

Beveridgean Unemployment Gap

Pascal Michaillat, Emmanuel Saez

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This paper estimates the unemployment gap (the actual unemployment rate minus the efficient unemployment rate) using the sufficient-statistic approach from public economics. While lowering unemployment puts more people into work, it forces firms to post more vacancies and devote more resources to recruiting. This unemployment-vacancy tradeoff, governed by the Beveridge curve, determines the efficient unemployment rate. Accordingly, in any model with a Beveridge curve, the unemployment gap depends on three sufficient statistics: elasticity of the Beveridge curve, social cost of unemployment, and cost of recruiting. Applying this novel formula to the United States, we find that the efficient unemployment rate varies between 3.0% and 5.4% since 1951, and has been stable between 3.8% and 4.6% since 1990. As a result, the unemployment gap is countercyclical, reaching 6 percentage points in deep slumps. Thus the US labor market is inefficient—especially inefficiently slack in slumps. The unemployment gap is in turn a crucial statistic to design labor-market and macroeconomic policies.

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1. Introduction

Research question. Does the labor market operate efficiently? If not, how far from efficiency is it? To answer these questions, we measure the unemployment gap—the actual unemployment rate minus the efficient unemployment rate. A reliable measure of the unemployment gap is necessary to a good understanding of the labor market and the macroeconomy. It guides how we model the labor market and the macroeconomy; and it is a key determinant of optimal labor-market policies, such as unemployment insurance, and of optimal macroeconomic policies, including monetary and fiscal policy.

Existing measures of the unemployment gap. Two measures of the unemployment gap are commonly used: the difference between the actual unemployment rate and its trend; and the difference between the actual unemployment rate and the non-accelerating-inflation rate of unemployment (NAIRU). But, although these two measures are easy to use, neither is an ideal measure of the unemployment gap because neither the unemployment-rate trend nor the NAIRU measure the efficient unemployment rate.

Our measure of the unemployment gap. This paper proposes a new measure of the unemployment gap. The measure builds upon the theory of efficiency in modern labor-market models (for example, Hosios 1990). Such models feature both unemployed workers and job vacancies, each associated with welfare costs: more unemployment means fewer people at work so less output; more vacancies mean more work effort devoted to recruiting and also less output. Furthermore, these models feature a Beveridge curve, which relates unemployment to vacancies. Because of the Beveridge curve, unemployment and vacancies cannot be simultaneously reduced: less unemployment requires more vacancies, and fewer vacancies create more unemployment. Our analysis resolves this unemployment-vacancy tradeoff, characterizing the efficiency point on the Beveridge curve.

To develop an unemployment-gap measure that is usable for policy work, we follow the sufficient-statistic method from public economics (Chetty 2009). A first advantage of the method is that it delivers a simple formula, which only involves three statistics. A second advantage is that the formula is easy to apply, because the three statistics are estimable. A third advantage is that the formula requires little theoretical structure. It applies to any labor market with a Beveridge curve, irrespective of the structure of the labor market, production, preferences, wage setting, and shocks; the relevant properties of the model are captured by the sufficient statistics. The formula therefore applies broadly because the Beveridge curve appears in many models,

including the widely used Diamond-Mortensen-Pissarides (DMP) model (Elsby, Michaels, and Ratner 2015).

We obtain the sufficient-statistic formula by solving the problem of a social planner who allocates labor between production, recruiting, and unemployment subject to the Beveridge curve. The formula gives the efficient unemployment rate as a function of actual unemployment and vacancy rates, and three statistics: the elasticity of the Beveridge curve, social cost of unemployment, and cost of recruiting.

Application to the United States. We provide estimates of the three sufficient statistics for the United States. Using standard unemployment and vacancy data, we estimate the elasticity of the Beveridge curve. Although the Beveridge curve is stable for long periods of time, it is also subjects to sudden shifts. To address this issue, we use the method proposed by Bai and Perron (1998, 2003), which allows us to estimate the elasticity while permitting multiple structural changes. We estimate that the Beveridge curve indeed experiences several structural breaks between 1951 and 2019, but the Beveridge elasticity remains fairly stable, in the 0.84–1.02 range. Next, we estimate the social cost of unemployment from the natural and field experiments analyzed by Borgschulte and Martorell (2018) and Mas and Pallais (2019). These experiments suggest that the value from home production and recreation during unemployment only replaces 26% of the marginal product of labor—implying a substantial social cost of unemployment. Last, we estimate the recruiting cost using evidence from the 1997 National Employer Survey conducted by the Census Bureau. The survey indicates that firms allocate 3.2% of labor to recruiting.

Using the estimated statistics, we compute the efficient unemployment rate in the United States between 1951 and 2019. We find that the efficient unemployment rate averages 4.3% between 1951 and 2019. It started around 3.5% in the 1950s, climbed to reach 5.4% in 1979, fell to 4.6% in 1990, and remained in the 3.8%–4.6% range until 2019.

Since the efficient unemployment rate is slow-moving while the actual unemployment rate is countercyclical, the unemployment gap is countercyclical. We infer that the US unemployment gap is almost never zero: the US labor market does not operate efficiently. In fact the unemployment gap is generally positive, averaging 1.4 percentage points over 1951–2019, so the US labor market is generally inefficiently slack. In slumps, the unemployment gap is especially high, reaching for instance 6.2 percentage points in 2010 in the aftermath of the Great Recession, so inefficiencies are exacerbated.

Robustness. We explore the sensitivity of the efficient unemployment rate to alternative values of the sufficient statistics. We find that for a range of plausible estimates of the sufficient statistics,

the unemployment gap remains within 1.2 percentage point of our baseline unemployment gap. This means that our substantive conclusions—that the US labor market is almost always inefficient, generally inefficient slack, and especially inefficiently slack in slumps—are robust to alternative calibrations.

Comparison with the Hosios (1990) condition. Finally, we apply our sufficient-statistic formula to the DMP model, and compare it with the formula arising from the well-known Hosios condition. The two formulas might be different because they solve different planning problems: in the Hosios planning problem unemployment follows a differential equation, whereas in ours unemployment is always on the Beveridge curve. Yet, we find that when the discount rate is zero, the two formulas are the same. And when the discount rate is positive, the two formulas are not the same, but quantitatively they generate almost identical solutions.

2. Beveridgean model of the labor market

We introduce the model of the labor market used to compute the unemployment gap. The model features both unemployed workers and job vacancies. The main ingredient is a Beveridge curve—a negative relation between unemployment and vacancies.

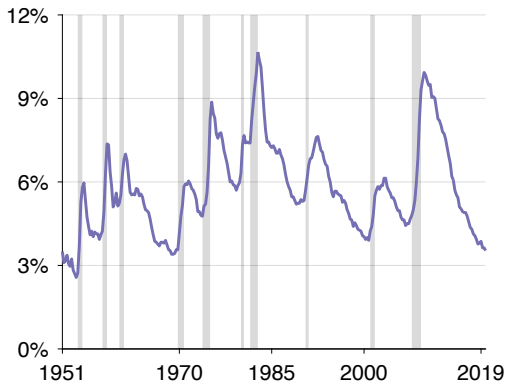
2.1. Notations

The unemployment rate is the number of unemployed workers divided by the size of the labor force; it is denoted by u . The vacancy rate is the number of vacancies divided by the size of the labor force; it is denoted by v . The labor-market tightness is the number of vacancies per unemployed worker; it is denoted by $\theta = v/u$. Last the employment rate is the number of employed workers divided by size of the labor force; it is denoted by n . Since the labor force is the sum of all employed and unemployed workers, employment and unemployment rates are related by $n = 1 - u$.

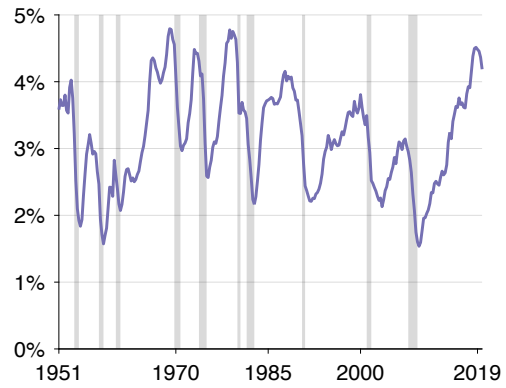
2.2. Beveridge curve

The structure of the labor market imposes a negative relation between unemployment and vacancies:

ASSUMPTION 1. *Unemployment and vacancy rates are related by a Beveridge curve. Thus, the vacancy rate is given by a strictly decreasing and convex function of the unemployment rate, $v(u)$.*



A. Unemployment rate



B. Vacancy rate



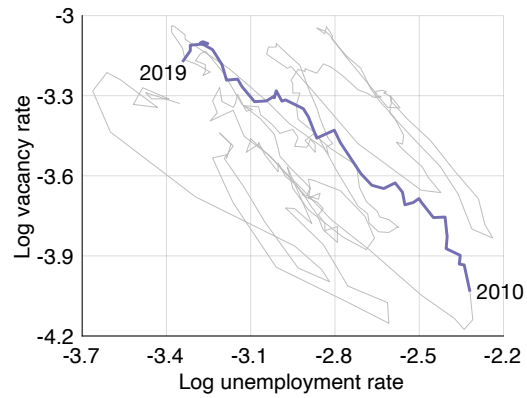
C. Beveridge curve, 1951-1969



D. Beveridge curve, 1970-1989



E. Beveridge curve, 1990-2009



F. Beveridge curve, 2010-2019

FIGURE 1. Beveridge curve in the United States, 1951-2019

Panel A: The unemployment rate is constructed by the BLS from the CPS. Panel B: For 1951-2000, the vacancy rate is constructed by Barnichon (2010) from the Conference Board help-wanted advertising index; for 2001-2019, the vacancy rate is the number of job openings measured by the BLS in JOLTS, divided by the civilian labor force constructed by the BLS from the CPS. Unemployment and vacancy rates are quarterly averages of seasonally adjusted monthly series. The shaded areas represent recessions, as identified by the National Bureau of Economic Research (NBER). Panels C-F display scatterplots of the unemployment and vacancy rates (from panels A and B) on a logarithmic scale. The four panels focus on consecutive periods: 1951-1969 in panel C, 1970-1989 in panel D, 1990-2009 in panel E, and 2010-2019 in panel F.

Beveridge curve in the data. Many countries exhibit a Beveridge curve (Jackman, Pissarides, and Savoury 1990; Nickell et al. 2002; Elsby, Michaels, and Ratner 2015), including the United States (Blanchard and Diamond 1989; Diamond and Sahin 2015; Elsby, Michaels, and Ratner 2015). Our methodology would apply in any of these countries.

As an illustration, we construct the US Beveridge curve. For the unemployment rate, we use the standard measure constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). This unemployment rate is plotted in panel A of figure 1; it averages 5.8% over the period.

For the vacancy rate, we use two different sources because there is no continuous national vacancy series over the period. For 1951–2000, we use the vacancy proxy constructed by Barnichon (2010). Barnichon starts from the help-wanted advertising index constructed by the Conference Board—a proxy for vacancies proposed by Abraham (1987) that has become standard (Shimer 2005, p. 29). He then corrects the Conference Board index, which is based on newspaper advertisements, to take into account the shift from print advertising to online advertising after 1995. Finally, he rescales the index into vacancies, and divides the vacancy number by the size of the labor force to obtain a vacancy rate. For 2001–2019, we obtain the vacancy rate from the number of job openings measured by the BLS in the Job Opening and Labor Turnover Survey (JOLTS), divided by the civilian labor force constructed by the BLS from the CPS. We then splice the Barnichon and JOLTS series to obtain a vacancy rate for 1951–2019. This vacancy rate is plotted in panel B of figure 1; it averages 3.2% over the period.

The Beveridge curve appears in scatterplots of the unemployment and vacancy rates (panels C–F of figure 1; for readability we separately plot the 1951–1969, 1970–1989, 1990–2009, and 2010–2019 periods). The Beveridge curve is stable for long periods, during which unemployment and vacancies moved up and down along a clearly defined branch. Furthermore, until the mid-1980s, the Beveridge curve shifts outward at the end of each period of stability. After the mid-1980s, the Beveridge curve shifts back inward to positions that were typical in the 1960s and 1970s.

Beveridge curve in macroeconomic models. Given the prevalence of the Beveridge curve in labor markets, it is not surprising that many modern labor-market models feature a Beveridge curve (Elsby, Michaels, and Ratner 2015). These models are nested into our framework.

Nested models include models of the labor market build around a matching function: the canonical DMP model (Pissarides 2000, chapter 1), and its variants with rigid wages (Hall 2005a; Hall and Milgrom 2008), large firms (Cahuc, Marque, and Wasmer 2008; Elsby and Michaels 2013), and job rationing (Michaillat 2012). It is true that in many matching models, unemployment follows a law of motion, and the Beveridge curve is defined as the locus of unemployment and

vacancies where the level of unemployment is steady. Yet Pissarides (2009a, p. 236) notes that

“Perhaps surprisingly at first, but on reflection not so surprisingly, we get a good approximation to the dynamics of unemployment if we treat unemployment as if it were always on the Beveridge curve.”

Hence, many matching models assume that the Beveridge curve holds at all times, as we do here (for example, Pissarides 1986, 2009b; Hall 2005a,b; Elsby, Michaels, and Solon 2009; Landais, Michailat, and Saez 2018b).¹

Even labor-market models without a matching function may feature a Beveridge curve: for instance, models of mismatch (Shimer 2007) and of stock-flow matching (Ebrahimi and Shimer 2010).

Beside matching models of the labor market, our framework also nest matching models of the entire economy in which labor market and goods market are combined into one market. In these models all workers are self-employed and sell labor services on a matching market. Examples of such models include the monetary model of Michailat and Saez (2019a) and the fiscal model of Michailat and Saez (2019b).

Sources of fluctuations along the Beveridge curve. Over the business cycle, unemployment and vacancy rates move along the Beveridge curve (figure 1) . What causes such fluctuations? The sources of fluctuations and mechanisms depend on the models.

In the DMP model, shocks to workers’ bargaining power lead to fluctuations along the Beveridge curve (Shimer 2005, table 6). Shocks to labor productivity also lead to fluctuations along the Beveridge curve, although these fluctuations are unrealistically small (Shimer 2005, table 3). In the variants of the DMP model proposed by Hall (2005a), Hall and Milgrom (2008), and Michailat (2012), real wages are rigid, so shocks to labor productivity generate realistic fluctuations along the Beveridge curve.

In the mismatch model proposed by Shimer (2007), shocks to aggregate productivity also generate sizable fluctuations along the Beveridge curve. The same is true in the stock-flow matching proposed by Ebrahimi and Shimer (2010).

Finally, in macroeconomic models, the fluctuations along the Beveridge curve are generated by aggregate demand shocks. In the monetary model of Michailat and Saez (2019a), these are shocks to households’ discount rate, to their marginal utility of wealth, or to their marginal utility of money balances. In the fiscal model developed by Michailat and Saez (2019b), these are shocks to households’ discount rate or to their’ marginal utility of land holdings.

¹In section 5, we apply our method to the DMP model and formally show that unemployment always remains close to the Beveridge curve (see in particular figure 10).

2.3. Beveridge elasticity

Plotted on a logarithmic scale, all branches of the Beveridge curve are close to linear, so each branch is close to isoelastic. A central statistic in measuring the unemployment gap is the elasticity of the Beveridge curve:

DEFINITION 1. *The Beveridge elasticity is the elasticity of the vacancy rate with respect to the unemployment rate along the Beveridge curve, normalized to be positive:*

$$\epsilon = -\frac{d \ln(v(u))}{d \ln(u)} = -\frac{u}{v(u)}v'(u).$$

2.4. Social welfare

The Beveridge curve governs the tradeoff between unemployment and vacancies. This tradeoff is central to the welfare analysis because both unemployment and vacancies influence welfare.

ASSUMPTION 2. *At any point in time, social welfare is a function of the employment rate, unemployment rate, and vacancy rate, $\mathcal{W}(n, u, v)$. The function \mathcal{W} is differentiable, strictly increasing in n , strictly decreasing in v , and less increasing in u than n ($\partial \mathcal{W} / \partial u < \partial \mathcal{W} / \partial n$). As a result, the alternate welfare function $\widehat{\mathcal{W}}(u, v) = \mathcal{W}(1 - u, u, v)$ is strictly decreasing in u and v . Furthermore, the function $\widehat{\mathcal{W}}$ is quasiconcave.*

Employed workers contribute to social welfare through market production, which is why $\partial \mathcal{W} / \partial n > 0$. Unemployed workers contribute to social welfare through home production and recreation (Aguiar, Hurst, and Karabarbounis 2013); this contribution is diminished if people suffer psychic pain from being unemployed (Brand 2015). However, unemployed workers contribute less to welfare than employed workers, so $\partial \mathcal{W} / \partial u < \partial \mathcal{W} / \partial n$. Vacancies lower social welfare because labor and other resources must be diverted away from market production and toward recruiting to fill each vacancy.

The alternate welfare function $\widehat{\mathcal{W}}(u, v)$ is obtained from the welfare function $\mathcal{W}(n, u, v)$ by substituting the employment rate n by $1 - u$. The fact that the alternate welfare function decreases with the unemployment and vacancy rates captures the social costs of unemployment and vacancies. We assume that the alternate welfare function is quasiconcave to ensure that the social planner's problem is well behaved.

2.5. Social value of nonwork

The effects of unemployment on welfare is given by the following statistic, which plays a key role in calculating the unemployment gap:

DEFINITION 2. *The social value of nonwork is the marginal rate of substitution between unemployment and employment in the welfare function:*

$$\zeta = \frac{\partial W / \partial u}{\partial W / \partial n} < 1.$$

The social cost of unemployment is

$$\frac{(\partial W / \partial n) - (\partial W / \partial u)}{\partial W / \partial n} = 1 - \zeta > 0.$$

The social value of nonwork ζ measures the marginal contribution of unemployed workers relative to that of employed workers. The social value of nonwork ζ is below 1 because unemployed workers' contribute less to welfare than employed workers (assumption 2). The social cost of unemployment $1 - \zeta > 0$ measures the social loss of having a person unemployed rather than employed. Such loss comprises foregone market production and psychological pain of being unemployed rather employed, net of the value home production and recreation when unemployed.

2.6. Recruiting cost

The effects of vacancies on welfare is given by the following statistic, which is also key in calculating the unemployment gap:

DEFINITION 3. *The recruiting cost is minus the marginal rate of substitution between vacancies and employment in the welfare function:*

$$\kappa = -\frac{\partial W / \partial v}{\partial W / \partial n} > 0.$$

The recruiting cost κ measures the resources absorbed by recruiting to fill a vacancy, expressed in terms of labor. The recruiting cost therefore measures the number of workers allocated to recruiting per vacancy. These workers are tasked with writing and advertising job vacancies; reading applications and screening suitable candidates; interviewing and evaluating selected candidates; and drafting and negotiating job offers.

3. Efficient unemployment rate and unemployment gap

We solve the problem of a social planner who maximizes welfare by allocating labor between employment and unemployment, subject to the Beveridge-curve constraint. The solution gives the

efficient unemployment rate. We then represent efficiency in a Beveridge diagram to understand the tradeoffs at play. Finally, we derive sufficient-statistic formulas for the efficient unemployment rate and unemployment gap.

3.1. Social planner's problem

We define efficiency as the solution to the problem of a social planner who is subject to the Beveridge-curve constraint:

DEFINITION 4. *The efficient unemployment and vacancy rates, denoted u^* and v^* , maximize social welfare $\widehat{W}(u, v)$ subject to the Beveridge-curve constraint $v = v(u)$. The efficient labor-market tightness is $\theta^* = u^*/v^*$, and the unemployment gap is $u - u^*$.*

The social planner's problem is similar in several ways to that introduced by Hosios (1990): the social planner maximizes welfare by allocating resources between production, recruiting, and unemployment; and the social planner is subject to a Beveridge-curve constraint (equation (3) in Hosios 1990). But the social planner's problem here is also more general: first, it applies to any labor market with a Beveridge curve, not just to those with a matching function; and second, it applies to any quasiconcave welfare function, not just to linear ones.

Comparison of the planning solution with the decentralized equilibrium. The planning solution is described by two variables, unemployment and vacancies, given by two equations: the Beveridge curve, and the first-order condition of the planner's problem. By contrast, the decentralized equilibrium is usually given by three variables: unemployment, vacancies, and wage. These three variables are usually given by three equations: the Beveridge curve; a wage equation; and an equation describing vacancy creation, such as the free-entry condition of the DMP model.

Moreover, in many Beveridgean models, unlike in Walrasian models, there is no guarantee that the decentralized equilibrium overlaps with the planning solution. This is because most wage mechanisms do not ensure efficiency. For instance, in the DMP model, the wage is determined in a situation of bilateral monopoly, so a range of wages is theoretically possible. A wage mechanism picks one wage among those possible. There is only an infinitesimal chance that the wage picked is the one wage that ensures efficiency, so there is no theoretical reason to believe that the unemployment gap is zero (Pissarides 2000, chapter 8).

3.2. Representation of efficiency in a Beveridge diagram

We now represent labor-market efficiency in a Beveridge diagram. This representation illustrates the tradeoffs facing the social planner.

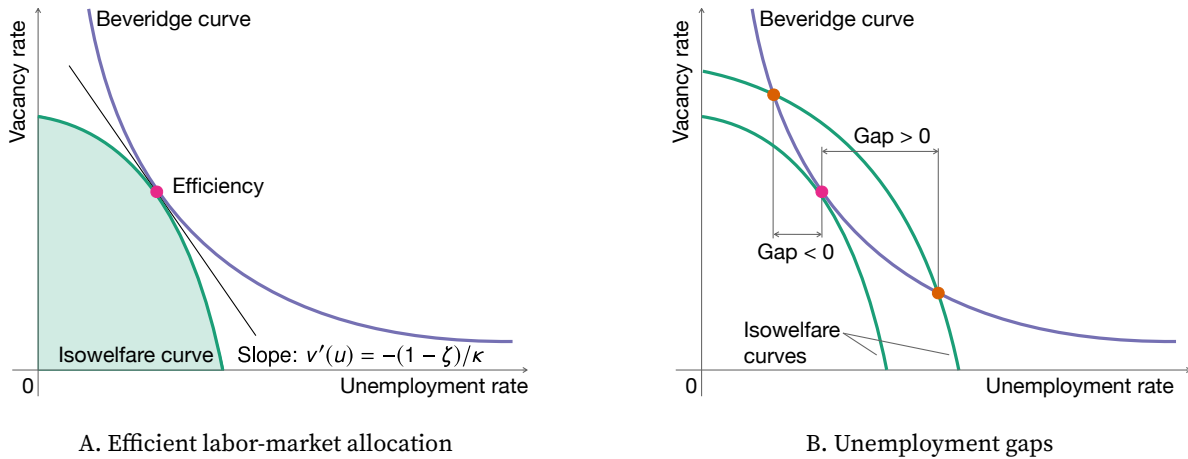


FIGURE 2. Efficient unemployment and unemployment gap in a Beveridge diagram

Panel A depicts an efficient labor market. The statistic κ is the recruiting cost, ζ is the social value of nonwork, and $v'(u)$ is the slope of the Beveridge curve. Panel B describes inefficient labor markets: either unemployment is inefficiently high, vacancies are inefficiently low, and the unemployment gap is positive; or unemployment is inefficiently low, vacancies are inefficiently high, and the unemployment gap is negative.

Beveridge diagram. The Beveridge diagram features unemployment rate on the x -axis and vacancy rate on the y -axis. It is depicted in figure 2, panel A.

In the diagram the Beveridge curve $v(u)$ is downward-sloping and convex. It gives the locus of unemployment and vacancy rates that are feasible in the economy.

The diagram also features an isowelfare curve: the locus of unemployment and vacancy rates such that social welfare $\widehat{W}(u, v)$ remains constant at some level (the equivalent of an indifference curve for a utility function or an isoquant for a production function). Since $\widehat{W}(u, v)$ is decreasing in unemployment and vacancies, all the points inside the isowelfare curve yield higher welfare, so the green area delineated by the isowelfare curve is an upper contour set of $\widehat{W}(u, v)$. Since the function \widehat{W} is quasiconcave, its upper contour sets are convex, which implies that the isowelfare curve must be concave.

Efficiency condition. The efficient unemployment and vacancy rates can easily be found in the Beveridge diagram. First, they have to lie on the Beveridge curve. Second, since both unemployment and vacancies impose a welfare cost, they must lie on the isowelfare curve that is as close to the origin as possible. The closest that the isowelfare curve can be while remaining in contact with the Beveridge curve is at the tangency point with the Beveridge curve. This is where the efficient unemployment and vacancy rates are found. The efficient labor-market tightness is also visible on the diagram: it is the slope of the origin line going through the tangency point.

As with indifference curves and isoquants, the slope of the isowelfare curve is minus the

marginal rate of substitution between unemployment and vacancies in the welfare function $\widehat{W}(u, v)$:

$$-\frac{\partial \widehat{W} / \partial u}{\partial \widehat{W} / \partial v} = -\frac{(\partial W / \partial u) - (\partial W / \partial n)}{\partial W / \partial v} = -\frac{1 - (\partial W / \partial u) / (\partial W / \partial n)}{-(\partial W / \partial v) / (\partial W / \partial n)} = -\frac{1 - \zeta}{\kappa}.$$

The efficient unemployment rate is found at the point where the Beveridge curve, with slope $v'(u)$, is tangent to isowelfare curve, with slope $-(1 - \zeta) / \kappa$. This yields a first result:

PROPOSITION 1. *In a Beveridge diagram, efficiency is achieved at the point where the Beveridge curve is tangent to an isowelfare curve. Hence, the efficient unemployment rate is implicitly defined by*

$$(1) \quad v'(u) = -\frac{1 - \zeta}{\kappa},$$

where $\zeta < 1$ is the social value of nonwork and $\kappa > 0$ the recruiting cost.

Formula (1) simply says that when the labor market operates efficiently, welfare costs and benefits from moving one worker from employment to unemployment are equalized. When one worker moves from employment to unemployment, the reduction in welfare is the social cost of unemployment, $1 - \zeta$. Having one more unemployed worker also means having $-v'(u) > 0$ fewer vacancies. Each vacancy reduces welfare by the recruiting cost, κ , so welfare improves by $-v'(u)\kappa$ through the reduction in recruiting activity. When welfare costs and benefits are equalized, we have $1 - \zeta = -v'(u)\kappa$, which is equivalent to (1).

Deviations from efficiency. There is no guarantee that the labor market operates efficiently (figure 2, panel B). The labor market may be above the efficiency point, where unemployment is too low, vacancies are too high, tightness is too high, and the unemployment gap is negative. It may also be below the efficiency point, where unemployment is too high, vacancies are too low, tightness is too low, and the unemployment gap is positive. As both situations are inefficient, they lie on a worse isowelfare curve than the efficiency point.

Comparative statics. We use the Beveridge diagram to derive several comparative statics about the efficient unemployment rate (figure 3).

We first consider an increase in the social cost of unemployment, from $1 - \zeta$ to $\sigma \cdot (1 - \zeta)$ with $\sigma > 1$. The isowelfare curve becomes steeper everywhere. In particular, at the previous efficiency point, the isowelfare curve is steeper than the Beveridge curve. This indicates that the new efficiency point is above the old one on the Beveridge curve, so the efficient unemployment rate is lower than previously (panel A). Intuitively, when unemployment is more costly, the

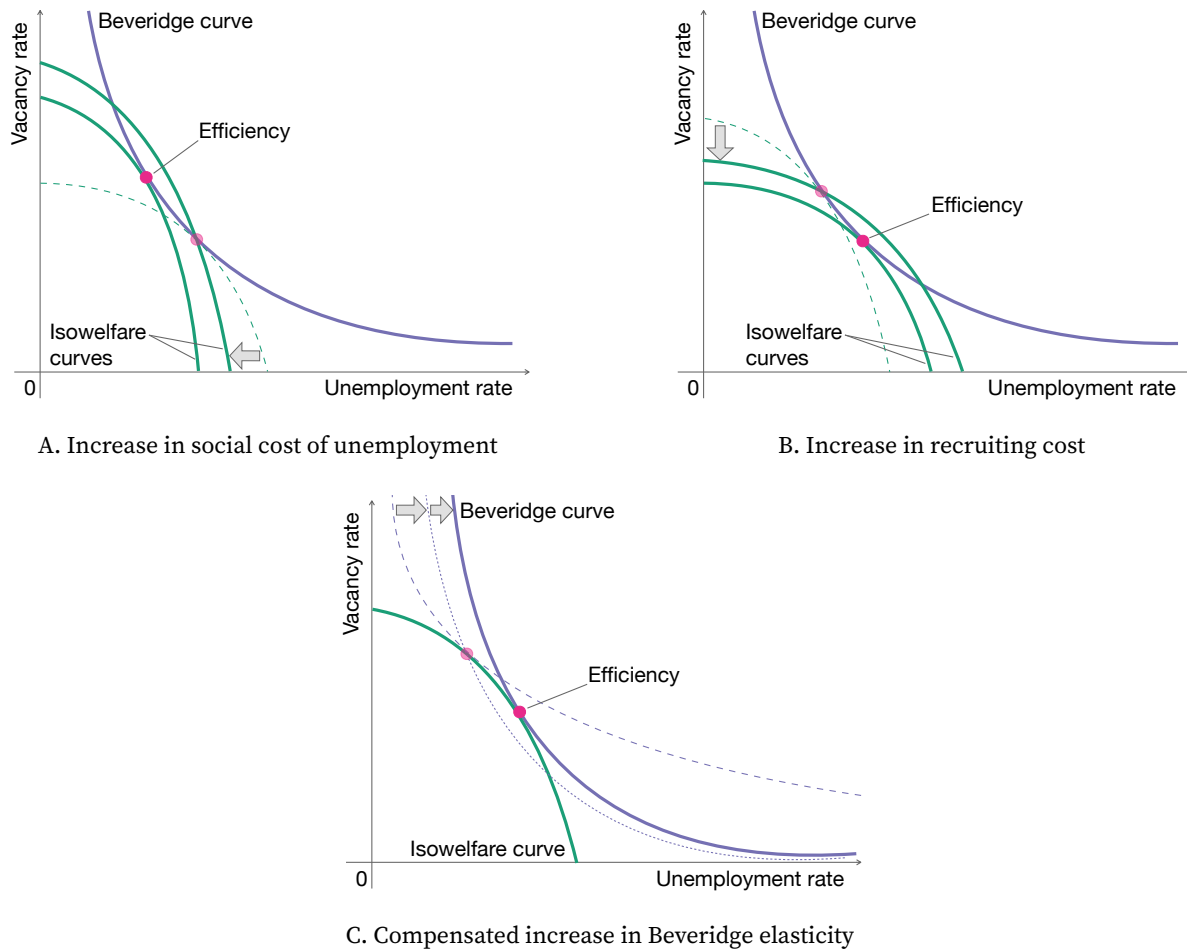


FIGURE 3. Comparative statics for the efficient unemployment rate

Panel A shows that the efficient unemployment rate decreases when the social cost of unemployment increases. Panel B shows that the efficient unemployment rate increases when the recruiting cost increases. Panel C shows that the efficient unemployment rate increases when the Beveridge elasticity increases, keeping welfare constant.

unemployment-vacancy tradeoff becomes less favorable to unemployment, and the efficient unemployment rate decreases.

We then consider an increase in recruiting cost, from κ to $\sigma \cdot \kappa$ with $\sigma > 1$. The isowelfare curve is now everywhere flatter. Following the opposite logic as in the previous case, the efficient unemployment rate is now lower (panel B). Intuitively, when recruiting is more costly, the unemployment-vacancy tradeoff becomes more favorable to unemployment, so the efficient unemployment rate increases.

Finally we consider a compensated increase in the Beveridge elasticity (analogous to a compensated price increase in the context of Hicksian demand). This is an increase in the Beveridge elasticity compensated by a change in the position of the curve so that the new Beveridge curve

remains tangent to the same isowelfare curve. Formally, the Beveridge curve changes from $v(u)$ to $\zeta \cdot (v(u))^\sigma$, where $\zeta > 0$ and $\sigma > 1$. The Beveridge elasticity increases from ϵ to $\sigma \cdot \epsilon$. Such a change steepens the Beveridge curve (panel C). At the previous efficiency point, the Beveridge curve is steeper than the isowelfare curve. This indicates that the new efficiency point is to the right of the old one on the isowelfare curve, and that the Beveridge curve must shift out to maintain welfare at the same level at efficiency (formally, $\zeta > 1$). Overall, the efficient unemployment rate is higher than previously. The intuition is simple: a rise in unemployment triggers a larger drop in vacancies, so the unemployment-vacancy tradeoff is more favorable to unemployment, and the efficient unemployment rate increases.

The corollary below summarizes the results.

COROLLARY 1. *The following comparative statics hold:*

- *An increase in the social cost of unemployment lowers the efficient unemployment rate and raises the efficient vacancy rate.*
- *An increase in recruiting cost raises the efficient unemployment rate and lowers the efficient vacancy rate.*
- *A compensated increase in the Beveridge elasticity (increase in elasticity keeping welfare constant) raises the efficient unemployment rate and lowers the efficient vacancy rate.*

3.3. Efficient labor-market tightness

We aim to obtain a sufficient-statistic formula for the unemployment gap. As an intermediate step, we rework the efficiency condition (1) to obtain a sufficient-statistic formula for the efficient labor-market tightness.

We begin by introducing the Beveridge elasticity:

$$\epsilon = -\frac{u}{v}v'(u) \quad \text{so} \quad \epsilon\theta = -v'(u).$$

With this result, we can re-express (1) as $\theta = (1 - \zeta)/(\kappa\epsilon)$.

In panel B of figure 2, we see that any point on the Beveridge curve above the efficiency point has $-v'(u) > (1 - \zeta)/\kappa$, and any point below it has $-v'(u) < (1 - \zeta)/\kappa$. Using again $\epsilon\theta = -v'(u)$, we infer that any point above the efficiency point satisfies $\theta > (1 - \zeta)/(\kappa\epsilon)$; and any point below the efficiency point has $\theta < (1 - \zeta)/(\kappa\epsilon)$.

Hence we can assess the efficiency of labor-market tightness from three sufficient statistics:

PROPOSITION 2. Consider a point on the Beveridge curve with labor-market tightness θ , Beveridge elasticity ϵ , recruiting cost κ , and social value of nonwork ζ . Then tightness is inefficiently high if $\theta > (1 - \zeta)/(\kappa\epsilon)$, inefficiently low if $\theta < (1 - \zeta)/(\kappa\epsilon)$, and efficient if

$$(2) \quad \theta = \frac{1 - \zeta}{\kappa\epsilon}.$$

Since the statistics ϵ , κ , and ζ generally depend on tightness θ , formula (2) characterizes the efficient tightness only implicitly. This limitation is typical of the sufficient-statistic approach (Chetty 2009), but it will complicate the task of computing the unemployment gap.

3.4. Unemployment gap

To compute the unemployment gap, we need to address the endogeneity of the sufficient statistics in (2). We use a workaround developed by Kleven (2020):

ASSUMPTION 3. The Beveridge elasticity (ϵ), recruiting cost (κ), and social value of nonwork (ζ) do not depend on the unemployment and vacancy rates.

How realistic is this assumption? Panels C–F in figure 1 suggest that the Beveridge curve is isoelastic, so the assumption on the Beveridge elasticity seems valid in US data. We do not have comparable evidence on the recruiting cost and social value of nonwork, but at least in the DMP model, these two statistics are independent of the unemployment and vacancy rates.

Under assumption 3, we obtain a simple formula for the unemployment gap. The assumption implies that the Beveridge curve is isoelastic:

$$(3) \quad v(u) = v_0 \cdot u^{-\epsilon},$$

where the parameter $v_0 > 0$ determines the location of the curve. On the Beveridge curve, tightness is related to unemployment by

$$\theta = \frac{v(u)}{u} = v_0 \cdot u^{-(1+\epsilon)} \quad \text{and} \quad \theta^* = v_0 \cdot (u^*)^{-(1+\epsilon)}.$$

We can therefore link the unemployment gap to the tightness gap:

$$(4) \quad \frac{u^*}{u} = \left(\frac{\theta}{\theta^*} \right)^{1/(1+\epsilon)}.$$

Under assumption 3, (2) gives $\theta^* = (1 - \zeta)/(\kappa\epsilon)$, which yields the following proposition:

PROPOSITION 3. *Under assumption 3, the efficient unemployment rate and unemployment gap can be measured from current unemployment rate (u), vacancy rate (v), Beveridge elasticity (ϵ), recruiting cost (κ), and social value of nonwork (ζ). The efficient unemployment rate satisfies*

$$(5) \quad u^* = \left(\frac{\kappa \epsilon}{1 - \zeta} \cdot \frac{v}{u} \right)^{1/(1+\epsilon)} u,$$

from which the unemployment gap $u - u^*$ follows.

The proposition gives an explicit formula for the unemployment gap, expressed in terms of observable sufficient statistics. The formula is valid in any Beveridgean model, irrespective of the structure of the labor and product markets, production, preferences, and wage setting. Another advantage of the formula is that we do not need to keep track of all the shocks disturbing the labor market—shocks to productivity, wages, preferences, labor-force participation, matching function, job separations, and so on. We only need to observe the sufficient statistics.²

4. Unemployment gap in the United States, 1951–2019

We apply formula (5) to measure the unemployment gap in the United States over the 1951–2019 period. The first step is to estimate the sufficient statistics: Beveridge elasticity, recruiting cost, and social value of nonwork.

4.1. Beveridge elasticity (ϵ)

We estimate the Beveridge elasticity with linear regressions of log vacancy rate on log unemployment rate. The data are displayed in figure 1. The sample contains $T = 276$ observations, since data are quarterly from 1951Q1 to 2019Q4. Since the Beveridge curve shifts over time, we need to allow for structural breaks in the estimation. We therefore use the algorithm proposed by Bai and Perron (1998, 2003) to estimate linear models with multiple structural changes.

Statistical model. The statistical model that we estimate has m breaks, and $m + 1$ regimes. It is given by

$$\ln(v(t)) = \beta_j + \epsilon_j \cdot \ln(u(t)) + z(t) \quad t = T_{j-1} + 1, \dots, T_j$$

for $j = 1, \dots, m + 1$. The observed dependent variable is the log of the vacancy rate, $\ln(v(t))$; the observed independent variable is the log of the unemployment rate, $\ln(u(t))$; $z(t)$ is the error at

²Without assumption 3, we could still obtain a formula for the unemployment gap, but it would require three additional statistics: the elasticities of ϵ , κ , and ζ with respect to the unemployment rate (Kleven 2020).

time t ; β_j is the intercept of the linear model in regime j ; ϵ_j is the Beveridge elasticity in regime j ; and T_1, \dots, T_m are the m break points (we use the convention that $T_0 = 0$ and $T_{m+1} = T$).

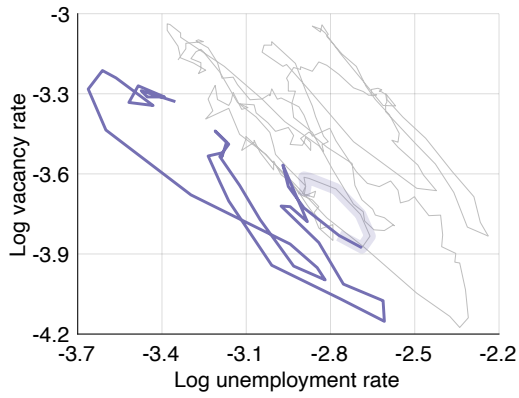
The regression coefficients $\beta_1, \dots, \beta_{m+1}$ and $\epsilon_1, \dots, \epsilon_{m+1}$, and the break points T_1, \dots, T_m , are unknown. They are jointly estimated with the Bai-Perron algorithm. The regression coefficients are estimated by least-squares while the break points are estimated by minimizing the sum of squared residuals. The algorithm also provides confidence intervals for the break dates and regression coefficients under various hypotheses about the structure of the data and the errors across regimes. Finally, the algorithm offers several ways to test for structural changes and to determine the number of breaks, m .

Algorithm setup. Before proceeding to the estimation, we make several modeling choices and calibrate the parameters of the algorithm. First, we allow for autocorrelation in the errors, and different variances of the errors across regimes. To obtain standard errors robust to autocorrelation and heteroskedasticity, the algorithm follows the method proposed by Andrews (1991), using a quadratic kernel with automatic bandwidth selection based on an AR(1) approximation. We also allow different distributions of the independent and dependent variables (unemployment and vacancy rates) across regimes. Next, we set the trimming parameter to $\epsilon = 0.15$, as suggested by Bai and Perron (2003, p. 15); hence each regime has at least $\epsilon \times T = 0.15 \times 276 = 41$ observations. As required by Bai and Perron (2003, p. 14), we set the maximum number of breaks to $M = 5$.

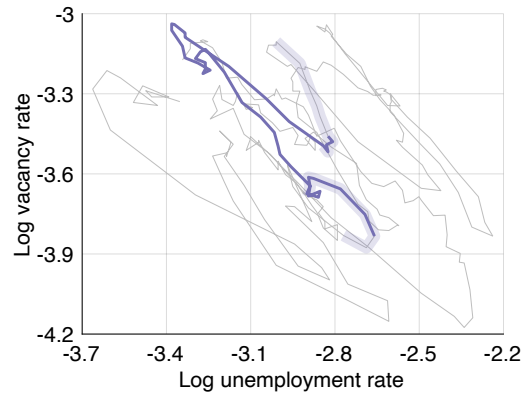
Existence and number of breaks. We first consider statistical tests for the existence of structural breaks. The algorithm provides supF tests of no structural break versus m breaks, for $m = 1, \dots, 5$. All such tests reject the null hypothesis of no breaks at the 1% significance level. The algorithm also provides two double-maximum tests of no structural break versus an unknown number of breaks below the upper bound $M = 5$. Again, both tests reject the null hypothesis of no breaks at the 1% significance level. Given these results, it is clear that at least one break is present.

To determine the number of structural breaks, we consider two information criteria: the Bayesian Information Criterion proposed by Yao (1988), and the modified Schwarz criterion proposed by Liu, Wu, and Zidek (1997). Both information criterion select 5 breaks.

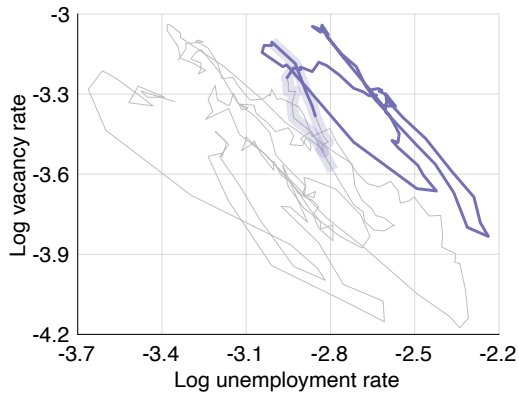
Break dates. Next we estimate the 5 break dates. The algorithm finds that the breaks in the Beveridge curve occur in 1961Q1, 1971Q4, 1989Q1, 1999Q2, and 2009Q3. The break dates are precisely estimated as all their 95% confidence intervals cover less than 2.5 years. The 6 branches of the Beveridge curve delineated by the break dates, together with the 95% confidence intervals for the dates, are represented on figure 4.



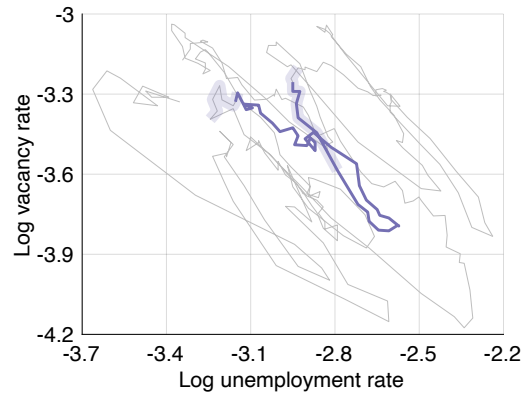
A. First branch: 1951Q1-1961Q1



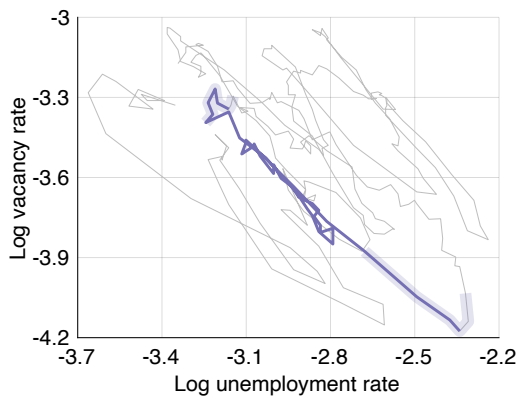
B. Second branch: 1961Q2-1971Q4



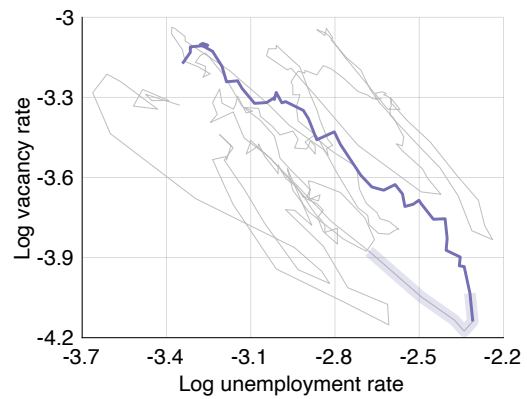
C. Third branch: 1972Q1-1989Q1



D. Fourth branch: 1989Q2-1999Q2



E. Fifth branch: 1999Q3-2009Q3



F. Sixth branch: 2009Q4-2019Q4

FIGURE 4. Beveridge-curve branches in the United States, 1951-2019

The figure is obtained by applying the methodology of Bai and Perron (1998, 2003) on log vacancy rate and log unemployment rate. The methodology is designed to estimate linear models with multiple structural changes. The figure reports the branches of the Beveridge curve between each estimated structural break (thin dark line), as well as the 95% confidence interval for each break date (wide transparent line).

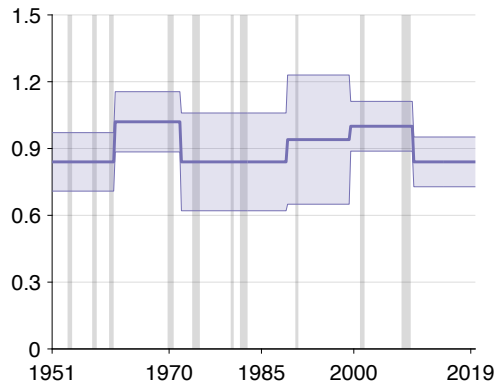


FIGURE 5. Beveridge elasticity and 95% confidence interval in the United States, 1951–2019

The figure is obtained by applying the methodology of Bai and Perron (1998, 2003) on log vacancy rate and log unemployment rate. The methodology is designed to estimate linear models with multiple structural changes. The figure reports the estimate of the Beveridge elasticity ϵ and its 95% confidence interval. The standard errors used to compute the confidence interval are corrected for autocorrelation in the errors, as well as heterogeneity in the data and errors across regimes. The shaded areas represent recessions, as identified by the NBER.

Beveridge elasticity. Finally, we estimate the Beveridge elasticities in the 6 different regimes. We find that from 1951 to 2019, the Beveridge elasticity fluctuated between 0.84 and 1.02, averaging 0.91. So overall, the elasticity remains quite stable over time. The elasticities are fairly precisely estimated. The standard errors—corrected for autocorrelation in the error term, as well as heterogeneity in the data and error term across regimes—vary between 0.06 and 0.15. The elasticities and their 95% confidence intervals are displayed in figure 5.³

The fit of the linear model with structural changes is good: $R^2 = 0.91$. The good fit confirms that unemployment and vacancy rates travel in the close vicinity of an isoelastic curve that shifts from time to time.

Comparison with estimates of the matching elasticity. Our estimates of the Beveridge elasticity are consistent with the estimates of the matching elasticity obtained by the empirical literature studying the matching function. In a DMP model, the Beveridge elasticity is related to the matching elasticity α by (16). Inverting this equation, we express the matching elasticity as a function of the Beveridge elasticity and prevailing unemployment rate:

$$(6) \quad \alpha = \frac{1}{1 + \epsilon} \left[\epsilon - \frac{u}{1 - u} \right].$$

³Any temporary deviation from the balanced-flow assumption in the data appears in the error term. This is why we allow the error term to be autocorrelated and heteroskedastic. Ahn and Crane (2020) refine the analysis by imposing more structure on labor-market flows. With the extra structure, they can quantify how much out-of-balance flows contribute to the movements in unemployment and vacancies. With this information, they are able to estimate to estimate the Beveridge elasticity more finely.

Over the 1951–2019 period, the average Beveridge elasticity is $\epsilon = 0.91$ and the average unemployment rate is $u = 5.8\%$. The matching elasticity consistent with these values is

$$\alpha = \frac{1}{1 + 0.91} \left[0.91 - \frac{0.058}{1 - 0.058} \right] = 0.44.$$

This value is in the range of estimates of the matching elasticity obtained with aggregate US data. Depending on the estimation method and specification, Blanchard and Diamond (1989, table 1) estimate α in the 0.32–0.60 range and Bleakley and Fuhrer (1997, table 1) in the 0.54–0.76 range. Shimer (2005, p. 32) estimate $\alpha = 0.72$, and Rogerson and Shimer (2011, p. 638) estimate $\alpha = 0.58$. Finally, Borowczyk-Martins, Jolivet, and Postel-Vinay (2013, p. 444) find a lower estimate than earlier work: $\alpha = 0.30$. Hence, the range of existing estimates for α in the United States is 0.30–0.76. The value of $\alpha = 0.44$ implied by the estimate of the Beveridge elasticity is squarely in that range.

4.2. Social value of nonwork (ζ)

To measure the social value of nonwork, we rely on the revealed-preference estimates provided by Borgschulte and Martorell (2018) and Mas and Pallais (2019).

Raw estimates. Using military administrative data covering 1993–2004, Borgschulte and Martorell study how servicemembers’ reenlistment choice is influenced by unemployment. This choice allows them to estimate the dollar value of the utility loss caused by higher unemployment during the transition to civilian life, and compare it to the earnings loss caused by higher unemployment. They find that during unemployment home production, recreation, and public benefits offset between 13% and 35% of lost earnings.

Using a large field experiment, Mas and Pallais study how unemployed job applicants choose between randomized wage-hour bundles. These choices imply that the value of home production and recreation during unemployment amounts to 58% of predicted earnings.

Translating raw estimates into social values of nonwork. Next, we translate these estimates into social values of nonwork. To simplify, we ignore the fact that unemployed workers are imperfectly insured, so employed and unemployed workers value consumption differently. By ignoring insurance issues, we can directly measure workers’ contribution to welfare from their productivity, at home or at work.⁴ For unemployed workers in particular, the contribution to social welfare

⁴See Landais, Michailat, and Saez (2018b) for a first attempt at measuring the social value of nonwork when unemployed workers are imperfectly insured. And see Landais and Spinnewijn (2020) for new, revealed-preference

should not include unemployment benefits, which are transfers from employed to unemployed workers.

The first step to translating the estimates is to express the estimates relative to the marginal product of labor rather than earnings. The marginal product of labor is higher than earnings for several reasons. First, the wage paid by firms is usually lower than the marginal product of labor. In a matching model the wedge amounts to the share of workers allocated to recruiting, so the marginal product is about 3% higher than the wage (Landais, Michaillat, and Saez 2018a, equation (1)). In a monopsony model, the wedge depends on the elasticity of the labor supply, and the marginal product may be 25% higher than the wage (Mas and Pallais 2019, p. 121). Second, the wage received by workers is lower than that paid by firms because of employer-side payroll taxes, which amount to 7.7%. Third, Mas and Pallais discount predicted earnings by 6% to capture the wage penalty incurred by workers who recently lost their jobs; we undo the discounting because the penalty does not seem to apply to the marginal product of labor (Davis and von Wachter 2011). To conclude, to obtain a marginal product of labor, Borgschulte and Martorell's earnings have to be adjusted by a factor between $1.03 \times 1.077 = 1.11$ and $1.25 \times 1.077 = 1.35$, and Mas and Pallais's earnings by a factor between $1.03 \times 1.077 \times 1.06 = 1.18$ and $1.25 \times 1.077 \times 1.06 = 1.43$. Accordingly, to obtain a value relative to the marginal product of labor, Borgschulte and Martorell's estimates must be adjusted by a factor between $1/1.35 = 0.74$ and $1/1.11 = 0.90$, and Mas and Pallais's estimates must be adjusted by a factor between $1/1.43 = 0.70$ and $1/1.18 = 0.85$.

The second step only applies to Borgschulte and Martorell's estimates, from which we subtract the value of public benefits. All servicemembers are eligible to unemployment insurance (UI). Chodorow-Reich and Karabarbounis (2016, pp. 1585–1586) find that UI benefits amount to 21.5% of the marginal product of labor. But this quantity has to be reduced for several reasons: the UI takeup rate is only 65%; UI benefits and consumption are taxed, imposing a factor of 0.83; the disutility from filing for benefits imposes a factor of 0.47; and UI benefits expire, imposing another factor of 0.83. In sum, the average value of UI benefits to servicemembers is $21.5 \times 0.65 \times 0.83 \times 0.47 \times 0.83 = 5\%$ of the marginal product of labor. Servicemembers are also eligible to other public benefits, which Chodorow-Reich and Karabarbounis quantify at 2% of the marginal product of labor. Hence, to account for benefits, we subtract $5\% + 2\% = 7\%$ of the marginal product of labor from Borgschulte and Martorell's estimates.

Combining these two steps, we find that Mas and Pallais's estimates imply a social value of nonwork between $0.58 \times 0.70 = 0.41$ and $0.58 \times 0.85 = 0.49$; and that Borgschulte and Martorell's estimates imply a social value of nonwork between $(0.13 \times 0.74) - 0.07 = 0.03$ and

evidence on the difference between the marginal value of consumption for unemployed workers and that for employed workers.

$(0.35 \times 0.90) - 0.07 = 0.25$. The range of plausible values for the social value of nonwork therefore is 0.03–0.49; we set the statistic to its midrange value, $\zeta = 0.26$.

Fluctuations of the social value of nonwork. In some models, the productivities of unemployed and employed workers do not move in tandem over the business cycle, which generates fluctuations in the social value of nonwork. However, Chodorow-Reich and Karabarbounis (2016, pp. 1599–1604) find no evidence of such fluctuations in US data. Instead, they establish that the utility derived by unemployed workers from recreation and home production moves proportionally to labor productivity—which implies that the social value of nonwork is acyclical. Accordingly, we keep the social value of nonwork constant over the business cycle.

The social value of nonwork could also exhibit medium-run fluctuations, but we omit them by lack of evidence.

Other possible contributors to the social value of nonwork. Measuring the social value of nonwork is complex. Here we rely on revealed-preference evidence to measure this statistic. This evidence captures the value of nonwork that transpires from people’s choices on the labor market.

While the revealed-preference approach is the gold standard to elicit how people value nonwork, it might miss other contributors to the social cost of unemployment that do not affect people’s choices. For instance, it has often been postulated that higher local unemployment rates lead to higher crime rates (a negative social externality). If unemployment generates a substantial increase in crime, the social cost of unemployment should be increased accordingly.

Many empirical studies have tried to measure the unemployment-crime relationship. The overall picture is murky: some studies find strong effects of unemployment on crime, but others do not. The surveys by Chiricos (1987) and Freeman (1999) conclude that unemployment stimulates crime, but not overwhelmingly so. In a meta-analysis of 214 empirical studies, Pratt and Cullen (2005) identify unemployment has one of the top macro-predictors of crime, but they warn that this result may not be robust because it is driven by a few studies finding very large effects.

If the unemployment-crime relationship was strong, the social cost of unemployment could be larger than that given by the revealed-preference approach. The current literature suggests that such relationship exists, but it may not be strong, so that our estimate of the social cost of unemployment may be a good first approximation.

4.3. Recruiting cost (κ)

Following Villena Roldan (2010), we measure the recruiting cost from the 1997 National Employer Survey. The survey was conducted by the Census Bureau in 1997 and asked thousands of

establishments about the recruiting and training of their workforce (Cappelli 2001). The establishments in the sample have at least 20 employees, covering manufacturing and non-manufacturing industries.

In the public-use files of the survey, 2007 establishments reported the percentage of labor costs devoted to recruiting. The mean response was 3.2%. Assuming that all workers are paid the same, we infer that firms allocate on average 3.2% of their labor to recruiting: $\kappa v = 3.2\% \times (1 - u)$. In 1997, the average vacancy rate is 3.3% and the average unemployment rate is 4.9% (figure 1). So the recruiting cost in 1997 is $\kappa = 3.2\% \times (1 - 4.9\%) / 3.3\% = 0.92$.

Unfortunately there is no other comprehensive measure of recruiting cost in the United States. However, in labor-market models, the recruiting cost is usually not assumed to be time-varying (for example, Pissarides 2000). Following this tradition, we assume that the recruiting cost remains at its 1997 value over the entire 1951–2019 period.

This lack of data is not ideal to assess past unemployment gaps, but it could easily be remedied in the future. To improve the measurement of the recruiting cost, the BLS would only need to add a new question into JOLTS—asking firms to report the number of man-hours devoted to recruiting in addition to the number of vacancies.

4.4. Unemployment gap

We now use our estimates of the Beveridge elasticity, recruiting cost, and social value of nonwork, as well as our unemployment and vacancy series, to measure the unemployment gap in the United States between 1951 and 2019.

Efficient labor-market tightness. We begin by computing the efficient labor-market tightness using formula (2) (figure 6, panel A). The efficient tightness fluctuates over time between 0.79 and 0.96, mirroring the movements of the Beveridge elasticity. Since actual and efficient tightness almost never coincide, the US labor market almost never operates efficiently.

Compared to its efficient level, actual tightness is almost always too low. During the period, the actual tightness averages 0.62 while the efficient tightness averages 0.89. There are only four episodes when tightness was inefficiently high: 1951–1953, during the Korean war; 1965–1970, at the peak of the Vietnam war; in 1999–2000, during the dot-com bubble; and in 2018–2019. Hence, the US labor market is inefficiently slack most of the time.

Efficient unemployment rate. Next we compute the efficient unemployment rate from formula (5) (figure 6, panel B). The efficient unemployment rate averages 4.3% between 1951 and 2019. It hovered around 3.5% in the 1950s, rose to 4.5% in the 1960s, and climbed to reach 5.4% in 1979.

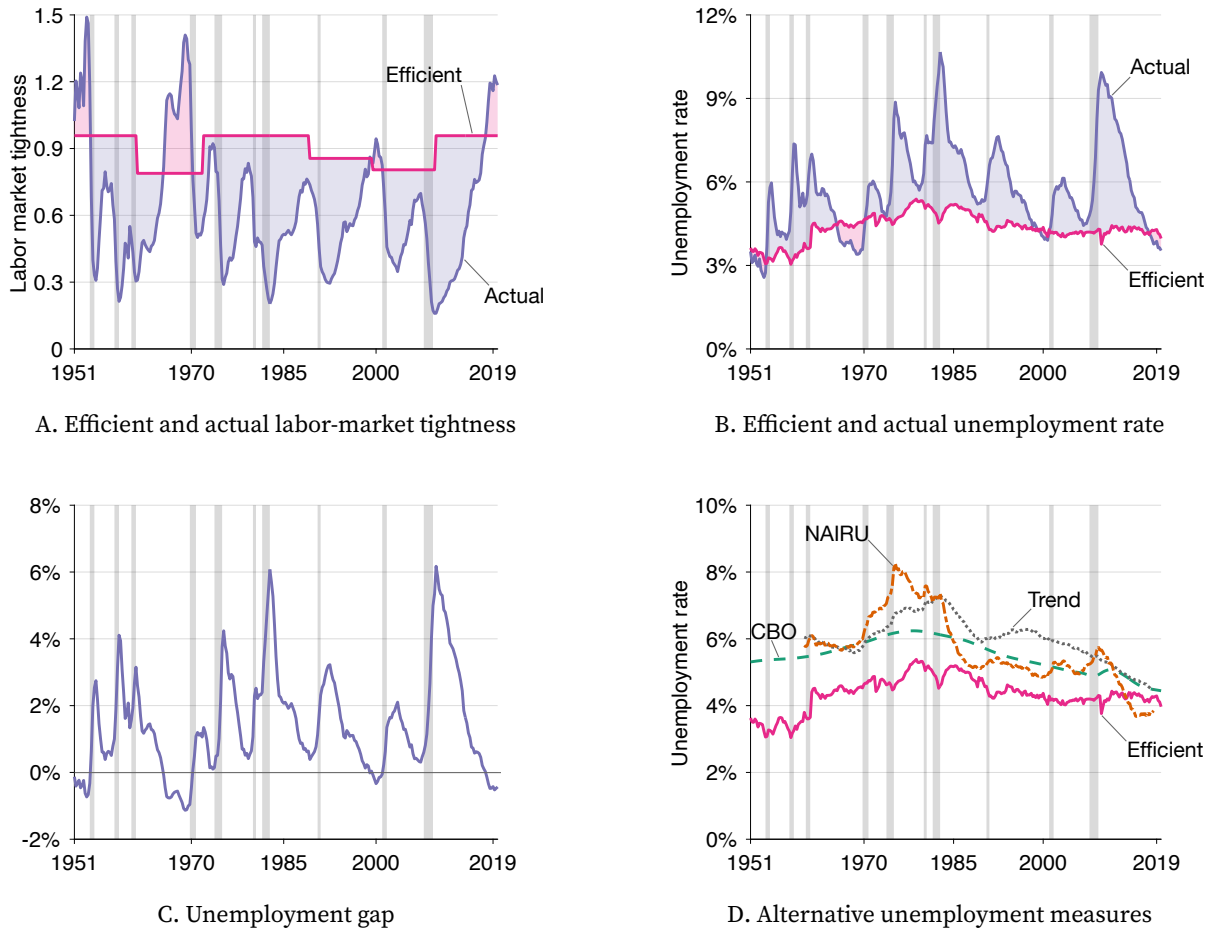


FIGURE 6. Unemployment gap in the United States, 1951–2019

Panel A: The actual labor-market tightness is vacancy rate (from figure 1, panel B) divided by unemployment rate (from figure 1, panel A). The efficient labor-market tightness is computed from (2) with $\kappa = 0.92$, $\zeta = 0.26$, and ϵ from figure 5. Panel B: The actual unemployment rate comes from figure 1. The efficient unemployment rate is computed from (5) with $\kappa = 0.92$, $\zeta = 0.26$, ϵ from figure 5, and the unemployment and vacancy rates from figure 1. Panel C: The unemployment gap is the difference between the actual and efficient unemployment rates from panel B. Panel D: The efficient unemployment rate come from panel B. CBO unemployment rate is the long-term natural rate of unemployment constructed by the Congressional Budget Office. NAIRU and trend unemployment rates come from Crump et al. (2019, figure 8B). The shaded areas represent recessions, as identified by the NBER.

The steady increase of the efficient unemployment rate between 1951 to 1979 was caused by a steady outward shift of the Beveridge curve (figure 4). Then, the efficient unemployment rate declined to reach 4.6% in 1990. The decline was caused by an inward shift of the Beveridge curve (figure 4, panels C–D). The efficient unemployment rate then remained stable through the 1990s, 2000s, and 2010s, hovering between 3.8% and 4.6%.

Interestingly the efficient unemployment rate did not increase in the aftermath of the Great Recession—despite the outward shift of the Beveridge curve (figure 4, panels E–F). This is because

the Beveridge curve also became flatter after 2009: it fell from 1.0 to 0.84 (figure 5). The flattening offset the outward shift, leaving the efficient unemployment rate almost unchanged by the recession.

Unemployment gap. Measuring the distance between the actual and the efficient unemployment rate, we obtain the unemployment gap (figure 6, panel C). The US labor market is almost never efficient, as the unemployment gap is almost never equal to 0.

The US unemployment rate is generally inefficiently high: between 1951 and 2019, the unemployment gap averages 1.4 percentage points. The unemployment gap is sharply countercyclical, which means that inefficiencies are exacerbated in slumps. The gap is close to zero at business-cycle peaks: sometimes negative (for instance, -1.1 percentage point in 1969 and -0.5 points in 2019), and sometimes positive (for instance, 0.4 points in 1979 and 0.3 points in 2007). And the unemployment gap is highly positive at business-cycle troughs: for instance, 6.1 points in 1982, 3.2 points in 1992, and 6.2 points in 2009. Unsurprisingly, the largest unemployment gaps occurred after the Volcker recession and Great Recession.

Relaxing assumption 3. Although assumption 3 is required to obtain precise values of the unemployment gap, it is possible to determine whether unemployment is inefficiently high or low without the assumption. Indeed, unemployment is inefficiently high whenever labor-market tightness is inefficiently low: $\theta < (1 - \zeta)/(\kappa\epsilon)$. Conversely, unemployment is inefficiently low whenever tightness is inefficiently high: $\theta > (1 - \zeta)/(\kappa\epsilon)$. But from (5), we know that $\theta < (1 - \zeta)/(\kappa\epsilon)$ whenever actual unemployment is above the efficiency line; and $\theta > (1 - \zeta)/(\kappa\epsilon)$ whenever actual unemployment is below the efficiency line. Hence, even if assumption 3 does not hold, the graph in panel C of figure 6 continues to be informative: it indicates whether unemployment is inefficiently high or low. In other words, without assumption 3, the size of the unemployment gap given by the graph may not be correct; but the sign of the unemployment gap remains valid.

Comparisons with other unemployment gaps. To provide some context, we compare our efficient unemployment rate to three other unemployment measures that are commonly used to construct unemployment gaps: trend unemployment, NAIRU, and the Congressional Budget Office's natural rate of unemployment, which features prominently in policy discussions (Dickens 2009).

These measures do not generally capture the efficient unemployment rate. In most models average unemployment is not efficient, so extracting the secular trend from the unemployment series does not provide a measure of efficient unemployment (Pissarides 2000, chapter 8; Hall 2005c). As for the NAIRU, obtained by estimating a Phillips curve, it was never meant to indicate labor-market efficiency (Rogerson 1997). The CBO's natural rate of unemployment is constructed

by blending trend and NAIRU considerations (Shackleton 2018, appendix B); it therefore cannot be expected to measure labor-market efficiency.

Our estimate of efficient unemployment is compared to trend unemployment, NAIRU, and the CBO's natural rate of unemployment in figure 6, panel D. Trend unemployment and the NAIRU are constructed by Crump et al. (2019, figure 8B) using state-of-the-art techniques. The main similarity between the four measures is that they are slow-moving. As the actual unemployment rate is sharply countercyclical, the unemployment gap constructed with either measure will be countercyclical. Another similarity is that the four unemployment measures were higher in the 1970s and 1980s, and lower after that. The main difference is that our measure is lower than the three others. On average the efficient unemployment rate is 1.6 percentage points below the CBO's natural rate of unemployment, 1.2 points below the NAIRU, and 1.5 points below trend unemployment. As a result, the unemployment gap constructed with the efficient unemployment rate will be higher than that constructed with the three other unemployment measures. However, the four measures converged in the 2010s, and as of 2019, they are close, between 4.0% and 4.5%.

4.5. Alternative calibrations of the sufficient statistics

We explore the sensitivity of the efficient unemployment rate and unemployment gap to alternative calibrations of the sufficient statistics.

Beveridge elasticity. We begin by reconstructing the efficient unemployment rate when the Beveridge elasticity takes any value in its 95% confidence interval (figure 7, panel A). When the estimated elasticity is higher, the efficient unemployment rate is also higher. Hence, when the Beveridge elasticity is at the top end of its 95% confidence interval, the efficient unemployment rate follows the same pattern as under the baseline calibration, but is on average 0.5 percentage point higher (top thin pink line). And when the Beveridge elasticity is at the bottom end of its 95% confidence interval, the efficient unemployment rate follows the same pattern as under the baseline calibration, but is on average 0.6 percentage point lower (bottom thin pink line). For any Beveridge elasticity inside the confidence interval, the efficient unemployment rate is somewhere between these two extremes (pink band).

To summarize, when the estimated elasticity ϵ spans its 95% confidence interval, the efficient unemployment rate remains contained in a band whose width averages 1.1 percentage point, and is always below 2.2 percentage points. Furthermore, since vacancy data from JOLTS have become available (2001), estimates of the Beveridge elasticity have become more precise, and the 95% confidence interval for ϵ has narrowed. As a result, the band of possible efficient unemployment rates has narrowed to a width of 0.7 percent points since 2001.

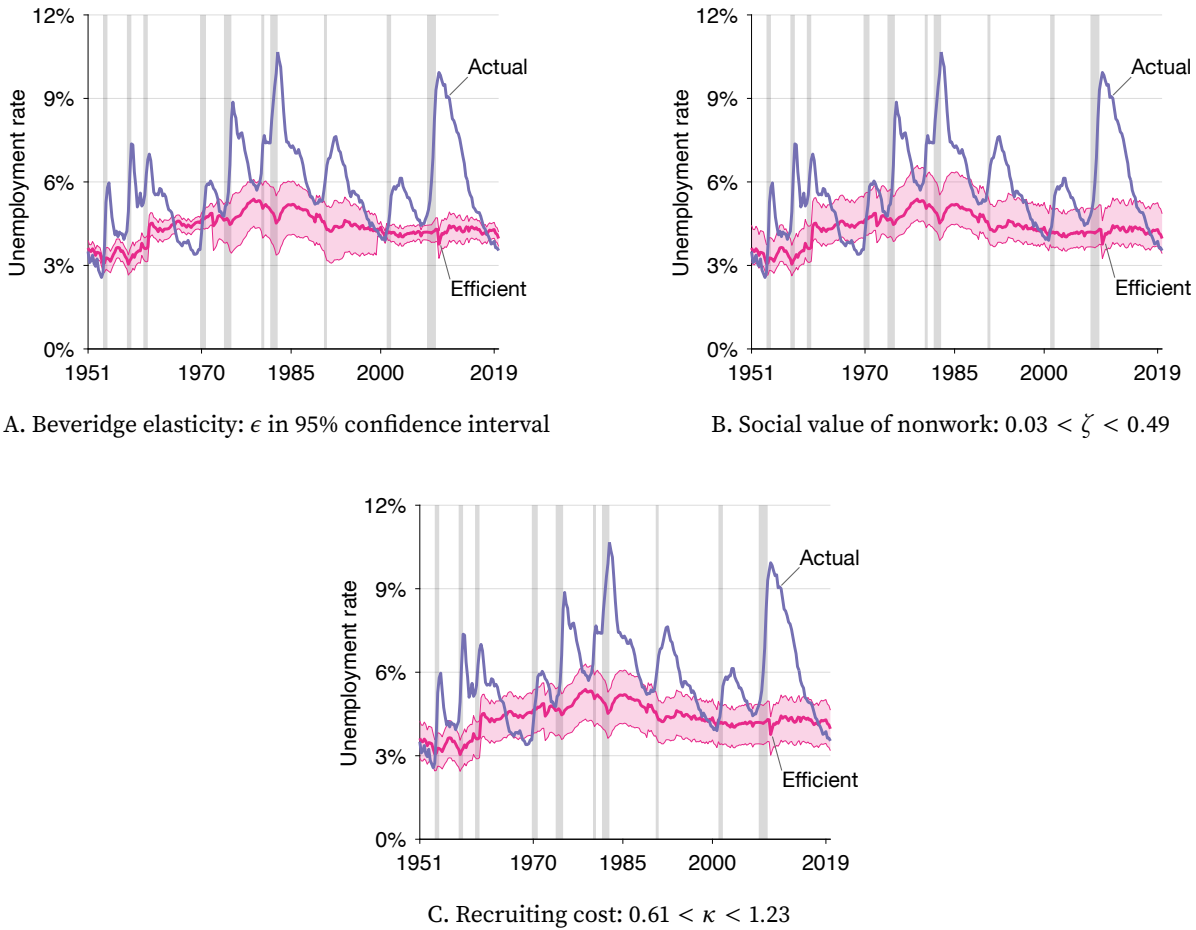


FIGURE 7. Efficient unemployment rate in the United States for a range of calibrations

The panels reproduce panel B of figure 6, and add ranges of efficient unemployment rates obtained when the sufficient statistics span ranges of plausible values (pink areas). In panel A, the Beveridge elasticity spans its 95% confidence interval (see figure 5). The top thin line is the efficient unemployment rate obtained with the top-range value of ϵ , and the bottom thin line is that obtained with the bottom-range value of ϵ . In panel B, the social value of nonwork spans the range of plausible values: $\zeta \in [0.03, 0.49]$. The top thin line is the efficient unemployment rate obtained with $\zeta = 0.49$, and the bottom thin line is that obtained with $\zeta = 0.03$. In panel C, the recruiting cost spans a plausible range: $\kappa \in [0.61, 1.23]$. The top thin line is the efficient unemployment rate obtained with $\kappa = 1.23$, and the bottom thin line is that obtained with $\kappa = 0.61$.

Social value of nonwork. Next we reconstruct the efficient unemployment rate when the social value of nonwork spans the range of plausible values given by Borgschulte and Martorell (2018) and Mas and Pallais (2019) (figure 7, panel B). First, we consider an estimate at the low end of the range of plausible values: $\zeta = 0.03$. Under this calibration, the efficient unemployment rate follows the same pattern as under the baseline calibration but is on average 0.6 percentage point lower (bottom thin pink line). Next, we consider an estimate at the high end of the range of plausible values: $\zeta = 0.49$, which is also consistent with estimates used in macro-labor literature (Chodorow-

Reich and Karabarbounis 2016, equation (30)). The efficient unemployment rate follows again the same pattern as under the baseline calibration, but it is on average 0.9 percentage point higher (top thin pink line).

Hence, around our baseline calibration of $\zeta = 0.26$, the efficient unemployment rate is fairly insensitive to the precise value of ζ . For any ζ in the 0.03–0.49 range, the efficient unemployment rate remains contained in a band that is never wider than 1.9 percentage points (pink band). This is reassuring as the range of plausible values for ζ is quite broad.

Recruiting cost. We do not have enough evidence to construct an interval of empirically plausible values for the recruiting cost. Instead we construct an artificial interval, and examine the sensitivity of the efficient unemployment rate to alternative values of the recruiting cost. We consider recruiting costs between two thirds and four thirds of our estimate, so between $2/3 \times 0.92 = 0.61$ and $4/3 \times 0.92 = 1.23$. When the low-end recruiting cost ($\kappa = 0.61$), the efficient unemployment rate follows the same pattern as under the baseline calibration but is on average 0.8 percentage point lower (bottom thin pink line). In contrast, with the high-end recruiting cost ($\kappa = 1.23$), the efficient unemployment rate follows the same pattern as under the baseline calibration but is on average 0.7 percentage point higher (top thin pink line).

Conclusion. For any plausible estimate of the sufficient statistics, the unemployment gap never departs from the baseline by more than 1 percentage point. This means that our substantive findings—that the US labor market is almost always inefficient, generally inefficient slack, and especially inefficiently slack in slumps—are robust to alternative calibrations.

Aside on some macro-labor calibrations of the social value of nonwork. Our baseline calibration implies that the social value of nonwork is much lower than labor productivity; in contrast, some macro-labor papers argue that the two are very close. A well-known calibration, due to Hagedorn and Manovskii (2008), is $\zeta = 0.96$. Such a calibration has a drastic impact: it pushes the efficient unemployment rate above 14%, and sometimes as high as 26%, with an average value of 20.2% (figure 8). Under this calibration, unemployment is always inefficiently low—even at the peak of the Great Recession. This result seems implausible, suggesting that such calibration understates the social cost of unemployment.

4.6. Inverse-optimum sufficient statistics

To provide further perspective, we compute the values of the sufficient statistics that arise under the assumption that US unemployment is efficient at all time. Such inverse-optimum sufficient

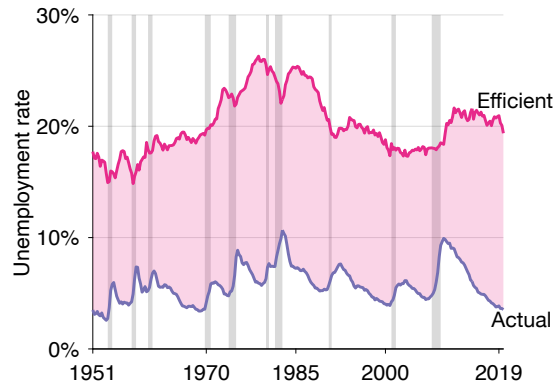


FIGURE 8. Unemployment gap in the United States when the social value of nonwork is $\zeta = 0.96$

The figure reproduces panel B of figure 6, but uses instead the social value of nonwork proposed by Hagedorn and Manovskii (2008): $\zeta = 0.96$. The shaded areas represent recessions, as identified by the NBER.

statistics are commonly computed in public economics to understand the conditions under which current policy would be optimal (for example, Hendren 2020). Here, the distance between the inverse-optimum values of the statistics and their calibrated values is another measure of the distance between current labor market conditions and efficiency.

Beveridge elasticity. We start with the inverse-optimum Beveridge elasticity. Proposition 2 shows that the value of the Beveridge elasticity under which actual tightness θ is efficient, given the other sufficient statistics, is

$$(7) \quad \epsilon^* = \frac{1 - \zeta}{\kappa\theta}.$$

To support labor market efficiency, the Beveridge elasticity would have to be strongly counter-cyclical, varying between 0.5 in booms and 5.0 during the Great Recession (figure 9, panel A). Furthermore, the Beveridge elasticity would need to be much higher than estimated, with an average value of 1.6 instead of 0.9. In sum, the inverse-optimum Beveridge elasticity is generally far above the 95% confidence interval for the estimated Beveridge elasticity.

Social value of nonwork. Next, we consider the inverse-optimum social value of nonwork. Once more, proposition 2 indicates that the social value of nonwork under which actual tightness is efficient is

$$(8) \quad \zeta^* = 1 - \kappa\epsilon\theta.$$

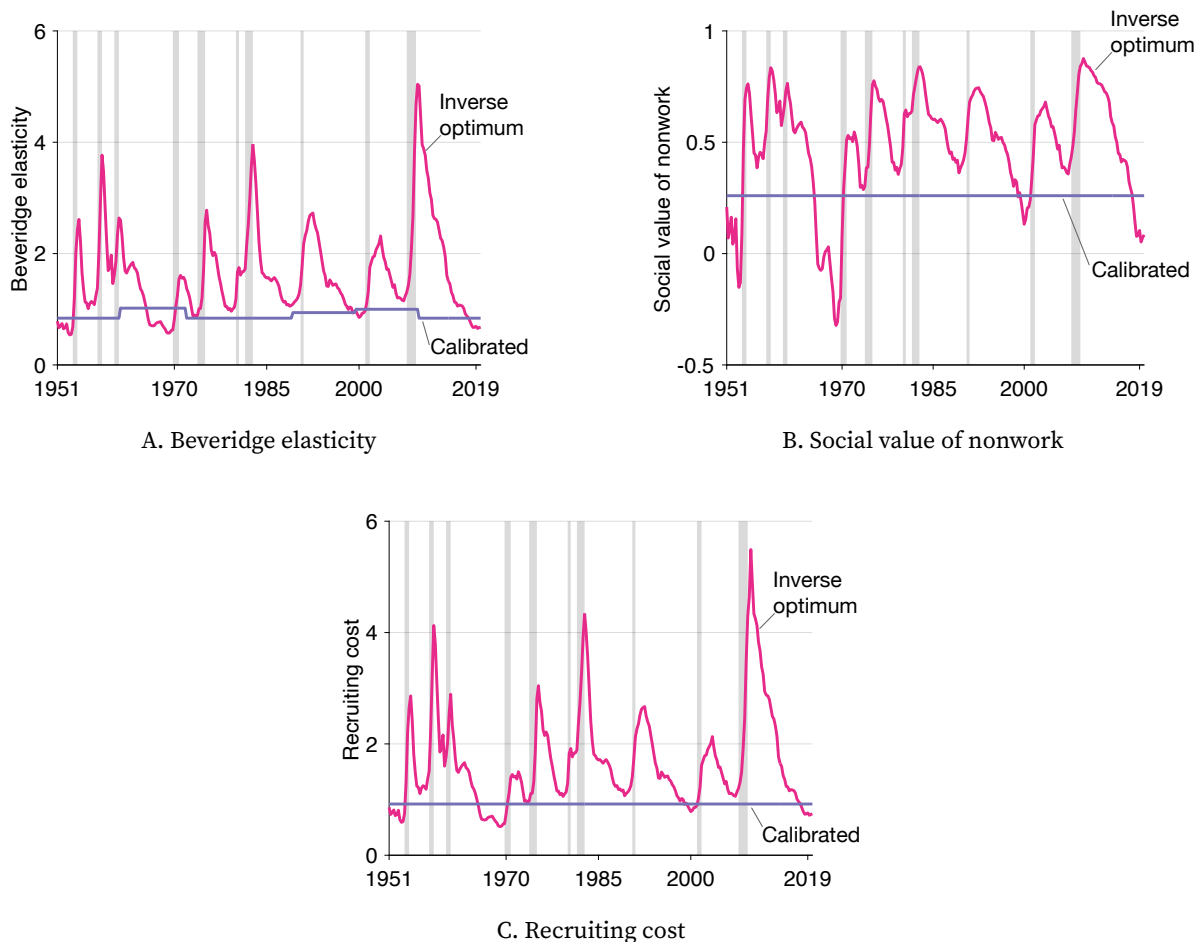


FIGURE 9. Inverse-optimum sufficient statistics

The figure displays the Beveridge elasticity (panel A), social value of nonwork (panel B), and recruiting cost (panel C) that would ensure that the observed unemployment rate is efficient at all time. These values are obtained from (7), (8), and (9). The figure also compares these inverse-optimum values with the calibrated values of the statistics. The shaded areas represent recessions, as identified by the NBER.

To sustain efficiency, the social value of nonwork would need to be immensely countercyclical, as low as -0.32 in booms and as high as 0.88 during the Great Recession, with an average value of 0.48 (figure 9, panel B). Under the inverse-optimum social value of nonwork, recessions would merely be vacations.

Recruiting cost. Last, we turn to the inverse-optimum recruiting cost. Again, from proposition 2, the value of the recruiting cost under which actual tightness is efficient is

$$(9) \quad \kappa^* = \frac{1 - \zeta}{\epsilon \theta}.$$

To support labor market efficiency, the recruiting cost would have to be strongly countercyclical, varying between 0.5 in booms and 5.5 during the Great Recession, with an average value of 1.7 (figure 9, panel C). Since the inverse-optimum recruiting cost is ten times higher in slumps than in booms, unemployment is continuously efficient only if recruiting demands ten times more labor in bad times than in good times, which seems implausible.

5. Application to the DMP model

The DMP model is the most widely used of all Beveridgean models of the labor market. Here we show how our sufficient-statistic formula for efficiency can be applied to the canonical DMP model presented in Pissarides (2000, chapter 1). We also compare the allocation given by our formula to that given by the well-known Hosios (1990) condition.

5.1. Beveridge curve

Matching function. Following common practice, we assume a Cobb-Douglas matching function

$$(10) \quad m(u, v) = \mu u^\alpha v^{1-\alpha},$$

where $\mu > 0$ is the matching efficacy and $\alpha \in (0, 1)$ is the matching elasticity.

Dynamics of the unemployment rate. The unemployment rate evolves according to the following differential equation:

$$(11) \quad \dot{u}(t) = s \cdot [1 - u(t)] - m(u(t), v(t)),$$

where s is the job-separation rate. The term $s \cdot [1 - u(t)]$ gives the number of workers who lose or quit their jobs and enter unemployment during a unit time. The term $m(u(t), v(t))$ gives the number of unemployed workers who find a job during a unit time. The difference between the inflows into unemployment and outflows from unemployment gives the change in the unemployment rate, \dot{u} .

The differential equation (11) can be expressed as a simple first-order homogeneous linear differential equation:

$$(12) \quad \dot{u}(t) + (s + f)[u(t) - u^b] = 0,$$

where $f = m(u, v)/u = \mu\theta^{1-\alpha}$ is the job-finding rate, and

$$(13) \quad u^b = \frac{s}{s+f}$$

is the Beveridgean unemployment rate—the unique unemployment rate at which inflows into unemployment equal outflows from unemployment, for given job-finding and job-separation rates. The Beveridgean unemployment rate is the critical point of differential equation (12).

We solve this differential equation by treating s and f as parameters. The solution is

$$(14) \quad u(t) - u^b = [u(0) - u^b]e^{-(s+f)t}.$$

Half-life of the deviation from Beveridgean unemployment. Equation (14) shows that the distance between the unemployment rate $u(t)$ and the Beveridgean unemployment rate u^b decays at an exponential rate. In the United States, labor-market flows are large, so the rate of decay $s + f$ is really fast. On average between 1951 and 2019, the job-separation rate is $s = 3.4\%$ per month, and the job-finding rate is $f = 58.7\%$ per month (appendix B). Hence, the rate of decay is $s + f = 62.1\%$ per month, and the half-life of the deviation from the Beveridgean unemployment rate, $u(t) - u^b$, is $\ln(2)/0.621 = 1.1$ month. Since about 50% of the deviation evaporates within one single month—and about 90% within one quarter—the unemployment rate is always close the Beveridgean unemployment rate, as previously noted by Elsby, Michaels, and Solon (2009, p. 88).

In practice, the unemployment rate is almost indistinguishable from the Beveridgean unemployment rate (see figure 10; see also Hall 2005b, figure 1). The correlation between the two series is 0.982. While the maximum absolute distance between the two series is 1.5 percentage points, the average absolute distance is only 0.2 points, and the average distance is 0.01 points.

Beveridge curve. Given such short half-life, it is accurate to assume that inflows into unemployment equal outflows from unemployment at all times: $s \cdot (1 - u) = m(u, v)$. Then the labor market is always on the Beveridge curve

$$(15) \quad v(u) = \left[\frac{s(1-u)}{\mu u^\alpha} \right]^{1/(1-\alpha)}.$$

Beveridge elasticity. From the Beveridge curve (15), we obtain the Beveridge elasticity:

$$(16) \quad \epsilon = \frac{1}{1-\alpha} \left(\alpha + \frac{u}{1-u} \right).$$

The Beveridge elasticity is closely related to the matching elasticity, α .

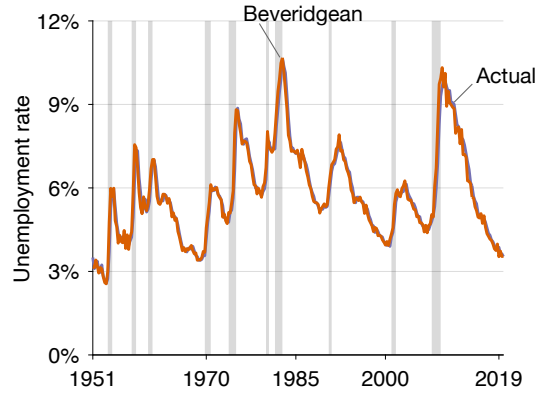


FIGURE 10. Accuracy of the Beveridgean model

The actual unemployment rate comes from panel A of figure 1. The Beveridgean unemployment rate is constructed from (13), using the job-separation and job-finding rates from figure A3. The shaded areas represent recessions, as identified by the NBER.

As the unemployment rate u is an order of magnitude smaller than the matching elasticity α , the term $u/(1-u)$ is an order of magnitude smaller than the term α . To a first-order approximation, the Beveridge elasticity is therefore given by

$$(17) \quad \epsilon \approx \frac{\alpha}{1 - \alpha}.$$

Hence, the Beveridge elasticity is approximately constant, in line with what we postulated in assumption 3.

5.2. Social welfare

Welfare function. The labor force is composed of L workers. Employed workers have a productivity $p > 0$. Unemployed workers have a productivity $p \cdot z < p$, where $z < 1$ is the relative productivity of unemployed workers.⁵ Finally, firms incur a resource cost $p \times c$ for each vacancy that they post.

Workers' utility function is linear, so the welfare function is

$$(18) \quad \mathcal{W}(n, u, v) = p \cdot (n + zu - cv) \cdot L.$$

⁵Pissarides (2000) initially specifies the productivity of unemployed workers as constant—independent of the productivity of employed workers (p. 13). But he also considers the specification that we use here (p. 74), and other specifications in which the productivity of unemployed workers is proportional to that of employed workers (p. 21 and p. 72). We opt to model the productivity of unemployed workers as proportional to that of employed workers to be consistent with the evidence presented by Chodorow-Reich and Karabarbounis (2016).

Social value of nonwork and recruiting cost. From the social welfare function (18), the social value of nonwork ζ and recruiting cost κ take a simple form:

$$(19) \quad \zeta = z \quad \text{and} \quad \kappa = c.$$

5.3. Efficiency condition

We now combine the values of the sufficient statistics with the efficiency condition (2) to determine the efficient labor-market tightness in the DMP model.

Simplified formula. Using the approximate expression of the Beveridge elasticity given by (17), we obtain a simple expression for the efficient labor-market tightness:

$$(20) \quad \theta^* = \frac{1 - \alpha}{\alpha} \cdot \frac{1 - z}{c}.$$

The efficient tightness depends only on the shape of the matching function, α , the recruiting cost, c , and the relative difference in productivity between employed and unemployment workers, $1 - z$.

Exact formula. We can also use the exact expression of the Beveridge elasticity, given by (16), to obtain a more accurate formula for the efficient labor-market tightness. This formula is also more cumbersome because it only defines the efficient tightness implicitly. Using the exact Beveridge elasticity, we find that the efficient tightness satisfies

$$\theta = \frac{1 - \alpha}{\alpha + u/(1 - u)} \cdot \frac{1 - z}{c}.$$

On the Beveridge curve, labor flows are balanced, so $s \cdot (1 - u) = f(\theta) \cdot u$, where $f(\theta) = \mu\theta^{1-\alpha}$ is the job-finding rate. This means that $u/(1 - u) = s/f(\theta)$. Accordingly, we obtain an implicit definition of the efficient tightness θ^* :

$$(21) \quad \alpha\theta^* + \frac{s}{q(\theta^*)} = (1 - \alpha) \cdot \frac{1 - z}{c},$$

where $q(\theta) = f(\theta)/\theta = \mu\theta^{-\alpha}$ is the vacancy-filling rate. The left-hand side of the equation is continuous and strictly increasing from 0 when $\theta^* = 0$ to $+\infty$ when $\theta^* \rightarrow +\infty$. By the intermediate-value theorem, there exists a unique θ^* that satisfies the equation (21).

5.4. Relation to the Hosios condition

In the DMP model, workers negotiate their wages with firms via Nash bargaining. And when workers' bargaining power β equals the matching elasticity α , the labor market is guaranteed to operate efficiently; that is, when $\beta = \alpha$, the decentralized equilibrium coincides with the planning solution (Hosios 1990).⁶

Here, we examine how the tightness given by the efficiency condition (21) relates to the tightness arising from the Hosios condition. These two tightnesses might be different because they solve different planning problems. In our planning problem the Beveridge curve holds at all times, whereas in the Hosios planning problem the unemployment rate follows the differential equation (11).

Tightness under the Hosios condition. We begin by describing the labor-market tightness in the DMP model. At any point in time, tightness is given by the job-creation curve:

$$(22) \quad (1 - \beta)(1 - z) - \left[\frac{r + s}{q(\theta)} + \beta\theta \right] c = 0,$$

where r is the discount rate, and β is workers' bargaining power (Pissarides 2000, equation (1.24)). This expression holds even if the labor market is temporarily away from the Beveridge curve (Pissarides 2000, chapter 1.7). It is obtained by combining the wage equation, which describes the wages obtained by Nash bargaining, and the free-entry condition, which says that vacancies are created until all profits opportunities from new jobs are exploited.

When the Hosios condition holds, $\beta = \alpha$, so tightness satisfies

$$(23) \quad \alpha\theta + \frac{r + s}{q(\theta)} = (1 - \alpha) \cdot \frac{1 - z}{c}.$$

Comparison with our efficiency condition. Comparing (23) with (21), we obtain the following results:

PROPOSITION 4. *In the DMP model with zero discount rate, the labor-market tightness given by the efficiency condition (2), θ^* , is exactly the same as the tightness given by the Hosios condition, θ^h . In the DMP model with positive discount rate, $r > 0$, the two tightnesses are not exactly the same, but the*

⁶Hosios proves the result by assuming the discount rate is zero and therefore that the social planner maximizes steady-state welfare (Hosios 1990, p. 281). But the result continues to hold when the discount rate is positive and the social planner maximizes the present-discounted sum of instantaneous social welfare (Pissarides 2000, chapter 8.1).

difference is minor as long as the discount rate is small. To a first-order approximation,

$$\frac{\theta^* - \theta^h}{\theta^*} = \frac{r}{\alpha \cdot (s + f)}.$$

Under the calibration in Shimer (2005, table 2), the relative deviation between the two tightnesses is commensurate to the quarterly discount rate r :

$$\frac{\theta^* - \theta^h}{\theta^*} = 0.96 \times r = 1.1\%.$$

The proof of the proposition is relegated to the appendix A, but the intuition is simple. When the discount rate is zero ($r = 0$), the equations (21) and (23) are the same, so they give the same tightness. When the discount rate is positive, the two tightnesses are not exactly the same, but the difference is small because r is small.

The proposition shows that in the DMP model, our efficiency condition is very accurate—even though it abstracts from unemployment dynamics.

5.5. Illustration of the results in Beveridge diagram

We now illustrate the efficiency properties of the DMP model in a Beveridge diagram (figure 11).

Efficiency. We combine our representation of labor-market efficiency with the standard representation of the DMP model's equilibrium. We first plot the Beveridge curve in the DMP model, given by (15). To find the efficient labor-market allocation, we add an isowelfare curve. Because the welfare function is given by (18), the isowelfare curve is linear with slope $-(1 - z)/c$. The efficient allocation is the point on the Beveridge curve that is tangent to the isowelfare curve.

In the DMP model the equilibrium is given by the intersection of the Beveridge curve and job-creation curve, which is given by (22). Since the job-creation curve determines a labor-market tightness θ , independent of unemployment or vacancies, it is represented by a ray through the origin, whose slope is θ .

When the equilibrium is efficient, the job-creation curve runs through the efficiency point on the Beveridge curve (panel A). Such job-creation curve appears when the Hosios condition holds, so workers' bargaining power satisfies $\beta = \alpha$.⁷

⁷Our efficient allocation and the Hosios equilibrium only exactly overlap when the discount rate is zero; when the discount rate is positive, they differ but the difference is minuscule.

Unemployment gaps. When the Hosios condition does not hold, the DMP model's equilibrium is inefficient. For instance, if workers' bargaining power is too high ($\beta > \alpha$), the job-creation curve is too low: unemployment is too high, vacancies are too low, and the unemployment gap is positive (panel B). Conversely, if workers' bargaining power is too low ($\beta < \alpha$), the job-creation curve is too high: unemployment is too low, vacancies are too high, and the unemployment gap is negative (panel C).

Business cycles under bargaining-power shocks. The evidence presented in section 4 indicates that in US business cycles, the efficient unemployment rate remains stable while the actual unemployment rate is sharply countercyclical, which generates countercyclical fluctuations in the unemployment gap. Such patterns are easily generated in the DMP model by introducing shocks to workers' bargaining power, as Shimer (2005, table 6) and Jung and Kuester (2015) do. Under such shocks, the job-creation curve rotates up and down over the business cycle, while the Beveridge and isowelfare curves are fixed. Accordingly, unemployment and vacancies travel up and down the Beveridge curve during the business cycle, while the efficient allocation is fixed.

Business cycles under labor-productivity shocks. Another simple way to generate the patterns observe in the United States is to replace Nash bargained wages by a fixed wage, as proposed by Hall (2005a), and to introduce shocks to labor productivity. In such a model, tightness is given by the job-creation curve

$$1 - \frac{w}{p} - \frac{(r+s)c}{q(\theta)} = 0,$$

where $w > 0$ is the fixed wage (Pissarides 2000, equation (1.22)). This expression is obtained by inserting a fixed wage into the DMP model's free-entry condition. The equilibrium in the fixed-wage model is given by this job-creation curve and the Beveridge curve, given by (15).

When labor productivity is low, the unit labor cost w/p is high, so the tightness given by the job-creation curve (5.5) is low (as in figure 11, panel B). Conversely, when labor productivity is high, the unit labor cost w/p is low, so the tightness given by the job-creation curve (5.5) is high (as in figure 11, panel C). At the same time, the Beveridge and isowelfare curves are unaffected by productivity. Thus, productivity shocks will generate the same fluctuations in the fixed-wage model as bargaining-power shocks in the DMP model.

5.6. Robustness and accuracy of the sufficient-statistic formula

Using the structure provided by the DMP model, we numerically assess the robustness and accuracy of the sufficient-statistic formula (5). We calibrate the DMP model to US data, 1951–2019, and

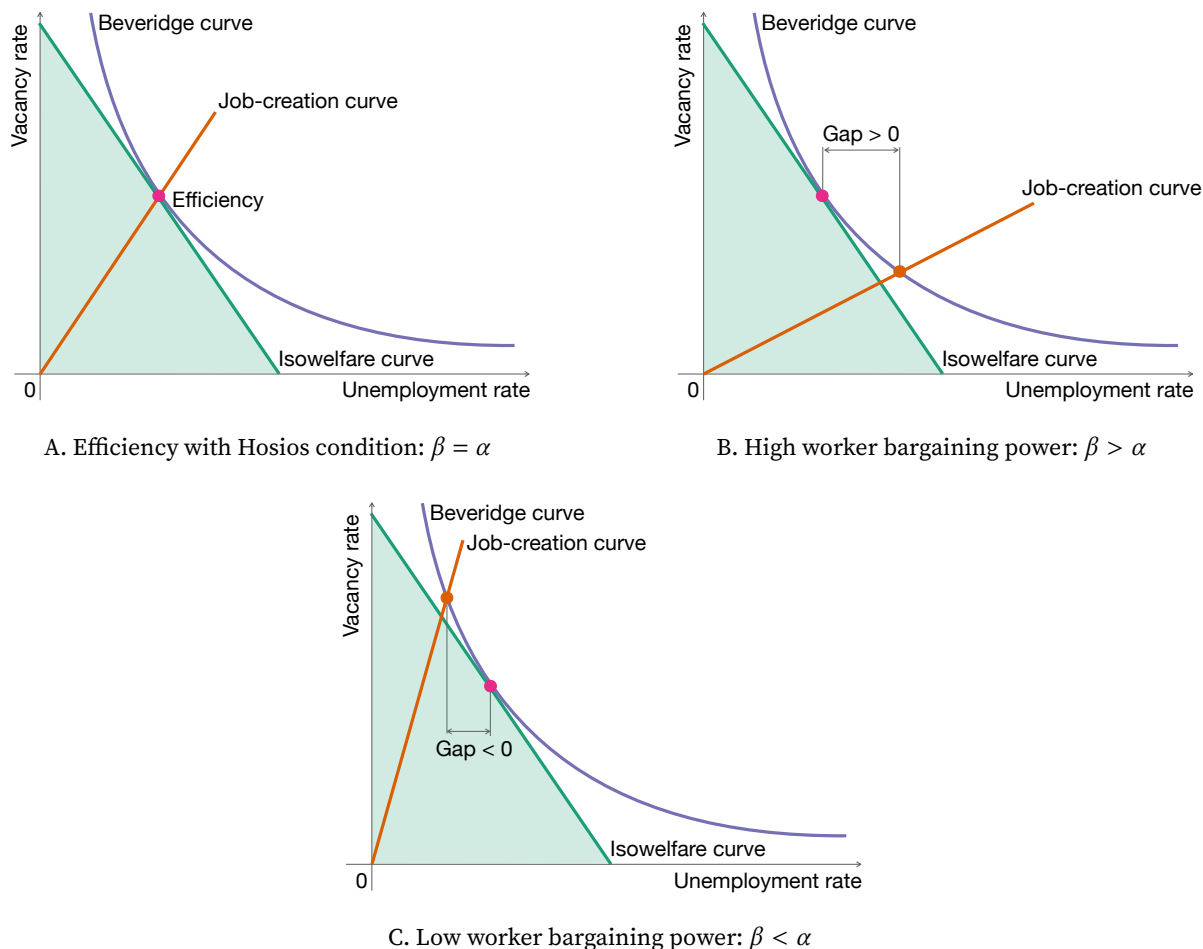


FIGURE 11. Efficient unemployment and unemployment gap in the DMP model

The Beveridge curve is given by (15). The job-creation curve is given by (22). Isowelfare curves are linear with slope $-(1-z)/c$, where z is the relative productivity of unemployment workers, and c is the recruiting cost. The equilibrium in the DMP model is at intersection of the Beveridge and job-creation curves. Panel A depicts an efficient equilibrium, which prevails when the Hosios condition holds: $\beta = \alpha$, where β is the worker bargaining power and α is the matching elasticity. Panel B depicts the equilibrium when workers' bargaining power is high ($\beta > \alpha$), so the job-creation curve is flat. Panel C depicts the equilibrium when workers' bargaining power is low ($\beta < \alpha$), so the job-creation curve is steep.

compare the efficient unemployment rate given by (5) to various alternatives. These computations confirm that despite its apparent simplicity, the sufficient-statistic formula (5) provides a robust and accurate measure of the efficient unemployment rate, and of the unemployment gap.

Baseline sufficient-statistic formula. We start by computing the efficient unemployment rate given by our sufficient-statistic formula in the context of the DMP model.

First, we translate the sufficient statistics in formula (2) in terms of parameters of the DMP model. We obtain formula (20).

Second, we calibrate the parameters to match the evidence presented in section 4. The parameters are related to the sufficient statistics by (17) and (19). We therefore set $\alpha = \epsilon / (1 + \epsilon)$, with ϵ given by figure 5; $z = \zeta = 0.26$; and $c = \kappa = 0.92$. We obtain the efficient labor-market tightness from formula (20) and these parameter values.

Third, we compute the efficient unemployment rate u^* from the efficient labor-market tightness θ^* and formula (4), which can be written

$$(24) \quad u^* = \left(\frac{\theta}{\theta^*} \right)^{1-\alpha} u.$$

This efficient unemployment rate gives the baseline plotted in all four panels of figure 12.

Nonneutral productivity fluctuations. When we apply our sufficient-statistic formula to the United States, we calibrate the social value of nonwork and recruiting cost to be constant (section 4). These two sufficient statistics are also constant in the DMP model when unemployed workers' productivity and vacancy cost are proportional to labor productivity p (equation (19)). While such proportionality necessarily holds in the the long run, it is sometimes assumed to fail in the short run (for example, Shimer 2005). In that case, the sufficient statistics and efficient unemployment rate respond to short-run productivity fluctuations. We quantify this response here.

To introduce productivity fluctuations that are nonneutral in the short run but neutral in the long run, we assume that unemployed workers' productivity and the vacancy cost are proportional to the trend of labor productivity, \bar{p} , instead of actual labor productivity. Under this alternative specification, the welfare function becomes

$$(25) \quad \mathcal{W}(n, u, v) = (p \cdot n + \bar{p} \cdot z \cdot u - \bar{p} \cdot c \cdot v) \cdot L,$$

so the social value of nonwork and recruiting cost become

$$(26) \quad \zeta = \frac{z}{\hat{p}} \quad \text{and} \quad \kappa = \frac{c}{\hat{p}},$$

where $\hat{p} = p/\bar{p}$ is detrended labor productivity. Formula (20) therefore becomes

$$(27) \quad \theta^* = \frac{1-\alpha}{\alpha} \cdot \frac{\hat{p} - z}{c}.$$

Next we measure labor productivity p as real output per worker in the nonfarm business sector, which is constructed by the BLS Major Sector Productivity and Costs (MSPC) program, and we compute the trend of productivity \bar{p} using a HP filter (figure A1, panel A). We divide p by

\bar{p} to obtain detrended labor productivity \hat{p} (figure A1, panel B).

Finally, we assess the impact of nonneutral productivity fluctuations on the efficient unemployment rate. Using (27), the same parameter values as for the baseline, and the series for detrended labor productivity in figure A1, we compute a new series for efficient labor-market tightness. We then translate it into an efficient unemployment rate using (24). We obtain the series in panel A of figure 12.

The efficient unemployment rates with and without nonneutral productivity fluctuations are indistinguishable. The correlation between the two series is 0.997; the maximum absolute distance between the two series is 0.13 percentage points. Hence, introducing nonneutral productivity fluctuations has virtually no effect on the efficient unemployment rate. This is not very surprising: Shimer (2005) shows that the efficient tightness and unemployment rate barely respond to productivity shocks.

Endogenous sufficient statistics. When we derive formula (5), we assume that the sufficient statistics do not depend on the unemployment and vacancy rates (assumption 3). Yet in the DMP model, the Beveridge elasticity does depend on the unemployment rate (equation (16)). We now recompute the efficient unemployment rate, taking into account the endogeneity of the Beveridge elasticity.

We start from formula (2), which is valid even if the sufficient statistics are endogenous. Using (6), we express the formula in terms of parameters of the DMP model. We obtain formula (21), which can be rewritten as

$$(28) \quad \alpha\theta^* + \frac{s}{\mu}(\theta^*)^\alpha = (1 - \alpha) \cdot \frac{1 - z}{c},$$

Second, we calibrate the parameters in (28). As in the baseline calculation, we set $z = 0.26$ and $c = 0.92$. We calibrate the matching elasticity α to a different value on each interval during which the Beveridge curve is stable (figure 4). On each interval, we use formula (6), the value of the Beveridge elasticity on that interval (figure 5), and the average unemployment rate on that interval (figure A2, panel A). The resulting matching elasticity is plotted in panel B of figure A2. Finally, we calibrate the ratio $s\mu$. On the Beveridge curve, the ratio, unemployment rate, and tightness are related by (13), which implies $s/\mu = \theta^{1-\alpha}u/(1 - u)$. We construct a series for $s\mu$ from this relation.

Third, we compute the efficient tightness from formula (28) and the parameter values.

Fourth, we compute the efficient unemployment rate u^* from the efficient tightness θ^* and

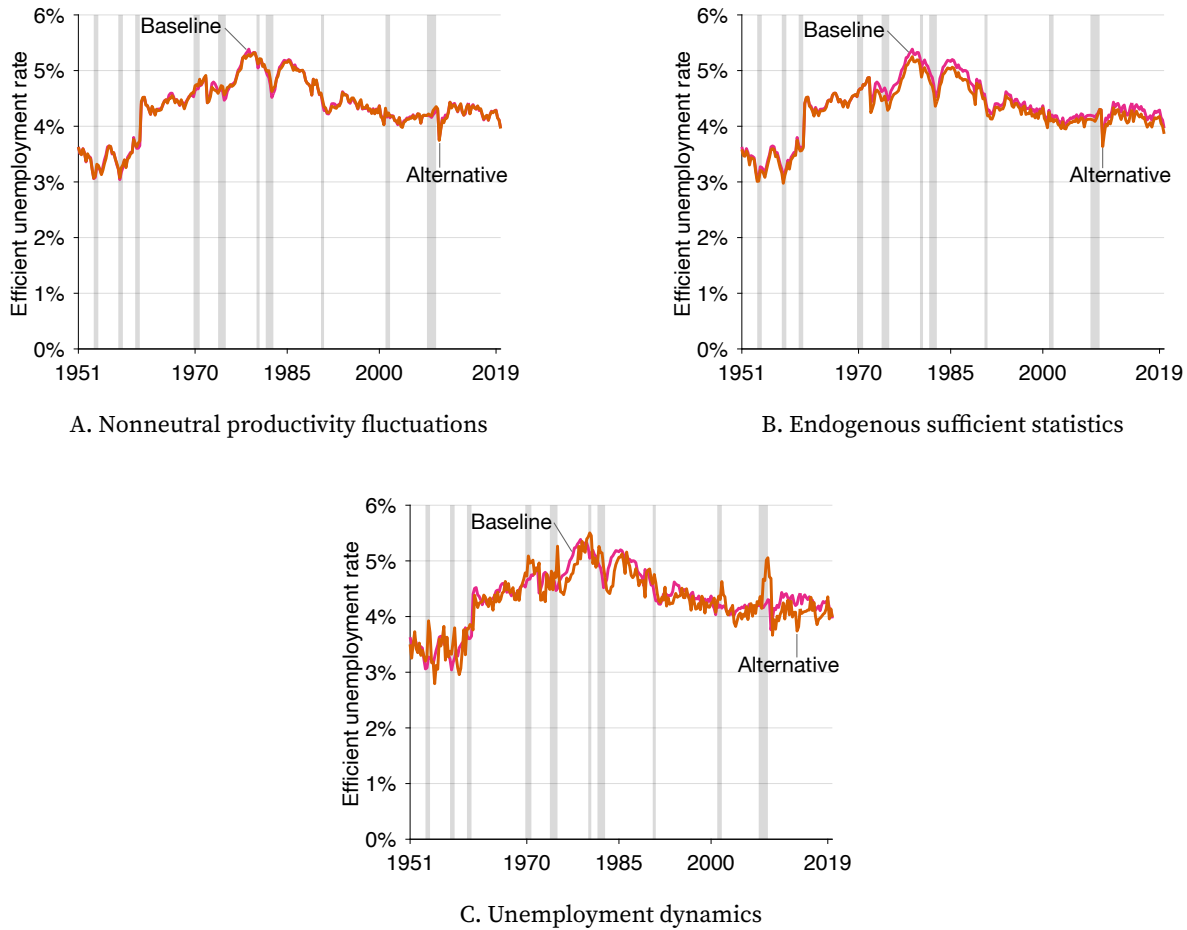


FIGURE 12. Robustness and accuracy of the sufficient-statistic formula for efficient unemployment

The baseline efficient unemployment rate in the three panels comes from panel B of figure 6. Panel A: The baseline is compared to an efficient unemployment rate that incorporates nonneutral fluctuations in labor productivity. This unemployment rate is constructed by combining (27) and (24). Panel B: The baseline is compared to an efficient unemployment rate that accounts for the endogeneity of the Beveridge elasticity in the DMP model (see (16)). This unemployment rate is constructed by solving (28) and then using (29). Panel C: The baseline is compared to an efficient unemployment rate that accounts for the dynamics of the unemployment rate in the DMP model (see (11)). This unemployment rate comes out of the Hosios condition. It is constructed by solving (30) and then using (31). The shaded areas represent recessions, as identified by the NBER.

formula (13), which can be written

$$(29) \quad u^* = \frac{(s/\mu)}{(s/\mu) + (\theta^*)^{1-\alpha}}.$$

(Here we cannot use (24) because we are not assuming that the Beveridge curve is isoelastic.) The resulting efficient unemployment rate is plotted in panel B of figure 12.

This efficient unemployment rate and the baseline are extremely close to each other. The

correlation between the two series is 0.995; the maximum absolute distance between the two series is 0.18 percentage points. Hence, accounting for the endogeneity of the Beveridge elasticity that appears in the DMP model has almost no effect on the efficient unemployment rate.

Unemployment dynamics. We derive formula (5) under the assumption that the Beveridge curve holds at all times (assumption 1). In contrast in the DMP model, unemployment dynamics are given by the differential equation (11). When unemployment dynamics are accounted for, the efficient tightness satisfies (23), which arises from the Hosios condition. We now examine how unemployment dynamics affect the efficient unemployment rate.

We begin by rewriting (23) as

$$(30) \quad \alpha\theta^* + \frac{s+r}{\mu}(\theta^*)^\alpha = (1-\alpha) \cdot \frac{1-z}{c},$$

We calibrate the parameters in (30) exactly the parameters in (28), with the exception of s and μ . Indeed, since we do not assume that unemployment is always the Beveridge curve, we cannot use that curve to measure s and μ . Instead, we use the method proposed by Shimer (2012) to compute s and μ from CPS data (appendix B). The resulting parameter values are plotted in panels C and D of figure A3. We also set $r = 0.012$, which corresponds to an annual discount rate of 5%, as in Shimer (2005, table 2).

Third, we compute the Hosios labor-market tightness from formula (30) and the parameter values.

Fourth, we compute the Hosios unemployment rate u^* from the Hosios labor-market tightness θ^* and the solution to differential equation (11), which is given by (14). We initialize $u^*(1) = u(1)$; we then construct the unemployment rate recursively, by iterating

$$(31) \quad u^*(t+1) = u^b(\theta^*(t)) + [u^*(t) - u^b(\theta^*(t))]e^{-[s+f(\theta^*(t))]},$$

where $f(\theta^*(t)) = \mu \cdot (\theta^*(t))^{1-\alpha}$ and $u^b(\theta^*(t)) = s/[s + f(\theta^*(t))]$. (We can use neither (24) nor (29) because we are not assuming that unemployment is on the Beveridge curve.) The resulting Hosios unemployment rate is plotted in panel C of figure 12.

The Hosios unemployment rate is close to the baseline efficient unemployment rate, although not as close as the previous series. The correlation between these two series is 0.887. And while the maximum absolute distance between the two series is 0.92 percentage points, the average absolute distance is only 0.19 points, and the average distance is only 0.05 points.

Actually the source of the distance between the two series is not the tightnesses, but the formulas used to convert tightness into unemployment, (24) and (31). In particular, the fluctuations in

the parameters s and μ (see figure A3) introduce additional volatility in the Hosios unemployment rate, especially at the onset of recessions.

6. Summary and Implications

To conclude, we summarize the results of the paper and discuss some of their implications.

6.1. Summary

This paper develops a new method to measure the unemployment gap—the difference between the actual and the socially efficient unemployment rate. We consider a labor-market model with only one structural element: a Beveridge curve relating unemployment and vacancies. This Beveridgean framework covers many modern labor-market models, including the DMP model. We show that the unemployment gap can be measured from current unemployment and vacancy rates, and three sufficient statistics: the elasticity of the Beveridge curve, cost of recruiting, and social cost of unemployment.

We apply our unemployment-gap formula to the United States, 1951–2019. We find that the US unemployment gap is countercyclical: the gap is close to zero in booms but is highly positive in slumps. We infer that the US unemployment rate is generally inefficiently high, and such inefficiency worsens in slumps.

6.2. Implications for labor-market models

Our Beveridgean model of the labor market is quite general, so it allows for a broad range of assumptions. But the evidence presented in the paper is not consistent with all of them. In particular, given that the unemployment rate is almost always inefficient, and sometimes sharply so, it might not be accurate to model the labor market as always efficient.

Nash bargaining with Hosios condition. In the DMP model, it is customary to set workers' bargaining power at the level given by the Hosios condition (for example, Mortensen and Pissarides 1994; Shimer 2005; Costain and Reiter 2008). This choice is convenient because the bargaining power is difficult to estimate empirically (Pissarides 2000, p. 229). But such calibration implies that unemployment is efficient at all times—which is at odds with this paper's findings. Instead, researchers working with DMP models should embrace the use of rigid wages proposed by Hall (2005a). Rigid wages generate realistic, countercyclical fluctuations in the unemployment gap.

Competitive-search equilibrium. In the DMP model matching is random. A popular alternative are models with directed search. In such models, jobseekers target submarkets that offer appealing employment conditions. The equilibrium concept used in most of these models is the competitive-search equilibrium developed by Moen (1997). This equilibrium concept is popular because it is theoretically appealing and very tractable. But because it predicts that unemployment is efficient at all times, it may not provide an accurate description of the labor market. This inaccuracy is particularly costly when the model is used to address policy questions.

6.3. Implications for policy

The finding that the unemployment gap is countercyclical has a range of policy implications.

Distance from full employment. Many governments are mandated to stabilize their economy at full employment. For instance, in the United States, the 1978 Humphrey-Hawkins Full Employment Act mandates that the government maintains the economy at full employment using monetary and fiscal policy. Because achieving zero unemployment is physically impossible, reaching full employment should not be interpreted as bringing unemployment to zero. Rather, it should be interpreted as reaching a socially efficient amount of unemployment. Viewed in this light, the mandate of US policymakers is to close the unemployment gap. Policymakers could therefore use our unemployment-gap measure—which can be calculated in real time—to monitor how far the economy still is from full employment.

Optimal monetary policy. Governmental mandates to eliminate the unemployment gap are consistent with optimal policy design if the government has access to stabilization policies that do not create secondary distortions. Monetary policy is such a policy when the divine coincidence holds—when closing the unemployment gap also brings inflation to its desired level, as in the New Keynesian model (Blanchard and Gali 2007).

For instance, Michaillat and Saez (2019a) develop a monetary model with unemployment and divine coincidence. In that model the central bank's optimal policy is to adjust the nominal interest rate to eliminate the unemployment gap. As the unemployment gap is countercyclical, and a reduction in interest rate lowers the unemployment rate (Bernanke and Blinder 1992; Coibion 2012), it is optimal to lower interest rates in bad times, when the unemployment gap is high, and to raise them in good times, as the unemployment gap turns negative.

Optimal fiscal policy. If the government only has access to stabilization policies that create secondary distortions—for instance because the zero lower bound is binding so conventional

monetary policy is unavailable—it is not optimal to eliminate the unemployment gap any more. Nevertheless, the unemployment gap remains a key input into optimal policy design.

Government spending is a policy that falls in this category. It can bring the unemployment rate closer to its efficient level; but in doing so it distorts households' consumption basket, as it increases the consumption of public goods at the expense of the consumption of private goods (Mankiw and Weinzierl 2011). In a model with unemployment and government spending, Michailat and Saez (2019b) indeed find that optimal government spending deviates from the Samuelson (1954) rule to reduce, but not eliminate, the unemployment gap. What fraction of the unemployment gap should be eliminated depends on the unemployment multiplier and the elasticity of substitution between public and private consumption—which measure the welfare cost of deviating from the Samuelson rule. Since the unemployment gap is countercyclical, and an increase in government spending reduces the unemployment rate (Ramey 2013, pp. 40–42), the optimal government-spending formula derived by Michailat and Saez implies that optimal government spending is countercyclical.

When the divine coincidence fails, monetary policy also falls in this category. For example, Blanchard and Gali (2010) embed a rigid-wage DMP model into a New Keynesian model. The divine coincidence fails in that model, so there is a tradeoff between closing the unemployment gap and bringing inflation to its target. Yet, the unemployment gap is a useful statistic for policymaking. Blanchard and Gali (2010, p. 23) find that a simple, linear monetary-policy rule responding to inflation and the unemployment gap achieves almost the same welfare as the optimal policy. According to that almost-optimal rule, the nominal interest rate should fall when the unemployment gap increases.

Optimal unemployment insurance. Even policies that aim to alleviate the cost of unemployment without directly reducing unemployment should be adjusted when the labor market departs from efficiency. Unemployment insurance is one such policy. Landais, Michailat, and Saez (2018a) show that when the labor market does not operate efficiently, optimal unemployment insurance deviates from the Baily (1978)-Chetty (2006) level so as to bring labor-market tightness closer to its efficient level. At the same time, most of the evidence suggests that an increase in unemployment insurance raises labor-market tightness (Landais, Michailat, and Saez 2018b, section 3). Our finding that the tightness gap is sharply procyclical therefore implies that the optimal generosity of unemployment insurance is countercyclical.

Policies for disaggregated labor markets. This paper computes the unemployment gap for the entire US labor market, but the method could also be applied to disaggregated labor markets. For

instance, with the local unemployment rates provided by the BLS and the local job vacancies measured from private-sector data by Chetty et al. (2020), it would be possible to construct local Beveridge curves and estimate local Beveridge elasticities. The data from the 1997 NES contain industry information for firms, so it would be possible to compute local recruiting cost by reweighting the NES data with local industrial compositions. Finally, we could assume that the social cost of unemployment is similar everywhere. Then we could compute local unemployment gaps and use them to guide local policies. Measuring local unemployment gaps would be helpful, for example, to target government spending to areas that need it most. They would also be helpful to tailor unemployment insurance to local labor-market conditions.

The same approach could also be applied to other disaggregated labor markets, such as labor markets for different education levels. With sufficient statistics by education level, it would be possible to compute unemployment gaps for each different education level, and to customize labor market policies to each education-specific submarket. Policies that could be targeted to each submarket include employment subsidies, hiring subsidies, and firing taxes. These policies effectively modulate labor demand (Pissarides 2000, chapter 9); therefore, they could be tailored to each submarket to close their respective unemployment gaps.

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Appendix A. Proof of proposition 4

This appendix proves proposition 4.

Definition of auxiliary function. First, we introduce a bivariate auxiliary function:

$$(A1) \quad F(r, \theta) = \alpha\theta + \frac{r+s}{q(\theta)},$$

where $r \geq 0$ is the discount rate, $\theta > 0$ is the labor-market tightness, and $q(\theta) = \mu \cdot \theta^{-\alpha}$ is the vacancy-filling rate. Note that for any r , the function $F(r, \theta)$ is strictly in θ .

From (23), we know that the tightness given by the Hosios condition, θ^h , satisfies

$$F(r, \theta^h) = (1 - \alpha) \cdot \frac{1 - z}{c}.$$

And from (21), we know that the tightness given by our efficiency condition, θ^* , satisfies

$$F(0, \theta^*) = (1 - \alpha) \cdot \frac{1 - z}{c}.$$

We infer that

$$(A2) \quad F(r, \theta^h) = F(0, \theta^*).$$

Zero discount rate. We begin by considering a zero discount rate. In that case, (A1) implies that $F(0, \theta^h) = F(0, \theta^*)$, so that $\theta^h = \theta^*$. We conclude that in the DMP model with zero discount rate, the labor-market tightness given by the Hosios condition is the same as the tightness given by the efficiency condition (2).

Positive discount rate. Next we consider a positive discount rate. We assess the gap between the tightnesses θ^h and θ^* by linearizing the function $F(r, \theta)$ around $(0, \theta^*)$. Up to a second-order term, the function $F(r, \theta)$ satisfies

$$(A3) \quad F(r, \theta) = F(0, \theta^*) + \frac{\partial F}{\partial r} \cdot r + \frac{\partial F}{\partial \theta} \cdot (\theta - \theta^*),$$

where the partial derivatives are evaluated at $(0, \theta^*)$. Using the definition of F given by (A1), we compute the partial derivatives at $(0, \theta^*)$:

$$\begin{aligned}\frac{\partial F}{\partial r} &= \frac{1}{q(\theta^*)} \\ \frac{\partial F}{\partial \theta} &= \alpha + \frac{s}{q(\theta^*)} \cdot \frac{\alpha}{\theta^*}.\end{aligned}$$

We now use the expressions of the partial derivatives to evaluate (A3) at (r, θ^h) :

$$F(r, \theta^h) = F(0, \theta^*) + \frac{r}{q(\theta^*)} + \alpha \cdot \left[\theta^* + \frac{s}{q(\theta^*)} \right] \cdot \frac{\theta^h - \theta^*}{\theta^*}$$

Given that $F(r, \theta^h) = F(0, \theta^*)$, we easily obtain the relative difference between θ^h and t^* :

$$(A4) \quad \frac{\theta^* - \theta^h}{\theta^*} = \frac{r}{\alpha \cdot (f + s)},$$

where $f = \theta^* q(\theta^*)$ is the job-finding rate at θ^* .

Applying the Shimer calibration. Last, we evaluate the relative difference between θ^h and t^* using the calibration provided by Shimer (2005, table 2): $\alpha = 0.72$, $r = 0.012$ per quarter, $s = 0.1$ per quarter, and $f = 1.35$ per quarter. Using these numbers and (A4), we conclude that the difference between θ^h and t^* is

$$\frac{\theta^* - \theta^h}{\theta^*} = \frac{r}{0.72 \cdot (1.35 + 0.1)} = 0.96 \times r = 1.1\%.$$

Appendix B. Parameters values in the DMP model

This appendix measures the parameters of the DMP model in US data, over the 1951–2019 period. We use these parameter values to compute the efficient unemployment rates displayed in figure 12.

Labor productivity (p). Following Shimer (2005), we measure labor productivity as the real output per worker in the nonfarm business sector constructed by the BLS Major Sector Productivity and Costs (MSPC) program (figure A1, panel A).

To apply the sufficient-statistic formula, we need a detrended measure of labor productivity. We compute the trend of productivity using a HP filter. Since the productivity series has a quarterly frequency, we set the filter's smoothing parameter to 1600 (Ravn and Uhlig 2002). We then compute

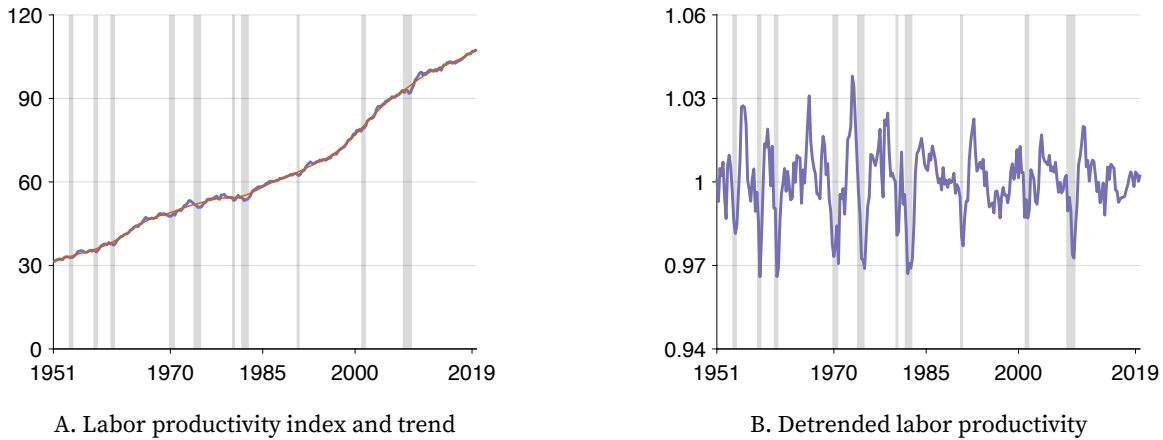


FIGURE A1. Labor productivity in the United States, 1951–2019

The labor productivity index is quarterly, seasonally adjusted, real output per worker in the nonfarm business sector, which is constructed by the BLS MSPC program and indexed to 100 in 2012. The trend of productivity is produced by a HP filter with smoothing parameter 1600. Detrended labor productivity is constructed by dividing the labor productivity index by its trend. The shaded areas represent recessions, as identified by the NBER.

detrended labor productivity by dividing the labor productivity index by its trend (figure A1, panel B).

Matching elasticity (α). We compute one matching elasticity for each interval during which the Beveridge curve is stable, as identified in figure 4. To compute the matching elasticity on each interval, we use formula (6), the value of the Beveridge elasticity on that interval (given in figure 5), and the average value of the unemployment rate on that interval (given in panel A of figure A2). The resulting matching elasticity is plotted in panel B of figure A2; it averages 0.44 over the period.

Matching efficacy (μ). With the Cobb-Douglas matching function (10), the job-finding rate is given by $f = \mu \cdot \theta^{1-\alpha}$. We already have measures of the labor-market tightness θ (figure A3, panel A) and of the matching elasticity α (figure A2, panel B). We therefore need a measure of the job-finding rate to infer the matching efficacy:

$$(A5) \quad \mu = \frac{f}{\theta^{1-\alpha}}.$$

To compute the job-finding rate, we apply the method developed by Shimer (2012, pp. 130–133).

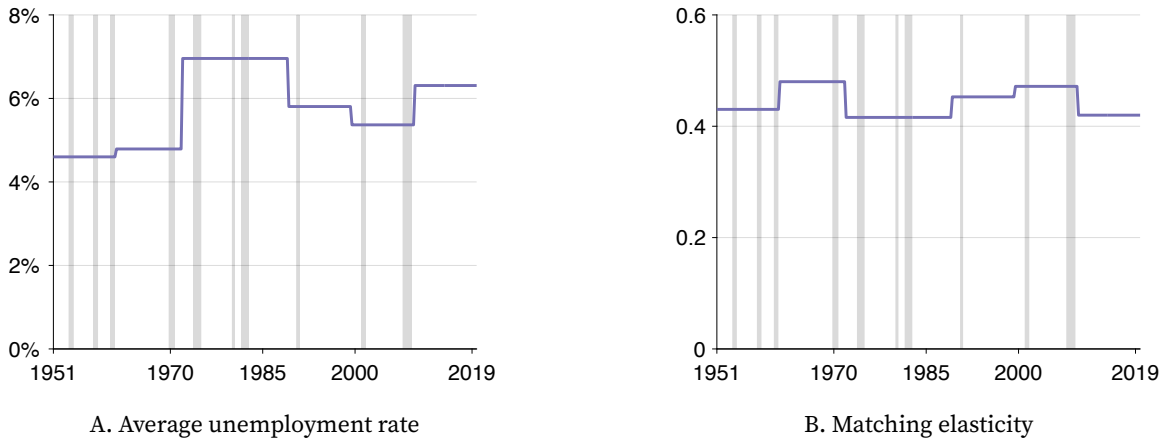


FIGURE A2. Matching elasticity in the United States, 1951–2019

The average unemployment rate is obtained by averaging the unemployment rate in panel A of figure 1 over each interval identified in figure 4. The matching elasticity is computed using (6), the Beveridge elasticity given in figure 5, and the unemployment rate in panel A. The shaded areas represent recessions, as identified by the NBER.

We first construct the monthly job-finding probability $F(t)$ in CPS data for 1951–2019:

$$(A6) \quad F(t) = 1 - \frac{u(t+1) - u^s(t+1)}{u(t)},$$

where $u(t)$ is the number of unemployed persons in month t , and $u^s(t)$ is the number of short-term unemployed persons in month t . The number of short-term unemployed persons is the number of unemployed persons with zero to four weeks duration, adjusted after 1994 as in Shimer (2012).

Assuming that unemployed workers find a job according to a Poisson process with monthly arrival rate $f(t)$, we infer the job-finding rate from the job-finding probability:

$$(A7) \quad f(t) = -\ln(1 - F(t)).$$

We multiply the monthly rate by 3 to translate it into a quarterly rate. The resulting quarterly job-finding rate is plotted in panel B of figure A3; it averages 1.76 over the period.

Finally, we construct the matching efficacy using (A5) and our measures of the job-finding rate, labor-market tightness, and matching elasticity. The resulting matching efficacy is plotted in panel C of figure A3; it averages 2.38 over the period.

Job-separation rate (s). To compute the job-separation rate, we apply the method developed by Shimer (2012, pp. 130–133) to CPS data for 1951–2019. The monthly job-separation rate $s(t)$ is

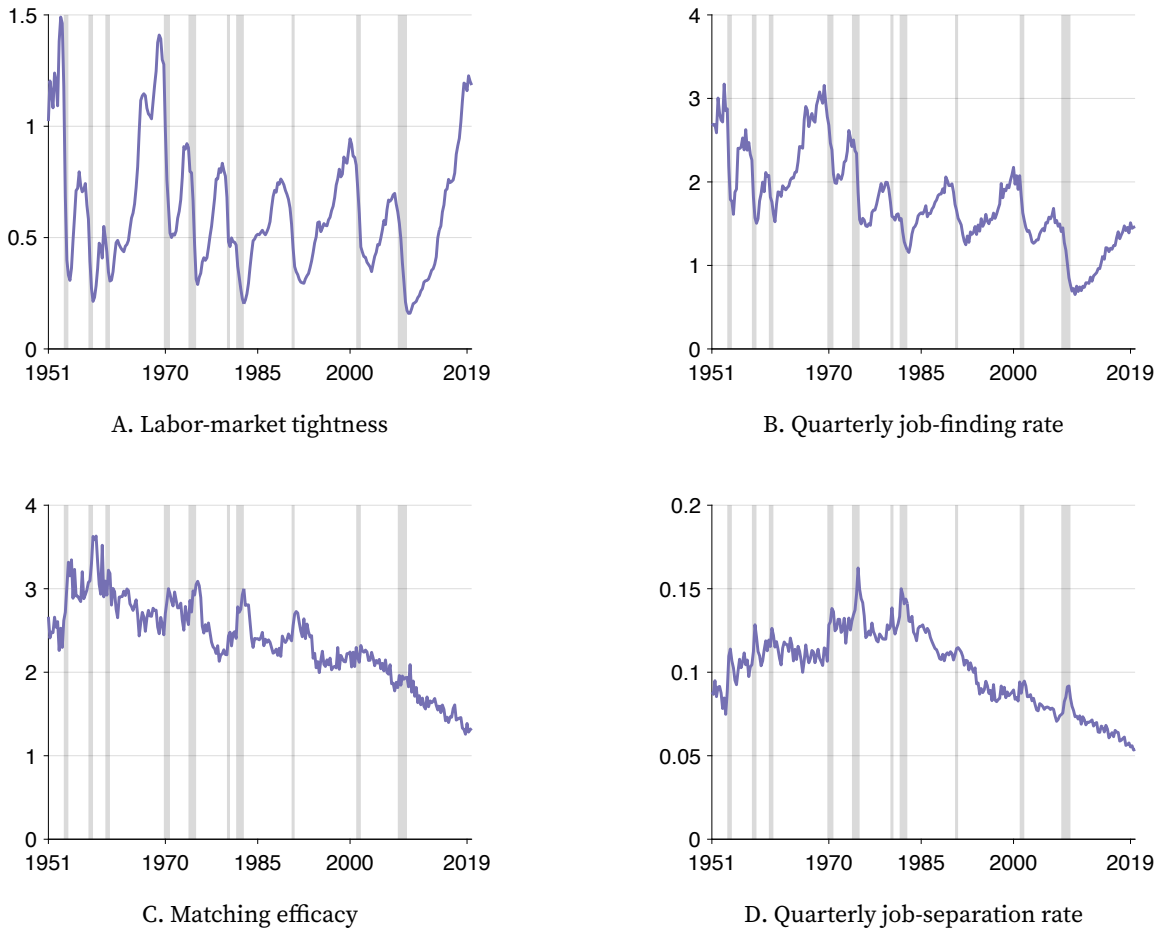


FIGURE A3. Matching efficacy and job-separation rate in the United States, 1951–2019

The labor-market tightness is vacancy rate (from figure 1, panel B) divided by unemployment rate (from figure 1, panel A). The quarterly job-finding rate is constructed from (A6) and (A7), using CPS data. The matching efficacy is constructed from (A5), using labor-market tightness from panel A and job-finding rate from panel B. The quarterly job-separation rate is constructed from (A8), using CPS data. The shaded areas represent recessions, as identified by the NBER.

implicitly defined by

$$(A8) \quad u(t+1) = \left[1 - e^{-(f(t)+s(t))} \right] \cdot \frac{s(t)}{f(t) + s(t)} \cdot h(t) + e^{-(f(t)+s(t))} \cdot u(t),$$

where $h(t)$ is the number of persons in the labor force, $u(t)$ is the number of unemployed persons, and $f(t)$ is the monthly job-finding rate, which we computed in (A7). Each month t , we solve (A8) to compute $s(t)$. We then multiply the monthly rate by 3 to translate it into a quarterly rate. The resulting job-separation rate is plotted in panel D of figure A3; it averages 0.10 over the period.