Short-Run Equilibrium of International Trade under Heterogeneous Discrete Firms with Multiple Continuous Varieties

By Gyu Hyun Kim[a]

This study demonstrates the impact of international trade on the lowering markups of multiple varieties produced by a discrete firm differentiated in productivity. Market liberalization promotes head-to-head competition among productive heterogeneous firms. Therefore, in this pro-competitive market, productive firms would survive by reducing markups and adjusting their range of products, while inefficient firms fail to survive. Despite the importance of this topic, most trade studies employing firm-level granularity and heterogeneity mute a change in markups in the short-run, while they identify properties in the long-run equilibrium in terms of prices, a range of varieties, and profits. With an assumption of fixed firm-level productivities across symmetric economies in the short-run, market liberalization introduces head-to-head competition. In this pro-competitive market, the productive firms survive by pricing competition with exporters with symmetric productivities. In the quantitative analysis, the results show that the survivors in the liberalized market lower their markups and change the range of varieties in the short-run equilibrium.

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1. Introduction

In liberalized markets, local firms respond to the introduction of productive exporters by lowering the markups of their products and adjusting varieties for sustaining their business. Within the exporters' expansion in the domestic market, the less productive firms struggle with a lower revenue because they cannot reduce markups of their product as much as efficient suppliers due to a higher cost. When those inefficient firms face negative profits in the integrated market, they suspend business and exit the market, resulting in the survival of only the

[a] Department of Economics, Iowa State University, Heady Hall, 518 Farm House LN, Ames, IA 50011, USA. (email: gyuhyun@iastate.edu).

efficient firms providing a broader range of varieties with lower markups. In sum, the welfare of an economy rises from international trade due to the increasing number of products and lower prices of each variety.

Previous studies in international trade do not discuss the change in markups in the short-run, while the models in those studies incorporate firm-level granularity and heterogeneity. For instance, Feenstra and Ma (2008) do not account for the movement of the margin after a bilateral trade under a heterogeneous and discrete firm environment. Feenstra and Ma (2008) show the integrated market of international trade lowers markups of varieties within discrete and symmetric firms. Given an identical distribution of productivity among a fixed number of firms, Feenstra and Ma (2008) demonstrate that both market shares and markups do not change in the short-run regardless of market integration. Additionally, they do not account for head-to-head competition between multiple numbers of firms in the short-run of international trade.

This paper demonstrates the impact of international trade on the lower markups of multiple varieties produced by a discrete firm differentiated in productivity. With bilateral trade between symmetric economies incorporated in a simple framework without fixed and transportation costs, this paper argues that productive firms in each economy survive from head-to-head competition by reducing markups and adjusting varieties. Accompanying a nested Constant Elasticity of Substitution (CES) demand system and a monopolistic competition environment, this paper investigates how a firm chooses the number of varieties, the price of each variety, and how they are related to the market share in the short-run equilibrium. Then, the quantitative results show the short-run effect of international trade on the markups, the number of varieties, prices, and profits.

There has been no previous research that provides quantitative results that markups fall when there is international trade under heterogeneous discrete firms with multiple varieties. In the case of a continuum of firms, a firm's markup is invariant as a result of the zero-measured market share. Although this simple framework shows a concentration of resources on the more productive firms and higher threshold productivity due to the exit of the least productive and active firms in autarky, it cannot sufficiently explain the firm's markup adjustment to changes in the market environment. Even though some studies account for all aspects of equilibrium from international trade except the changes in markups, they do not compare an individual firm's optimal choice between ex-ante and ex-post. For example, they verify only the firm's ex-post behavior; more productive firms surviving in the liberalized market have lower markups, higher expenditure, and higher profit.

Given the firm-level heterogeneity, only productive firms survive by lowering markups and adjusting their range of varieties in the short-run equilibrium of bilateral trade between symmetric economics. As head-to-head competition expels the least productive firms from the market, the number of domestic firms serving local markets is reduced compared to autarky, which raises the intensity of a small number of productive firms after bilateral trade. Specifically, in the short-run equilibrium, the most productive firms take over the market share of the less productive firms active in the integrated market. Moreover, an increase in the range of varieties expanded by the survivors is not proportional to an increase in the size of the integrated market.

Section 2 reviews previous literature related to this paper. Then, Section 3 outlines the theoretical framework and analytically demonstrates a firm's choice rule in equilibrium. In Section 4, the quantitative results show intensive competition when the bilateral trade begins, which causes the markups to fall. Finally, 5 offers concluding remarks.

2. Related literature

This study is mainly associated with previous pivotal studies on the behavior of the heterogeneous firm producing multiple varieties in the global market. Therefore, a framework incorporated in this research allows discrete firms (granularity) differentiated in productivity (heterogeneity) to produce multiple varieties. Nevertheless, this paper is innovative in that it explores head-to-head competition resulting in changes in markups and a range of varieties in the short-run equilibrium of international trade. Therefore, this research defines the environment of international trade as the active firms in autarky that retain their productivities and face the integrated market in the short-run. As active firms are discrete, they have market power on their varieties [1].

2.1 Firm Heterogeneity in Production Efficiency

This paper is in the line of substantial research regarding heterogeneous firms in productivity and their behavior in a monopolistic competitive market, à la Melitz (2003). Melitz (2003) begins its framework that a continuum of potential firms decide whether to enter the market by comparing its expected payoff to entry cost. The entry cost is assumed as a sunk cost. Unless the sunk cost exceeds the expected profit, the firm participates in the monopolistic competitive market, and it is given a draw of productivity. In autarky, given a set of random productivity within firms that join the market, the zero-cutoff profit productivity is determined uniquely, then each firm has to make the second-stage decision about whether it proceeds to produce by paying both fixed and variable costs or not. If the productivity of a firm is higher than the zero-cutoff one, the firm is qualified to survive at the second stage and take advantage of a non-negative profit from producing its single variety. In the multilateral trade between symmetric economies,

[1] A general form of nested Constant Elasticity of Substitution (CES) utility function allows an endogenous sector-level share. In the case of the Cobb-Douglas utility function on the top-tier preference, the sectoral share is fixed by an exogenous parameter.

Melitz (2003) assumes that a fixed cost for exporting a variety is higher than one for serving to a local market, which guarantees higher zero-cutoff productivity of exporters. As a result, a productive firm serves its product not only for the domestic market but also for the foreign markets. Moreover, the existence of productive exporters in each market results in a higher zero-cutoff in equilibrium. This higher cutoff in equilibrium expels the lower productive domestic firms out of the market. Melitz (2003) suggests a static equilibrium of international trade by incorporating differential productivity across firms. Given the heterogeneous firms in productivity, Ghironi and Melitz (2005), Das, Roberts, and Tybout (2007), Alessandria and Choi (2014) and Alessandria, Choi, and Ruhl (2014) investigate how firms respond to international trade dynamically. Konings and Vandenbussche (2008) suggests that trade policy can have a heterogeneous impact on firms relying on their assigned productivity.

2.2 Multiple Varieties Produced by a Firm

Maintaining a continuum of firms with differentiated in productivity in the economy, Bernard, Redding, and Schott (2011) allow the zero-measured firm to produce multiple varieties. Bernard, Redding, and Schott (2011) consider two stochastic environments, firm-level random productivity (ability) and firm-product-level random variable (attribute). If a firm decides to enter the market by paying the sunk cost as demonstrated in Melitz (2003), it receives random draws of firm-level productivity and product-level consumer taste. Each firm can determine the range of a continuum of varieties and their prices. In equilibrium, a high-ability firm introduces high-attribute products in the global market. This product replaces low-attribute domestic goods. This product-level reallocation raises zero-cutoff productivity in equilibrium in the integrated market between symmetric economies.

Feenstra and Ma (2008) and Dhingra (2013) depart from the zero-measured firms assumed in Melitz (2003) and Bernard, Redding, and Schott (2011). Instead, Feenstra and Ma (2008) and Dhingra (2013) account for multiple varieties within a relatively large firm in the market. Since each firm is large enough to be positively measured, it has pricing power on products. Along with a firm's ability to choose its range of varieties, those studies incorporate the cannibalization effect, which stands for the firm's opportunity costs from expanding a range of varieties. The existence of the cannibalization effect limits firm's choice for a range of varieties in a finite number. Dhingra (2013) utilizes a linear demand system and lets the cost of production depend on the investment in innovation. Dhingra (2013) argues that gains from trade depend on the focus of innovation: either expanding varieties or production processing. Although Dhingra (2013) concentrates on the theoretical framework of the industrial organization, Feenstra and Ma (2008) conducts structural experimentation to find the heterogeneous firm's dynamics in the integrated market between symmetric economies. Feenstra and Ma (2008) offer meaningful results that are consistent with the granular environment in the global economy: As the link between the market size and the number of firms is weak, the market integration does not necessarily yield a marginal increase in the number of survivors. The small number of market survivors respond to the global market by expanding their range of varieties proportional to the aggregated market size. Since there is no change in the number of survivors in the market with the same productivity, the market share is persistent as in the long-run equilibrium. Therefore, markups are constant regardless of the changes in the exogenous market size.

2.3 Discrete Firm

In international trade investigating individual firm behavior, granularity consists of the following two necessary features: 1) a fat-tailed distribution of firms and 2) discreteness of firms. Those two features let a small number of productive firms control markups and product scopes in response to changes in both domestic and foreign markets. Moreover, as the law of large numbers is not valid under granularity, a single exporter can affect the aggregate economy, such as price indexes.

Using Compustat data, Gabaix (2011) finds sales from the top 50 firms in the United States share 24% of Gross Domestic Product (GDP) and that the next top 50 firms share 5% of GDP. It suggests the hypothesis that firm-level shocks can be associated with aggregate economic phenomena. The main empirical results indicate that the firm-level shocks to the top 100 firms account for one-third of GDP fluctuations.

Di Giovanni and Levchenko (2012) expand the granular model in the spirit of Melitz (2003) while the markups derived from their model are constant regardless of productivity. According to Bernard et al. (2018), Di Giovanni and Levchenko (2012) focus on a particular case of monopolistic competition. Di Giovanni and Levchenko (2012) suggest that the concept of granularity can help explain the high sensitivity of a small open economy to the aggregate fluctuations from firm-level idiosyncratic shock.

Employing granularity, Eaton, Kortum, and Sotelo (2012) also construct a granular model expanded from Melitz (2003) and Chaney (2008). They analytically compare how the discrete firm model is different from that of a continuum firm model in terms of optimal pricing.

Edmond, Midrigan, and Xu (2015) and Gaubert and Itskhoki (2018) amend the constant markup in Di Giovanni and Levchenko (2012). According to Amiti, Itskhoki, and Konings (2019), there are strategic complementarities when firms set their price. Gaubert and Itskhoki (2018) strictly follow the concept of granularity in developing their theoretical framework. Accounting for granularity and Zipf's law, they show how activities of a few discrete firms are associated with the sector-level comparative advantage. In general equilibrium combining two random draws in both sectoral and firm-level productivities, Gaubert and Itskhoki (2018) find that granularity can explain 20% of the export volatility [2]. Moreover, they suggest mean reversion in the comparative advantage related to granularity [3].

Feenstra and Ma (2008), Hottman, Redding, and Weinstein (2016) and Bernard et al. (2018) maintain various markups consistent with the concept of granularity. They allow firms to internalize firm-level impacts on macro-level aggregates. Based on a nested CES demand system, they offer the pricing rule of a product, which depends not only on marginal costs but also on producer status in each market. Therefore, the price of a product varies relying on the market status of each region despite that the marginal cost is identical. They suggest that, given the equilibrium factor and product prices, productive firms set a higher markup and a broader range of varieties among survivors. The drawback of those studies is that they do not investigate how an individual firm changes its optimal choices in the transition from autarky to international trade.

2.4 Contributions

This paper accounts for both granularity and multiple products within firms. Even though Melitz (2003) and Gaubert and Itskhoki (2018) suggest meaningful academic insights on new trade theory employing firm-level heterogeneity, they fail to account for the cannibalization effect from producing multiple products. The single-variety framework has its advantage in that one can reach the general equilibrium easily with the concept of average productivity representing surviving firms. Also, this concept of average productivity is proven to be unique. In reality, most firms produce multiple products that are imperfect substitutes for each other. Therefore, the optimal range of variety is a crucial element in firm-level decision making, because adding the marginal number of varieties gives both benefit and cost. From the marginal extension in the range of varieties, the firm can earn a higher market share while it experiences fewer revenues from existing products.

Moreover, a conventional environment of international economics in the short-run is amended to show a change in the markup of a variety. According to the conventional short-run concept, a new stage of the pricing competition begins at the onset of international trade. Therefore, as all of the potential firms in each economy have to be assigned their productivity again; it is not available to find out whether active firms in autarky survive in the integrated market or whether those firms would change their markup of a variety. In this study, all survivors in autarky are assumed to maintain their productivity and compete with exporters

[2] Two random draws concerning firm-level productivity and sector-level number of the promising entrants are introduced like Eaton, Kortum, and Sotelo (2012).

[3] For example, Gaubert and Itskhoki (2018) point that the death of a granular firm can change the comparative advantage significantly.

from the symmetric economy in the short-run. This new definition of the short-run enables us to explain the lower markups after international trade.

3. Framework

This section introduces the theoretical framework and derives analytic properties in equilibrium.

3.1 Preference

The postulated demand system employs a nested CES utility function with discrete firms and a continuum of varieties. As in Bernard et al. (2018) and Feenstra and Ma (2008), the CES preference nested in the Cobb-Douglas utility function has two advantages. First, the insight of firms' behavior from the Cobb-Douglas preference is consistent with granularity; firms are large enough to affect the sector but sufficiently small not to have an impact on the aggregate economy. Second, it gives convenience in computation to find the firm-level optimal allocations as they are independent of the sectoral share in the Cobb-Douglas utility function. Therefore, one can calculate computational solutions more straightforwardly with a fixed level of sector-level expenditure [4]. In the appendix, a CES utility function at the top-level demand is incorporated as a generalized demand system.

Figure 1 shows the overview of the demand system of a represented consumer, a CES preference nested to the Cobb-Douglas utility function. Consumers in an economy choose the amount of sectoral consumption to maximize their utility measured as the aggregate demand. Moreover, symmetric economies have an identical amount of labor (L), and the wage (w) fixed as one in both economies.

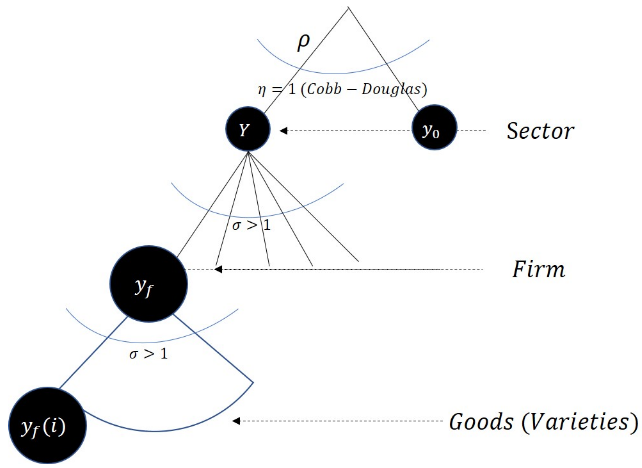
There are two sectors nested to the aggregate demand. The sector supplying a single type of homogeneous product is denoted as sector 0, and the heterogeneous sector provides differentiated goods. The weight on demand for the heterogeneous sector is ρ, which is also the elasticity of sectoral demand. The unity in the elasticity of substitution in the sectoral demand implies the top-tier demand is the Cobb-Douglas preference. In that case, the expenditure share on the heterogeneous sector is invariant as ρ. The aggregate demand can be specified as Equation (1).

$$U = y_0 + \rho \, ln \, (Y) \tag{1}$$

where U is the utility of the representative consumer. y_0 is the consumption of a homogeneous good, and Y the an aggregate consumption of heterogeneous goods. The aggregate demand of the heterogeneous sector, Y, is a CES aggregate across discrete firm-level demands as Equation (2):

[4] A Cobb-Douglas utility is a particular case of CES demand by setting the elasticity of substitution as one.

Figure 1. A nested CES Demand System



$$Y = \left[\sum_{f \in \Omega} \left(y_f \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \tag{2}$$

where y_f is a consumption index for a firm f in the heterogeneous sector. $\sigma\,(>1)$ is the elasticity of substitution in the firm-level demand within the heterogeneous sector. Ω is a set of active firms in the heterogeneous sector in equilibrium. Also, there is no difference in taste appeals across firm-level demands. Therefore, the firm-level aggregate demand is a CES aggregate across continuous variety-level demand as Equation (3):

$$y_f = \left[\int_{i \in \Omega_f} \left(y_f\,(i) \right)^{\frac{\sigma - 1}{\sigma}} di \right]^{\frac{\sigma}{\sigma - 1}} \tag{3}$$

where $y_f\,(i)$ is consumption of variety i produced by firm f in the heterogeneous sector. If the same elasticity of substitution (σ) is applied to the variety-level demand, these identical elasticities of substitution reduce the computational burden. Especially, given the symmetric production technology across the varieties within the firm, the variety-level elasticity of substitution is redundant in the firm's optimal pricing rule. Ω_f is a set of index of varieties that firm f in the heterogeneous sector produces, $\left(\Omega_f = \left\{ i \in \mathbb{R} \,|\, y_f(i) > 0 \right\}\right)$.

The number of varieties produced by firm f in heterogeneous sector is mathematically denoted as $N_f = \int_{i \in \Omega_f} \mathbf{1}_f\,(i)\,di$ where $\mathbf{1}_f\,(i) = 1$ if $i \in \Omega_f$ and zero otherwise.

Equation (4) describes the budget constraint of a representative consumer.

$$P_0 y_0 + \sum_{f \in \Omega} \left[\int_{i \in \Omega_f} \left[P_f\left(i\right) \times y_f\left(i\right) \right] di \right] \leq I \tag{4}$$

where I is the income of a representative consumer in the economy.

Utility maximization yields a sector-level price index, which is dual to a sectoral demand function. In the heterogeneous sector, the sectoral price index (P) dual to the sectoral demand in Equation (2) is a CES aggregate across price indexes of the firm-level composite, such as Equation (5):

$$P = \left[\sum_{f \in \Omega} \left(P_f\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{5}$$

The price of firm-level consumption, P_f, consists of prices of varieties produced by firm f as in Equation (6):

$$P_f = \left[\int_{i \in \Omega_f} \left(P_f\left(i\right)\right)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}. \tag{6}$$

In equilibrium, the optimal allocations determine the firm-level expenditure share within the heterogeneous sector [5]. Equation (7) shows the expenditure share of the heterogeneous sector in the economy, which is fixed at ρ due to the Cobb-Douglas preference at the top-level demand:

$$S = \frac{\sum_{f \in \Omega} \left[\int_{i \in \Omega_f} \left\{ P_f\left(i\right) y_f\left(i\right) \right\} di \right]}{I} = \rho. \tag{7}$$

Moreover, Equation (8) offers the firm-level expenditure share within the heterogeneous sector

$$S_f = \frac{\int_{i \in \Omega_f} \left\{ P_f\left(i\right) y_f\left(i\right) \right\} di}{\sum_{g \in \Omega} \left[\int_{j \in \Omega_g} \left\{ P_g\left(j\right) y_g\left(j\right) \right\} dj \right]} \tag{8}$$

where R is the expenditure on the heterogeneous sector, $R = \rho I$. Note that a summation symbol is applied to firm-level aggregation because the firms are treated

[5] In the nested CES demand to the Cobb-Douglas utility function, the sector-level expenditure share within the economy is assumed as fixed as ρ. In the generalized utility function, this sector-level expenditure share is also determined in equilibrium. The sectoral expenditure share is shown as an endogenous factor in the CES utility function at the top-level demand in the appendix.

as discrete. In contrast, as the varieties are supposed to be continuous, an integral is taken for aggregation. The expenditure on firm-level consumption in the heterogeneous sector is $E_f = \int_{i \in \Omega_f} \{P_f(i)\, y_f(i)\}\, di = RS_f = \rho I S_f$.

The expenditures and prices in equilibrium determine the Marshallian demand function at each level of consumption. In the heterogeneous sector, the sectoral demand function is

$$Y = \rho I \frac{1}{P} = R \frac{1}{P}. \tag{9}$$

The demand function of firm-level composite is

$$\begin{aligned} y_f &= \rho I S_f \frac{1}{P_f} \qquad (10) \\ &= \rho I \frac{P_f^{1-\sigma}}{P^{1-\sigma}} \frac{1}{P_f} \end{aligned}$$

where $S_f = \frac{P_f^{1-\sigma}}{P^{1-\sigma}}$ from the property of the CES utility function.

It is worth documenting the variety-level demand in the case of symmetric technologies across the varieties within firms, while the variety-level consumption is continuous, zero-measured. The firm-level demand in Equation (3) is converted to Equation (11).

$$\left(y_f\right)^{\frac{\sigma-1}{\sigma}} = \int_{i \in \Omega_f} \left(y_f(i)\right)^{\frac{\sigma-1}{\sigma}} di \tag{11}$$

Applying Equation (10) to (11), one can derive Equation (12):

$$\left[\rho I \frac{P_f^{1-\sigma}}{P^{1-\sigma}} \frac{1}{P_f}\right]^{\frac{\sigma-1}{\sigma}} = \int_{i \in \Omega_f} \left(y_f(i)\right)^{\frac{\sigma-1}{\sigma}} di\ . \tag{12}$$

When the technology is symmetric across varieties within firm f, then the right-hand side of equation (12) can be represented as the mass of variety, such as

$$\left[\rho I \frac{P_f^{1-\sigma}}{P^{1-\sigma}} \frac{1}{P_f}\right] = N_f \left[y_f(i)\right]^{\frac{\sigma-1}{\sigma}} \tag{13}$$

as $y_f(i)$ does not depend on i anymore and $N_f = \int_{i \in \Omega_f} \mathbf{1}_f(i)\, di$ where $\mathbf{1}_f(i) = 1$ if $i \in \Omega_f$ and zero otherwise. Then, Equation (14) offers the variety-level demand:

$$y_f(i) = y_{fi} = \rho I S_f \frac{1}{P_f} (N_f)^{\frac{\sigma}{1-\sigma}} \;\; \forall i \in \Omega_f \qquad (14)$$
$$= \rho I S_f \frac{1}{N_f} \frac{1}{p_{fi}} .$$

3.2 Technologies

The homogeneous good is produce using a constant return to scale (CRS) technology with a unit of labor, and it is traded without any trade costs. There is no fixed cost for either entry or production. As the homogeneous good is treated as the numeraire ($P_0 = 1$), the price of this good is one. Therefore, the supplied quantity of the homogeneous good is w across economy because of $P_0 \times y_0 = y_0 = w \times 1$. Besides, as the wage in both symmetric economies is assumed to be one, the expenditure on this homogeneous good is one across economies. This paper accounts for the equilibria that all economies produce some of the homogeneous goods to simplify the analysis.

In the heterogeneous sector, there are M_e entrants in the market. In the short-run, the number of entrants (M_e) is fixed, so it is assumed that there is no sunk cost of entering the market. Each of them is given random productivity drawn from the Pareto distribution, φ_f, following the cumulative distribution function of Equation (15) [6] [7]:

$$Prob\left[\varphi_f < \varphi\right] = 1 - \left(\frac{\varphi^{min}}{\varphi}\right)^{\gamma} \qquad (15)$$

where γ is a shape parameter of the Pareto distribution, and φ^{min} is the lower support of random productivities. A higher γ implies that all productivities drawn are close to the lower bound φ^{min}. Productivities drawn are common knowledge among M_e firms; firms can determine whether to initiate production or exit the market depending on the expected profit. Surviving firms are assigned higher productivities so that they start production and benefit from a non-negative profit.

For those survivors, they must account for two types of fixed costs related to producing the varieties. K_0 is a fixed cost of initiating the production. K_1 is a fixed

[6] As in Chaney (2008), assuming a Pareto distribution on firm sizes (productivities) has two advantages: (1) it is tractable analytically, and (2) it approximates the firm sizes in the United States.

[7] According to Eaton, Kortum, and Sotelo (2012), it is not necessary to set the restriction on the relationship between γ and σ $(\gamma > \sigma - 1)$ in the discrete-firm case. It is because the discrete case has no point for the integration over the distribution of prices. Also, as in Feenstra and Ma (2008), It is not possible to get a stationary equilibrium. In the granular environment, there is a set of productivity for each random draw, and there are potentially infinite draws, which implies different ZCP condition for each random productivity set.

cost incurred when a firm adds a marginal variety to the production. Both K_0 and K_1 are measured by the unit of labor and identical across economies, firms, and varieties.

A heterogeneous firm f that produces multiple varieties is defined by random productivity, φ_f. As the only factor for the production is the labor, firm f uses $(1/\varphi_f)$ units of labor to produce a unit of variety. Therefore, when firm f produces $y_f(i)$ units of an individual variety i, the required units of labor for this variety is $(y_f(i)/\varphi_f)$. Considering the wage (w), the marginal cost of producing a variety is $\frac{w}{\varphi_f}$ for firm f. It is also the variable cost for producing a unit of a variety. Combining both fixed and variable costs, Equation (16) offers the total cost function:

$$C_f = w\left[\frac{1}{\varphi_f}\left\{\int_{i\in\Omega_f} y_f(i)di\right\} + K_1\left\{\int_{i\in\Omega_f}\mathbf{1}_f(i)di\right\} + K_0\right] \tag{16}$$
$$where\ \ \mathbf{1}_f(i) = \begin{cases} 1 & if\ i\in\Omega_f \\ 0 & Otherwise \end{cases} .$$

In the case of the symmetric technology across varieties within a firm, the supply of a variety i is independent of the index i. Therefore, one can denote the quantity of a variety as $y_f(i) = y_{fi}$. Moreover, the price of a variety is also the same across all varieties within a firm $(P_f(i) = P_{fi})$. Therefore, the total cost of firm f in the heterogeneous sector consists of:

$$C_f = w\left[\frac{1}{\varphi_f}N_f y_{fi} + K_1 N_f + K_0\right] \tag{17}$$

where N_f is the number of varieties produced by firm f in the heterogeneous sector. Then, Equation (18) specifies the profit function of firm f in the heterogeneous sector:

$$\begin{aligned}\Pi_f &= \int_{i\in\Omega_f}\left[P_f(i)\times y_f(i)\right]di - C_f \\ &= N_f\left(P_{fi} - \frac{w}{\varphi_f}\right)y_{fi} - w\left[K_1 N_f + K_0\right] .\end{aligned} \tag{18}$$

3.3 Optimal Conditions

In the monopolistic competitive market, active firms have market power on their varieties. Those active firms set their optimality consisting of the range of varieties (N_f) and the price of each variety (P_{fi}).

Given the optimal range of varieties (N_f), the First-Order Necessary Condition (FONC) to the price of a variety (P_{fi}) is a derivative of the profit function with respect to P_{fi} as in Equation (19):

$$y_{fi} + P_{fi}\frac{\partial y_{fi}}{\partial P_{fi}} = \frac{w}{\varphi_{zf}}\frac{\partial y_{fi}}{\partial P_{fi}} \, . \tag{19}$$

The left-hand side of Equation (19) is an additional benefit (or revenue) from a marginal increase in the price of a variety, which should be equal to the marginal cost on the right-hand side.

Combining the demand function of a variety, Equation (20) offers the optimal rule of pricing an individual variety produced by firm f

$$P_{fi} = \frac{w}{\varphi_f}\mu_f = \frac{w}{\varphi_f}\frac{\epsilon_f}{\epsilon_f - 1} \ \forall i \in \Omega_f \tag{20}$$

where $\epsilon_f \ (= \sigma + (1-\sigma) S_f)$ is the demand elasticity for the firm-level composite good. It is also the demand elasticity of a variety in the symmetric technology case. Likewise, $\mu_f \ (= \frac{\epsilon_f}{\epsilon_f - 1})$ is the markup of a variety, which is also the markup of the firm-level composite good. If a firm sets its variety price larger than the right-hand side of Equation (20) $(P_{fi} > \frac{w}{\varphi_f}\frac{\epsilon_f}{\epsilon_f-1})$, the firm must lower its price as the marginal cost exceeds the marginal benefit. As the demand elasticity is higher than one $(\epsilon_f > 1)$, a one percent decrease in price results a more than one percent increase in the quantity demanded, which results in higher revenue [8] [9].

Given the optimal price of the varieties (P_{fi}), the first-order condition corresponding to the number of varieties (N_f) is

$$\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi} + N_f \left(P_{fi} - \frac{w}{\varphi_f}\right) \frac{\partial y_{fi}}{\partial N_f} = w K_1 \, . \tag{21}$$

The left-hand side of Equation (21) is the benefit from a marginal increase in the range of varieties, and the right-hand side offers the marginal cost of the range of varieties. Firms exiting the market face a marginal cost of adding a variety (wK_1) larger than the marginal benefit at $N_f = 0$.

Note that the active firms producing varieties internalize a cannibalization effect in their marginal benefit of the range of varieties. Equation (21) shows the partial derivative of the Marshallian demand of a variety (y_{fi}) from Equation (14) with respect to the number of varieties (N_f):

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[8] According to Equation (20), firms that exit from the market set their price at $P_{fi} = \frac{w}{\varphi_f}\frac{\sigma}{\sigma-1}$, which is identical to the optimal pricing in a continuum of firms model like Melitz (2003).
[9] Zero pricing, $P_{fi} = 0$, holds only if $\frac{w}{\varphi_f}\frac{\epsilon_f}{\epsilon_f-1}$ is non-positive.

$$\frac{\partial y_{fi}}{\partial N_f} = -\frac{y_{fi}}{N_f} + y_{fi}\frac{1}{S_f}\frac{\partial S_f}{\partial N_f} \ . \tag{22}$$

Equations (8) and (14) simplify $\frac{\partial S_f}{\partial N_f}$ as $\frac{S_f}{N_f}(1 - S_f)$. Then, Equation (21) is represented as Equation (23), and the second term of Equation (23) shows the cannibalization effect from the multiple varieties:

$$\underbrace{\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi}}_{Marginal\ benefit} - \underbrace{\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi} S_f}_{Cannibalization\ effect} = wK_1 \ . \tag{23}$$

A marginal increase in the range of variety raises the market share, resulting in a higher price and markup. As the varieties are substitutes for each other $(\sigma > 1)$, this higher markup reduces the revenue from each variety. Cannibalization is the effect of lowering revenue from the introduction of a new variety, and the heterogeneous firms internalize this effect [10]. Equation (24) is a concise form of Equation (23):

$$\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi} \frac{\epsilon_f - 1}{\sigma - 1} = wK_1 \ . \tag{24}$$

Applying the optimal pricing rule of Equation (20) to Equation (24) results in Equation (25), which represents the quantity of a variety that an active firm supplies.

$$y_{fi} = \varphi_f\,(\sigma - 1)\,K_1 \ . \tag{25}$$

Equation (26) is the expanded form of the heterogeneous firm's market share. It is computed by applying the pricing rule of Equation (20) and the supply of a variety of Equation (25) to the firm-level market share described in Equation (8):

$$\begin{aligned} S_f &= \frac{E_f}{R} \\ &= \frac{N_f P_{fi} y_{fi}}{R} \\ &= \underbrace{\frac{w}{\varphi_f}\frac{\epsilon_f}{\epsilon_f - 1}}_{=P_{fi}} \frac{N_{fi}}{\rho I} \underbrace{\varphi_f\,(\sigma - 1)\,K_1}_{=y_{fi}} \ . \end{aligned} \tag{26}$$

[10] In the general form of CES preference of the Appendix, there is an indirect cannibalization effect that reduces a firm's revenue from the interaction with the aggregate price index. The interaction with the price (P_{fi}) and number of varieties (N_f) within firms makes the sectoral shares determined endogenously.

Accompanying the wage as unity $(w = 1)$, the optimal range of varieties of an active firm f in the heterogeneous sector is

$$N_f = \frac{R}{K_1}\frac{1}{\sigma - 1}\frac{\epsilon_f - 1}{\epsilon_f}S_f > 0 \tag{27}$$

where R is a sectoral expenditure as $R = \rho I$ and $\epsilon_f = \sigma + (1-\sigma)\, S_f$ [11].

3.4 Zero-cutoff Profit Condition and Short-run Equilibrium

3.4.1 Zero-Cutoff Profit Condition

In the existence of the fixed costs (K_0 and K_1), not all M_e differentiated firms in the market are guaranteed to survive in equilibrium. Less productive firms with higher marginal costs may want to stop production instead of experiencing a negative profit from supplying a variety. Suppose that $M_o\,(\leq M_e)$ firms survive with non-negative profits in equilibrium. Given the aggregate income (I), Bertrand-Nash equilibrium offers the optimal price and number of varieties for M_o survivors in the short-run equilibrium [12].

Since the set of productivities randomly drawn determines a firm's decision on producing a variety, the number of active firms (M_o) depends on cutoff productivity as in Melitz (2003) and Feenstra and Ma (2008) [13].

Among the M_o survivors, the cutoff (or zero-cutoff profit) productivity or threshold productivity refers to the productivity of the least productive firm(s). The inefficient firms whose productivity is lower than the cutoff productivity exit the market, and those firms are not an element of the M_o survivors.

This zero-cutoff profit (ZCP) productivity forms the following condition (hereafter ZCP condition). Given the set of M_e productivities, the Bertrand competition offers all M_o survivors to earn non-negative profits. In contrast, the participation of the $M_o + 1^{th}$ productive firm in the production has at least one firm face a negative profit. The ZCP condition determines the maximum number of surviving firms (M_o) that have non-negative profits. When the productivity sorts the

[11] If other things are fixed except the market size (I), N_f is proportional to the market size. Feenstra and Ma (2008) show the proportionality in the short-run effect of international trade between symmetric economies when 'same' number of entered firms have 'identical' productivities regardless of the multilateral trade.

[12] In the long-run, the number of entering firm (M_e) is endogenous, and it is determined by free-entry condition. Therefore, M_e is determined at which an ex-ante average profit equals the entry fee as a sunk cost (Melitz (2003) and Feenstra and Ma (2008))

[13] In symmetric economies without government interventions, a cutoff productivity corresponds to a threshold marginal cost as in Melitz (2003) and Feenstra and Ma (2008). Therefore, the cutoff productivity straightforwardly determines the number of surviving firms (M_o). If there are trade costs and government barriers such as tariffs, M_o does not rely on the productivities, but on a threshold marginal cost.

entrants in descending order ($\varphi_1 \geq \varphi_2 \geq \cdots \geq \varphi_{M_e}$), the ZCP productivity belongs to the M_o^{th} productive firm when there are no trade costs or government interventions. Concisely, the ZCP condition is defined as:

$$\begin{cases} \Pi_f \geq 0 \ and \ S_f > 0 & for \ f = 1, \cdots, M_o \\ \Pi_f = 0 \ and \ S_f = 0 & for \ f = M_o + 1, \cdots, M_e \end{cases} \tag{28}$$

where Π_fs and S_fs are the profits and the market share of a surviving firm f in the heterogeneous sector. In the symmetric economies without trade costs or barriers, the ZCP condition directly determines the ZCP productivity (φ^{ZCP}) and the market share of the marginal firm (S^{ZCP}) in the short-run equilibrium.

3.4.2 Short-run Equilibrium and Mechanism to Find equilibrium

Given the set of productivities of the M_e firms, the ZCP condition offers of the ZCP productivity (φ^{ZCP}) and the market share of the marginal firm (S^{ZCP}). This condition determines 1) which firms will proceed to produce based on the ZCP productivity, and 2) market shares of the survivors (S_f). They affect the optimality of M_o survivors, such as the price of a variety (P_{fi}) and range of the varieties (N_f). The Short-run equilibrium is defined as:

Definition. (Short-run Equilibrium) *Given the set of productivity for M_e entrants in an economy with income (I), the Bertrand-Nash equilibrium consists of*

1) a set of information about the ZCP condition in the heterogeneous sector: $\{S^{ZCP}, \ \varphi^{ZCP}\}$,
2) a vector of the optimality set by the M_o surviving firms, including the price of a variety, the range of varieties, and the firm-level market share: $\{P_{fi}, \ N_f, \ S_f\}_{f=1}^{M_o}$, and
3) a sectoral price index within the economy: P,

which solves both utility and profit maximization simultaneously.

Feenstra and Ma (2008) introduce a mechanism to find the unique equilibrium in terms of φ^{ZCP} [14]. As the survivors produce their varieties, the optimal range of varieties (N_f) should be positive for those firms. From Equations (14) and (25), the firm-level productivity can be converted to a function of the firm's properties such as

$$\varphi_f = \left[\frac{\sigma - 1}{R} K_1\right]^{\frac{1}{\sigma-1}} \frac{1}{P} \left[\frac{\epsilon_f}{\epsilon_f - 1}\right]^{\frac{\sigma}{\sigma-1}} . \tag{29}$$

Using ZCP productivity (φ^{ZCP}), an inverse of the relative productivity of a survivor (τ_f) can be calculated, such as

[14] This mechanism is also applied to the generalized CES demand system environment in the appendix.

$$\tau_f = \frac{\varphi^{ZCP}}{\varphi_f} = \left[\frac{\frac{\epsilon_f}{\epsilon_f - 1}}{\frac{\epsilon^{ZCP}}{\epsilon^{ZCP} - 1}} \right]^{-\frac{\sigma}{\sigma-1}} \tag{30}$$

where ϵ^{ZCP} is demand elasticity for the marginal firm's variety. Then, the survivor's market share becomes a function of τ_f and the market share of the marginal firm (S^{ZCP}) [15]:

$$\begin{aligned} S_f &= S\left(\tau_f\right) \\ &= 1 - \frac{1}{\left\{\left(\sigma - 1 + \frac{1}{1 - S^{ZCP}}\right)\left(\tau_f\right)^{-\frac{\sigma-1}{\sigma}} - (\sigma - 1)\right\}} \ . \end{aligned} \tag{31}$$

Among the M_e firms, firms with $\tau_f > 1$ are the less productive firms than the marginal firms, so $S_f = 0$ if $\tau_f > 1$. Assuming the market share of the marginal firm (S^{ZCP}) as an arbitrary value, the profit of each surviving firm is computed as:

$$\begin{aligned} \Pi_f &= \Pi(\tau_f) \\ &= \frac{\left\{S(\tau_f)\right\}^2}{1 + (\sigma - 1)\left\{1 - S(\tau_f)\right\}} R - wK_0 \ . \end{aligned} \tag{32}$$

All M_e elements in the productivity set are candidates of φ^{ZCP}. Therefore, an output table contains τ_fs, S_fs, and Π_fs for each candidate of φ^{ZCP}. Beginning with the highest productivity candidate (i.e. $\varphi^{ZCP} = \varphi_1$), one can calibrate the solution S^{ZCP} satisfying the sum of the market share across the survivors as unity. Then, there is the least (the M_o^{th}) productivity $(\varphi^{ZCP} = \varphi_{M_o})$ satisfying the sum of the market share as unity [16]. This M_o^{th} productive firm is the marginal firm in equilibrium, and S^{ZCP} denotes its market share. The market share of the marginal firm is the smallest among the survivors. In the cases of other candidate of φ^{ZCP}, such as $\varphi^{ZCP} = \varphi_j \ j \in \{M_o + 1, \cdots, M_e\}$, there is no solution to satisfy the sum of the market share among the survivors as one [17].

3.5 Properties of a Firm's Behavior

The short-run equilibrium determines the survivor's market share in the heterogeneous sector (S_f), the optimal price of a variety (P_{fi}), and the optimal range

[15] As in Feenstra and Ma (2008), if there is no change in φ^{ZCP} and τ_f , a higher productive firm's benefit from a higher market share because $\frac{\partial S_f(\tau_f)}{\partial \tau_f} < 0$.

[16] This firm has the highest marginal cost for producing a variety among the M_o survivors.

[17] In the Cobb-Douglas utility function at the top-level demand, one can disregard the finding optimal sectoral share process because it is fixed as ρ.

of varieties (N_f).

Regarding the optimal pricing (P_{fi}), the short-run equilibrium offers 1) an increasing markup (μ_f) and 2) a decreasing price of a variety (P_{fi}) in productivity (φ_f) among the M_0 survivors, and those results are consistent with the literature. Given the fixed sectoral share (ρ), the survivor's market share (S_f) is decreasing in the inverse of relative productivity to the ZCP productivity (τ_f), which leads to a lower demand elasticity for a firm-level composite good (ϵ_f). This lower demand elasticity leads to a higher markup of the varieties [18].

The optimal range of varieties (N_f) is proportional to the market size (I) only if there is no update on the ZCP condition [19]. Given the market size (I), a productive firm with a higher market share produces a broader range of varieties. In contrast, as in the Equation (27), the existence of the inverse of markup prevents the optimal range of varieties from expanding proportionally to the firm-level market share [20].

3.6 Short-run Equilibrium of International Trade

International trade, or market liberalization, offers an integrated market. This part introduces the environment of bilateral trade in the short-run, then defines the short-run equilibrium in the integrated market in the context of this framework. Finally, using the concept of the short-run equilibrium, the effects of international trade on the firm's optimality are identified.

3.6.1 The Environment of International Trade in the Short-run

In this paper, international trade is assumed as bilateral trade between symmetric economies. Both economies share labor income $(I = wL)$, with wage as unity $(w = 1)$, and the set of the firm-level productivity. When the bilateral trade begins, each heterogeneous survivor in the autarky faces a larger market from I to $I^W = 2I$.

---

[18] In contrast, as in the appendix, the generalized nested CES preference can not guarantee a decreasing price of varieties in productivity. A change in sectoral share (S_z) can amplify the markup of the varieties for a higher productive firm. If this amplification affects the price of varieties, a higher price of varieties for a higher productive firm is possible when this amplification dominates the effect from the lower marginal cost (or, the higher relative productivity). The computational solution in the appendix demonstrates this case.

[19] When there is no update on the ZCP condition, both the firm-level market share (S_f) and the markup of varieties (μ_f) are fixed. In contrast, the international trade allowing head-to-head competition results in updating both the market size (I) and the ZCP condition. The combination of those two effects prevents the optimal range of varieties from expanding proportionally.

[20] In the extreme case of monopoly, the monopolist minimizes its range of variety despite the only firm serving in the market. As this monopolist already excise the highest markup on its existing products, it does not have an incentive to internalize the cannibalization effect from various products. It is consistent with an inverted U-shape relationship between the number of varieties and the firm's market share in Feenstra and Ma (2008).

As in Feenstra and Ma (2008), the environment of international trade is simplified so that the fixed cost is only for expanding a marginal range of variety (wK_1), which is K_1 when the wage is one. And the market is perfectly integrated between two symmetric economies. All survivors do not have to pay a fixed cost to introduce their product to a foreign market. Besides, the varieties can be freely traded without iceberg trade costs or duties. Moreover, the one-time fixed cost for producing a variety (K_0) is zero.

Unlike the settings of previous studies, including those in Feenstra and Ma (2008), the survivors in autarky are assumed to maintain their own productivity in the short-run after the markets are liberalized [21]. The survivors in autarky are treated as the only entrants at the time of market liberalization. It is because the inefficient entrants in autarky must be already expelled if those productivities are lower than the ZCP productivity. If M_e^W is the number of entrants at the time of international trade, M_e^W is double the number of survivors in the autarky $(M_e^W = 2M_o)$. The M_o firms from each economy should account for the existence of M_o competitors from the trade partner, and a pair of two entrants from two different economies share productivity. This head-to-head competitive environment promotes intensive competition in the integrated market.

In the environment described above, the M_e^W entrants should decide whether to produce or exit and set their optimality, such as the price and range of the varieties.

3.6.2 Short-run Equilibrium of Market Liberalization and Firm's Behavior

When the markets are liberalized, the intensive competition among M_e^W $(= 2M_o)$ entrants in the integrated market updates the ZCP condition. The updated ZCP condition indicates the ZCP productivity $(\varphi^{W,ZCP})$ and the market share of the marginal firm $(S^{W,ZCP})$ in the integrated market. Based on the updated ZCP condition of the global market, the short-run equilibrium of international trade can be defined as:

Definition. (Short-run Equilibrium of the Trade Liberalization) *Given the fixed firm-level productivity of $M_e^W = 2M_o$ firms surviving in autarky of the symmetric economies with the income of I, the M_e^W firms face head-to-head competition in the integrated market with I^W $(= 2I)$. The Bertrand-Nash equilibrium of trade liberalization*

[21] Feenstra and Ma (2008) assume that the environment of the integrated market is identical to the one in autarky. Both the integrated market and autarky share the number of entrants $(M_e^W = M_e)$. It is because Feenstra and Ma (2008) focus on the steady-state. Feenstra and Ma (2008) do not account for the fact that multiple firms with the same productivity can exist right after the market liberalization. Therefore, each firm may be exclusively assigned one new productivity, and the distribution of firm-level productivity among those firms is invariant regardless of international trade. It results in a constant markup and proportional expansion of variety range to market size as there is no update in the ZCP condition.

in the short-run consists of

1) a set of information about the ZCP condition in the heterogeneous sector: $\{S^{W,ZCP},\ \varphi^{W,ZCP}\}$
2) a vector of the optimality set by the M_o^W *surviving firms in the integrated market, including the price of a variety, the range of varieties, and the firm-level market share:* $\{P_{fi},\ N_f,\ S_f\}_{f=1}^{M_o^W}$*, and*
3) a sectoral price index within the integrated economy: P*,*

which solves both utility and profit maximization simultaneously in the integrated market.

Despite the doubled market size, the integrated market is more pro-competitive. Head-to-head competition in the integrated market updates the productivity distribution among the M_e^W entrants, and it implies that the productive firms must compete with each other. It renews the ZCP condition in the integrated market.

The intensive competition among the productive firms in the integrated market raises the ZCP productivity ($\varphi^{W,ZCP} > \varphi^{ZCP}$). According to the environment of international trade described in the former part, all M_e^W entrants have higher productivity than the ZCP productivity in autarky (φ^{ZCP}). In this environment of market liberalization with the intensive competition, negative expected profits may expel the least productive firms among the M_e^W entering ones. It implies that the range of productivity for those expelled firms is between φ^{ZCP} and $\varphi^{W,ZCP}$.

This higher ZCP productivity in the integrated market affects the firm-level market shares and markups of the varieties through various channels. As a direct effect, a higher ZCP productivity raises the inverse of relative productivities as in Equation (30), which results in the lower market share and markups of the varieties. In contrast, note that there are indirect changes in the market shares among the M_o^W survivors in the integrated market. Those survivors increase their range of variety when they take over the market shares of the dropouts. This expansion of the range of varieties results in indirectly increasing the market share as well as the cannibalization effect. Besides, the intensive competition among the survivors allows the most productive firms to take over the share of less productive survivors. Accompanying the fact that a productive firm has to account for head-to-head competition with the same productive firm from the trade partner, the direct impact may dominate the indirect effects. In sum, the updated ZCP productivity of market liberalization lowers the firm-level market share. Further, it lowers markups of the varieties among the survivors in the integrated market.

Those changes in market shares and markups update the optimal range of varieties. The update of the ZCP condition in the integrated market prevents the range of varieties from expanding proportionally to the market size.

In contrast, Feenstra and Ma (2008) argue that there is no update on the ZCP condition despite the market liberalization. In Feenstra and Ma (2008), the cannibalization effect is marginal to the firm's behavior compared to the impact of expanding the market size. Therefore, in the short-run equilibrium of international trade, the firm-level market share is not changed at the onset of international trade. Moreover, the survivors are allowed to expand the range of varieties proportional to the market size.

4. Quantitative Analysis

The heterogeneous firms are treated as discrete ones, and they are not small enough relative to the market. Therefore, this discreteness assumption prevents the application of the law of large numbers to the theoretical framework.

The major drawback of the granular heterogeneous firm environment is unavailability of closed-form solutions in equilibrium. According to Feenstra and Ma (2008), the discreteness in treating the heterogeneous firms fails to offer the closed-form of average productivity or revenue among the survivors in equilibrium. Therefore, quantitative analysis is an alternative way to identify the properties of the short-run equilibrium in international trade.

The quantitative analysis starts by replicating the results of Feenstra and Ma (2008) in which the market environment is invariant except for the market size. Then, given the relevant parameters and firm-level productivities in Feenstra and Ma (2008), the quantitative analysis follows the framework and procedure in Feenstra and Ma (2008) except for the environment of market liberalization applied to the framework. This means that the revised environment described in "The Environment of International Trade in the Short-run" of the previous section is applied to the framework. In the next step, the different productivity sets, the samples drawn from a Pareto distribution, offer the quantitative results in the short-run equilibrium of international trade.

4.1 Quantitative Results of the Short-run Equilibrium in Feenstra and Ma (2008)

As the Cobb-Douglas utility function at the top-level preference fixes the sectoral share as ρ, Feenstra and Ma (2008) defines the bilateral trade between two symmetric economies as only doubling the market size in the heterogeneous sector from $R\ (= \rho I = 1000)$ in autarky to $R^W\ (= \rho I^W = 2000)$.

In the equilibrium of international trade, the firm-level market share can be achieved by applying the ZCP condition of market liberalization to Equation (31). Equation (33) represents the firm-level market share in the integrated market.

$$S_f = S\left(\tau_f\right) = 1 - \frac{1}{\left\{\left(\sigma - 1 + \frac{1}{1-S^{W,ZCP}}\right)\left(\tau_f\right)^{-\frac{\sigma-1}{\sigma}} - (\sigma - 1)\right\}} \tag{33}$$

Application of this share to Equation (32) offers the profit of the survivors in the integrated market.

A quantitative analysis follows the mechanism of Feenstra and Ma (2008) to find the short-run equilibrium, which was explained in the previous part defining the short-run equilibrium. Moreover, in this analysis, the random sample of the

Table 1. Relative Ratio of a Firm's Productivity to the productivity of a threshold firm (τ_f)

Firm	Productivity	τ_f						
		$\varphi^{ZCP} =$ 0.202	$\varphi^{ZCP} =$ 0.224	$\varphi^{ZCP} =$ 0.2345	$\varphi^{ZCP} =$ 0.235	$\varphi^{ZCP} =$ 0.237	$\varphi^{ZCP} =$ 0.267	$\varphi^{ZCP} =$ 0.279
1	0.279	0.723	0.802	0.840	0.841	0.850	0.956	1
2	0.267	0.756	0.839	0.879	0.880	0.889	1	1.046
3	0.237	0.851	0.944	0.988	0.990	1	1.125	1.177
4	0.235	0.859	0.953	0.998	1	1.010	1.136	1.189
5	0.235	0.861	0.955	1	1.002	1.012	1.138	1.191
6	0.224	0.901	1	1.047	1.049	1.060	1.192	1.247
7	0.202	1	1.109	1.162	1.164	1.175	1.322	1.384

productivities utilized in Feenstra and Ma (2008) is maintained until the novel samples are introduced later.

First, given the set of productivity of M_e entrants randomly drawn, Equation (30) offers the ratio of marginal costs (or the inverse of relative productivities, ($\tau_f s$)) for each candidate of the ZCP productivity. Table 1 shows the computed inverse of relative productivities for each candidate of the ZCP productivity (φ^{ZCP}).

Second, given the arbitrary marginal firm's market share as the initial point (S^{ZCP}), Equation (31) gives the tentative firm-level market share as a function of the inverse of relative productivities ($\tau_f s$) [22]. According to the ZCP condition, the firm-level market share with inverse of relative productivities higher than one ($\tau_f > 1$) is zero ($S_f = 0$). Table 2 shows the tentative market share when the marginal firm's market share is assumed as 0.5% ($S^{ZCP} = 0.5\%$).

Third, starting with the highest marginal productivity candidate (e.g.,$\varphi^{ZCP} = 0.279$), the sequential non-linear programming provides the solution (S^{ZCP}) that satisfies the sum of the survivor's market share as unity for each ZCP productivity candidate. In the short-run equilibrium, among M_e entrants, there is the least productive firm whose market share is S^{ZCP} as the results from non-linear solver with the restriction of the sum of the survivors' market shares as unity. The non-linear programming fails to find a valid S^{ZCP} for the lower ZCP productivity candidates than φ^{ZCP} in the equilibrium. In the case of Feenstra and Ma (2008) with seven entrants, the short-run equilibrium indicates that five survivors are in the market, and the ZCP productivity that the marginal firm assigned is 0.2345 ($\varphi^{ZCP} = 0.2345$).

Last, the value in the short-run equilibrium derived in the previous step de-

[22] In this step, the marginal firm's market share (S^{ZCP}) may be set as any number between zero and one.

Table 2. Example of Firms' Market Shares by the Productivity Thresholds at Step 2

Firm	Productivity	Market Shares						
		$\varphi^{ZCP} =$ 0.202	$\varphi^{ZCP} =$ 0.224	$\varphi^{ZCP} =$ 0.2345	$\varphi^{ZCP} =$ 0.235	$\varphi^{ZCP} =$ 0.237	$\varphi^{ZCP} =$ 0.267	$\varphi^{ZCP} =$ 0.279
1	0.279	65.2%	54.9%	48.6%	48.3%	46.8%	19.1%	0.5%
2	0.267	61.2%	48.7%	40.8%	40.4%	38.4%	0.5%	0.0%
3	0.237	46.5%	23.2%	6.0%	5.2%	0.5%	0.0%	0.0%
4	0.235	44.9%	20.0%	1.4%	0.5%	0.0%	0.0%	0.0%
5	0.235	44.5%	19.4%	0.5%	0.0%	0.0%	0.0%	0.0%
6	0.224	35.4%	0.5%	0.0%	0.0%	0.0%	0.0%	0.0%
7	0.202	0.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Sum of Market Shares		298.2%	166.7%	97.4%	94.5%	85.7%	19.6%	0.5%

note. Initial market share of the threshold firm is assumed as 0.5%.

Table 3. Firms' Market Shares by the Productivity Thresholds

Firm	Productivity	Market Shares						
		$\varphi^{ZCP} =$ 0.279	$\varphi^{ZCP} =$ 0.267	$\varphi^{ZCP} =$ 0.237	$\varphi^{ZCP} =$ 0.235	$\varphi^{ZCP} =$ 0.2345	$\varphi^{ZCP} =$ 0.224	$\varphi^{ZCP} =$ 0.202
1	0.279	100.0%	53.3%	49.5%	49.0%	**48.9%**	Fail to find S^{ZCP}	Fail to find S^{ZCP}
2	0.267	0.0%	46.7%	41.9%	41.3%	**41.1%**		
3	0.237	0.0%	0.0%	8.6%	7.1%	**6.7%**		
4	0.235	0.0%	0.0%	0.0%	2.6%	**2.1%**		
5	0.235	0.0%	0.0%	0.0%	0.0%	**1.2%**		
6	0.224	0.0%	0.0%	0.0%	0.0%	**0.0%**		
7	0.202	0.0%	0.0%	0.0%	0.0%	**0.0%**		
$S^{ZCP} = S^{W,ZCP}$		100.0%	46.7%	8.6%	2.6%	**1.2%**		
Sum of Market Shares		100.0%	100.0%	100.0%	100.0%	**100.0%**		

termines the surviving firm's optimality in terms of the price and the range of variety in the market [23].

Feenstra and Ma (2008) assume that international trade still retains the number of entrants ($M_e = M_e^W = 7$) and an identical set of productivity as in autarky. Therefore, at the beginning of international trade, seven new firms enter the integrated market, and the productivity distribution across the entrants is the same as in autarky. The concept in Feenstra and Ma (2008) fails to account for the head-to-head competition between the two entrants assigned the same productivity. Therefore, the ZCP condition is stable regardless of the market liberalization ($\varphi^{W,ZCP} = \varphi^{ZCP}$). It implies that the market share of the five survivors ($S^f = S^{W,f}$) is the same in autarky, including the marginal firm ($S^{ZCP} = S^{W,ZCP}$).

As a result, for each corresponding firm-level productivity, the markup of the varieties is the same in autarky despite the market liberalization due to the stable market share. Also, as there is no update on the ZCP condition, the profits and the number of varieties are doubled in the bilateral trade between symmetric economies.

4.2 Quantitative Results with the Revised Environment of Market Liberalization and Comparison

In contrast to the claim of Feenstra and Ma (2008) that the number of entrants in the integrated market is the same as in autarky, this study accounts for the more rigorous environment in which the survivors in both symmetric economies maintain their own productivity and those firms become the entrants of the liberalized market in the short-run.

This new environment of international trade revises the distribution of firm-level productivity. Given the example in Feenstra and Ma (2008), as five firms in each economy served in autarky, the number of entrants in the integrated market is ten ($M_e^W = 2M_o = 10$). Figure 2 compares the environment of international trade to the one in Feenstra and Ma (2008) in terms of the distribution of productivity among entrants in the integrated market.

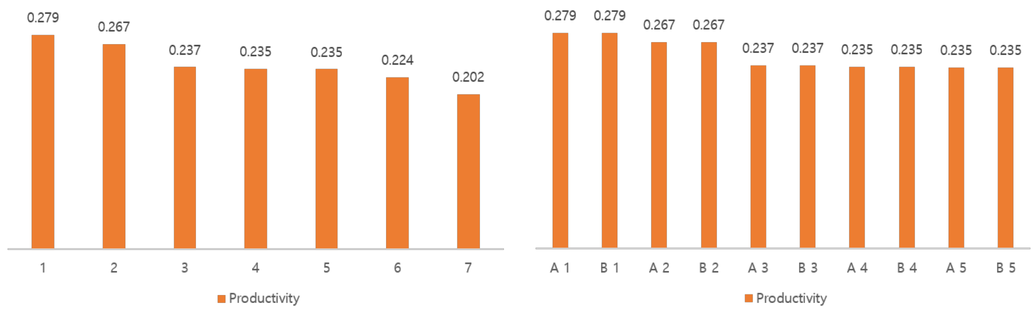
Although there is a change in the environment of international trade, other features representing the market are the same as in Feenstra and Ma (2008). The partial equilibrium analysis treats the market size defined as sectoral market size as an exogenous variable, 2,000 in the integrated market ($R^W = \rho I^W = 2000$).

Moreover, this quantitative analysis maintains the parameters. For example, the elasticity of substitution across varieties is six ($\sigma = 6$). Moreover, the fixed cost of marginally expanding the range of the varieties is five ($K_1 = 5$).

Under the revised environment of international trade, ten entrants, five pro-

[23] As in Feenstra and Ma (2008), the sectoral market size is assumed as 1,000 in autarky and 2,000 in the integrated market. Also, the fixed cost of extending a range of the varieties is set as five ($K_1 = 5$). The elasticity of substitution employed is six ($\sigma = 6$).

Figure 2. Productivity Distribution in the Integrated Market in Feenstra and Ma (2008) (Left) and Based on the Revised Environment of Market Liberalization (Right)



ductive firms from each economy, compete with each other. This intensive head-to-head competition among ten entrants may expel the least productive firms, which results in a higher ZCP productivity ($\varphi^{W,ZCP} > \varphi^{ZCP}$). For example, in the case of bilateral trade between the symmetric economy with the productivity set in Feenstra and Ma (2008), only four out of ten entrants survive shortly after the market liberalization. It means that the most and the second-most productive firms survive in each economy. The introduction of foreign competitors sharing the same productivity reduces the market share of the local survivors. It lowers markups of the varieties in international trade. Table 4 compares the firm-level market share in the short-run equilibrium of international trade.

The detailed information about the short-run equilibrium of international trade is shown in Table 5.

Intensive competition among the ten productive entrants in the integrated market prevents Firm 3˜5s in each economy from getting a non-negative profit in the short-run equilibrium of international trade. The ZCP productivity increases from $\varphi^{ZCP} = 0.202$ to $\varphi^{W,ZCP} = 0.267$, and it implies the concentration of the resources on the more productive firms in each economy. As in Figure 3, the bilateral trade updates the market share of the marginal firms from $S^{ZCP} = 1.2\%$ to $S^{W,ZCP} = 18.3\%$. Accompanying head-to-head competition among the productive firms, the update of the ZCP condition reduces the market share of the survivors in the integrated market. In this example, the market share of Firm 1 in each economy falls to 32%. Also, Firm 2s' market share is reduced to 18%, and the decreasing rate is higher compared to the change rate in Firm 1s' share. It suggests that the loss of market share is higher for less productive survivors because the introduction of a foreign productive competitor takes over the market share from which a less productive local firm benefits in autarky.

The profits of the survivors are affected by the market liberalization through the two channels: International trade expands the market size that increases the

Table 4. Summary - Comparison of Effect of the International Trade between Environments

Firm	Productivity	Revised Environment			Feenstra and Ma (2008)	
		Autarky	Trade	Active	Autarky = Trade	Active
1	0.279	48.9%	31.7%	Yes	48.9%	Yes
2	0.267	41.1%	18.3%	Yes	41.1%	Yes
3	0.235	6.7%	0.0%	No	6.7%	Yes
4	0.235	2.1%	0.0%	No	2.1%	Yes
5	0.202	1.2%	0.0%	No	1.2%	Yes
6	0.224	0.0%	Already Stop in Autarky		0.0%	No
7	0.202	0.0%	Already Stop in Autarky		0.0%	No
	S^{ZCP}	1.2%	18.3%		1.2%	

Table 5. Summary - Short-run Equilibrium of the International Trade in Each Economy

	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
Before	1	0.279	48.9%	67.12	14.05	4.98	1.39	Yes
	2	0.267	41.1%	42.80	12.27	5.02	1.34	Yes
	3	0.235	6.7%	0.79	2.21	5.12	1.21	Yes
	4	0.235	2.1%	0.08	0.71	5.13	1.20	Yes
	5	0.202	1.2%	0.03	0.41	5.13	1.20	Yes
	6	0.224	0.0%	0.00	0.00	-	-	No
	7	0.202	0.0%	0.00	0.00	-	-	No

	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
After	1	0.279	31.7%	45.47	19.61	4.63	1.29	Yes
	2	0.267	18.3%	13.20	11.77	4.66	1.24	Yes
	3	0.235	0.0%	0.00	0.00	-	-	No
	4	0.235	0.0%	0.00	0.00	-	-	No
	5	0.202	0.0%	0.00	0.00	-	-	No
	6	0.224			Already Stop in Autarky			
	7	0.202			Already Stop in Autarky			

survivors' profit; in contrast, the smaller market size from the head-to-head competition reduces the profits. Figure 4 describes the changes in profit in the short-run equilibrium of market liberalization. In this example, all survivors in the integrated market face a lower profit after the bilateral trade. It implies that the decreasing effect from a smaller market share dominates the increasing effect of larger market size. Firm 1s' profit falls to 45.47 while it is higher than half of the profit in autarky. Firm 2s' profit falls to a third of their profit in autarky, which relates to the loss of market share due to the operation of more productive foreign competitors.

Although the optimal range of varieties depends on the changes in the market size and the firm-level market share, the aggregate range of varieties in the market is expanded after the market liberalization. As in Equation (27) including the cannibalization effects that all the survivors internalize, a larger market size (R) raises the optimal range of varieties while the smaller market share from head-to-head competition prevents the survivors from expanding the range. In this example, the dominating effect from two factors is different depending on the firm's productivity. As in Figure 5, the more productive firms (Firm 1s) expand their range of variety to 19.6 while the less productive firms (Firm 2s) reduce the range to 11.8. Also, due to the existence of two opposite impacts, Firm 1s' expanding their range of varieties is not proportional to the market size. The introduction of productive competitors raises the aggregate range of variety to 62.76 in the integrated market despite the loss of varieties from the expelled survivors in autarky.

Figure 6 describes the changes in prices and markups of varieties in the short-run equilibrium of international trade. The market liberalization reduces both the variety-level price and markups for both survivors. In this example of the Cobb-Douglas utility function at top-level preference, the higher productive survivor enjoys a lower price and higher productivity, which is consistent with the previous studies. Also, the less productive firms struggle with a higher rate of decrease in the markups of varieties.

In sum, the quantitative analysis incorporating the revised environment offers more reasonable gains from trade. In the short-run, international trade reallocates resources to more productive firms, which intensifies head-to-head competition. Therefore, the survivors in the liberalized market set a lower markup and price of varieties. Although the firm-level range of varieties is determined by the changes in both market size and firm-level market share, international trade extends the aggregate range of variety, which results in the love of variety. As a result, each economy gains from trade.

4.3 Sampling for the New Sets of Firm-level Productivity

Under granularity, the specific sample of the productivity set is not able to represent a continuous-form of a Pareto distribution with the shape of γ and the lower support of φ^{min}. The short-run equilibrium is not stationary, so each

Figure 3. Short-run Equilibrium of the Market Share after the International Trade Based on the Revised Environment of Market Liberalization

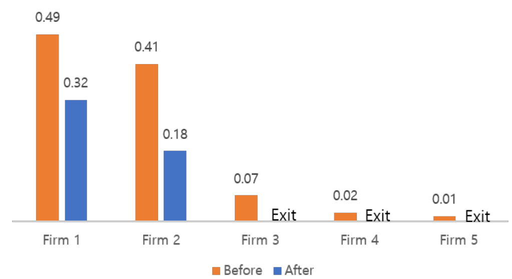


Figure 4. Short-run Changes of the Profits and the Threshold Productivity Based on the Revised Environment of Market Liberalization

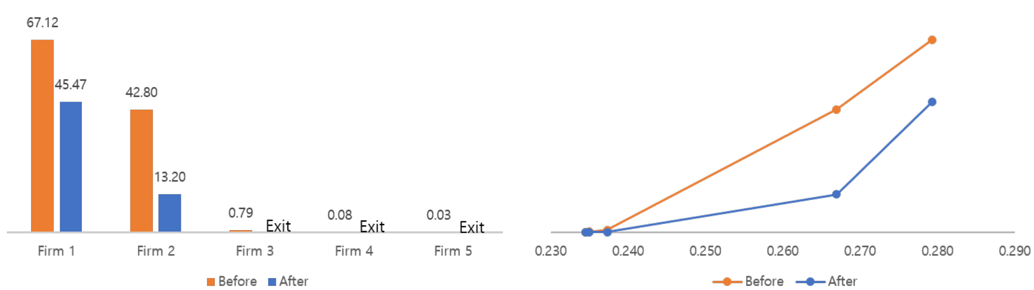


Figure 5. Short-run Changes of Firm's Variety (Left) and Total Number of Varieties in Each Economy (Right) based on the Revised Environment of Market Liberalization

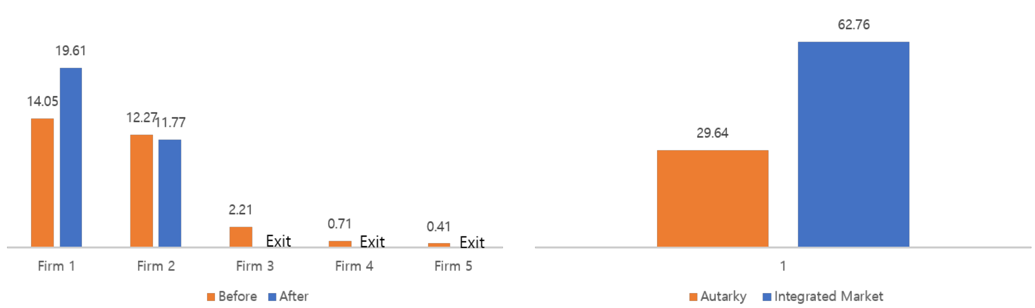
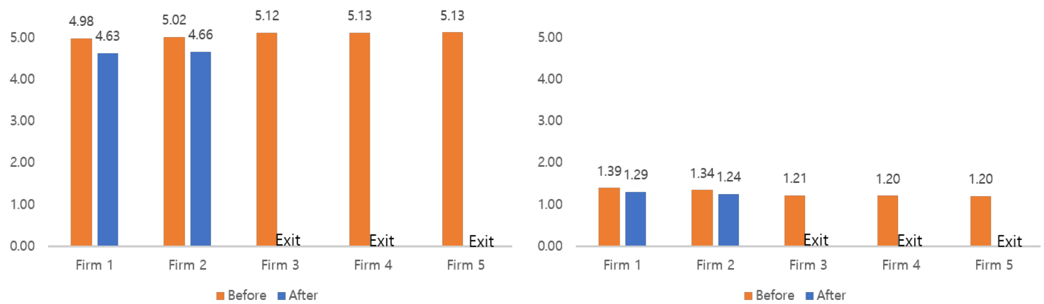


Figure 6. Short-run Changes of Prices (Left) and Markups (Right) of a Variety Based on the Revised Environment of Market Liberalization



quantitative analysis relies on the random productivity set randomly drawn.

From this part, this study conducts the quantitative analysis using three new productivity sets drawn by the clustered and systematic sampling method. The samples are generated from Pareto distribution with the same parameters $(\gamma,\ \varphi^{min})$, and each of the three new sets consist of productivity for ten entrants $(M_e = 10)$ in autarky.

Using the Pareto distribution with $\gamma = 5$ (Shape) and $\varphi^{min} = 0.2$ (Lower bound) as in Feenstra and Ma (2008), 500,000 random productivities are generated. After being sorted by decreasing order, the generated sample can be clustered into ten groups consisting of 50,000 productivities. For instance, the first group consists of the 50,000 highest productivities, while the tenth group does the 50,000 smallest productivities among the 500,000 aggregate samples.

A new productivity set is formed by ten productivities, each of which are achieved from each cluster. The first set (Example 1) is the productivity set in which each element is the highest within the clusters. A higher shape parameter $(\gamma > 1)$ in the Pareto distribution may render an extremely highest random productivity as an outlier. Therefore, the variance of this new set is highest among the three new productivity sets. In contrast, the elements of the second set (Example 2) are the smallest productivities in each group, which gives the smallest variance within the example. In the third set (Example 3), the elements are the median productivities in each cluster. Table 6 and Figure 7 show the summary of the three new sets of productivities for ten entrants.

Assuming the 500,000 samples form a continuous form of the Pareto distribution, Example 3 is the discrete sample set that could represent the continuous form of the distribution in terms of mean and variance. For Example 1, the mean and variance are higher due to the extreme element of 2.406. In the case of Example 2 set, the variance is lower than the one from the continuous distribution because the productivities are concentrated on the smallest support, $\varphi^{min} = 0.2$.

Table 6. Example Sets of Productivities for the Quantitative Analysis

Firm	Productivity		
	Example 1 (The highest in each cluster)	Example 2 (The smallest in each cluster)	Example 3 (The median in each cluster)
1	2.406	0.317	0.364
2	0.317	0.276	0.293
3	0.276	0.254	0.264
4	0.254	0.240	0.247
5	0.240	0.230	0.235
6	0.230	0.221	0.225
7	0.221	0.215	0.218
8	0.215	0.209	0.212
9	0.209	0.204	0.207
10	0.204	0.200	0.202
Variance	0.470	0.001	0.002
Mean	0.457	0.237	0.247

note. We assume $\gamma = 5$ (Shape) and $\varphi^{min} = 0.2$ (Lower bound). For the set of 50,000 productivities, the variance is 0.004 and the mean is 0.250.

Figure 7. Sample Productivities for the Quantitative Analysis Mapped on the Kernel of Pareto Distribution with $\gamma = 5$ (Shape) and $\varphi^{min} = 0.2$ (Lower bound)

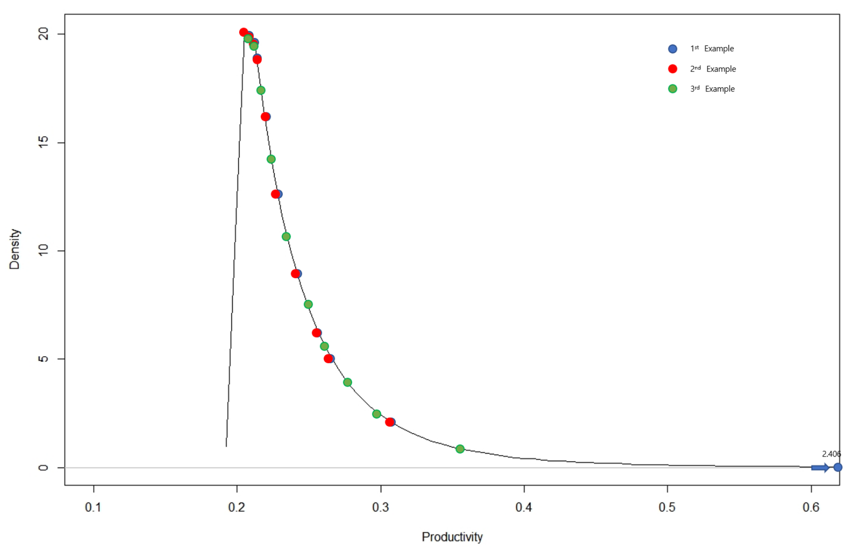


Table 7. Summary - Short-run Equilibrium of the International Trade in Each Economy (Example 1 Set of Productivities)

	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
Before	1	2.406	96.4%	787.0	5.9	2.72	6.54	Yes
	2	0.317	3.6%	0.2	1.2	3.81	1.21	Yes
	3	0.276	0.0%	0.0	0.0	-	-	No
	4	0.254	0.0%	0.0	0.0	-	-	No
	5	0.240	0.0%	0.0	0.0	-	-	No
	6	0.230	0.0%	0.0	0.0	-	-	No
	7	0.221	0.0%	0.0	0.0	-	-	No
	8	0.215	0.0%	0.0	0.0	-	-	No
	9	0.209	0.0%	0.0	0.0	-	-	No
	10	0.204	0.0%	0.0	0.0	-	-	No
After	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
	1	2.406	50.0%	142.9	28.6	0.58	1.40	Yes
	2	0.317	0.0%	0.0	0.0	-	-	No

4.4 Quantitative Analysis with the Three New Productivity Sets

Accompanying those three new sets of productivities formed by the clustered and systematic sampling method, the quantitative analysis employs the same parameters as in the previous analysis, such as the elasticity of substitution across varieties ($\sigma = 6$), and the fixed cost of marginally expanding the range of the varieties ($K_1 = 5$). Also, the wage is fixed as unity ($w = 1$).

From the competition among the entrants ($M_e = 10$) in autarky with the sectoral market size of 1,000 ($R = \rho I = 1000$), the short-run equilibrium suggests the number of survivors (M_o) in autarky in each economy and the corresponding productivities of the survivors ($\varphi_f s$). When the bilateral trade begins, those M_o survivors in autarky of each economy become the entrants of the integrated market with doubled market size ($R^W = \rho I^W = 2000$). Therefore, the number of entrants in the integrated market is the sum of the survivors in autarky ($M_e^W = 2M_o$). The competition among M_e^W offers the short-run equilibrium of international trade.

As shown in Tables 7 ˜9, the results from the quantitative analysis also support the pro-competitive liberalized market from the head-to-head competition when the new environment is employed into the computation model. In the short-run, international trade makes resources concentrated on the more productive firms so that the least productive survivors in autarky are expelled.

In each example, the pro-competitive market environment and the introduction

Table 8. Summary - Short-run Equilibrium of the International Trade in Each Economy (Example 2 Set of Productivities)

	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
Before	1	0.317	56.9%	102.7	15.5	4.62	1.46	Yes
	2	0.276	34.4%	27.6	10.5	4.73	1.30	Yes
	3	0.254	8.7%	1.4	2.9	4.79	1.22	Yes
	4	0.240	0.0%	0.0	0.0	-	-	No
	5	0.230	0.0%	0.0	0.0	-	-	No
	6	0.221	0.0%	0.0	0.0	-	-	No
	7	0.215	0.0%	0.0	0.0	-	-	No
	8	0.209	0.0%	0.0	0.0	-	-	No
	9	0.204	0.0%	0.0	0.0	-	-	No
	10	0.200	0.0%	0.0	0.0	-	-	No
	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
After	1	0.317	44.4%	104.6	26.14	4.29	1.36	Yes
	2	0.276	5.6%	1.1	3.67	4.39	1.21	Yes
	3	0.254	0.0%	0.0	0.00	-	-	No

Table 9. Summary - Short-run Equilibrium of the International Trade in Each Economy (Example 3 Set of Productivities)

	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
Before	1	0.364	64.9%	153.1	16.54	4.31	1.57	Yes
	2	0.293	35.0%	28.8	10.70	4.47	1.31	Yes
	3	0.264	0.1%	0.0	0.03	4.55	1.20	Yes
	4	0.247	0.0%	0.0	0.00	-	-	No
	5	0.235	0.0%	0.0	0.00	-	-	No
	6	0.225	0.0%	0.0	0.00	-	-	No
	7	0.218	0.0%	0.0	0.00	-	-	No
	8	0.212	0.0%	0.0	0.00	-	-	No
	9	0.207	0.0%	0.0	0.00	-	-	No
	10	0.202	0.0%	0.0	0.00	-	-	No
	Firm	Productivity	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active
After	1	0.364	50.0%	142.86	28.57	3.84	1.40	Yes
	2	0.293	0.0%	0.00	0.00	-	-	No
	3	0.264	0.0%	0.00	0.00	-	-	No

of foreign productive competitors reduce the firm-level market share, resulting in the lower markups and prices of varieties. While the optimal range of varieties is determined by the interaction between the changes in market size and firm-level market share, the aggregate range of variety is extended, which results in gains from trade. For example, according to the results from Example 2, the productive firms set a lower price and higher markup of varieties than the less efficient competitors. Moreover, the rate of a falling markup is smaller for the most productive firms.

5. Conclusion

This study demonstrates the lowering of markup in the short-run equilibrium of international trade when discrete firms are heterogeneous and allowed to produce multiple varieties. This paper departs from the environment defined in Feenstra and Ma (2008). Therefore, the survivors in autarky are assumed to maintain their productivity in the short-run at the onset of international trade. Since the least productive firms in autarky are already expelled from the local market, those survivors in autarky are considered as the entrants in the liberalized market.

In setting the framework, a nested constant elasticity of substitution (CES) demand system and a monopolistic competition across the heterogeneous firms are employed. With an assumption of symmetric technology across the varieties within firms, the firm's profit maximization offers analytic forms of the optimal price and range of varieties. Incorporating the ZCP condition, the market share of the survivors suggests how the survivor's optimality is related to the firm-level market share. Since granularity prevents the derivation of closed-forms of the firm-level optimality, this study utilizes a quantitative analysis incorporating the revised environment of international trade. The results from the quantitative analysis show the effect of international trade on the markups, the number of varieties, prices, and profits.

The results suggest that the pro-competitive market environment and the introduction of foreign productive competitors reduce the firm-level market share, which results in the lower markups and prices of varieties. While the optimal range of varieties is determined by the interaction between the changes in market size and firm-level market share, the aggregate range of variety is extended, which results in gains from trade.

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Appendix. Case of Generalized Demand System

The Cobb-Douglas utility function at the top-tier preference is a particular case of the general form of CES preference at this level. It offers three benefits to assume the Cobb-Douglas because the sectoral may be treated as an exogenous parameter. Firstly, the optimal allocations set by the survivors are independent of the sectoral share. Secondly, the Cobb-Douglas utility function reduces the computational burden in quantitative analysis. Finally, the Cobb-Douglas utility assumption for the aggregate demand fits the spirit of the granularity; a firm has market power within the sector while it is small in the aggregate economy.

In the appendix, this paper relaxes the strict assumption of the Cobb-Douglas utility function at the top-tier preference. Given the previous assumptions and parameters across the symmetric economies, a generalized CES demand system is introduced to replace the Cobb-Douglas utility function. It implies that the sectoral share is not exogenous anymore as in the current trade studies but determined endogenously. In the generalized CES demand system, a firm's choice may affect the aggregate economy. The introduction of this generalized demand system may have implications for analyzing the oligopoly market.

The generalized case starts with specifying the CES preference for the aggregate demand. The demonstration of a firm's optimality defines the equilibrium, including the endogenous sectoral share within the aggregate economy. Like the Cobb-Douglas set up, this paper relies on the quantitative analysis as the firms are not continuous. The computational solution offers intensive competition when the bilateral trade begins, resulting in gains from trade.

Demand

As mentioned in the previous section, international trade is the bilateral trade between symmetric economies. Therefore, both economies have the same amount of labor size (L) and wage as unity $(w = 1)$. The labor income stands for the market size I, which is different from the previous section in which the heterogeneous sector size is utilized as the market size. Figure A.1 outlines a generalized demand system.

As in Section 3, two sectoral demands form the aggregate demand. In the generalized case, there are two types of elasticities of substitution. η denotes the elasticity of substitution across the sectoral composite goods, and $\eta > 1$ while $\eta < \sigma$. Like the Cobb-Douglas utility at the top-tier demand, σ (> 1) measures the elasticity of substitution across both the firm-level and variety demands. Given the budget constraint of Equation (4), the utility maximization offers 1) price index, 2) Marshallian demand function, and 3) expenditure share at each level of demand. Table A.1 summarizes the demand system of this framework.

Unlike the Cobb-Douglas utility at the top-tier level, a firm is large from the perspective of the aggregate economy. Therefore, firm-level choices affect the aggregate economy. The sectoral share (Ψ) is endogenous as a function of the

Figure A.1. A Generalized Nested CES Demand System

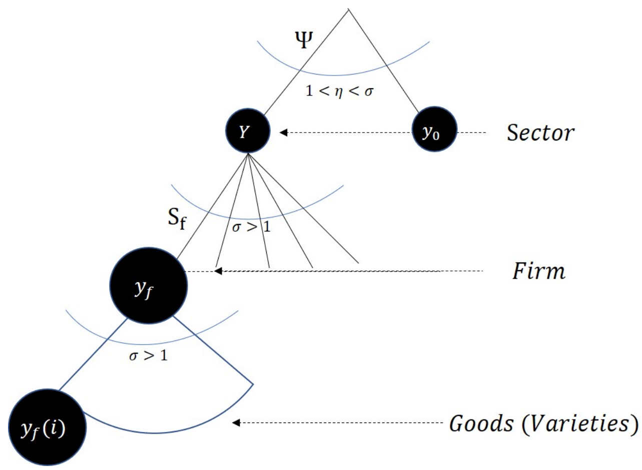

Table A.1. Demand System of a General Form of CES Preference

Level	Demand	Price Index
Aggregate	$U = \left[(y_0)^{\frac{\eta-1}{\eta}} + (Y)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$	$\mathbb{P} = \left[P_0^{1-\eta} + (P)^{1-\eta} \right]^{\frac{1}{1-\eta}}$
Sectoral	$Y = \left[\sum_{f \in \Omega} \left(y_f \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$	$P = \left[\sum_{f \in \Omega} \left(P_f \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$
Firm	$y_f = \left[\int_{i \in \Omega_f} \left(y_f(i) \right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$	$P_f = \left[\int_{i \in \Omega_f} \left(P_f(i) \right)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$
Variety	$y_f(i)$	$P_f(i)$
Variety (Symmetric Technology)	y_{fi}	P_{fi}

Level	Marshallian Demand	Expenditure Share
Aggregate	$U = \frac{wL}{\mathbb{P}}$	-
Sectoral	$Y = I \frac{P^{1-\eta}}{\mathbb{P}^{1-\eta}} \frac{1}{P}$	$\Psi = \frac{P^{1-\eta}}{\mathbb{P}^{1-\eta}} = \frac{P^{1-\eta}}{P_0^{1-\eta} + P^{1-\eta}}$
Firm	$y_f = I \frac{P^{1-\eta}}{\mathbb{P}^{1-\eta}} \frac{P_f^{1-\sigma}}{P^{1-\sigma}} \frac{1}{P}$	$S_f = \frac{P_f^{1-\sigma}}{P^{1-\sigma}} = \frac{\int_{i \in \Omega_f} \left\{ P_f(i) y_f(i) \right\} di}{\sum_{g \in \Omega} \left[\int_{j \in \Omega_g} \left\{ P_g(j) y_g(j) \right\} dj \right]}$
Variety	$y_f(i) = I \frac{P^{1-\eta}}{\mathbb{P}^{1-\eta}} \frac{P_f^{1-\sigma}}{P^{1-\sigma}} \frac{\left\{ P_f(i) \right\}^{1-\sigma}}{P_f^{1-\sigma}} \frac{1}{P_f(i)}$	-
Variety (Symmetric Technology)	$y_{fi} = I \frac{P^{1-\eta}}{\mathbb{P}^{1-\eta}} \frac{P_f^{1-\sigma}}{P^{1-\sigma}} \frac{1}{N_f} \frac{1}{P_{fi}}$	-

price index of sectoral demand (P). Also, the Ψ and P relate to the price index for the aggregate demand $(\mathbb{P})$. An example of firm-level decision making that may have an impact on the economies is Samsung-Korea.

Technologies

Technologies follow the details described in Section 3. The homogeneous sector relies on CRS technology and produces a numeraire using a unit of labor. The M_e heterogeneous entrants are assigned a random productivity according to Equation (15). With the total cost as Equation (17), the profit function for a firm in the heterogeneous sector is described as Equation (18).

In the monopolistic competition among the discrete firms, each firm chooses the optimal range of varieties (N_f) and the price of varieties (P_{fi}) that maximizes the profits. The optimal pricing rule is the same form as in Equation (20), except the demand elasticity for a firm-level good or a variety (ϵ_f). The optimal pricing rule can be rewritten as:

$$P_{fi} = \frac{w}{\varphi_f}\mu_f = \frac{w}{\varphi_f}\frac{\epsilon_f}{\epsilon_f - 1} \; \forall i \in \Omega_f \tag{A.1}$$

where $\epsilon_f(= \sigma + (\eta - \sigma)S_f + (1-\eta)S_f\Psi)$ is the elasticity of substitution in this case. When the sectoral goods are substituted for each other $(\eta > 1)$, an increase in sectoral share (Ψ) reduces the demand elasticity (ϵ_f) if the firm's market share S_f is not changed. It relates to a higher markup of the firm's varieties.

In the optimal condition for the range of varieties, the cannibalization effect can be decomposed into direct and indirect effects, such as:

$$\begin{aligned} &\underbrace{\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi}}_{Direct\ marginal\ benefit} + \underbrace{\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi} \frac{1-\eta}{1-\sigma} S_f}_{Indirect\ marginal\ benefit} \\ &- \underbrace{\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi} S_f}_{Direct\ cannibalization} - \underbrace{\left(P_{fi} - \frac{w}{\varphi_f}\right) y_{fi} \frac{1-\eta}{1-\sigma} S_f \Psi}_{Indirect\ cannibalization} = wK_1 \ . \end{aligned} \tag{A.2}$$

In Equation (A.2), direct cannibalization relates to the reduced revenue within the heterogeneous sector. Besides, the indirect cannibalization reduces the firm's revenue through the interaction with the aggregate price index $(\mathbb{P})$ due to a change in the endogenous sectoral share (Ψ) [24].

[24] When the sectoral goods are complementary for each other $(0 < \eta < 1)$, a sign of the net indirect effect is negative, which means the indirect cannibalization effect dominates the indirect marginal benefit from extending a range of the varieties. In sum, the net marginal benefit from extending the variety range is ambiguous at some N_fs as the sign depends on the parameters.

Following the procedure between Equations (24) and (26) suggests the optimal number of varieties that active firms produce in the market, such as:

$$N_f = \frac{I\Psi}{K_1}\frac{1}{\sigma-1}\frac{\epsilon_f-1}{\epsilon_f}S_f > 0\,. \tag{A.3}$$

From Equation (A.3), a change in the heterogeneous sectoral share (Ψ) affects the range of varieties. Unlike the Cobb-Douglas utility at the top-tier preference, a change in the market environment affects the number of variety (N_f) through three channels: 1) the revised market environment alters the firm's market share (S_f), 2) the market share associates to an inverse of the markup, and 3) the sectoral share (Ψ) is also changed as the firm is large.

ZCP condition

As discussed in Section 3, the ZCP condition offers information to reach the short-run equilibrium in which survivors do not struggle with a negative profit. Given the productivity set for the M_e entrants, the Bertrand competition gives the M_o survivors to have non-negative profits. In contrast, the inclusion of the $M_o + 1^{\,th}$ productive firm into the survivors results in a negative profit for at least one survivor. If the ZCP condition is satisfied, M_o gives the maximum number of survivors with non-negative profits in the market. As a result, M_o^{th} productivity is the ZCP productivity when we sort firms' productivity by descending order, such as $\varphi_1 \geq \varphi_2 \geq \cdots \geq \varphi_{M_e}$. The zero-cutoff profit (ZCP) condition is:

$$\begin{cases} \Pi_f \geq 0 \; and \; S_f > 0 & for\; f \;=\; 1, \cdots, M_o \\ \Pi_f = 0 \; and \; S_f = 0 & for\; f \;=\; M_o+1, \cdots, M_e \end{cases} \tag{A.4}$$

where Π_fs and S_fs are the profits and the market share of the heterogeneous firm f surviving in the market.

In the generalized CES demand system, the endogenous sectoral share (Ψ) in equilibrium relates to the threshold condition. Therefore, it is complicated to obtain the ZCP condition. For example, the sectoral market share (Ψ) also affects the market share of the marginal firm S^{ZCP}. Moreover, this sectoral share (Ψ) is associated with the firm's optimality in equilibrium, such as the optimal price (P_f) and range of varieties (N_f). Therefore, in contrast to the Cobb-Douglas case in which the equilibrium can follow the ZCP condition, the elements of the ZCP condition $(\varphi^{ZCP},\, S^{ZCP})$ and the equilibrium, including the sectoral share (Ψ), are simultaneously determined.

Short-run Equilibrium and Mechanism

Due to simultaneity in determining both the ZCP condition and equilibrium, the short-run equilibrium should be redefined as:

Definition. (Short-run Equilibrium - A Generalized CES Utility Function) *Given the set of productivity for M_e entrants in an economy with income (I), the Bertrand-Nash equilibrium consists of*

1) a set of information about the ZCP condition in the heterogeneous sector: $\{S^{ZCP},\ \varphi^{ZCP}\}$,
2) a vector of the optimality set by M_o surviving firms, including the price of a variety, the range of varieties, and the firm-level market share: $\{P_{fi},\ N_f,\ S_f\}_{f=1}^{M_o}$,
3) a set of a sectoral price index and a sector-level expenditure share within an economy: $\{P, \Psi\}$, and
4) an aggregate price index $\mathbb{P}$,

which solves both utility and profit maximization simultaneously.

Because of this simultaneity, the generalized CES demand system also requires extra steps to determine both the ZCP condition and equilibrium. The mechanism starts with introducing an inverse of the relative productivity of a survivor (τ_f) as in Equations (29) and (30). Then, Equation (A.5) suggests the firm's market share as a function of the inverse of the relative productivity (τ_f), the market share of the marginal firm (S^{ZCP}), and the sectoral share (Ψ):

$$\begin{aligned} S_f &= S\left(\tau_f\right) \\ &= \frac{1}{(\sigma-\eta)+(\eta-1)\Psi}\left[\sigma-\frac{\sigma+(\eta-\sigma)S^{ZCP}+(1-\eta)\Psi S^{ZCP}}{1+\{\sigma+(\eta-\sigma)S^{ZCP}+(1-\eta)\Psi S^{ZCP}\}\left\{\left(\tau_f\right)^{-\frac{\sigma-1}{\sigma}}-1\right\}}\right] . \end{aligned} \tag{A.5}$$

Among the M_e entered firms, the firms with $\tau_f > 1$ are less productive than the marginal firm, so they suspend producing goods $(S_f = 0\ if\ \tau_f > 1)$. Given arbitrary S^{ZCP} and Ψ, the profit of each survivor can be described as:

$$\begin{aligned} \Pi_f &= \Pi\left(\tau_f\right) \\ &= \frac{\left(S\left(\tau_f\right)\right)^2}{\Psi S\left(\tau_f\right)+(\sigma-\eta)\left(1-S\left(\tau_f\right)\right)+\eta\left(1-\Psi S\left(\tau_f\right)\right)}\frac{(\sigma-\eta)+(\eta-1)\Psi}{\sigma-1}I\Psi \end{aligned} \tag{A.6}$$

when the fixed cost for inaugurating production is assumed as zero $(K_1 = 0)$.

Including the market-clear condition, there are two constraints that the mechanism for the generalized CES demand system must consider. The first constraint is the market-clear condition associated with the market share of the marginal firm (S^{ZCP}): the sum of the survivors' market shares should be equal to one. The second constraint is regarding the equivalency of the sectoral market share to the computed one: $\Psi = \frac{P^{1-\eta}}{1+P^{1-\eta}}$.

Accounting for those two constraints, the non-linear programming suggests two solutions $(S^{ZCP},\ \Psi)$ satisfying both the ZCP condition and the definition of equilibrium.

Market Liberalization

As in the example with the Cobb-Douglas utility function at the top-tier preference, the revised environment of market liberalization is adopted to define the equilibrium of international trade. Two symmetric economies, with identical income (I) and firm-level productivity set, begin bilateral trade, which forms the liberalized (or integrated) market with a doubled market size $(I^W = 2I)$. The bilateral trade scenario is simplified by assuming that there is no additional fixed cost for exporting goods, fixed cost for the initial production $(K_0 = 0)$, iceberg trade cost, nor duties such as tariffs.

Regarding the entrants, the M_o survivors in autarky are assumed to maintain their original productivity when they enter the liberalized market. As a result, the $M_e^W (= 2M_o)$ entrants determine whether to produce in the integrated market. As it is bilateral trade among the two symmetric economies, two firms from different regions share productivity.

Moreover, the sectoral share within the economy (Ψ) and the productivity of marginal firm (S^{ZCP}) are assumed to have less impact on the survivor's market share S_f than the updated inverse of the relative productivity of international trade. First, the changes in Ψ and S^{ZCP} depend on the random set of productivity among the survivors. Secondly, the results from the quantitative analysis support this assumption.

The market liberalization between the two symmetric economies updates both the ZCP condition and the short-run equilibrium simultaneously, which determines the optimality of the survivors in the integrated market. The updated ZCP condition consists of the ZCP productivity $(\varphi^{W,ZCP})$ and the market share of the marginal firm $(S^{W,ZCP})$ in the liberalized market. Also, the updated short-run equilibrium offers the revised sectoral share of expenditure (Ψ^W). In the general case, the short-run equilibrium of the trade liberalization is defined as:

Definition. (Short-run Equilibrium of the Trade Liberalization - A Generalized CES Utility Function) *Given the fixed firm-level productivity of $M_e^W = 2M_o$ firms surviving in autarky of the symmetric economies with the income of I, the M_e^W firms face head-to-head competition in the integrated market with $I^W (= 2I)$. The Bertrand-Nash equilibrium of trade liberalization in the short-run consists of*

1) a set of information about the ZCP condition in the heterogeneous sector: $\{S^{W,ZCP}, \varphi^{W,ZCP}\}$,
2) a vector of the optimality set by the M_o^W surviving firms in the integrated market, including the price of a variety, the range of varieties, and the firm-level market share: $\{P_{fi}, N_f, S_f\}_{f=1}^{M_o}$,
3) a set of a sectoral price index and a sector-level expenditure share within an economy: $\{P, \Psi^W\}$, and
4) an aggregate price index $\mathbb{P}$,

which solves both utility and profit maximization simultaneously.

Quantitative Analysis

Quantitative analysis numerically demonstrates the partial equilibrium of international trade in the short-run. The analysis utilizes the three new productivity sets already drawn in Section 4 through the clustered and systematic sampling method. Moreover, most of the analysis settings in the generalized case incorporate those in the Cobb-Douglas utility function at the top-tier preference [25].

The distinctive features in the generalized case are the indication of the market size and the introduction of the elasticity of substitution across the sectoral composite goods. As the sectoral market share (Ψ) is endogenous, international trade doubles not the sectoral market size but the aggregate market size. Therefore, the sectoral market size cannot represent the market size in an economy. Instead, the generalized case accounts for the aggregate market size measured by labor income. In this quantitative analysis, the employed market sizes are $I = 2000$ $(= wL)$ in autarky and $I^W = 4000$ $(= 2I = 2wL)$ in the integrated market.

Moreover, the sector-level elasticity of substitution is assumed as two ($\eta = 2$). It is reasonable in that the sectoral elasticity of substitution is usually less than the firm-level elasticity of substitution within a sector.

Tables A.2~A.4 report the short-run equilibrium of international trade. Like the Cobb-Douglas utility function case, the liberalized market is more pro-competitive. Head-to-head competition reallocates resources to more productive firms among the M_e^W entrants, which deprives monopolistic market power of the most productive firms (Example 1) or expels the less productive firms from the liberalized market (Examples 2 and 3) in the short-run. As a result, the quantitative analysis offers that only the most productive firms survive in the liberalized market.

The market liberalization results in a higher expenditure share in the heterogeneous sector (Ψ). The head-to-head competition lowers the optimal price of a variety, which offers a lower sectoral price index. As the sectoral goods are substitutes for each other with a higher elasticity of substitution ($\eta > 1$), a lower sectoral price index suggests increases in both market size and share in the heterogeneous sector. From Examples 2 and 3, sectoral shares increase from 26.9% to 32.8% and from 27.7% to 36.3%, respectively.

In the liberalized market, the survivors reduce markup and price of a variety, and the aggregate range of variety is expanded. Notably, in Example 1, the pro-competitive market makes the former monopolists (Firm 1s) expand the range of variety to maintain market power and revenue in the integrated market.

In contrast, there is a difference from previous studies: when there is a higher

[25] As in Section 4. The fixed cost for initiating production is zero ($K_0 = 0$), and the cost for marginally expanding the range of varieties is five ($K_1 = 5$). The elasticity of substitution across the firm-level demands is assumed as six ($\sigma = 6$).

elasticity of substitution across the sectoral composite goods ($\eta > 1$), a higher productive firm may set a 'higher' price and markup. A higher sectoral share (Ψ) in the liberalized market lowers the firm-level demand elasticity (ϵ_f). Therefore, productive firms set a higher markup and price when the difference in productivity (or marginal cost) is less than the difference in markups among the survivors.

Table A.2. Summary - Short-run Equilibrium of the International Trade in Each Economy (CES, Example 1 Set of Productivities)

	Firm	Prod.	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active	Sectoral Share	Sectoral Price Index
Before	1	2.406	100.0%	699.8	13.7	1.35	3.25	Yes	55.5%	0.80
	2	0.317	0.0%	0.0	0.0	-	-	No		
	3	0.276	0.0%	0.0	0.0	-	-	No		
	4	0.254	0.0%	0.0	0.0	-	-	No		
	5	0.240	0.0%	0.0	0.0	-	-	No		
	6	0.230	0.0%	0.0	0.0	-	-	No		
	7	0.221	0.0%	0.0	0.0	-	-	No		
	8	0.215	0.0%	0.0	0.0	-	-	No		
	9	0.209	0.0%	0.0	0.0	-	-	No		
	10	0.204	0.0%	0.0	0.0	-	-	No		
After	Firm	Prod.	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active	Sectoral Share	Sectoral Price Index
	1	2.406	50.0%	217.1	46.8	0.58	1.39	Yes	81.1%	0.23

Table A.3. Summary - Short-run Equilibrium of the International Trade in Each Economy (CES, Example 2 Set of Productivities)

	Firm	Prod.	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active	Sectoral Share	Sectoral Price Index
Before	1	0.317	70.3%	75.8	10.9	4.73	1.50	Yes	26.9%	2.72
	2	0.276	29.7%	8.5	3.4	4.59	1.27	Yes		
	3	0.254	0.0%	0.0	0.0	-	-	No		
	4	0.240	0.0%	0.0	0.0	-	-	No		
	5	0.230	0.0%	0.0	0.0	-	-	No		
	6	0.221	0.0%	0.0	0.0	-	-	No		
	7	0.215	0.0%	0.0	0.0	-	-	No		
	8	0.209	0.0%	0.0	0.0	-	-	No		
	9	0.204	0.0%	0.0	0.0	-	-	No		
	10	0.200	0.0%	0.0	0.0	-	-	No		
After	Firm	Prod.	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active	Sectoral Share	Sectoral Price Index
	1	0.317	50.0%	74.0	19.40	4.27	1.35	Yes	32.8%	2.05
	2	0.276	0.0%	0.0	0.00	-	-	No		

Table A.4. Summary - Short-run Equilibrium of the International Trade in Each Economy (CES, Example 3 Set of Productivities)

	Firm	Prod.	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active	Sectoral Share	Sectoral Price Index
Before	1	0.364	80.9%	122.2	10.87	4.53	1.65	Yes	27.7%	2.61
	2	0.293	19.1%	3.3	3.41	4.24	1.24	Yes		
	3	0.264	0.0%	0.0	0.00	-	-	No		
	4	0.247	0.0%	0.0	0.00	-	-	No		
	5	0.235	0.0%	0.0	0.00	-	-	No		
	6	0.225	0.0%	0.0	0.00	-	-	No		
	7	0.218	0.0%	0.0	0.00	-	-	No		
	8	0.212	0.0%	0.0	0.00	-	-	No		
	9	0.207	0.0%	0.0	0.00	-	-	No		
	10	0.202	0.0%	0.0	0.00	-	-	No		
After	Firm	Prod.	Share	Profit	Number of Varieties	Price of a Variety	Markup	Active	Sectoral Share	Sectoral Price Index
	1	0.364	50.0%	82.95	21.43	3.72	1.35	Yes	36.3%	1.75
	2	0.293	0.0%	0.00	0.00	-	-	No		