Announcements, Expectations, and Stock Returns with Asymmetric Information*

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January 3, 2021

Abstract: Revisions of consensus forecasts of macroeconomic variables positively predict announcement day forecast errors, whereas stock market returns on forecast revision days negatively predict announcement day returns. A dynamic noisy rational expectations model with periodic macroeconomic announcements quantitatively accounts for these findings. Under asymmetric information, average beliefs are not Bayesian: they underweight new information and positively predict subsequent belief errors. In addition, stock prices are partly driven by noise, and therefore negatively predict returns on announcement days when noise is revealed and the market corrects itself.

Keywords: Expectations Formation, Noisy Rational Expectations, Macroeconomic Announcement, Asymmetric Information.

JEL Code: D83, D84, G11, G12, G14

*I'm grateful to Yulei Luo, Hengjie Ai, Kenneth Kasa, Thomas Maurer, Lei Zhang, and Yu Xu for their continuous guidance and support. I also appreciate the helpful comments from Daniel Andrei (WFA discussant), Ravi Bansal, Anmol Bhandari, Xiang Fang, Dunhong Jin, Shiyang Huang, Amit Kumar, Alan Kwan, Yang Liu, Xuewen Liu, Pooya Molavi, Jun Nie, Simon Xuguang Sheng, Andrew Sinclair, Wing Suen, Chang Sun, Lai Xu, Ming Yang, Penghui Yin, and from conference and seminar participants at the 2021 American Economic Association, 2020 Western Finance Association, 2020 Econometric Society World Congress, 2020 European Finance Association, 2020 European Economic Association annual meetings, and the University of Hong Kong.

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1 Introduction

In representative agent rational expectations models, beliefs must be martingales, and forecast errors must be unpredictable. Survey evidence measuring investor beliefs, however, often shows that consensus forecast errors are *positively* correlated with forecast revisions — see, for example, Coibion and Gorodnichenko (2015). I further document that stock market returns on forecast revision days *negatively* predict subsequent returns on macroeconomic announcement days when forecast errors are realized. This stock market return predictability is particularly strong in periods of high economic uncertainty. The joint evidence of the positive predictability of forecast errors and the negative predictability of stock market reactions to them pose a challenge to rational expectations models. The purpose of this paper is to develop a dynamic noisy rational expectations equilibrium model (NREE) with periodic macroeconomic announcements to provide a unified explanation for the above facts.

Using data from the Survey of Professional Forecasters (SPF) and Consensus Economics (CE), I first show that consensus forecast revisions positively predict subsequent forecast errors. Both surveys ask participants to report, on a quarterly basis, forecasts of key macroeconomic variables such as GDP, unemployment and Consumer Price Index (CPI) for the next four quarters. The difference between the current forecast for this quarter and the previous forecast submitted last quarter provides a measure of forecast revisions concerning the current quarter. Consistent with Coibion and Gorodnichenko (2015), I show that revisions of consensus forecasts are positively correlated with their errors. I next show that the stock market returns on forecast revision days negatively predict announcement day returns, especially during periods of high economic uncertainty. Stock market returns over the forecast revision periods represent market reactions to forecast revisions and announcement day returns represent market reactions to forecast errors. Taken together, my evidence suggests that the stock market's responses to these forecast revisions negatively predict market responses to forecast errors.

The main contribution of my paper is to develop a dynamic NREE model to account for the dynamics of beliefs, returns and trading patterns around macroeconomic announcements. My model builds on the continuous-time setup of Wang (1993). It features two groups of investors, the informed and uninformed. I further assume that dividends are driven by a hidden state, unobservable to both investors. This generates uncertainty for both investors due to hidden state estimation errors. Informed investors observe a noisy signal about the latent state variable, whereas the uninformed do not. In addition, pre-scheduled macroeconomic announcements fully reveal the true value of the latent state, thereby periodically resolving the uncertainty. Finally, both investors update their beliefs based on observed equilibrium stock prices.

Both groups of investors continuously revise their beliefs rationally using the Kalman filter as new information arrives. Forecast errors are revealed periodically upon macroeconomic announcements. I demonstrate that revisions of the consensus forecast, defined as the average forecast of both investors, positively predict the forecast errors realized on announcement days. Intuitively, arrivals of the private information not only contribute to revisions of informed investors' belief but also lead to errors of the uninformed investors, who cannot update their beliefs accordingly. As a result, forecast revisions of informed investors positively predict the errors of the rational but uninformed investors. Because the consensus forecast is an average of both informed and uninformed investors' forecasts, its revisions positively predict subsequent forecast errors.

To study the predictability of announcement returns in my model, I solve for the noisy rational expectations equilibrium under periodically scheduled macroeconomic announcements. As in standard NREE models, the equilibrium price can be represented as a linear function of two state variables, the posterior belief about the state variable that governs dividend dynamics (fundamentals) and the posterior belief about the noisy supply of the stock, where posterior beliefs are computed under all publicly available information. I establish three results in this context.

First, I show that if the pricing coefficients are continuous over time, then announcement day pricing errors are unpredictable by past returns. Pricing errors are unpredictable because in equilibrium, price is a linear combination of Bayesian beliefs, and the errors of rational Bayesian beliefs cannot be predicted by their past revisions. Traditional dynamic NREE models such as Wang (1993) feature continuous (in fact, constant) pricing coefficients. Therefore, they cannot generate the observed predictability of announcement-day pricing errors.

Second, I show that the pricing coefficient on noisy supply in my model is an increasing function of time in between announcements, and then jumps downward discontinuously at announcements. I demonstrate that this feature of the pricing function generates negative predictability of pricing errors. Intuitively, following announcements, equilibrium prices become increasingly sensitive to noisy supply as the uncertainty about fundamentals accumulates. For example, a negative supply shock raises the stock price. When uninformed investors are uncertain about fundamentals, rational learning from prices implies that they will attribute part of the price increase to positive news about fundamentals, and respond by purchasing more stocks. This produces a positive forecast revision from the uninformed investors, and accordingly, the market reacts positively. As a result, the price must increase further to clear the market. In periods without macroeconomic announcements, uncertainty accumulates since the underlying state has yet to be revealed. Consequently, the above effect becomes stronger and the pricing function's noisy supply coefficient gradually rises over time.

Upon announcements, however, the true state is revealed, and the market must correct itself. Because there is a discrete reduction in uncertainty upon announcements, the equilibrium price jumps instantaneously upon announcements to reflect the revealed fundamental value of the asset. Increases in the stock price between announcements due to accumulated noise, for example, will be associated with a downward adjustment upon announcements as the market corrects itself. I provide a proposition that formally establishes that the above mechanism generates negative predictability of pricing errors. The above effect is quantitatively small if the uncertainty about the hidden state and therefore the degree of asymmetric information is small, but stronger in periods of heightened uncertainty and information asymmetry.

Third, I show that my model predicts a sharp increase in the trading volume upon announcements and an immediate drop afterwards, which is also consistent with empirical evidence. While the discontinuity in the pricing function accounts for the pricing error predictability, the discontinuity in demand functions accounts for the spikes in trading volume.

In my model, the pricing coefficients on the state variables, as well as the value and policy functions of investors, are time varying. They are jointly characterized by a system of ordinary differential equations (ODEs) subject to boundary conditions at the announcement dates. I develop a recursive method to simultaneously solve the system of ODEs numerically to obtain equilibrium prices and policy functions. I calibrate my model to match general asset market moments and replicate the regressions I conducted using the actual data. I demonstrate that my model can *quantitatively* account for the the predictability of consensus forecast errors, the predictability of announcement returns, and the dynamics of trading around pre-scheduled macroeconomic announcements as in the data.

Related Literature This paper builds on the noisy rational expectations literature pioneered by Grossman and Stiglitz (1980), Grossman (1981), and Hellwig (1980). Breon-Drish (2015) extends this class of models to allow for general information structures. Bond and Goldstein (2015) study government intervention in a model in which prices aggregate private information. Banerjee and Green (2015) analyze an environment in which uninformed investors are uncertain about whether other traders are informed or not. Han, Tang, and Yang (2016) and Goldstein and Yang (2017) analyze public information disclosure in financial markets. Gao, Sockin, and Xiong (2020) study information aggregation in the housing market.

Several recent papers study information aggregation on financial markets using this framework. Within this literature, my model is more closely related to dynamic NREE models such as Wang (1993, 1994). Relatedly, Sockin (2019) studies the feedback between financial investor trading behavior and real investment. Buffa, Vayanos, and Woolley (2019) analyze equilibrium pricing in an environment with delegated asset management. More recently, the paper by Andrei and Cujean (2017) show that an increasing rate of information acquisition can generate both momentum and reversals on financial markets. Andrei, Cujean, and Wilson (2018) incorporates time-varying public information into a NREE model to provide a novel explanation of the empirical fact that the capital asset pricing model holds on macroeconomic announcement days but not on non-announcement days. In contrast, my focus here is on the time variation in the price sensitivity to news generated by periodic macroeconomic announcements and its implications for pricing error predictability.

This paper is also related to the growing literature on measuring and explaining the expectations formation process. My evidence for belief error predictability is closely related to Coibion and Gorodnichenko (2012, 2015), who attribute the positive predictability of consensus forecast errors to the existence of information frictions.[1] This paper focuses on belief formation in the content of the asset market environment, which is similar in spirit to Allen, Morris, and Shin (2006) and

[1]By comparing surveys of return expectations and realized returns, Adam, Matveev, and Nagel (2018) find that expected stock market returns are unconditionally unbiased. Greenwood and Shleifer (2014) argue that investor expectations from data measurements are strongly negatively correlated with expected returns implied by rational expectations representative investor models.

Jouini and Napp (2007) on analyzing the consensus beliefs in the stock market and shows the predictability in terms of asymmetric information. Allen, Morris, and Shin (2006) show that prices exhibit short-run positive correlation, whereas Banerjee, Kaniel, and Kremer (2009) demonstrate that the correlation will be negative if agents learn from prices. None of the above papers provide a quantitative study for belief error predictability, announcement return predictability, as well as the dynamics of trading volume around announcements in an infinite horizon setup as I do.

A large literature has addressed the importance of heterogeneous expectations in macroeconomics and finance. Besides heterogeneous information (e.g., Wang, 1994; He and Wang, 1995; Goldstein and Yang, 2015), investors might simply hold divergent opinions even when all information is publicly available (Mayshar, 1983; Kandel and Pearson, 1995). Another stream of the literature uses survey data to directly study belief heterogeneity in asset markets. For example, Giglio, Maggiori, Stroebel, and Utkus (2019) demonstrate that beliefs are mostly characterized by individual heterogeneity, which affects trading and expected returns. Das, Kuhnen, and Nagel (2020) show that heterogeneity in individuals' socioeconomic status (income and education) influences investors' expectations and stock market participation. Piazzesi and Schneider (2009) study heterogeneity in households' beliefs and housing prices. This paper argues that heterogeneity arising from asymmetric information is crucial in explaining the positive predictability of consensus forecast errors under the assumption of rational expectations. In addition, this information asymmetry is eliminated periodically following macroeconomic announcements, producing negative predictability of price responses to the information revelation.

The remainder of the paper is organized as follows. Section 2 provides empirical evidence on the relationships between forecast revisions and forecast errors and the stock market's reactions to them. Section 3 develops a dynamic continuous-time general equilibrium model featuring periodic macroeconomic announcements. Section 4 provides a quantitative analysis of the model. It shows that data generated from the model can explain the empirical evidence documented in Section 2. Finally, Section 5 concludes by discussing a few possible extensions. A technical appendix contains robustness checks, proofs, and derivations.

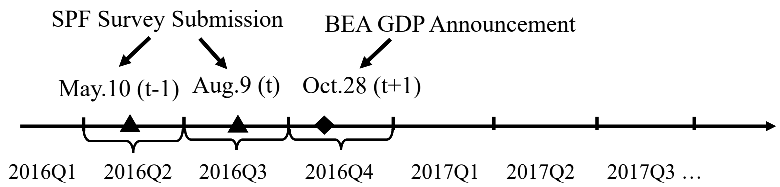
2 Empirical Evidence

In this section, I present two sets of related empirical evidence about the expectations formation process on macroeconomic variables and investors' reactions on financial markets. First, revisions of consensus forecasts for key macroeconomic variables, such as GDP, unemployment and CPI, positively predict the errors of consensus forecasts. Second, the stock market returns on forecast revision days negatively predict the stock market returns on GDP, unemployment and CPI announcements days, and this effect is stronger in periods with high economic uncertainty. I provide the details of the data construction and robustness analysis in Appendix 5.1.

2.1 Survey Forecasts and Macroeconomic Announcements

The Survey of Professional Forecasters (SPF) and Consensus Economics (CE) collect professional quarterly forecasts of macroeconomic variables, for example quarterly GDP, unemployment and CPI, whereby panelists are asked to provide quarter-by-quarter forecasts over five horizons. The realizations of these macroeconomic variables are eventually published by government agencies. For example, GDP is announced by the Bureau of Economic Analysis (BEA) quarterly, and the unemployment rate and CPI are published by the Bureau of Labor Statistics (BLS) on a monthly basis.

Figure 1: Timeline



This figure illustrates an example of the underlying timing of the data. The triangle displays SPF survey submission dates and the diamond shows the GDP announcement date. After the BEA's announcement for the 2016Q2 GDP, panelists are asked to submit the survey on August 9 (time t). Panelists must forecast quarter by quarter over five horizons, from the current quarter, 2016Q3 to 2017Q3. On October 28 (time $t+1$), the BEA announced an advance (first) estimate of 2016Q3 realized GDP. The same procedure also took place in the last quarter (2016Q2), when panelists needed to forecast from 2016Q2 to 2017Q2, and they submitted on May 10 (time $t-1$).

I illustrate the timing of professional forecasts and macroeconomic announcements in Figure 1 using the SPF data as an example. In this example, the macroeconomic variable under consideration, which I denote as x, is the third quarter GDP of 2016. On May 10 of 2016, which I denote as time $t-1$, survey forecasts for the current and next four quarters (2016Q2 to 2017Q2) are submitted, which include the third quarter GDP of 2016. On August 9, 2016 (time t), the GDP forecast of the current quarter, which is the third quarter of 2016, along with that of the next four quarters are submitted. The third quarter GDP is eventually announced by BEA, in this case, on Oct 28, 2016 (time $t+1$).

In general, for any macroeconomic variable x, I define the forecast revision of x at time t, $Frev_t(x)$, to be the revision of the consensus forecast for x submitted at time t relative to that from time $t-1$:

$$Frev_t(x) = \bar{\mathbb{E}}_t(x) - \bar{\mathbb{E}}_{t-1}(x), \tag{1}$$

where $\bar{\mathbb{E}}_t(x)$ is the consensus forecast (the cross-sectional average forecast) of x made at time t. Hence, forecast revisions reflect the new information obtained and processed by agents from $t-1$ to t. In the above example, it is the difference between the consensus forecast for the third quarter GDP submitted on August 9 and the previous forecast made on May 10.

For a macroeconomic variable x announced at $t+1$, I define the forecast error of x as the difference between the realized value and its most recent consensus forecast made at t:

$$Ferr_{t+1}(x) = x - \bar{\mathbb{E}}_t(x). \tag{2}$$

2.2 Empirical Evidence

My first empirical evidence is the predictability of the consensus forecast errors. This follows and confirms the result of Coibion and Gorodnichenko (2015). Following their paper, I regress consensus forecast errors of the real GDP growth rate, unemployment rate, and CPI, respectively, on their forecast revisions:

$$Ferr_{t+1}(x) = \alpha + \beta_F Frev_t(x) + \varepsilon_{t+1}, \tag{3}$$

where x is either the quarter-t real GDP growth rate, the unemployment rate or the CPI. Under the representative agent rational expectations hypothesis, nothing should predict the error of a rational belief, βs should not be significantly different from zero. However, using the data from SPF, the regression coefficient β_F is 0.39 for the real GDP growth rate with a Newey-West t-statistics of 2.29, significantly different from zero. Likewise, the point estimates of β_F are 0.42 and 0.58 for unemployment and CPI forecasts, with the respective t-statistics of 5.15 and 4.11, again significantly positive. This result implies that the consensus forecast does not respond sufficiently to newly-arrived information relative to the representative agent rational expectations benchmark.

Evidently, the stock market responds to macroeconomic forecasts. I use the cumulative return between current and last quarter forecast submission days to measure the stock market reaction to forecast revisions: $Rrev_t = \frac{P_t - P_{t-1}}{P_{t-1}}$, where P stands for the closing price of the S&P 500 ETF (SPY), and t and $t-1$ are two consecutive forecast submission dates. In the above example, it is the cumulative return from May 10 to August 9, 2016. Furthermore, I use the return on SPY generated during the 30 minutes around the announcement at 8:30 a.m. (8:15 - 8:45 a.m.) from CRSP Millisecond Trade and Quote (TAQ) dataset to measure the stock market reaction to forecast errors. I use $Rerr_{t+1}(x)$ to denote the announcement return of x.[2]

My second empirical evidence is the predictability of the stock market announcement day returns. I show that stock market return over forecast revision days *negatively* predicts the return on announcement days, especially during periods of high economic uncertainty. I regress the 30 minutes window announcement return on the return during the forecast revision period:

$$Rerr_{t+1}(x) = \alpha + \beta_P Rrev_t(x) + \varepsilon_{t+1}. \tag{4}$$

Using the Consensus Economics data, for example, the estimate β_P, is -0.011 of GDP with a Newey-West t-statistic of -2.71. Similarly, the same regression produces a β_P of -0.009 for the unemployment and -0.005 for the CPI announcement with t-statistics of -1.90 and -2.16, respectively. Positive returns on revision days predict negative returns on the announcement day, which

[2]Specifically, $Rerr_{t+1}(x)$ is defined as the difference between the stock price at 8:45 am and 8:15 am divided by the price at 8:15 am. The result is robust to different specifications of announcement-day returns, for example, one-hour return around the announcement or 8:25 am-to-8:55 am return.

implies that the stock market responds too much to information relative to the representative agent rational expectations benchmark.

To further examine the impact of economic uncertainty on the predictability of announcement returns, I use the CBOE Volatility Index (VIX) to measure the economic uncertainty and create an indicator variable $I_L(t)$ which equals one if the VIX is below its median at time t and zero otherwise.[3] I conduct the following regression:

$$Rerr_{t+1}(x) = \alpha + \beta_H Rrev_t(x) + \beta_{Dif} I_L(t) \times Rrev_t(x) + \beta_{Dummy} I_L(t) + \varepsilon_{t+1}, \quad (5)$$

where the coefficient β_H measures the predictability of the announcement return by the cumulative return over belief revision days during periods of high economic uncertainty, and β_{Dif} measures the difference between the predictability of high and low economic uncertainty periods.

I report the key regression coefficients, β_H and β_{Dif} in Table 1, where I use two measures of forecast revision days from SPF and CE. As I show in Table 1, the regression coefficients β_Hs are significantly negative for GDP and unemployment announcements at the 1% level and significant for the CPI announcement at the level of 10%. This predictability is insignificant or even positive for periods of low economic uncertainty.

Table 1: Announcement Return Predictability

	Panel A: SPF			*Panel B: CE*		
	GDP	UE	CPI	GDP	UE	CPI
β_H	-0.0181***	-0.0203***	-0.00626*	-0.0139***	-0.0121***	-0.00424*
	(-3.70)	(-3.09)	(-1.65)	(-4.15)	(-3.74)	(-1.81)
β_{Dif}	0.0184*	0.0455*	0.0126	0.0167***	0.00828	-0.00256
	(1.75)	(1.89)	(1.40)	(3.20)	(0.45)	(-0.54)

This table reports the coefficient estimates of equation (5) for real GDP growth rate, unemployment rate (UE) and Consumer Price Index (CPI), respectively. Panel A and B use survey submission dates from Survey of Professional Forecasters (SPF) and Consensus Economics (CE), respectively. The variable $Rrev_t$ is the close-to-close returns on SPDR S&P 500 ETF (SPY) between two consecutive forecast submission days, and $Rerr_{t+1}$ represents the announcement return, which is the cumulative return earned in the 30-minutes window, 8:15 a.m. to 8:45 a.m., of the announcement at 8:30 a.m. The full sample period is from 2003Q1 to 2019Q4. Newey-West t-statistics (with 5 lags) are in parentheses. Note: *$p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Taken together, consensus forecast revisions positively predict subsequent forecast errors realized on announcement days. In addition, the stock market return over revision days negatively predicts announcement day returns, especially during periods of high economic uncertainty. Both shreds of evidence suggest a rejection of the representative agent rational expectations hypothesis. The literature (for example, Coibion and Gorodnichenko, 2015) typically interprets the forecast error predictability as a rejection of the full information rational expectations hypothesis. In this

[3] In Appendix 5.1, I show that my results are robust if using the daily news-based U.S. Economic Policy Uncertainty (EPU) index from Baker, Bloom, and Davis (2016) as an alternative measure of economic uncertainty. My results are also robust if excluding the 2007-2008 financial crisis period or controlling for the crisis dummy.

paper, I show that a deviation from the representative agent setup, but not the rational expectations assumption, is enough to reconcile the above evidence. In the rest of the paper, I present a noisy rational expectations model to jointly account for both empirical facts.

3 The Dynamic Model

In this section, I develop a continuous-time NREE model with periodic macroeconomic announcements to explain the above facts on the predictability of forecast errors and the stock market's reaction to them. The model is based on Wang (1993), except that here I incorporate periodic macroeconomic announcements following Ai and Bansal (2018). The dynamic setup allows me to study the predictability of forecast and pricing errors under a coherent equilibrium framework. It also allows me to calibrate the model and examine its quantitative implications. I choose the continuous-time setup because it is particularly suitable to capture the fact that macroeconomic announcements happen within a short window of time and the stock market reacts *instantaneously*. The model can therefore capture the high-frequency return and trading dynamics that occurred at the announcement.

3.1 Model Setup

Preference and Endowment There is a unit measure of investors who maximize identical CARA utilities represented by $\left[\mathbb{E}\int_0^\infty -e^{-\rho t - C_t} dt\right]$, where C_t is the consumption at time t and ρ is the subjective time discount rate. For simplicity, I assume that the absolute risk aversion is 1. The model can easily accommodate more general CARA preferences.

There are two assets available for trading, a stock and a risk-free bond. I assume that the risk-free return r is constant. The stock is the claim to the following dividend process:

$$dD_t = (x_t - D_t)\, dt + \sigma_D dB_{D,t}, \tag{6}$$

where D_t is the dividend flow, x_t is the long-run trend for the dividend flow, σ_D is the volatility of the dividend flow, and $dB_{D,t}$ is an i.i.d. shock to the dividend payment modeled as a standard Brownian motion. I model the expected dividend flow as $x_t - D_t$, so that the dividend process is stationary. The assumption that the mean revision rate equals 1 is not important and can be relaxed without affecting most parts of the model. The long-run trend of the dividend flow, x_t, is itself mean reverting, modeled as an Ornstein-Uhlenbeck (OU) process:

$$dx_t = b\,(\bar{x} - x_t)\, dt + \sigma_x dB_{x,t}, \tag{7}$$

where $\bar{x}$ is the long-run mean of x_t, b is the rate of mean reversion, σ_x is the volatility of the hidden state x_t, and $B_{x,t}$ is a standard Brownian motion. In addition, as is standard in the NREE literature, I assume that the total equity supply is a stochastic process and denote it as θ_t:

$$d\theta_t = -a\theta_t dt + \sigma_\theta dB_{\theta,t}. \tag{8}$$

In the above equation, a is the rate of mean reversion for θ_t and σ_θ is the noisy supply volatility. Assume that Brownian motions $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$ are mutually independent. For tractability, I assume that the long-run average of θ_t is zero.

Information Structure The dividend is observable to all investors. However, its long-run trend x_t and the total risky asset supply θ_t are not. Assume that a fraction $(1-\omega)$ of investors are informed, meaning that they observe a noisy signal about x_t, denoted as s_t:

$$ds_t = x_t dt + \sigma_s dB_{s,t}, \tag{9}$$

where σ_s is the signal volatility and $B_{s,t}$ is a Brownian motion independent of $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$. The standard Kalman-Bucy filter implies that the informed investor's belief about x_t can be summarized by a posterior mean and a posterior variance, $\hat{x}_t \equiv \hat{\mathbb{E}}_t[x_t]$ and $\hat{q}(t) \equiv \hat{\mathbb{E}}_t\left[(\hat{x}_t - x_t)^2\right]$, respectively, where $\hat{\mathbb{E}}_t$ stands for the conditional expectation under the informed investor's information set at time t.

Uninformed investors do not observe the signal s_t but can update their beliefs based on information in the equilibrium asset price. Denote $\tilde{x}_t \equiv \tilde{\mathbb{E}}_t[\hat{x}_t]$ and $\tilde{q}(t) \equiv \tilde{\mathbb{E}}_t\left[(\tilde{x}_t - \hat{x}_t)^2\right]$ as the uninformed investor's posterior mean and variance of the informed investor's posterior belief about x_t, and $\tilde{\theta}_t \equiv \tilde{\mathbb{E}}_t[\theta_t]$ for their posterior mean of the total equity supply. From here on, denote $\tilde{\mathbb{E}}_t$ as the conditional expectation under the uninformed investor's information set at time t.

In addition, at predetermined times, macroeconomic announcements are made by the government. As in Ai and Bansal (2018), I assume that macroeconomic announcements are made every T periods. That is, for $n = 1, 2, \cdots$, at time $t = nT$, a macroeconomic announcement is made and assumed to reveal the true value of x_t. Because announcements fully reveal x_t, investors' beliefs about it instantaneously reset to its true value right after announcements: $\hat{x}_{nT}^{+} = \tilde{x}_{nT}^{+} = x_{nT}$ for all n, where I use superscript $+$ to denote quantities right after the announcements: $\hat{x}_T^{+} = \lim_{t \to T^{+}} \hat{x}_t$. Similarly, I will use superscript $-$ for quantities right before the announcements, for example, $\hat{x}_T^{-} = \lim_{t \to T^{-}} \hat{x}_t$. After the announcement, information about x_t starts to become imprecise, and both $\hat{x}_t$ and $\tilde{x}_t$ drift away from the true value of x_t. Uncertainties start to build up as the estimation errors $\hat{q}(t)$ and $\tilde{q}(t)$ accumulate over time.

Learning from Prices As in standard NREE models, the informed investors observe more information than the uninformed and try to profit from it by trading competitively and non-strategically in the stock market. Because their trading affects asset demand and therefore the stock price, their information is reflected in the price. Because of the presence of the noisy asset supply, the equilibrium price is only partially revealing and contains noisy information about the fundamentals, x_t. I conjecture and later verify that the equilibrium stock price takes the following linear form:

$$P_t = \phi + \phi_D D_t - \phi_\theta (t) \theta_t + \phi_x (t) \hat{x}_t + \phi_\Delta (t) \tilde{x}_t \tag{10}$$

$$= \phi + \phi_D D_t - \phi_\theta (t) \tilde{\theta}_t + \bar{\phi}_x \tilde{x}_t, \tag{11}$$

where $\phi_\theta (t)$, $\phi_x (t)$, and $\phi_\Delta (t)$ are time-varying sensitivities of price to θ_t, $\hat{x}_t$, and $\tilde{x}_t$, respectively. As I guess and verify in Appendix 5.2, the price coefficient on dividends ϕ_D is time invariant, and so is the sum of $\phi_x (t)$ and $\phi_\Delta (t)$, denoted as

$$\bar{\phi}_x = \phi_x (t) + \phi_\Delta (t) . \tag{12}$$

The informed investors observe the realizations of D_t, s_t, P_t, and the pre-scheduled announcements. At announcements, $\hat{x}_t$ is set to the true value of x_t, and $\hat{q}(t)$ goes to zero. After announcements, $\hat{q}(t)$ increases above zero as the uncertainty about x_t accumulates, and informed investors use the standard Kalman-Bucy filter to compute their posterior beliefs, $\{\hat{x}_t, \hat{q}(t)\}$. Because the uninformed investors' information is a subset of the informed investors' information, the informed can perfectly infer the posterior belief of the uninformed, $\tilde{x}_t$. Thus, informed investors can perfectly compute the total equity supply θ_t from the price (10). In this linear pricing framework, the private information of the informed investors on asset prices is completely summarized in the posterior mean, $\hat{x}_t$. The conditional expectation can therefore be summarized as $\hat{\mathbb{E}}_t \equiv \mathbb{E}_t \left[\cdot \, |\mathcal{F}^i_\tau \right]$, where $\mathcal{F}^i_\tau = \{D_\tau, s_\tau, P_\tau, \tilde{x}_\tau; \tau \leq t\}$, or equivalently, $\mathcal{F}^i_\tau = \{D_\tau, \theta_\tau, \hat{x}_\tau, \tilde{x}_\tau; \tau \leq t\}$.

On the other hand, the uninformed investors do not observe s_t. They observe D_t, the equilibrium price P_t, and the announcements. Unlike the informed, observing the price only reveals a noisy combination of $\hat{x}_t$ and θ_t to the uninformed. As x_t, $\hat{x}_t$, and θ_t are unknown to the uninformed investors, in general, one needs to characterize the posterior beliefs of all three variables in order to solve the optimization problem of the uninformed. However, it turns out that characterizing the dynamics of $\tilde{x}_t$ and $\tilde{q}(t)$ is sufficient to compute the posteriors of all three variables. To see this, first, $\tilde{\mathbb{E}}_t [x_t] = \tilde{\mathbb{E}}_t [\hat{x}_t]$, by the law of iterated expectations. Second, because uninformed investors observe the price, $P_t = \tilde{\mathbb{E}}_t [P_t]$ must hold. Computing the conditional expectation of price in (10) gives (11). Because the uninformed investor observes P_t and D_t, given the pricing function (10), the posterior belief about θ_t can be computed as

$$\tilde{\theta}_t = \frac{1}{\phi_\theta (t)} \left(\phi + \phi_D D_t + \bar{\phi}_x \tilde{x}_t - P_t \right) = \theta_t - \frac{\phi_x (t)}{\phi_\theta (t)} (\hat{x}_t - \tilde{x}_t) . \tag{13}$$

Third, as shown in equation (63) in Appendix 5.2, the posterior variance-covariance matrix can be further inferred from the above equation.

To formulate the uninformed investors' learning problem, it is useful to define $\xi_t = \phi_x (t) \hat{x}_t - \phi_\theta (t) \theta_t - \frac{\hat{q}(t)}{\sigma_D^2} \phi_x (t) D_t$. As shown in Appendix 5.2, the dynamic of ξ_t is conditionally independent of D_t, which gives the filtering problem a more intuitive interpretation. From the perspective of the uninformed investors, observing the price and the dividend process is equivalent to observing ξ_t and D_t because the mapping between P_t and ξ_t is one to one. Hence, I call ξ_t the information content of the price. It is now clear that the conditional expectation can be defined in terms of the uninformed investors' information set as $\tilde{\mathbb{E}}_t \equiv \mathbb{E}_t [\cdot\, | \mathcal{F}^u_\tau]$, where $\mathcal{F}^u_\tau = \{D_\tau, \tilde{x}_\tau, P_\tau; \tau \leq t\}$, or equivalently, $\mathcal{F}^u_\tau = \{D_\tau, \tilde{x}_\tau, \xi_\tau; \tau \leq t\}$. This easily shows that informed investors observe all information from the uninformed, $\tilde{\mathbb{E}}_t \subseteq \hat{\mathbb{E}}_t$, whereas the uninformed only observe a noisy information set of what informed investors know.

3.2 Dynamics of Posterior Beliefs

This section characterizes the dynamics of informed and uninformed investors' posterior beliefs. Then I discuss the implications for forecast error predictabilities. I demonstrate that in my model, consensus forecast revisions positively predict subsequent consensus forecast errors.

Forecast Error Predictability The posterior beliefs of the informed and uninformed investors can be computed using the standard Kalman-Bucy filter (see, for example, Liptser and Shiryaev (2001), Theorem 10.3). The following lemma summarizes the dynamics of posterior beliefs.

Lemma 1. *In the interior, $t \in ((n-1)T, nT)$, $n = 1, 2, \ldots$, the posterior mean of informed investors, $\hat{x}_t$ satisfies*

$$d\hat{x}_t = b(\bar{x} - \hat{x}_t)\,dt + \frac{\hat{q}(t)}{\sigma_D} d\hat{B}_{D,t} + \frac{\hat{q}(t)}{\sigma_s} d\hat{B}_{s,t}, \tag{14}$$

where $d\hat{B}_{D,t} = \frac{1}{\sigma_D}\left(dD_t - \hat{\mathbb{E}}_t[dD_t]\right)$ and $d\hat{B}_{s,t} = \frac{1}{\sigma_s}\left(ds_t - \hat{\mathbb{E}}_t[ds_t]\right)$ are innovations in the dividend flow and informed investors' signal process, respectively. The dynamic for the posterior variance $\hat{q}(t)$ is given in equation (51) in Appendix 5.2.

The posterior mean of uninformed investors, $\tilde{x}_t$, is given by

$$d\tilde{x}_t = b(\bar{x} - \tilde{x}_t)\,dt + \frac{\hat{q}(t) + \tilde{q}(t)}{\sigma_D} d\tilde{B}_{D,t} + \nu(t)\,\sigma_\xi(t)\,d\tilde{B}_{\xi,t}, \tag{15}$$

where $d\tilde{B}_{D,t} = \frac{1}{\sigma_D}\left(dD_t - \tilde{\mathbb{E}}_t[dD_t]\right)$ and $d\tilde{B}_{\xi,t} = \frac{1}{\sigma_\xi(t)}\left(d\xi_t - \tilde{\mathbb{E}}_t[d\xi_t]\right)$ are innovations in the dividend flow and the information content of price under the uninformed investor's information set, respectively. The function $\nu(t)$ is defined in equation (58) and the volatility of ξ_t, $\sigma_\xi(t)$ is defined in (57) in Appendix 5.2. The law of motion for the posterior variance $\tilde{q}(t)$ is given in equation (61) in the appendix.

Proof. See Appendix 5.2 for the proof. □

Using the above formula, I can derive the optimal forecasts of the informed and uninformed investors. Suppose the true value of x_T is announced at time T. The consensus forecast at t, as in the data, is defined as the average of the informed and the uninformed investors' beliefs:

$$\bar{\mathbb{E}}_t [x_T] = (1-\omega) \hat{\mathbb{E}}_t [x_T] + \omega \tilde{\mathbb{E}}_t [x_T] . \tag{16}$$

Since both $\hat{\mathbb{E}}_t [x_T]$ and $\tilde{\mathbb{E}}_t [x_T]$ are derived from the optimal Kalman filter with respect to the informed and the uninformed investors' information set, rationality implies that forecast revisions of informed and uninformed investors cannot predict their *own* forecast errors, that is,

$$\text{Cov}\left(\hat{\mathbb{E}}_t [x_T] - \hat{\mathbb{E}}_0 [x_T] , x_T - \hat{\mathbb{E}}_t [x_T]\right) = 0, \tag{17}$$

$$\text{and Cov}\left(\tilde{\mathbb{E}}_t [x_T] - \tilde{\mathbb{E}}_0 [x_T] , x_T - \tilde{\mathbb{E}}_t [x_T]\right) = 0, \tag{18}$$

where $\hat{\mathbb{E}}_t [x_T] - \hat{\mathbb{E}}_0 [x_T]$ (or $\tilde{\mathbb{E}}_t [x_T] - \tilde{\mathbb{E}}_0 [x_T]$) is the revision of the informed (or uninformed) investor's forecast from time 0 to t, and $x_T - \hat{\mathbb{E}}_t [x_T]$ (or $x_T - \tilde{\mathbb{E}}_t [x_T]$) is the error of the informed (or uninformed) investor's forecast realized at the announcement T. The revision of the consensus forecast could be represented as

$$Frev_t = (1-\omega)\left(\hat{\mathbb{E}}_t [x_T] - \hat{\mathbb{E}}_0 [x_T]\right) + \omega\left(\tilde{\mathbb{E}}_t [x_T] - \tilde{\mathbb{E}}_0 [x_T]\right), \tag{19}$$

and the error of the consensus forecast can be written as

$$Ferr_{t+1} = (1-\omega)\left(x_T - \hat{\mathbb{E}}_t [x_T]\right) + \omega\left(x_T - \tilde{\mathbb{E}}_t [x_T]\right). \tag{20}$$

In this model, the consensus forecast revision predicts the consensus forecast error because the forecast revision of the informed investors contains their private information and therefore predicts the forecast error of the uninformed. This is intuitive, as informed investors know all that uninformed investors know, whereas the uninformed do not observe the informed investors' private information. Hence, the consensus forecast puts less weight on the informed investor's private signals and more weight on the priors, apparently deviating from the Bayesian optimality. Relative to the Bayesian belief, it assigns too much weight to prior information because the uninformed do not take informed investors' information into account when updating their own beliefs. This creates the positive predictability of the consensus forecast errors. I summarize this result in the following proposition.

Proposition 1. *Let the optimal filtering equations be given in Lemma 1, then*

$$Cov\Big(\underbrace{\hat{\mathbb{E}}_t [x_T] - \hat{\mathbb{E}}_0 [x_T]}_{\textit{informed's forecast revision}} , \underbrace{x_T - \tilde{\mathbb{E}}_t [x_T]}_{\textit{uninformed's forecast error}}\Big) > 0, \tag{21}$$

and

$$Cov(Frev_t, Ferr_{t+1}) > 0. \tag{22}$$

Proof. See Appendix 5.2 for the proof. □

It is also important to notice that the consensus belief cannot be captured by any representative agent's belief, since a Bayesian investor's belief is a martingale. Consequently, *heterogeneity*, in this context arising from asymmetric information, is crucial under this rational framework to generate the observed empirics.

Difference in Beliefs The dynamics of belief differences, $\Delta_t \equiv \hat{\mathbb{E}}_t [x_t] - \tilde{\mathbb{E}}_t [x_t] = \hat{x}_t - \tilde{x}_t$, are important in understanding the key results of the model. From equations (14) and (15), we have, on non-announcement days,

$$d\Delta_t = -a_\Delta (t) \Delta_t dt - \sigma_{\Delta D} (t) d\hat{B}_{D,t} + \sigma_{\Delta s} (t) d\hat{B}_{s,t} + \sigma_{\Delta \theta} (t) dB_{\theta,t}, \tag{23}$$

where all coefficients, $a_\Delta (t)$, $\sigma_{\Delta D} (t)$, $\sigma_{\Delta s} (t)$, and $\sigma_{\Delta \theta} (t)$, which are given in Appendix 5.2, are strictly positive. At the announcements, the true value of x_t is revealed, and Δ_t is set to zero. After the announcements, the uninformed investor observes less information than the informed, and their disagreement Δ_t evolves according to (23). Differences in beliefs are driven by several sources of information. First, Δ_t has a negative loading on dividend innovations: the uninformed investor responds more to $d\hat{B}_{D,t}$ than the informed. By comparing (15) with (14), we can see that the uninformed investor has a larger loading on dD_t. Both types of investors observe the dividend process and try to extract information about x_t from it. The informed investor has additional information s_t, whereas the uninformed has to guess at what the informed knows from observing the changes in dividends. Because the uninformed knows the informed has more precise information, his or her sensitivity to $d\hat{B}_{D,t}$ has an additional term, coming from his or her estimation of the informed investor's belief $\hat{x}_t$. Second, $\sigma_{\Delta s} (t) > 0$, the uninformed investor's belief responds less to innovations in the informed investor's private signal s_t. The intuition is as follows. Since the uninformed investors do not observe s_t directly, they can only infer information about it by observing the stock price. Because the stock price is merely a noisy signal of s_t, the informed investor's private signal has a smaller impact on the belief of the uninformed. Finally, $\sigma_{\Delta \theta} (t) > 0$. Seeing that the informed investor perfectly knows θ_t, a positive shock to $dB_{\theta,t}$ lowers the equilibrium price, and therefore the uninformed investor's belief $\tilde{x}_t$, without affecting the informed investor's belief $\hat{x}_t$.

Using equation (13), one can represent the difference in investors' beliefs about θ_t as a function of Δ_t:

$$\theta_t - \tilde{\theta}_t = \frac{\phi_x (t)}{\phi_\theta (t)} \Delta_t. \tag{24}$$

Intuitively, after a sequence of positive shocks increases the level of noisy supply θ_t, the market price declines. Because uninformed investors do not observe $\hat{x}_t$ and θ_t separately, but only a combination of them, they rationally interpret the decline in price as partly reflecting increases in θ_t and partly deteriorations in $\hat{x}_t$. As a result, they downwardly revise their beliefs about $\hat{x}_t$ so that $\tilde{x}_t$ drops. And $\tilde{\theta}_t$ does not rise as fast as θ_t. The difference in beliefs about $\hat{x}_t$ therefore translates into a difference in beliefs about θ_t.

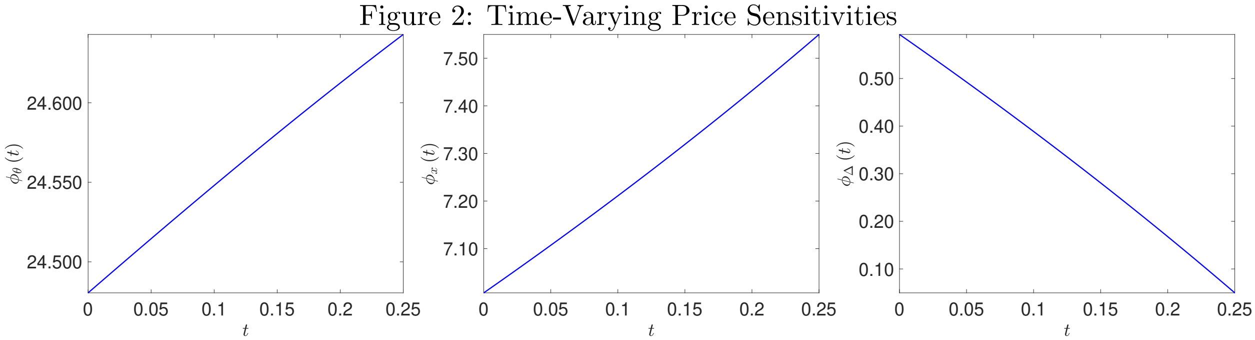
3.3 Equilibrium

Let's now turn to the model solution. I follow the standard approach in the NREE literature. First, given the conjectured functional form for the stock price, (10), solve the optimal learning problem of both informed and uninformed investors. Second, given investors' beliefs, solve their optimization problems. Finally, impose market clearing conditions to determine the conjectured coefficients $\phi_\theta(t)$, $\phi_x(t)$, and $\phi_\Delta(t)$ in the pricing function and investors' value functions. In contrast to previous dynamic NREE papers in the literature, here the coefficients in the pricing function and the value functions are assumed to be time varying. The time variation in the price sensitivity to information is the key for my model to account for the predictability of price corrections upon announcements.

For simplicity, I focus on an equilibrium in which all announcement cycles are identical. That is, the coefficients on the pricing function depend only on the timing of the announcements. For example, $\phi_\theta(t) = \phi_\theta(t \; mod \; T)$ for all t, where *mod* is the modulo operator that returns the remainder after division of t by T. Under this assumption, I only need to characterize the time-varying coefficients on one representative announcement cycle, $[0, T]$. In addition, although announcements are made at both time 0 and time T, I will use the convention that 0 (or T^+) denotes the time right after announcements and T (or T^-) refers to the moment right before announcements.

Equilibrium Pricing Figure 2 plots the pricing function coefficients $\phi_\theta(t)$, $\phi_x(t)$, and $\phi_\Delta(t)$ for one announcement cycle (recall equations (10) and (11)). The price sensitivity to noise, $\phi_\theta(t)$, is positive and increasing over time. The function $\phi_\theta(t)$ is positive because increases in the aggregate equity supply θ_t lower the price. It is monotonically increasing over time because the asymmetric information amplifies the sensitivity of price with respect to noisy supply, and this effect is stronger as uncertainty builds up over time.

Figure 2: Time-Varying Price Sensitivities



This figure plots the time-varying pricing function coefficients $\phi_\theta(t)$, $\phi_x(t)$, and $\phi_\Delta(t)$ for one announcement cycle under the benchmark parameter values in Table 2. Here, $t = 0$ stands for the time right after the announcements and $t = T$ refers to the moment right before the announcements.

In my model, increases in θ_t lower price for two reasons. First, increases in supply lower the equilibrium price due to a downward sloping demand curve as in standard equilibrium models. This effect does not depend on the uncertainty or the asymmetric information. Second, the information asymmetry and learning amplify the responses of prices to supply shocks, therefore an increase in θ_t further lowers the price. Because the uninformed investors cannot infer the true value of θ_t and x_t from prices, they attribute part of the price drop as deteriorations in fundamentals and downwardly revise their beliefs about x_t. The uninformed investors reduce their holdings of the stock because of their pessimistic beliefs. This lowers the demand of the asset and price has to drop further to clear the market. Clearly, the second effect is stronger when uninformed investors are more uncertain about x_t. At time $t = 0$, right after an announcement, uninformed investors know the true value of x_0 and the information asymmetry is temporarily eliminated. As t increases, x_t is evolving, the uninformed investors are less certain about it and start to learn from prices. As the uncertainty and information asymmetry about x_t build up over time, changes in prices have stronger impacts on the beliefs of the uninformed investors. Therefore, prices become more sensitive to supply shocks. In another word, the price sensitivity to noise, $\phi_\theta(t)$ increases. At time T when the next announcement approaches, the uncertainty about x_T instantaneously resolves, the price sensitivity to noise jumps down discontinuously from $\phi_\theta(T)$ to $\phi_\theta(0)$, and a new announcement cycle starts.As I will prove formally below, the monotonicity and discontinuity of the $\phi_\theta(t)$ function in my model is the key to explain the patterns of the predictability of announcement returns.

The function $\phi_x(t)$ is positive and increasing, and $\phi_\Delta(t)$ is positive and decreasing, whereas the sum of the two, $\phi_x(t) + \phi_\Delta(t) = \bar{\phi}_x$, is a constant. First, naturally, $\bar{\phi}_x$ is positive because a more optimistic outlook about dividend flow raises the price. Moreover, the value of $\bar{\phi}_x$ must be constant over time. Because the aggregate risky asset supply is θ_t, market clearing implies that the total asset demand cannot depend on beliefs about x_t. Although beliefs about x_t affect stock prices, they cannot change expected returns. In other words, it cannot introduce a predictable component in stock prices. This requirement alone implies that $\bar{\phi}_x$ must be a constant. Second, the term $\phi_x(t)$, representing the price impact from the informed investor, is increasing over time; this is intuitive. Informed investors observe additional signals. If, for example, news has been favorable, they increase holdings of the stock and benefit from the superior information. This drives up the price. As t increases, this information advantage builds up as uncertainty increases. Immediately upon the announcement, information becomes symmetric, and $\phi_x(t)$ drops from $\phi_x(T)$ to $\phi_x(0)$. Third, $\phi_\Delta(t)$, reflecting the price sensitivity to the uninformed investor's belief, peaks right after the announcement and decreases later on. Again, the intuition is straightforward. As a result of the announcement, uninformed investors suddenly become informed and information becomes symmetric. Hence, they correct for their beliefs of the mistaken accumulated noise that was previously perceived as profitable fundamentals. Afterward, they become less and less willing to trade against informed investors' private information, as uncertainty slowly rises following the announcement.

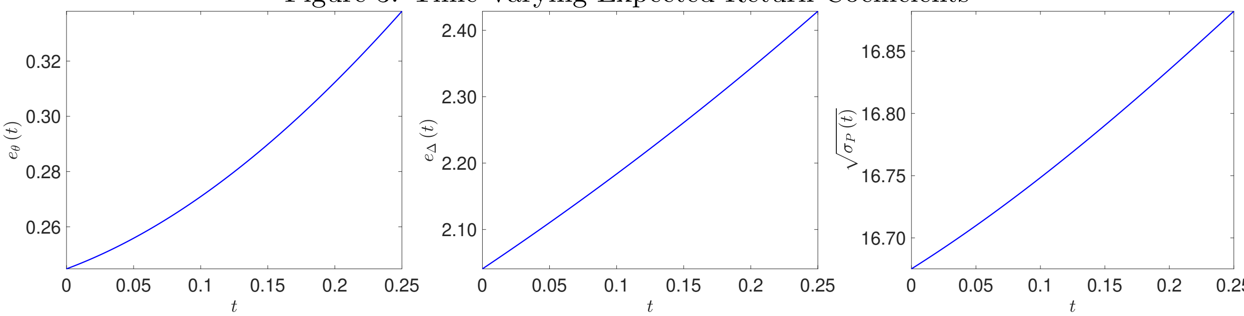
Expected Returns To understand investors' portfolio allocations, it is useful to first compute expected stock returns for the informed and uninformed investors. Using the conjectured equilibrium price in (10) and (11), expected excess returns under the informed and uninformed investors' beliefs can be written respectively as

$$\hat{\mathbb{E}}_t [dP_t + D_t dt - rP_t dt] = [e_\theta (t) \theta_t + e_\Delta (t) \Delta_t] dt, \quad (25)$$

$$\tilde{\mathbb{E}}_t [dP_t + D_t dt - rP_t dt] = \left[e_\theta (t) \tilde{\theta}_t\right] dt, \quad (26)$$

where the coefficients $e_\theta (t)$ and $e_\Delta (t)$ are given in Appendix 5.2.

Figure 3: Time-Varying Expected Return Coefficients



This figure plots the coefficients $e_\theta (t)$ and $e_\Delta (t)$ in expected returns, as well as the volatility of the excess return $\sqrt{\sigma_P (t)}$ over one announcement cycle from the calibrated parameter values in Table 2.

Figure 3 plots the coefficients $e_\theta (t)$ and $e_\Delta (t)$, as well as the variance of the excess return, denoting $\sigma_P (t)$ from the calibrated model. The term $e_\theta (t)$ is positive and increasing. To understand the pattern of $e_\theta (t)$, note that the equilibrium price in (10) depends negatively on the total asset supply, θ_t. Because the process θ_t is mean reverting, higher values of θ_t are associated with higher levels of price in the future, and this effect is stronger when the price sensitivity to noise $\phi_\theta (t)$ is larger.[4] Note that the pattern of $e_\theta (t)$ mimics the pattern of $\phi_\theta (t)$ in Figure 2 very well. The fact that $e_\theta (t)$ is positive and increasing is also important in market clearing. Because in equilibrium θ_t equals the total portfolio holdings of informed and uninformed investors, expected returns must be increasing in θ_t in order to induce investors to hold the existing stock of the asset. Intuitively, since investors are risk averse, as t increases and both types of investors become more uncertain about x_t, expected returns must become more sensitive to θ_t to clear the market.

The coefficient $e_\Delta (t)$ is positive and increasing over time. As $\Delta_t = \hat{x}_t - \tilde{x}_t$ is only within the informed investor's information set, equation (10) can be rewritten as $P_t = \phi + \phi_D D_t - \phi_\theta (t) \theta_t + \bar{\phi}_x \hat{x}_t - \phi_\Delta (t) \Delta_t$. Ceteris paribus, the equilibrium price is a decreasing function of Δ_t. Lower values of $\tilde{x}_t$ indicate that the uninformed are less optimistic about the fundamentals and would like to reduce their stock holdings. As a result, the price must drop to clear the market. However, informed investors observe additional information and understand that the lower price is not due to deteriorations of fundamentals but rather to a sequence of negative shocks in the price signals that produce the undue pessimism of uninformed investors. Therefore, from the perspective of the informed, expected returns are high when Δ_t is large. They increase their holdings of the stock and take advantage of the difference in beliefs. As t increases from 0 to T, the information advantage

[4] In fact, as shown in equation (70) in Appendix 5.2, $e_\theta (t) = (a + r) \phi_\theta (t) - \phi'_\theta (t)$. However, the second term is quantitatively small, and $e_\theta (t)$ is mainly determined by $\phi_\theta (t)$.

of the informed investor rises and the difference in beliefs, Δ_t, has an increasingly strong impact on expected returns. As a result, $e_\Delta(t)$ is strictly increasing in t.

The variance of return, $\sigma_P(t)$, is also increasing over time. Recall that with the exception of $\phi_\theta(t)$, all coefficients are constants from the equilibrium price in equation (11), which is increasing over time, as shown in Figure 3. Intuitively, as uncertainty about x_t builds up over time, the equilibrium price becomes more sensitive to asset supply, and stock returns become more volatile.

Expected returns at time T, right before announcements, however, are different from (25) and (26) because prices are expected to jump discontinuously upon announcements. Using the pricing function (11), the return upon announcements can be computed as

$$P_T^+ - P_T^- = \bar{\phi}_x(x_T - \hat{x}_T) + \phi_\Delta(T)\Delta_T + [\phi_\theta(T) - \phi_\theta(0)]\theta_T. \tag{27}$$

Here P_T^+ is the stock price right after the announcement at time T, and P_T^- stands for the stock price right before the announcement. Therefore, the expected return at announcements from the informed and uninformed investors' perspective can be written as

$$\hat{\mathbb{E}}_t\left[P_T^+ - P_T^-\right] = \phi_\Delta(T)\Delta_T + [\phi_\theta(T) - \phi_\theta(0)]\theta_T, \tag{28}$$

$$\tilde{\mathbb{E}}_t\left[P_T^+ - P_T^-\right] = [\phi_\theta(T) - \phi_\theta(0)]\tilde{\theta}_T, \tag{29}$$

respectively.[5] Note that the expected returns at the announcements are positively correlated with both types of investors' beliefs about θ_T. This is because at announcements, this correlation is completely determined by the jump of $\phi_\theta(T)$ to $\phi_\theta(T^+) = \phi_\theta(0)$, which is positive as $\phi_\theta(T) - \phi_\theta(0) > 0$. When the supply shock at the announcement is unfavorable, $\theta_T > 0$, for example, expected returns are high since investors face higher risks and require positive risk compensation. For the same reason as in the non-announcement days, for informed investors, expected returns are positively correlated with differences in beliefs, Δ_T, and this is the source of profit for informed investors.

Optimal Portfolio Holdings I guess and verify in Appendix 5.3 that the informed investor's value function $J(t, W_t, \theta_t, \Delta_t)$ takes a quadratic form, where W_t denotes the financial wealth. In the interior of $(0, T)$, the informed investor's optimization problem is

$$J(t, W_t, \theta_t, \Delta_t) = \max_{\{\alpha_t, C_t\}} \hat{\mathbb{E}}_t\left[\int_0^{T-t} -e^{-\rho z - C_{t+z}} dz + J\left(T, W_T^-, \theta_T, \Delta_T\right)\right], \tag{30}$$

subject to the following budget constraint,

$$dW_t = (W_t r - C_t)\,dt + \alpha_t\left[dP_t + (D_t - rP_t)\,dt\right], \tag{31}$$

[5] Although I use the terminology of expected returns for both the interior case (equations (25) and (26)) and the boundary case (equations (28) and (29)), it is important to note that (25) and (26) are expected returns per unit of time, and (25) and (26) are expected returns at the instant of announcements.

where r is the exogenous risk-free rate, C_t denotes the consumption, and α_t is the portfolio holding of the risky asset. In the above problem, the law of motion for the state variables is defined in (8) and (23). Accordingly, my timing convention, $J\left(T, W_T^-, \theta_T, \Delta_T\right)$, is the value function at time T right before the announcements.

At time T, however, the portfolio choice problem is different. Because the stock price jumps from P_T^- to P_T^+, wealth jumps from W_T^- to W_T^+ accordingly. The optimization problem of the informed investor at the instant of the announcement is

$$\begin{aligned} J\left(T, W_T^-, \theta_T, \Delta_T\right) &= \max_{\alpha_T^-} \left\{ \hat{\mathbb{E}}_T \left[J\left(0, W_T^+, \theta_T, 0\right)\right]\right\} \\ \text{s.t. } W_T^+ &= W_T^- + \alpha_T^- \left(P_T^+ - P_T^-\right). \end{aligned} \tag{32}$$

Because θ_t is a continuous process, it has the same value before and after announcements. The belief difference, Δ_t, is set to 0 upon the announcement because the true value of x_T is revealed so that investors' beliefs converge.

The uninformed investor's optimization problem takes a similar form, except the state variable is $\tilde{\theta}_t$. Denote β_t as the uninformed investor's risky asset holdings. In Appendix 5.3, I conjecture and prove the form of the uninformed investor's value function $V\left(t, W_t, \tilde{\theta}_t\right)$. The following lemma summarizes the investors' optimal portfolio decisions.

Lemma 2. *The informed and uninformed investors' optimal risky asset holdings are*

$$\alpha_t = \alpha_\theta(t)\,\theta_t + \alpha_\Delta(t)\,\Delta_t, \tag{33}$$

$$\beta_t = \beta_\theta(t)\,\tilde{\theta}_t, \tag{34}$$

where $\alpha_\theta(t)$, $\alpha_\Delta(t)$, and $\beta_\theta(t)$ are defined in Appendix 5.3, with the boundary values $\alpha_\theta(T)$, $\alpha_\Delta(T)$, and $\beta_\theta(T)$ defined in Appendix 5.4.

Proof. See Appendix 5.3 and 5.4 for the derivations. □

Note that the market clearing condition requires that the total risky asset demand equals the aggregate supply θ_t, that is, $(1-\omega)\alpha_t + \omega\beta_t = \theta_t$. Using (13) to replace $\tilde{\theta}_t$, this can be written as

$$\left[(1-\omega)\alpha_\theta(t) + \omega\beta_\theta(t)\right]\theta_t + \left[(1-\omega)\alpha_\Delta(t) - \omega\beta_\theta(t)\frac{\phi_x(t)}{\phi_\theta(t)}\right]\Delta_t = \theta_t. \tag{35}$$

The above equation implies two conditions,

$$(1-\omega)\alpha_\theta(t) + \omega\beta_\theta(t) = 1, \tag{36}$$

$$\alpha_\Delta(t) = \frac{\omega}{1-\omega}\beta_\theta(t)\frac{\phi_x(t)}{\phi_\theta(t)}. \tag{37}$$

The first equation shows that informed and uninformed investors must hold the total supply of the stock θ_t together. In fact, both $\alpha_\theta(t)$ and $\beta_\theta(t)$ are close to one, that is, the demand of the asset in a model with symmetric information. The term $\alpha_\Delta(t)$ reflects the informed investors' portfolio holding owing to their information advantage. The term $\alpha_\Delta(t) > 0$ because whenever the informed investors are more optimistic about the fundamental x_t because of the private information, they increase their positions in the stock and trade more aggressively. Over time, this information advantage increases as uncertainty builds up on non-announcement days; therefore, informed investors trade more against the difference in beliefs. At the announcements, $\alpha_\Delta(t)$ jumps downward because information becomes symmetric and beliefs converge among investors.

Predictability of Pricing Errors While the predictability of forecast errors is a property of rational beliefs and does not depend on the detailed functional form of equilibrium prices, the predictability of pricing errors is a unique implication of my model that depends crucially on the time variation in pricing coefficients due to periodic announcements. In my model, price innovations on non-announcement days negatively predict price reactions to announcements. The key to this result is the time variation in the price reactions to noise, captured by the $\phi_\theta(t)$ function.

Note that in my model, the equilibrium price can be written as a linear function of posterior beliefs with respect to publicly available information: $P_t = \phi + \phi_D D_t - \phi_\theta(t)\tilde{\theta}_t + \bar{\phi}_x \tilde{x}_t$. If all pricing functions are continuous functions of time, then innovations in price at the announcement must come from innovations in the uninformed investors' posterior beliefs, $\tilde{\theta}_t$ and $\tilde{x}_t$. Because innovations in rational beliefs cannot be predictable, neither announcement returns. I summarize this observation in the following lemma.

Lemma 3. *Suppose $\phi_\theta(t)$ is continuous over time, then*

$$Cov\left(P_{t+\delta} - P_t, P_T^+ - P_T^-\right) = 0. \tag{38}$$

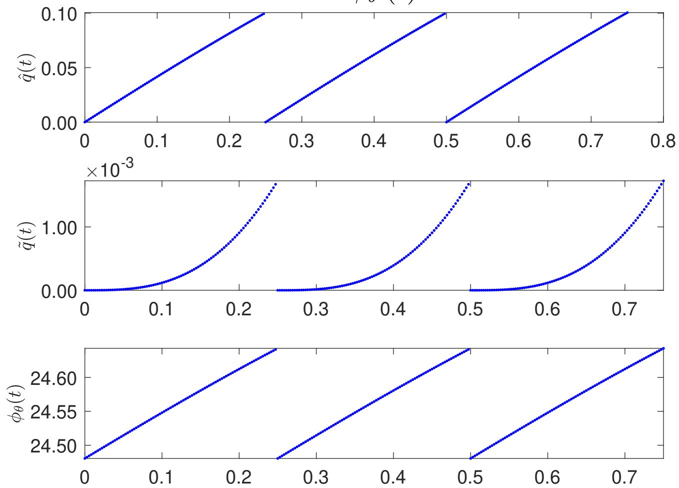
Proof. See Appendix 5.5 for the proof. □

In standard NREE models, such as Wang (1993, 1994), returns may be predictable over longer horizons because dividends or the noisy supply or both are mean-reverting processes. These models can potentially generate predictability of returns over longer horizons. However, as shown in Lemma 3, as long as the pricing coefficient is continuous over time, pricing errors realized at a high frequency, for examples, in minutes around macroeconomic announcements, are not predictable. The key implication of Lemma 3 is that to understand the pricing error predictability in the model, I need to focus on the time variation in the price coefficient $\phi_\theta(t)$ and, in particular, its discontinuity at announcements.

In Figure 4, I plot the posterior variances of the informed and uninformed investors, respectively, and the $\phi_\theta(t)$ function over multiple announcement cycles.[6] Posterior variances are zeros upon the announcements because announcements reveal the true value of x_t. After announcements,

[6]Note that $\hat{q}(t)$ is the informed investor's posterior variance of x_t, and $\tilde{q}(t)$ is the uninformed investor's posterior variance of $\hat{x}_t$. Equation (63) in Appendix 5.2 shows that the uninformed investor's posterior variance of x_t equals $\hat{q}(t) + \tilde{q}(t)$. This is intuitive because uninformed investors face higher estimation errors about x_t than the informed because of the lack of private information.

Figure 4: Posterior Variances and $\phi_\theta(t)$ over Announcement Cycles



This figure plots the model-implied posterior variances of the informed investor $\hat{q}(t)$ (top panel) and uninformed investor $\tilde{q}(t)$ (middle panel) and the price sensitivity to noise $\phi_\theta(t)$ (bottom panel) over three quarterly announcement cycles. The benchmark parameter values are given in Table 2.

uncertainties slowly build up because x_t is time varying and neither group observes the true value of x_t. Until the next announcement, estimation errors drop to zero again after announcements resolve the uncertainties. In my model, a negative shock to the total asset supply increases the stock price. When information is imprecise, there is a feedback mechanism from learning: this price increase triggers more purchases from the uninformed investors because they interpret it as good news for fundamentals x_t and upwardly revise their beliefs $\tilde{x}_t$. As a result, the equilibrium price has to rise further to clear the market. After an announcement, as uncertainty builds up, the feedback mechanism is stronger, and the uninformed investors rely more and more on the market price to learn about x_t. Therefore, as plotted in the bottom panel of Figure 4, the price sensitivity to noise as captured by the pricing function $\phi_\theta(t)$ is increasing between announcements.

The predictability of the pricing error in my model comes from the pattern of the $\phi_\theta(t)$ function over announcement cycles, in particular, the monotonicity of $\phi_\theta(t)$ in the interior of $(0, T)$ and its discontinuity at T. To see this, I can write the pricing error realized on announcement days as

$$P_T^+ - P_T^- = \bar{\phi}_x(x_T - \tilde{x}_T) - \phi_\theta(T^+)\left[\theta_T - \tilde{\theta}_T\right] \tag{39}$$

$$- \underbrace{\left[\phi_\theta(T^+) - \phi_\theta(T^-)\right]}_{<0} \tilde{\theta}_T, \tag{40}$$

where the last term reflects the adjustment for the changes in $\phi_\theta(T^-)$ to $\phi_\theta(T^+)$ upon announcements. Note that all terms in (39) are errors of rational beliefs relative to true information. Therefore, these innovations cannot be predicted by functions of histories of public information. Because the function $\phi_\theta(t)$ is monotonically increasing in $(0, T)$, and $\phi_\theta(T^+) - \phi_\theta(T^-) < 0$ at T, the last term in (40) is predictable, as $\tilde{\theta}_T$ is. The proposition below provides a sufficient condition for the predictability of pricing errors in my model.

Proposition 2. *Under the condition (*) in Appendix 5.5, for all t and $\delta > 0$,*

$$Cov\left(P_{t+\delta} - P_t, P_T^+ - P_T^-\right) < 0. \tag{41}$$

Proof. See Appendix 5.5 for the proof. □

To understand the above proposition, consider an econometrician who tries to predict price changes on announcement days $P_T^+ - P_T^-$ by regressing them on reactions of price to news between t to $t+\delta$ before the announcement, $P_{t+\delta} - P_t$. One can represent the price revision as

$$\begin{aligned} P_{t+\delta} - P_t &= \phi_D \left[D_{t+\delta} - D_t\right] + \bar{\phi}_x \left[\tilde{x}_{t+\delta} - \tilde{x}_t\right] - \phi_\theta\left(t+\delta\right)\left[\tilde{\theta}_{t+\delta} - \tilde{\theta}_t\right] \\ &\quad - \underbrace{\left[\phi_\theta\left(t+\delta\right) - \phi_\theta\left(t\right)\right]}_{>0} \tilde{\theta}_t. \end{aligned} \tag{42}$$

The first line of equation (42) shows the price adjustment if regarding the price sensitivity to noise, $\phi_\theta\left(t\right)$, as constant over time. The second line reflects the adjustment due to the changes in $\phi_\theta\left(t\right)$. Since $\phi_\theta\left(t+\delta\right) - \phi_\theta\left(t\right) > 0$, the term $\left[\phi_\theta\left(t+\delta\right) - \phi_\theta\left(t\right)\right]\tilde{\theta}_t$ can be interpreted as the accumulation of noise in price due to the accumulation of uncertainties over time. Clearly, the last terms in (42) and (40) are negatively correlated, $\text{Cov}\left(\left[\phi_\theta\left(t+\delta\right) - \phi_\theta\left(t\right)\right]\tilde{\theta}_t, \left[\phi_\theta\left(0\right) - \phi_\theta\left(T\right)\right]\tilde{\theta}_T\right) < 0$, because the belief of θ_t is persistent and mean reverts to zero in the long run. Therefore, the predictability of pricing errors in my model comes from the property of the $\phi_\theta\left(t\right)$ function, which reflects the time-varying changes in the information structure. In general, the terms in the first line of (42) can also be correlated with $\left[\phi_\theta\left(0\right) - \phi_\theta\left(T\right)\right]\tilde{\theta}_T$. The condition (*) in Proposition 2 is sufficient but not necessary to guarantee that the correlation between other terms will be dominated by the sign of $\text{Cov}\left(\left[\phi_\theta\left(t+\delta\right) - \phi_\theta\left(t\right)\right]\tilde{\theta}_t, \left[\phi_\theta\left(0\right) - \phi_\theta\left(T\right)\right]\tilde{\theta}_T\right)$.

Intuitively, the term $\left[\phi_\theta\left(t+\delta\right) - \phi_\theta\left(t\right)\right]\tilde{\theta}_t$ captures noise accumulations in price before announcements, which is corrected upon announcements. As explained above, on non-announcement days, the uninformed investor relies on prices to learn about the private information of informed investors. As uncertainty builds up, both $\hat{q}_t$ and $\tilde{q}_t$ rise, and stock prices become increasingly more sensitive to noise. As a result, $\phi_\theta\left(t\right)$ increases and the noise component of stock price accumulates over time. At announcements, the true value of x_t is revealed and the accumulated noise in the stock price must be corrected. As a result, positive realizations of noise on non-announcement days must predict a negative correction on announcement days, which accounts for the announcement return predictability in my model.[7,8]

In my model, the monotonicity of the $\phi_\theta\left(t\right)$ function is due to the effect of learning and asym-

[7]Note that the predictability of pricing errors in my model is an equilibrium compensation for risk. Announcement returns are predictable, as shown in equations (25) and (26), but they are risk premiums and not arbitrage opportunities.

[8]The mechanism here is similar to the "wisdom after the fact" models of Romer (1993) and Caplin and Leahy (1994). The key difference is that in those models, fixed costs or externalities prevent fundamental information from getting revealed to the market until a critical threshold is reached. In contrast, here it is the noise that accumulates, and the threshold is exogenously determined by data announcement dates.

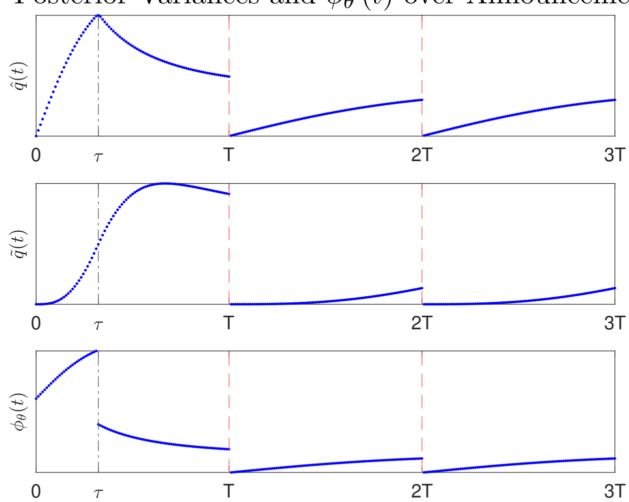
metric information. Without the time-varying information structure, $\phi_\theta(t)$ will be a constant, and announcement returns will not be predictable. Naturally, a higher uncertainty in my model should amplify the impact of asymmetric information and result in a steeper slope of the pricing function $\phi_\theta(t)$. As I show in the quantitative exercise in the following section, as in the data, the predictability of announcement return in my model is stronger in periods of high uncertainty due to the amplification effect of learning.

The mechanism for the negative predictability in my model is in contrast with He and Wang (1995), who study a finite horizon model in which investors learn the value of the asset which will be revealed at the terminal date T. As $t \to T$, the posterior variance about the asset value decreases towards zero and reaches the minimum before T, whereas in my model, the underlying state variable is time-varying. As a result, the posterior variance increases as t approaches the announcement time and drops right after the announcement. This pattern is consistent with the empirical facts documenting the dynamics of implied volatility at the aggregate level (Hu, Pan, Wang, and Zhu, 2020) and those at the firm level (Ai, Han, Pan, and Xu, 2019). The increasing pattern of uncertainty before the announcement is the key reason for the monotonicity and jump of $\phi_\theta(t)$ in my model. In addition, the incentive for trading here is also very different from finite horizon models in which the only uncertainty is the value of the asset at the terminal date. In these setups, trading gradually increases before the announcement because of the reduction in remaining trading opportunities towards the terminal date and peaks before the announcement. This is inconsistent with the data I document in Figure 6 in Section 4, where trading only jumps after the announcement. My model is designed to capture the cyclical dynamics of returns and trading around macroeconomic announcements. In my model, trading moves slowly before the announcement but sharply spikes right after the announcement because both groups of investors unwind their positions right after the announcement resolves the uncertainty. Therefore, my model not only has predictions on belief errors and announcement returns, but also on the patterns of trading volume before and after announcements. In the next section, I calibrate my model and evaluate the ability of the model to quantitatively account for the predictabilities of belief errors and announcement returns as well as trading volumes around announcements.

An Unexpected Shock to Volatility In the data, the negative predictability of announcement day returns is strongest in periods where macroeconomic volatility is high and insignificant or even positive in periods with low volatility. In fact, from Table 1, the point estimate for the predictability regression is slightly positive in low volatility periods. This feature of the data is also consistent with my model. The key implication of my model is that noisy supply has a higher impact on equilibrium price when investors are more uncertain about the underlying state of the economy. In periods with high volatility, uncertainty builds up over time during the revision days, and the equilibrium price becomes increasingly sensitive to noisy supply. This pattern generates the negative predictability of announcement day returns, because the accumulated noise is partially eliminated from price at the announcements.

However, when an economy switches from high volatility to low volatility, uncertainty accumulated in high volatility periods starts to reduce over time, and can result in a lower sensitivity of price to noisy supply. In Figure 5, I assume that there is an unexpected reduction in volatility at time τ and plot the posterior variance for informed investors (top panel), that for uninformed investors (middle panel) and the pricing coefficient on noisy supply, $\phi_\theta(t)$ (bottom panel) as functions of time. After the previous announcement at time 0, both $\hat{q}_t$ and $\tilde{q}_t$ increase quickly until time τ, when the economy suddenly transits into a low volatility state. As a result, the posterior variance of informed investors starts to reduce. It takes some time for the posterior variance for the uninformed to drop, but $\tilde{q}_t$ eventually drops. From 0 to τ, $\phi_\theta(t)$ increases as uncertainty builds up, but drops afterwards until the next announcement due to reductions in uncertainty $\hat{q}_t$ and $\tilde{q}_t$. Because $\phi_\theta(t)$ reduces over time, the same argument as in the the last section implies that the returns during the forecast revision period positively predicts announcement returns. I confirm this implication of the model in the next section.

Figure 5: Posterior Variances and $\phi_\theta(t)$ over Announcement Cycles



This figure plots the model-implied posterior variances of the informed investor $\hat{q}(t)$ (top panel) and uninformed investor $\tilde{q}(t)$ (middle panel) and the price sensitivity to noise $\phi_\theta(t)$ (bottom panel) over three quarterly announcement cycles. At time τ, there is an expected MIT shock. The benchmark parameter values are given in Table 2.

4 Quantitative Results

This section presents a quantitative analysis and demonstrates that my model can account for the stylized empirical facts documented in Section 2.

Numerical solution In my model, the equilibrium is characterized by a set of pricing functions $\{\phi_\theta(t), \phi_x(t), \phi_\Delta(t)\}$, the portfolio demand functions $\{\alpha_\theta(t), \alpha_\Delta(t), \beta_\theta(t)\}$, and the value functions of informed and uninformed investors. These functions must be jointly determined by the optimality and market clearing conditions. In Appendix 5.3 and 5.4, I show that these conditions boil down to a system of ODEs subject to boundary conditions at the announcements. I describe a recursive method that simultaneously solves the system of ODEs that characterize the equilibrium. Using these solutions, I simulate my model, compute relevant moments to calibrate my model, and replicate the regressions I conducted using the actual data. The model is simulated at a daily frequency, and announcements are made at the end of each quarter. In total, there are four announcements every year as in the data. I assume that investors start to revise their beliefs after each announcement until the survey is submitted at the middle of each quarter. The numerical method is discussed in details in Appendix 5.7. Below, I first report my calibration procedure and then evaluate the quantitative implications of the model.

Estimates Table 2 contains calibrated and estimated benchmark parameter values. First, preference parameters are chosen to be consistent with the literature: $\rho = 0.03$. I choose the steady-state level of dividend, $\bar{x} = 10$ so that together with the choice of the volatility parameters of the model, it implies a volatility of the aggregate consumption growth, i.e., that of the informed and uninformed investors of 3% per year. Under this choice, the implied relative risk aversion of both groups of investors is about 10 in steady state. I set $\omega = \frac{1}{2}$ so that half of the investors are informed and half are uninformed. Second, several parameters are calibrated to match outside data. The annual risk-free interest rate $r = 2.8\%$ and the dividend persistence $b = 0.1$ are set to match the mean and autocorrelation of the price-to-dividend ratio, respectively.[9] The model produces a mean of 36.16 and a first-order autocorrelation of 0.96 for the price-to-dividend ratio, compared to 35.51 and 0.93 in the data. The price-to-dividend ratio and dividend growth data are formulated from CRSP value-weighted NYSE/Amex index annual returns for the period 1968-2019.

To capture the impact of the time-varying uncertainty, I allow the volatility of the hidden state variable x_t, σ_x to be a two-state Markov chain with state space $\{\sigma_H, \sigma_L\}$, where $\sigma_H = 0.65$ and $\sigma_L = 0.05$. I assume that the the volatility shocks are unexpected. The transition matrix for σ_x is $\begin{bmatrix} 1-\kappa\delta & \kappa\delta \\ \kappa\delta & 1-\kappa\delta \end{bmatrix}$ during an infinitesimal time interval δ. I choose $\kappa = 0.20$ so that together with $\sigma_H = 0.65$ and $\sigma_L = 0.05$, the three parameters jointly match the mean, the standard deviation, and the first-order autocorrelation of the daily VIX index for the period 2003-2019. The model produces 15%, 7.8%, and 0.95 for the above three moments, with the data counterparts of 18.44%, 8.47%, and 0.98. As an un-targeted moment, my model generates a volatility of the log price-to-dividend ratio of 30% per year, close to its empirical counterpart of 35%.

I choose $\sigma_d = 1$ to match the volatility of the dividend growth rate, which is 6.46% from the model and 6.26% from the data. The parameter σ_θ is chosen to match the annual realized return volatility of S&P 500 index. The model gives a return volatility of 14.95%, which is close to 16.25% in the data. Third, the remaining parameters $a = 0.01$ and $\sigma_s = 0.7$ are calibrated to match the regression coefficient β_P of -0.013 and β_F of 0.390 in the data (their model counterparts are -0.014 and 0.343, respectively). Proposition 1 demonstrates that, the informed investor's forecast revision could predict the uninformed investor's forecast error because of the private information

[9] In the model, the stationary price-to-dividend ratio can be directly calculated as follows: $\frac{\bar{P}}{\bar{D}} = \frac{\phi+\phi+(\phi_D+\bar{\phi}_x)\bar{x}}{\bar{x}} = \frac{1}{1+r}\left(\frac{1}{r}+1\right)$, which is uniquely determined by the risk-free rate.

from the additional signals. Therefore, $1/\sigma_s$, the signal precision of the informed investor, mainly drives the predictability of the consensus forecast error. Furthermore, a captures the persistence of the aggregate equity supply, which is important in generating the negative predictability of pricing errors, as implied by Proposition 2.

Table 2: Parameters

Para.	Value	Description	Para.	Value	Description
r	0.028	risk-free rate	σ_s	0.7	inverse of signal precision
ρ	0.03	time discount factor	σ_H	0.65	high volatility of hidden state
$\bar{x}$	10	mean level of dividend flow	σ_L	0.05	low volatility of hidden state
b	0.1	persistence of hidden state	σ_θ	0.68	volatility of total equity supply
a	0.01	persistence of total equity supply	κ	0.2	transition rate from high to low state
σ_d	1	dividend flow volatility	ω	0.5	fraction of uninformed investor

This table displays annualized parameter values used in the simulations. Appendix 5.7 summarizes the details of the numerical and calibration procedures.

Quantitative Results I first construct the consensus forecast revision and consensus forecast error from my model. Note that announcements are made periodically at nT, where n is an integer and $T = \frac{1}{4}$ indicates quarterly announcements. For any announcement scheduled at $(n+1)\,T$, I assume that the first forecast is made right after the previous announcement at nT and a revision of the forecast is made at $nT+\delta$, and I set $\delta = \frac{1}{2}T$ so that revisions are made in the middle of two consecutive announcements as in the actual data. Let x be the forecast variable announced at time $(n+1)\,T$, using definition (19), the consensus forecast revision made at time $nT+\delta$ is defined as:

$$Frev_{nT+\delta}\,(x) = \bar{\mathbb{E}}_{nT+\delta}[x] - \bar{\mathbb{E}}_{nT}[x], \tag{43}$$

where $\bar{\mathbb{E}}_t[x] = (1-\omega)\,\hat{\mathbb{E}}_t[x] + \omega\tilde{\mathbb{E}}_t[x]$ is the average belief of informed and uninformed investors. Similarly, applying equation (20), forecast error of x is defined as

$$Ferr_{(n+1)T}\,(x) = x - \bar{\mathbb{E}}_{nT+\delta}[x]. \tag{44}$$

My definitions of consensus forecast revision and error are therefore identical to those in the empirical exercise I presented in Section 2, as equations (1) and (2).

As in the data, I define the return responses to forecast revisions as $Rrev_{nT+\delta} = \frac{P_{nT+\delta}-P_{nT}}{P_{nT}}$ and the return reactions to forecast errors upon the announcements as $Rerr_{(n+1)T} = \frac{P^{+}_{(n+1)T}-P^{-}_{(n+1)T}}{P^{-}_{(n+1)T}}$. I regress the consensus forecast errors on consensus forecast revisions and regress announcement returns on returns during the revision periods as specified in equations (3) and (4) using the model simulated data. I report these regression results in Table 3. As model generated moments are estimated from long samples, they are all statistically significant and I do not report their corresponding t-statistics. Consensus forecast revisions positively predict forecast errors in my model with a slope coefficient of $\beta_F = 0.343$, close to its empirical counterpart. As I explained in Section 3, in my model, the private information of informed investors is not observed by the uninformed therefore accounts for the forecast errors made by uninformed investors. As a result, the revision of informed investors, based on their private information, positively predicts the error of the uninformed. Because the consensus forecast is the average of the forecasts of both groups of investors, revisions of the consensus forecast positively predict the errors of the forecast so that $\beta_F > 0$.

Table 3: Goodness of Fit

Panel A: Regression Coefficients

	Data	Model
β_F	0.390**	0.343***
β_P	-0.013***	-0.014***
β_H	-0.018***	-0.019***
β_{Dif}	0.018*	0.018***

This table presents regression coefficients based on equations (3), (4) and (5) from the data and model, respectively. I simulate the model for 20,000 years with quarterly macroeconomic announcements and daily stock prices. I discard the first 2,000 years and keep the remaining 18,000 years to ensure stationarity and then run OLS regressions. The maximum ODE convergence tolerance is 1.6e-13 for the simulation. Note: $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$. Appendix 5.7 summarizes the details for the numerical exercise.

It is clear from Table 3 that returns over forecast revision days negatively predict returns on announcement days with $\beta_P = -0.013$. This negative predictability comes from the cyclical patterns of the announcements. On non-announcement days, for example, an increase in the noisy supply θ_t lowers the stock price and learning amplifies this impact because the estimation-error-induced uncertainty builds up over time and the information becomes more asymmetric due to the increased information gap from the private information. Upon announcements, however, this asymmetric information-induced price reductions reverse itself as the true value of x_{nT} is revealed and the uncertainty is resolved, generating a negative predictability of the announcement returns.

From the above discussion, higher uncertainty will be associated with stronger impacts from learning and asymmetric information hence a stronger negative predictability of announcement-day returns. To test this unique implication of my model, I construct a forward looking measure of stock return volatility, which is the counterpart of the VIX index. This allows me to define periods of high uncertainty and low uncertainty as in the data, and replicate the regression specification (5) presented in Section 2. I provide details of the calculation of model implied volatility in Appendix 5.6 and report the regression results in the last two rows of Table 3. Evidently, the predictability is stronger in periods of higher uncertainty, and the difference between the predictability in high uncertainty and low uncertainty periods, β_{Dif} is positive and significant.

To further confirm the basic intuition in my model, I decompose the pricing error upon the announcement into three components as in equation (40), and regress them separately on price changes over revision days as follows:

$$\bar{\phi}_x \left[x_{(n+1)T} - \tilde{x}_{(n+1)T}\right] = \alpha_{P1} + \beta_{P1} \left(P_{nT+\delta} - P_{nT}\right) + \varepsilon, \quad (45)$$

$$-\phi_\theta(0) \left[\theta_{(n+1)T} - \tilde{\theta}_{(n+1)T}\right] = \alpha_{P2} + \beta_{P2} \left(P_{nT+\delta} - P_{nT}\right) + \varepsilon, \quad (46)$$

$$-\left[\phi_\theta(0) - \phi_\theta(T)\right] \tilde{\theta}_{(n+1)T} = \alpha_{P3} + \beta_{P3} \left(P_{nT+\delta} - P_{nT}\right) + \varepsilon. \quad (47)$$

My model predicts that the only component of the announcement return that is predicable comes from $\left[\phi_\theta(0) - \phi_\theta(T)\right] \tilde{\theta}_{(n+1)T}$. My simulation confirms this result: β_{P1} and β_{P2} are essentially zero and insignificant in long sample regressions while $\beta_{P3} = -0.010$ and is highly significant.

Implications for Trading Volume Given the investors' optimal trading strategies in equations (33) and (34), I can characterize the equilibrium trading volume implied by the model. Since the two types of investors trade against each other, the trading volume can be calculated as changes in the portfolio holdings of either group of investors. Define the trading volume from t to $t+\delta$ as the uninformed investor's turnover rate (see Wang, 1994):

$$M(t, t+\delta) = \omega \left|\beta_{t+\delta} - \beta_t\right| = \omega \left|\beta_\theta(t+\delta)\, \tilde{\theta}_{t+\delta} - \beta_\theta(t)\, \tilde{\theta}_t\right|. \quad (48)$$

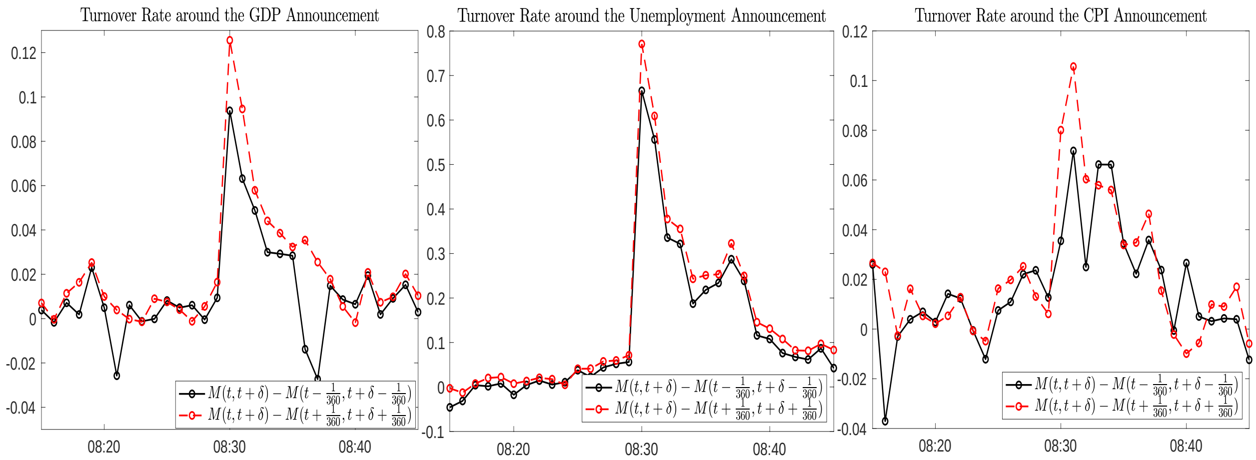
At the announcement, the trading volume between T^- and T^+ can be calculated as

$$M\left(T^-, T^+\right) = \omega \left|\beta_T^+ - \beta_T^-\right| = \omega \left|\beta_\theta(0)\, \theta_T - \beta_\theta(T)\, \tilde{\theta}_T\right|. \quad (49)$$

On non-announcement days, trading evolves slowly, as both $\beta_\theta(t)$ and $\tilde{\theta}_t$ are continuous functions of time in equation (48). At the announcement, however, both $\beta_\theta(t)$ and $\tilde{\theta}_t$ have discontinuous jumps so that a large trading volume is realized. The fact that trading peaks right upon the announcement in my model is consistent with the empirical evidence. In Figure 6, I plot the turnover rate for SPY over a 30-minute interval on announcement days for three types of announcements, GDP, unemployment, and CPI. For comparison purpose, the solid line plots the trading volume difference between the announcement day and one day before, $M(t, t+\delta) - M\left(t - \frac{1}{360}, t+\delta - \frac{1}{360}\right)$, where t is a time on the announcement day, δ is 30 minutes, and $t - \frac{1}{360}$ is the same time on the day before the announcement. Similarly, the dashed line plots the trading volume difference between the announcement day and one day afterwards, $M(t, t+\delta) - M\left(t + \frac{1}{360}, t+\delta + \frac{1}{360}\right)$. Evidently, relative to non-announcement days, the announcement triggers a spike in trading as implied by my model.

To formally evaluate the quantitative implications of my model, in Table 4, I compute the average changes in trading volume one day before and after the announcement and compare my model implications with the data. Here $M\left(T, T + \frac{1}{360}\right)$ is the announcement day turnover rate, and $M\left(T - \frac{1}{360}, T\right)$ and $M\left(T + \frac{1}{360}, T + \frac{2}{360}\right)$ are the turnover rate on the day before and after the

Figure 6: Trading Volume around Announcement Days



This figure plots the SPY turnover rate in the 30-minutes window, 8:15 am to 8:45 am, of the GDP, unemployment and CPI announcement at 8:30 am, respectively. The high frequency trading uses CRSP Millisecond Trade and Quote (TAQ) dataset, averaged over all the respective announcements from 2003Q1 to 2019Q4. The turnover rate is calculated as the total number of shares traded divided by the total number of shares outstanding, as in Lo and Wang (2000). The solid (dashed) line depicts the turnover rate difference between the announcement day and one day before (after) the announcement.

announcement, respectively. In the data, the difference in trading volume is statistically significant, and the point estimates are fairly close to the same moments implied by my model.

Table 4: Trading Volume Change around Announcement Days

	Data			*Model*
	GDP	Unemployment	CPI	
$\ln M\left(T,T+\frac{1}{360}\right)-\ln M\left(T-\frac{1}{360},T\right)$	13.71%	12.21%	8.70%	10.65%
	(2.58)	(2.44)	(1.68)	
$\ln M\left(T,T+\frac{1}{360}\right)-\ln M\left(T+\frac{1}{360},T+\frac{2}{360}\right)$	−10.43%	−20.74%	−11.66%	−10.27%
	(−2.47)	(−3.87)	(−3.42)	

This table reports changes in log trading volume one day before and after the advance (first) GDP estimates, unemployment and CPI announcement days and their time-series Newey-West t-statistics (in parenthesis) when testing whether the change is significantly different from zero. I also report the model-implied log change in volume around announcement days. The sample period is from 1993Q1 to 2019Q4. The trading volume is defined as the turnover rate and calculated as the total number of shares traded divided by the total number of shares outstanding, as in Lo and Wang (2000).

5 Conclusion

In this paper, I develop a noisy rational expectations model to jointly account for the positive predictability of consensus forecast errors and the negative predictability of pricing errors realized upon macroeconomic announcements. The announcement return predictability is particularly strong in high economic uncertainty periods. The key new ingredients in my model are asymmetric information and periodic macroeconomic announcements. I provide a full characterization of equilibrium prices and quantities. They are shown to be the solution to a system of ODEs subject to boundaries conditions at the announcement. I calibrate the model, and demonstrate its ability to quantitatively account for the predictabilities that I document in the data.

To simplify the computation of the equilibrium, this paper relies on exponential preferences, which deliver a convenient linear pricing function (albeit with time-varying coefficients). It may be possible to extend the current model to allow for generalized risk-sensitive preferences (Ai and Bansal (2018)) to account for the macroeconomic announcement premium as well as the belief and return dynamics documented in this paper in a unified setup. Another possible extension would be to exploit other data sources on survey expectations. For example, rather than focus on the aggregate stock market using macroeconomic surveys, the analysis here could be replicated using I/B/E/S data on analyst forecasts of the earnings of individual firms. I leave these interesting directions for future research.

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Appendix

5.1 Data and Measurements

Forecasts Following most of the literature on measuring expectations formation, I use historical survey data from the Survey of Professional Forecasters (SPF) and Consensus Economics (CE) forecasts for the United States. I focus on forecasts of real GDP, the unemployment rate and Consumer Price Index (CPI).[10] SPF is a quarterly survey containing approximately 40 professional forecasters, beginning in 1968Q4. Since 1990 it has been run by the Federal Reserve Bank of Philadelphia. Panelists are largely from the business world and Wall Street, spanning different sectors (e.g., banks, consulting firms, universities, private firms). Each forecaster is asked to forecast at horizons from the current quarter to four quarters later. The data are reported at both the individual level and the consensus level, computed as the cross-sectional mean from the individual level forecasts at a point in time.[11] The SPF survey is distributed after the release of last quarter's first GDP estimates, and roughly one or two weeks later, the panelists are asked to submit the survey before the submission deadline. Since 1990Q2, the survey has been conducted in the second month of the quarter, and the deadline for submitting forecasts is around the middle of the survey month.[12,13] Similarly, CE conduct quarterly survey for many countries including the United States since 1989Q4 and the panelists are also professional forecasters. However, the survey submission dates are different from SPF, typically at the end of each quarter on March, June, September and December.

Announcements I collect GDP announcement dates from the Bureau of Economic Analysis (BEA)'s website, which reports quarterly GDP, inflation, and consumption at the end of each month.[14] GDP announcements are made monthly, so each quarter contains three announcements: advance (first), second, and third estimate. For example, in April the advance estimate for the Q1 GDP growth rate is released, followed by a second estimate of the same Q1 GDP growth rate in May, and a third estimate given in June. I focus on the advance estimates for three reasons. First, the advance estimates are believed to reveal the most information, thus resolving most of the uncertainty. Second, the subsequent revisions may not reflect the initial reactions to the surprises

[10] Coibion and Gorodnichenko (2015) show that the predictability of consensus forecast errors holds for both dataset and for many other macroeconomic variables, including the inflation rate (GDP/GNP deflator) and real consumption expenditures. I focus on the GDP announcement because the inflation rate and disposable personal income are announced together with GDP by the BEA at the same time. However, unemployment/non-farm payrolls and CPI are announced at different dates by the BLS.

[11] The cross-sectional mean could change because of a change in forecaster composition. Coibion and Gorodnichenko (2015) only include forecasters that participate in two consecutive surveys and find robust results. The results are also robust for cross-sectional median forecasts.

[12] The detailed deadline dates can be found at: https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-release-dates.txt?la=en.

[13] The survey first gets published (open to the public) around one week later than the submission day. Since this paper mainly focuses on stock market reactions to market participants' forecast revisions, the information those panelists use to update their beliefs has been updated until the day they submit the survey. Therefore, the data publication day does not matter because it could not represent the right timing for panelists' belief revisions.

[14] https://www.bea.gov/data/gdp/gross-domestic-product.

in GDP growth rate announcements. Third, Gilbert (2011) and Gilbert, Scotti, Strasser, and Vega (2017) show that only the advance release has the significant impact on asset prices, whereas the final release almost has no price impact. Therefore, in this paper, the forecast revision at quarter t is associated with the realization of the quarter-t GDP growth rate announced at $t+1$, and forecast errors are calculated using advance estimates. I further collect quarterly unemployment rate and CPI announcement dates from the Bureau of Labor Statistics (BLS).[15] While monthly unemployment situation/non-farm payrolls and CPI are released in the following month, quarterly realizations are released around January, April, July, and October, about one month after each quarter.

Stock Market Returns First, I use realized close-to-close returns on adjacent survey submission days to measure the cumulative returns over the time panelists revise their beliefs, i.e., $Rrev_t(x)$ defined in the main text. This is because information arrives continuously, and beliefs are accordingly being continuously revised. I assume that revisions start right after the last survey submission day until the current submission day, so forecast revisions defined in equation (3) should reflect the cumulative revisions in response to newly obtained information between two adjacent forecast quarters. Therefore, the cumulative returns over the forecast revision periods reflect the stock market reactions to forecast revisions. Second, I use returns over the 30-minutes interval, 8:15 am to 8:45 am, because GDP, the employment situation, and CPI are all released at 8:30 am. before the stock market opens. This high-frequency window provides a fairly clean way to measure the stock market reactions to forecast errors realized upon macroeconomic announcements. Third, instead of using the S&P 500 index, I calculate returns on revision days based on the SPDR S&P 500 ETF (SPY) dataset, which is available starting from January 1993. The high frequency SPY is from CRSP Millisecond Trade and Quote (TAQ) dataset, dating back to 2003.[16]

Table 5 summarizes the data used in the empirical analysis. Several things are worth noting. First, no abnormal returns are observed on survey submission days. As the average return is about 1 bps from SPF and 4 bps from CE, both statistically insignificant different from zero, it is reasonable to believe that on average no special news announcements or events happen on forecast submission days. Second, the announcement returns earned in the 30-minutes window are -0.75, 3.98, and 6.81 bps for GDP, unemployment, and CPI, respectively, while only CPI announcement return is significantly different from zero. Third, one can see that professional forecasters do not have significant forecasting biases, at least at one-quarter-ahead horizons. Therefore, I assume they are marginal investors who can generally represent the stock market participants.[17]

[15] https://www.bls.gov/schedule/news_release/empsit.htm.

[16] I do this because the S&P 500 calculates its opening price at 9:31 a.m. when many stocks are not open. As a result, the opening price for the S&P 500 is often the same as the previous trading day's closing price, which produces many zero overnight returns when using S&P 500 data. In contrast, the SPY is calculated based on S&P 500 futures, which is always open by 9:31 a.m. and traded overnight. See Lou, Polk, and Skouras (2019) and Hendershott, Livdan, and Rösch (2020) for how returns behave differently between daytime and overnight.

[17] Adam, Matveev, and Nagel (2018) use various surveys of return forecasts and find that investors' aggregate return expectations are unconditionally unbiased. Stark (2010) analyzes the accuracy of forecasts and find that the SPF forecasts outperform benchmark projections from univariate autoregressive time-series models at short

Table 5: Summary Statistics

Variable	Mean (%)	s.d.(%)	Obs.	Time	Variable	Mean (%)	s.d.(%)	Obs.	Time
Panel A: Return reaction to forecast revisions									
Data Source: SPF					*CE*				
$Rrev_t$	1.895	0.796	77	1993Q1-2019Q4	$Rrev_t$	2.215	0.792	105	1993Q1-2019Q4
Rv_t	0.011	0.131	88	1993Q1-2019Q4	Rv_t	0.041	0.098	107	1993Q1-2019Q4
Panel B: Announcement returns, forecast revisions and forecast errors									
Variable	Mean (%)	s.d.(%)	Obs.	Time	Variable	Mean (%)	s.d.(%)	Obs.	Time
Real GDP Growth Rate					*UE*				
$Rerr_{t+1}$	-0.0075	0.032	65	2003Q1-2019Q4	$Rerr_{t+1}$	0.0398	0.057	62	2003Q1-2019Q4
$Frev_t$	-0.277	0.085	204	1969Q1-2019Q4	$Frev_t$	-0.014	0.019	204	1969Q1-2019Q4
$Ferr_{t+1}$	0.091	0.126	204	1969Q1-2019Q4	$Ferr_{t+1}$	-0.033	0.018	204	1969Q1-2019Q4
CPI									
$Rerr_{t+1}$	0.0681	0.034	64	2003Q1-2019Q4					
$Frev_t$	-0.112	0.079	153	1981Q4-2019Q4					
$Ferr_{t+1}$	-0.016	0.093	153	1981Q4-2019Q4					

This table reports summary statistics for the main variables used in the empirical tests. Panels A reports $Rrev_t$ (the close-to-close cumulative returns between adjacent SPF submission deadline days) and Rv_t (the daily returns of survey submission deadline days) from Survey of Professional Forecasters (SPF) and Consensus Economics (CE), respectively. Both $Rrev_t$ and Rv_t are calculated based on the SPDR S&P 500 ETF (SPY) dataset. Panels B displays the summary statistics for real GDP growth rate, unemployment rate (UE), and Consumer Price Index (CPI) respectively. $Rerr_{t+1}$ represents the announcement return, which is the cumulative return earned in the 30-minutes window, 8:15 am to 8:45 am, of the announcement at 8:30 am. The forecast error ($Ferr_{t+1}$) equals initial released values minus the most recent forecasts. The forecast revision ($Frev_t$) is defined as the difference between the forecast of a variable at this quarter and the forecast of the same variable made at last quarter. All returns exclude observations on non-trading days.

Robustness Check In this section, I conduct robustness tests for empirical results in the main text. Panel A in Table 6 presents the regression coefficients from equation (5) using SPF and CE dataset, respectively. The magnitudes for GDP, unemployment and CPI are similar from both dataset. Unemployment and CPI have low t-statistics in SPF. This is likely due to the fact that there are several SPF surveys conducted on non-trading days, which results in a smaller and discontinuous sample compared to CE. Panel B displays the regression results of equation (4) using daily news-based U.S. economic policy uncertainty indices (EPU) from Baker, Bloom, and Davis (2016) as an alternative measure of uncertainty. In order to show the results are not driven by extreme events during the financial crisis, in Table 7 I show the results are robust after excluding the 2007-2008 financial crisis period.

horizons. The special survey of analyzing the panelists' forecasting methods shows that "20 of 25 respondents said they use a combination of mathematical/computer models *plus* subjective adjustments to that model in reporting their projections."

Table 6: Announcement Return Predictability (Robustness Check)

	SPF			*CE*		
	GDP	UE	CPI	GDP	UE	CPI
Panel A. $Rerr_{t+1}(x) = \alpha + \beta_P Rrev_t(x) + \varepsilon_{t+1}$.						
β_P	-0.0133***	-0.0115	-0.00429	-0.0105***	-0.00889*	-0.00501**
	(-3.12)	(-1.33)	(-1.20)	(-2.71)	(-1.90)	(-2.16)
Panel B. $Rerr_{t+1}(x) = \alpha + \beta_H Rrev_t(x) + \beta_{Dif} I_L(t) \times Rrev_t(x) + \beta_{Dummy} I_L(t) + \varepsilon_{t+1}$.						
β_H	-0.0172***	-0.0192***	-0.000153	-0.0162***	-0.0126**	-0.00282
	(-5.37)	(-2.65)	(-0.04)	(-3.64)	(-2.03)	(-0.98)
β_{Dif}	0.0137*	0.0297*	-0.0192	0.0225***	0.0146	-0.00842**
	(1.67)	(1.66)	(-0.72)	(3.97)	(1.17)	(-2.04)
obs.	57	54	56	62	60	61

Panel A and B report the coefficient estimates of equation (5) and (4) for real GDP growth rate, unemployment rate (UE) and Consumer Price Index (CPI) using data from Survey of Professional Forecasters (SPF) and Consensus Economics (CE), respectively. Panel B uses U.S. economic policy uncertainty (EPU) from Baker, Bloom, and Davis (2016) to measure uncertainty. The full sample period is from 2003Q1 to 2019Q4. Newey-West t-statistics (with 5 lags) are in parentheses. Note: $*p < 0.1$, $**\ p < 0.05$, $***\ p < 0.01$.

Table 7: Excluding 2007-2008 Financial Crisis

	SPF			*CE*		
	GDP	UE	CPI	GDP	UE	CPI
Panel A. Using VIX to measure uncertainty						
β_H	-0.0103***	-0.0275***	-0.00629**	-0.00837**	-0.0102**	-0.00465***
	(-3.29)	(-3.63)	(-2.22)	(-2.47)	(-2.42)	(-3.91)
β_{Dif}	0.0103	0.0531**	0.0119	0.0112**	0.00642	-0.00215
	(1.10)	(2.29)	(1.30)	(2.14)	(0.34)	(-0.52)
Panel B. Using EPU to measure uncertainty						
β_H	-0.0107***	-0.0302***	-0.00923**	-0.00993*	-0.0178***	-0.00164*
	(-3.17)	(-4.17)	(-2.35)	(-1.68)	(-2.81)	(-1.68)
β_{Dif}	0.0114	0.0489***	0.0163**	0.0131**	0.0245**	-0.00945***
	(1.51)	(3.51)	(2.45)	(2.01)	(2.16)	(-3.34)
obs.	49	46	48	55	52	54

This table reports the robustness check for the regression specified in equation (4) for real GDP growth rate, unemployment rate (UE) and Consumer Price Index (CPI) using data from Survey of Professional Forecasters (SPF) and Consensus Economics (CE), respectively. Panel A and B use VIX index and U.S. economic policy uncertainty (EPU) to measure uncertainty, respectively. The full sample period is from 2003Q1 to 2019Q4, excluding 2007-2008 financial crisis. Newey-West t-statistics (with 6 lags) are in parentheses. Note: $*p < 0.1$, $**\ p < 0.05$, $***\ p < 0.01$.

5.2 Investors' Learning Problems

In this section, I compute the both types of investors' optimal learning problems. I first prove Lemma 1. Then I prove Proposition 1, which summarizes one main result that the informed investor's forecast revision could positively predict the forecast error of the uninformed.

Proof for Lemma 1 The optimal learning for the informed investor is a standard Kalman-Bucy filter problem with the unobserved state variable given in (7) and the observed processes (6), (8), and (9). Applying Theorem 10.3 from Liptser and Shiryaev (2001), it is straightforward to show that the law of motion of the posterior mean satisfies (14) where the innovation processes for (6) and (9) are given by

$$d\hat{B}_{D,t} = \frac{1}{\sigma_D}\left[dD_t - \left(\hat{x}_t - D_t\right)dt\right], \text{and } d\hat{B}_{s,t} = \frac{1}{\sigma_s}\left(ds_t - \hat{x}_t dt\right). \tag{50}$$

The law of motion of the conditional variance $\hat{q}_t$ must satisfy the Riccati equation

$$d\hat{q}\left(t\right) = \left[\sigma_x^2 - 2b\hat{q}\left(t\right) - \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\hat{q}^2\left(t\right)\right]dt. \tag{51}$$

Because the stock price is a function of $\hat{x}_t$ and θ_t (see equation (10)), whereas both of which are unobserved, the uninformed investors need to compute the posterior distributions for both. Equation (13) implies that the posterior distribution of θ_t can be inferred from that of $\hat{x}_t$. Therefore, I focus on the learning problem for $\hat{x}_t$, whose law of motion is given by (14). Informed investors observe two sources of information for $\hat{x}_t$, the dividend process and the equilibrium price. Rewrite equation (50) in terms of the innovation process $d\hat{B}_{D,t}$ as

$$dD_t = \left(\hat{x}_t - D_t\right)dt + \sigma_D d\hat{B}_{D,t}. \tag{52}$$

Note that observing the price is equivalent to observing $\zeta_t = \phi_x\left(t\right)\hat{x}_t - \phi_\theta\left(t\right)\theta_t$, because all other variables in (10) are known to the uninformed investors. Applying Ito's lemma, ζ_t can be represented as a Markov process given the state variables $\hat{x}_t$ and ζ_t itself:

$$\begin{aligned} d\zeta_t \;=\; & \left[b\bar{x}\phi_x\left(t\right) + \left(\left(a - b - \frac{\phi_\theta'\left(t\right)}{\phi_\theta\left(t\right)}\right)\phi_x\left(t\right) + \phi_x'\left(t\right)\right)\hat{x}_t + \left(\frac{\phi_\theta'\left(t\right)}{\phi_\theta\left(t\right)} - a\right)\zeta_t\right]dt \\ & + \frac{\hat{q}\left(t\right)}{\sigma_D}\phi_x\left(t\right)d\hat{B}_{D,t} + \frac{\hat{q}\left(t\right)}{\sigma_s}\phi_x\left(t\right)d\hat{B}_{s,t} - \sigma_\theta\phi_\theta\left(t\right)dB_{\theta,t}. \end{aligned} \tag{53}$$

It is useful to define $\xi_t = \zeta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x\left(t\right)D_t$ so that $\left(\hat{x}_t, D_t, \xi_t\right)$ has a state space representation and the innovations of dD_t and $d\xi_t$ are mutually independent. The dynamics of ξ_t is

$$d\xi_t = \left[b\bar{x}\phi_x\left(t\right) + m_x\left(t\right)\hat{x}_t + \left(\frac{\phi_\theta'\left(t\right)}{\phi_\theta\left(t\right)} - a\right)\xi_t + m_D\left(t\right)D_t\right]dt + \frac{\hat{q}\left(t\right)}{\sigma_s}\phi_x\left(t\right)d\hat{B}_{s,t} - \sigma_\theta\phi_\theta\left(t\right)dB_{\theta,t}, \tag{54}$$

where the coefficients $m_x\left(t\right)$ and $m_D\left(t\right)$ are defined as

$$m_x\left(t\right) = \left(a - b - \frac{\phi_\theta'\left(t\right)}{\phi_\theta\left(t\right)} - \frac{\hat{q}\left(t\right)}{\sigma_D^2}\right)\phi_x\left(t\right) + \phi_x'\left(t\right), \tag{55}$$

$$m_D\left(t\right) = \frac{1}{\sigma_D^2}\left[\hat{q}\left(t\right)\phi_x\left(t\right)\left(1 - a + \frac{\phi_\theta'\left(t\right)}{\phi_\theta\left(t\right)}\right) - \hat{q}'\left(t\right)\phi_x\left(t\right) - \hat{q}\left(t\right)\phi_x'\left(t\right)\right]. \tag{56}$$

It is convenient to define

$$\sigma_\xi\left(t\right) = \sqrt{\frac{\hat{q}^2\left(t\right)}{\sigma_s^2}\phi_x^2\left(t\right) + \sigma_\theta^2\phi_\theta^2\left(t\right)}, \tag{57}$$

as the volatility of ξ_t, and define

$$\nu(t) = \frac{1}{\sigma_\xi^2(t)} \left[\frac{\phi_x(t)}{\sigma_s^2} \hat{q}^2(t) + m_x(t)\tilde{q}(t) \right]. \tag{58}$$

I will call ξ_t the information content of price, as observing price is equivalent to observing ξ_t. To apply the Kalman-Bucy filter, I will treat (14) as the unobserved state variable and (52) and (54) as the observations. The filtering equation in (15) can therefore be derived by applying Theorem 10.3 in Liptser and Shiryaev (2001), where the innovation processes are standard Brownian motions with respect to the uninformed investor's information set which are defined as

$$\begin{aligned} d\tilde{B}_{D,t} &= \frac{1}{\sigma_D}\left[dD_t - (\tilde{x}_t - D_t)\,dt\right], & (59)\\ d\tilde{B}_{\xi,t} &= \frac{1}{\sigma_\xi(t)}\left[d\xi_t - \left[b\bar{x}\phi_x(t) + m_x(t)\tilde{x}_t + \left(\frac{\phi_\theta'(t)}{\phi_\theta(t)} - a\right)\xi_t + m_D(t)D_t\right]dt\right]. & (60)\end{aligned}$$

The posterior variance, $\tilde{q}(t)$ satisfies the following Riccati equation

$$d\tilde{q}_t = \left[\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\hat{q}_t^2 - 2b\tilde{q}_t - \left(\frac{\hat{q}_t + \tilde{q}_t}{\sigma_D}\right)^2 - \nu^2(t)\,\sigma_\xi^2(t)\right]dt. \tag{61}$$

Applying Ito's lemma to equation (13), the law of motion for $\tilde{\theta}_t$ is therefore derived as

$$d\tilde{\theta}_t = -a\tilde{\theta}_t dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}d\tilde{B}_{D,t} + \frac{1}{\phi_\theta(t)}\sigma_\xi(t)\left[\nu(t)\phi_x(t) - 1\right]d\tilde{B}_{\xi,t}. \tag{62}$$

The following variances and covariances can be computed from the law of total covariance:

$$\begin{aligned} \tilde{\mathrm{Var}}(x_t) &= \tilde{\mathbb{E}}[\hat{\mathrm{Var}}(x_t)] + \tilde{\mathrm{Var}}(\hat{\mathbb{E}}[x_t]) = \hat{q}_t + \tilde{q}_t,\ \tilde{\mathrm{Cov}}(x_t, \hat{x}_t) = \tilde{\mathbb{E}}[\hat{\mathrm{Cov}}(x_t, \hat{x}_t)] + \tilde{\mathrm{Cov}}(\hat{\mathbb{E}}[x_t], \hat{\mathbb{E}}[\hat{x}_t]) = \tilde{q}_t,\\ \tilde{\mathrm{Cov}}(x_t, \theta_t) &= \tilde{\mathrm{Cov}}[x_t, \frac{1}{\phi_{\theta,t}}(\phi_{x,t}\hat{x}_t - \zeta_t)] = \frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t,\ \tilde{\mathrm{Cov}}(\hat{x}_t, \theta_t) = \frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t,\ \text{and } \tilde{\mathrm{Var}}(\theta_t) = \frac{\phi_{x,t}^2}{\phi_{\theta,t}^2}\tilde{q}_t. \quad (63)\end{aligned}$$

Difference in Beliefs Next, I characterize the dynamics of difference in beliefs between informed and uninformed investors, $\Delta \equiv \hat{x}_t - \tilde{x}_t$. The stochastic process for Δ_t can be derived directly from equations (14) and (15). Equations (52) and (54) allow me to replace dD_t and $d\xi_t$ in the definition of the innovation processes, (59) and (60), respectively to write $d\Delta_t$ in terms of Brownian motions with respect to the informed investors' information set, $\hat{B}_{D,t}$, $\hat{B}_{s,t}$, and $B_{\theta,t}$ as in (23). The coefficients are: $a_\Delta(t) = b + \frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D^2} + m_x(t)\nu(t)$, $\sigma_{\Delta D}(t) = \frac{\tilde{q}(t)}{\sigma_D} > 0$, $\sigma_{\Delta s}(t) = \frac{\hat{q}(t)}{\sigma_s}[1 - \phi_x(t)\nu(t)]$, and $\sigma_{\Delta\theta}(t) = \sigma_\theta\phi_\theta(t)\nu(t)$, where $\sigma_\xi(t)$ and $\nu(t)$ are defined in (57) and (58), respectively. In the calibrated example, $a_\Delta(t) > 0$, $\sigma_{\Delta s}(t) > 0$ and $\sigma_{\Delta\theta}(t) > 0$.

Proof for Proposition 1 Now I provide a proof for Proposition 1. The forecast error of the uninformed could be decomposed as: $x_T - \tilde{x}_T = (x_T - \hat{x}_T) + (\hat{x}_T - \tilde{x}_T)$. The standard optimal filtering implies that the optimal forecast revision of informed investors could not predict their own forecast error, i.e., $\text{Cov}\left(x_T - \hat{x}_T, \hat{\mathbb{E}}_t[x_T] - \hat{\mathbb{E}}_0[x_T]\right) = 0$. Therefore, to establish $\text{Cov}\left(x_T - \tilde{x}_T, \hat{\mathbb{E}}_t[x_T] - \hat{\mathbb{E}}_0[x_T]\right) > 0$, it is sufficient to show $\text{Cov}\left(\hat{x}_T - \tilde{x}_T, \hat{\mathbb{E}}_t[x_T] - \hat{\mathbb{E}}_0[x_T]\right) > 0$. Note that $\hat{\mathbb{E}}_t[x_T] = \hat{\mathbb{E}}_t[\hat{x}_T]$ and $\hat{\mathbb{E}}_0[x_T] = \hat{\mathbb{E}}_0[\hat{x}_T]$. Therefore,

$$\text{Cov}\left(\hat{x}_T - \tilde{x}_T, \hat{\mathbb{E}}_t[x_T] - \hat{\mathbb{E}}_0[x_T]\right) = \text{Cov}\left(\hat{x}_T - \tilde{x}_T, \hat{\mathbb{E}}_t[\hat{x}_T] - \hat{\mathbb{E}}_0[\hat{x}_T]\right) \tag{64}$$

$$= \text{Cov}\left(\hat{x}_T - \tilde{x}_T, \hat{\mathbb{E}}_t[\hat{x}_T] - \tilde{\mathbb{E}}_t[\hat{x}_T] + \tilde{\mathbb{E}}_t[\hat{x}_T] - \tilde{\mathbb{E}}_0[\hat{x}_T]\right) = \text{Cov}\left(\hat{x}_T - \tilde{x}_T, \hat{\mathbb{E}}_t[\hat{x}_T] - \tilde{\mathbb{E}}_t[\hat{x}_T]\right), \tag{65}$$

where the second equality is true because at time 0, both groups of investors have the same belief about x_T after the announcement, $\tilde{\mathbb{E}}_0[x_T] = \hat{\mathbb{E}}_0[x_T]$. The last equality holds because the belief revision of the uninformed should not predict his or her belief error: $\text{Cov}\left(\hat{x}_T - \tilde{x}_T, \tilde{\mathbb{E}}_t[\hat{x}_T] - \tilde{\mathbb{E}}_0[\hat{x}_T]\right) = 0$. Hence, it is equivalent to show $\text{Cov}\left(\Delta_T, \hat{\mathbb{E}}_t[\Delta_T]\right) > 0$.

Further, equation (23) yields,

$$\Delta_T = e^{-\int_t^{T-t} a_\Delta(z)dz}\Delta_t + e^{-\int_0^{T-t} a_\Delta(z)dz}\int_t^T e^{\int_0^u a_\Delta(z)dz}\hat{\sigma}(u)\,dB_{\Delta,u}, \tag{66}$$

where $B_{\Delta,t} = \left[\hat{B}_{D,t}, \hat{B}_{s,t}, B_{\theta,t}\right]$ is a vector Brownian motion, and $\hat{\sigma}(t) = \left[-\sigma_{\Delta D}(t), \sigma_{\Delta s}(t), \sigma_{\Delta\theta}(t)\right]^\top$. Clearly $\hat{\mathbb{E}}_t[\Delta_T] = e^{-\int_t^{T-t} a_\Delta(z)dz}\Delta_t$ is positively correlated with Δ_T.

Excess Returns In this subsection, I use the results from the filtering problem derived above to derive the excess return of the stock as diffusion processes under both types of investors' beliefs. The equilibrium pricing function (10) can be written as:

$$P_t = \phi + \phi_D D_t - \phi_\theta(t)\theta_t + \bar{\phi}_x\hat{x}_t - \phi_\Delta(t)\Delta_t, \tag{67}$$

$$= \phi + \phi_D D_t - \phi_\theta(t)\tilde{\theta}_t + \bar{\phi}_x\tilde{x}_t, \tag{68}$$

where $\bar{\phi}_x = \phi_x(t) + \phi_\Delta(t)$. In equation (67), all variables are observable to informed investors, whereas equation (68) represents the price as a function of the state variables measurable within the uninformed investors' information set.

Define the instantaneous excess return as $dQ_t = dP_t + D_t dt - rP_t dt$. Consider first the informed investors. Equations (52), (8), (14), and (23) represent the variables D_t, θ_t, $\hat{x}_t$, and Δ_t in terms of Brownian motions with respect to their information set. These give

$$\begin{aligned} dQ_t = {} & \left\{\left[-r\phi + b\bar{x}\bar{\phi}_x\right] + \left[1 - (1+r)\phi_D(t)\right]D_t + e_\theta(t)\theta_t + \left[\phi_D - (b+r)\phi_x\right]\hat{x}_t + e_\Delta(t)\Delta_t\right\}dt \\ & + \varrho_D(t)\,d\hat{B}_{D,t} + \varrho_s(t)\,d\hat{B}_{s,t} + \varrho_\theta(t)\,dB_{\theta,t}, \end{aligned} \tag{69}$$

where $e_\Delta(t) = \left[r + b + \frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D^2} + m_x(t)\nu(t)\right]\phi_\Delta(t) - \phi'_\Delta(t)$, and

$$e_\theta(t) = (a+r)\phi_\theta(t) - \phi'_\theta(t), \tag{70}$$

and the diffusion coefficients are given by $\varrho_D(t) = \frac{1}{\sigma_D}\left[\phi_D \sigma_D^2 + \bar{\phi}_x \hat{q}(t) + \phi_\Delta(t)\tilde{q}(t)\right]$, $\varrho_s(t) = [1+\phi_\Delta(t)\nu(t)]\phi_x(t)\frac{\hat{q}(t)}{\sigma_s}$, and $\varrho_\theta(t) = -[1+\phi_\Delta(t)\nu(t)]\phi_\theta(t)\sigma_\theta$. Further define the variance of excess return as

$$\sigma_P(t) = \varrho_D^2(t) + \varrho_s^2(t) + \varrho_\theta^2(t). \tag{71}$$

The market clearing condition implies that the expected return of the stock cannot depend on D_t, $\hat{x}_t$ and the constant. As a result, the coefficients them must be 0, implying

$$\phi_D = \frac{1}{1+r},\ \bar{\phi}_x = \frac{1}{r(b+r)},\ \text{and } \phi = \frac{b\bar{x}\bar{\phi}_x}{r}. \tag{72}$$

Similarly, I can use equations (59), (62), and (15) to write the excess return in terms of Brownian motions with respect to the uninformed investor's information set. This gives $dQ_t = \left[e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)\,d\tilde{B}_{D,t} + \sigma_\xi(t)[1+\phi_\Delta(t)\nu(t)]\,d\tilde{B}_{\xi,t}$.

5.3 Solving for Optimization Problems in the Interior

For illustration purpose, in this appendix I use the superscript i to indicate variables of the informed investor and u of the uninformed. For example, W^i stands for the informed investor's wealth and W^u is the uninformed's. The following lemma summarizes the solutions to both types of investors' optimization problems. I drop the time scripts for simplicity.

Lemma 4. *In the interior $(0,T)$, the informed and uninformed investor's value function takes the form of*

$$J\left(t, W^i, \theta, \Delta\right) = -e^{-\rho t - rW^i - g(t,\theta,\Delta)}, \tag{73}$$

$$V\left(t, W^u, \tilde{\theta}\right) = -e^{-\rho t - rW^u - f\left(t,\tilde{\theta}\right)}, \tag{74}$$

respectively, where $g(t,\theta,\Delta)$ and $f(t,\tilde{\theta})$ are quadratic forms of

$$g(t,\theta,\Delta) = g(t) + \frac{1}{2}g_{\theta\theta}(t)\theta_t^2 + \frac{1}{2}g_{\Delta\Delta}(t)\Delta_t^2 + g_{\theta\Delta}(t)\theta_t\Delta_t, \tag{75}$$

$$f\left(t,\tilde{\theta}\right) = f(t) + \frac{1}{2}f_{\theta\theta}(t)\tilde{\theta}_t^2, \tag{76}$$

where the coefficients are time varying and satisfy the ODEs system defined as follows:

$$
\begin{aligned}
g'(t) &= r-\rho-r\ln r+rg(t)-\frac{1}{2}\sigma_\theta^2 g_{\theta\theta}(t)-\frac{1}{2}\sigma_\Delta(t)\,g_{\Delta\Delta}(t)-\sigma_\theta\sigma_{\Delta\theta}(t)\,g_{\theta\Delta}(t),\\
g'_{\theta\theta}(t) &= rg_{\theta\theta}(t)-r^2\sigma_P(t)\,\alpha_\theta^2(t)+2ag_{\theta\theta}(t)+\sigma_\theta^2 g_{\theta\theta}^2(t)+\sigma_\Delta^2(t)\,g_{\theta\Delta}^2(t)+2\sigma_\theta\sigma_{\Delta\theta}(t)\,g_{\theta\theta}(t)\,g_{\theta\Delta}(t),\\
g'_{\Delta\Delta}(t) &= rg_{\Delta\Delta}(t)-r^2\sigma_P(t)\,\alpha_\Delta^2(t)+2a_\Delta(t)\,g_{\Delta\Delta}(t)+\sigma_\theta^2 g_{\theta\Delta}^2(t)+\sigma_\Delta(t)\,g_{\Delta\Delta}^2(t)\\
&\quad+2\sigma_\theta\sigma_{\Delta\theta}(t)\,g_{\theta\Delta}(t)\,g_{\Delta\Delta}(t),\\
g'_{\theta\Delta}(t) &= rg_{\theta\Delta}(t)-r^2\sigma_P(t)\,\alpha_\theta(t)\,\alpha_\Delta(t)+ag_{\theta\Delta}(t)+a_\Delta(t)\,g_{\theta\Delta}(t)+\sigma_\theta^2 g_{\theta\theta}(t)\,g_{\theta\Delta}(t)\\
&\quad+\sigma_\Delta(t)\,g_{\Delta\Delta}(t)\,g_{\theta\Delta}(t)+\sigma_\theta\sigma_{\Delta\theta}(t)\left[g_{\theta\theta}(t)\,g_{\Delta\Delta}(t)+g_{\theta\Delta}^2(t)\right]; \qquad (77)\\
f'(t) &= r-\rho-rlnr+rf(t)-\frac{1}{2}\sigma_{\theta\theta}(t)\,f_{\theta\theta}(t),\\
f'_{\theta\theta}(t) &= rf_{\theta\theta}(t)-r^2\sigma_P(t)\,\beta_\theta^2(t)+2af_{\theta\theta}(t)+\sigma_{\theta\theta}(t)\,f_{\theta\theta}^2(t). \qquad (78)
\end{aligned}
$$

Proof. Conjecture the informed investor's value function takes the form of equation (73), where $g(t,\theta,\Delta)$ is of the form (75). Using Ito's Lemma, the HJB equation is

$$
\begin{aligned}
\rho J &= -e^{-\rho t-C^i}+J_t+J_W\left[rW^i-C^i+\alpha\left(e_\theta(t)\,\theta+e_\Delta(t)\,\Delta\right)\right]+\frac{1}{2}J_{WW}\alpha^2\sigma_P(t)+\alpha J_{W\theta}\sigma_\theta\varrho_\theta(t)\\
&\quad+\alpha J_{W\Delta}\sigma_{Q\Delta}(t)-J_\theta a\theta-J_\Delta a_\Delta(t)\,\Delta+\frac{1}{2}J_{\theta\theta}\sigma_\theta^2+\frac{1}{2}J_{\Delta\Delta}\sigma_\Delta(t)+J_{\theta\Delta}\sigma_\theta\sigma_{\Delta\theta}(t), \qquad (79)
\end{aligned}
$$

where $\sigma_\Delta(t)=\sigma_{\Delta D}^2(t)+\sigma_{\Delta s}^2(t)+\sigma_{\Delta\theta}^2(t)$ and

$$
\sigma_{Q\Delta}(t) = -\varrho_D(t)\,\sigma_{\Delta D}(t)+\varrho_s(t)\,\sigma_{\Delta s}(t)+\varrho_\theta(t)\,\sigma_{\Delta\theta}(t), \qquad (80)
$$

The first order condition (FOC) with respect to C^i is: $rW^i-C^i=\ln r-g(t,\theta,\Delta)$. Under the guessed value function form, HJB can be rewritten as

$$
\begin{aligned}
0 &= r-2\rho-\frac{\partial g}{\partial t}-r\left[\ln r-g+\alpha\left(e_\theta(t)\,\theta+e_\Delta(t)\,\Delta\right)\right]+\frac{1}{2}r^2\alpha^2\sigma_P(t)+\alpha r\frac{\partial g}{\partial\theta}\sigma_\theta\varrho_\theta(t)+\alpha r\frac{\partial g}{\partial\Delta}\sigma_{Q\Delta}(t)\\
&\quad+\frac{\partial g}{\partial\theta}a\theta+\frac{\partial g}{\partial\Delta}a_\Delta(t)\,\Delta+\frac{1}{2}\left[\left(\frac{\partial g}{\partial\theta}\right)^2-\frac{\partial^2 g}{\partial\theta^2}\right]\sigma_\theta^2+\frac{1}{2}\left[\left(\frac{\partial g}{\partial\Delta}\right)^2-\frac{\partial^2 g}{\partial\Delta^2}\right]\sigma_\Delta(t)+\left(\frac{\partial g}{\partial\theta}\frac{\partial g}{\partial\Delta}-\frac{\partial^2 g}{\partial\theta\partial\Delta}\right)\sigma_\theta\sigma_{\Delta\theta}(t) \qquad (81)
\end{aligned}
$$

The FOC with respect to α gives $\alpha=\frac{e_\theta(t)\theta+e_\Delta(t)\Delta-\frac{\partial g}{\partial\theta}\varrho_\theta(t)\sigma_\theta-\frac{\partial g}{\partial\Delta}\sigma_{Q\Delta}(t)}{r\sigma_P(t)}$. Under the guessed form of $g(t,\theta,\Delta)$, substituting expressions in (80) yields: $\alpha_t=\alpha_\theta(t)\,\theta_t+\alpha_\Delta(t)\,\Delta_t$, where

$$
\begin{aligned}
\alpha_\theta(t) &= \frac{1}{r\sigma_P(t)}\left[e_\theta(t)-\varrho_\theta(t)\,\sigma_\theta g_{\theta\theta}(t)-\sigma_{Q\Delta}(t)\,g_{\theta\Delta}(t)\right], \qquad (82)\\
\alpha_\Delta(t) &= \frac{1}{r\sigma_P(t)}\left[e_\Delta(t)-\varrho_\theta(t)\,\sigma_\theta g_{\theta\Delta}(t)-\sigma_{Q\Delta}(t)\,g_{\Delta\Delta}(t)\right]. \qquad (83)
\end{aligned}
$$

Similar to informed investor's problems defined in (30), the uninformed investor's optimization problem is characterized as

$$V\left(t,W_t^u,\tilde{\theta}_t\right) = \max_{\beta_t,C_t^u} \tilde{\mathbb{E}}\left[\int_0^{T-t} -e^{-\rho s - C_{t+s}^u} ds + V\left(T,W_T^u,\tilde{\theta}_T\right)\right], \tag{84}$$
$$s.t.\ dW_t^u = \left(W_t^u r - C_t^u\right)dt + \beta_t\left\{\left[e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)\, d\tilde{B}_{D,t} + \sigma_\xi(t)\left[1+\phi_\Delta(t)\nu(t)\right]d\tilde{B}_{\xi,t}\right\},$$

and the state variable $\tilde{\theta}_t$ satisfies (62). The HJB equation is

$$\rho V = -e^{-C^u} + V_t + V_W\left[rW^u - C^u + \beta e_\theta(t)\tilde{\theta}\right] + \frac{1}{2}V_{WW}\beta^2\sigma_P(t) + \beta V_{W\theta}\sigma_{Q\theta}(t) - V_\theta a\tilde{\theta} + \frac{1}{2}V_{\theta\theta}\sigma_{\theta\theta}(t), \tag{85}$$

where $\sigma_{Q\theta}(t) = \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}\varrho_D(t) + \frac{1}{\phi_\theta(t)}\sigma_\xi^2(t)\left(\nu(t)\phi_x(t)-1\right)\left[1+\phi_\Delta(t)\nu(t)\right]$ and $\sigma_{\theta\theta}(t) = \left[\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}\right]^2 + \left[\frac{1}{\phi_\theta(t)}\sigma_\xi(t)\left(\nu(t)\phi_x(t)-1\right)\right]^2$. Furthermore, conjecture the uninformed investor's value function would be of the form (74) , where $f\left(t,\tilde{\theta}\right)$ satisfies (76). Substituting the guessed forms into HJB, the FOC with respect to C^u is: $rW^u - C^u = \ln r - f\left(t,\tilde{\theta}\right)$, and β_t gives: $\beta_t = \beta_\theta(t)\tilde{\theta}_t$, where

$$\beta_\theta(t) = \frac{1}{r\sigma_P(t)}\left[e_\theta(t) - \sigma_{Q\theta}(t) f_{\theta\theta}(t)\right]. \tag{86}$$

Finally, substituting the optimal policy functions back into the HJB equations, and matching coefficients of the value functions would give the ODEs system of two types of investors' value function coefficients stated in Lemma 4. □

After obtaining the optimal portfolio holdings and solved value functions, one could finally pin down the solutions to the postulated pricing function coefficients. By substituting equations (82), (83) and (86) back into the market clearing conditions (36) and (37), one would obtain the ODEs for pricing coefficients $\phi_\theta(t)$ and $\phi_\Delta(t)$. The following lemma summarizes the result.

Lemma 5. *The ODEs for $\phi_\theta(t)$ and $\phi_\Delta(t)$ can be characterized as follows*

$$\phi_\theta'(t) = (a+r)\phi_\theta(t) - r\sigma_P(t) - (1-\omega)\left[\varrho_\theta(t)\sigma_\theta g_{\theta\theta}(t) + \sigma_{Q\Delta}(t) g_{\theta\Delta}(t)\right] - \omega\sigma_{Q\theta}(t) f_{\theta\theta}(t), \tag{87}$$
$$\phi_\Delta'(t) = \left(a_\Delta(t)+r\right)\phi_\Delta(t) - \varrho_\theta(t)\sigma_\theta g_{\theta\Delta}(t) - \sigma_{Q\Delta}(t) g_{\Delta\Delta}(t)$$
$$-\left[\frac{\omega}{1-\omega}r\sigma_P(t) + \omega\left[\varrho_\theta(t)\sigma_\theta g_{\theta\theta}(t) + \sigma_{Q\Delta}(t) g_{\theta\Delta}(t) - \sigma_{Q\theta}(t) f_{\theta\theta}(t)\right]\right]\frac{\phi_x(t)}{\phi_\theta(t)}. \tag{88}$$

5.4 Proof for Equilibrium Conditions on the Boundary

Solving for Boundary Conditions for Value Function Coefficients The boundary conditions for value functions coefficients and price sensitivities can be summarized as follows:

Lemma 6. *At the pre-determined announcement T, the boundary conditions for the informed* *investor's value function coefficients could be characterized by*

$$\begin{aligned} g(T) - g(0) &= 0, \quad g_{\theta\theta}(T) - g_{\theta\theta}(0) = \frac{[\phi_\theta(T) - \phi_\theta(0)]^2}{\hat{q}_T \bar{\phi}_x^2}, \\ g_{\Delta\Delta}(T) &= \frac{\phi_\Delta^2(T)}{\hat{q}_T \bar{\phi}_x^2}, \quad g_{\theta\Delta}(T) = \frac{\phi_\Delta(T)[\phi_\theta(T) - \phi_\theta(0)]}{\hat{q}_T \bar{\phi}_x^2}, \end{aligned} \tag{89}$$

and the boundary conditions for the uninformed investor's value function coefficients could be characterized by

$$\begin{aligned} f(T) - f(0) &= \frac{1}{2} ln\left(\phi_\theta^2(T) + f_{\theta\theta,0}\phi_x^2(T)\tilde{q}_T\right) - ln\phi_\theta(T), \\ f_{\theta\theta}(T) &= \frac{\phi_{\theta,T}^2\left[f_{\theta\theta,0}\left(\tilde{q}_T\left(\phi_{x,T} - \bar{\phi}_x\right)^2 + \hat{q}_T\bar{\phi}_x^2\right) + \left(\phi_{\theta,0} - \phi_{\theta,T}\right)^2\right]}{\tilde{q}_T\left[f_{\theta\theta,0}\hat{q}_T\phi_{x,T}^2\bar{\phi}_x^2 + \left(\phi_{\theta,0}\phi_{x,T} - \phi_{\theta,T}\bar{\phi}_x\right)^2\right] + \hat{q}_T\phi_{\theta,T}^2\bar{\phi}_x^2}. \end{aligned} \tag{90}$$

Proof. First, I derive boundary conditions for the informed investor's value function coefficients. The informed investor's optimization problem at the boundary can be written as

$$\begin{aligned} -e^{-rW^{i-} - g(T,\theta_T,\Delta_T)} &= \max_{\alpha_T}\left\{-\hat{\mathbb{E}}_T\left[e^{-rW^{i+} - g(0,\theta_T,0)}\right]\right\} \\ &= e^{-rW^{i-}}\max_{\alpha_T}\left\{-\hat{\mathbb{E}}_T\left[e^{-r\alpha_T\left(P_T^+ - P_T^-\right) - g(0,\theta_T,0)}\right]\right\}, \end{aligned} \tag{91}$$

where $x_T \sim \mathcal{N}(\hat{x}_T, \hat{q}_T)$. Solving the exponent part within the expectation operator yields: $-r\alpha_T\left(P_T^+ - P_T^-\right) - g(0, \theta_T, 0) = -\Phi_0 - \Phi_1 x_T$, where $\Phi_0 = r\alpha_T\{-[\phi_\theta(0) - \phi_\theta(T)]\theta_T - \bar{\phi}_x\hat{x}_T + \phi_\Delta(t)\Delta_T\} + g(0) + \frac{1}{2}g_{\theta\theta}(0)\theta_T^2$, $\Phi_1 = r\alpha_T\bar{\phi}_x$. Then

$$\hat{\mathbb{E}}_T\left[e^{-r\alpha_T\left(P_T^+ - P_T^-\right) - g(0,\theta_T,0)}\right] = e^{-\Phi_0 - \left(\Phi_1\hat{x}_T - \frac{1}{2}\Phi_1^2\hat{q}_T\right)} = e^{Term^i},$$

where $Term^i = -r\alpha_T\{-[\phi_\theta(0) - \phi_\theta(T)]\theta_T + \phi_\Delta(t)\Delta_T\} - g(0) - \frac{1}{2}g_{\theta\theta}(0)\theta_T^2 + \frac{1}{2}r^2\alpha_T^2\bar{\phi}_x^2\hat{q}_T$. Optimization implies

$$\alpha_T = \alpha_\theta(T)\theta_T + \alpha_\Delta(T)\Delta_T, \tag{92}$$

where

$$\alpha_\theta(T) = \frac{\phi_\theta(T) - \phi_\theta(0)}{r\bar{\phi}_x^2\hat{q}_T}, \text{ and } \alpha_\Delta(T) = \frac{\phi_\Delta(T)}{r\bar{\phi}_x^2\hat{q}_T}. \tag{93}$$

Therefore, $g(T, \theta_T, \Delta_T) = -Term^i$ gives

$$\begin{aligned} & g(T) + \frac{1}{2}g_{\theta\theta}(T)\theta_T^2 + \frac{1}{2}g_{\Delta\Delta}(T)\Delta_T^2 + g_{\theta\Delta}(T)\theta_T\Delta_T \\ = & \frac{[\phi_\Delta(T)\Delta_T + (\phi_\theta(T) - \phi_\theta(0))\theta_T]^2}{\hat{q}_T\bar{\phi}_x^2} + \frac{1}{2}g_{\theta\theta}(0)\theta_T^2 + g(0). \end{aligned} \tag{94}$$

Matching the coefficients yields the boundary conditions for the uninformed investors' value functions summarized in Lemma 6.

Second, I derive boundary conditions for the uninformed investor's value function coefficients. The uninformed investor's optimization problem at the boundary is

$$\begin{aligned} -e^{-rW^{u-}-f\left(T,\tilde{\theta}_T\right)} &= \max_{\beta_T}\left\{\tilde{\mathbb{E}}_T\left[-e^{-rW_T^{u+}-f(0,\theta_T)}\right]\right\} \\ &= e^{-rW^{u-}}\max_{\beta_T}\tilde{\mathbb{E}}_T\left[-e^{-r\beta_T\left(P_T^+-P_T^-\right)-f(0,\theta_T)}\right], \end{aligned} \tag{95}$$

where $\begin{pmatrix} x_T \\ \theta_T \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \tilde{x}_T \\ \tilde{\theta}_T \end{pmatrix}, \begin{pmatrix} \hat{q}_T+\tilde{q}_T & \frac{\phi_x(t)}{\phi_\theta(T)}\tilde{q}_T \\ \frac{\phi_x(t)}{\phi_\theta(T)}\tilde{q}_T & \frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T \end{pmatrix}\right)$, in which I use the variance-covariance relationship derived in equation (63). Solving the exponent part within the expectation operator gives: $-r\beta_T\left(P_T^+-P_T^-\right)-f\left(0,\theta_T\right)=\Psi_0+\Psi_1 x_T+\Psi_2\theta_T-\frac{1}{2}f_{\theta\theta}\left(0\right)\theta_T^2$, where $\Psi_0=r\beta_T\left[\bar{\phi}_x\tilde{x}_T-\phi_\theta\left(T\right)\tilde{\theta}_T\right]-f\left(0\right)$, $\Psi_1=-r\beta_T\bar{\phi}_x$, $\Psi_2=r\beta_T\phi_\theta\left(0\right)$. Given $\phi_\theta\left(t\right)>0$, log multivariate normal distribution implies

$$\tilde{\mathbb{E}}_T\left[e^{-r\beta_T\left(P_T^+-P_T^-\right)-f(0,\theta_T)}\right] = \frac{\phi_{\theta,T}}{\sqrt{\phi_{\theta,T}^2+f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T}}e^{\Psi_0+\frac{\bar{\Psi}}{2\left(\phi_{\theta,T}^2+f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T\right)}} = e^{Term^u}, \tag{96}$$

where $Term^u=\ln\phi_\theta\left(T\right)-\frac{1}{2}\ln\left(\phi_\theta^2\left(T\right)+f_{\theta\theta,0}\phi_x^2\left(T\right)\tilde{q}_T\right)+\Psi_0+\frac{\bar{\Psi}}{2\left(\phi_{\theta,T}^2+f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T\right)}$, and

$$\begin{aligned} \bar{\Psi} &= r^2\beta_T^2\left[\tilde{q}_T\left(f_{\theta\theta,0}\hat{q}_T\phi_{x,T}^2\bar{\phi}_x^2+\left(\phi_{\theta,0}\phi_{x,T}-\phi_{\theta,T}\bar{\phi}_x\right)^2\right)+\phi_{\theta,T}^2\hat{q}_T\bar{\phi}_x^2\right]-f_{\theta\theta,0}\phi_{\theta,T}^2\tilde{\theta}_T^2 \\ &\quad +2r\beta_T\left[\phi_{\theta,T}\tilde{\theta}_T\left(f_{\theta\theta,0}\phi_{x,T}\bar{\phi}_x\tilde{q}_T+\phi_{\theta,0}\phi_{\theta,T}\right)-\bar{\phi}_x\tilde{x}_T\left(f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T+\phi_{\theta,T}^2\right)\right]. \end{aligned} \tag{97}$$

The FOC with respect to β_T gives

$$\beta_T = \beta_\theta\left(T\right)\tilde{\theta}_T=\beta_\theta\left(T\right)\theta_T-\beta_\theta\left(T\right)\frac{\phi_{x,T}}{\phi_{\theta,T}}\Delta_T, \tag{98}$$

where the second equality uses equation (13) to replace $\tilde{\theta}_T$, and

$$\beta_\theta\left(T\right)=\frac{\phi_{\theta,T}\left[f_{\theta\theta,0}\phi_{x,T}\tilde{q}_T\left(\phi_{x,T}-\bar{\phi}_x\right)+\phi_{\theta,T}\left(\phi_{\theta,T}-\phi_{\theta,0}\right)\right]}{r\tilde{q}_T\left[f_{\theta\theta,0}\hat{q}_T\phi_{x,T}^2\bar{\phi}_x^2+\left(\phi_{\theta,0}\phi_{x,T}-\phi_{\theta,T}\bar{\phi}_x\right)^2\right]+r\hat{q}_T\phi_{\theta,T}^2\bar{\phi}_x^2}. \tag{99}$$

Substituting this into $f\left(T,\tilde{\theta}_T\right)=-Term^u$ and matching the coefficients completes the proof for Lemma 6. In addition, combining the ODEs system in Lemma 5, Figure 7 in Internet Appendix B displays the calibrated time-varying value functions coefficients. □

Solving for boundary conditions for Time-Varying Price Sensitivities Note that market clearing requires: $(1-\omega)\,\alpha_T+\omega\beta_T=\theta_T$. This implies

$$(1-\omega)\,\alpha_\Delta\left(T\right)-\omega\beta_\theta\left(T\right)\frac{\phi_{x,T}}{\phi_{\theta,T}} = 0, \tag{100}$$

$$(1-\omega)\,\alpha_\theta\left(T\right)+\omega\beta_\theta\left(T\right) = 1. \tag{101}$$

Substituting expressions in equations (93) and (99) eventually pins down the boundary conditions for the pricing function coefficients. The following lemma summarizes the result.

Lemma 7. *At the pre-determined announcement T, the equilibrium pricing function coefficients satisfy*[18]

$$\begin{aligned} \phi_\Delta(T) &= \bar{\phi}_x - \frac{(1-\omega)\bar{\phi}_x\phi_{\theta,T}}{(1-\omega)\phi_{\theta,0} + r\bar{\phi}_x^2\hat{q}_T}, \qquad (102) \\ \phi_\theta(T) &= \frac{1}{2}\tilde{q}_T\left(\phi_{\Delta,T} - \bar{\phi}_x\right)^2 \left\{\frac{r\omega\bar{\phi}_x^2\hat{q}_T}{(1-\omega)\phi_{\Delta,T}\left(\bar{\phi}_x - \phi_{\Delta,T}\right)\tilde{q}_T} \right. \\ &\left. + \frac{\sqrt{r^2\omega^2\hat{q}_T^2\bar{\phi}_x^4 - 4(1-\omega)\phi_{\Delta,T}^2\left[r^2\hat{q}_T\bar{\phi}_x^2 + (1-\omega)f_{\theta\theta,0}\right]\tilde{q}_T}}{(1-\omega)\phi_{\Delta,T}\left(\bar{\phi}_x - \phi_{\Delta,T}\right)\tilde{q}_T}\right\}. \qquad (103) \end{aligned}$$

Note that the above boundary conditions imply: $\phi_\theta(T) - \phi_\theta(0) = \frac{r\bar{\phi}_x^2}{1-\omega}\hat{q}_T - \frac{\phi_{\Delta,T}\phi_{\theta,T}}{\bar{\phi}_x - \phi_{\Delta,T}}$.

5.5 Proof for Pricing Error Predictability

Proof for Lemma 3 When $\phi_\theta(t)$ is continuous, at the announcement T, $\phi_\theta(0) = \phi_\theta(T) = \phi_\theta$. Using (11), pricing errors realized on announcement can be written as $P_T^+ - P_T^- = \bar{\phi}_x(x_T - \tilde{x}_T) - \phi_\theta\left(\theta_T - \tilde{\theta}_T\right)$. Note that both $x_T - \tilde{x}_T$ and $\theta_T - \tilde{\theta}_T$ are errors of rational Bayesian beliefs, therefore, they cannot be predicted by price reactions to revisions, $P_{t+\delta} - P_t = \phi_D(D_{t+\delta} - D_t) - \phi_\theta\left(\tilde{\theta}_{t+\delta} - \tilde{\theta}_t\right) + \bar{\phi}_x(\tilde{x}_{t+\delta} - \tilde{x}_t)$, which is adapted to the uninformed investor's information set (i.e., $\mathrm{Cov}_t(P_{t+\delta} - P_t, x_T - \tilde{x}_T) = 0$ and $\mathrm{Cov}_t(P_{t+\delta} - P_t, \theta_T - \tilde{\theta}_T) = 0$).

Proof for Proposition 2 When $\phi_\theta(t)$ is time-varying, from equation (42), $P_T^+ - P_T^-$ has an extra term $[\phi_\theta(T) - \phi_\theta(0)]\tilde{\theta}_T$. In order to show Proposition 2, one needs to show $\mathrm{Cov}_t\left(P_{t+\delta} - P_t, \tilde{\theta}_T\right) < 0$, which is equivalent as showing $\mathrm{Cov}_t\left(P_{t+\delta} - \mathbb{E}_t[P_{t+\delta}], \tilde{\theta}_{t+\delta} - \mathbb{E}_t\left[\tilde{\theta}_{t+\delta}\right]\right) < 0$. First, $\tilde{x}_t$, $\tilde{\theta}_t$ and D_t can be written as integrals of Brownian motions by applying stochastic integration using equations (15), (62) and (52)

$$\begin{aligned} \tilde{x}_{t+\delta} &= e^{-b\delta}\tilde{x}_t + \left(1 - e^{-b\delta}\right)\bar{x} + \int_0^\delta e^{-b(\delta-z)}\left(\frac{\hat{q}_{t+z} + \tilde{q}_{t+z}}{\sigma_D}d\tilde{B}_{D,t+z} + \nu_{t+z}\sigma_{\xi,t+z}d\tilde{B}_{\xi,t+z}\right), \qquad (104) \\ \tilde{\theta}_{t+\delta} &= e^{-a\delta}\tilde{\theta}_t + \int_0^\delta e^{-a(\delta-z)}\left(\frac{\phi_{x,t+z}}{\phi_{\theta,t+z}}\frac{\tilde{q}_{t+z}}{\sigma_D}d\tilde{B}_{D,t+z} + \frac{1}{\phi_{\theta,t+z}}\sigma_{\xi,t+z}\left(\nu_{t+z}\phi_{x,t+z} - 1\right)d\tilde{B}_{\xi,t+z}\right) \qquad (105) \\ D_{t+\delta} &= e^{-\delta}\left[D_t + \int_0^\delta e^z\tilde{x}_{t+z}dz + \int_0^\delta e^z\sigma_D d\tilde{B}_{D,t+z}\right]. \qquad (106) \end{aligned}$$

[18] Note that the solution to the market clearing condition is $f_{\theta\theta}(0) = \frac{r\hat{q}_T\bar{\phi}_x^2}{1-\omega}\left[\frac{\omega\phi_\theta(T)}{\tilde{q}_T\left(\bar{\phi}_x - \phi_\Delta(T)\right)\phi_\Delta(T)} - r\right] - \frac{\phi_\theta^2(T)}{\tilde{q}_T\left(\bar{\phi}_x - \phi_\Delta(T)\right)^2}$, where $\phi_\theta(T)$ has two roots. Imposing the condition that $\phi_\theta(0) < \phi_\theta(T)$, one could exclude one root and obtain the unique solution.

Then substitute the expression of $\tilde{x}_{t+\delta}$ into $\int_0^\delta e^z \tilde{x}_{t+z} dz$ gives

$$\int_0^\delta e^z \tilde{x}_{t+z} dz = X + \int_0^\delta e^s \int_0^s e^{-b(s-z)} \frac{\hat{q}_{t+z} + \tilde{q}_{t+z}}{\sigma_D} d\tilde{B}_{D,t+z} ds + \int_0^\delta e^s \int_0^s e^{-b(s-z)} \nu_{t+z} \sigma_{\xi,t+z} d\tilde{B}_{\xi,t+z} ds, \quad (107)$$

where $X = \int_0^\delta e^z \left(e^{-bz}\tilde{x}_z + \left(1 - e^{-bz}\right)\bar{x}\right) dz$. Note here, one only needs to focus on the Brownian motion terms, as $P_{t+\delta} - \mathbb{E}_t\left[P_{t+\delta}\right]$ only has the Brownian motion terms. Apply Fubini's Theorem,

$$\int_0^\delta e^s \left(\int_0^s e^{-b(s-z)} \frac{\hat{q}_{t+z} + \tilde{q}_{t+z}}{\sigma_D} d\tilde{B}_{D,t+z}\right) ds = \int_0^\delta e^{bz} \left(\int_z^\delta e^{(1-b)s} \frac{\hat{q}_{t+z} + \tilde{q}_{t+z}}{\sigma_D} ds\right) d\tilde{B}_{D,t+z}, \quad (108)$$

$$\int_0^\delta e^s \left(\int_0^s e^{-b(s-z)} \nu_{t+z}\sigma_{\xi,t+z} d\tilde{B}_{\xi,t+z}\right) ds = \int_0^\delta e^{bz} \left(\int_z^\delta e^{(1-b)s} \nu_{t+z}\sigma_{\xi,t+z} ds\right) d\tilde{B}_{\xi,t+z}. \quad (109)$$

Therefore, denote $\tau_D(z) = \int_z^\delta e^{(1-b)s} \frac{\hat{q}_{t+z}+\tilde{q}_{t+z}}{\sigma_D} ds > 0$,

$$D_{t+\delta} = \mathbb{E}_t\left[D_{t+\delta}\right] + e^{-\delta}\int_0^\delta \left[e^z \sigma_D + \tau_D(z)\right] d\tilde{B}_{D,t+z} + e^{-\delta}\int_0^\delta \tau_\xi(z)\, d\tilde{B}_{\xi,t+z}, \quad (110)$$

where $\tau_\xi(z) = e^{bz}\int_z^\delta e^{(1-b)s}\nu_{t+z}\sigma_{\xi,t+z} ds = e^{bz}\frac{1}{1-b}\nu_{t+z}\sigma_{\xi,t+z}\left(e^{(1-b)\delta} - e^{(1-b)z}\right)$.

Second, one can write

$$P_{t+\delta} - \mathbb{E}_t\left[P_{t+\delta}\right] = \int_0^\delta \sigma_{P,D}(t+z)\, d\tilde{B}_{D,t+z} + \int_0^\delta \sigma_{P,\xi}(t+z)\, d\tilde{B}_{\xi,t+z}, \quad (111)$$

$$\tilde{\theta}_{t+\delta} - \mathbb{E}_t\left[\tilde{\theta}_{t+\delta}\right] = \int_0^\delta \sigma_{\theta,D}(t+z)\, d\tilde{B}_{D,t+z} + \int_0^\delta \sigma_{\theta,\xi}(t+z)\, d\tilde{B}_{\xi,t+z}, \quad (112)$$

where $\sigma_{\theta,D}(t+z) = e^{-a(\delta-z)}\frac{\phi_{x,t+z}}{\phi_{\theta,t+z}}\frac{\tilde{q}_{t+z}}{\sigma_D} > 0$, $\sigma_{\theta,\xi}(t+z) = e^{-a(\delta-z)}\frac{1}{\phi_{\theta,t+z}}\sigma_{\xi,t+z}\left(\nu_{t+z}\phi_{x,t+z} - 1\right)$, $\sigma_{P,D}(t+z) = \phi_D e^{-\delta}\left(e^z\sigma_D + \tau_D(z)\right) + \bar{\phi}_x e^{-b(\delta-z)}\frac{\hat{q}_{t+z}+\tilde{q}_{t+z}}{\sigma_D} - \phi_\theta(t+\delta)\,\sigma_{\theta,D}(t+z)$, and $\sigma_{P,\xi}(t+z) = \phi_D e^{-\delta}\tau_\xi(z) + \bar{\phi}_x e^{-b(\delta-z)}\nu_{t+z}\sigma_{\xi,t+z} - \phi_\theta(t+\delta)\,\sigma_{\theta,\xi}(t+z)$. Finally, compute:

$$\begin{aligned} \mathrm{Cov}_t\left[P_{t+\delta}, \tilde{\theta}_{t+\delta}\right] &= \mathrm{Cov}_t\left(P_{t+\delta} - \mathbb{E}_t\left[P_{t+\delta}\right], \tilde{\theta}_{t+\delta} - \mathbb{E}_t\left[\tilde{\theta}_{t+\delta}\right]\right) \\ &= \int_0^\delta \sigma_{P,D}(t+z)\,\sigma_{\theta,D}(t+z)\, dz + \int_0^\delta \sigma_{P,\xi}(t+z)\,\sigma_{\theta,\xi}(t+z)\, dz. \end{aligned} \quad (113)$$

Denote $\Phi(\delta) = \mathrm{Cov}_t\left[P_{t+\delta}, \tilde{\theta}_{t+\delta}\right]$, then $\Phi(0) = 0$, and $\Phi'(\delta) = \sigma_{P,D}(t+\delta)\,\sigma_{\theta,D}(t+\delta) + \sigma_{P,\xi}(t+\delta)\,\sigma_{\theta,\xi}(t+\delta)$. Then obviously a sufficient condition for $\mathrm{Cov}_t\left[P_{t+\delta}, \tilde{\theta}_{t+\delta}\right] < 0$ is

$$\sigma_{P,D}(t)\,\sigma_{\theta,D}(t) + \sigma_{P,\xi}(t)\,\sigma_{\theta,\xi}(t) < 0 \quad (*)$$

for all t.

5.6 Implied Volatility

In order to compute the forward looking implied variance $Var_0[P_t - P_0] = Var_0[P_t]$, I first consider the case in which $t < T$ and solve the three components in the pricing function separately. From equation (104), $\tilde{x}_t$ can be written as

$$\tilde{x}_t = \left(1 - e^{-bt}\right)\bar{x} + e^{-bt}\int_0^t e^{bz}\left(\frac{\hat{q}_z + \tilde{q}_z}{\sigma_D}d\tilde{B}_{D,z} + \nu(z)\,\sigma_\xi(z)\,d\tilde{B}_{\xi,z}\right). \tag{114}$$

Therefore, with an abuse of notation, I use $DF[X]$ to denote the diffusion part of X,

$$DF\left[\bar{\phi}_x\tilde{x}_t\right] = \bar{\phi}_x\int_0^t e^{b(z-t)}\frac{\hat{q}_z + \tilde{q}_z}{\sigma_D}d\tilde{B}_{D,z} + \bar{\phi}_x\int_0^t e^{b(z-t)}\nu(z)\,\sigma_\xi(z)\,d\tilde{B}_{\xi,z}. \tag{115}$$

Secondly, from equation (106), D_t can be solved as

$$D_t = e^{-t}\left(D_0 + \int_0^t e^z\tilde{x}_z dz + \int_0^t e^z\sigma_D d\tilde{B}_{D,z}\right). \tag{116}$$

wherein the term

$$\int_0^t e^u\tilde{x}_u du = \int_0^t e^{(1-b)u}\int_0^u\left\{e^{bz}(b\bar{x})\,dz + \int_0^u e^{bz}\frac{\hat{q}_z + \tilde{q}_z}{\sigma_D}d\tilde{B}_{D,z} + \int_0^u e^{bz}\nu(z)\,\sigma_\xi(z)\,d\tilde{B}_{\xi,z}\right\}du. \tag{117}$$

so that the diffusion part

$$\begin{aligned}\int_0^t\int_0^u e^{bz+(1-b)u}\frac{\hat{q}_z + \tilde{q}_z}{\sigma_D}d\tilde{B}_{D,z}du &= \int_0^t\int_z^t e^{bz+(1-b)u}\frac{\hat{q}_z + \tilde{q}_z}{\sigma_D}dud\tilde{B}_{D,z}\\ &= \frac{1}{(1-b)\,\sigma_D}\int_0^t\left[e^{(1-b)t+bz} - e^z\right](\hat{q}_z + \tilde{q}_z)\,d\tilde{B}_{D,z}.\end{aligned} \tag{118}$$

Similarly,

$$\int_0^t\int_0^u e^{bz+(1-b)u}\nu(z)\,\sigma_\xi(z)\,d\tilde{B}_{\xi,z}du = \frac{1}{(1-b)}\int_0^t\left[e^{(1-b)t+bz} - e^z\right]\nu(z)\,\sigma_\xi(z)\,d\tilde{B}_{\xi,z}. \tag{119}$$

Therefore, the diffusion part of D_t is

$$DF[\phi_D D_t] = \phi_D\int_0^t\left[\left(e^{b(z-t)} - e^{z-t}\right)\frac{\hat{q}_z + \tilde{q}_z}{(1-b)\,\sigma_D} + e^{z-t}\sigma_D\right]d\tilde{B}_{D,z} + \phi_D\int_0^t\left[e^{b(z-t)} - e^{z-t}\right]\frac{\nu(z)\,\sigma_\xi(z)}{1-b}d\tilde{B}_{\xi,z} \tag{120}$$

Finally, from equation (105),

$$DF\left[-\phi_\theta(t)\,\tilde{\theta}_t\right] = -\phi_\theta(t)\left\{\int_0^t e^{a(z-t)}\frac{\phi_x(z)}{\phi_\theta(z)}\frac{\tilde{q}(z)}{\sigma_D}d\tilde{B}_{D,z} + \int_0^t e^{a(z-t)}\left[\phi_x(z)\,\nu(z) - 1\right]\frac{\sigma_\xi(z)}{\phi_\theta(z)}d\tilde{B}_{\xi,z}\right\}. \tag{121}$$

Summing up (115), (120), and (121), I can represent price in the form of

$$DF\left[P_t\right]=\int_0^t Term_D\left(z\right)d\tilde{B}_{D,z}+\int_0^t Term_\xi\left(z\right)d\tilde{B}_{\xi,z}, \tag{122}$$

where $Term_D\left(z\right)=\phi_D\left[\left(e^{b(z-t)}-e^{z-t}\right)\frac{\hat{q}_z+\tilde{q}_z}{(1-b)\sigma_D}+e^{z-t}\sigma_D\right]-\phi_\theta\left(t\right)e^{a(z-t)}\frac{\phi_x(z)}{\phi_\theta(z)}\frac{\tilde{q}_z}{\sigma_D}+\bar{\phi}_x e^{b(z-t)}\frac{\hat{q}_z+\tilde{q}_z}{\sigma_D}$ and $Term_\xi\left(z\right)=\phi_D\left[e^{b(z-t)}-e^{z-t}\right]\frac{\nu(z)\sigma_\xi(z)}{1-b}-\phi_\theta\left(t\right)e^{a(z-t)}\left[\phi_x\left(z\right)\nu\left(z\right)-1\right]\frac{\sigma_\xi(z)}{\phi_\theta(z)}+\bar{\phi}_x e^{b(z-t)}\nu\left(z\right)\sigma_\xi\left(z\right)$. The variance can be computed as:

$$Var_0\left[P_t\right]=\int_0^t Term_D^2\left(z\right)dz+\int_0^t Term_\xi^2\left(z\right)dz. \tag{123}$$

Next, consider the general case of $Var_t\left[P_{t+\tau}\right]$. If $t+\tau<T$, that is, if computing implied variance within an announcement cycle, use the above formula. If $t+\tau>T$, first compute $Var_t\left[P_{T^-}\right]$ using the above formula and then compute $Var_{T^+}\left[P_{t+\tau}\right]$:

$$Var_t\left[P_{t+\tau}\right]=\int_t^{t+\tau} Term_D^2\left(z\right)dz+\int_t^{t+\tau} Term_\xi^2\left(z\right)dz. \tag{124}$$

Because the different components are independent, I can compute $Var_{T^-}\left[P_{T^+}-P_{T^-}\right]$ simply by adding up the two components together:

$$Var_{T^-}\left[P_{T^+}-P_{T^-}\right] = \bar{\phi}_x^2\left(\hat{q}_T+\tilde{q}_T\right)+\phi_\theta^2\left(0\right)\frac{\phi_x^2\left(T\right)}{\phi_\theta^2\left(T\right)}\tilde{q}_T-2\bar{\phi}_x\phi_\theta\left(0\right)\frac{\phi_x\left(T\right)}{\phi_\theta\left(T\right)}\tilde{q}_T, \tag{125}$$

where $P_{T^+}-P_{T^-}$ comes from equation (39). Therefore, the total implied variance is obtained by

$$\begin{aligned}&Var_t\left[P_{T^-}\right]+Var_{T^-}\left[P_{T^+}-P_{T^-}\right]+Var_{T^+}\left[P_{t+\tau}\right]\\ =&\int_t^{T^-} Term_D^2\left(z\right)dz+\int_t^{T^-} Term_\xi^2\left(z\right)dz+\int_{T^+}^{t+\tau} Term_D^2\left(z\right)dz+\int_{T^+}^{t+\tau} Term_\xi^2\left(z\right)dz\\ &+\bar{\phi}_x^2\left(\hat{q}_T+\tilde{q}_T\right)+\phi_\theta^2\left(0\right)\frac{\phi_x^2\left(T\right)}{\phi_\theta^2\left(T\right)}\tilde{q}_T-2\bar{\phi}_x\phi_\theta\left(0\right)\frac{\phi_x\left(T\right)}{\phi_\theta\left(T\right)}\tilde{q}_T\end{aligned} \tag{126}$$

5.7 Numerical Solutions

The numerical challenge is to solve the ten ordinary differential equations (ODEs) system subject to the boundary conditions defined as a combination of initial conditions, terminal conditions and the distance between the initial and terminal values. The ODEs are defined in equations (51), (61), (77), and (78), with the boundary conditions $\hat{q}\left(0\right)=0$, $\tilde{q}\left(0\right)=0$, and the rest given in (89), (90), (102) and (103). First, it is important to specify the initial guessed values close enough to the true values so that the system would converge by itself to the stationary values. In order to determine the initial guess, I first solve a time-invariant stationary model to obtain the initial conditions for those ten coefficients. Based on those initializations, I use Matlab build-in solver 'ode45' to solve an initial value problem. From the model intuitions, I have established that $\phi_\theta\left(t\right)$

and $f_{\theta\theta}(t)$ are positive and increasing functions, and $\phi_{\Delta}(t)$ is positive and decreasing with t. Using these conditions, I update the critical initial guess for $\phi_{\theta}(0)$, $\phi_{\Delta}(0)$ and $f_{\theta\theta}(0)$ whenever these conditions are violated. Then I use Matlab '*bvp4c*' solver to solve the ODEs system with the requested convergence accuracy of 1e-10. Finally, if the convergence accuracy is too high, I update the initial guess again until it satisfies.

It is easy to generate simulated time paths using the dynamic general equilibrium model. I use the simulation-based estimation methodology of indirect inference to calibrate the parameter values. Basically, I calculate the model implied unconditional moments based on simulated sample paths. The model is then evaluated based on how close the averaged estimated moments are from the actual data. The minimum distance/weighting matrix is used to test the null that the structural model is correctly specified. In this paper, the distance is simply defined as: $distance = \sum_{i=1}^{n}\left(\frac{z_{i,simulated}-z_{i,data}}{z_{i,data}}\right)^2$, where $z_{i,simulated}$ denotes the simulated moment and $z_{i,data}$ is the actual data. The calibrated parameters and their targets are illustrated in Section 4.

For each candidate parameter set I simulate 20,000 years, and use the final 18,000 years to compute population moments and the regression coefficients. The simulation results are robust if I vary the simulation years or seeds in generating random variables. Then I compute the distance based on the time path and update the parameters based on the distance. Finally, the calibrated values in Table 2 gives the minimum distance.

Figure 7 displays the calibrated time-varying value function coefficients.

Figure 7: Time Varying Value Function Coefficients