Does Unobservable Heterogeneity Matter for Portfolio-Based Asset Pricing Tests?*

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Abstract

We show that portfolio sorts, as widely used in empirical asset pricing, tend to misattribute cross-sectional return predictability to the firm characteristic underlying the sort. Such misattribution arises if the sorting variable correlates with a firm-specific effect capturing unobservable heterogeneity across firms. We propose a new, firm-level regression approach that can reproduce the results from standard portfolio sorts. Besides, our method handles multivariate firm characteristics and, if firm fixed effects are included, is robust to misattributing cross-sectional return predictability. Our empirical results confirm that portfolio sorts have limited power in detecting abnormal returns: Several characteristics-based factors lose their predictive power when we control for unobservable heterogeneity across firms.

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1. Introduction

Portfolio sorts are a standard research methodology in empirical asset pricing. The estimation procedure consists of two steps. In the first step, individual assets are sorted into quintile or decile portfolios based on a certain firm characteristic. The second step regresses the return spread between the top portfolio and the bottom portfolio on a set of factors (e.g., the three Fama-French factors). If the regression intercept, or "alpha", of the second step time-series regression is statistically significant, this is considered evidence that the firm characteristic underlying the portfolio sort predicts the cross-section of average stock returns. This type of analysis, to which we henceforth refer to as the "portfolio sorts approach", was introduced by Black, Jensen, and Scholes (1972), and has been widely used in empirical asset pricing and other areas of empirical finance research ever since.[1]

In this paper, we contribute to the sparse literature analyzing the econometric properties of the portfolio sorts approach.[2] Our analysis builds on a key assumption of the portfolio sorts approach, which requires that firm-specific factors do not exert any meaningful impact on asset pricing tests. This assumption seems innocuous and intuitive since it is well known that a broadly diversified portfolio only bears systematic risk, that is non-diversifiable and priced, but no unsystematic (or firm-specific) risk, which is diversifiable and thus not priced. Our paper challenges this assumption. We formally show and empirically validate that unobservable heterogeneity across firms, which is unsystematic by nature, may significantly impact the results of empirical asset pricing tests based on portfolio sorts. Specifically, we demonstrate that the portfolio sorts approach is vulnerable to misattributing cross-sectional return predictability to the firm characteristic underlying the sort.

To gain some intuition on why the portfolio sorts approach is prone to suffer from such misattribution, we casually observe that the stellar performance of firms like Google or Apple is unlikely to be the result of a single firm characteristic X. It is more likely that such outstanding performance is due to a successful combination of a multitude of observable and potentially *unobservable* firm characteristics. This, however, stands at odds with the portfolio sorts approach for at least two reasons. First, the portfolio sorts approach assumes by construction that the alpha of the top-versus-bottom portfolio is fully

[1] Recent examples of empirical asset pricing studies using the portfolio sorts approach are Novy-Marx (2013), Frazzini and Pedersen (2014), Ball, Gerakos, Linnainmaa, and Nikolaev (2015), Fama and French (2015), Gerakos and Linnainmaa (2017), Bali, Brown, and Tang (2017), Weber (2018), and van Binsbergen and Opp (2019). The portfolio sorts approach is also widely used in other areas of empirical finance research, such as household finance (e.g., Barber and Odean, 2000, 2001; Ivkovic, Sialm, and Weisbenner, 2008; Seasholes and Zhu, 2010; Korniotis and Kumar, 2013), research on insider trading (e.g., Jeng, Metrick, and Zeckhauser, 2003), and studies analyzing the performance of mutual funds and hedge funds (e.g., Kacperczyk, Sialm, and Zheng, 2008; Fung, Hsieh, Naik, and Ramadorai, 2008).

[2] Research covering methodological and econometric aspects of the portfolio sorts approach includes Fama (1998), Lyon, Barber, and Tsai (1999), Loughran and Ritter (2000), Kothari and Warner (2008), and Cochrane (2011). More recently, Cattaneo, Crump, Farrell, and Schaumburg (2020) analyze the statistical properties of portfolio sorts and apply a nonparametric estimator to obtain optimal choices for the number of portfolios.

attributable to the *observable* firm characteristic(s) underlying the sort. Second, the portfolio sorts approach assumes that all the sorted portfolios are *well diversified* such that any firm-specific factors are purely random and unsystematic. While this assumption is admissible for the *market portfolio*, it turns out to be problematic for individual quintile or decile portfolios resulting from a standard portfolio sorts approach. Specifically, a certain firm characteristic X, which effectively does not predict the cross-section of average stock returns, may accidentally pack a few stocks with particularly good (poor) performance into the top (bottom) portfolio. If this is the case, the top portfolio may well outperform the bottom portfolio. However, the difference between the top portfolio's alpha and the bottom portfolio's alpha is then the result of unobservable heterogeneity across firms and is thus unrelated to the firm characteristic underlying the sort. By focusing on performance differences across *seemingly* well-diversified portfolios, while *neglecting* firm-specific effects, the portfolio sorts approach may thus misattribute cross-sectional return predictability to the firm characteristic underlying the sort. As a result, empirical asset pricing tests that rely on the portfolio sorts approach may have limited power in detecting abnormal returns.

To test this empirically, we propose a new, alternative framework for analyzing stock returns. Our methodology relies on estimating a firm-level panel regression.[3] The model specification is such that the individual firms' monthly excess returns over the risk-free rate are regressed on a set of market factors (e.g., the three Fama-French factors), a series of firm characteristics (e.g., gross profitability, volatility, etc.), and all interaction terms between the market factors and the firm characteristics. By relying on standard econometrics, our technique easily handles multiple dimensions and, if firm-specific fixed effects are included in the analysis, is robust to misattribution of cross-sectional return predictability. Our firm-level regression approach nests all variants of the standard portfolio sorts approach as a special case. In fact, we prove theoretically and confirm empirically that the proposed regression approach can be specified such that it *exactly* reproduces the alpha and factor exposure estimates from all variants of sorting assets into portfolios. As a result, the proposed methodology shares all the statistical properties and the straightforward economic interpretation of the portfolio sorts approach. We therefore refer to our methodology as the "Generalized Portfolio Sorts" approach, or "GPS-model".

Using our GPS-model, we are able to show *why* portfolio sorts are prone to misattributing cross-sectional return predictability to the firm characteristic underlying the sort. To this end, we first show mathematically that our GPS-model can reproduce the alpha and factor exposure estimates from standard portfolio sorts *if and only if* it is estimated with pooled OLS. From textbook econometrics it is,

[3] For ease of exposition, but without loss of generality, we simply refer to "firms" throughout the paper. However, note that our methodology applies to all research questions in empirical finance, where portfolio sorts have become a standard methodology (e.g., mutual or hedge funds, private or institutional investors, countries or districts, etc.). Furthermore, our technique can be used for evaluating the long-term performance of firms, funds, or investors.

however, well-known that the pooled OLS estimator only produces consistent coefficient estimates, if the so-called random effects (RE) assumption holds. As a result, empirical asset pricing tests relying on portfolio sorts are only meaningful if firm-specific fixed effects (if present) are uncorrelated with the firm characteristic(s) underlying the sort. Otherwise, standard portfolio sorts will produce alpha and factor exposure estimates that on average differ from their true value and, hence, are biased.

Second, we show that firm characteristics, which successfully predict the cross-section of average stock returns but do *not* predict the time-series of returns, are particularly likely to suffer from a violation of the RE assumption. Thus, a firm characteristic that predicts the cross-section of average stock returns well should only be considered a good predictor of asset returns, if it also predicts the time-series of returns. This finding supports Cochrane's (2011, p. 1062) claim that "*time-series forecasting regressions, cross-sectional regressions, and portfolio mean returns are really the same thing*". Utilizing our GPS-model, we then develop a Hausman (1978) type "portfolio sorts specification test". This statistical test allows us to investigate whether the results from standard portfolio sorts are likely affected by a violation of the RE assumption.

Our paper also contributes to the growing literature addressing the factor zoo issue raised by Cochrane (2011).[4] The portfolio sorts specification test provides a simple tool for distinguishing between "valid" factors, which are robust to violations of the RE assumption, and "spurious" factors, which depend on the RE assumption to hold. Consequently, our portfolio sorts specification test complements the result of Harvey, Liu, and Zhu (2016). They argue that a new factor needs to clear a significant hurdle (such as having a *t*-statistic greater than 3.0) in order to be considered a significant determinant for predicting the cross-section of average stock returns. However, because the results from standard portfolio sorts depend on the RE assumption to hold, our formal analysis implies that a large *t*-statistic is insufficient to identify relevant factors. To qualify as a reliable predictor for stock returns, a characteristic-based factor also needs to be robust to violations of the RE assumption and, hence, predict the cross-section of stock returns even in the presence of unobserved heterogeneity across firms, i.e., even if firm fixed effects are included in the analysis.[5] As a consequence, empirical asset pricing tests that rely on fixed effects (FE) estimation are statistically more powerful than asset pricing tests relying on pooled OLS estimation. While standard portfolio sorts belong to the pooled OLS class of asset pricing

[4] Recent research addressing the factor zoo issue includes Bryzgalova (2016), Pukthuanthong, Roll, and Subrahmanyam (2018), Harvey and Liu (2019), and Huang, Li, and Zhou (2019). Recently machine learning techniques are increasingly used to tame the factor zoo, see, e.g., Kozak, Nagel, and Santosh (2018); Bryzgalova, Pelger, and Zhu (2019), Feng, Giglio, and Xiu (2020), and Freyberger, Neuhierl, and Weber (2020). Related to the factor zoo literature, Harvey (2017) points out that empirical finance research relies too much on p-values, and that researchers are incentivized to engage in "p-hacking" in order to get their research published. Hou, Xue, and Zhang (2017) and Chordia, Goyal, and Saretto (2017) analyze "p-hacking" empirically. Their results imply that p-hacking is a serious issue, and that financial markets are more efficient than commonly assumed.

[5] Note that from econometrics it is well-known that the fixed effects (FE) estimator is robust to violations of the RE assumption. For details, see Cameron and Trivedi (2005, chapter 21).

tests, it is straightforward to estimate our GPS-model with a FE estimator. Hence, our approach responds to the concern of Gormley and Matsa (2014) who point out that unobserved heterogeneity constitutes a fundamental challenge for empirical finance research. Compared to standard portfolio sorts, which to date serve as a major workhorse methodology in empirical asset pricing research, the GPS-model therefore allows for implementing more robust asset pricing tests.[6]

In addition, our GPS-model also resolves another major shortcoming of the portfolio sorts approach: Portfolio sorts are generally limited to the analysis of a small number of firm characteristics. Cochrane (2011, p. 1061), for instance, argues that while researchers often "*sort assets into portfolios based on a characteristic […] we cannot do this with 27 variables*". A related concern is that the portfolio sorts approach impedes an analysis of the functional relationship across multiple portfolios. In fact, researchers often just compare the top and bottom group portfolios (Patton and Timmermann, 2010). This, however, involves a loss of potentially valuable information when analyzing the return predictability of a certain firm characteristic.[7] By allowing for the inclusion of multivariate and continuous firm characteristics in the analysis, the GPS-model overcomes these limitations of standard portfolio sorts.[8]

In the empirical part of the paper, we investigate to what extent standard portfolio sorts tend to misattribute cross-sectional return predictability. To this end, we study the return predictability of four randomly chosen firm characteristics that have been shown to predict asset returns. The first two characteristics measure a firm's profitability. Novy-Marx (2013) shows that "gross profitability", defined as gross profit scaled by the book value of total assets, is a better predictor for the cross-section of average stock returns than alternative measures based on bottom line net income, cash flows, or dividends. Ball, Gerakos, Linnainmaa, and Nikolaev (2015) challenge the findings of Novy-Marx (2013). They find operating profitability to predict the cross-section of average stock returns even better than gross profitability.[9] Hence, gross and operating profitability are the first two firm characteristics considered in our empirical analysis. The other two characteristics are the stocks' 52-week rolling beta and 52-week rolling volatility. Ang, Hodrick, Xing, and Zhang (2006, 2009) and Baker, Bradley, and Wurgler (2011), among others, show that stocks with high volatility tend to deliver low risk-adjusted returns. Baker,

[6] The Fama and MacBeth (1973) procedure represents another major workhorse for analyzing stock returns in empirical asset pricing studies. Its econometric properties are studied in Petersen (2009), Vogelsang (2012), Kamstra (2017), and Jegadeesh, Noh, Pukthuanthong, Roll, and Wang (2019) among others.

[7] Cattaneo, Crump, Farrell, and Schaumburg (2020) develop a general framework for portfolio sorting and, based on a nonparametric estimator, show that the optimal number of portfolios is often larger than the common choices of five or 10.

[8] By estimating a *single* firm-level regression, the GPS-model can reproduce the results from multiple portfolio sorts at once. Therefore, a standard Wald test can be applied to test whether the alphas of a series of sorted portfolios are jointly equal to zero. Such a Wald test constitutes an easy-to-implement, yet econometrically robust, alternative to the popular "GRS-test" of Gibbons, Ross, and Shanken (1989).

[9] Ball, Gerakos, Linnainmaa, and Nikolaev (2015) define operating profitability as gross profit minus selling, general, and administrative expenses (excluding R&D expenditures), deflated by the book value of total assets.

Bradley, and Wurgler (2011) and Frazzini and Pedersen (2014) show that high-beta stocks, on average, deliver low risk-adjusted returns.

We start our empirical analysis by evaluating the performance difference between the top and bottom quintile portfolios of stocks sorted on operating profitability. Consistent with Ball, Gerakos, Linnainmaa, and Nikolaev (2015), we find that the portfolio of stocks with high operating profitability outperforms the low profitability portfolio by about 0.5% per month, statistically significant at the 1% level. We then show that our GPS-model estimated with (weighted) pooled OLS exactly replicates both the coefficient estimates and *t*-statistics from the standard portfolio sorts approach. However, when reestimating our GPS-regression model with the fixed effects (FE) estimator rather than with pooled OLS, we find that operating profitability loses its predictive power. This finding suggests that operating profitability is correlated with an unobservable firm-specific factor (or firm-specific effect). Using standard portfolio sorts, this correlation is picked up by the regression intercept or alpha, which is biased upwards. Hence, it is not operating profitability *per se* that has true predictive power for the cross-section of average stock returns, but some persistent firm-specific characteristic, which is captured econometrically by the inclusion of firm fixed effects. As a consequence, operating profitability will have out-of-sample predictive power for the cross-section of average stock returns *if and only if* its correlation with the firm fixed effects persists beyond the estimation period.

When estimating a GPS model for the other three firm characteristics using (weighted) pooled OLS, we again find results very similar to those reported in prior research. Specifically, we find the Fama-French three-factor model alpha to be significantly positively related to gross profitability and negatively related to both volatility and beta. However, as explained above, these results are only valid if the RE assumption holds. To empirically test for the validity of the RE assumption, we use our GPS-model to perform a Hausman (1978) type specification test. The null hypothesis of the test assumes that the RE assumption holds. With the exception of gross profitability, this "portfolio sorts specification test" rejects the null hypothesis of the RE assumption at the 5% level or better for each individual firm characteristic. This implies that three of the four characteristics are likely to be strongly correlated with a persistent, firm-specific factor. Figure 1 confirms this empirically. The figure plots GPS-model-estimated firm fixed effects versus operating profitability (Panel A) and gross profitability (Panel B). While operating profitability is strongly positively correlated with the firm fixed effects ($\rho = 0.36$), gross profitability is only weakly correlated with the firm fixed effects ($\rho = 0.08$). Hence, we would expect gross profitability to predict the cross-section of average stock returns more robustly than operating profitability. This is confirmed when re-estimating the GPS-models using the FE estimator. Operating profitability loses its predictive power when firm fixed effects are included in the analysis. In contrast, gross profitability remains a significant predictor for the cross-section of average stock returns. When analyzing the return predictability of volatility and stock beta using a FE estimator, we find volatility to lose

its predictive power while stock beta turns out to be a remarkably robust predictor for the cross-section of average stock returns. As a result, only the low-beta part of the low-risk anomaly withstands tests that include firm fixed effects.

Note that our empirical results also shed new light on the findings in McLean and Pontiff (2016). They analyze the out-of-sample return predictability of 97 variables, which have been documented to predict the cross-section of stock returns in prior academic research. Their results show that for many variables the return predictability declines significantly after publication. Furthermore, the post-publication predictability declines are more pronounced for variables with particularly high in-sample predictive power. McLean and Pontiff (2016) attribute this decline in predictive power to publication-induced trading. Our empirical evidence offers an alternative explanation for their finding. Firm characteristics with high in-sample predictive power may be highly correlated with firm-specific fixed effects and thus violate the RE assumption. This correlation, however, between the firm characteristics and the fixed effects is expected to eventually dissipate, resulting in a significant decline in predictive power out-of-sample. Hence, the findings in McLean and Pontiff (2016) are not only consistent with publication-induced trading, but also with a violation of the RE assumption by some predictor variables.

In summary, our empirical analysis shows that the estimation of our GPS-model with pooled OLS, which is equivalent to running a standard portfolio sorts analysis, is prone to misattribute cross-sectional return predictability to firm characteristics lacking true predictive power. Contrary to the fundamental assumption of the portfolio sorts approach, our theoretical and empirical analysis thus demonstrates that firm-specific effects may have a significant impact on the results of empirical asset pricing tests.

2. A regression-based approach to analyzing asset returns

In this section, we first describe the general model setup, economic interpretation, and the statistical properties of our regression-based approach to analyzing asset returns. We then demonstrate the methodology's flexibility in handling multiple dimensions and continuous firm, fund, or investor characteristics. Next, we develop a framework for analyzing the cross-sectional versus time-series predictability of asset returns and show that such an analysis is closely related to Hausman's (1978) specification test. Specifically, we show that a firm characteristic that predicts the cross-section of stock returns well only qualifies as a good predictor for asset returns if it also has predictive power for the time-series of returns.

2.1 Model setup

We propose to analyze the cross-section of stock returns using the following firm-level regression model, and to draw statistical inference based on Driscoll and Kraay (1998) standard errors that are

robust to heteroskedasticity and general forms of cross-sectional and temporal dependence (Driscoll and Kraay, 1998; Hoechle, 2007):

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + c_i + v_{it} \qquad (i = 1, \dots, N, \quad t = 1, \dots, T) \tag{1}$$

The dependent variable r_{it} is the period t (excess) return of firm i. Vector $\boldsymbol{z}_{it} = [1 \;\; z_{2,it} \;\; \dots \;\; z_{M,it}]$ comprises a constant and a set of firm characteristics $z_{m,it}$ $(m = 2, \dots, M)$ which may vary across both firms and time. Vector $\boldsymbol{x}_t = [1 \;\; x_{1t} \;\; \dots \;\; x_{Kt}]$ consists of a constant and a series of market-level factor variables $x_{k,t}$ $(k = 1, \dots, K)$ which only vary over time but not across firms. Popular examples of variables in vector $\boldsymbol{x}_t$ are the market excess return, the Fama and French (1993, 2015) size, value, profitability and investment factors, and the Carhart (1997) momentum factor. With $\otimes$ denoting the Kronecker product, regression (1) comprises three types of explanatory variables: individual firm characteristics, market-level factor variables, and all interactions between firm characteristics and factor variables. Fully interacted regression model (1) thus consists of $M \times (K + 1)$ explanatory variables whose regression coefficients are stored in coefficient vector $\boldsymbol{\theta}$. Finally, c_i is an unobserved firm-specific effect with $E(c_i) = 0$, and v_{it} is the regression disturbance that is assumed to be strictly exogenous (i.e. $E[v_{it} \mid c_i, (\boldsymbol{z}_{i1} \otimes \boldsymbol{x}_1), \dots, (\boldsymbol{z}_{iT} \otimes \boldsymbol{x}_T)] = 0$ for all i and t).

If there is no unobserved heterogeneity across firms (i.e., if $c_i = 0$ for all firms) or if the firm-specific effects c_i are uncorrelated with the regressors (i.e., $E[c_i|(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)] = 0$), then regression model (1) can be estimated consistently with (weighted) pooled OLS estimation.[10] As we detail in section 2.2 below, regression (1) – with firm-specific effects c_i excluded – can be specified such that it *exactly* reproduces the results from all types of conventional portfolio sorts. We therefore refer to regression (1) as the "Generalized Portfolio Sorts" approach or, in short, the *GPS*-model.

Unfortunately, the random effects assumption $E[c_i|(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)] = 0$ cannot be generally justified. Given all the heterogeneity across firms in terms of size, investments, financing, business culture, or management quality, among others, it may well be that firm-specific effects exist, and that they are correlated with the explanatory variables of regression (1). In the case of $E[c_i|(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)] \neq 0$, the pooled OLS estimator, however, suffers from an omitted variables bias and, hence, should *not* be used for estimating regression (1). By contrast, the fixed effects (FE) estimator is robust to violations of the random effects assumption. The FE model treats firm-specific effects (c_i) as unobservable, random variables that may or may not be correlated with the explanatory variables in the regression. Therefore, the

[10] Note that the pooled OLS estimator is consistent under both the constant coefficients model and the random effects (RE) model. However, under the RE model pooled OLS is inefficient compared to the FGLS random effects estimator. For details, see Cameron and Trivedi (2005, chapter 21).

FE estimator provides a means for consistently estimating GPS-model (1) even if the firm-specific effects (c_i) are present and correlated with the regressors ($\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$).

2.2 GPS-model vs. portfolio sorts

Popularized by Fama and French's (1993, 1996) influential research, the portfolio sorts methodology became a major workhorse in empirical finance. The method offers an intuitive economic interpretation and ensures robust statistical inference even in the presence of cross-sectional and temporal dependence (Lyon, Barber, and Tsai, 1999). In this section, we demonstrate that GPS-model (1) is able to reproduce the results of all variants of conventional portfolio sorts. The GPS-model proposed in this paper therefore has a sound theoretical and statistical foundation and the coefficient estimates of regression (1) offer a straightforward economic interpretation.

2.2.1 Formal exposition of the portfolio sorts approach

The portfolio sorts methodology involves two steps. In the first step, for each period t, the portfolio (excess) return r_{pt} is computed for a group of firms i as follows:

$$r_{pt} = \sum_{i=1}^{N_t} w_{it} r_{it} \tag{2}$$

w_{it} denotes the beginning of period t portfolio weight of firm i ($i = 1, \dots, N_t$), and r_{it} refers to the firm's stock (excess) return in period t. The second step of the portfolio sorts approach then evaluates the (risk-adjusted) performance of portfolio p by estimating a linear K-factor time-series regression with r_{pt} from (2) as the dependent variable:

$$r_{pt} = \beta_0 + \beta_1 x_{1t} + \dots + \beta_K x_{Kt} + \varepsilon_t \tag{3}$$

In most applications, variables x_{kt} ($k = 1, \dots, K$) are specified such that (3) represents a Jensen (1968), Fama and French (1993, 2015), or Carhart (1997) type factor model. The intercept term (β_0), which in this case is often referred to as the "alpha", then measures the risk-adjusted performance of portfolio p.

2.2.2 Evaluating the performance of a single portfolio with the GPS-model

By estimating firm-level GPS-model (1) with pooled OLS, we can reproduce the results of time-series regression (3). To this end, we specify $\boldsymbol{z}_{it} = [\,1\,]$ and $\boldsymbol{x}_t = [\,1 \;\; x_{1t} \;\ldots\; x_{Kt}]$ to obtain the following regression model:[11]

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} = ([\,1\,] \otimes [\,1 \;\; x_{1t} \;\; \ldots \;\; x_{Kt}\,])\boldsymbol{\theta} + \nu_{it} \\ &= \theta_0 + \theta_1\, x_{1t} + \ldots + \theta_K\, x_{Kt} + \nu_{it} \end{aligned} \tag{4}$$

For ease of mathematical tractability, but without loss of generality, we limit our formal analysis to the case of a balanced panel with *N* firms, *T* time periods, and portfolio weights $w_{it} = 1/N$ (i.e., all firms are equally weighted).[12] Under these assumptions, the following result holds true:

Proposition 1 (Single Portfolio)

- ***Part A – Coefficient estimates.*** *Estimating linear regression (4) with pooled OLS yields coefficient estimates identical to estimating time-series regression (3) with OLS, i.e.,* $\hat{\theta}_k \equiv \hat{\beta}_k \quad (\forall\, k = 0, 1, \ldots, K)$.
- ***Part B – Standard errors.*** *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates* $\hat{\theta}_k$ *in pooled OLS regression (4) are identical to Newey and West (1987) standard errors for coefficient estimates* $\hat{\beta}_k$ *in time-series regression (3), i.e.,* $\mathrm{SE}(\hat{\theta}_k) \equiv \mathrm{SE}(\hat{\beta}_k) \quad (\forall\, k = 0, 1, \ldots, K)$.

Proof: See Appendix A.1.

Part A of Proposition 1 is an application of a well-known property from portfolio theory which says that the portfolio beta is equal to the weighted sum of individual asset betas. Part B of Proposition 1 is intuitive since the Driscoll and Kraay (1998, p. 552) "covariance matrix estimator is precisely the standard Newey and West (1987) heteroskedasticity and serial correlation consistent covariance matrix estimator, applied to the sequence of cross-sectional averages" of the moment conditions.

2.2.3 Using the GPS-model to obtain portfolio sorts

In applications of the portfolio sorts approach, the analysis is usually not restricted to a single group of firms as discussed in Section 2.2.2 above. Rather, firms are sorted into a series of five, ten, or more portfolios with predefined properties. In many cases, the portfolios are constructed on the basis of a

[11] For consistency with the portfolio sorts approach, throughout Section 2.2 we assume $c_i = 0$ (for all i) and omit the firm-specific effects c_i from the analysis. This allows us to estimate GPS-model (1) with (weighted) pooled OLS.

[12] In our empirical analysis, we also consider unbalanced panels (i.e., time-varying cross-sections) and value weighted portfolios. In Section 3, we demonstrate that our theoretical results also hold in this more general setup.

single firm characteristic, such as the book-to-market ratio. Occasionally, researchers also form double (or higher dimensional) sorts in which portfolios are constructed based on multiple firm characteristics such as the book-to-market ratio *and* firm size. For each portfolio, the researcher then evaluates the risk-adjusted performance individually by aid of a Jensen (1968), Fama and French (1993, 2015), or Carhart (1997) type time-series regression as described in Section 2.2.1 above.

Formally, in this more general setup, the first step of the portfolio sorts approach groups the firms into characteristics-based portfolios p for which the average month t excess return, r_{pt}, is equal to

$$r_{pt} = \sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} r_{it} \tag{5}$$

As before, r_{it} denotes the period t excess return of firm i, and $w_{it}^{(p)}$ is the firm's beginning-of-period t weight in portfolio p (with $p = 1, \ldots, P$). $z_{it}^{(p)}$ is a dummy variable which is equal to one if firm i belongs to portfolio p, and zero otherwise. For each portfolio p, the weights sum up to $\sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} = 1$, and the period t cross-section comprises $N_t = \sum_{p=1}^{P} \sum_{i=1}^{N_t} z_{it}^{(p)}$ firms.

The second step of the portfolio sorts approach then evaluates the (risk-adjusted) performance $\beta_{p,0}$ of portfolio p by aid of a linear K-factor time-series regression with r_{pt} from (5) as the dependent variable:

$$r_{pt} = \beta_{p,0} + \beta_{p,1} x_{1t} + \ldots + \beta_{p,K} x_{Kt} + \varepsilon_{pt} \tag{6}$$

If the coefficient estimate for $\beta_{p,0}$ is positive (negative) and statistically significantly different from zero, then portfolio p has a positive (negative) alpha.

With GPS-model (1) it is possible to reproduce the results from time-series regression (6) for each and every portfolio p (with $p = 1, \ldots, P$) by estimating a single firm-level regression with pooled OLS. As before, we specify $\boldsymbol{x}_t = [\, 1 \quad x_{1t} \quad \ldots \quad x_{Kt} \,]$. When forming vector $\boldsymbol{z}_{it}$, we recognize that the analysis needs to include a full set of P portfolio dummies $z_{it}^{(p)}$. To avoid the dummy variables trap, we omit the constant from vector $\boldsymbol{z}_{it}$ and, hence, set $\boldsymbol{z}_{it} = [\, z_{it}^{(1)} \quad z_{it}^{(2)} \quad \ldots \quad z_{it}^{(P)} \,]$ to obtain

$$\begin{aligned} r_{it} = & \left(\left[z_{it}^{(1)} \quad z_{it}^{(2)} \quad \ldots \quad z_{it}^{(P)} \right] \otimes [\, 1 \quad x_{1t} \quad \ldots \quad x_{Kt} \,] \right) \boldsymbol{\theta} + \nu_{it} \\ = \; & \theta_{1,0}\, z_{it}^{(1)} + \theta_{1,1}\, x_{1t} z_{it}^{(1)} + \ldots + \theta_{1,K}\, x_{Kt} z_{it}^{(1)} \\ + \; & \theta_{2,0}\, z_{it}^{(2)} + \theta_{2,1}\, x_{1t} z_{it}^{(2)} + \ldots + \theta_{2,K}\, x_{Kt} z_{it}^{(2)} \\ + \; & \ldots \\ + \; & \theta_{P,0}\, z_{it}^{(P)} + \theta_{P,1}\, x_{1t} z_{it}^{(P)} + \ldots + \theta_{P,K}\, x_{Kt} z_{it}^{(P)} + \nu_{it} \end{aligned} \tag{7}$$

Under the assumptions of Proposition 1, and provided that portfolios $p = 1, \dots, P$ are constant over time (i.e., $z_{it}^{(p)} = z_i^{(p)}$ for all t), the following result holds true:[13]

Proposition 2 (Portfolio sorts)

- ***Part A – Coefficient estimates.*** *For each portfolio p, pooled OLS coefficient estimates for $\theta_{p,k}$ in GPS-model (7) coincide with OLS coefficient estimates for $\beta_{p,k}$ from time-series regression (6), i.e.,* $\hat{\theta}_{p,k} \equiv \hat{\beta}_{p,k}$ $(\forall\ k = 0, 1, \dots, K \ \ and \ \ p = 1, \dots, P)$.
- ***Part B – Standard errors.*** *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{p,k}$ in GPS-model (7) coincide with Newey and West (1987) standard errors of portfolio p's coefficient estimates $\hat{\beta}_{p,k}$ from time-series regression (6), i.e.,* $\text{SE}(\hat{\theta}_{p,k}) \equiv \text{SE}(\hat{\beta}_{p,k})$ $(\forall\ k = 0, 1, \dots, K \ \ and \ \ p = 1, \dots, P)$.

Proof: See Appendix A.2.

According to Proposition 2, the coefficient estimates of GPS-model (7) have a straightforward economic interpretation: Coefficient estimate $\hat{\theta}_{p,0}$ $(p = 1, \dots, P)$ measures the alpha of portfolio p and coefficient estimate $\hat{\theta}_{p,k}$ (with $k = 1, \dots, K$) represents portfolio p's exposure versus factor k. Note that GPS-model (7) reproduces the results of a set of P independent time-series regressions (6) by aid of a *single* linear regression on the firm level. As a result, a standard Wald test can be applied to test whether the risk-adjusted performance of the P portfolios is *jointly* equal to zero:

$$\text{H}_0\text{: } \theta_{1,0} = \theta_{2,0} = \ldots = \theta_{P,0} = 0 \qquad \text{vs.} \qquad \text{H}_1\text{: } \theta_{p,0} \neq 0 \ \text{ for at least one } p \text{ in } 1, \dots, P \tag{8}$$

The multiple hypothesis test in (8) offers an alternative to the widely applied Gibbons, Ross, and Shanken (1989) or "GRS" test, a finite-sample *F*-test commonly used to test the joint significance of the alphas across a set of (e.g., decile) portfolios. Estimating GPS-model (7) with Driscoll and Kraay (1998) standard errors ensures that the Wald test in (8) allows for valid statistical inference if the error terms (v_{it}) of the regression are heteroskedastic, autocorrelated, and cross-sectionally dependent.

2.2.4 Performance differences between two portfolios

The portfolio sorts approach is widely used to test whether the alpha of the top group portfolio differs significantly from the alpha of the bottom group portfolio. When investigating whether, say, firms with

[13] For Proposition 2 to hold in the general case of an unbalanced panel with time-varying portfolios, GPS-model (7) needs to be estimated with weighted pooled OLS. Details on the weighting scheme reproducing the results of portfolio sorts with value-weighted portfolios are provided in Section 3.1.

top-quintile book-to-market ratios (so-called "value stocks") outperform firms with bottom-quintile book-to-market ratios (so-called "growth stocks"), the first step of the analysis computes average month t excess returns for both the top and bottom group portfolios as follows:

$$r_{p,t} = \sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} r_{it} \tag{9}$$

As before, r_{it} is firm i's period t excess return and $z_{it}^{(p)}$ is a dummy variable with value one if firm i belongs to group p (with $p = high,\ low$), and zero otherwise.[14] For both portfolios the beginning-of-period t portfolio weights $w_{it}^{(p)}$ sum up to $\sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} = 1$, and the cross-section considered in the analysis comprises a total of $N_t^* = \sum_{i=1}^{N_t} z_{it}^{(low)} + \sum_{i=1}^{N_t} z_{it}^{(high)}$ firms. The period t return difference between the two portfolios is thus equal to

$$\Delta r_{p,t} = r_{high,t} - r_{low,t} \tag{10}$$

The second step of the portfolio sorts approach then evaluates the risk-adjusted performance of zero investment portfolio (10) based on a K-factor time-series regression as follows:

$$\Delta r_{p,t} = \beta_{\Delta 0} + \beta_{\Delta 1} x_{1t} + \ldots + \beta_{\Delta K} x_{Kt} + \varepsilon_{\Delta t} \tag{11}$$

If the coefficient estimate for $\beta_{\Delta 0}$ is positive (negative) and significantly different from zero, then portfolio "$high$" is considered to outperform (underperform) portfolio "low".

With GPS-model (1), again estimated with pooled OLS, it is possible to reproduce the results of time-series regression (11). For this purpose, we set $\boldsymbol{z}_{it} = [\,1 \;\; z_{it}^{(high)}\,]$ and $\boldsymbol{x}_t = [\,1 \;\; x_{1t} \;\; \ldots \;\; x_{Kt}\,]$ to obtain the following firm-level regression model:

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} \\ &= \theta_{low,0} + \theta_{low,1}\, x_{1t} + \ldots + \theta_{low,K}\, x_{Kt} \\ &+ \theta_{\Delta 0}\, z_{it}^{(high)} + \theta_{\Delta 1}\, x_{1t} z_{it}^{(high)} + \ldots + \theta_{\Delta K}\, x_{Kt} z_{it}^{(high)} + \nu_{it} \end{aligned} \tag{12}$$

Under the assumptions of Proposition 1, and provided that portfolios "$high$" and "low" are constant over time, the following result holds true:[15]

[14] For simplicity but without loss of generality, we label the portfolios as "high" and "low" here. However, subscript p could also refer to "IPO firms" and "mature firms", "firms with a female CEO" and "firms with a male CEO", or any other set of two portfolios that are meant to be compared.

[15] For Proposition 3 to hold in the general case of an unbalanced panel with time-varying portfolios, regression (12) has to be estimated with weighted pooled OLS. See Section 3.3 for details.

Proposition 3 (Performance difference between two groups)

Part A – Coefficient estimates.

- *Pooled OLS coefficient estimates for $\theta_{\Delta k}$ in GPS-model (12) are identical to OLS coefficient estimates for $\beta_{\Delta k}$ in time-series regression (11), i.e., $\hat{\theta}_{\Delta k} \equiv \hat{\beta}_{\Delta k}$ ($\forall\, k = 0, 1, \dots, K$).*
- *Pooled OLS coefficient estimates for $\theta_{low,k}$ in GPS-model (12) are identical to OLS coefficient estimates for $\beta_{low,k}$ in time-series regression (6) for portfolio p="low", i.e., $\hat{\theta}_{low,k} \equiv \hat{\beta}_{low,k}$ ($\forall\, k = 0, 1, \dots, K$).*

Part B – Standard errors.

- *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{\Delta k}$ in GPS-model (12) are identical to Newey and West (1987) standard errors for coefficient estimates $\hat{\beta}_{\Delta k}$ of time-series regression (11), i.e., $SE(\hat{\theta}_{\Delta k}) \equiv \mathrm{SE}(\hat{\beta}_{\Delta k})$ ($\forall\, k = 0, 1, \dots, K$).*
- *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{low,k}$ in GPS-model (12) are identical to Newey and West (1987) standard errors for coefficient estimates $\hat{\beta}_{low,k}$ of time-series regression (6) for portfolio p="low", i.e., $\mathrm{SE}(\hat{\theta}_{low,k}) \equiv \mathrm{SE}(\hat{\beta}_{low,k})$ ($\forall\, k = 0, 1, \dots, K$).*

Proof: See Appendix A.3.

Proposition 3 shows how to specify the GPS-model when analyzing the relative performance of two portfolios. This result can be further generalized to the comparison of a certain base portfolio's (e.g., portfolio $p = 1$) performance with the performance of each other (e.g., quintile or decile) portfolio. Assuming that portfolio $p = 1$ is the base portfolio, we set $\boldsymbol{z}_{it} = [\,1 \;\; z_{it}^{(2)} \;\dots\; z_{it}^{(P)}\,]$ and $\boldsymbol{x}_t = [\,1 \;\; x_{1t} \;\dots\; x_{Kt}\,]$ to obtain the following firm-level regression:

$$
\begin{aligned}
r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} \\
&= \theta_{1,0} + \theta_{1,1}\, x_{1t} + \dots + \theta_{1,K}\, x_{Kt} \\
&+ \theta_{\Delta(2\to1),0}\, z_{it}^{(2)} + \theta_{\Delta(2\to1),1}\, x_{1t} z_{it}^{(2)} + \dots + \theta_{\Delta(2\to1),K}\, x_{Kt} z_{it}^{(2)} \\
&+ \dots \\
&+ \theta_{\Delta(P\to1),0}\, z_{it}^{(P)} + \theta_{\Delta(P\to1),1}\, x_{1t} z_{it}^{(P)} + \dots + \theta_{\Delta(P\to1),K}\, x_{Kt} z_{it}^{(P)} + \nu_{it}
\end{aligned}
\tag{13}
$$

A direct consequence of Propositions 2 and 3 is as follows:

Corollary 1 (Relative performance versus a base portfolio)

- *The pooled OLS coefficient estimate for $\theta_{\Delta(p\to1),k}$ in GPS-model (13) coincides with the OLS coefficient estimate for $\beta_{\Delta k}$ in time-series regression (11) where portfolio p (with $p = 2, \dots, P$) is compared with portfolio 1, i.e., $\hat{\theta}_{\Delta(p\to1),k} \equiv \hat{\beta}_{\Delta k}$ ($\forall\ k = 0, 1, \dots, K$).*
- *For a given lag length H, the Driscoll and Kraay (1998) standard error for coefficient estimate $\hat{\theta}_{\Delta(p\to1),k}$ in GPS-model (13) coincides with the Newey and West (1987) standard error for coefficient estimate $\hat{\beta}_{\Delta k}$ of time-series regression (11) where portfolio p (with $p = 2, \dots, P$) is compared with portfolio 1, i.e., $SE(\hat{\theta}_{\Delta(p\to1),k}) \equiv \mathrm{SE}(\hat{\beta}_{\Delta k})$ ($\forall\ k = 0, 1, \dots, K$).*

■

2.3 Applications of the GPS-model beyond the scope of traditional portfolio sorts

Conventional portfolio sorts have a series of drawbacks. First, they are generally limited to the analysis of a small number of firm characteristics (Cochrane, 2011). Hence, using portfolio sorts, it is challenging to test for competing hypotheses. Second, it is difficult to assess the functional relationship across multiple portfolios. Researchers applying the portfolio sorts approach therefore often focus on a comparison of the top and bottom group portfolios for simplicity (Patton and Timmermann, 2010). Third, the results in Section 2.2 show that the portfolio sorts approach crucially depends on the random effects (RE) assumption to hold. This directly follows from the fact that GPS-model (1) needs to be estimated with pooled OLS (which is only consistent under the RE assumption) to replicate the results from portfolio sorts. Therefore, statistical results from portfolio sorts are biased when firm-specific effects are present and correlated with the explanatory variables.

The GPS-model proposed in this paper has no such limitations and, hence, facilitates the analysis of research questions that are beyond the scope of conventional portfolio sorts. We now consider a series of such applications. Our analysis starts by discussing how to interpret the results from GPS-models that include multivariate and continuous firm characteristics. Next, we apply GPS-model (1) to study the cross-section versus time-series predictability of stock returns and demonstrate that such an analysis is closely related to performing a Hausman (1978) type specification test. Finally, we discuss the consistent estimation of GPS-models in the presence of firm-specific effects, and we derive a statistical test for analyzing whether the results from portfolio sorts are valid.

2.3.1 Multivariate and continuous firm characteristics

GPS-model (1) can be specified such that vector $\boldsymbol{z}_{it}$ contains multivariate binary or continuous firm characteristics. As a result, the GPS-model offers a natural solution to the "multidimensional challenge" of conventional portfolio sorts (Cochrane, 2011), and it provides a simple framework for analyzing formal tests of competing hypotheses as well as for implementing robustness checks.

Notwithstanding this flexibility, the GPS-model retains a clear-cut economic interpretation even if multiple firm characteristics are included in the analysis. To demonstrate this, we consider the case of a CAPM factor structure and two firm characteristics.[16] Denoting the period t market return in excess of the risk-free return by r_{mt}, we specify $\boldsymbol{x}_t = [\,1 \quad r_{mt}]$ and $\boldsymbol{z}_{it} = [\,1 \quad z_{1,it} \quad z_{2,it}\,]$. Absent firm-specific effects, we thus obtain the following GPS-model:

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} = \left([\,1 \quad z_{1,it} \quad z_{2,it}\,] \otimes [\,1 \quad r_{mt}\,]\right)\boldsymbol{\theta} + \nu_{it} \\ &= \left(\theta_{\alpha,0} + \theta_{\alpha,1} z_{1,it} + \theta_{\alpha,2} z_{2,it}\right) + \left(\theta_{\beta,0} + \theta_{\beta,1} z_{1,it} + \theta_{\beta,2} z_{2,it}\right) \times r_{mt} + \nu_{it} \\ &= \alpha_{it} + \beta_{it} \times r_{mt} + \nu_{it} \end{aligned} \tag{14}$$

where $\alpha_{it} = \theta_{\alpha,0} + \theta_{\alpha,1} z_{1,it} + \theta_{\alpha,2} z_{2,it}$ and $\beta_{it} = \theta_{\beta,0} + \theta_{\beta,1} z_{1,it} + \theta_{\beta,2} z_{2,it}$. The last two rows in (14) show that the GPS-model linearly decomposes the risk-adjusted performance (α_{it}) and the factor exposure (β_{it}) with respect to the firm characteristics in $\boldsymbol{z}_{it}$. The Jensen alpha (α_{it}) and beta (β_{it}) in GPS-model (14) therefore represent *conditional* measures. A simple example can illustrate this. Assume that an estimation of regression (14) yields the following result:

$$\begin{aligned} r_{it} &= \hat{\alpha}_{it} + \hat{\beta}_{it} \times r_{mt} + \nu_{it} \\ &= (0.2 + 0.5 z_1 - 0.8 z_2) + (0.8 - 0.3 z_1 + 0.1 z_2) \times r_{mt} + \nu_{it} \end{aligned}$$

In this particular case, the Jensen alpha loads positively on firm characteristic z_1 and negatively on z_2 such that the higher the value of z_1 and the lower the value of z_2 the higher is $\hat{\alpha}_{it}$. The (conditional) Jensen alpha for company A with $z_1 = 1$ and $z_2 = 0.5$ is equal to $\hat{\alpha}_A = 0.2 + 0.5 \times 1 - 0.8 \times 0.5 = 0.3$. The alpha for firm B, with $z_1 = -0.2$ and $z_2 = 0.5$, is $\hat{\alpha}_B = 0.2 + 0.5 \times (-0.2) - 0.8 \times 0.5 = -0.3$. The (conditional) beta for firm i is derived analogously.

When firm-specific effects are present and correlated with the firm characteristics in vector $\boldsymbol{z}_{it}$, (weighted) pooled OLS estimation produces biased, invalid statistical results. In this case, GPS-model (14) needs to be estimated with the fixed effects (FE) estimator to allow for valid statistical inference.

[16] Note that the specification of vectors $\boldsymbol{x}_t$ and $\boldsymbol{z}_{it}$ can easily be extended to comprise multiple factor variables or firm characteristics without changing the logic of how to interpret the results from estimating a GPS-model.

This, however, does *not* affect the interpretation of the GPS-model results. To see this, note that the analysis in GPS-model (14) focuses on slope coefficients $\theta_{\alpha,1}$ and $\theta_{\alpha,2}$ which measure by how much the Jensen alpha changes if firm characteristics $z_{1,it}$ and $z_{2,it}$ change by one unit. By contrast, the coefficient estimate for $\theta_{\alpha,0}$ depends on the sample means of $z_{1,it}$ and $z_{2,it}$ and, hence, is of minor interest. When regression (14) is estimated with firm fixed effects, the intercept term cannot be identified and "drops out" of the regression as part of the within-transformation. However, the slope coefficients, which matter for the analysis, are consistently estimated. Therefore, interpretation of GPS-model coefficients remains the same, irrespective of whether the model is estimated with or without firm fixed effects.

2.3.2 Time-series versus cross-section predictability

In empirical asset pricing, an important question concerns the time-series versus cross-sectional predictability of asset returns (Cochrane, 2011). We now apply GPS-model (1) to formally test how well firm characteristic z_{it} predicts the time-series of asset returns as compared to the cross-section of returns. For this purpose, we start by decomposing firm characteristic z_{it} as

$$z_{it} = \bar{z}_i + \tilde{z}_{it} \tag{15}$$

where $\bar{z}_i = T_i^{-1} \sum_{t=1}^{T_i} z_{it}$ refers to firm i's time-series average of characteristic z_{it}, and $\tilde{z}_{it} = z_{it} - \bar{z}_i$ quantifies by how much the firm's period t value of z_{it} deviates from $\bar{z}_i$. Econometrically speaking, $\tilde{z}_{it}$ represents the within-transformed (or time-series demeaned) version of z_{it}. Based on (15), we therefore set vector $\boldsymbol{z}_{it}$ to $\boldsymbol{z}_{it} = [1 \quad \bar{z}_i \quad \tilde{z}_{it}]$. When specifying vector $\boldsymbol{x}_t$, we account for the fact that in the application at hand we focus on analyzing asset *returns* (rather than risk-adjusted performance) and, hence, set $\boldsymbol{x}_t = [1]$. Assuming $c_i = 0$ for all i, we obtain the following GPS-model:

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} = ([1 \quad \bar{z}_i \quad \tilde{z}_{it}] \otimes [1])\,\boldsymbol{\theta} + \nu_{it} = \theta_0 + \theta_B \bar{z}_i + \theta_W \tilde{z}_{it} + \nu_{it} \tag{16}$$

Note that in regression (16), θ_B measures pure cross-sectional return predictability of firm characteristic z_{it} and θ_W quantifies the characteristic's time-series return predictability. In fact, GPS-model (16) is structured as Neuhaus and Kalbfleisch's (1998) variant of Mundlak's (1978) approach. Pooled OLS estimation of regression (16) thus yields the between estimate for coefficient θ_B and the within (or fixed effects) estimate for coefficient $\hat{\theta}_W$.[17] GPS-model (16) thus constitutes a "hybrid" model that combines

[17] Note that in the case of a balanced panel, pooled OLS estimation of regression (16) yields results identical to estimation with the (efficient) FGLS random effects estimator. If the panel is unbalanced, however, estimation results from pooled OLS for $\hat{\theta}_B$ differ slightly from those of the FGLS random effects estimator. The reason is that the between estimator, contained in the FGLS random effects estimator, weights each firm i equally, independently

the between (BE) estimator with the fixed effects (FE) estimator in a single regression (Allison, 2009). For our purposes, this setup is useful as it allows us to formally test how the time-series predictability ($\hat{\theta}_W$) of firm characteristic z_{it} relates to the cross-sectional predictability ($\hat{\theta}_B$).

Testing for $\hat{\theta}_B = \hat{\theta}_W$ is important for at least two reasons. First, referring to the managed-portfolio theorem, Cochrane (2011, p. 1062) argues that "*time-series forecasting regressions, cross-sectional regressions, and portfolio mean returns are really the same thing. […] An instrument* z_t *in a time-series test* $0 = E[(m_{t+1}R^e_{t+1})z_t]$ *corresponds to a managed-portfolio return* $R^e_{t+1}z_t$ *in an unconditional test* $0 = E[m_{t+1}(R^e_{t+1}z_t)]$." As a result, testing for $\hat{\theta}_B = \hat{\theta}_W$ is of economic relevance. Second, the test is important from an econometric point of view. If the hypothesis of $\hat{\theta}_B = \hat{\theta}_W$ is rejected, then the difference between the within estimate ($\hat{\theta}_W$) and the between estimate ($\hat{\theta}_B$) is statistically significant such that the random effects (RE) assumption *cannot* be assumed to hold. GPS-model (16) therefore provides an alternative to Hausman's (1978) specification test. With $\hat{\theta}_B \neq \hat{\theta}_W$, variable z_{it} is likely to be correlated with other firm characteristics not included in the regression. As a result, rejection of hypothesis $\hat{\theta}_B = \hat{\theta}_W$ implies that pooled OLS estimation of GPS-model $r_{it} = ([1 \;\; z_{it}] \otimes [1])\,\boldsymbol{\theta} + \nu_{it} = \theta_0 + \theta_1 z_{it} + \nu_{it}$ suffers from an omitted variables bias and, hence, produces biased coefficient estimates for θ_0 and θ_1. An analysis of return predictability of firm characteristic z_{it} in this case needs to account for firm fixed effects in order to ensure valid statistical inference. Put differently, a firm characteristic that predicts the cross-section of returns well should only be considered a good predictor for expected returns if it also successfully predicts the time-series of asset returns. We therefore conclude that asset pricing tests should focus on time-series return predictability, which can consistently be estimated with the within-estimator, rather than on cross-sectional return predictability, which is at risk of suffering from an omitted variables bias.

2.3.3 GPS-model specification test for the presence of fixed effects

Building on our analysis in Section 2.3.2, we now derive a Hausman (1978) type specification test, which allows us to investigate whether the random effects (RE) assumption for any given specification of a GPS-model holds. Our testing procedure starts with GPS-model (1):

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + c_i + \nu_{it} = \left([1 \;\; z_{2,it} \;\; \dots \;\; z_{M,it}] \otimes [1 \;\; x_{1t} \;\; \dots \;\; x_{Kt}]\right)\boldsymbol{\theta} + c_i + \nu_{it}$$

Relying on Mundlak's (1978) correlated RE assumption we model firm-specific effect c_i as

of the number of observations in the sample. By contrast, in pooled OLS, the weight of firm i depends on the length of its time-series.

$$c_i = \left(\sum_{m=2}^{M} \xi_{m,0}\, \bar{z}_{m,it} \;+\; \sum_{m=2}^{M} \sum_{k=1}^{K} \xi_{m,k}\, \bar{q}_i^{(m,k)} \right) + u_i \equiv \overline{\boldsymbol{q}}_{\boldsymbol{i}} \boldsymbol{\xi} + u_i \tag{17}$$

where u_i is a mean zero firm-specific effect that is assumed to be uncorrelated with explanatory variables $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$. $\bar{z}_{m,it}$ refers to firm i's time-series average of characteristic $z_{m,it}$, and $\bar{q}_i^{(m,k)} = T_i^{-1} \sum_{t=1}^{T_i} (z_{m,it} x_{kt})$ represents the firm's time-series average of the interaction term between firm characteristic $z_{m,it}$ and factor variable x_{kt}. For ease of notation, we collect all the time-series averages in row vector $\overline{\boldsymbol{q}}_{\boldsymbol{i}}$ and store the $\xi_{m,k}$ coefficients in column vector $\boldsymbol{\xi}$.[18]

Replacing firm-specific effect c_i in GPS-model (1) by the expression in (17), we obtain the following regression model:

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\, \boldsymbol{\theta} + \overline{\boldsymbol{q}}_{\boldsymbol{i}} \boldsymbol{\xi} + u_i + v_{it} \tag{18}$$

GPS-model (18) is structured as a correlated RE model (Mundlak, 1978). Therefore, estimating (18) with pooled OLS will yield the fixed effects (or "within") coefficient estimates for vector $\boldsymbol{\theta}$. Likewise, pooled OLS coefficient estimates for vector $\boldsymbol{\xi}$ quantify by how much the between estimator differs from the fixed effects estimator. Following Wooldridge (2010, p. 332), we can therefore test for

$$\mathrm{H}_0\colon \; \boldsymbol{\xi} = 0 \qquad \text{vs.} \qquad \mathrm{H}_1\colon \; \boldsymbol{\xi} \neq 0 \tag{19}$$

to obtain a regression-based variant of Hausman's (1978) specification test for GPS-model (1). To ensure robust statistical inference of our GPS-model specification test, we estimate regression (18) with Driscoll and Kraay (1998) standard errors that are robust to cross-sectional dependence, autocorrelation, and heteroskedasticity. If the null hypothesis of $\boldsymbol{\xi} = 0$ cannot be rejected, the random effects (RE) assumption is considered to hold. In this case, estimating GPS-model (1) with (weighted) pooled OLS allows for valid statistical inference. However, if the Wald test in (19) rejects the null hypothesis of $\boldsymbol{\xi} = 0$, estimating the GPS-model with (weighted) pooled OLS is likely to produce inconsistent results. In the latter case of $\hat{\boldsymbol{\xi}} \neq 0$, GPS-model (1) should be estimated with the fixed effects estimator to ensure valid statistical inference.

[18] Note that we assume that all characteristics $z_{m,it}$ vary both across firms as well as over time. If $z_{m,it}$ is time-invariant, then GPS-model (1) can only be estimated with (weighted) pooled OLS and, hence, needs to rely on the RE assumption. This is a direct consequence of the fact that the within estimator is unable to estimate the regression coefficients of time-invariant variables since time-invariant variables are absorbed by the firm-specific effect. For details, see Cameron and Trivedi (2005, Chapter 21).

2.3.4 Testing the validity of conventional portfolio sorts

The results from Section 2.2 demonstrate that GPS-model (1) estimated with pooled OLS nests all variants of conventional portfolio sorts as a special case. As the pooled OLS estimator is only consistent under the random effects (RE) assumption, this implies that the portfolio sorts approach also depends on the RE assumption to hold. Put differently, if firm-specific effects are present and correlated with the characteristic underlying the portfolio sort, the results from conventional portfolio sorts may inadvertently misattribute part of the alpha to the firm characteristic underlying the portfolio sort.

Based on our results from Section 2.3.3, we propose a Hausman (1978) type test for analyzing whether results from conventional portfolio sorts are statistically valid. To this end, we rely on GPS-model (13) which compares the performance of a certain base portfolio to the performance of each other portfolio in the sort. Correspondingly, we specify $\boldsymbol{x}_t = [\,1 \;\; x_{1t} \;\ldots\; x_{Kt}\,]$ and $\boldsymbol{z}_{it} = [\,1 \;\; z_{it}^{(2)} \;\ldots\; z_{it}^{(P)}]$. Modelling the firm-specific effects as in (17) and storing the variables containing the firm-specific time-series averages in vector $\overline{\boldsymbol{q}}_{\boldsymbol{i}}$, we estimate regression (18) with (weighted) pooled OLS and perform the Wald test in (19). If the null hypothesis of $\boldsymbol{\xi} = 0$ cannot be rejected, the random effects (RE) assumption underlying the sorted portfolios is likely to hold and, hence, the statistical results of the portfolio sorts approach can be considered valid. However, if null hypothesis $\boldsymbol{\xi} = 0$ is rejected, the portfolio sorts approach is likely to suffer from an omitted variables bias. In this latter case, estimation of a GPS-model with firm fixed effects should be preferred to performing a portfolio sorts analysis.

Note that the "portfolio sorts specification test" proposed here offers a novel approach for addressing the factor-zoo-issue raised by Cochrane (2011). This test allows to distinguish between valid factors (where the RE assumption underlying the sorted portfolios is likely to hold) and invalid factors (which are vulnerable to an omitted variables bias).

3. Empirical Analysis using the GPS-model

In this section, we illustrate the importance of accounting for firm fixed effects in empirical asset pricing tests. To this end, we study the return predictability of four randomly chosen firm characteristics that are widely used in recent asset pricing studies: gross profitability (Novy-Marx, 2013), operating profitability (Ball, Gerakos, Linnainmaa, and Nikolaev, 2015), the stocks' 52-week rolling volatility (Ang, Hodrick, Xing, and Zhang, 2006, 2009; Baker, Bradley, and Wurgler, 2011), and 52-week rolling beta (Baker, Bradley, and Wurgler, 2011; Frazzini and Pedersen, 2014).

Novy-Marx (2013) shows that gross profit scaled by the book value of total assets, henceforth referred to as "gross profitability", is a better predictor of the cross-section of average stock returns than alternative measures that are based on bottom line net income, cash flows, or dividends. He argues that the good performance of gross profitability in predicting the cross-section of average stock returns is mainly due to its numerator, gross profit, being a cleaner measure of economic profitability than, say, net income. Ball, Gerakos, Linnainmaa, and Nikolaev (2015), henceforth abbreviated as BGLN, challenge the findings of Novy-Marx (2013). Their critique centers on the observation that Novy-Marx (2013) deflates net income by the book value of equity while deflating gross profit by the book value of total assets. BGLN demonstrate that the predictive power of net income and gross profit is comparable if the same deflator is used. Furthermore, they suggest an alternative profitability measure, operating profitability, which more closely relates current expenses to current revenues. Defining operating profitability as gross profit minus selling, general, and administrative expenses (excluding R&D expenditures) deflated by the book value of total assets, BGLN find this profitability measure to predict the cross-section of average stock returns even better than gross profitability.

Which characteristic is a better predictor of asset returns, gross profitability or operating profitability? We contribute to this debate by utilizing several variants of GPS-model (1). After an empirical validation of Propositions 1 to 3, we show that the inclusion of firm fixed effects in the analysis may significantly impact the result. We also perform Hausman (1978) type specification tests to analyze whether the results from the portfolio sorts approach, which assumes the random effects assumption to hold, allow for valid statistical inference. Next, we demonstrate the flexibility of GPS-model (1) in handling multivariate and continuous firm characteristics. At this point, we introduce the other two characteristics-based factors, the stocks' 52-week rolling volatility (Ang, Hodrick, Xing, Zhang, 2006, 2009; Baker, Bradley, and Wurgler, 2011) and 52-week rolling beta (Baker, Bradley, and Wurgler, 2011; Frazzini and Pedersen, 2014). Finally, we conduct a horse race to examine which firm characteristics withstand a multivariate test and are robust to the inclusion of firm fixed effects. Our empirical analysis relies on the CRSP-Compustat merged database and spans the sample period from July 1963 through December 2016. We prepare the sample data as described in BGLN.

3.1 The performance of a single portfolio

To validate Proposition 1 empirically, we start with the portfolio sorts approach. Each year at the end of June, we sort the stocks into quintiles based on NYSE breakpoints and hold the portfolios for the subsequent year. For each portfolio p ($p = 1, \dots, 5$), we then compute monthly value-weighted portfolio excess returns (r_{pt}) as

$$r_{pt} = \sum_{i=1}^{N_t} w_{it}^{(p)} \, OA_{it}^{(p)} \, r_{it} \tag{20}$$

where $w_{it}^{(p)}$ refers to the beginning-of-month t portfolio weight of stock i in quintile portfolio p, r_{it} denotes stock i's month t excess return, $OA_{it}^{(p)}$ is a dummy variable with value one if stock i in month t belongs to operating profitability portfolio p, and N_t refers to the overall month t number of stocks in the sample.

Using r_{pt} from (20) as the dependent variable, we then estimate the Fama and French (1993) three-factor model as follows:

$$r_{pt} = \alpha + \beta_{RMRF}\, RMRF_t \; + \; \beta_{SMB}\, SMB_t \; + \; \beta_{HML}\, HML_t \; + \varepsilon_t \tag{21}$$

where $RMRF_t$ is the market excess return, SMB_t denotes the return of a zero-investment size portfolio, and HML_t refers to the return of a zero-investment book-to-market portfolio.

According to Proposition 1, the results from estimating time-series regression (21) for portfolio p can be reproduced by aid of GPS-model (1) being specified as follows:

$$r_{it} = \theta_\alpha \; + \; \theta_{RMRF}\, RMRF_t \; + \; \theta_{SMB}\, SMB_t \; + \; \theta_{HML}\, HML_t \; + \; \nu_{it} \tag{22}$$

Due to the focus on a single portfolio p, GPS-model (22) only considers the subset of observations for which $OA_{it}^{(p)} = 1$ (i.e., observations on stocks that are included in operating profitability portfolio p). To reproduce the results of time-series regression (21) with GPS-model (22), we need to account for two things. First, the portfolio sorts approach typically considers value-weighted rather than equal-weighted portfolios. Second, the CRSP-Compustat database constitutes an unbalanced panel with time-varying cross-sections. Therefore, GPS-model (22) needs to be estimated with *weighted* pooled OLS, where observation weights are fixed such that they match the (implicit) weighting scheme underlying portfolio p's value-weighted return from expression (20). Consequently, we set the weight of observation it equal to the beginning-of-month t value weight of stock i in operating profitability portfolio p:

$$w_{it} = \frac{OA_{it}^{(p)} ME_{it}}{\sum_{i=1}^{N_t} \left(OA_{it}^{(p)} ME_{it} \right)} \tag{23}$$

where ME_{it} refers to stock i's beginning-of-month t market value of equity.

The results of the top-quintile portfolio sorted on operating profitability are reported in Table 1. The first row ("Portfolio sorts approach") reports the results from estimating portfolio-level time-series regression (21) with OLS. Statistical inference is based on Newey and West (1987) standard errors with a lag-length of three months. In line with BGLN, the top quintile portfolio has a statistically significantly positive Fama and French (1993) alpha of +0.225% per month, or +2.7% per year. In the second row, labelled as "GPS-model (weighted pooled OLS)", we report the results of estimating GPS-model (22)

with weighted pooled OLS, where observation weights are set equal to the stocks' beginning-of-month t value-weights. Statistical inference relies on Driscoll and Kraay (1998) standard errors with a lag-length of three months. The results from estimating GPS-model (22) are identical with those from estimating time-series regression (21) reported in the first row.[19] This empirically validates our Proposition 1 from Section 2.2.2.

The third row, labelled "GPS-model (standard pooled OLS)", estimates GPS-model (22) with standard pooled OLS, where all stocks are equally weighted such that microcaps receive the same weight as large caps. As a consequence, the SMB factor loading in this case is large (+0.79) and statistically significant. Moreover, the change in weights also affects the alpha which increases to +0.31% per month (or +3.75% per year). This confirms Fama and French's (2008) concern that small- and microcap stocks can be influential for the results when observationsare equally-weighted.

3.2 Analyzing portfolio sorts with the GPS-model

According to Proposition 2, GPS-model (1) can be specified such that it reproduces the results of multiple sorted portfolios with a single regression on the firm-level. To validate Proposition 2 empirically, we apply the portfolio sorts procedure from Section 3.1 to each of the five quintile portfolios sorted on operating profitability. The results are reported in Panel A of Table 2. In line with BGLN, the alphas of the sorted portfolios monotonically increase with operating profitability. While the low profitability ($p = 1$) portfolio has a significantly negative alpha of -0.32% per month, the top-quintile ($p = 5$) portfolio shows a significantly positive alpha of +0.23% per month.

To reproduce the results of the portfolio sorts analysis with the GPS-model, we estimate regression (7) with weighted pooled OLS, where the weight of observation it is set equal to stock i's period t value-weight in the respective operating profitability portfolio. Panel B of Table 2 reports the results in a two-dimensional matrix. The portfolio dummy variables in vector $\boldsymbol{z}_{it} = \left[OA_{it}^{(1)} \quad OA_{it}^{(2)} \quad \cdots \quad OA_{it}^{(5)} \right]$ define the columns and the factor variables in vector $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$ identify the rows. All elements in the results matrix thus represent the coefficient estimates (and t-statistics) of the interaction term between firm-characteristic $OA_{it}^{(p)}$ and factor variable (or constant) x_{kt}. A comparison of the results in Panels A and B of Table 2 shows that both the coefficient estimates and t-statistics

[19] Note that the number of observations included in the estimation differs across models. The time-series regression of the portfolio sorts approach comprises 642 monthly observations whereas the weighted pooled OLS estimation of the firm-level GPS-model includes 412,443 firm-month observations.

(based on Driscoll-Kraay standard errors) of the GPS-model coincide with those from applying the portfolio sorts approach (independently) to the five quintile portfolios sorted on operating profitability. This empirically confirms the theoretical result stated in Proposition 2 in Section 2.2.3.

The GPS-model can also be specified such that it reproduces the results from "two-way portfolio sorts" by aid of a single regression on the firm-level. To illustrate this, we reproduce parts of the analysis in Table 8 of BGLN. Relying on NYSE breakpoints, we sort the stocks into quintiles based on operating profitability and market capitalization.[20] As before, we form the portfolios at the end of each June and then hold them for the subsequent year. Panel A of Table 3 reports the results for the two-way sorted portfolios. For brevity, we only show the coefficient estimates (and *t*-values) of the alpha from estimating time-series regression (21) for each of the 25 two-way sorted portfolios. Despite a slightly different sample period, the coefficient estimates and *t*-values match closely with those in BGLN.

Next, we turn to GPS-model (1) and show how to replicate the results of all the two-way sorted portfolios by estimating a single regression on the firm-level. To this end, we specify $\boldsymbol{z}_{it}$ as follows:

$$\boldsymbol{z}_{it} = \left[\left[OA_{it}^{(1)} \quad \dots \; OA_{it}^{(5)}\right] \otimes \left[ME_{it}^{(1)} \quad \dots \quad ME_{it}^{(5)}\right]\right]$$

Here, $OA_{it}^{(p)}$ $(p = 1, \dots, 5)$ refers to a dummy variable with value one if stock i in month t belongs to operating profitability portfolio p, and $ME_{it}^{(q)}$ $(q = 1, \dots, 5)$ is a dummy variable with value one if stock i in month t belongs to market capitalization quintile q. As before, vector $\boldsymbol{x}_t$ includes a constant and the three Fama-French factors and, hence, is specified as $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$. To reproduce the results of the portfolio sorts approach, we estimate the GPS-model with weighted pooled OLS, where observation weights are fixed such that they match the value-weights of the stocks in the two-way sorted portfolios.

Panel B of Table 3 reports the regression coefficients decomposing the alpha, i.e., for the coefficient estimates (and *t*-statistics) of the variables in vector $\boldsymbol{z}_{it}$.[21] The results are reported in a two-dimensional matrix with the operating profitability quintile dummies, $OA_{it}^{(p)}$, identifying the rows and the market capitalization quintile dummies, $ME_{it}^{(q)}$, defining the columns. Element (p, q) in the results matrix thus represents the coefficient estimate (and *t*-statistic) of the interaction term between operating profitability quintile dummy p and market cap quintile dummy q. When comparing the results from estimating the

[20] Following BGLN, the sorts are carried out independently of each other. BGLN use these two-way portfolio sorts to investigate whether the predictive power of operating profitability for the cross-section of average returns is a market-wide phenomenon or if it is confined to certain size groups.

[21] Note that the characteristics in vector $\boldsymbol{z}_{it}$ in fact represent the interaction terms of the variables in $\boldsymbol{z}_{it}$ with the constant in $\boldsymbol{x}_t$.

GPS-model (Panel B) with the results from the two-way sorted portfolios (Panel A), it is evident that they match perfectly. This provides further empirical evidence for Proposition 2.

3.3 Top versus bottom portfolio performance

To validate Proposition 3 empirically, we first apply the portfolio sorts approach to evaluate the performance difference between the top ($p = 5$) and bottom ($p = 1$) quintile portfolios of stocks sorted on operating profitability. To this end, we first compute the month t excess returns (r_{pt}) for the top and bottom quintile portfolios as outlined in expression (20) above. We then evaluate the performance difference between the two portfolios by estimating portfolio-level time-series regression (21) with $\Delta r_{pt} = r_{5,t} - r_{1,t}$ as the dependent variable. Panel A of Table 4 reports the results. On a risk-adjusted basis, the portfolio of stocks with high operating profitability outperforms the low profitability portfolio by a significant +0.54% per month (or +6.48% per year). Despite a slightly different sample period, the coefficient estimates and *t*-values are very similar to those reported in Table 6 of BGLN.

To replicate the results of the portfolio sorts approach with a regression on the individual firm-level, we estimate GPS-model (12) with weighted pooled OLS. In this analysis, we only consider observations on stocks with top ($p = 5$) or bottom ($p = 1$) quintile operating profitability and set observation weights equal to the stocks' value-weights in their operating profitability portfolio. Panel B of Table 4 displays the results in a two-dimensional matrix. The elements of firm characteristics' vector $\boldsymbol{z}_{it} = \left[\, 1 \quad OA_{it}^{(5)} \right]$ define the columns while the factor variables in $\boldsymbol{x}_t = [\, 1 \quad RMRF_t \quad SMB_t \quad HML_t \,]$ identify the rows. All elements in the results matrix thus represent the coefficient estimates (and *t*-statistics based on Driscoll-Kraay standard errors) for the interactions between the elements in vector $\boldsymbol{z}_{it}$ and those in vector $\boldsymbol{x}_t$. The results from estimating GPS-model (12) with weighted pooled OLS (Panel B) coincide with the results of the portfolio sorts approach in Panel A. This empirically confirms Proposition 3.

3.4 Do firm fixed effects matter for the prediction of stock returns?

The analysis in Panel A of Table 4, and by extension in Panel B of Table 4, depends on the assumption that firm-specific effects (if present) are uncorrelated with the explanatory variables in $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$, i.e., that the random effects (RE) assumption holds. In Panel C of Table 4, we reproduce the analysis from Panel B, but estimate the GPS-models with the fixed effects (or "within") estimator that is robust to violations of the RE assumption.

Most important, we find that operating profitability no longer statistically significantly predicts the cross-section of average stock returns when firm fixed effects are accounted for. Hence, the results in

Panel C suggest that operating profitability is correlated with the firm fixed effects resulting in a significant fraction of the alpha reported in Panels A and B being misattributed to operating profitability. In other words, it is not operating profitability that has predictive power, but a time-invariant component of it that is absorbed by the firm fixed effects. As a consequence, operating profitability is expected to have out-of-sample predictive power only if its correlation with the unobservable firm-specific factor persists beyond our sample period, such that operating profitability remains correlated with the fixed effects that drive the results in Panels A and B. In summary, the results in Table 4 suggest that firm fixed effects can have a major impact on the results from empirical asset pricing tests.

3.5 Using the GPS-model to test the validity of conventional portfolio sorts

As shown above, GPS-model (1), estimated with (weighted) pooled OLS, is able to reproduce the results from conventional portfolio sorts. This, however, implies that the results from conventional portfolio sorts are valid if and only if the random effects assumption holds. We now investigate whether firm-specific effects (if present) indeed are uncorrelated with the characteristic(s) underlying the portfolio formation. To examine whether the results from conventional portfolio sorts are valid, we rely on the portfolio sorts specification test developed in Section 2.3.4.

We first analyze whether portfolio sorts on operating profitability are sensitive to the random effects assumption. To this end, we specify GPS-model (1) with $\boldsymbol{z}_{it} = [\,1 \quad OA_{it}^{(2)} \quad \dots \quad OA_{it}^{(5)}\,]$ and $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$. We then add the firm-specific time-series averages (which we store in vector $\bar{\boldsymbol{q}}_{\boldsymbol{i}}$) of all variables in $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$ that vary across both firms and time to the regression. The resulting regression model $r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \bar{\boldsymbol{q}}_{\boldsymbol{i}}\boldsymbol{\xi} + \varepsilon_{it}$ constitutes our first specification of regression (18), which forms the basis of our portfolio sorts specification test. Panel A of Table 5 reports the results from estimating the respective regression with pooled OLS, where observation weights are set equal to the beginning-of-month t value-weights of the stocks in the quintile portfolios sorted on operating profitability. The results are displayed in matrix form. The first block of results, labeled as "Coefficient estimates on explanatory variables $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$", contains the estimation results for vector $\boldsymbol{\theta}$. With the portfolio dummies from $\boldsymbol{z}_{it}$ defining the columns and the factor variables in $\boldsymbol{x}_t$ defining the rows, element (p, k) in the results matrix represents the coefficient estimates (and *t*-statistics) for the interaction term between firm characteristic (or constant) $z_{p,it}$ and factor variable (or constant) x_{kt}. Hence, the results in the first column are for the bottom-quintile profitability portfolio while the remaining columns show by how much the results of profitability quintile portfolio p $(p = 2, 3, 4, 5)$ differ from those of the bottom-quintile portfolio. As explained in Section 2.3, the coefficient estimates for $\boldsymbol{\theta}$ represent fixed effects (FE) estimates that allow for valid statistical inference even if firm-specific effects are correlated with the variables in $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$. The results in row "1 (Intercept)" show that when controlling for firm

fixed effects, the Fama-French three-factor model alpha of the top-quintile portfolio sorted on operating profitability is no longer statistically significantly different from that of the bottom-quintile profitability portfolio. The difference between the two portfolios' alpha now amounts to only +0.02% per month, which is consistent with results in Panel C of Table 4, but stands in stark contrast to the highly significant +0.54% per month alpha-difference obtained in a conventional portfolio sorts analysis that ignores firm fixed effects (see Panels A and B of Table 4).

The second block of Panel A in Table 5, labeled as "Coefficient estimates on time-series averages", contains the estimation results for vector $\boldsymbol{\xi}$. The structure of the estimation results in this block is similar to that of $\boldsymbol{\theta}$ discussed before. The coefficient estimates for the elements in $\boldsymbol{\xi}$ quantify by how much the between (BE) estimator differs from the fixed effects (FE) estimator. The results show that the difference between BE and FE coefficient estimates is particularly pronounced for the coefficient estimates quantifying the difference of quintile portfolio p's alpha ($p = 2,3,4,5$) versus the bottom-quintile portfolio. For instance, the BE estimate for the alpha-difference between the top- and bottom-quintile portfolios deviates by a significant +1.10% per month from the respective FE estimate. As a consequence, the evaluation of operating profitability as a predictor of asset returns critically hinges on the random effects (RE) assumption. To test for the validity of the RE assumption, we perform the Wald test in (19). This portfolio sorts specification test has an F-statistic of 4.61 and rejects the null hypothesis of $\boldsymbol{\xi} = 0$ at all conventional levels of statistical significance. For portfolio sorts on operating profitability the random effects assumption can therefore not be considered to hold, invalidating the results from conventional portfolio sorts.

Panel B of Table 5 reproduces the analysis from Panel A for portfolio sorts on gross profitability, defined as gross profit divided by total assets (Novy-Marx, 2013). We specify $\boldsymbol{z}_{it} = [\,1 \quad GA_{it}^{(2)} \quad \dots \quad GA_{it}^{(5)}\,]$, where $GA_{it}^{(p)}$ represents a dummy variable with value of one if stock i in month t belongs to quintile portfolio p sorted on gross profitability. The results are quite different from those on operating profitability. Specifically, the coefficient estimate measuring by how much the alpha of the top-quintile portfolio differs from that of the bottom-quintile portfolio of +0.42% per month is positive and statistically significant. Furthermore, the differences between BE and FE coefficient estimates (i.e., the coefficient estimates for $\boldsymbol{\xi}$) are less pronounced than in Panel A when sorting on operating profitability. Correspondingly, the "portfolio sorts specification test" no longer rejects the null hypothesis of $\boldsymbol{\xi} = 0$. The respective Wald test has an F-statistic of 1.19 which corresponds to a p-value of 0.27. We therefore conclude that the statistical results from portfolio sorts on gross profitability are reliable. All else equal, stocks with high gross profitability outperform stocks with low gross profitability.

3.6 Firm fixed effects and continuous and multivariate firm characteristics

We now demonstrate that the GPS-model provides a versatile framework for investigating the predictability of stock returns as well as for analyzing the role of firm fixed effects. To determine which profitability and volatility measures offer return predictability, we estimate (subsets of) the following GPS-model:

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + e_{it} \\ &= ([1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}] \otimes [\,1 \quad RMRF_t \quad SMB_t \quad HML_t])\,\boldsymbol{\theta} + e_{it} \end{aligned} \tag{24}$$

where OA_{it} is operating profitability, GA_{it} measures gross profitability, $Vola_{it}$ is the standard deviation of 52 weekly stock returns ending on the last Friday prior to the end of month t, and $Beta_{it}$ is the CAPM-beta of weekly stock returns estimated over the 52 weeks ending on the last Friday prior to the end of month t. The variables in vector $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$ represent a constant and the three Fama and French (1993) factors.

Panel A of Table 6 presents the results from estimating GPS-model (24) with weighted pooled OLS, where observation weights are set equal to the firms' month t value weights. For brevity, only the estimates for the regression coefficients decomposing the Fama-French three-factor model alpha are reported.[22] In column (1), we estimate regression (24) with $\boldsymbol{z}_{it} = [1 \quad OA_{it}]$. The results show that alpha increases with operating profitability. The coefficient estimate is positive (+1.74) and highly statistically significant. A qualitatively similar result is obtained when estimating GPS-model (24) with $\boldsymbol{z}_{it} = [1 \quad GA_{it}]$ (column 2). By contrast, the alpha decreases with volatility (column 3) and the stock beta (column 4). In summary, these univariate results are in line with the findings from previous research. Columns (5) and (6) take advantage of the GPS-model's capability to handle multivariate firm characteristics. When estimating GPS-model (24) with $\boldsymbol{z}_{it} = [1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}]$, we find that only operating profitability (coefficient estimate: +1.11) and the stock beta (coefficient estimate: -0.30) statistically significantly predict stock returns while the coefficient estimates on gross profitability and volatility are insignificant.

The analysis in Panel A of Table 6 depends on the random effects assumption to hold. Next, we use our GPS-model specification test developed in Section 2.3.3 to test for the existence of firm fixed effects in each regression specification of Panel A. For this purpose, we extend regression (24) with the firm-level time-series averages of all variables that vary cross-sectionally and over time. We then employ a Wald test to examine whether the coefficient estimates for the time-series average variables are jointly equal to zero. If this null hypothesis cannot be rejected, the RE assumption can be assumed to

[22] Note that conditional on firm i's characteristics in period t, the Fama-French three-factor model alpha is obtained as $\hat{\alpha}_{it} = \hat{\theta}_0 + \hat{\theta}_1 \times OPAT_{it} + \hat{\theta}_2 \times GPAT_{it} + \hat{\theta}_3 \times Vola_{it} + \hat{\theta}_4 \times Beta_{it}$. For details, see Section 2.3.1.

hold and GPS-model (24) can be estimated with (weighted) pooled OLS. The results from the GPS-model specification test are reported in Panel B of Table 6. With the exception of the GPS-model specification in column (2), where vector $\boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_{it} = [1 \;\; GA_{it}]$, the GPS-model specification test rejects the RE assumption for all specifications in Panel A at the 5% confidence level or better. As a result, the statistical results from estimating GPS-model (24) with pooled OLS are likely to be invalid.

In Panel C of Table 6, we reestimate the analysis in Panel A using the fixed effects estimator, which allows firm-specific effects to be correlated with the explanatory variables in $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$. For some of the specifications, we observe considerable differences between the results in Panel C and those in Panel A. Specifically, in column (1), we find that operating profitability no longer statistically significantly predicts the cross-section of average stock returns when firm fixed effects are accounted for. Furthermore, in the multivariate analysis of column (6), which specifies vector $\boldsymbol{z}_{it}$ as $\boldsymbol{z}_{it} = [1 \;\; OA_{it} \;\; GA_{it} \;\; Vola_{it} \;\; Beta_{it}]$, the coefficient estimate of operating profitability changes sign and assumes a significantly negative value of -1.58. This result for operating profitability stands in contrast to the findings for gross profitability, whose coefficient estimate remains positive and statistically significant even when estimating regression (24) with firm fixed effects. These findings suggest that gross profitability is a more robust predictor for stock returns than operating profitability. When comparing the return predictability of volatility with that of the stock beta, we find the stock beta to be a remarkably robust predictor for the cross-section of average stock returns. Irrespective of the regression specification in Panel C of Table 6, the coefficient estimate for the stock beta is around -0.40, and statistically significant at the 1% level. In contrast, when accounting for firm fixed effects, volatility no longer has predictive power for the Fama-French three-factor model alpha. We therefore conclude that the 52-week rolling stock beta is a more robust predictor for stock returns than 52-week rolling volatility. More generally, we conclude that firm fixed effects can have a major impact on the results from empirical asset pricing tests.

4. Conclusion

Addressing recent concerns about the robustness of abnormal returns, we propose a novel, regression-based alternative to the popular portfolio sorts approach for analyzing asset returns. Our "GPS-model" relies on estimating a linear panel regression on the individual firm level, and on drawing statistical inference based on Driscoll and Kraay (1998) standard errors that are robust to heteroskedasticity as well as cross-sectional and temporal dependence. Our technique easily handles continuous and multivariate firm characteristics. Most importantly, it allows for the inclusion of firm fixed effects. Using formal econometric analysis, we show that our approach nests all variants of the widely-applied portfolio sorts approach. Moreover, we prove formally that alpha and factor exposure estimates from standard

portfolio sorts are valid *if and only if* the random effects (RE) assumption holds. This is a direct consequence of the fact, that to exactly reproduce the results from portfolio sorts, our GPS-model needs to be estimated with pooled OLS (which depends on the RE assumption). Using our methodology, we also develop a Hausman (1978) type specification test that allows us to analyze whether the results from portfolio sorts are likely to be affected by unobservable heterogeneity across firms.

In the empirical part of the paper, we examine the relevance of accounting for firm fixed effects in empirical asset pricing tests. We do so by considering four randomly chosen characteristics that have been shown to predict asset returns: operating profitability, gross profitability, volatility, and beta. Our empirical results reveal that two of the four tested characteristics-based factors, operating profitability and low volatility, do not withstand tests accounting for firm fixed effects. We therefore conclude that the RE assumption implicitly underlying the portfolio sorts approach in fact matters. Specifically, our results suggest that operating profitability and (low) volatility are significantly correlated with an unobservable, firm-specific component. As a result, the predictive power for the cross-section of average stock returns of these two characteristics-based factors will only continue beyond our sample period if operating profitability and (low) volatility keep their correlation pattern with the firm-specific effects.

In summary, our results suggest that firm-specific effects can have a major impact on the results from empirical asset pricing tests. The GPS-model proposed in this paper addresses and resolves this issue. Relying on a simple yet econometrically robust framework, the GPS-model can easily be estimated with the fixed effects estimator, ensuring valid statistical inference even in the case of the RE assumption being violated.

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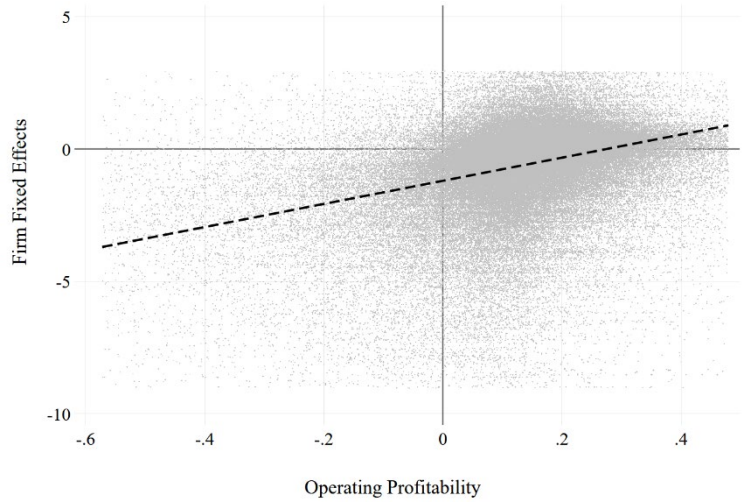
Figure 1: Relationship between firm-specific fixed effects and firm characteristics

This figure plots firm fixed effects versus operating profitability (Panel A) and gross profitability (Panel B). The sample period goes from July 1963 through December 2016. The firm fixed effects are inferred from estimating a firm-level panel regression as shown in equation (1) with the fixed effects (FE) estimator. The market-level variables in vector $\boldsymbol{x}_t$ comprise a constant and the three Fama and French (1993) factors, i.e., RMRF (market excess return), SMB (small minus big), and HML (high minus low). In Panel A, regression (1) is estimated with vector $\boldsymbol{z}_{it}$ containing five quintile dummy variables for operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. The quintile dummy variables for operating profitability are based on NYSE breakpoints at the end of each June and then remain constant for the subsequent year. The correlation between the (estimated) firm fixed effects and operating profitability is 0.36. In Panel B, vector $\boldsymbol{z}_{it}$ contains five quintile dummy variables for gross profitability, defined as gross profit deflated by the book value of total assets. The quintile dummy variables for gross profitability are based on NYSE breakpoints at the end of each June and remain constant for the subsequent year. The correlation between the (estimated) firm fixed effects and gross profitability is 0.08. The charts do not display the top and bottom 1% of observations for the firm fixed effects, as well as for operating profitability and gross profitability, respectively.

Panel A: Firm fixed effects versus operating profitability



Panel B: Firm fixed effects versus gross profitability

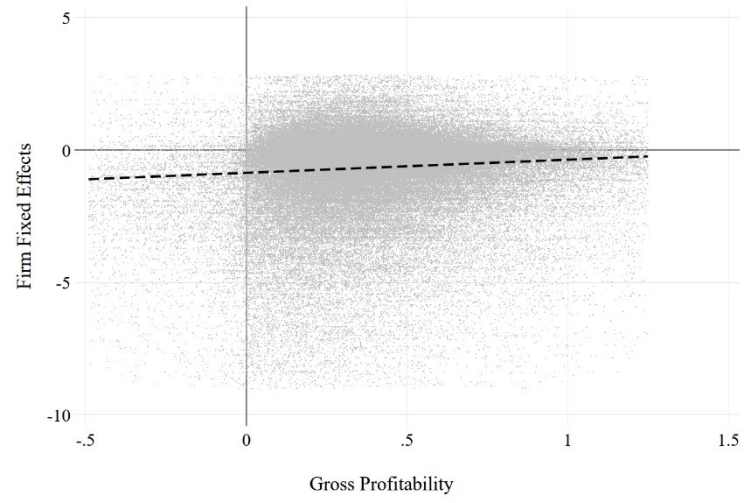


Table 1: Single portfolio analysis

	Operating profitability: Top-quintile portfolio							
	Average Return	Three-factor model				Three-factor model statistics		
		a	b_{RMRF}	b_{SMB}	b_{HML}	*R-squared*	*N Obs.*	*N Stocks.*
(1) Portfolio sorts approach	0.574***	0.225***	0.950***	-0.075***	-0.316***	0.949	642	
	(3.10)	(4.92)	(73.78)	(-4.44)	(-18.60)			
(2) GPS-model (weighted pooled OLS)	0.574***	0.225***	0.950***	-0.075***	-0.316***	0.261	412,443	6,831
	(3.10)	(4.92)	(73.78)	(-4.44)	(-18.60)			
(3) GPS-model (standard pooled OLS)	1.070***	0.313***	1.037***	0.790***	-0.028	0.150	412,443	6,831
	(3.92)	(4.73)	(50.53)	(13.08)	(-0.63)			

This table reports the average return as well as the 3-factor model alpha along with RMRF (market), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. The portfolio sort is based on NYSE breakpoints at the end of each June and the portfolio is held for the subsequent year. The sample period goes from July 1963 through December 2016. All results are for the quintile 5 (high) portfolio comprising the stocks with top quintile operating profitability. Row (1) reports the results from a conventional portfolio sort where the portfolio's excess return is regressed on the three Fama and French (1993) factors. Rows (2) and (3) present the results from estimating GPS-model (22). In Row (2), the regression is estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time t value-weights of the stocks. In Row (3), the regression is estimated with standard pooled OLS, where all observations are equally weighted. t-statistics from testing for significance against a value of zero are presented in parentheses. Statistical inference for the portfolio sorts approach in Row (1) is based on Newey and West (1987) standard errors with a lag-length of three. The GPS-models in Rows (2) and (3) are estimated with Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 2: Analysis of quintile portfolios sorted on operating profitability

Panel A: Conventional portfolio sorts

	Quintile Portfolio				
	1 (low)	2	3	4	5 (high)
a	-0.318***	-0.116*	0.038	0.046	0.225***
	(-4.26)	(-1.86)	(0.73)	(0.88)	(4.92)
b_{RMRF}	1.092***	0.957***	0.943***	1.009***	0.950***
	(48.10)	(48.78)	(58.71)	(63.60)	(73.78)
b_{SMB}	0.212***	0.061	-0.054*	-0.048*	-0.075***
	(7.65)	(1.37)	(-1.73)	(-1.66)	(-4.44)
b_{HML}	0.175***	0.256***	0.135***	0.047	-0.316***
	(4.24)	(5.80)	(5.27)	(1.61)	(-18.60)
R-squared	0.897	0.891	0.916	0.933	0.949
N Obs.	642	642	642	642	642

Panel B: GPS-model

Vector $\boldsymbol{z}_t$ $\rightarrow$ Vector $\boldsymbol{x}_t$ $\downarrow$	$OA_{it}^{(1)}$	$OA_{it}^{(2)}$	$OA_{it}^{(3)}$	$OA_{it}^{(4)}$	$OA_{it}^{(5)}$
1 (Intercept)	-0.318***	-0.116*	0.036	0.046	0.225***
	(-4.26)	(-1.86)	(0.73)	(0.88)	(4.92)
$RMRF_t$	1.092***	0.957***	0.943***	1.009***	0.950***
	(48.10)	(48.78)	(58.71)	(63.60)	(73.78)
SMB_t	0.212***	0.061	-0.054*	-0.048*	-0.075***
	(7.65)	(1.37)	(-1.73)	(-1.66)	(-4.44)
HML_t	0.175***	0.256***	0.135***	0.047	-0.316***
	(4.24)	(5.80)	(5.27)	(1.61)	(-18.60)
R-squared	0.230				
N Obs.	1,969,221				
N Stocks	16,244				

This table reports value-weighted 3-factor model alphas along with RMRF (market excess return), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A reports the results from conventional portfolio sorts where a portfolio's excess return is regressed on the three Fama and French (1993) factors. Panel B presents the results from estimating a single GPS-model with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks. Coefficient estimates and *t*-statistics (in parentheses) in Panel B (GPS-model) are for the interaction of the (market-level) variables and a constant contained in vector $\boldsymbol{x}_t$ (displayed on the vertical axis) and the quintile dummy variables $OA_{it}^{(q)}$ $(q = 1, \ldots, 5)$ for operating profitability in vector $\boldsymbol{z}_{it}$ (displayed on the horizontal axis). The quintile portfolios (Panel A) and dummy variables (Panel B) are formed based on NYSE breakpoints at the end of each June and then remain unchanged throughout the subsequent year. The sample period goes from July 1963 through December 2016. Statistical inference on the portfolio sorts (Panel A) is based on Newey and West (1987) standard errors with a lag-length of three. The GPS-model in Panel B is estimated with Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 3: Two-way portfolio sorts on operating profitability and market capitalization

Panel A: Conventional portfolio sorts

Operating profitability	Market capitalization				
	Q1	Q2	Q3	Q4	Q5
Q1	-0.40***	-0.38***	-0.30***	-0.26**	-0.25**
	(-3.76)	(-4.10)	(-3.26)	(-2.57)	(-2.43)
Q2	-0.04	-0.05	-0.06	-0.05	-0.14*
	(-0.60)	(-0.69)	(-0.66)	(-0.67)	(-1.84)
Q3	0.08	0.08	-0.01	-0.02	0.06
	(1.20)	(1.09)	(-0.15)	(-0.21)	(1.08)
Q4	0.10*	-0.01	0.10	0.09	0.04
	(1.70)	(-0.04)	(1.39)	(1.21)	(0.70)
Q5	0.31***	0.25***	0.18***	0.28***	0.23***
	(3.72)	(3.50)	(2.59)	(3.43)	(4.28)

Panel B: GPS-model

Operating profitability	Market capitalization				
	$ME_{it}^{(1)}$	$ME_{it}^{(2)}$	$ME_{it}^{(3)}$	$ME_{it}^{(4)}$	$ME_{it}^{(5)}$
$OA_{it}^{(1)}$	-0.40***	-0.38***	-0.30***	-0.26**	-0.25**
	(-3.76)	(-4.10)	(-3.26)	(-2.57)	(-2.43)
$OA_{it}^{(2)}$	-0.04	-0.05	-0.06	-0.05	-0.14*
	(-0.60)	(-0.69)	(-0.66)	(-0.67)	(-1.84)
$OA_{it}^{(3)}$	0.08	0.08	-0.01	-0.02	0.06
	(1.20)	(1.09)	(-0.15)	(-0.21)	(1.08)
$OA_{it}^{(4)}$	0.10*	-0.01	0.10	0.09	0.04
	(1.70)	(-0.04)	(1.39)	(1.21)	(0.70)
$OA_{it}^{(5)}$	0.31***	0.25***	0.18***	0.28***	0.23***
	(3.72)	(3.50)	(2.59)	(3.43)	(4.28)
R-squared	0.207				
N Obs.	1,969,221				
N Stocks	16,244				

This table reports value-weighted three-factor model alphas and *t*-statistics (in parentheses) for portfolios sorted by market capitalization and operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A reports the results from conventional portfolio sorts where a portfolio's excess return is regressed on the Fama and French (1993) market (RMRF), size (SMB), and value (HML) factors. Panel B presents the results from estimating a single GPS-model with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks. Coefficient estimates and *t*-statistics (in parentheses) for the GPS-model are for the product of the market capitalization quintile dummy variables and quintile dummy variables for operating profitability. The dummy variables (Panel B) and two-way sorted portfolios (Panel A) are formed based on NYSE breakpoints at the end of each June and remain unchanged throughout the subsequent year. The market capitalization and operating profitability sorts are independent of each other. The sample period is from July 1963 through December 2016. Statistical inference for the portfolio sorts approach (Panel A) is based on Newey and West (1987) standard errors with a lag-length of three. The GPS-model in Panel B is estimated with Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 4: Comparison of high vs. low quintile portfolios sorted on operating profitability

Panel A: Conventional portfolio sorts		
	Q1 (low)	Q5 - Q1
a	-0.318***	0.542***
	(-4.26)	(5.48)
b_{RMRF}	1.092***	-0.143***
	(48.10)	(-4.99)
b_{SMB}	0.212***	-0.287***
	(7.65)	(-8.19)
b_{HML}	0.175***	-0.491***
	(4.24)	(-10.35)
R-squared	0.897	0.335
N Obs.	642	642

Panel B: GPS-model (Pooled WLS estimation)			
	Vector $\boldsymbol{z}_{it}$ →	1	$OA_{it}^{(5)}$
← Vector $\boldsymbol{x}_t$	1 (Intercept)	-0.318***	0.542***
		(-4.26)	(5.48)
	$RMRF_t$	1.092***	-0.143***
		(48.10)	(-4.99)
	SMB_t	0.212***	-0.287***
		(7.65)	(-8.19)
	HML_t	0.175***	-0.491***
		(4.24)	(-10.35)
R-squared		0.224	
N Obs.		1,025,809	
N Stocks		14,705	

Panel C: GPS-model (weighted FE estimation)			
	Vector $\boldsymbol{z}_{it}$ →	1	$OA_{it}^{(5)}$
← Vector $\boldsymbol{x}_t$	1 (Intercept)	-0.064	0.031
		(-0.66)	(0.19)
	$RMRF_t$	1.091***	-0.142***
		(48.90)	(-5.03)
	SMB_t	0.209***	-0.283***
		(7.69)	(-8.24)
	HML_t	0.187***	-0.503***
		(4.86)	(-11.25)
within R-squared		0.225	
N Obs.		1,025,809	
N Stocks		14,705	

This table reports value-weighted 3-factor model alphas along with RMRF (market excess return), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted on operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A reports the results from conventional portfolio sorts where the bottom quintile portfolio's excess return (first column) or the return difference between the top quintile and the bottom quintile profitability portfolio (second column) is regressed on the three Fama and French (1993) factors. Statistical inference for the conventional portfolio sorts approach is based on Newey and West (1987) standard errors with a lag-length of three. Panel B reproduces the results from the conventional portfolio sorts by aid of GPS-model (12). The GPS-model is estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks in the bottom- and the top-quintile profitability portfolio, respectively. Panel C presents the results from estimating GPS-model (12) with firm fixed effects (i.e., with the fixed effects estimator). Observation weights are set equal to those in Panel B. Coefficient estimates and *t*-statistics (in parentheses) for the GPS-models in Panels B and C are for the product of the market-level factor variables (plus a constant) contained in vector $\boldsymbol{x}_t$ (displayed on the vertical axis) and firm-characteristics vector $\boldsymbol{z}_{it}$. Vector $\boldsymbol{z}_{it}$ comprises a constant and dummy variable $OA_{it}^{(5)}$, which is one for stocks with top-quintile profitability (based on NYSE breakpoints at the end of each June) and zero otherwise. The GPS-models in Panels B and C only include firms, which belong to the top- or bottom-quintile groups of firms sorted on operating profitability. Statistical inference on the GPS-models is based on Driscoll and Kraay (1998) standard errors with a lag-length of three. The sample period is from July 1963 through December 2016. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 5: Portfolio sorts specification test

Panel A: Operating profitability

	Coefficient estimates on explanatory variables $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$ (coefficient vector $\boldsymbol{\theta}$)					Coefficient estimates on time-series averages (coefficient vector $\boldsymbol{\xi}$)			
	1	$OA_{it}^{(2)}$	$OA_{it}^{(3)}$	$OA_{it}^{(4)}$	$OA_{it}^{(5)}$	$OA_{it}^{(2)}$	$OA_{it}^{(3)}$	$OA_{it}^{(4)}$	$OA_{it}^{(5)}$
1 (Intercept)	-0.715***	0.034	0.055	-0.073	0.018	0.983***	0.712**	1.041***	1.099***
	(-4.79)	(0.33)	(0.53)	(-0.59)	(0.13)	(2.98)	(2.41)	(3.68)	(3.42)
RMRF	1.092***	-0.135***	-0.149***	-0.083***	-0.143***	-0.210	-0.138	-0.183	-0.122
	(47.89)	(-4.31)	(-5.12)	(-2.71)	(-4.97)	(-1.16)	(-0.83)	(-0.78)	(-0.43)
SMB	0.213***	-0.153***	-0.269***	-0.262***	-0.288***	0.407**	0.511**	0.393	0.285
	(7.66)	(-2.90)	(-6.35)	(-6.20)	(-8.18)	(2.09)	(2.00)	(1.46)	(0.90)
HML	0.175***	0.083	-0.037	-0.128**	-0.491***	-0.693***	-0.581**	-0.016	-0.244
	(4.23)	(1.21)	(-0.69)	(-2.08)	(-10.39)	(-2.80)	(-2.04)	(-0.05)	(-0.72)
R-squared	0.228					Portfolio sorts specification test:		*F(16, 641)*	4.612
N Obs.	1,969,221							*p-value*	0.000

Panel B: Gross profitability

	Coefficient estimates on explanatory variables $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$ (coefficient vector $\boldsymbol{\theta}$)					Coefficient estimates on time-series averages (coefficient vector $\boldsymbol{\xi}$)			
	1	$GA_{it}^{(2)}$	$GA_{it}^{(3)}$	$GA_{it}^{(4)}$	$GA_{it}^{(5)}$	$GA_{it}^{(2)}$	$GA_{it}^{(3)}$	$GA_{it}^{(4)}$	$GA_{it}^{(5)}$
1 (Intercept)	-0.265***	-0.074	0.078	0.130	0.420**	0.325	0.075	0.332	0.277
	(-2.96)	(-0.69)	(0.61)	(0.82)	(2.56)	(1.37)	(0.34)	(0.97)	(0.82)
RMRF	0.940***	0.077***	0.062**	0.074***	-0.029	0.092	0.064	-0.140	-0.070
	(51.13)	(2.85)	(2.28)	(2.76)	(-0.91)	(0.46)	(0.36)	(-0.56)	(-0.24)
SMB	0.007	-0.069	0.029	-0.013	-0.050	0.517**	0.772***	0.534**	0.282
	(0.26)	(-1.37)	(0.69)	(-0.29)	(-1.13)	(2.49)	(3.05)	(2.29)	(0.95)
HML	0.197***	-0.020	-0.148***	-0.431***	-0.487***	-0.556**	-0.141	-0.295	-0.449
	(5.87)	(-0.35)	(-3.37)	(-9.94)	(-8.71)	(-2.02)	(-0.46)	(-0.80)	(-1.17)
R-squared	0.236					Portfolio sorts specification test:		*F(16, 641)*	1.194
N Obs.	1,969,221							*p-value*	0.267

Table 5 – continued

This table reports the results from estimating regression $r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \bar{\boldsymbol{q}}_i\boldsymbol{\xi} + \varepsilon_{it}$. Panel A contains the results for portfolio sorts on operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Vector $\boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_{it} = [\,1 \quad OA_{it}^{(2)} \quad \dots \quad OA_{it}^{(5)}\,]$, where $OA_{it}^{(p)}$ $(p = 2, \dots, 5)$ is a dummy variable with value one if firm i belongs to operating profitability quintile portfolio p. Panel B presents the results for portfolio sorts on gross profitability, defined as gross profit deflated by the book value of total assets. Here, vector $\boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_{it} = [\,1 \quad GA_{it}^{(2)} \quad \dots \quad GA_{it}^{(5)}\,]$ where $GA_{it}^{(p)}$ $(p = 2, \dots, 5)$ is a dummy variable with value one if firm i belongs to gross profitability quintile portfolio p. In both panels, vector $\boldsymbol{x}_t$ is set to $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t\,]$ with RMRF (market excess return), SMB (small minus big), and HML (high minus low) representing monthly Fama and French (1993) factor returns. Vector $\bar{\boldsymbol{q}}_i$ comprises the firm-specific time-series averages from all variables in $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$ that vary across both firms and time. Estimation results for coefficient vectors $\boldsymbol{\theta}$ and $\boldsymbol{\xi}$ are reported in matrix form. Each coefficient estimate and *t*-statistic (in parentheses) is for the product of the market-level factor variable in vector $\boldsymbol{x}_t$ (displayed on the vertical axis) and the portfolio dummy variable (or constant) in vector $\boldsymbol{z}_{it}$ (left panel), or for the time-series average of the interaction between the factor variable in $\boldsymbol{x}_t$ and the portfolio dummy variable in $\boldsymbol{z}_{it}$ (right panel). The regressions are estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks in the quintile portfolios sorted on operating profitability (Panel A) and gross profitability (Panel B), respectively. The portfolio sorts specification test displayed in the lower right part of each Panel is a Wald-test on $H_0\colon \boldsymbol{\xi} = 0$. Statistical inference is based on Driscoll and Kraay (1998) standard errors with a lag-length of three months. The sample period is from July 1963 through December 2016. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 6: Continuous and multivariate firm characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: GPS-models estimated with weighted pooled OLS						
Constant	-0.317***	-0.248***	0.275**	0.343***	0.096	-0.008
	(-4.22)	(-3.46)	(2.54)	(3.02)	(0.80)	(-0.06)
Operating Profitability	1.743***					1.105***
	(4.97)					(2.61)
Gross Profitability		0.778***			0.719***	0.338
		(4.47)			(4.14)	(1.58)
Volatility			-5.423*			-0.142
			(-1.94)			(-0.05)
Beta				-0.290**	-0.315***	-0.303**
				(-2.57)	(-2.75)	(-2.52)
R-squared	0.236	0.235	0.264	0.281	0.286	0.295
N Obs.	2,115,518	2,115,518	2,289,867	2,275,370	2,073,983	2,059,734
N Stocks	17,008	17,008	19,109	19,124	16,949	16,908
Panel B: GPS-model specification test						
F(4, 641)	4.429***	2.110*	3.486***	2.400**	2.957***	5.466***
p-value of specification test	0.002	0.078	0.008	0.049	0.003	0.000
Panel C: GPS-models including firm fixed effects (weighted FE estimation)						
Constant	-0.0826	-0.335***	0.0440	0.422***	0.093	-0.063
	(-0.82)	(-3.32)	(0.33)	(3.05)	(0.58)	(-0.37)
Operating profitability	0.614					-1.583***
	(1.31)					(-2.63)
Gross profitability		1.003***			0.913***	1.744***
		(3.99)			(3.75)	(5.40)
52w rolling Vola			0.269			4.672
			(0.08)			(1.60)
52w rolling Beta				-0.370***	-0.385***	-0.420***
				(-2.67)	(-2.72)	(-2.95)
within R-squared	0.237	0.236	0.265	0.282	0.287	0.296
N Obs.	2,115,518	2,115,518	2,289,867	2,275,370	2,073,983	2,059,734
N Stocks	17,008	17,008	19,109	19,124	16,949	16,908

Table 6 – continued

This table reports the coefficient estimates and *t*-statistics (in parentheses) from GPS-models with multivariate and continuous firm characteristics in vector $\boldsymbol{z}_{it}$. The GPS-model is specified as

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} = ([1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}] \otimes [\,1 \quad RMRF_t \quad SMB_t \quad HML_t])\,\boldsymbol{\theta} + \nu_{it}$$

OA_{it} is operating profitability (defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets). GA_{it} is gross profit deflated by the book value of total assets. OA_{it} and GA_{it} are formed at the end of each June and then remain unchanged throughout the subsequent year. $Vola_{it}$ refers to the standard deviation of weekly returns measured over rolling 52 weeks ending on the last Friday (or, in case of a bank holiday, the subsequent trading day) prior to the end of month t. Likewise, $Beta_{it}$ is the CAPM-beta of weekly returns measured over rolling 52 weeks ending on the last Friday (or, in case of a bank holiday, the subsequent trading day) prior to the end of month t. The table only displays the results for coefficient estimates (of subsets) of the variables in vector $\boldsymbol{z}_{it}$. The Fama-French three-factor model alpha (as a measure of the risk-adjusted performance) conditional on firm i's characteristics in period t is obtained as $\hat{\alpha}_{it} = \hat{\theta}_0 + \hat{\theta}_1 \times OPAT_{it} + \hat{\theta}_2 \times GPAT_{it} + \hat{\theta}_3 \times Vola_{it} + \hat{\theta}_4 \times Beta_{it}$. The results for the interactions of vector $\boldsymbol{z}_{it}$ firm characteristics with the market ($RMRF_t$), SMB_t (small minus big), and HML_t (high minus low) factors are not shown in the table. The sample period is from July 1963 through December 2016. The GPS-models in Panel A are estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks. Panel B displays the results from the GPS-model specification test discussed in Section 2.3.3. To this end, the GPS-models from Panel A are extended with a series of control variables. These control variables are the firm-level time averages for all variables that vary over both the cross-section and time. The GPS-model specification test then tests by aid of a standard Wald test whether the coefficient estimates for the control variables are jointly equal to zero. Panel C reports the results from estimating the GPS-models from Panel A with the fixed effects (FE) estimator, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks. *t*-statistics test for significance against a value of zero. Statistical inference is based on Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Appendix A: Proof of Propositions 1 to 3

In this appendix, we first set the ground by rewriting both the GPS-model and the portfolio sorts approach in matrix notation. We then perform a series of basic transformations that apply to all three propositions stated in Section 2. Finally, we proof Propositions 1 to 3 mathematically. For ease of mathematical tractability and as outlined in Section 2, we thereby restrict our formal analysis to the case of a balanced panel (N firms with T regularly spaced observations), time-constant firm characteristics (i.e., $\boldsymbol{z}_{it} \equiv \boldsymbol{z}_i$), and equally weighted portfolios (i.e., $w_{it} = 1/N$). Under these simplifying assumptions, the GPS-model can reproduce the results of the portfolio sorts approach by aid of standard pooled OLS where all observations are equally weighted.

A.1 Matrix notation and proof of Proposition 1

A.1.1 Coefficient estimates and standard errors for the GPS-model

Applying matrix notation, we can write the GPS-model in (1) as[23]

$$\begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{1T} \\ r_{21} \\ \vdots \\ r_{NT} \end{bmatrix} = \left(\begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \vdots \\ \boldsymbol{z}_N \end{bmatrix} \otimes \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix} \right) \boldsymbol{\theta} + \begin{bmatrix} \nu_{11} \\ \nu_{12} \\ \vdots \\ \nu_{1T} \\ \nu_{21} \\ \vdots \\ \nu_{NT} \end{bmatrix} \tag{A1}$$

or, more briefly:

$$vec(\boldsymbol{R}) = \begin{bmatrix} \boldsymbol{\gamma}_1 \\ \boldsymbol{\gamma}_2 \\ \vdots \\ \boldsymbol{\gamma}_N \end{bmatrix} = (\boldsymbol{Z} \otimes \boldsymbol{X})\, \boldsymbol{\theta} + \boldsymbol{\nu} \tag{A2}$$

where $vec(\boldsymbol{R})$ represents a $(NT \times 1)$-vector of the firms' period t (excess) returns, $\boldsymbol{z}_i$ is a $(1 \times M)$-vector of firm characteristics (which are assumed to remain constant over time), $\boldsymbol{x}_t = [1 \;\; x_{1t} \;\; \cdots \;\; x_{Kt}]$ refers to a $(1 \times (K+1))$-vector of market-level variables (which apart from the constant change over time but do not vary in the cross-section), and $(\boldsymbol{Z} \otimes \boldsymbol{X})$ denotes the Kronecker product of $(N \times M)$-dimensional matrix $\boldsymbol{Z} = [\boldsymbol{z}_1' \;\; ... \;\; \boldsymbol{z}_N']'$ with $(T \times (K+1))$-dimensional matrix $\boldsymbol{X} = [\boldsymbol{x}_1' \;\; ... \;\; \boldsymbol{x}_T']'$. Estimating regression model (A2) with pooled OLS, and applying the calculus rules for the Kronecker product, yields the following coefficient estimates for $\boldsymbol{\theta}$:

$$\begin{aligned} \widehat{\boldsymbol{\theta}} &= \left((\boldsymbol{Z} \otimes \boldsymbol{X})' (\boldsymbol{Z} \otimes \boldsymbol{X}) \right)^{-1} (\boldsymbol{Z} \otimes \boldsymbol{X})' vec(\boldsymbol{R}) \\ &= (\boldsymbol{Z}'\boldsymbol{Z} \otimes \boldsymbol{X}'\boldsymbol{X})^{-1} (\boldsymbol{Z}' \otimes \boldsymbol{X}') vec(\boldsymbol{R}) \\ &= ((\boldsymbol{Z}'\boldsymbol{Z})^{-1} \boldsymbol{Z}' \otimes (\boldsymbol{X}'\boldsymbol{X})^{-1} \boldsymbol{X}') vec(\boldsymbol{R}) \end{aligned} \tag{A3}$$

[23] Throughout Appendix A, we assume $c_i = 0$ (for all i) and omit the firm-specific effects c_i from the analysis.

Next, we use the following Lemma from linear algebra (e.g., see Sydsaeter, Strom, and Berck, 2000, p. 146):

Lemma 1. *For any three matrices* $\boldsymbol{A} \in \mathbb{R}^{r,r}$, $\boldsymbol{B} \in \mathbb{R}^{r,s}$, *and* $\boldsymbol{C} \in \mathbb{R}^{s,s}$ *it holds true that* $vec(\boldsymbol{ABC}) = (\boldsymbol{C}'\otimes\boldsymbol{A})vec(\boldsymbol{B})$.

Applying Lemma 1 to expression (A3) above yields

$$\widetilde{\boldsymbol{\theta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1} \tag{A4}$$

Here, $\widetilde{\boldsymbol{\theta}}$ refers to the $((K+1)\times M)$-dimensional matrix of coefficient estimates $\hat{\theta}_{k,m}$ for the interaction of firm characteristic m (with $m = 1, \dots, M$) from vector $\boldsymbol{z}_i$ and market-level factor variable k (with $k = 0, \dots, K$) in vector $\boldsymbol{x}_t$.

We now turn to the Driscoll and Kraay (1998) covariance matrix estimator for the pooled OLS regression model in (A1). For H lags, it has the following structure:

$$\begin{aligned}\widetilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}}) &= \left((\boldsymbol{Z}\otimes\boldsymbol{X})'(\boldsymbol{Z}\otimes\boldsymbol{X})\right)^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\left((\boldsymbol{Z}\otimes\boldsymbol{X})'(\boldsymbol{Z}\otimes\boldsymbol{X})\right)^{-1} \\ &= ((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1})\,\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\,((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1})\end{aligned} \tag{A5}$$

with $\quad \tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \widehat{\boldsymbol{\Omega}}_0 + \sum_{j=1}^{H}\omega_{j,H}\left(\widehat{\boldsymbol{\Omega}}_j + \widehat{\boldsymbol{\Omega}}_j'\right), \quad \widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T}\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}})\,\boldsymbol{h}_{\tau-j}'(\widehat{\boldsymbol{\theta}})$,

and $\quad \boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}}) = (\boldsymbol{Z}\otimes\boldsymbol{x}_\tau)'\,\hat{\boldsymbol{v}}$

The modified Bartlett weights $\omega_{j,H} = 1 - j/(H+1)$ ensure positive semi-definiteness of $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ and smooth the sample autocovariance function such that higher order lags receive less weight.

A.1.2 Coefficient estimates and standard errors for the portfolio sorts approach

The portfolio sorts approach comprises two steps. The first step involves computing the month t average return for portfolio p as outlined in Equation (5). In our case of a balanced panel, time-constant firm characteristics, and equally weighted portfolios (i.e., $w_{it} = N_p^{-1} = \left(\sum_{i=1}^{N} z_i^{(p)}\right)^{-1}$), we can rewrite Equation (5) as follows:

$$r_{pt} = \frac{1}{N_p}\sum_{i=1}^{N} z_i^{(p)} r_{it} = \boldsymbol{r}_t'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A6}$$

with $\boldsymbol{r}_t' = [r_{1t} \quad r_{2t} \quad \dots \quad r_{Nt}]$ and $\boldsymbol{d}_p' = \left[z_1^{(p)} \quad z_2^{(p)} \quad \dots \quad z_N^{(p)}\right]$.

Here, $z_i^{(p)}$ is a dummy variable with value one if firm i belongs to portfolio p, and zero otherwise. In the second step of the procedure, r_{pt} from (A6) is regressed on a constant and the K factor variables as outlined in Equation (6). This yields OLS coefficient estimates as follows:

$$\widehat{\boldsymbol{\beta}}_p = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} r_{p1} \\ \vdots \\ r_{pT} \end{bmatrix} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} \boldsymbol{r}_1'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \\ \vdots \\ \boldsymbol{r}_T'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \end{bmatrix}$$

$$= (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} \boldsymbol{r}_1' \\ \vdots \\ \boldsymbol{r}_T' \end{bmatrix}\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'[\boldsymbol{\gamma}_1 \;\; \boldsymbol{\gamma}_2 \;\; \ldots \;\; \boldsymbol{\gamma}_N]\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}$$

$$= (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A7}$$

The formula for computing the Newey and West (1987) covariance matrix with lag length H for the regression model in Equation (6) has the following structure:

$$\widehat{\boldsymbol{V}}\left(\widehat{\boldsymbol{\beta}}_p\right) = (\boldsymbol{X}'\boldsymbol{X})^{-1}\widehat{\boldsymbol{S}}_{\boldsymbol{T}}(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

$$\text{with} \qquad \widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T}\hat{\varepsilon}_{pt}^2\,\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\hat{\varepsilon}_{p\tau}\hat{\varepsilon}_{p\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \tag{A8}$$

Restricting the sample to two portfolios (or groups of firms), arbitrarily denoted as $p =$ "$high$" and $p =$ "low", we can use matrix notation to rewrite Equation (10) computing the month t return difference as follows:

$$\Delta r_{p,t} = r_{high,t} - r_{low,t} = \boldsymbol{r}_t'\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_2 = \begin{bmatrix} r_{low,t} & \Delta r_{p,t} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{A9}$$

where matrix $\boldsymbol{Z}$ is specified as $\boldsymbol{Z} = [\iota \quad \boldsymbol{d}_{high}]$ and ι is a $(N \times 1)$-dimensional vector of ones. When regressing $\Delta r_{p,t}$ from (A9) on a constant and the K factor variables according to Equation (11), one obtains the following OLS coefficient estimates:

$$\widehat{\boldsymbol{\beta}}_\Delta = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} \Delta r_{p,1} \\ \vdots \\ \Delta r_{p,T} \end{bmatrix} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_2 \tag{A10}$$

The Newey and West (1987) covariance matrix estimator for the coefficient estimates in (A10) has the same structure as the one displayed in Equation (A8), with $\hat{\varepsilon}_{\Delta t}$ replacing $\hat{\varepsilon}_t$ in the formula.

A.1.3 Proof of Proposition 1

Proposition 1 states that the GPS-model can reproduce the results of the portfolio sorts approach for the case of a single portfolio if vector $\boldsymbol{z}_i \equiv \boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_i = [\,1\,]$. In this case, matrix $\boldsymbol{Z}$ in Equation (A2) is given as $\boldsymbol{Z} = \iota$. As a result, the coefficient estimates of the GPS-model in this case are

$$\widetilde{\boldsymbol{\theta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\iota(\iota'\iota)^{-1} \tag{A11}$$

When there is only a single subject group, then $z_i^{(p)}$ for all firms i is equal to 1, i.e., $\boldsymbol{d}_p = \iota$. Consequently, the coefficient estimates for the portfolio sorts approach in Equation (A7) are equal to

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\iota(\iota'\iota)^{-1} \tag{A12}$$

As stated in Part A of Proposition 1, we thus have $\widetilde{\boldsymbol{\theta}} \equiv \widehat{\boldsymbol{\beta}}$. This completes the first part of the proof. ■

We next turn to the standard errors for the coefficient estimates. With $r_{pt} = N^{-1}\sum_{i=1}^{N} r_{it}$ and $\widetilde{\boldsymbol{\theta}} = \widehat{\boldsymbol{\beta}}$, the (estimated) residual $\hat{\varepsilon}_t$ in Equation (3) is equal to

$$\hat{\varepsilon}_t = N^{-1}\sum_{i=1}^{N} \hat{v}_{it} \equiv N^{-1}V_t \tag{A13}$$

where $\hat{v}_{it}$ is the (estimated) residual from pooled OLS regression (4). Replacing $\hat{\varepsilon}_t$ in (A8) by the corresponding term from (A13) yields

$$N^2\,\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} V_t^2\,\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_\tau V_{\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \tag{A14}$$

In case of the GPS-model, we plug in $\boldsymbol{Z} = \iota$ in Equation (A5). This gives

$$\widetilde{\boldsymbol{V}}\left(\widehat{\boldsymbol{\theta}}\right) = ((\iota'\iota)^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1})\,\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\,((\iota'\iota)^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}) = (\boldsymbol{X}'\boldsymbol{X})^{-1}\frac{\tilde{\boldsymbol{S}}_{\boldsymbol{T}}}{N^2}(\boldsymbol{X}'\boldsymbol{X})^{-1} \tag{A15}$$

Comparing $\widetilde{\boldsymbol{V}}\left(\widehat{\boldsymbol{\theta}}\right)$ in (A15) with $\widehat{\boldsymbol{V}}\left(\widehat{\boldsymbol{\beta}}\right)$ from (A8) in case of a single subject group, we hence have to show that $N^{-2}\tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \widehat{\boldsymbol{S}}_{\boldsymbol{T}}$. Rewriting $\boldsymbol{h}_\tau\left(\widehat{\boldsymbol{\theta}}\right)$ in (A5), we obtain:

$$\boldsymbol{h}_\tau\left(\widehat{\boldsymbol{\theta}}\right) = (\iota\otimes\boldsymbol{x}_\tau)'\,\hat{\boldsymbol{v}} = [\boldsymbol{x}_\tau' \quad \cdots \quad \boldsymbol{x}_\tau']\hat{\boldsymbol{v}} = \begin{bmatrix} \sum_{i=1}^{N}\hat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N}\hat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N}\hat{v}_{i\tau} \end{bmatrix} = \boldsymbol{x}_\tau' V_\tau \tag{A16}$$

From (A16) it follows for $\widehat{\boldsymbol{\Omega}}_j$ in (A5) that $\widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T}\boldsymbol{h}_\tau\left(\widehat{\boldsymbol{\theta}}\right)\boldsymbol{h}_{\tau-j}'\left(\widehat{\boldsymbol{\theta}}\right) = \sum_{\tau=j+1}^{T} V_\tau V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j}$, and consequently

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} V_t^2\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_\tau V_{\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \equiv N^2\,\widehat{\boldsymbol{S}}_{\boldsymbol{T}} \tag{A17}$$

This completes the proof. ■

A.2 Proof of Proposition 2

Proposition 2 states that the GPS-model can reproduce the results of the portfolio sorts approach for multiple sorted portfolios by estimating a single pooled OLS regression on the individual firm level. To this end, we specify vector $\boldsymbol{z}_i$ as $\boldsymbol{z}_i = [\,z_i^{(1)} \;\; z_i^{(2)} \;\ldots\; z_i^{(P)}]$. Using the definition of $\boldsymbol{d}_p$ in (A6), $(N \times P)$-matrix $\boldsymbol{Z}$ in equation (A2) is given as follows:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_N \end{bmatrix} = \begin{bmatrix} z_1^{(1)} & \cdots & z_1^{(P)} \\ \vdots & & \vdots \\ z_N^{(1)} & \cdots & z_N^{(P)} \end{bmatrix} = [\boldsymbol{d}_1 \quad \cdots \quad \boldsymbol{d}_P] \tag{A18}$$

The coefficient estimates for the GPS-model are derived in (A4). Specifying matrix $\boldsymbol{Z}$ according to (A18) thus results in a $((K+1) \times P)$-dimensional matrix of coefficient estimates $\widetilde{\boldsymbol{\theta}}$. The p-th column of results matrix $\widetilde{\boldsymbol{\theta}}$ can be obtained as follows:

$$\widetilde{\boldsymbol{\theta}}_p = \widetilde{\boldsymbol{\theta}}\boldsymbol{e}_p = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p \tag{A19}$$

where $\boldsymbol{e}_p$ is a P-dimensional vector of zeroes with a one on position p. Proposition 2 claims that $\widetilde{\boldsymbol{\theta}}_p \equiv \widehat{\boldsymbol{\beta}}_p$ where $\widehat{\boldsymbol{\beta}}_p$ refers to the portfolio p coefficient estimates from estimating the second-step time-series regression of the portfolio sorts approach. The respective coefficient estimates have been derived in Equation (A7). We thus have to proof that $\widetilde{\boldsymbol{\theta}}_p \equiv \widehat{\boldsymbol{\beta}}_p = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}$ or, equivalently, that

$$\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p \equiv \boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A20}$$

To show that (A20) indeed is an identity, we note that $\boldsymbol{d}_p = \boldsymbol{Z}\boldsymbol{e}_p$ and that $(\boldsymbol{Z}'\boldsymbol{Z})^{-1}$ is a diagonal matrix with element $(\boldsymbol{Z}'\boldsymbol{Z})_{p,p}^{-1} = \left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}$. With $(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p = \left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}\boldsymbol{e}_p = \boldsymbol{e}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}$ it therefore holds true that

$$\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p = \underbrace{\boldsymbol{Z}\boldsymbol{e}_p}_{\boldsymbol{d}_p}\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} = \boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A21}$$

This shows that $\widetilde{\boldsymbol{\theta}}_p \equiv \widehat{\boldsymbol{\beta}}_p$ and, hence, completes the first part of the proof of Proposition 2. ■

Next, we turn to the standard errors for the coefficient estimates. With $r_{pt} = N_p^{-1}\sum_{i=1}^{N} z_i^p r_{it}$ and $\widetilde{\boldsymbol{\theta}}_p = \widehat{\boldsymbol{\beta}}_p$, the (estimated) residual $\hat{\varepsilon}_{pt}$ for the portfolio sorts approach in equation (6) is equal to

$$\hat{\varepsilon}_{pt} = N_p^{-1}\sum_{i=1}^{N} z_i^{(p)}\hat{v}_{it} \equiv N_p^{-1}V_{pt} \tag{A22}$$

where $\hat{v}_{it}$ is the (estimated) residual from pooled OLS regression (7). Replacing $\hat{\varepsilon}_{pt}$ in (A8) by the corresponding term from (A22) yields

$$N_p^2\,\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} V_{pt}^2\,\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{p\tau}V_{p,\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \equiv \widehat{\boldsymbol{S}}_{\boldsymbol{T}}^{(p)} \tag{A23}$$

As a consequence, we finally obtain the Newey and West (1987) standard errors in case of the portfolio sorts approach as follows:

$$\widehat{\boldsymbol{V}}\left(\widehat{\boldsymbol{\beta}}_p\right) = N_p^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\widehat{\boldsymbol{S}}_{\boldsymbol{T}}^{(p)}(\boldsymbol{X}'\boldsymbol{X})^{-1} \tag{A24}$$

We now consider the GPS-model with matrix $\boldsymbol{Z}$ being defined according to Equation (A18). The $\big(P \times (K+1)\big)$-dimensional column vector $\boldsymbol{h}_\tau\big(\widehat{\boldsymbol{\theta}}\big)$ from (A5) in this case is equal to

$$\boldsymbol{h}_\tau\big(\widehat{\boldsymbol{\theta}}\big) = (\boldsymbol{Z}' \otimes \boldsymbol{x}_\tau')\,\hat{\boldsymbol{v}} = \begin{bmatrix} \sum_{i=1}^{N} z_i^{(1)} \hat{v}_{i\tau} \\ x_{1\tau} \sum_{i=1}^{N} z_i^{(1)} \hat{v}_{i\tau} \\ \vdots \\ x_{K\tau} \sum_{i=1}^{N} z_i^{(1)} \hat{v}_{i\tau} \\ \sum_{i=1}^{N} z_i^{(2)} \hat{v}_{i\tau} \\ x_{1\tau} \sum_{i=1}^{N} z_i^{(2)} \hat{v}_{i\tau} \\ \vdots \\ x_{K\tau} \sum_{i=1}^{N} z_i^{(P)} \hat{v}_{i\tau} \end{bmatrix} = \begin{bmatrix} V_{1\tau} \\ x_{1\tau} V_{1\tau} \\ \vdots \\ x_{K\tau} V_{1\tau} \\ V_{2\tau} \\ x_{1\tau} V_{2\tau} \\ \vdots \\ x_{K\tau} V_{P\tau} \end{bmatrix} = (\boldsymbol{I}_P \otimes \boldsymbol{x}_\tau') \begin{bmatrix} V_{1\tau} \\ \vdots \\ V_{P\tau} \end{bmatrix} \equiv (\boldsymbol{I}_P \otimes \boldsymbol{x}_\tau')\boldsymbol{V}_\tau \qquad \text{(A25)}$$

where $\boldsymbol{I}_P$ is the P-dimensional identity matrix. With (A25) it follows for $\widehat{\boldsymbol{\Omega}}_j$ in Equation (A5) that

$$\widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T} (\boldsymbol{I}_P \otimes \boldsymbol{x}_\tau')\boldsymbol{V}_\tau \boldsymbol{V}_{\tau-j}' \big(\boldsymbol{I}_P \otimes \boldsymbol{x}_{\tau-j}\big) = \sum_{\tau=j+1}^{T} \big(\boldsymbol{V}_\tau \boldsymbol{V}_{\tau-j}'\big) \otimes \big(\boldsymbol{x}_\tau' \boldsymbol{x}_{\tau-j}\big) \qquad \text{(A26)}$$

As a result, matrix $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ in (A5) can be written as follows:

$$\begin{aligned} \tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \; & \sum_{t=1}^{T} (\boldsymbol{V}_t \boldsymbol{V}_t') \otimes (\boldsymbol{x}_t' \boldsymbol{x}_t) \\ & + \sum_{j=1}^{H} \omega_{j,H} \sum_{\tau=j+1}^{T} \Big(\big(\boldsymbol{V}_\tau \boldsymbol{V}_{\tau-j}'\big) \otimes \big(\boldsymbol{x}_\tau' \boldsymbol{x}_{\tau-j}\big) + \big(\boldsymbol{V}_{\tau-j} \boldsymbol{V}_\tau'\big) \otimes \big(\boldsymbol{x}_{\tau-j}' \boldsymbol{x}_\tau\big) \Big) \end{aligned} \qquad \text{(A27)}$$

We next define $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,q)}$ as follows

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,q)} \equiv \sum_{t=1}^{T} V_{pt} V_{qt} (\boldsymbol{x}_t' \boldsymbol{x}_t) + \sum_{j=1}^{H} \omega_{j,H} \sum_{\tau=j+1}^{T} \Big(V_{p\tau} V_{q,\tau-j} \big(\boldsymbol{x}_\tau' \boldsymbol{x}_{\tau-j}\big) + V_{p,\tau-j} V_{q\tau} \big(\boldsymbol{x}_{\tau-j}' \boldsymbol{x}_\tau\big) \Big) \qquad \text{(A28)}$$

where V_{pt} is a scalar as in (A22) above. Consequently, matrix $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ is equal to

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \begin{bmatrix} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} & \cdots & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,P)} \\ \vdots & \ddots & \vdots \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,1)} & \cdots & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,P)} \end{bmatrix} \qquad \text{(A29)}$$

Moreover, with matrix $\boldsymbol{Z}$ being defined according to (A18), $\boldsymbol{Z}'\boldsymbol{Z}$ now is a $(P \times P)$-dimensional diagonal matrix with element (p,p) equal to N_p and all off-diagonal elements equal to zero. With $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ structured according to Expression (A29), we can thus rewrite $\widetilde{\boldsymbol{V}}\big(\widehat{\boldsymbol{\theta}}\big)$ in (A5) as follows:

$$\widetilde{\boldsymbol{V}}\big(\widehat{\boldsymbol{\theta}}\big) = \begin{bmatrix} N_1^{-2} (\boldsymbol{X}'\boldsymbol{X})^{-1} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} (\boldsymbol{X}'\boldsymbol{X})^{-1} & \cdots & N_1^{-1} N_P^{-1} (\boldsymbol{X}'\boldsymbol{X})^{-1} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,P)} (\boldsymbol{X}'\boldsymbol{X})^{-1} \\ \vdots & \ddots & \vdots \\ N_1^{-1} N_P^{-1} (\boldsymbol{X}'\boldsymbol{X})^{-1} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,1)} (\boldsymbol{X}'\boldsymbol{X})^{-1} & \cdots & N_P^{-2} (\boldsymbol{X}'\boldsymbol{X})^{-1} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,P)} (\boldsymbol{X}'\boldsymbol{X})^{-1} \end{bmatrix} \qquad \text{(A30)}$$

The second part of Proposition 2 claims that $\mathrm{SE}(\hat{\theta}_{p,k}) = \mathrm{SE}(\hat{\beta}_{p,k})$ for $k = 0, 1, \ldots, K$ and $p = 1, \ldots, P$. To proof that this holds true, it is sufficient to show that $N_p^{-2} (\boldsymbol{X}'\boldsymbol{X})^{-1} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,p)} (\boldsymbol{X}'\boldsymbol{X})^{-1}$ in Expression (A30)

is identical to $\widehat{\boldsymbol{V}}\left(\widehat{\boldsymbol{\beta}}_p\right)$ in Equation (A24) for every $p = 1, \dots, P$. This in turn is equivalent to demonstrating that $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}^{(p)}$ from (A23) coincides with $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,p)}$ in (A28). By comparing the two expressions we see that this holds true, which completes the proof. ■

A.3 Proof of Proposition 3

Proposition 3 states that the GPS-model can reproduce the results of the portfolio sorts approach for the case of performance differences between two portfolios. In order to compare the performance of firms in group "high" with that of firms belonging to group "low", we have to specify vector $\boldsymbol{z}_i$ as $\boldsymbol{z}_i = [\,1 \;\; z_i^{(high)}\,]$ such that the $(N \times 2)$-dimensional matrix $\boldsymbol{Z}$ comprising the characteristics of all N firms is equal to $\boldsymbol{Z} = [\iota \quad \boldsymbol{d}_{high}]$. This matches the definition of matrix $\boldsymbol{Z}$ in Expression (A9) which is used for deriving the results of performance differences in case of the portfolio sorts approach. Estimating the GPS-model in (A2) with pooled OLS yields the coefficient estimates, structured as a $((K+1) \times 2)$-dimensional matrix, in Equation (A4).

The second column of $\widetilde{\boldsymbol{\theta}}$ in (A4) contains the coefficient estimates for the interaction terms between dummy variable $z_i^{(high)}$ and the factor variables in vector $\boldsymbol{x}_t$. In Equation (12), those coefficient estimates are named $\hat{\theta}_{\Delta k}$. To extract the coefficient estimates for $\hat{\theta}_{\Delta k}$ from $\widetilde{\boldsymbol{\theta}}$, we post-multiply Expression (A4) with $\boldsymbol{e}_2$. The resulting term coincides with the one in Equation (A10) for the portfolio sorts approach. This shows that $\hat{\beta}_{\Delta\, k} \equiv \hat{\theta}_{\Delta\, k}$ $(\forall\; k = 0, 1, \dots, K)$.

The first column of $\widetilde{\boldsymbol{\theta}}$ in (A4) comprises the coefficient estimates for subject group "low". In Equation (12), the respective coefficient estimates are labeled as $\hat{\theta}_{low,k}$ (with $k = 0, 1, \dots, K$). The coefficient estimates for $\theta_{low,k}$ are retrieved by post-multiplying (A4) with $\boldsymbol{e}_1$. In case of the portfolio sorts approach, we obtain the coefficient estimates for the "low" portfolio ($\hat{\beta}_{low,k}$) by repeating the analysis of (A9) and (A10) with $\boldsymbol{e}_1$ replacing $\boldsymbol{e}_2$. The resulting expressions for the portfolio sorts approach and the GPS-model again coincide. This demonstrates $\hat{\beta}_{low,k} \equiv \hat{\theta}_{low,k}$ $(\forall\; k = 0, 1, \dots, K)$ and, hence, completes the first part of the proof. ■

For the second part of the proof, we note that due to $r_{high,t} = r_{low,t} + \Delta\, r_{p,t}$ the following corollary holds true:

Corollary 2. $\qquad \hat{\beta}_{high,k} = \hat{\beta}_{low,k} + \hat{\beta}_{\Delta\, k} = \hat{\theta}_{low,k} + \hat{\theta}_{\Delta\, k}$ for all $k = 0, 1, \dots, K.$ (A31)

Based on Corollary 2 and Proposition 2, and because of $r_{pt} = N_p^{-1} \sum_{i=1}^{N} z_i^{(p)} r_{it}$ (for p = "low", "high"), residual $\hat{\varepsilon}_{\Delta t}$ in the second-step time-series regression (11) of the portfolio sorts approach is equal to

$$\hat{\varepsilon}_{\Delta t} = N_{high}^{-1} \sum_{i=1}^{N} z_i^{(high)} \hat{v}_{it} - N_{low}^{-1} \sum_{i=1}^{N} z_i^{(low)} \hat{v}_{it} \equiv N_{high}^{-1} V_{high,t} - N_{low}^{-1} V_{low,t} \qquad \text{(A32)}$$

where $\hat{v}_{it}$ is the (estimated) residual from pooled OLS regression (7). Replacing $\hat{\varepsilon}_{\Delta t}$ in the Newey and West (1987) covariance matrix estimator (A8) for the coefficient estimates in (A10) by the respective expression in (A32) yields

$$\begin{aligned}\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = &\sum_{t=1}^{T}\left(\frac{V_{high,t}}{N_{high}} - \frac{V_{low,t}}{N_{low}}\right)^2 \boldsymbol{x}_t'\boldsymbol{x}_t \\ &+ \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\left(\frac{V_{high,\tau}}{N_{high}} - \frac{V_{low,\tau}}{N_{low}}\right)\left(\frac{V_{high,\tau-j}}{N_{high}} - \frac{V_{low,\tau-j}}{N_{low}}\right)\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right)\end{aligned} \tag{A33}$$

We now turn to the GPS-model with matrix $\boldsymbol{Z}$ being defined as $\boldsymbol{Z} = [\iota \quad \boldsymbol{d}_{high}]$. The $\big(2 \times (K+1)\big)$-dimensional column vector $\boldsymbol{h}_\tau\big(\widehat{\boldsymbol{\theta}}\big)$ from (A5) in this case is equal to

$$\boldsymbol{h}_\tau\big(\widehat{\boldsymbol{\theta}}\big) = (\boldsymbol{Z}'\otimes\boldsymbol{x}_\tau')\,\hat{\boldsymbol{v}} = \begin{bmatrix} \sum_{i=1}^{N}\hat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N}\hat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N}\hat{v}_{i\tau} \\ \sum_{i=1}^{N} z_i^{(high)}\hat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N} z_i^{(high)}\hat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N} z_i^{(high)}\hat{v}_{i\tau} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_\tau' & \boldsymbol{x}_\tau' \\ \boldsymbol{x}_\tau' & \boldsymbol{0}' \end{bmatrix}\begin{bmatrix} V_{high,\tau} \\ V_{low,\tau} \end{bmatrix} \equiv \begin{bmatrix} \boldsymbol{x}_\tau' & \boldsymbol{x}_\tau' \\ \boldsymbol{x}_\tau' & \boldsymbol{0}' \end{bmatrix}\boldsymbol{V}_\tau \tag{A34}$$

From (A34) and $V_{high,\tau} + V_{low,\tau} \equiv V_\tau$ it follows for $\widehat{\boldsymbol{\Omega}}_j$ in Equation (A5) that

$$\widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T}\begin{bmatrix} V_\tau V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} & V_\tau V_{high,\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} \\ V_{high,\tau}V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} & V_{high,\tau}V_{high,\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} \end{bmatrix} \tag{A35}$$

As a result, matrix $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ in (A5) can be written in block form as follows:

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \begin{bmatrix} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,2)} \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,1)} & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,2)} \end{bmatrix} \tag{A36}$$

where

$$\begin{aligned}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} &= \sum_{t=1}^{T} V_t^2(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_\tau V_{\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,2)} &= \sum_{t=1}^{T} V_t V_{high,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_\tau V_{high,\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + V_{high,\tau}V_{\tau-j}\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right) \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,1)} &= \sum_{t=1}^{T} V_t V_{high,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{high,\tau}V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + V_\tau V_{high,\tau-j}\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right) \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,2)} &= \sum_{t=1}^{T} V_{high,t}^2(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{high,\tau}V_{high,\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right)\end{aligned}$$

Next, we rewrite matrix $(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}$ in (A5) as

$$(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1} = \begin{bmatrix} N_{low}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1} & -N_{low}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1} \\ -N_{low}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1} & \left(N_{low}^{-1} + N_{high}^{-1}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1} \end{bmatrix} \tag{A37}$$

and insert (A37) into the Driscoll and Kraay (1998) covariance matrix estimator of (A5) to obtain

$$\widetilde{\boldsymbol{V}}\left(\widehat{\boldsymbol{\theta}}\right) = \left((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}\right)\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\left((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}\right) = \begin{bmatrix}\widetilde{\boldsymbol{V}}^{(1,1)} & \widetilde{\boldsymbol{V}}^{(1,2)}\\ \widetilde{\boldsymbol{V}}^{(2,1)} & \widetilde{\boldsymbol{V}}^{(2,2)}\end{bmatrix} \tag{A38}$$

where

$$\widetilde{\boldsymbol{V}}^{(1,1)} = N_{low}^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\left(\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} + \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

$$\widetilde{\boldsymbol{V}}^{(1,2)} = N_{low}^{-1}N_{high}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1}\left(\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1} - \widetilde{\boldsymbol{V}}^{(1,1)}$$

$$\widetilde{\boldsymbol{V}}^{(2,1)} = N_{low}^{-1}N_{high}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1}\left(\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1} - \widetilde{\boldsymbol{V}}^{(1,1)}$$

$$\widetilde{\boldsymbol{V}}^{(2,2)} = N_{high}^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}(\boldsymbol{X}'\boldsymbol{X})^{-1} - \widetilde{\boldsymbol{V}}^{(1,1)} - \widetilde{\boldsymbol{V}}^{(1,2)} - \widetilde{\boldsymbol{V}}^{(2,1)}$$

According to the second part of Proposition 3, we have to show that $\widetilde{\boldsymbol{V}}^{(2,2)}$ in (A38) coincides with the Newey-West covariance matrix estimator in (A10) with $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ specified according to (A33). Therefore, we simplify the "sandwich" expressions in (A38) as follows

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} + \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})} = \sum_{t=1}^{T} V_{low,t}^{2}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{low,\tau}V_{low,\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})} = \sum_{t=1}^{T} V_{low,t}V_{high,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\left(V_{low,\tau}V_{high,\tau-j}\right)\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \left(V_{low,\tau-j}V_{high,\tau}\right)\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})} = \sum_{t=1}^{T} V_{high,t}V_{low,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\left(V_{high,\tau}V_{low,\tau-j}\right)\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \left(V_{high,\tau-j}V_{low,\tau}\right)\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)$$

and insert the resulting expressions into $\widetilde{\boldsymbol{V}}^{(2,2)}$ from (A38). This finally yields

$$\widetilde{\boldsymbol{V}}^{(2,2)} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\,\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{\boldsymbol{A}}\,(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

with

$$\begin{aligned}\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{\boldsymbol{A}} = &\sum_{t=1}^{T}\left(\frac{V_{high,t}}{N_{high}} - \frac{V_{low,t}}{N_{low}}\right)^{2}\boldsymbol{x}_t'\boldsymbol{x}_t\\ &+\sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\left(\frac{V_{high,\tau}}{N_{high}} - \frac{V_{low,\tau}}{N_{low}}\right)\left(\frac{V_{high,\tau-j}}{N_{high}} - \frac{V_{low,\tau-j}}{N_{low}}\right)\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right)\end{aligned} \tag{A39}$$

Since $\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{\boldsymbol{A}}$ in (A39) and $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ in (A33) coincide, this shows that $\mathrm{SE}(\hat{\theta}_{\Delta k}) = \mathrm{SE}(\hat{\beta}_{\Delta k})$ for all $k = 0, 1, \ldots, K$.

The last part of Proposition (3) claims that $\widetilde{\boldsymbol{V}}^{(1,1)}$ in (A38) coincides with the Newey-West covariance matrix estimator for the second-step time-series regression of the portfolio sorts approach applied to portfolio "low". The respective Newey-West covariance estimator has been derived in Expression (A24) above with p = "low". By replacing $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} + \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}$ with the corresponding term derived above, we finally obtain the following expression for $\widetilde{\boldsymbol{V}}^{(1,1)}$:

$$\widetilde{\boldsymbol{V}}^{(1,1)} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\,\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{high}\,(\boldsymbol{X}'\boldsymbol{X})^{-1} \tag{A40}$$

with

$$\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{high} = \sum_{t=1}^{T} N_{low}^{-2} V_{low,t}^{2}\, \boldsymbol{x}_t' \boldsymbol{x}_t + \sum_{j=1}^{H} \omega_{j,H} \sum_{\tau=j+1}^{T} \left(N_{low}^{-2} V_{low,\tau} V_{low,\tau-j} \left(\boldsymbol{x}_\tau' \boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}' \boldsymbol{x}_\tau \right) \right)$$

Since $\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{high}$ in (A40) and $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ for portfolio p = "low" in (A23) are identical, this demonstrates that $\mathrm{SE}(\hat{\theta}_{\mathrm{low},k}) = \mathrm{SE}(\hat{\beta}_{\mathrm{low},k})$ for all $k = 0, 1, \ldots, K$. This completes the proof of Proposition 3. ■