank of Canada, they produce some real outcomes that are close to what happened in the data. Second, we counterfactually change the magnitude of QE done by the Fed to see how it would have changed the outcomes for the Canadian economy. Third, we run a counterfactual experiment in which we allow the Bank of Canada to engage in QE in the wake of 2008 crisis. For ease of exposition, we divide these steps into three separate subsections.

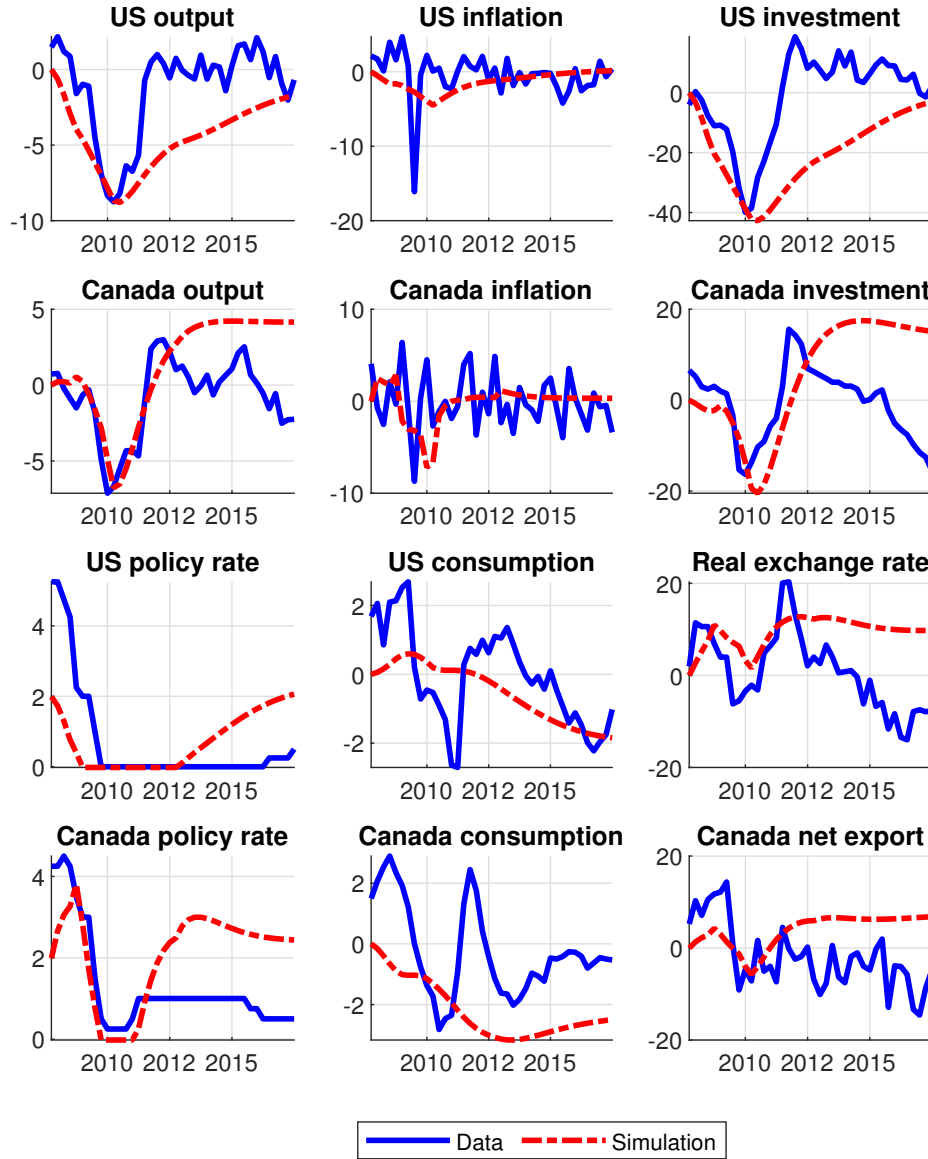
### A. *Benchmark*

There were steep drops in output and investment in both the US and Canada between 2008 and 2010. We use these drops as our targets and choose the liquidity shocks (shocks to  $\theta_t$ ) for the two economies such that the simulated drops in output and investment are very close to what we observe in the data.<sup>14</sup> In Figure 4, we plot actual versus simulated time series for selected variables. We are able to closely match the output and investment drops in both the US and Canada. This is not surprising as we choose the liquidity shocks to match these drops. We also show a number of other variables in the figure to give the reader some idea about the fit of our model to data for this particular benchmark. A few comments are in order.

The recovery in output and investment for the US is much slower in the model. The

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<sup>14</sup>We introduce the following exogenous shock to  $\theta_t$ . A 1.5 standard deviation negative shock hits the US economy each period from periods 2 to 6. A 1.0 standard deviation negative shock hits both the US and Canadian economies from periods 7 to 11.



*Notes:* We use the filter proposed by Hamilton (2018) to detrend output, investment and consumption series for both US and Canada. We use 8 lags in estimating the trend.

Figure 4: Benchmark, 2008 crisis and its aftermath (model vs. data)

model does a much better job in matching the recovery in output and investment for Canada. The time that the US economy spends at the ZLB is much shorter in the simulation. The model fails to replicate the initial drop in US consumption but matches the subsequent drop (from 2012 onwards) better.

The model does well to capture the initial contours of the real exchange rate in the data, though it does not do so well in the case of net exports. The primary purpose of Figure 4 is not to match the data, which would be very difficult given that we introduce only one shock to both economies. Instead, the purpose of the figure is to give some perspective to our benchmark simulation and set the stage for the subsequent counterfactual experiments.

### ***B. Counterfactual Changes in QE by the Fed***

In the aftermath of the 2008 crisis, the Fed increased its balance sheet from 6% of US GDP to 25%. In this set of counterfactual experiments, we change the magnitude of Fed's QE to see how the spillovers to the Canadian economy change.

In Figure 5, we compare the benchmark with a counterfactual scenario of no QE by the Fed (Counterfactual 1). If the Fed did not engage in QE, the desired policy rate given by the Taylor rule would be around  $-6\%$ . Given the ZLB restriction, this would imply a deeper recession in the US. In particular, the US output and investment would drop by almost twice as much as in the benchmark scenario. Other effects on the US would include a sharper increase in the bond yield and a steeper drop in inflation. In short, had the Fed not engaged in any QE at all, the US great recession would have been much worse.

No QE by the Fed would lead to somewhat better outcomes for Canada compared to the benchmark. For example, the yield in Canada would not go up by as much as in the benchmark scenario. This would result in a smaller drop and earlier recovery in Canada's output and investment. Because of a higher relative yield in the US and a lower relative yield in Canada, the Canadian real exchange rate would depreciate even more and the drop in net exports (after the initial increase) would be milder. The effect on consumption would be mixed. Consumption would drop more in the first few quarters in the counterfactual scenario and then drop at a slower rate.

Note that these different effects on the Canadian economy in the counterfactual scenario are driven by the spillovers from the US. There is no difference in shocks to the Canadian economy between the benchmark and the counterfactual scenarios. Similarly, there is no change in the Canadian monetary policy beyond the endogenous response of the Taylor-rule policy rate. These results are in line with the findings of a large literature on international

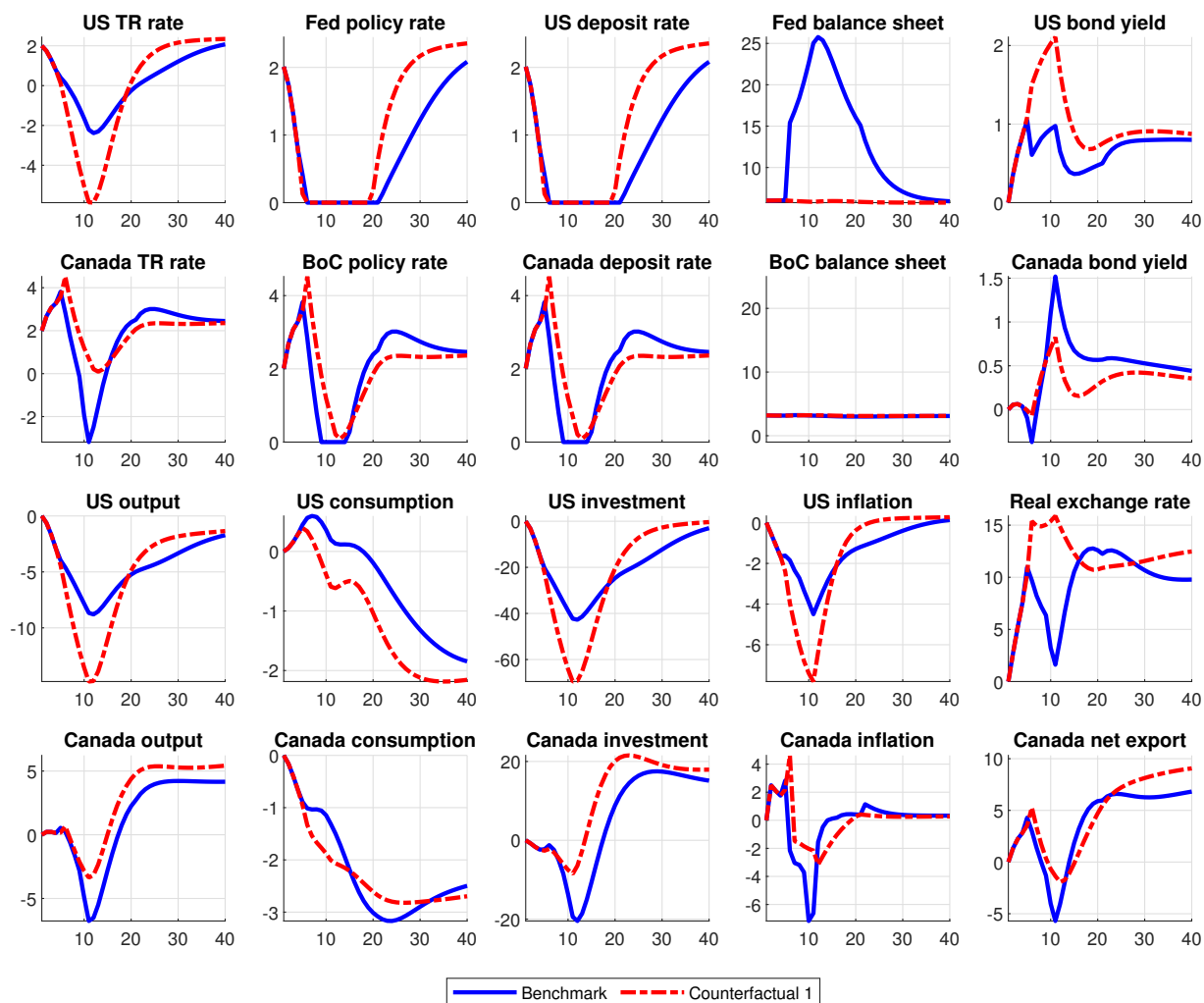


Figure 5: Benchmark vs. no QE by the Fed (Counterfactual 1)

monetary policy spillovers that documents substantial effects on small open economies of changes in the monetary policy of their large trading partner(s).

What if we allowed the Fed to do QE but not by as much as in the benchmark scenario? We show the results of this experiment in Figures 7 in Appendix C. For this counterfactual, we allow the Fed to engage in QE and increase the size of its balance sheet from the steady-state level of 6% of GDP to 16%. In the benchmark scenario, the increase in the balance sheet is from 6% of GDP to 25%. As one would expect, the outcomes of a milder QE policy (Figure 7) are somewhere in the middle of no QE (Figure 5) and the benchmark.

Here a natural question arises: Are there constant returns to QE? If the QE of a certain magnitude affects a variable by  $x\%$ , will doubling the size of QE affect the same variable by  $2x\%$ . The answer varies by variable but our experiments show that the QE follows diminishing returns to scale in affecting real variables that we care about the most (output, investment, consumption, net exports, etc.). This is true for both foreign country doing the QE and the home country being affected by the spillovers from that policy. One can see this by comparing our Figures 7 and 8 in Appendix C. To save on space, we only show the effects of varying QE on output and investment in the countries in Table 3.

Table 3: Diminishing returns to QE

Scenario	Change in Fed balance sheet	Maximum change in			
		US output	US investment	Canada output	Canada investment
Counterfactual 2	6% to 16%	-13.0%	-60.0%	-3.5%	-11.0%
Benchmark	6% to 25%	-8.5%	-41.0%	-7.0%	-20.0%
Counterfactual 3	6% to 36%	-8.5%	-42.0%	-8.5%	-23.0%

As we increase the size of Fed’s QE, we expect the drops in US output and investment (caused by the negative liquidity shocks) to become smaller. This does happen when we move from Counterfactual 2 to the benchmark. However, when we move from the benchmark to Counterfactual 3, there is no significant decrease in the size of the drop. In the case of Canada, a more aggressive QE by the Fed increases the size of drops in output and investment. But again, the increase in drops is bigger when we move from Counterfactual 2 to the benchmark and smaller when we move from the benchmark to Counterfactual 3.

The reason for these decreasing returns to QE is the response of investment to changes in cost of borrowing. In our model, QE affects the real economy by reducing the wholesale firm’s cost of borrowing (the yield on private bonds). However, the effects of a change in the cost of borrowing on investment are nonlinear and become smaller when the cost of borrowing is very low. The economic rationale is that even if borrowing is almost free, the wholesale firm has real constraints on its investment and would not issue more bonds beyond its investment needs. This reason is different from those highlighted by Bernanke (2020) to explain the smaller effects of the second and third waves of QE in the US compared to the

first wave. Bernanke (2020) attributes smaller later effects to market expectations. Because the later rounds of QE were anticipated by the markets, the surprise element, which was there in the case of the first wave of QE, was mostly lost in the later waves. In our model, however, the reason is grounded in the real economy. In fact, to achieve the 36% balance sheet, we need to add exogenous unanticipated shocks to QE. Regardless of these shocks, the wholesale firm does not find it optimal to increase investment.

### *C. Counterfactual QE by the Bank of Canada*

When the Fed implemented its QE policies between 2008 and 2014, the Bank of Canada did not follow. It did slash its policy rate from more than 4% to almost zero within a few quarters (see subplot in row 4 column 1 of Figure 4) but it did not engage in QE and kept its balance sheet small. In our next counterfactual experiment, we allow the Bank of Canada to engage in QE on top of the Fed's QE. The results of this experiment are in Figure 6.

The first thing to note in Figure 6 is that all the US variables remain unaffected by the QE by the Bank of Canada. This is because of our assumption that Canada is a small open economy and has no effect on the US.

The effects of this counterfactual experiment on the Canadian economy are quite predictable. The Canadian bond yield does not initially increase by as much as in the benchmark scenario. And once the QE in Canada really picks up (see subplot (2,4) in Figure 6), the Canadian bond yield drops below zero. Canada's real exchange rate depreciates more when the Bank of Canada engages in QE. The ultimate effects on real variables are generally positive. Output, investment and net exports drop by much less and recover quickly. The drop in consumption is milder and slower.

This counterfactual experiment suggests that had the Bank of Canada followed the Fed and engaged in QE, the real economic outcomes would have been better for Canada. There are a couple of obvious reasons why the Bank of Canada decided against the QE. First, the US experiment with QE was new and it was not clear how it would pan out. As Bernanke (2020) discusses in detail, there was a lot of skepticism even among the Fed officials about the effectiveness of QE. Second, the Canadian financial sector was in a much better shape than its US counterpart so the Bank of Canada felt confident that it could get the Canadian

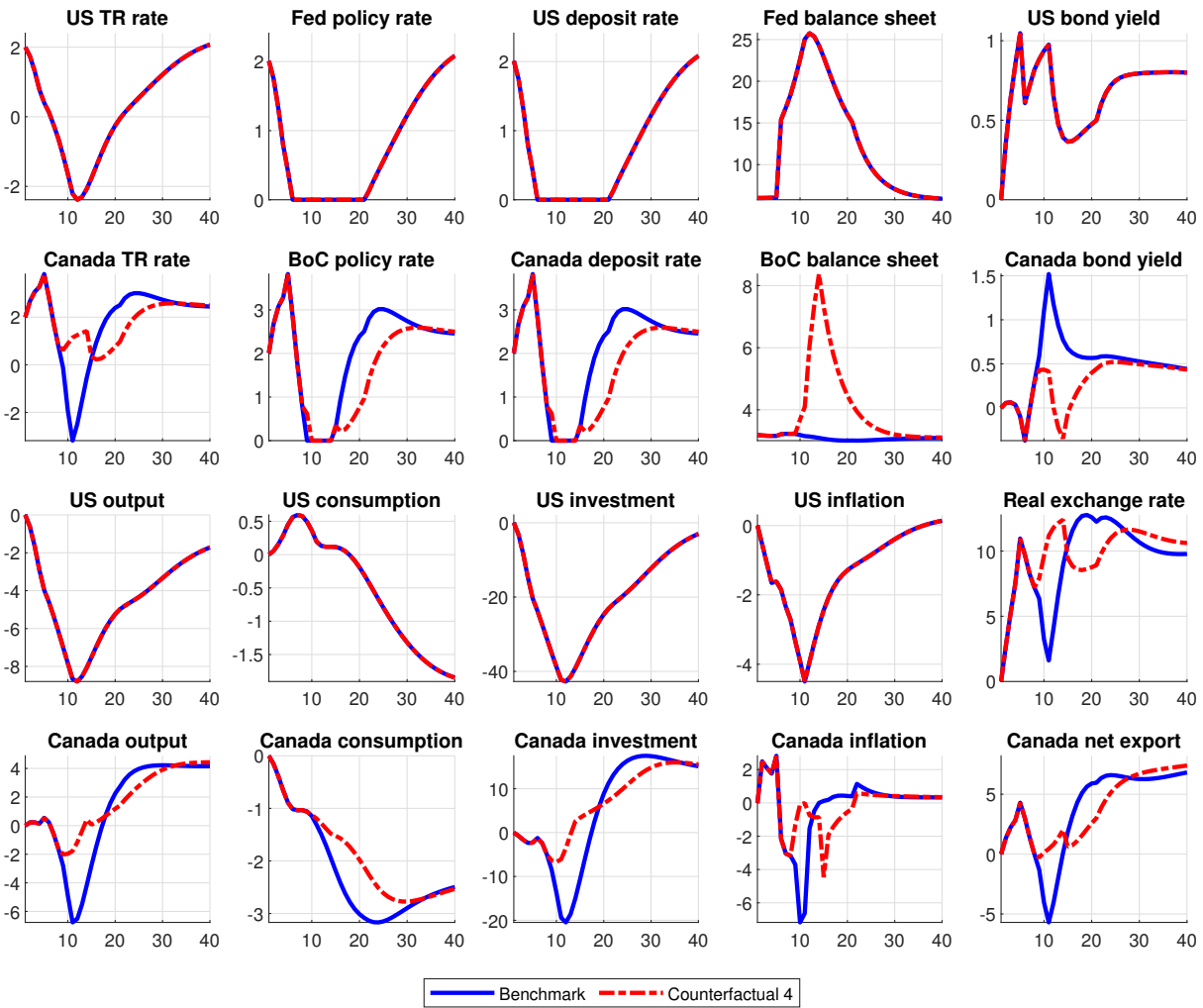


Figure 6: Benchmark vs. Bank of Canada doing QE (Counterfactual 4)

economy back on track without doing any QE. However, had the magnitude of negative shocks hitting the Canadian economy in 2008 been bigger, the Bank might have taken the QE route. For example, in the wake of the current new corona virus pandemic, the Bank of Canada did not hesitate to top up its near zero policy rate with a healthy dose of QE.

## V. Sensitivity Analysis

In our sensitivity analysis, we focus on the ten openness parameters that are new to our model compared to the model in SW. Among the ten, five, namely  $\eta_1, \eta_2, \eta_3, \eta_4$  and  $\eta_5$ , are elasticity parameters and the other five, namely  $\nu_1, \nu_2, \nu_3, \nu_4$  and  $\nu_5$ , are share parameters.

In the benchmark scenario, for the elasticity parameters, we assumed  $\eta_1 = \eta_2 = \eta_3 = \eta_4 = \eta_5 = 1$ . We now change one of these parameters at a time and compare the results with the benchmark. For the parameter that we change, we try two alternative values: 0.5 and 2.0. To save on space, the full set of sensitivity figures is in the online appendix. Here, we briefly comment on the results.

Parameter  $\eta_1$  captures the elasticity of imports with respect to the real exchange rate. If  $\eta_1$  is high, we expect imports to respond more to a given change in the real exchange rate. This, in turn, implies that the equilibrium real exchange rate does not have to move much to get the same change in imports. This is exactly what we see in our sensitivity experiments (see Figure 1 in the online appendix). When  $\eta_1 = 2$ , compared to the benchmark  $\eta_1 = 1$ , the equilibrium real exchange rate fluctuates less and the equilibrium net exports are slightly lower because imports are slightly higher. Consumption is the only other variable that is significantly affected. A higher  $\eta_1$  leads to a higher overall consumption. The intuition is that due to the expansionary monetary policy in the US, the real exchange rate depreciates and imports respond a lot because  $\eta_1$  is higher. At the same time, lower relative price causes the consumer to increase the consumption of home good. The net effect on total consumption is positive. Output and investment are slightly lower but the effect is negligible. To sum up, a higher  $\eta_1$  leads to less fluctuations in the real exchange rate and consumption, and more fluctuations in net exports. The effects on output and investment are negligible.

The effects on spillovers of changing the export elasticity parameter,  $\eta_2$ , are qualitatively similar to those of changing  $\eta_1$  but quantitatively they are more pronounced. We report the results of these sensitivity experiments in Figure 2 in the online appendix. A higher value of  $\eta_2$  leads to more fluctuations in net exports and less fluctuations in the real exchange rate. Consumption also fluctuates less. However, unlike the change in  $\eta_1$ , a higher  $\eta_2$  leads to slightly lower output and investment in the medium run. Another important difference is



that the effects of changing  $\eta_2$  are asymmetrical. A drop in  $\eta_2$  to 0.5 from the benchmark value of 1, leads to a much higher depreciation of the real exchange rate, a much higher increase in net exports and a much lower consumption. However, it also leads to a lower bond yield, higher investment and higher output.

Parameter  $\eta_3$  determines the response of the wholesale firm's relative supply of foreign-currency bonds to changes in their relative price. The results of these sensitivity experiments are in Figure 3 in the online appendix. A higher  $\eta_3$  leads to slightly more fluctuations in a number of Canadian variables. These includes the Taylor-rule policy rate, the bond yield, inflation, real exchange rate, net exports, consumption, investment and output. A lower  $\eta_3$  leads to slightly less fluctuations. However, a quick glance at Figure 3 of the online appendix suggests that these effects are quantitatively small. Although the changes in  $\eta_3$  affect the spillovers in predictable ways, i.e., a higher  $\eta_3$  leads to more fluctuations, the effects are very small.

Parameter  $\eta_4$  determines the response of the financial intermediaries' relative demand for foreign private bonds (relative to home private bonds) to their relative price. In our baseline calibration, the financial intermediaries hold only 7.85% of their private bond holdings in the form of foreign bonds. Because the foreign bond share is so small, it is not surprising that we see in Figure 4 in the online appendix that changing the value of  $\eta_4$  hardly has any effect on the spillovers.

Parameter  $\eta_5$  determines the response of the financial intermediaries' relative demand for foreign government bonds (relative to home government bonds) to their relative price. The effects of changing  $\eta_5$  are similar to those of changing  $\eta_3$  but in the opposite direction (see Figure 5 in the online appendix). A higher  $\eta_5$  leads to smaller fluctuations in the real exchange rate, net exports, investment, consumption and output. The intuition is that if the financial intermediaries respond more to changes in relative prices of foreign government bonds, they mitigate the spillovers. However, just as in the case of changes in  $\eta_3$ , changes in  $\eta_5$  lead to spillovers that are both qualitatively and quantitatively similar to those in the benchmark scenario.

A holistic glance at Figures 1 to 5 in the online appendix suggests that changes in  $\eta_1$  and  $\eta_2$  have sizeable effects on spillovers to the real exchange rate, net exports and consumption.

Changes in  $\eta_3$  and  $\eta_5$  affect a number of variables but these effects are small and these changes lead to qualitatively similar spillovers. Changes in  $\eta_4$  have almost no effects in our sensitivity experiments because of the low weight on foreign private bonds in our baseline calibration.

The shares of imports and exports in GDP are observed in the data and help us pin down  $\nu_1$  and  $\nu_2$ . In our sensitivity analysis, we leave these two parameters unchanged. The other three share parameters ( $\nu_3$ ,  $\nu_4$  and  $\nu_5$ ) depend on foreign bias of the wholesale firm when issuing bonds and the foreign bias of the financial intermediaries when buying bonds. Specifically:  $\nu_3$  is the share of foreign currency bonds in total bonds (issued in home and foreign currencies) issued by the wholesale firm;  $\nu_4$  is the share of foreign currency private bonds in total private bonds held by the financial intermediaries; and  $\nu_5$  is the share of foreign currency government bonds in total government bonds held by the financial intermediaries. When we calibrate  $\nu_3$ ,  $\nu_4$  and  $\nu_5$ , we also need to make sure that the balance of payment equation is satisfied in the steady state. With this additional restriction, we need two other targets to pin down these three parameters. In the benchmark scenario, we pin down  $\nu_5$  to the observed share of foreign currency government bonds in total government bonds held by the five largest banks in Canada. We then adjust  $\nu_3$  and  $\nu_4$  to make sure that the ratio of the foreign government bonds held by the financial intermediaries to the foreign private bonds held by the financial intermediaries matches the data and the balance of payment equation is also satisfied. In the following sensitivity analysis, we try three other calibration strategies for these parameters. See Table 4 for the alternative parameter values.

Table 4: Sensitivity Analysis: Parameters  $\nu_3$ ,  $\nu_4$  and  $\nu_5$

Parameter	Banchmark value	Sensitivity Experiments		
		(1)	(2)	(3)
$\nu_3$	0.58	0.48	0.54	0.48
$\nu_4$	0.08	0.02	0.00	0.00
$\nu_5$	0.43	0.18	0.43	0.23

In our first sensitivity experiment, instead of matching  $\nu_5$  to data, we match  $\nu_3$  to data and then adjust  $\nu_4$  and  $\nu_5$  to make sure that the ratio of the foreign government bonds held by

the financial intermediaries to the foreign private bonds held by the financial intermediaries matches the data and the balance of payment equation is also satisfied. This effectively leads to smaller values for all three parameters. In other words, this experiment reduces the degree of foreign bias (or increases the home bias) in the financial asset markets. We report the results in Figure 6 in the online appendix. With less openness in financial markets, the Canadian bond yield is higher compared to the benchmark. The real exchange rate and net exports fluctuate more because more adjustment takes place on the real side of international transactions. Consumption drops more. Output and investment drop slightly more but also recover faster. Because of the faster recovery, the Canadian Taylor-rule policy rate has to increase faster during the recovery phase.

In the next sensitivity experiment, we fix  $\nu_5$  to match the data (just like the benchmark) but do not match the ratio of the foreign government bonds held by the financial intermediaries to the foreign private bonds held by the financial intermediaries. Instead, we try to get  $\nu_3$  as close to its data target as possible. As we lower  $\nu_3$  from its benchmark value towards its data target, because we are keeping  $\nu_5$  fixed, we need to lower  $\nu_4$  to make sure that the balance of payment equation is satisfied in the steady state. Before we reach the  $\nu_3$  target,  $\nu_4$  becomes zero. In this case, both  $\nu_3$  and  $\nu_4$  are slightly lower than their benchmark values ( $\nu_5$  remains at its benchmark value). Once again, in this experiment, the degree of home bias increases, but this time only slightly. We report the results in Figure 7 in the online appendix. The changes in Figure 7 (relative to the benchmark) are qualitatively similar to the changes in Figure 6 but quantitatively they are very small.

In our last sensitivity experiment, we fix  $\nu_3$  to match the data and try to get  $\nu_5$  as close to the target as possible until  $\nu_4$  becomes zero. In this experiment, all three parameters are again lower than their benchmark values. We report the results in Figure 8 in the online appendix. Results in Figure 8 are both qualitatively and quantitatively similar to those in Figure 6. Once again the message is that if the home bias in financial markets increases, the adjustment in the real exchange rate and net exports has to be larger in response to foreign monetary policy shocks.

## VI. Conclusion

We contribute to the ongoing debate on international spillovers of new monetary policy. We extend the New Keynesian DSGE model in SW to an open economy setting. The model features an SOE (Canada) that has close trade and financial links to a large economy (US). We calibrate the steady state of the model to Canadian and US economies just before the financial crisis of 2008.

In our first set of experiments, we choose exogenous monetary policy shocks such that each new monetary policy tool (QE, FG and NIRP) has the same effect on US output as the conventional monetary policy. We then compare the spillovers to Canadian economy of both new and conventional monetary policies. An expansionary monetary policy in the US, if not accompanied by a similar policy in Canada, lowers Canada's output mainly through the exchange rate channel. In the case of conventional monetary policy, the yield on Canadian bonds increases. This increases the cost of borrowing in Canada and depresses its investment. However, this yield channel operates differently in the case of new monetary policy. QE in the US, lowers the bond yield in Canada and stimulates investment. This positive yield effect mitigates the negative exchange rate effect on Canadian output. However, the other two tools of new monetary policy in our model, FG and NIRP, increase Canadian yield even more than does the conventional monetary policy. As a result, investment in Canada takes a deeper dip and the negative effect on output is worse. The new insight from our work is that it is not enough just to compare conventional and new monetary policy spillovers. Even among the new monetary policy tools, there are big differences in the ways their effects spillover to other economies.

In our second set of experiments, we first construct a benchmark scenario by introducing credit shocks to both Canadian and US economies to the extent that the drops in output in both countries mimic the drops in the data. In the benchmark scenario, when the Taylor-rule policy rates in the two economies hit the zero lower bound, the Bank of Canada does not engage in any QE and the Fed engages in QE by expanding the size of its balance sheet to 25% of US GDP. After constructing a reasonable benchmark, we do a number of counterfactual experiments. The main insight from these counterfactual experiments is that

there are diminishing returns to QE. As we increase the size of QE done by the Fed, the spillovers to the Canadian economy increase but at a decreasing rate. We also find that had the Bank of Canada engaged in QE of its own, on top of what the Fed did, the real economic outcomes for Canada would have been better after the 2008 financial crisis. For example, the drop in Canadian GDP, instead of being 8% in the benchmark scenario, would have been just 2%.

Our main contribution is that we take a state-of-the-art dynamic New Keynesian model that combines both conventional and new monetary policy tools in a unified framework, and extend it to an open-economy model. This modification allows us to not only compare the international spillovers of conventional monetary policy and quantitative easing as others have done before us, but also to compare the international spillovers caused by forward guidance and negative interest rate policy, something that is new to the literature.

The open-economy framework in this paper is rich enough to be used in a number of other applications. For example, instead of thinking about an SOE and its interactions with a large economy, the framework can be easily modified to think about the monetary-policy interactions between two large economies like the US and the European Union. The suitability of this framework to study such interactions has further increased in recent years when both the Fed and the European Central Bank have significantly expanded their use of new monetary policy tools.

The framework in this paper also provides a rich environment to quantitatively assess Rey's hypothesis about the lack of monetary policy independence in small open economies Rey (2016). We are currently working on this idea in a related paper.

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## A Notation

In this section, we collect all variables and parameters used in the paper in one place. We follow the following order in both lists: the uppercase English symbols appear first followed by the lowercase English, uppercase Greek and lowercase Greek symbols. Within each category, the variables and parameters are listed in lexicographical order.

### A. Variables

The price of home consumption good,  $P_{C,t}$ , is the numeraire for the home economy. We do not list it as a separate variable. The euro sign (€) before a variable means that the variable is in terms of foreign currency.

1.  $A_t$  : Technology/Total factor productivity
2.  $C_t$  : Total consumption, CES aggregate of  $C_{H,t}$  and  $C_{F,t}$
3.  $C_{F,t}$  : Consumption of foreign produced goods (imports)
4.  $C_{H,t}$  : Consumption of home produced goods
5.  $E_t$  : Nominal exchange rate, units of home currency needed to buy one unit of foreign currency
6.  $G_t$  : Government expenditure
7.  $I_t$  : Investment before adjustment cost
8.  $\hat{I}_t$  : Investment net of adjustment cost, gross addition to capital stock
9.  $K_t$  : Capital stock
10.  $L_{1,t}$  : Aggregate employment (labor supplied by the household to labor unions)
11.  $L_{2,t}$  : Final labor bundle sold by the labor packer to the wholesale firm
12.  $M_{1,t}$  : Auxiliary variable, a function of the Lagrange multiplier in the wholesale firm's optimization problem
13.  $M_{2,t}$  : Auxiliary variable, a function of the Lagrange multiplier, in the wholesale firm's optimization problem
14.  $MU_{C,t}$  : Marginal utility of consumption
15.  $Q_t^{Gov}$  : CES aggregate of  $Q_{F,t}^{Gov}$  and  $Q_{H,t}^{Gov}$
16.  $Q_{F,t}^{Gov} \equiv E_t \in Q_{F,t}^{Gov}$  : Price of a foreign government bond in home currency
17.  $Q_{H,t}^{Gov}$  : Price of a home government bond
18.  $Q_t^{Pvt}$  : CES aggregate (based on parameters  $\eta_4$  and  $\nu_4$ ) of  $Q_{H,t}^{Pvt}$  (the price of home private bond) and  $Q_{F,t}^{Pvt}$  (the price of foreign private bond in home currency)
19.  $Q_{F,t}^{Pvt} \equiv E_t \in Q_{F,t}^{Pvt}$  : Price of a foreign private bond in home currency
20.  $Q_{H,t}^{Pvt}$  : Price of a home private bond
21.  $\bar{Q}_{H,t}^{Pvt}$  : CES aggregate (based on parameters  $\eta_3$  and  $\nu_3$ ) of  $Q_{H,t}^{Pvt}$  (the price of home private bond) and  $Q_{F,t}^{Pvt}$  (the price of foreign private bond in home currency)
22.  $R_t^D$  : Gross nominal interest rate on deposits
23.  $R_t^{Gov}$  : CES aggregate of  $R_{H,t}^{Gov}$  (the gross nominal return on home government bonds) and  $R_{F,t}^{Gov}$  (the gross nominal return on foreign government bonds)
24.  $R_{F,t}^{Gov} \equiv E_t \in R_{F,t}^{Gov} / E_{t-1}$  : Gross nominal return (in home currency) on a foreign government bond

25.  $R_{H,t}^{Gov}$  : Gross nominal return on a home government bond
26.  $R_t^{Pol}$  : Policy rate implied by the Taylor rule
27.  $R_t^{Pvt}$  : CES aggregate of  $R_{H,t}^{Pvt}$  (the gross nominal return on home private bonds) and  $R_{F,t}^{Pvt}$  (the gross nominal return, in home currency, on foreign private bonds)
28.  $R_{F,t}^{Pvt} \equiv E_t \in R_{F,t}^{Pvt} / E_{t-1}$  : Gross nominal return (in home currency) on a foreign private bond
29.  $R_{H,t}^{Pvt}$  : Gross nominal return on a home private bond
30.  $R_t^S$  : Gross interest rate on reserves
31.  $T_t$  : Lump sum tax paid by the household to the fiscal authority
32.  $X_t$  : Exports
33.  $Y_t$  : Final-good output
34.  $Y_{2,t}$  : Output of the wholesale firm
35.  $Z_t$  : Preference shock
36.  $b_t^{Gov}(fi)$  : CES aggregate of  $b_{H,t}^{Gov}(fi)$  (home government bonds) and  $b_{F,t}^{Gov}(fi)$  (foreign government bonds) held by the financial intermediaries
37.  $b_{F,t}^{Gov}(fi) \equiv B_{F,t}^{Gov}(fi) / P_{C,t}$  : Real holdings of  $B_{F,t}^{Gov} \equiv \int B_{F,t}^{Gov}(j) dj$  (foreign government bonds) by the financial intermediaries
38.  $b_{H,t}^{Gov}(fi) \equiv B_{H,t}^{Gov}(fi) / P_{C,t}$  : Real holdings of  $B_{H,t}^{Gov}(fi) \equiv \int B_{H,t}^{Gov}(j) dj$  (home government bonds) by the financial intermediaries
39.  $b_{H,t}^{Gov}(ma)$  : Monetary authority's real holdings of home government bonds
40.  $b_t^{Pvt}(fi)$  : CES aggregate of  $b_{H,t}^{Pvt}(fi)$  (home government bonds) and  $b_{F,t}^{Pvt}(fi)$  (foreign government bonds) held by the financial intermediaries
41.  $b_{F,t}^{Pvt}(fi)$  : Real holdings of  $B_{F,t}^{Pvt}(fi) \equiv \int B_{F,t}^{Pvt}(j) dj$  (foreign private bonds) by the financial intermediaries
42.  $b_{H,t}^{Pvt} = b_{H,t}^{Pvt}(fi) + b_{H,t}^{Pvt}(ma)$  : Real outstanding private bonds issued by the wholesale firm in home currency
43.  $b_{H,t}^{Pvt}(fi)$  : Real holdings of  $B_{H,t}^{Pvt}(fi) \equiv \int B_{H,t}^{Pvt}(j) dj$  (home private bonds) by the financial intermediaries
44.  $b_{H,t}^{Pvt}(ma)$  : Monetary authority's real holdings of home private bonds
45.  $b_{H,FC,t}^{Pvt} \equiv B_{H,FC,t}^{Gov} / P_{C,t}$  : Real outstanding bonds issued by the home wholesale firms in foreign currency
46.  $\bar{b}_{H,t}^{Pvt}$  : CES aggregate of  $b_{H,t}^{Pvt}$  (real outstanding bonds issued by the wholesale firm in home currency) and  $b_{H,FC,t}^{Pvt}$  (real outstanding bonds issued by the wholesale firm in foreign currency)
47.  $d_t \equiv D_t / P_{C,t}$  : Real household deposits at financial intermediaries
48.  $f_{1,t}$  : An auxiliary variable in the optimal real wage equation
49.  $f_{2,t}$  : An auxiliary variable in the optimal real wage equation
50.  $n_t \equiv N_t / P_{C,t}$  : Real networth of a financial intermediary
51.  $p_{2,t} \equiv P_{2,t} / P_{C,t}$  : Relative price of wholesale firm's output
52.  $p_{F,t} \equiv P_{F,t} / P_{C,t} = E_t \in P_{F,t} / P_{C,t}$  : Real exchange rate
53.  $p_{H,t} \equiv P_{H,t} / P_{C,t}$  : Relative home price

54.  $p_{H,t}^* \equiv P_{H,t}^*/P_{C,t}$  : Relative optimal price chosen by a retail firm
55.  $p_{K,t} \equiv P_{K,t}/P_{C,t}$  : Relative price of capital
56.  $s_t \equiv S_t/P_{C,t}$  : Real monetary authority reserves (held by financial intermediaries)
57.  $u_t$  : Capital utilization
58.  $w_{1,t} \equiv W_{1,t}/P_{C,t}$  : The real wage that the household receives from labor unions
59.  $w_{2,t} \equiv W_{2,t}/P_{C,t}$  : The real wage that the wholesale firm pays to the labor packer
60.  $w_{2,t}^* \equiv W_{2,t}^*/P_{C,t}$  : real optimal wage chosen by a trade union
61.  $x_{1,t}$  : An auxiliary variable in the optimal real price equation
62.  $x_{2,t}$  : An auxiliary variable in the optimal real price equation
63.  $\Lambda_{t,t+1} = \beta \text{MU}_{C,t+1}/\text{MU}_{C,t}$  : Stochastic discount factor
64.  $\Pi_{ma,t}^{\text{real}}$  : Profit of the monetary authority
65.  $\Omega_t$  : An auxiliary variable in the first-order conditions of a financial intermediary
66.  $\theta_t$  : Liquidity shock
67.  $\lambda_{1,t}$  : The Lagrange multiplier on the costly enforcement constraint in the problem of a financial intermediary
68.  $\lambda_{2,t}$  : The Lagrange multiplier on the reserve requirement constraint in the problem of a financial intermediary
69.  $\pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$  : CPI inflation
70.  $v_t^p$  : Price dispersion
71.  $v_t^w$  : Wage dispersion
72.  $\phi_t$  : An auxiliary variable in the first-order conditions of a financial intermediary (endogenous leverage)

### **B. Shocks**

There are seven shocks in the model: (1)  $\varepsilon_{1,t}$  in eqn. 66/72 for the monetary authority's government bond holdings; (2)  $\varepsilon_{2,t}$  in eqn. 67/72 for the monetary authority's private bond holdings; (3)  $\varepsilon_{A,t}$  in eqn. 17/72 for  $A_t$ ; (4)  $\varepsilon_{G,t}$  in eqn. 69/72 for  $G_t$ ; (5)  $\varepsilon_{R,t}$  in eqn. 63/72 for Taylor-rule policy rate  $R_t^{\text{Pol}}$ ; (6)  $\varepsilon_{Z,t}$  in eqn. 02/72 for  $Z_t$ ; and (7)  $\varepsilon_{\theta,t}$  in eqn. 46/72 for  $\theta_t$ .

### **C. Foreign Variables**

These are the foreign variables that appear in the home-economy equations. The euro sign ( $\text{€}$ ) before a variable means that the variable is in terms of foreign currency.

1.  $\text{€}Q_{F,t}^{\text{Gov}}$  : Price of a foreign government bond
2.  $\text{€}Q_{F,t}^{\text{Pvt}}$  : Price of a foreign private bond
3.  $\text{€}R_{F,t}^{\text{Gov}} = (\text{€}1 + \kappa \text{€}Q_{F,t}^{\text{Gov}}) / \text{€}Q_{F,t-1}^{\text{Gov}}$  : Gross nominal return on a foreign government bond
4.  $\text{€}R_{F,t}^{\text{Pvt}} = (\text{€}1 + \kappa \text{€}Q_{F,t}^{\text{Pvt}}) / \text{€}Q_{F,t-1}^{\text{Pvt}}$  : Gross nominal return on a foreign private bond
5.  $Y_{F,t}$  : Foreign income
6.  $\text{€}\pi_{F,t} \equiv \text{€}P_{F,t}/\text{€}P_{F,t-1}$  : Inflation in foreign country in foreign currency ( $\text{€}P_{F,t}$  is the numeraire for foreign economy)

## B Non-Linear Equilibrium Conditions

In our model of the small open economy, there are 72 non-linear equilibrium equations in 72 endogenous variables. In this section, we list these equations. We have already listed the 72 endogenous variables in Section A.. We have divided these equations into 7 blocks: (1) Household; (2) Labor market; (3) Non-financial firms; (4) Financial intermediaries; (5) Monetary authority; (6) Fiscal authority; and (7) Foreign exchange market. We has assigned each equation unique serial number for easy reference. The numbers range from eqn. 01/72 to eqn. 72/72.

### A. Household

$$(eqn. 01/72) \quad MU_{C,t} \equiv Z_t [C_t - hC_{t-1}]^{-\sigma} - \beta h \mathbb{E}_t Z_{t+1} [C_{t+1} - hC_t]^{-\sigma}$$

$$(eqn. 02/72) \quad \ln Z_t = \rho_Z \ln Z_{t-1} + s_Z \varepsilon_{Z,t}$$

$$(eqn. 03/72) \quad \Lambda_{t,t+1} \equiv \frac{\beta \mathbb{E}_t MU_{C,t+1}}{MU_{C,t}}$$

$$(eqn. 04/72) \quad \omega Z_t L_{1,t}^\varphi = MU_{C,t} w_{1,t}$$

$$(eqn. 05/72) \quad \mathbb{E}_t \Lambda_{t,t+1} \pi_{C,t+1}^{-1} R_t^D = 1$$

$$(eqn. 06/72) \quad Y_t = C_{H,t} + I_t + G_t + X_t$$

$$(eqn. 07/72) \quad C_{H,t} = (1 - \nu_1) p_{H,t}^{-\eta_1} C_t,$$

where

$$C_t = \begin{cases} \left( (1 - \nu_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1-1}{\eta_1}} + \nu_1^{\frac{1}{\eta_1}} (C_{F,t})^{\frac{\eta_1-1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1-1}} & \text{if } \eta_1 \neq 1 \\ \frac{C_{H,t}^{1-\nu_1} C_{F,t}^{\nu_1}}{(1-\nu_1)^{1-\nu_1} \nu_1^{\nu_1}} & \text{if } \eta_1 = 1 \end{cases}$$

$$(eqn. 08/72) \quad C_{F,t} = \nu_1 p_{F,t}^{-\eta_1} C_t$$

$$(eqn. 09/72) \quad \begin{aligned} 1 &= (1 - \nu_1) p_{H,t}^{1-\eta_1} + \nu_1 p_{F,t}^{1-\eta_1} & \text{if } \eta_1 \neq 1 \\ 1 &= p_{H,t}^{1-\nu_1} p_{F,t}^{\nu_1} & \text{if } \eta_1 = 1 \end{aligned}$$

### B. Labor Market

**Labor Unions.**—

$$(eqn. 10/72) \quad w_{2,t}^* = \frac{\epsilon_w}{\epsilon_w - 1} \frac{f_{1,t}}{f_{2,t}}$$

$$(eqn. 11/72) \quad f_{1,t} = w_{1,t} w_{2,t}^{\epsilon_w} L_{2,t} + \theta_w \pi_{C,t}^{-\epsilon_w \gamma_w} \mathbb{E}_t \Lambda_{t,t+1} f_{1,t+1} \pi_{C,t+1}^{\epsilon_w}$$

$$(eqn. 12/72) \quad f_{2,t} = w_{2,t}^{\epsilon_w} L_{2,t} + \theta_w \pi_{C,t}^{(1-\epsilon_w) \gamma_w} \mathbb{E}_t \Lambda_{t,t+1} f_{2,t+1} \pi_{C,t+1}^{\epsilon_w - 1}$$

**Labor Packer.**—

$$(eqn. 13/72) \quad L_{1,t} = L_{2,t} v_t^w$$

$$(eqn. 14/72) \quad v_t^w = (1 - \theta_w) \left( \frac{w_{2,t}^*}{w_{2,t}} \right)^{-\epsilon_w} + \theta_w \left( \frac{\pi_{C,t}}{\pi_{C,t-1}^{\gamma_w}} \right)^{\epsilon_w} \left( \frac{w_{2,t}}{w_{2,t-1}} \right)^{\epsilon_w} v_{t-1}^w$$

$$(eqn. 15/72) \quad w_{2,t}^{1-\epsilon_w} = \left( \frac{\pi_{C,t}^{\gamma_w}}{\pi_{C,t}} \right)^{1-\epsilon_w} \theta_w w_{2,t-1}^{1-\epsilon_w} + (1 - \theta_w) (w_{2,t}^*)^{1-\epsilon_w}$$

### C. Non-Financial Firms

#### Wholesale.—

$$(eqn. 16/72) \quad Y_{2,t} = A_t (u_t K_t)^\alpha L_{2,t}^{1-\alpha}$$

$$(eqn. 17/72) \quad \ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t}$$

$$(eqn. 18/72) \quad K_{t+1} = \hat{I}_t + (1 - \delta(u_t)) K_t,$$

where  $\delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2$ .

$$(eqn. 19/72) \quad \psi p_{K,t} \hat{I}_t = \bar{Q}_{H,t}^{Pvt} (\bar{b}_{H,t}^{Pvt} - \kappa \bar{b}_{H,t-1}^{Pvt} \pi_{C,t}^{-1})$$

$$(eqn. 20/72) \quad b_{H,t}^{Pvt} = b_{H,t}^{Pvt} (fi) + b_{H,t}^{Pvt} (ma)$$

$$(eqn. 21/72) \quad w_{2,t} = (1 - \alpha) p_{2,t} A_t (u_t K_t)^\alpha L_{2,t}^{-\alpha}$$

$$(eqn. 22/72) \quad p_{K,t} M_{1,t} \delta'(u_t) = \alpha p_{2,t} A_t (u_t K_t)^{\alpha-1} L_{2,t}^{1-\alpha},$$

where  $\frac{\partial \delta(u_t)}{\partial u_t} \equiv \delta'(u_t) = \delta_1 + \delta_2 (u_t - 1)$ .

$$(eqn. 23/72) \quad p_{K,t} M_{1,t} = \mathbb{E}_t \Lambda_{t,t+1} [\alpha p_{2,t+1} A_{t+1} K_{t+1}^{\alpha-1} u_{t+1}^\alpha L_{2,t+1}^{1-\alpha} + (1 - \delta(u_{t+1})) p_{K,t+1} M_{1,t+1}]$$

$$(eqn. 24/72) \quad \bar{Q}_{H,t}^{Pvt} M_{2,t} = \mathbb{E}_t \Lambda_{t,t+1} \pi_{C,t+1}^{-1} [1 + \kappa \bar{Q}_{H,t+1}^{Pvt} M_{2,t+1}]$$

$$(eqn. 25/72) \quad \frac{M_{1,t} - 1}{M_{2,t} - 1} = \psi$$

$$(eqn. 26/72) \quad b_{H,FC,t}^{Pvt} = v_3 \left( \frac{Q_{F,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} \right)^{\eta_3} \bar{b}_{H,t}^{Pvt}$$

$$(eqn. 27/72) \quad b_{H,t}^{Pvt} = (1 - v_3) \left( \frac{Q_{H,t}^{Pvt}}{\bar{Q}_{H,t}^{Pvt}} \right)^{\eta_3} \bar{b}_{H,t}^{Pvt}$$

$$(eqn. 28/72) \quad \bar{Q}_{H,t}^{Pvt} \equiv \left[ (1 - v_3) (Q_{H,t}^{Pvt})^{1+\eta_3} + v_3 (Q_{F,t}^{Pvt})^{1+\eta_3} \right]^{\frac{1}{1+\eta_3}}$$

#### Retail.—

$$(eqn. 29/72) \quad p_{H,t}^* = \frac{\epsilon_p}{\epsilon_p - 1} \frac{x_{1,t}}{x_{2,t}}$$

$$(eqn. 30/72) \quad x_{1,t} = p_{2,t} p_{H,t}^{\epsilon_p} Y_t + \theta_p \pi_{C,t}^{-\gamma_p \epsilon_p} \mathbb{E}_t \Lambda_{t,t+1} x_{1,t+1} \pi_{C,t+1}^{\epsilon_p}$$

$$(eqn. 31/72) \quad x_{2,t} = p_{H,t}^{\epsilon_p} Y_t + \theta_p \pi_{C,t}^{\gamma_p (1-\epsilon_p)} \mathbb{E}_t \Lambda_{t,t+1} x_{2,t+1} \pi_{C,t+1}^{\epsilon_p - 1}$$

#### Final-Good Producer.—

$$(eqn. 32/72) \quad Y_{2,t} = Y_t v_t^p,$$

where  $v_t^p \equiv \int_0^1 \left( \frac{p_{H,t}(f)}{p_{H,t}} \right)^{-\epsilon_p} df$ .

$$(eqn. 33/72) \quad v_t^p = \left( \frac{p_{H,t-1} \pi_{C,t-1}^{\gamma_p}}{p_{H,t} \pi_{C,t}} \right)^{-\epsilon_p} \theta_p v_{t-1}^p + (1 - \theta_p) \left( \frac{p_{H,t}^*}{p_{H,t}} \right)^{-\epsilon_p}$$

$$(eqn. 34/72) \quad p_{H,t}^{1-\epsilon_p} = \left( \frac{\pi_{C,t-1}^{\gamma_p}}{\pi_{C,t}} \right)^{1-\epsilon_p} \theta_p p_{H,t-1}^{1-\epsilon_p} + (1 - \theta_p) (p_{H,t}^*)^{1-\epsilon_p}$$

**Capital-Good Producer.** —

$$(eqn. 35/72) \quad \hat{I}_t = \left[ 1 - \frac{\kappa_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t$$

$$(eqn. 36/72) \quad p_{K,t} \frac{\partial \hat{I}_t}{\partial I_t} + \mathbb{E}_t \Lambda_{t,t+1} p_{K,t+1} \frac{\partial \hat{I}_{t+1}}{\partial I_t} = p_{H,t}$$

**Exporter.** —

$$(eqn. 37/72) \quad X_t = \nu_2 \left( \frac{p_{H,t}}{p_{F,t}} \right)^{-\eta_2} Y_{F,t}$$

#### D. Financial Intermediaries

$$(eqn. 38/72) \quad Q_t^{Pvt} b_t^{Pvt}(fi) + Q_t^{Gov} b_t^{Gov}(fi) + s_t = d_t + n_t$$

$$(eqn. 39/72) \quad n_t = \vartheta \pi_{C,t}^{-1} \left[ (R_t^{Pvt} - R_{t-1}^D) Q_{t-1}^{Pvt} b_{t-1}^{Pvt}(fi) + (R_t^{Gov} - R_{t-1}^D) Q_{t-1}^{Gov} b_{t-1}^{Gov}(fi) + (R_{t-1}^s - R_{t-1}^D) s_{t-1} + R_{t-1}^D n_{t-1} \right] + \chi$$

$$(eqn. 40/72) \quad \phi_t n_t = Q_t^{Pvt} b_t^{Pvt}(fi) + \Delta Q_t^{Gov} b_t^{Gov}(fi)$$

$$(eqn. 41/72) \quad \mathbb{E}_t \Omega_{t+1} \Lambda_{t,t+1} (R_{t+1}^{Pvt} - R_t^D) \pi_{C,t+1}^{-1} = \frac{\lambda_{1,t}}{1 + \lambda_{1,t}} \theta_t$$

$$(eqn. 42/72) \quad \mathbb{E}_t \Omega_{t+1} \Lambda_{t,t+1} (R_{t+1}^{Gov} - R_t^D) \pi_{C,t+1}^{-1} = \frac{\lambda_{1,t}}{1 + \lambda_{1,t}} \Delta \theta_t$$

$$(eqn. 43/72) \quad \mathbb{E}_t \Omega_{t+1} \Lambda_{t,t+1} (R_{t+1}^s - R_t^D) \pi_{C,t+1}^{-1} = -\frac{\lambda_{2,t}}{1 + \lambda_{1,t}}$$

$$(eqn. 44/72) \quad \Omega_t \equiv 1 - \vartheta + \vartheta \theta_t \phi_t$$

$$(eqn. 45/72) \quad \theta_t \phi_t = (1 + \lambda_{1,t}) \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} R_t^D \pi_{C,t+1}^{-1} - \frac{\lambda_{2,t} s_t}{n_t}$$

$$(eqn. 46/72) \quad \ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + s_\theta \varepsilon_{\theta,t}$$

$$(eqn. 47/72) \quad b_{H,t}^{Pvt}(fi) = (1 - \nu_4) \left( \frac{Q_{H,t}^{Pvt}}{Q_t^{Pvt}} \right)^{-\eta_4} b_t^{Pvt}(fi),$$

where  $b_t^{Pvt}(fi) \equiv \left[ (1 - \nu_4)^{1/\eta_4} (b_{H,t}^{Pvt}(fi))^{1-1/\eta_4} + (\nu_4)^{1/\eta_4} (b_{F,t}^{Pvt}(fi))^{1-1/\eta_4} \right]^{\eta_4/(\eta_4-1)}$  if  $\eta_4 \neq 1$  and  $b_t^{Pvt}(fi) \equiv \frac{1}{(1-\nu_4)^{1-\nu_4} \nu_4^{\nu_4}} (b_{H,t}^{Pvt}(fi))^{1-\nu_4} (b_{F,t}^{Pvt}(fi))^{\nu_4}$  if  $\eta_4 = 1$ .

$$(eqn. 48/72) \quad b_{F,t}^{Pvt}(fi) = \nu_4 \left( \frac{Q_{F,t}^{Pvt}}{Q_t^{Pvt}} \right)^{-\eta_4} b_t^{Pvt}(fi)$$

$$(eqn. 49/72) \quad b_{H,t}^{Gov}(fi) = (1 - \nu_5) \left( \frac{Q_{H,t}^{Gov}}{Q_t^{Gov}} \right)^{-\eta_5} b_t^{Gov}(fi),$$

where  $b_t^{Gov}(fi) \equiv \left[ (1 - \nu_5)^{1/\eta_5} (b_{H,t}^{Gov}(fi))^{1-1/\eta_5} + (\nu_5)^{1/\eta_5} (b_{F,t}^{Gov}(fi))^{1-1/\eta_5} \right]^{\eta_5/(\eta_5-1)}$  if  $\eta_5 \neq 1$  and  $b_t^{Gov}(fi) = \frac{1}{(1-\nu_5)^{1-\nu_5}\nu_5^{\nu_5}} (b_{H,t}^{Gov}(fi))^{1-\nu_5} (b_{F,t}^{Gov}(fi))^{\nu_5}$  if  $\eta_5 = 1$ .

$$(eqn. 50/72) \quad b_{F,t}^{Gov}(fi) = \nu_5 \left( \frac{Q_{F,t}^{Gov}}{Q_t^{Gov}} \right)^{-\eta_5} b_t^{Gov}(fi)$$

$$(eqn. 51/72) \quad Q_t^{Pvt} = \left[ (1 - \nu_4) (Q_{H,t}^{Pvt})^{1-\eta_4} + \nu_4 (Q_{F,t}^{Pvt})^{1-\eta_4} \right]^{\frac{1}{1-\eta_4}}$$

if  $\eta_4 \neq 1$  and  $Q_t^{Pvt} = (Q_{H,t}^{Pvt})^{1-\nu_4} (Q_{F,t}^{Pvt})^{\nu_4}$  if  $\eta_4 = 1$ .

$$(eqn. 52/72) \quad Q_t^{Gov} = \left[ (1 - \nu_5) (Q_{H,t}^{Gov})^{1-\eta_5} + \nu_5 (Q_{F,t}^{Gov})^{1-\eta_5} \right]^{\frac{1}{1-\eta_5}}$$

if  $\eta_5 \neq 1$  and  $Q_t^{Gov} = (Q_{H,t}^{Gov})^{1-\nu_5} (Q_{F,t}^{Gov})^{\nu_5}$  if  $\eta_5 = 1$ .

$$(eqn. 53/72) \quad R_{t+1}^{Pvt} = R_{H,t+1}^{Pvt} \frac{Q_{H,t}^{Pvt} b_{H,t}^{Pvt}}{Q_t^{Pvt} b_t^{Pvt}} + R_{F,t+1}^{Pvt} \frac{Q_{F,t}^{Pvt} b_{F,t}^{Pvt}}{Q_t^{Pvt} b_t^{Pvt}}$$

$$(eqn. 54/72) \quad R_{t+1}^{Gov} = R_{H,t+1}^{Gov} \frac{Q_{H,t}^{Gov} b_{H,t}^{Gov}}{Q_t^{Gov} b_t^{Gov}} + R_{F,t+1}^{Gov} \frac{Q_{F,t}^{Gov} b_{F,t}^{Gov}}{Q_t^{Gov} b_t^{Gov}}$$

$$(eqn. 55/72) \quad R_{H,t}^{Gov} = \frac{1 + \kappa Q_{H,t}^{Gov}}{Q_{H,t-1}^{Gov}}$$

$$(eqn. 56/72) \quad R_{H,t}^{Pvt} = \frac{1 + \kappa Q_{H,t}^{Pvt}}{Q_{H,t-1}^{Pvt}}$$

$$(eqn. 57/72) \quad R_{F,t}^{Gov} = \frac{E_t}{E_{t-1}} \epsilon R_{F,t}^{Gov},$$

where  $\epsilon R_{F,t}^{Gov} = \frac{\epsilon_{1+\kappa} \epsilon Q_{F,t}^{Gov}}{\epsilon Q_{F,t-1}^{Gov}}$ .

$$(eqn. 58/72) \quad R_{F,t}^{Pvt} = \frac{E_t}{E_{t-1}} \epsilon R_{F,t}^{Pvt}$$

$$(eqn. 59/72) \quad Q_{F,t}^{Gov} = E_t \epsilon Q_{F,t}^{Gov}$$

$$(eqn. 60/72) \quad Q_{F,t}^{Pvt} = E_t \epsilon Q_{F,t}^{Pvt}$$

### E. Monetary Authority

$$(eqn. 61/72) \quad Q_{H,t}^{Pvt} b_{H,t}^{Pvt}(ma) + Q_{H,t}^{Gov} b_{H,t}^{Gov}(ma) = s_t$$

$$(eqn. 62/72) \quad \Pi_{ma,t}^{real} \equiv \frac{\Pi_{ma,t}}{P_{C,t}} = \pi_{C,t}^{-1} [R_{H,t}^{Pvt} Q_{H,t-1}^{Pvt} b_{H,t-1}^{Pvt}(ma) + R_{H,t}^{Gov} Q_{H,t-1}^{Gov} b_{H,t-1}^{Gov}(ma) - R_{t-1}^S s_{t-1}]$$

$$(eqn. 63/72) \quad \ln R_t^{Pol} = (1 - \rho_r) \ln (R^{Pol})^{SS} + \rho_r \ln R_{t-1}^{Pol} + (1 - \rho_r) [\phi_\pi (\ln \pi_{C,t} - \ln \pi_{C,t}^{SS}) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_r \epsilon_{r,t}$$

$$(eqn. 64/72, Scenario 1) \quad R_t^S = R_t^{Pol}$$

(eqn. 64/72, Scenario 2)

$$R_t^S = \max \{1, R_t^{Pol}\}$$

(eqn. 64/72, Scenario 3)

$$R_t^S = \max \{\underline{R}, R_t^{Pol}\}$$

(eqn. 65/72)

$$R_t^D = \max \{1, R_t^S\}$$

(eqn. 66/72, Endo.)

$$b_{H,t}^{Pvt}(ma) = (1 - \rho_1) (b_H^{Pvt}(ma))^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) \\ + (1 - \rho_1) \Psi_1 [\phi_\pi (\ln \pi_{C,t} - \ln \pi_C^{SS}) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_1 \varepsilon_{1,t}$$

(eqn. 66/72, Exog.)

$$b_{H,t}^{Pvt}(ma) = (1 - \rho_1) (b_H^{Pvt}(ma))^{SS} + \rho_1 b_{H,t-1}^{Pvt}(ma) + s_1 \varepsilon_{1,t}$$

(eqn. 67/72, Endo.)

$$b_{H,t}^{Gov}(ma) = (1 - \rho_2) (b_H^{Gov}(ma))^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) \\ + (1 - \rho_2) \Psi_2 [\phi_\pi (\ln \pi_{C,t} - \ln \pi_C^{SS}) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_2 \varepsilon_{2,t}$$

(eqn. 67/72, Exog.)

$$b_{H,t}^{Gov}(ma) = (1 - \rho_2) (b_H^{Gov}(ma))^{SS} + \rho_2 b_{H,t-1}^{Gov}(ma) + s_2 \varepsilon_{2,t}$$

### F. Fiscal Authority

(eqn. 68/72)

$$G_t + \bar{b}_H^{Gov} \pi_{C,t}^{-1} = T_t + \Pi_{ma,t}^{real} + Q_{H,t}^{Gov} \bar{b}_H^{Gov} (1 - \kappa \pi_{C,t}^{-1})$$

(eqn. 69/72)

$$\ln G_t = (1 - \rho_G) \ln G^{SS} + \rho_G \ln G_{t-1} + s_G \varepsilon_{G,t}$$

(eqn. 70/72)

$$\bar{b}_H^{Gov} = b_{H,t}^{Gov}(fi) + b_{H,t}^{Gov}(ma),$$

where  $b_{H,t}^{Gov}(fi) = \int b_{H,t}^{Gov}(j) dj$ .

### G. Foreign Exchange Market

(eqn. 71/72)

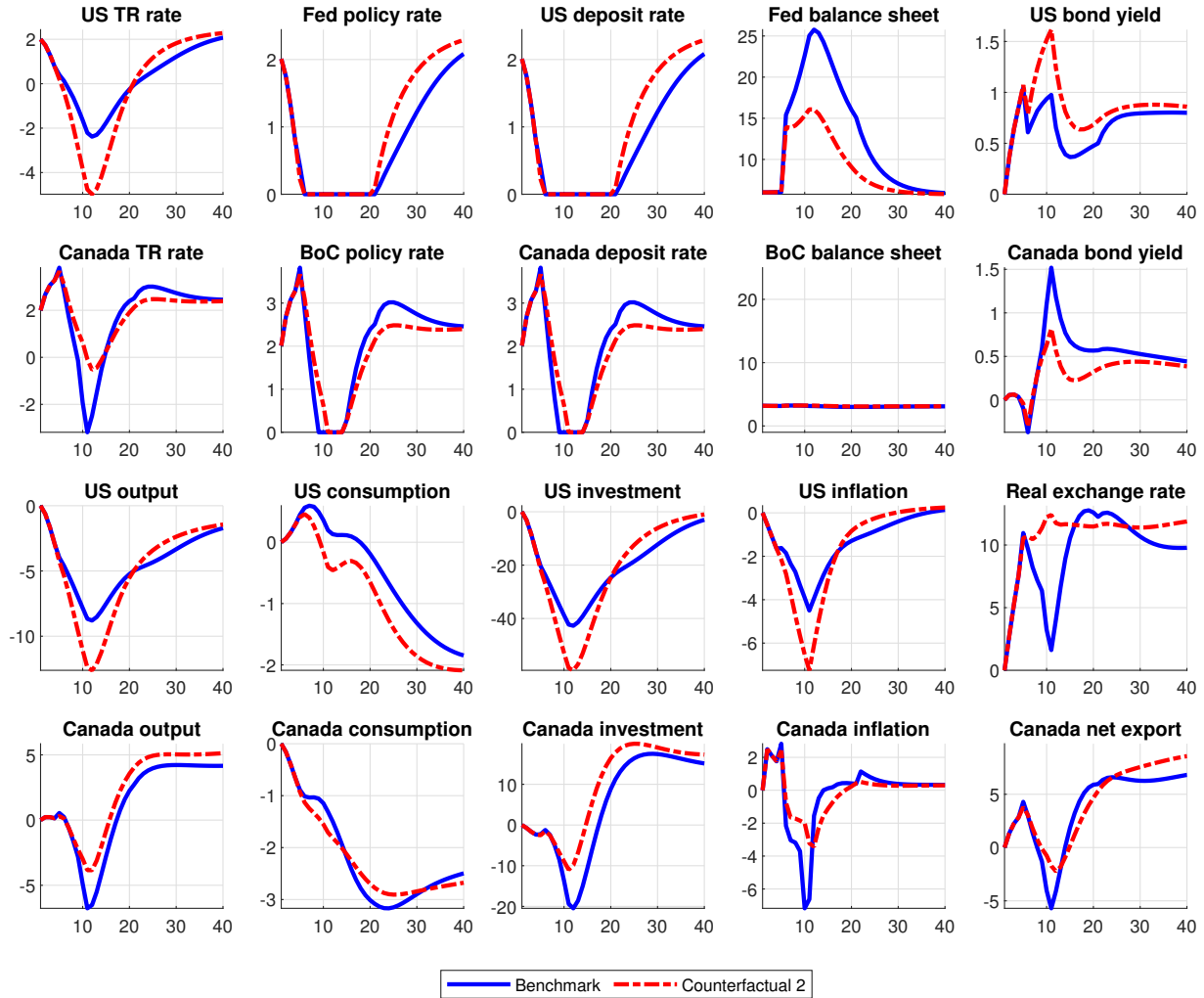
$$E_t = \frac{p_{F,t}}{p_{F,t-1}} \frac{\pi_{C,t}}{\pi_{F,t}} E_{t-1}$$

(eqn. 72/72)

$$p_{H,t} X_t - p_{F,t} C_{F,t} = E_t b_{H,FC,t-1}^{Pvt} \pi_{C,t}^{-1} - (b_{F,t-1}^{Pvt}(fi) + b_{F,t-1}^{Gov}(fi)) \pi_{C,t}^{-1} E_t \\ + Q_{F,t}^{Pvt} (b_{F,t}^{Pvt}(fi) - \kappa b_{F,t-1}^{Pvt}(fi) \pi_{C,t}^{-1}) + Q_{F,t}^{Gov} (b_{F,t}^{Gov}(fi) - \kappa b_{F,t-1}^{Gov}(fi) \pi_{C,t}^{-1}) \\ - Q_{F,t}^{Pvt} (b_{H,FC,t}^{Pvt} - \kappa b_{H,FC,t-1}^{Pvt} \pi_{C,t}^{-1})$$

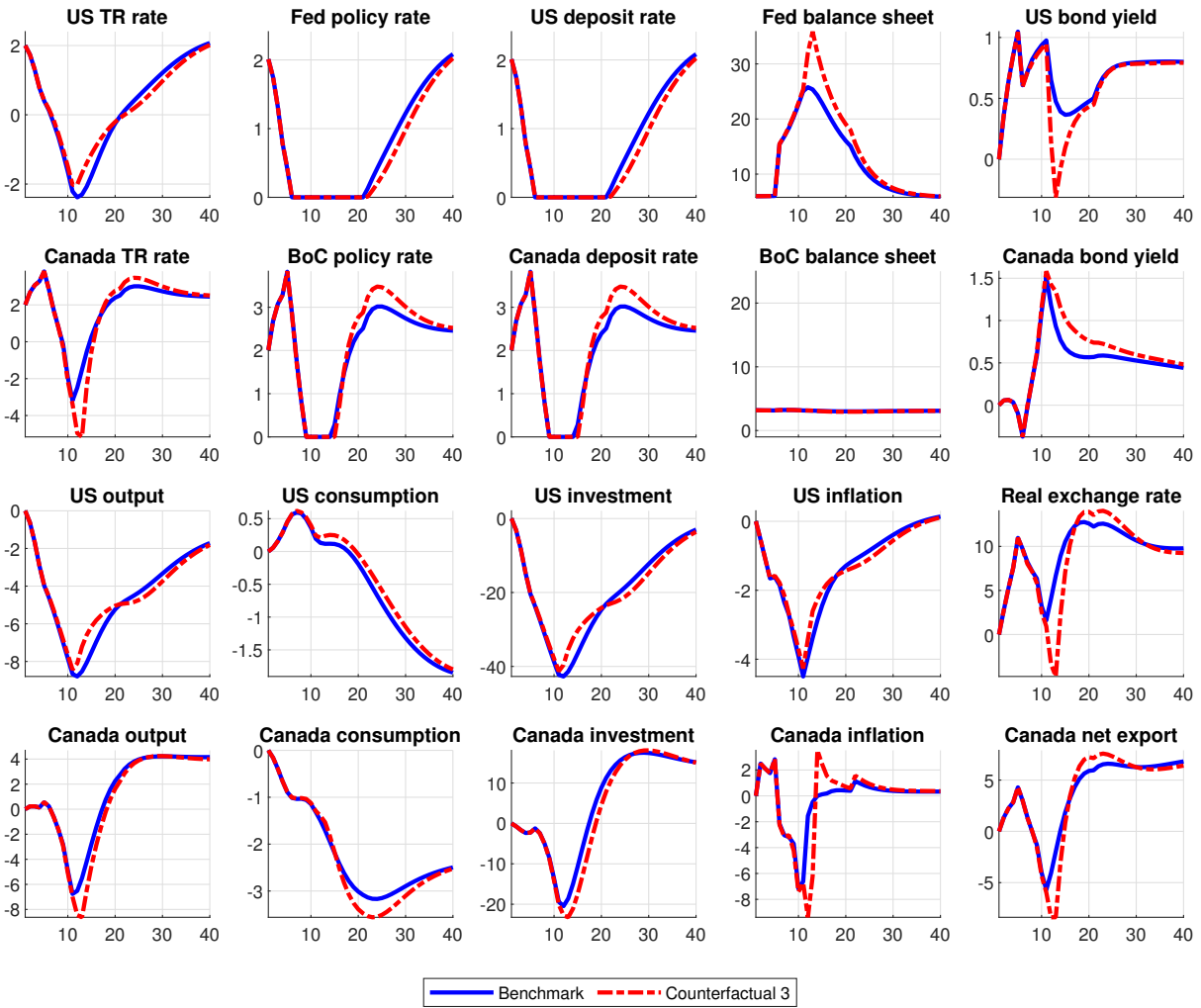


### C Figures



*Notes:* In this counterfactual experiment the Fed increases its balance sheet from the steady-state level of 6% of GDP to 16% of GDP. See the subplot in row 1 column 4 of this figure.

Figure 7: Benchmark vs. mild QE by the Fed (Counterfactual 2)



*Notes:* In this counterfactual experiment the Fed increases its balance sheet from the steady-state level of 6% of GDP to 36% of GDP. See the subplot in row 1 column 4 of this figure.

Figure 8: Benchmark vs. more aggressive QE by the Fed (Counterfactual 3)

## D Tables

Table 5: Standard Deviation and Persistence of Shocks

Parameter	Symbol	Value
SD of shock to MA's govt. bond holdings	$s_1$	0.14
SD of shock to MA's pvt. bond holdings	$s_2$	0.14
SD of shock to technology	$s_A$	0.0065
SD of shock to government expenditure	$s_G$	0.01
SD of shock to Taylor-rule policy rate	$s_R$	0.01
SD of preference shock	$s_Z$	0.01
SD of liquidity shock	$s_\theta$	0.04
Degree of persistence in the MA's purchase of govt. bonds	$\rho_1$	0.8
Degree of persistence in the MA's purchase of private bonds	$\rho_2$	0.8
Degree of persistence of technology shock	$\rho_A$	0.95
Degree of persistence of government expenditure shock	$\rho_G$	0.95
Degree of persistence of the Taylor-rule rate	$\rho_R$	0.8
Degree of persistence of preference shock	$\rho_Z$	0.8
Degree of persistence of a liquidity shock	$\rho_\theta$	0.95

Table 6: Calibrated Parameters or Targets

Parameter	Symbol	Value/Target
SS government expenditure <i>Target: Govt. expenditure to GDP ratio</i>	$G$	0.2
Neutral policy rate	$R^{Pol}$	1.005
SS real govt. bond holdings of MA <i>Target: Bond value to GDP ratio</i>	$b_H^{Gov} (ma)$	0.0319
SS real pvt. bond holdings of MA <i>Target: Bond value to GDP ratio</i>	$b_H^{Pvt} (ma)$	0
Fixed real govt. debt <i>Target: Govt. debt to GDP ratio</i>	$\bar{b}_H^{Gov}$	0.4
Habit parameter	$h$	0.7
Govt. bond recovery <i>Target: Excess return govt. vs pvt. bonds</i>	$\Delta$	1/3
Parameter on endogenous component of QE (pvt. bonds)	$\Psi_1$	-20
Parameter on endogenous component of QE (govt. bonds)	$\Psi_2$	-20
Share of capital in output	$\alpha$	0.33
Discount factor	$\beta$	0.995
Backward price indexation parameter	$\gamma_p$	0
Backward wage indexation parameter	$\gamma_w$	0
Quarterly depreciation of capital in the steady state	$\delta_0$	0.025
Coefficient of linear term in depreciation function <i>Target: <math>u^{SS}</math></i>	$\delta_1$	1
Coefficient of squared term in depreciation function	$\delta_2$	0.01
Elasticity of substitution between any two retail goods	$\epsilon_p$	11
Elasticity of substitution between any two labor types	$\epsilon_w$	11
Fraction of pvt. bonds kept by FI <i>Target: Pvt. bond excess return</i>	$\theta$	0.0075
Degree of price rigidity	$\theta_p$	0.75
Degree of wage rigidity	$\theta_w$	0.75
Fraction of FI's that survive each period	$\vartheta$	0.95
Depreciation rate of coupon payment on bonds <i>Target: Bond duration</i>	$\kappa$	40
Adjustment cost of investment parameter	$\kappa_I$	2
Inverse of the intertemporal elasticity of substitution	$\sigma$	1.4
Parameter on output growth in the Taylor rule	$\phi_y$	0.25
Parameter on inflation gap in the Taylor rule	$\phi_\pi$	1.25
Inverse Frisch elasticity	$\varphi$	1
Minimum share of borrowing to finance investment	$\psi$	0.81
Lumpsum transfer from household to entering FI's <i>Target: Leverage ratio</i>	$\chi$	4
Weight on disutility of work <i>Target: <math>L_1^{SS}</math></i>	$\omega$	1