Large Shareholders and Financial Distress

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Abstract

Blockholders play a prominent role in distressed firms' access to finance. I develop a dynamic model of the interaction between these investors and distressed firms to examine blockholders' impact on efficiency and the distribution of value. The model captures key empirical facts on distressed equity issuances, including the provision of substantial discounts to large investors. Blockholders' impact on debt overhang problems is generically non-monotone. Whereas inefficiencies are exacerbated for intermediate levels of distress, they are alleviated in deep distress, when blocks are acquired in last-minute interventions. The paper proposes a novel set of modeling tricks that yield global solutions in environments with optimal default and learning, while only requiring the inversion of sparse matrices.

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1. Introduction

Large shareholdings by wealthy individuals and financial institutions are a pervasive empirical phenomenon. According to Holderness (2009), 96% of U.S. firms contain at least one shareholder who holds more than 5% of outstanding shares. A key channel differentiating blockholders from small shareholders is that large exposures to a firm alleviate free-rider problems among investors (see, e.g., Grossman and Hart, 1980, Shleifer and Vishny, 1986). In financial distress, these free-rider problems can be particularly consequential. Lacking sufficient incentives to acquire information about a firm's fundamental solvency, each individual small investor may refuse to provide funds when doing so is crucial for firm survival. In contrast, blockholders have larger incentives to be informed about a firm's conditions, and thus are likely better equipped to distinguish solvent from insolvent firms. Consistent with this advantage, firms in practice increasingly rely on large investors when raising external finance in financial distress, with a majority of funding being raised via private issuances of public equity (Park, 2011). The prominent investors involved in these transactions — often hedge funds or private equity firms — have, however, also attracted scrutiny, as firms' performance following their involvement appears to be poor on average (Hertzel, Lemmon, Linck, and Rees, 2002, Brophy, Ouimet, and Sialm, 2009, Park, 2011, Lim, Schwert, and Weisbach, 2019).

Motivated by these observations, I develop a tractable dynamic model of distressed firms' interactions with large shareholders that matches key empirical facts related to block acquisitions in financial distress. The framework facilitates counterfactual analyses that allow distilling blockholders' effect on other claimants and on inefficiencies associated with financial distress. These effects tend to be difficult to discern based on reduced-form analyses that are subject to selection effects and limited availability of relevant shocks. In particular, the model allows evaluating the impact of regulatory changes, such as increased regulatory hurdles for private transactions in distress. Moreover, the paper provides a methodological contribution by proposing a set of modeling tricks that ensure that solving a setting with heterogeneous agents, information acquisition, and free boundary problems associated with optimal default involves only inverting sparse matrices and yields exact global solutions.

In the presented model, large shareholders acquire information on the firm's earnings prospects as doing so improves future funding decisions and increases the value of their block positions. As a result, blockholders' decisions act as a signal and play a pivotal role in distressed firms' access to finance. Atomistic investors infer information from a blockholder's willingness to provide additional funds to a distressed firm, and optimally follow this investor's lead. For instance, when a large investor like Warren Buffet supports a firm in financial distress, this action is a strong signal to other investors, increasing their willingness to provide finance as well. Large shareholders thus change firms' default risk dynamics, creating an externality that affects total firm value and its distribution across claims.

In practice, large investors often acquire substantial stakes in distressed firms' private placements of public equity, which are also referred to as PIPEs. Consistent with empirical evidence, existing shareholders in the model favor issuing new equity to large investors at *discounted* prices when a firm requires new funds and existing shareholders are uncertain about firm solvency. Despite the discounted offer, involving a large investor can be beneficial, since atomistic investors can free-ride on a blockholder's information acquisition efforts going forward. The model endogenizes the incidence of PIPE transactions and reveals that these transactions are more likely to occur for low interest coverage ratios and in states with substantial uncertainty about the firm's future growth prospects, mirroring the findings of the empirical literature.[1]

While a blockholder's presence can be beneficial for small shareholders, the analysis reveals under what circumstances blockholders' involvement increases financial distress costs, which generically arise due to conflicts of interest between debt and equity holders.[2] Due to these

[1] Brophy, Ouimet, and Sialm (2009) document that hedge funds tend to finance companies that have poor fundamentals and pronounced informational frictions, and require substantial discounts.

[2] After all, if such conflicts were not present, high financial leverage could never interfere with firm value

conflicts, large shareholders' efforts might exacerbate financial distress costs by facilitating strategies that benefit equity holders at the expense of debt holders and total surplus. While the analysis reveals that blockholders' information production may indeed have this negative effect, it predicts that there is a clear non-monotone impact as a function of a firm's distance to default. Whereas inefficiencies are amplified for low to medium levels of distress, they are alleviated in deep distress, when large equity investors join the firm in "last-minute" rescue interventions.

Quantitatively, the calibrated model suggests that externalities of large shareholders' involvement can be substantial. For intermediate levels of distress, firm value is reduced by up to about 4%, whereas in deep distress, large shareholders can increase firm value by approximately 10%, with even higher values in recessions. All effects are amplified in recessions, since distress costs conditional on going into bankruptcy are calibrated to be higher in these times, following results of the existing macro and credit risk literature.[3] Correspondingly, any changes in default risk dynamics due to the presence of a blockholder are more consequential in recessions.

To understand why blockholders generically have a non-monotone effect on financial distress costs, consider first a firm that is so deeply indebted that it is effectively destined to default at the next debt payment date absent a large investor. In this case, involving a large investor who acquires information can provide positive or negative incremental information, but the implications of this information are asymmetric. If the blockholder uncovers negative information, default is merely confirmed. In contrast, when she uncovers positive information and chooses to inject more funds, other investors regain confidence and follow the blockholder. On average, the likelihood with which the firm enters bankruptcy thus declines, reducing the present value of associated inefficiencies.

maximization, as the firm could always readjust its leverage so as to avoid any direct or indirect costs of financial distress. See also Jensen and Meckling (1976) and Myers (1977).

[3] See, e.g., Caballero and Hammour (2005) and Eisfeldt and Rampini (2006) for evidence on greater frictions in reallocating capital in recessions. Chen (2010) shows the importance of cyclical bankruptcy frictions for explaining credit spreads.

Next, consider a firm that is in a better financial position, that is, current beliefs about its prospects are positive enough to warrant funding by investors in case it is needed to make an upcoming debt payment. In this case, the presence of a blockholder increases default risk at that payment date. The blockholder's incremental information introduces the possibility that negative information is uncovered, and now, this negative information is not irrelevant, but rather causes a firm to default at the next payment date that otherwise would not have defaulted. While this exercising of limited liability is optimal for equity holders, given the incremental information, it triggers bankruptcy costs that are borne by debt holders and that would have been avoided without the presence of the blockholder.

Due to this non-monotone effect, the model predicts that frictions that increase the hurdles for PIPE transactions can have positive implications for overall firm value. Such frictions may, for example, be due to transaction costs or legal and regulatory compliance requirements. The model reveals that these costs can ensure that new block acquisitions occur only in deep distress, where the efficiency-enhancing effects of large shareholders' actions dominate the value-destroying rent-seeking effects. At the same time, regulations should not impose prohibitively high costs, since last-minute rescue interventions do have a positive effect on total firm value.

Large shareholders' impact on debt values is shaped by both the non-monotone implications for total firm value and redistributional effects. Large shareholders' information facilitates the optimal exercising of equity holders' put option, in which debt investors hold a short position. That is, large shareholders' increased attention to firm conditions generates a redistribution of value from debt to equity holders. Incorporating the effect on both total surplus and its distribution, the calibrated model predicts that in booms, debt value changes exhibit an inverse hump-shaped pattern as a function of the interest coverage ratio. In these states, debt values are consistently negatively affected by the actions of a large shareholder. In contrast, in recessions, debt values are positively affected for deeply distressed firms with uncertain growth prospects, as large shareholders tend to reduce default risk for these firms. Involving a blockholder can therefore positively affect debt values in deep distress, in particular when the deadweight losses from bankruptcy are material.

The paper proposes a set of novel modeling tricks that ensure that a structural credit risk model in the Leland (1994) tradition remains tractable when introducing a blockholder, dynamic information acquisition, and learning. A key implication of these modeling choices is that both earnings and beliefs follow processes that feature stochastic lumpy adjustments (SLA). Granularity can be freely adjusted, nesting continuous adjustments as a limiting case. This modeling approach yields distributions and valuations in closed-form for any given policy function, side-stepping the need for simulations. Global solutions to the model are obtained by merely inverting sparse matrices. In the considered setting, global solutions are particularly valuable, as they reveal large shareholders' highly non-linear effects close to the default boundaries. Moreover, by nesting standard quantitative credit risk models as a limiting case and yielding solutions for conditional distributions, the model lends itself to both qualitative and quantitative analyses. Counterfactual analyses based on the calibrated model help overcome some of the challenges of typical reduced-form approaches, such as selection effects and limited availability of shocks related to frictions changing the incidence of block acquisitions.

Literature. Following Grossman and Hart (1980) and Shleifer and Vishny (1986), an influential theoretical literature has analyzed channels through which large shareholders affect corporate governance. In these studies, blockholders overcome free-rider problems and can alleviate conflicts of interest between shareholders and management (see, e.g., Maug, 1998, Edmans and Manso, 2011) but may also seek private benefits and harm small shareholders (see, e.g., Burkart, Gromb, and Panunzi, 1997, Pagano and Rell, 1998).[4] In contrast to this literature, I study large shareholders' externalities in financial distress, when conflicts of interest between debt and eq-

[4]See Edmans (2014) for a comprehensive review of theoretical and empirical papers in this literature. See also Brav, Jiang, Partnoy, and Thomas (2008) for evidence on channels through which large investors may affect conflicts of interest between shareholders and management, including proxy fights and shareholder proposals to replace management.

uity holders are generically most severe. My analysis focuses on these conflicts of interest rather than conflicts of interest between equity holders and management, which are not specific to financial distress. In fact, by having a disciplining effect on managers, high levels of leverage are typically seen as a channel mitigating the agency costs of free cash flow (Jensen, 1986). In my analysis, blockholders affect other investors' willingness to support the firm, which is needed to avert bankruptcy and associated inefficiencies. Through this channel, blockholders effectively shape the severity of financial distress costs.

In building on a structural credit risk model and examining conflicts of interest between debt and equity holders my paper is related to quantitative analyses of the agency costs of debt, such as those considering debt overhang problems (see, e.g., Moyen, 2007, Hackbarth, 2009). I contribute to this literature by analyzing the effects of large shareholders and their endogenous information acquisition. In contrast to risk shifting and debt overhang, blockholders' externalities on the agency costs of debt do not require changes in the composition of a firm's assets; rather they operate via their impact on strategic default decisions. Moreover, whereas the agency problems of risk shifting and debt overhang are generally predicted to increase with a firm's indebtedness, the analysis reveals that blockholders' impact on the agency costs of debt is generically non-monotone as a function of a firm's distance to default. These elements also differentiate my paper from the existing literature focusing on the relationship between capital structure and asset pricing (e.g., Hackbarth, Miao, and Morellec, 2006, Gomes and Schmid, 2010). Following the structural credit risk models of Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010), my model features business cycle fluctuations that play a quantitatively relevant role for valuations and default costs.

In addition, several papers in the credit risk literature feature notions of passive learning, but do not consider blockholders and their efficiency implications in the context of the strategic timing of default. Duffie and Lando (2001) study the implications of imperfect information for credit spreads, assuming segmented markets where bond investors cannot observe information available to equity holders directly, and receive instead only periodic and imperfect accounting reports. David (2008) features learning about aggregate dynamics and assumes the same default rules as in Merton (1974) in order to solve for prices. This assumption implies that a key channel of my paper — that optimal strategic timing decisions depend on the speed with which information is expected to arrive in the future — is absent. Similarly, this channel is absent in Johnson's (2004) analysis of a Merton (1974) type setting with noise.

2. The Model

The model is in continuous time. To facilitate the exposition, I will present the environment in two steps, following the principle of backward induction used in the model analysis: first, I consider a firm that already has a blockholder, that is, a block position was already established in the past. Second, I discuss the process by which a block acquisition can occur in the first place. I will make three specific modeling choices that, together, will ensure that solving the model involves only inverting sparse matrices. Throughout the exposition, I will highlight these three tricks. I will begin the model description by detailing the processes governing aggregate dynamics. Afterwards, I describe the market structure, technology, and the objectives of firms and large investors.

2.1. Aggregate Processes and Stochastic Discount Factor

Following much of the asset pricing literature and the existing structural credit risk literature, I introduce a persistent macroeconomic process that affects both firm dynamics and the dynamics of the stochastic discount factor. The existing structural credit risk literature has highlighted that the pricing of these persistent macroeconomic fluctuations significantly improves models' ability to capture empirical facts about the pricing of debt claims (see, e.g., Bhamra, Kuehn, and Strebulaev, 2010, Chen, 2010). Moreover, this process will allow the model to capture cyclical variation in distress costs, which yields relevant predictions for blockholders' differential impact on allocative efficiency over the business cycle.

Let Z denote this aggregate state that governs persistent, mean-reverting variation in the macroeconomic environment (e.g., booms vs. recessions). Z follows a continuous-time Markov chain that takes values in the discrete set Ω_Z and has the generator matrix Λ_Z, which collects transition rates between states $Z \in \Omega_Z$. Let $\mathbf{N}_{Z,t}$ denote a matrix that collects counting processes that keep track of all jumps between the states $Z \in \Omega_Z$. To balance increases in the off-diagonal elements, the diagonal elements of this matrix count down by one each time a given state Z is left for another state Z'. I also define the matrix $\mathbf{M}_{Z,t}$ collecting the compensated processes:

$$d\mathbf{M}_{Z,t} = d\mathbf{N}_{Z,t} - \Lambda_Z dt. \tag{1}$$

Throughout, for any given matrix $\mathbb{M}$, I will use the notation $\mathbb{M}(x)$ to indicate the x-th row of that matrix, and $\mathbb{M}(x, x')$ to represent the (x, x')-th element.

In addition to the persistent, mean-reverting process Z, I allow for a stochastic trend that scales all variables in the economy and can capture common growth in firm-specific and aggregate variables. Specifically, the state Y, which could represent aggregate consumption growth, follows a geometric Brownian motion:

$$\frac{dY_t}{Y_t} = \mu_Y(Z_t)dt + \sigma_Y(Z_t)dB_t. \tag{2}$$

Agents can observe the aggregate states Z and Y.

SDF dynamics. Let m denote the stochastic discount factor (SDF) reflecting agents' marginal utility. I specify a flexible process for m that can capture the pricing properties of a variety of benchmark asset pricing models. The SDF follows a Markov-modulated jump diffusion process:

$$\frac{dm(Z_t)}{m(Z_t)} = -r_f\left(Z_t\right)dt - \nu(Z_t)dB_t + \sum_{Z' \neq Z_t} (e^{\phi(Z_t,Z')} - 1)d\mathbf{M}_{Z,t}\left(Z_t, Z'\right), \tag{3}$$

where r_f denotes the risk free rate, ν is the price of risk for aggregate Brownian shocks, and $\phi(Z, Z')$ determines the jumps in m conditional on a change in the state Z. Let $\bar{\Lambda}_Z$ denote the generator matrix under the risk neutral measure, which collects the risk neutral transition rates $\bar{\Lambda}_Z(Z, Z') = e^{\phi(Z,Z')}\Lambda_Z(Z, Z')$.

2.2. The Firm

As commonly assumed in the structural credit risk literature, the earnings of a firm are split between contractual debt payments promised to debt holders and a dividend paid to equity holders, that is, the firm does not hold excess cash (see, e.g., Goldstein, Ju, and Leland, 2001, Hackbarth, Hennessy, and Leland, 2007, Strebulaev, 2007, Bhamra, Kuehn, and Strebulaev, 2010, Chen, 2010).[5] Throughout, management acts in the interest of existing shareholders. I denote the corporate tax rate by τ.

Earnings dynamics. The first unconventional modeling choice contributing to the model's tractability pertains to the specification of earnings dynamics. Contrary to standard structural credit risk models in the Leland (1994) tradition, earnings dynamics are primarily governed by stochastic lumpy adjustments.[6] Let $X_t \equiv Y_t \cdot e^{x_t} \geq 0$ denote the firm's before-interest

[5] Apart from tax implications, cash holdings would not affect the analysis if the firm could pay out cash at any time before default is triggered.

[6] These earnings dynamics can be endogenized with the SLA investment technology proposed in Binsbergen and Opp (2019). Yet, as firms in financial distress tend to have a Tobin's Q less than one and face material frictions when attempting to divest capital (due to asymmetric adjustment costs), there is typically not much action on the investment side for these firms.

earnings rate. Earnings are affected linearly by the aggregate trend factor Y_t and the firm-specific earnings state x_t. The state x_t follows the jump-process:

$$dx_t = \Delta_x \cdot (dN^+_{x,t} - dN^-_{x,t}), \tag{4}$$

where $N^+_{x,t}$ and $N^-_{x,t}$ denote Poisson processes keeping track of the number of upward and downward innovations to x_t. Given these dynamics, x_t takes values in a discrete set Ω_x, the elements of which constitute a grid with increments of size $\Delta_x > 0$,[7] and the earnings rate X_t then follows the jump-diffusion process:

$$\frac{dX_t}{X_t} = \mu_Y(Z_t)dt + \sigma_Y(Z_t)dB_t + (e^{\Delta_x} - 1)dN^+_{x,t} + (e^{-\Delta_x} - 1)dN^-_{x,t}, . \tag{5}$$

reflecting the effects of both trend growth dY/Y and firm-specific growth dx.[8]

This specification for earnings dynamics captures discrete shocks, as, for example, caused by the arrival of new sales transactions in practice. Moreover, as further discussed below, it nests, as a limiting case, the conventional specification where earnings follow a geometric Brownian motion (for $\Delta_x \downarrow 0$). More generally, any desired degree of granularity can be accommodated.

The arrival intensities of innovations to the state x are affected by a firm-specific state $\theta_t \in \Omega_\theta = \{h, l\}$ that is not directly observable. The possible θ-values h and l refer to a high-growth state and a low-growth state, respectively. At date $t = 0$, nature determines the value of θ, where h is drawn with probability $\hat{\pi}$, and l is drawn with complimentary probability $(1-\hat{\pi})$. Let $\lambda^+_x(\theta, Z)$ and $\lambda^-_x(\theta, Z)$ denote the Poisson arrival rates of upward and downward innovations to x, which depend on the hidden state θ and the observable aggregate state Z.

[7]This modeling approach for earnings is very flexible in that it can be adjusted to accommodate a range of possible jump sizes, reflecting for example both small and large shocks.

[8]The presence of the stochastic trend Y that scales all variables is not essential for the key results of the paper. The general modeling approach for the x-process is in principle sufficiently flexible to capture rich patterns in earnings dynamics.

The second essential modeling choice ensuring tractability is the assumption that the volatility of x does not contain incremental information about the hidden state θ. This is the case if the *total* Poisson arrival rate with which any adjustment to x occurs (either an upward or a downward adjustment) does not depend on the hidden state θ, although it may vary with the aggregate state Z or any other observable state or firm characteristic. Specifically, the total jump intensity is given by:

$$\lambda_x(Z) \equiv \lambda_x^+(\theta, Z) + \lambda_x^-(\theta, Z) \quad \text{for all } \theta, Z. \tag{6}$$

This specification implies that the mean and the volatility of adjustments to x are given by:

$$\mu_x(\theta, Z) = \Delta_x(\lambda_x^+(\theta, Z) - \lambda_x^-(\theta, Z)), \tag{7}$$

$$\sigma_x(Z) = \Delta_x\sqrt{\lambda_x(Z)}, \tag{8}$$

that is, as intended, the drift μ_x depends on the hidden state θ_t, whereas the volatility σ_x does not. As in models where earnings follow a geometric Brownian motion with hidden drift, the dynamics for x are fully described by the drifts $\mu_x(\theta, Z)$ and the volatilities $\sigma_x(Z)$; given these objects, the Poisson intensities $\lambda_x^+(\theta, Z)$ and $\lambda_x^-(\theta, Z)$ are uniquely pinned down, given any choice for Δ_x. Correspondingly, I will calibrate the model by directly choosing values for the drifts $\mu_x(\theta, Z)$ and the volatilities $\sigma_x(Z)$, and back out the associated Poisson intensities. Crucially, this second modeling choice will imply that beliefs about the state θ_t exhibit stochastic lumpy adjustments after earnings innovations are realized and stay constant otherwise.

It is worth noting that the restriction that volatility is not informative about the hidden state is an implicit assumption of all models where earnings follow a geometric Brownian motion with hidden drift. In those settings, if volatility were related to the hidden state, agents would instantaneously learn the state's value. Imposing this restriction in the considered environment with Poisson shocks has the additional desirable feature that learning remains imperfect even in the limiting case where the x-process approaches a geometric Brownian motion.[9]

Debt obligations. At date $t = 0$, the firm can issue debt obligations that involve lumpy payments to debt holders.[10] Payment dates associated with debt obligations arrive with Poisson intensity λ_C and require the firm to make lumpy payments of size $C_t = Y_t e^c$, where c is a constant. That is, the firm's required debt payments are indexed to the observable aggregate trend state Y, but not to the firm-specific earnings state x_t.[11] The expected rate of debt payments is then equal to $\lambda_C C_t$. It is convenient to introduce the state variable

$$\rho_t \equiv \log \left[\frac{X_t}{\lambda_C C_t} \right], \tag{9}$$

which can be interpreted as the log interest coverage ratio. Let $N_{C,t}$ denote the Poisson process that keeps track of the number of debt payments since date 0, and let $\delta_t \in \{0, 1\}$ denote equity holders' decision whether to make a debt payment at time t if one comes due at that time. In the event of default, debt holders recover a fraction $\alpha(Z)$ of the value of the firm's unlevered assets.

The structure for debt payments can capture any degree of lumpiness in debt payments. For high levels of λ_C, debt payments occur frequently, and the time to the next debt payment has a tight distribution, e.g., with 99% probability, the next payment might be due over the next 24 hours. In fact, in the limiting case $\lambda_C \to \infty$, payments are made continuously. Yet lumpy debt payments are in fact a plausible feature empirically. While the random nature of the exact timing

[9]Suppose we specify, $\lambda_x(Z) = \frac{\sigma_x(Z)^2}{\Delta_x^2}$ for some fixed value $\sigma_x(Z)$. Then, for $\Delta_x \searrow 0$, the x-process approaches a Brownian motion with volatility $\sigma_x(Z)$.

[10]Given this paper's focus on blockholders' impact in financial distress, I will abstract from the possibility of additional debt issuances at a later point in time, which would be suboptimal in distressed states. As in Chen (2010), such issuances would become optimal only when the firm's equity capitalization reaches an upper bound (i.e., when the firm has low leverage). Extending the model to account for such additional issuances would, however, be feasible.

[11]While the presence of the trend Y can simplify the model calibration, it is not an essential model component. In particular, we can set $Y_t = 1$, which implies that payments are not indexed to an aggregate variable.

of debts payments ensure tractability, it might also be interpreted as a reduced-form approach of capturing uncertainty regarding the ability to roll-over debt.

2.3. Blockholders

As noted above, I present blockholders' interactions with the firm in two steps. First, I describe the scenario where a firm already has a *legacy* blockholder that established its position in the past, possibly a long time before the firm entered financial distress. For example, founder family members may be such large blockholders. Second, I describe how a new investor can endogenously establish a block, which will tend to happen in financial distress.

2.3.1. Legacy Blockholder

Suppose the firm already has an existing blockholder holding a fraction $\omega > 0$ of the firm's equity. The blockholder maximizes the market value of her position accounting for the costs associated with information acquisition. The blockholder can produce incremental information on the firm's hidden state θ while she is "matched" with a firm. By incurring costs at a rate $I_t \geq 0$ the large shareholder's investigation yields her a private signal with a Poisson arrival rate $\lambda_B = \psi \tilde{I}^{\eta}$, where $\psi > 0$, $\eta \in (0, 1)$, and $\tilde{I} \equiv I/X$. To keep the analysis parsimonious, the signal is assumed to reveal the firm's state θ_t without noise, although relaxing this assumption is feasible. The specification for information acquisition costs implies cointegration with total firm size as measured by the variable X. I define $\{N_{B,t}\}_{t=0}^{\infty}$ as a counting process that keeps track of the number of signals generated since date 0.

One may wonder why blockholders could generate information that is not available to a firm's management team. First, when it comes to a firm in financial distress, it is quite likely that the existing management is of mediocre quality, implying that its forecasting ability may be inferior to that of specialized financial investors. Second, management may have biased incentives to always report that the firm has good prospects (e.g., due to managers' career concerns), implying that regular investors view such reports as uninformative cheap talk. In contrast, a blockholder's investment decisions can generate a credible signal about the firm's prospects.

Arrangement between a blockholder and the firm. The firm and an existing blockholder have the following rights offering arrangement. Whenever the firm seeks to raise additional external funds via equity, it first asks the large shareholder to contribute a fraction ω of those funds.[12] If the blockholder agrees, she obtains a fraction ω of the total number of newly issued shares. Consequently, the blockholder has the option to keep her ownership share constant.[13] The blockholder can also reject such a request, but that observable action provides a signal to other agents and breaks the match with the firm. As shown below, these implications of rejections will imply that it is suboptimal for the blockholder to reject funding unless default is indeed optimal. Let $\delta_{B,t} \in \{0,1\}$ denote the large shareholder's decision when asked to provide co-financing at time t. I introduce the firm-specific state variable $b_t \in \{0,1\}$ that indicates whether a firm is currently matched with a blockholder ($b = 1$) or not ($b = 0$).

Trading environment in secondary markets. The trading environment in secondary markets abstracts from noise traders (as, e.g., in Grossman and Stiglitz, 1980). As a result, all market participants understand that a blockholder would trade only for informational reasons, implying that she cannot extract additional information rents with such trades (as in Milgrom and Stokey, 1982). The resulting lack of incentives to trade in secondary markets implies that unless the blockholder rejects a funding request ($\delta_{B,t} = 0$), she maintains an ω-stake in the firm's equity (until default occurs). Nonetheless, a blockholder will still have incentives to acquire information, as doing so improves her funding decisions and the value of her equity stake.

[12] This arrangement is related to what is known is a "right of first refusal" in practice.

[13] This arrangement will simplify the analysis in that it will imply that ω will not be a state variable that we need to keep track of.

Apart from maintaining focus and tractability, abstracting from noise is a plausible modeling choice in light of regulations in the United States. First, investors acquiring more than 5% of a firm's equity (with the intent to exert control) have to file a Schedule 13D with the SEC. As noted by Brav, Jiang, Partnoy, and Thomas (2008), given that such an investor *"needs to file an amendment to its Schedule 13D reflecting any material change in its position, including a reduction of its position to below 5%, "promptly" after the change (some law firms recommend filing within one business day), it would have very little time to sell its block before making a public statement."* Second, as documented by Lim, Schwert, and Weisbach (2019), 81% of PIPE transactions involve issuances of unregistered shares that cannot be freely traded until they are registered with the SEC, which happens 100 days after issuance on average. Moreover, when a blockholder still attempts to unwind her position of at least 5% in the short time period before filing a Schedule 13D amendment, she faces large price impact, especially since financially distressed stocks are typically thinly traded and thus highly illiquid. Such trades then effectively reveal the negative information of the blockholder even before the amendment is filed. These mechanisms of real-world markets ensure that other investors learn about a blockholder's position adjustments at a high enough frequency to inform their own funding decisions, which is the essential externality channel operating in the model.

Finally, when it comes to the *acquisition* of block positions, it is generally much more advantageous for large investors to build up positions in negotiated PIPE transactions (given the empirically documented large discounts) than to acquire exposures with open market trades, which occur at going market prices and further push up prices. The next section describes how such transactions occur in the model.

2.3.2. Block Acquisitions

A continuum of *large investors* may potentially establish a block position. When a firm that currently is not matched with a large investor has a payment date (when $dN_{C,t} = 1$), one investor from this set is randomly chosen to obtain the opportunity to negotiate a transaction with the firm with probability $\kappa \in (0, 1]$. Here, the parameter κ governs frictions in finding and matching with a firm. The larger these search frictions, the more does the market structure deviate from one with competitive free entry, which affects the magnitude of rents obtained by a large investor. If a negotiation opportunity is obtained, matching requires an investor to incur a fixed cost $\chi_t = X_t\tilde{\chi}$, capturing costs related to legal and regulatory fees. The presence of these fixed costs implies that transactions do not only have benefits for the parties involved — in equilibrium, a transaction will occur only if a large investor obtains a surplus greater than the fixed cost χ_t.

Features of PIPE transactions. The transactions negotiated between a large investor and a firm mirror standard private investments in public equity, which involve an investor purchasing a block of newly issued equity shares from a firm. In particular, the investor acquires an ownership share $\omega \in (0, 1]$ at an endogenously determined purchase price P_t^B.[14] When negotiating the purchase price P_t^B, the large investor is assumed to make a take-it-or-leave-it offer to management. When choosing whether to accept the offered price, management takes into account its outside option to remain a firm without a large shareholder and the possibility of matching with a different investor in the future.

I specify the large investor's ownership share ω as an exogenous parameter. In practice, this ownership share is affected by a variety of forces that are not in the focus of this paper, such as regulations, financial institution's assets under management and investment mandate, and capital constraints. For instance, NASDAQ Rule 5635(d) requires issuers to obtain prior approval of shareholders when an issuance below market value represents 20% or more of com-

[14] As detailed above, I follow the common assumption in the literature that the firm does not hold excess cash (see, e.g., Goldstein, Ju, and Leland, 2001, Hackbarth, Hennessy, and Leland, 2007, Strebulaev, 2007, Bhamra, Kuehn, and Strebulaev, 2010, Chen, 2010). Thus, if the purchase price P_t^B is insufficient to cover the firm's external cash needs when the transaction with the large investor occurs, other shareholders inject the remaining funds at the competitive market price. Conversely, if the purchase price exceeds the cash needed (if $\max(P_t^B - C_t, 0) > 0$, the extra funds are settled with existing equity holders.

mon shares.[15] The corresponding increased transaction costs and delays of such transactions imply that the empirical distribution of PIPE transactions involving financially distressed firms features significant bunching just below the regulatory 20% cutoff (see Park, 2011)

Summary of the timeline. At the time of the initial transaction, the firm and the large investor bargain under symmetric information.[16] On payment dates *after* the initial transaction, the less informed party (management) sends requests to the large shareholder to co-finance a fraction ω of external funds needed to make the debt payment. The large shareholder can respond to each request by either accepting or rejecting it.

3. Analysis

In this section, I characterize the equilibrium dynamics of a perfect Bayesian Nash equilibrium with the following signaling mechanism: conditional on a rejection of a co-financing request, agents other than the blockholder infer that the blockholder has received a signal revealing that $\theta = b$, unless the firm is already unambiguously insolvent without that additional signal. Given this signaling mechanism, I start by establishing how small investors optimally use the signals conveyed by the large shareholder's co-financing decisions. Afterwards, I characterize the evolution of agents' beliefs on the equilibrium path. Given this belief evolution, I solve for large investors' optimal information investment and co-financing decisions, as well as for large shareholders' value and the value of claims to the firm. The characterization of a firm's optimal ex ante leverage choice is relegated to Appendix B.

LEMMA 1 (Optimal default rule with a large shareholder)**.** *In equilibrium, if a firm is matched*

[15]NYSE rule 312.03 and NYSE Amex Equities Sec. 713 specify similar rules for discounted private offerings involving more than 20% of existing shares.

[16]The model can be extended to feature an initial due diligence process that can uncover incremental information. This additional feature would not affect the main insights of the paper, as it would also generate the informational externality at the core of the existing analysis.

with a large shareholder ($b_t = 1$)*, the firm enters default if and only if the large shareholder rejects a co-financing request on a debt payment date. As a result, firm default and co-financing rejections coincide,* $\delta_t = \delta_{B,t}$*.*

Proof. See Appendix A.1. ■

While I provide a detailed proof of Lemma 1 in Appendix A.1, I discuss the intuition underlying this result here in the main text. Between debt payment dates, net-payout to shareholders is non-negative, implying that defaulting is never optimal at those times. Moreover, when the large shareholder is willing to co-finance a fraction ω of the external funds needed on a debt payment date, it must be the case that co-financing the remaining fraction $(1-\omega)$ is also optimal for the other investors. Since atomistic shareholders can free-ride on the large shareholder's information without incurring information production cost, the per-share value of equity to passive investors is weakly larger than the per-share value the large shareholder assigns after accounting for her information acquisition cost. Conversely, if the large shareholder is not willing to co-finance while anticipating that this rejection will trigger firm default, then the value of the large shareholder's ω-stake in the equity must be worth less than the funds that would be raised from her to meet the required debt payment (which are also an ω-share of the total funds needed). Given that other agents infer that the large shareholder already has received a negative signal when she rejects co-financing and know that the match with her is broken (implying no future information production), they also conclude that making the debt payment is suboptimal, triggering firm default.

The following lemma further streamlines the characterization of equilibrium beliefs.

LEMMA 2. *It is weakly optimal for the large shareholder to truthfully reveal any information she obtains.*

Proof. The large shareholder can in principle benefit from maintaining asymmetric information (by not disclosing or disclosing non-truthfully) in two types of markets: in the primary market (when the firm issues new equity) and in the secondary market (through trading with other investors). Regarding the primary market, note that the equilibrium financing and default strategy described in Lemma 1 is optimal for shareholders no matter if the large shareholder discloses additional information or not. Given the rights offering arrangement and equilibrium beliefs' dependence on the large shareholder's responses to co-financing requests, the large shareholder cannot extract additional profits in the process of new equity issuances, no matter if the large shareholder discloses its signals or not. Regarding the secondary market, the above-stated assumption of lack of noise in the trading system implies that any trades initiated by the large shareholder are interpreted as informational trades, leading the no-trade theorem to apply (Milgrom and Stokey, 1982).[17] In sum, under the described market arrangements, the large investor cannot extract additional information rents by keeping her information private. ■

Lemma 2 shows that truthful revelation of information by the large shareholder is consistent with optimality in equilibrium. While the firm's optimal default policy (Lemma 1) does not require these additional information releases, considering the case where the large shareholder indeed reveals its information significantly streamlines the remaining analysis. As the equilibrium information sets of all agents are identical in this case, the analysis requires keeping track of only one set of beliefs on the equilibrium path. Going forward, I denote by π_t the probability under agents' common filtration $\mathcal{F}_t$ that the firm is in the good state, $\pi_t \equiv \Pr[\theta_t = h | \mathcal{F}_t]$. I now proceed to characterizing the evolution of these beliefs.

LEMMA 3 (Bayesian updating)**.** *The log-odds ratio $o_t \equiv \log[\pi_t/(1-\pi_t)]$ evolves according to the following process:*

$$do_t = f^+(Z_t)dN^+_{x,t} + f^-(Z_t)dN^-_{x,t} + (\infty \mathbb{1}_{\theta=h} - \infty \mathbb{1}_{\theta=l} - o_t)dN_{A,t}, \tag{10}$$

[17] As highlighted above, this mechanism is empirically plausible at the relevant frequencies, both due to regulatory disclosures and price impact in thinly traded distressed stocks.

where I define the log-Bayes factors associated with positive and negative innovations to x:

$$f^+(Z) \equiv \log\left[\frac{\lambda_x^+(h,Z)}{\lambda_x^+(l,Z)}\right] = \log\left[\frac{1+\Delta_x\frac{\mu_x(h,Z)}{\sigma_x(Z)^2}}{1+\Delta_x\frac{\mu_x(l,Z)}{\sigma_x(Z)^2}}\right], \tag{11}$$

$$f^-(Z) \equiv \log\left[\frac{\lambda_x^-(h,Z)}{\lambda_x^-(l,Z)}\right] = \log\left[\frac{1-\Delta_x\frac{\mu_x(h,Z)}{\sigma_x(Z)^2}}{1-\Delta_x\frac{\mu_x(l,Z)}{\sigma_x(Z)^2}}\right]. \tag{12}$$

Proof. See Appendix A.2. ■

Agents update their beliefs after observing innovations to the log interest coverage ratio x and after the large shareholder obtains and reveals signals. Bayesian updating commands that the log-odds ratio increases by the log-Bayes factor $f^+(Z)$ after a positive innovation to x, and decreases by the log-Bayes factor $f^-(Z)$ after a negative innovation. The lemma also reveals how these log-Bayes factors are uniquely determined by the grid increments Δ_x, the drifts $\mu_x(\theta, Z)$, and the volatility $\sigma_x(Z)$. As the *total* arrival intensity $\lambda_x(Z)$ is independent of the hidden state θ, agents do not obtain additional information from the amount of time that passes between innovations to x. Note that $o = +\infty$ and $o = -\infty$ are well-defined states where agents know the current value of $\theta_t \in \{h, l\}$ with certainty.

In general, the log-odds ratio o_t can take values anywhere on the real line. Yet, imposing the following arbitrarily mild parameter restriction ensures that o_t only attains values on a grid with increments of size $\Delta_o > 0$. This is the third significant modeling choice that is necessary to generate the model's tractability.

ASSUMPTION 1. *Parameters satisfy the restriction that $\frac{f^+(Z)}{\Delta_o}$ and $\frac{f^-(Z)}{\Delta_o}$ are natural numbers.*

As Δ_o can be chosen to be arbitrarily small, this assumption imposes effectively no constraints on the parameter choices determining earnings dynamics. Yet this trick ensures the tractability of the setup in the presence of learning and free boundary problems associated with optimal default. In particular, it ensures that obtaining precise global solutions only requires inverting matrices.

Following the principle of backward induction, I proceed by first analyzing the optimal behavior of large shareholders after matching with a firm, that is, for $b_t = 1$. Conditional on being matched with a firm, the relevant state variables for a large shareholder are (ρ, o, Z, Y). Let Π_t denote the cumulative after-tax net payout to shareholders of the firm between dates 0 and t. Absent default, net payout to shareholders over an instant $[t, t+dt)$ is given by:

$$d\Pi_t = (1-\tau)(X_t dt - C_t dN_{C,t}) \tag{13}$$

The large shareholder dynamically maximizes the market value of its position by optimally choosing information investments I_t and co-financing rejections $\delta_{B,t}$. The following proposition characterizes the large shareholder's value and optimal policy functions.

PROPOSITION 1 (Blockholder value and policies)**.** *The market value of the large shareholder is given by*

$$V(\rho_t, o_t, Z_t, X_t) = \max_{\{I_s\}_{s=t}^{\infty} \geq 0,\ \{\delta_{B,s}\}_{s=t}^{\infty} \in \{0,1\}} \mathbb{E}\left[\int_t^{s_B^*} \frac{m_s}{m_t} \left(\omega d\Pi_s - I_s ds\right) \middle| \mathcal{F}_t \right], \tag{14}$$

where the time of default is $s_B^* \equiv \inf\{s \geq t : \delta_{B,s} dN_{C,s} = 1\}$. The Hamilton-Jacobi-Bellman (HJB) equation associated with the maximization problem (14) implies that the scaled value function $\tilde{V}(\rho, o, Z)$ solves the following set of equations for all $(\rho, o, Z) \in \Omega_\rho \times \Omega_o \times \Omega_Z$:

$$\begin{aligned}
0 = \max_{\tilde{I} \geq 0,\ \delta_B \in \{0,1\}} \Big\{ & \omega(1-\tau)(1 - e^{-\rho}(1 - \delta_B(\rho, o, Z))) - \tilde{I}(\rho, o, Z) \\
& - (r_f(Z) + rp_B(\rho, o, Z) - \mu_Y(Z) + \lambda_C \delta_B(\rho, o, Z)) \cdot \tilde{V}(\rho, o, Z) \\
& + \psi \tilde{I}(\rho, o, Z)^\eta \cdot (\pi(o) \cdot \tilde{V}(\rho, \infty, Z) + (1 - \pi(o)) \cdot \tilde{V}(\rho, -\infty, Z) - \tilde{V}(\rho, o, Z)) \\
& + \Lambda_{\rho,o}(\rho, o, Z) \tilde{\mathbf{V}}_{\rho,o}(\rho, o, Z) + \Lambda_Z(Z) \tilde{\mathbf{V}}_Z(\rho, o, Z) \Big\},
\end{aligned} \tag{15}$$

where I define $\pi(o) \equiv \frac{e^o}{1+e^o}$, where $\tilde{\mathbf{V}}_{\rho,o}(\rho, o, Z)$ indicates a vector that collects the values of *the function $\tilde{V}(\rho, o, Z)$ evaluated at all $(\rho, o) \in \Omega_\rho \times \Omega_o$ while keeping the other arguments fixed, and where the matrix $\Lambda_{\rho,o}(\rho, o, Z)$ reflects that the states (ρ, o) move simultaneously respecting the evolution of the log-odds ratio o stated in equation (3). The expression for a large shareholder's risk premium rp_B is given in Appendix A.3. The optimal controls solving (15) are given by:*

$$\tilde{I}(\rho, o, Z) = \max\left\{\psi\eta[\pi(o)\cdot\tilde{V}(\rho,\infty,Z) + (1-\pi(o))\cdot\tilde{V}(\rho,-\infty,Z) - \tilde{V}(\rho,o,Z)], 0\right\}^{\frac{1}{1-\eta}}, \quad (16)$$

$$\delta_B(\rho, o, Z) = \mathbb{1}_{\{\tilde{V}(\rho,o,Z)<\omega e^c(1-\tau)\}}. \quad (17)$$

Proof. See Appendix A.3. ■

The blockholder's choice for I (see 16) reveals that the optimal amount of information acquisition increases with the convexity of the value function, which, in turn, is due to the default option that is embedded in the equity claim.

The tractability of the proposed setup follows from the fact that the highlighted modeling choices ensure that conditional on *any* policy functions $\tilde{I}(\rho, o, Z)$ and $\delta_B(\rho, o, Z)$, the value function $\tilde{V}(\rho, o, Z)$ is available in closed form; equation (15) represents a linear system that can be solved by inverting a *sparse* matrix. The system is generically sparse since in continuous time, the firm's state can transition only to a relatively small set of neighboring states, implying that most values in the transition matrix are equal to zero. Sparsity, in turn, ensures that even very large matrices can be inverted quickly and efficiently. As a result, obtaining precise solutions to the free-boundary problem with endogenous information acquisition is straightforward. Policy function iteration can be applied by relying on exact solutions for value functions in each step.

Next, I turn to characterizing the value of the firm's equity. The relevant state vector for the equity value is (b, ρ, o, Z, X).

PROPOSITION 2 (Equity value)**.** *The total equity market value is given by:*

$$P\left(b_t, \rho_t, o_t, Z_t, X_t\right) = \max_{\{\delta_s\}_{s=t}^{\infty} \in \{0,1\}} \mathbb{E}\left[\int_t^{s^*} \frac{m_s}{m_t} d\Pi_s \middle| \mathcal{F}_t \right], \tag{18}$$

where $s^* \equiv \inf\{s \geq t : \delta_s dN_{C,s} = 1\}$*. The HJB equation associated with the maximization problem in (18) implies that the scaled value function* $\tilde{P}(b, \rho, o, Z)$ *solves the following set of equations for all* $(b, \rho, o, Z) \in \Omega_b \times \Omega_\rho \times \Omega_o \times \Omega_Z$*:*

$$\begin{aligned} 0 = \big\{ & (1-\tau)(1 - e^{-\rho}(1 - \delta(b, \rho, o, Z))) \\ & - (r_f(Z) + rp(b, \rho, o, Z) - \mu_Y(Z) + \lambda_C \delta(b, \rho, o, Z)) \cdot \tilde{P}(b, \rho, o, Z) \\ & + a\psi \tilde{I}(\rho, o, Z)^{\eta} \cdot (\pi(o) \cdot \tilde{P}(a, \rho, \infty, Z) + (1 - \pi(o))\tilde{P}(a, \rho, -\infty, Z) - \tilde{P}(b, \rho, o, Z)) \\ & + \Lambda_{\rho,o}(b, \rho, o, Z) \tilde{\mathbf{P}}_{\rho,o}(b, \rho, o, Z) + \Lambda_Z(Z) \tilde{\mathbf{P}}_Z(b, \rho, o, Z) \Big\}. \end{aligned} \tag{19}$$

The expression for a firm's equity risk premium rp_P *is given in Appendix A.4. The optimal controls are given by:*

$$\delta(1, \rho, o, Z) = \delta_B(\rho, o, Z), \tag{20}$$

$$\delta(0, \rho, o, Z) = \mathbb{1}_{\{\tilde{P}(0,\rho,o,Z) < e^c(1-\tau)\}}. \tag{21}$$

Lemma 1 implies that conditional on a match, $(b = 1)$, the firm optimally defaults when the large shareholder rejects to co-finance. Otherwise, for $b = 0$, the firm defaults when the equity continuation value is below the required payment to debt holders. Equation (37) does not explicitly reflect the arrival rate with which a firm matches with a large shareholder (that is, b switching from 0 to 1), since these shocks are value-neutral to existing equity holders — given the large investor's take-it-or-leave-it offer, the large investor extracts the incremental equity value created by her future efforts. The firm has the outside option to reject and wait for the next large investor, but the next investor will again make a take-it-or-leave it offer. As a result, the systems of equations determining the values $\tilde{P}(1, \rho, o, Z)$ and $\tilde{P}(0, \rho, o, Z)$ can be solved separately. Conditional on the policy functions, value functions are again available in closed-form due to the linearity of the system (37) in $\tilde{P}(b, \rho, o, Z)$.

The following lemma characterizes large investors' decisions on when to establish a match with a firm.

LEMMA 4 (Matching between large investors and firms)**.** *Conditional on obtaining a negotiation opportunity with a firm, a large investor implements a transaction if and only if the value created exceeds the fixed negotiation costs:*

$$\underbrace{\tilde{V}(\rho, o, Z) + (1-\omega)\tilde{P}(1, \rho, o, Z)}_{\textit{Total value with large shareholder}} \quad - \quad \underbrace{\tilde{P}(0, \rho, o, Z)}_{\textit{Total value without large shareholder}} \quad > \; \tilde{\chi}. \tag{22}$$

The ex ante value of surplus extracted by the group of large investors with a given firm is characterized in Appendix A.6.

Given these endogenous matching decisions, we can now proceed to characterizing the value of debt in all states.

PROPOSITION 3 (Debt value)**.**

$$D(b_t, \rho_t, o_t, Z_t, X_t) = \mathbb{E}\left[\int_t^{s^*} \frac{m_s}{m_t} C_t dN_{C,s} \middle| \mathcal{F}_t\right] \tag{23}$$

The HJB equation associated with (23) implies that the scaled debt value $\tilde{D}(b, \rho, o, Z)$ solves *the following set of equations for all $(b, \rho, o, Z) \in \Omega_b \times \Omega_\rho \times \Omega_o \times \Omega_Z$:*

$$
\begin{aligned}
0 = \Big\{ & e^{-\rho}(1 - \delta(b, \rho, o, Z)) - (r_f(Z) + rp_D(b, \rho, o, Z) - \mu_Y(Z)) \cdot \tilde{D}(b, \rho, o, Z) \\
& + \lambda_C \delta(b, \rho, o, Z) \cdot (\alpha(Z) \cdot \tilde{U}(o, Z) - \tilde{D}(b, \rho, o, Z)) \\
& + \lambda_C \cdot (1 - b) \cdot \kappa \cdot \mathbb{1}_{\{\tilde{V}(\rho,o,Z) + (1-\omega)\tilde{P}(1,\rho,o,Z) - \tilde{P}(0,\rho,o,Z) > \tilde{\chi}\}} (\tilde{D}(1, \rho, o, Z) - \tilde{D}(0, \rho, o, Z)) \\
& + b\psi \tilde{I}(\rho, o, Z)^\eta \cdot (\pi(o) \cdot \tilde{D}(1, \rho, \infty, Z) + (1 - \pi(o)) \cdot \tilde{D}(1, \rho, -\infty, Z) - \tilde{D}(1, \rho, o, Z)) \\
& + \Lambda_{\rho,o}(\rho, o, Z) \tilde{\mathbf{D}}_{\rho,o}(b, \rho, o, Z) + \Lambda_Z(Z) \tilde{\mathbf{D}}_Z(b, \rho, o, Z) \Big\},
\end{aligned}
\tag{24}
$$

where the expression for the debt risk premium rp_D is given in Appendix A.4, and where U denotes the value of the unlevered firm, which is characterized in Appendix A.5.

In states where a firm is not yet matched with a large shareholder ($b = 0$), the debt value encodes a potential future match (a switch to $b = 1$), which affects default policies and thus also debt holders' value. Moreover, equation (24) reflects that in default, debt holders recover a fraction $\alpha(Z)$ of the firm's unlevered assets.

4. Calibration and Evaluation

In this section, I calibrate the model and evaluate its predictions.

4.1. Choosing Parameters

In the following, I discuss the parameter choices determining the dynamics of exogenous macroeconomic processes, and the technologies of firms and large investors. Parameter values are listed in Table 1. I use values from the existing literature to calibrate the dynamics of the macreconomic processes Z and Y and of the stochastic discount factor (Chen, Xu, and Yang, 2013, Binsbergen and Opp, 2019). This calibration considers two aggregate states $Z \in$

$\{\text{boom}, \text{recession}\}$. Transition rates between these aggregate states are set such that expansions and recessions last, on average, 10 years and 2 years, respectively.

Conditional on the dynamics of the state variables Z and Y, a firm's earnings dynamics are fully determined by the drifts $\mu_x(\theta, Z)$ and the volatilities $\sigma_x(Z)$. I choose the expected earnings growth rate differentials across aggregate states to match those estimated in Bhamra, Kuehn, and Strebulaev (2010). Moreover, the differences in growth rates across firms in states g and b are chosen to ensure that under the stationary distribution, the mapping between firm default rates and interest coverage ratios is consistent with the data (Moody's, 2017, 2018). These earnings dynamics ensure that firms earn an equity risk premium of about 6% at a leverage of 30%. Overall earnings volatility is specified to match estimates in Bhamra, Kuehn, and Strebulaev (2010).[18] Similarly, recovery rates in default, $\alpha(Z)$, are taken from Bhamra, Kuehn, and Strebulaev (2010).

Given the fact that the empirical distribution of equity stakes acquired in PIPE transactions of distressed firms exhibits significant bunching at the regulatory 20% threshold (Park, 2011), I calibrate the acquisitions stake $\omega = 0.2$. I set the parameters κ and χ such that under the stationary distribution, matching with large investors occurs when a firm's interest coverage ratio is typically between 0.5 and 0.7, corresponding to the average and median interest coverage ratio of firms rated Caa-C (Moody's, 2017). These interest coverage values are also representative of firms categorized as financially distressed in Andrade and Kaplan (1998). I choose a decreasing returns to scale parameter of 0.6, similar to standard assumptions on decreasing returns at the firm level. I set the information investment efficiency parameter ψ to match a typical gain of about 20% for large investors from PIPE transactions, consistent with empirical estimates by Park (2011).

[18] In Bhamra, Kuehn, and Strebulaev (2010) local idiosyncratic volatility is 22.6 percent and local systematic volatility is on average 10.1 percent.

Table 1

Parameters. The three panels list parameters of the macroeconomy, firms, and large investors. The grid increments for the log interest coverage ratio and the log-odds ratio are given by $\Delta_x = 0.095$ and $\Delta_o = 0.018$, respectively. The parameter choices imply that the log-Bayes factor associated with a positive earnings innovation, $f^+(Z)$, is $8\Delta_o$ in booms and $9\Delta_o$ in recessions. Similarly, the log-Bayes factor for negative innovations, $f^-(Z)$, is $9\Delta_o$ in booms and $8\Delta_o$ in recessions. The arrival intensity of interest payments implies that payments occur on average every quarter.

Macroeconomy			
Parameter	Variable	Boom	Recession
Physical transition rates for aggregate states	λ_Z	0.100	0.500
Risk neutral transition rates for aggregate states	$\bar{\lambda}_Z$	0.200	0.250
Trend growth	μ_Y	0.025	-0.015
Trend volatility	σ_Y	0.029	0.029
Risk free rate	r_f	0.050	0.050
Local risk price	ν	0.700	1.300
Tax rate	τ	0.150	

Firms			
Parameter	Variable	Boom	Recession
Drift of x in good firm state ($\theta = g$)	μ_x	0.080	0.010
Drift of x bad firm state ($\theta = b$)	μ_x	-0.010	-0.080
Volatility of x	σ_x	0.240	0.240
Arrival rate of interest payments	λ_C	4.000	

Large Investors		
Parameter	Variable	Values
Probability of matching opportunity on payment dates	κ	0.500
Fixed costs of negotiation	$\tilde{\chi}$	0.300
Information investment efficiency	ψ	5.000
Decreasing returns to scale parameter	η	0.600
Ownership stake	ω	0.200

4.2. Results of the Calibration

In this section, I analyze the predictions of the calibrated model. Throughout, the presented figures plot outcomes as a function of the state variables. Given the high dimensionality of the state space, the graphs focus on three belief levels $\pi \in \{0.07, 0.22, 0.39\}$ that correspond to the 20th, 50th, and 80th percentile under the stationary distribution, conditional on having an interest coverage ratio below 1. The reason for these relatively pessimistic beliefs is that earnings innovations and beliefs are correlated. When moving from an interest coverage ratio of around 10, which is representative for A-rated firms (Moody's, 2017), to an interest coverage ratio below 1, typical for C-rated firms, firms have experienced a series of negative earnings innovations that have caused agents to update their beliefs downwards. The horizontal axes of the figures represent the interest coverage ratio e^{ρ}. Finally, the figures consistently feature two panels that separately illustrate outcomes in booms and recessions.

Information acquisition. Figure I illustrates the endogenous arrival rate of information obtained by a large shareholder conditional on having matched with a firm. The figure reveals the highly non-linear behavior of large shareholders' efforts, which are intimately linked to the convexity of equity claims. For sufficiently high earnings, the firm is very likely to be solvent and deserving of continued support from shareholders, implying that incentives for incremental information acquisition are low. Yet, for lower interest coverage ratios, attention to the firm's conditions increases strongly, peaking at an interest coverage ratio that implies high one-year default rates (see Panels (a) and (b) of Figure II). If interest coverage declines even further, incentives for information acquisition decline again as the chances of firm solvency become minimal. In sum, when the firm is very likely to be either solvent or insolvent, incentives for further information acquisition are low. In contrast, when there is uncertainty about firm solvency and shareholders' benefits from providing continued support to the firm, the large investor pays significantly more attention to evaluating the firm's prospects.

Default rates. Panels (a) and (b) of Figure II illustrate the one-year default rates of a firm that has a large shareholder. Given the model solution, these default rates are determined exactly, based on the equilibrium generator matrix describing firm dynamics, without the need for simulations. The Panels show that apart from the interest coverage ratio, beliefs and the state of the

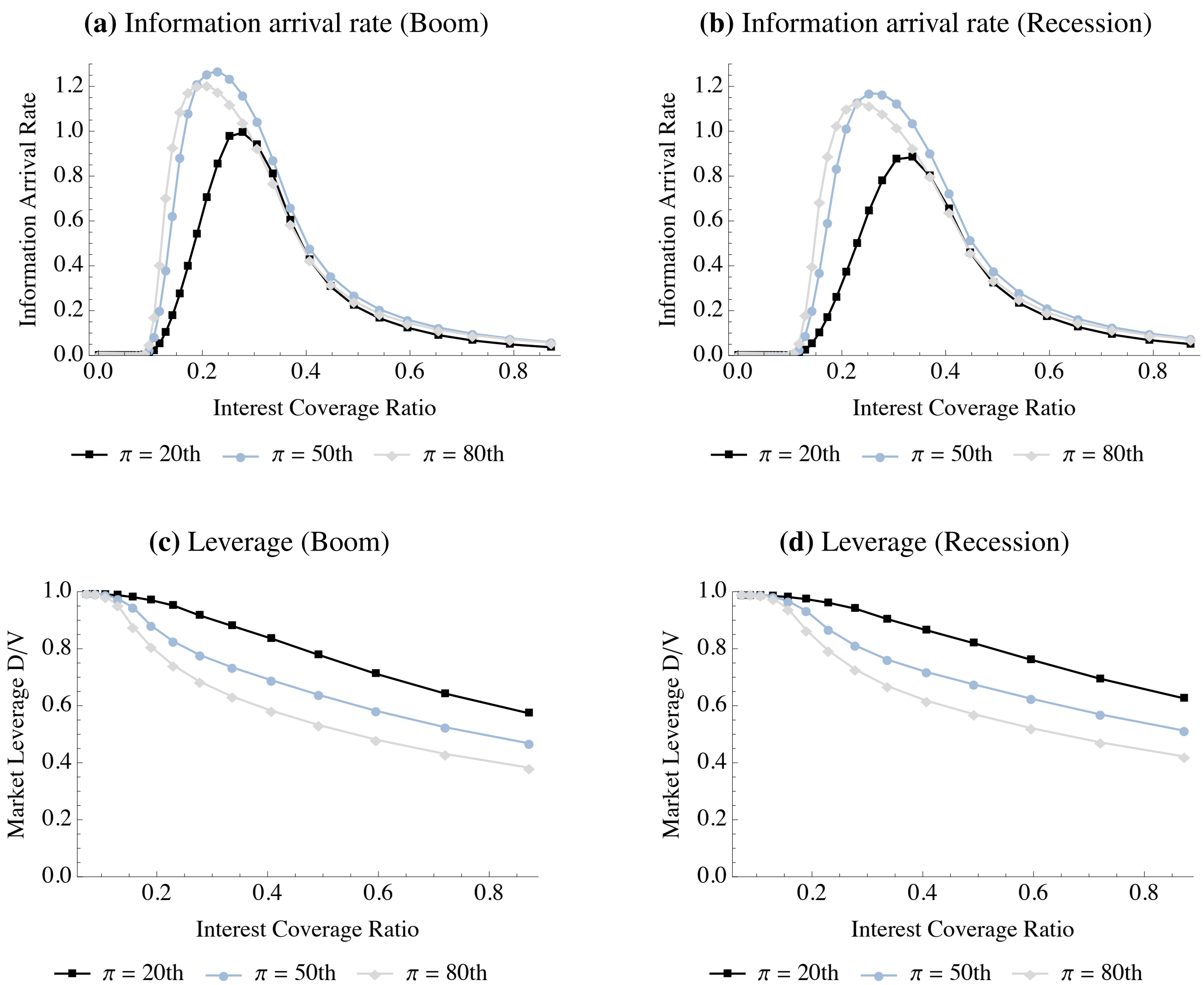

FIGURE I

Information acquisition and leverage. Panels (a) and (b) plot the arrival intensity of signals obtained by the large shareholder in booms and in recessions, respectively. Panels (c) and (d) illustrate market leverage in booms and recessions, respectively. The graphs consider three belief levels that are representative of the 20th, 50th, and 80th percentile of beliefs under the stationary distribution. The horizontal axis represents the firm's interest coverage ratio $\tilde{X}$. All parameter values are provided in Table 1.

business cycle play an important role in determining default risk. Moreover, Panels (c) and (d) reveal the non-monotone impact of the large shareholder's information acquisition on default rates. These Panels compare the state-contingent one-year default rates with a large shareholder to those in a counterfactual economy without large shareholders. This counterfactual can be informative in the context of debates on regulatory changes that could increase the costs of

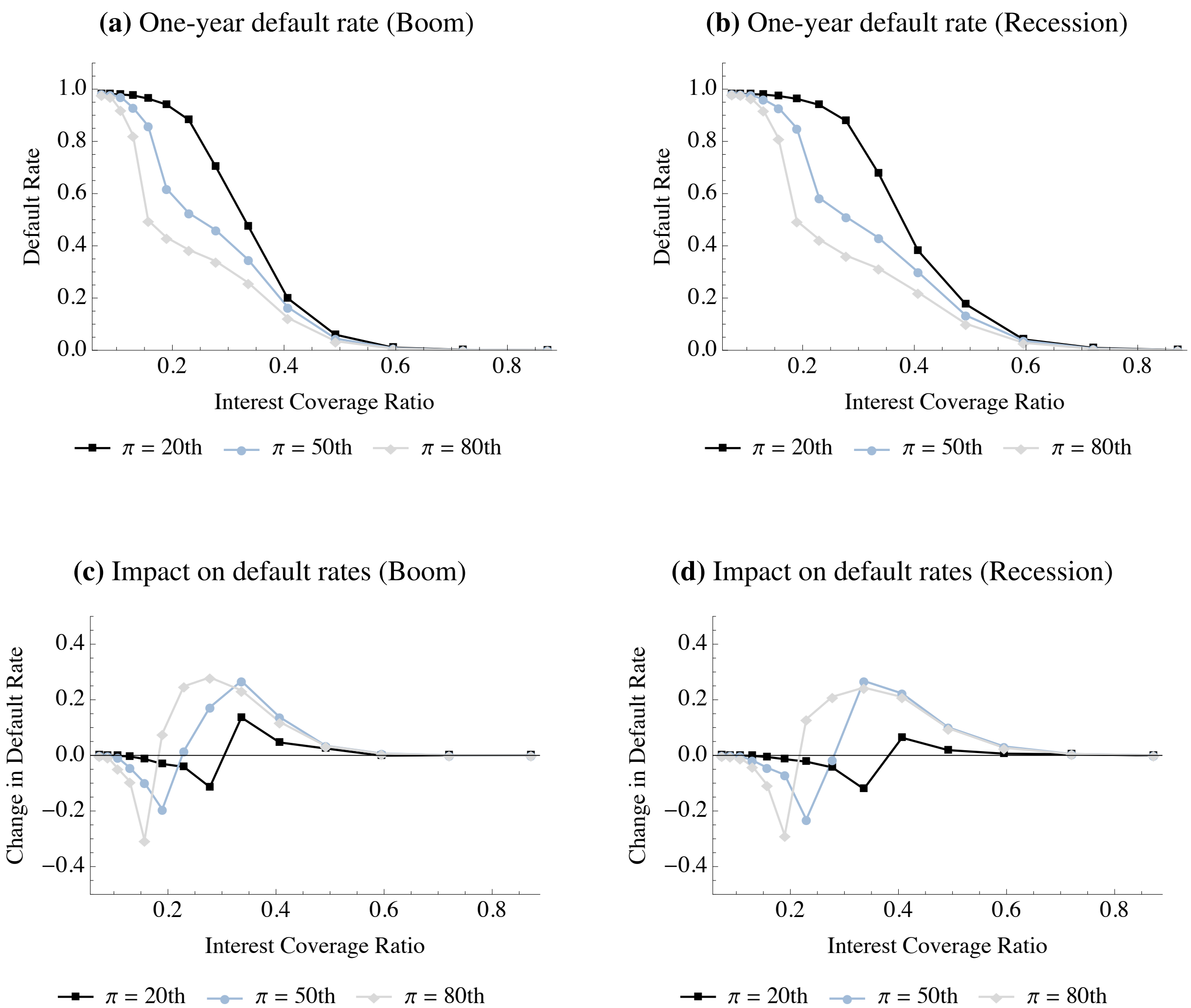

FIGURE II

One-year default rates and the impact of blockholders. The figure illustrates the conditional one-year default rates (Panels (a) and (b)) and the change in these default rates relative to those obtaining in a counterfactual economy without blockholders (Panels (c) and (d)). The panels plot these measures in booms (Panels (a) and (c)) and in recessions (Panels (b) and (d)) as a function of the interest coverage ratio $\tilde{X}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

completing PIPE transactions (as captured by the parameter $\tilde{\chi}$), thereby reducing the incidence of block acquisitions.

The analysis reveals that as a result of the presence of a large shareholder, conditional default rates are substantially increased for low and intermediate levels of distress, but they are decreased close to default. For low and intermediate levels of distress, the large shareholder frequently obtains information confirming the firm's insolvency, causing shareholders to withdraw their support more quickly. In contrast, for extremely distressed firms, the involvement of a large shareholder tends to prolong the firm's life expectancy — whereas default is almost certain absent a large shareholder in those states, there is a chance of resurrection when a blockholder is present and may uncover incremental positive information in the short-run. This non-monotone impact of large investors on default risk is of first-order importance for externalities on equity, debt, and firm value, which I discuss in more detail next.

Impact on equity values. Figure III illustrates large investors' impact on state-contingent equity and debt values for a matched firm $(b = 1)$ relative to a counterfactual economy without large shareholders. All value changes are scaled by the state-contingent firm values in the counterfactual economy. Panels (a) and (b) illustrate the impact on equity values, showing a hump-shaped pattern as a function of the interest coverage ratio, which is qualitatively similar to the pattern obtained for the large shareholder's information acquisition (Figure I). Comparing equity value gains across different belief levels confirms the intuition that gains are limited if agents are already quite certain that the firm is in a bad state (at the 20th percentile of beliefs the chance of being in a bad state is 93%).

It is worth highlighting that the illustrated *ratios* of gains are affected by both the equity gains (the numerator) and the "autarky" firm value (the denominator). The autarky firm value declines with lower interest coverage ratios not only because of lower earnings, but also because of a higher chance of default and associated distress costs (which include tax shield losses given the standard assumption that debt holders recover a fraction $\alpha(Z)$ of the *unlevered* firm value).

When interpreting the illustrated value gains, it is important to keep in mind that they differ fundamentally from "announcement returns" that obtain for existing equity holders conditional

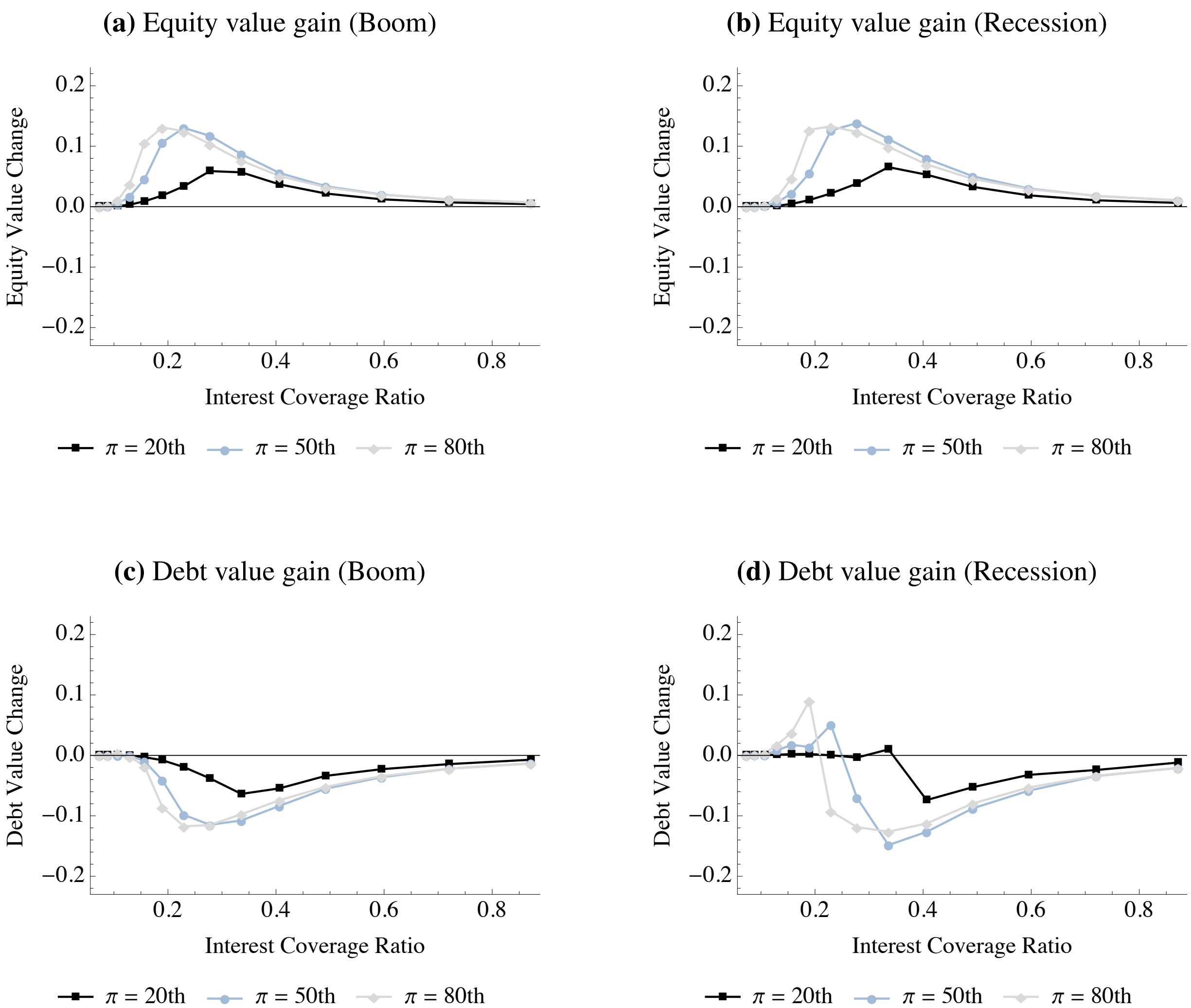

FIGURE III

Impact on equity and debt values. The figure illustrates the gain in equity and debt values for a firm that has a large shareholder, relative to a counterfactual economy without large shareholders. The panels plot these value gains for equity and debt in booms (Panels (a) and (c)) and in recessions (Panel (b) and (d)) as a function of the interest coverage ratio $\tilde{X}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

on observing a new match between a firm and a large investor. These announcement returns are in fact negative for two reasons. First, equity value gains from information acquisition are extracted by large investors through negotiated PIPE transactions in which existing atomistic shareholders are pushed to their outside option. Second, transactions coincide with payment shocks $dN_C = 1$ that are negative news for equity values. Negative returns on announcement dates can therefore not be taken as evidence that large shareholders destroy equity value or that existing equity holders are worse off when a large investor acquires an equity stake.

Impact on debt values. Large shareholders' impact on debt values is a priori theoretically ambiguous. On the one hand, large shareholders' information facilitates the optimal exercising of equity holders' put option, in which debt investors hold a short position. That is, large shareholders' increased attention to firm conditions generates a redistribution of value from debt to equity holders. On the other hand, the total value to be distributed between debt and equity holders — that is, firm value — may increase as the put value gain for equity holders weakly lowers the incidence of default in any *given* state of the world (ρ, o, Z). Yet, as large shareholders' information acquisition also affects the probability distribution of states, it generally has an ambiguous effect on default probabilities, as demonstrated by the above-discussed one-year default rates (Figure II). The possibility of lower default risk, in turn, implies the possibility of gains for debt holders.

Reflecting these competing effects, Panels (c) and (d) of Figure III reveal distinct patterns for debt value changes in booms and recessions. In booms, debt value changes exhibit an inverse hump-shaped pattern as a function of the interest coverage ratio. In these states, debt values are consistently negatively affected by the presence of the large shareholder. In contrast, in recessions, debt values are positively affected for highly distressed firms. As highlighted above, the large shareholder tends to reduce default risk for highly distressed firms with uncertain growth prospects. Moreover, since default leads to greater deadweight losses in recessions (recovery rates are lower), these reductions in default risk have a larger positive effect on overall firm value. In net, a large shareholder can therefore positively affect debt values in highly distressed states, in particular when the deadweight losses from bankruptcy are material.

Again, these illustrated value changes differ fundamentally from announcement returns.

Prior to matching with a large shareholder, debt market prices already encode the possibility of future investor involvement (a switch from $b = 0$ to $b = 1$). When the matching with a large investor becomes highly likely in the near-term, debt trades already close to the value that obtains once a PIPE transaction has occurred. Thus, announcement returns are generally a poor measure of the impact of large shareholders on debt values.

Impact on overall firm value. Panels (a) and (b) of Figure IV combine the discussed value changes for equity and debt to evaluate the implications for overall firm value. The graphs again reveal a non-monotone pattern, consistent with the above-discussed implications for default risk illustrated in Figure II. Large shareholders affect firm value through their impact on the efficiency of default decisions. In recessions, when default risk leads to greater inefficiencies, any changes in default rates have a more material effect on overall firm value. As a result, firm value gains can be quite substantial for highly distressed firms in recessions, reaching values in excess of 10%. In contrast, in booms, the effects are dampened by smaller default costs. In this context it is useful to highlight that in the limit, when default does not lead to any inefficiencies (for $\tau = 0$ and $\alpha(Z) = 0$), the value implications of large shareholders for overall *firm* value are nil — in this case, large shareholders only cause a redistribution of value from debt to equity claims.[19]

Endogenous timing of PIPE transactions. Figure V illustrates the probabilities with which a large investor will match with the firm the next time the firm needs to raise external funds. The graphs reveal that PIPE transactions are more likely to occur when there is substantial uncertainty about the firm's prospects, which is the case at the illustrated 50th and 80th percentiles of beliefs. In contrast, at the 20th percentile of beliefs, agents know with 93% probability that the firm is in the bad state, implying that PIPE transactions are not negotiated unless the firm is very

[19] Taxes affect losses conditional on default as debt holders are assumed to collect a fraction $\alpha(Z)$ of the *unlevered* firm value, implying that losses are due to both bankruptcy costs and reduced future interest tax shields.

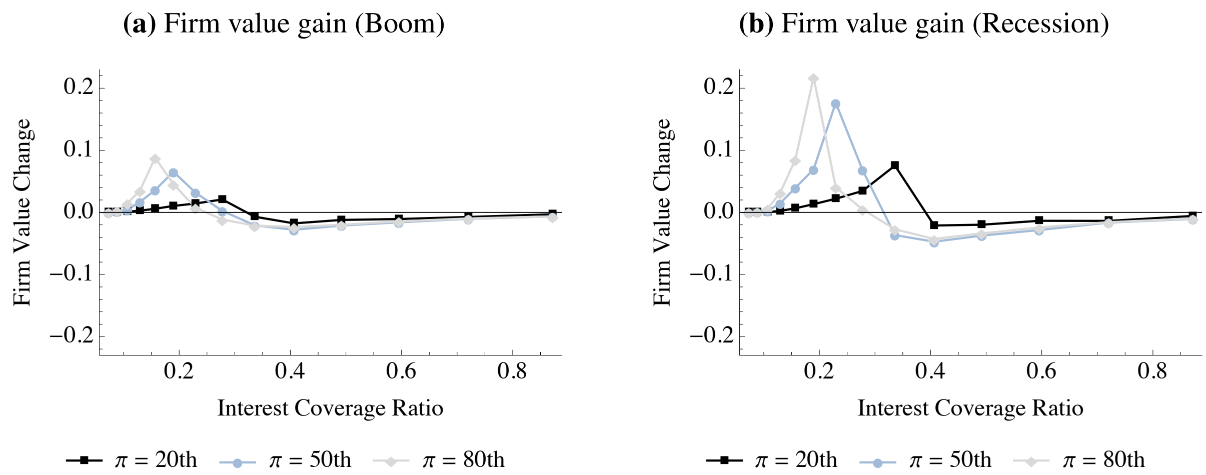

FIGURE IV

Impact on firm value. The figure illustrates the gain in firm value for a firm that has a large shareholder, relative to a counterfactual economy without large shareholders. The panels plot these value gains in booms (Panel (a)) and in recessions (Panel (b)) as a function of the interest coverage ratio $\tilde{X}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

close to default. Interestingly, matching and negotiation frictions causing such delays in large shareholder involvement can have positive implications for overall firm value. As highlighted in the previous discussion of Figure IV, the efficiency-enhancing effects of large shareholders' actions dominate the value-destroying rent-seeking effects for ”last minute” interventions, that is, interventions in states in which the firm would be very likely to go bankrupt on the next payment date unless a PIPE transaction occurs. These results suggest that frictions causing PIPE transactions to predominantly occur in the form of such last-minute interventions can have positive implications for overall efficiency.

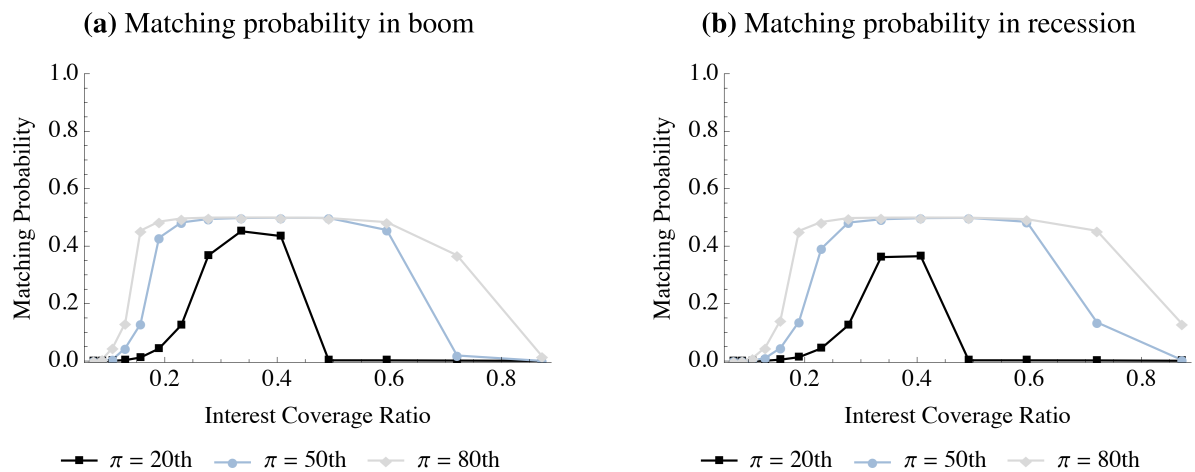

FIGURE V

Probability of matching when next debt payment is due. The figure illustrates the probability with which a large investor matches with an unmatched firm ($b = 0$) when the next debt payment is due. The two panels plot these probabilities in booms (Panel (a)) and in recessions (Panel (b)) as a function of the interest coverage ratio $\tilde{X}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

5. Conclusion

Large shareholdings are a pervasive empirical phenomenon that can have first-order effects on investors' efforts to influence the decisions of firms and other investors. In this paper, I analyze the externalities posed by large shareholders in the context of financial distress where the quality of investors' information about firm solvency is essential for firm survival. I propose a tractable dynamic model of the interplay between large shareholders and indebted firms' atomistic equity holders that nests central features of standard structural credit risk models. The model yields global solutions that reveal the highly nonlinear behavior of large shareholders' attention and its associated externalities. Information acquisition on the firm's prospects causes blockholders to take a pivotal role in distressed firm's financing, materially affecting not only the distribution of value across different claims but also overall efficiency. Large shareholders' impact on firm value is generically non-monotone in financial distress, turning positive only in deeply distressed states. The model sheds light on the implications of equity issuances via private investments in public equity (PIPE transactions) and frictions making such transactions more likely to occur as “last minute” interventions that support firms in deep distress.

A. Proofs

A.1. Proof of Lemma 1

Between payment dates, net-payout to shareholders is non-negative implying that shutting down the firm is never optimal at those times. Suppose that the firm indeed defaults if and only if the large shareholder refuses to co-finance on a payment date. Then, the large shareholder knows that it effectively controls default decisions with its accept/reject response in those states. Thus, the large shareholder maximizes its value by choosing the information investment rate I and whether to effectively trigger default $\delta_B \in \{0, 1\}$ conditional on the arrival of a payment date:

$$\max_{\{I_s\}_{s=t}^{\infty} \geq 0,\ \{\delta_{B,s}\}_{s=t}^{\infty} \in \{0,1\}} \mathbb{E}\left[\int_t^{s^*} \frac{m_s}{m_t} \left(\omega d\Pi_s - I_s ds \right) \middle| \mathcal{F}_{A,t} \right], \tag{25}$$

where $\mathcal{F}_{A,t}$ denotes the large shareholder's filtration, and where the time of default is $s^* \equiv \inf\{s \geq t : \delta_{B,s} dN_{C,s} = 1\}$. Note that after the initial PIPE transaction, the large shareholder by construction consistently collects a fraction ω of net-payout up until it refuses to co-finance and triggers default. Next, we need to verify whether the firms default policy is indeed optimal for small shareholders if the large shareholder chooses $\{I_s\}_{s=t}^{\infty} \geq 0,\ \{\delta_{B,s}\}_{s=t}^{\infty} \in \{0, 1\}$ to maximize (25). First, if the large shareholder accepts to co-finance a required debt payment at date t, then it must be the case that the maximized objective (25) is weakly larger than ωC_t. If (25) is weakly larger than ωC_t, then it must also be the case that:

$$\mathbb{E}\left[\int_t^{s^*} \frac{m_s}{m_t} (1 - \omega) d\Pi_s \middle| \mathcal{F}_{A,t} \right] > (1 - \omega) C_t, \tag{26}$$

since (25) also includes the investment cost $I \geq 0$ (which lowers the objective). Given the inequality (26), small investors indeed optimally keep the firm afloat when the large shareholder provides co-financing (notice that (25) conditions on the filtration of the large shareholder $\mathcal{F}_{A,t}$, which is weakly larger than other agents' filtration). Second, if the large shareholder does not accept to co-finance at a payment date t then it must be the case that (25) is smaller than ωC_t. Notice that by optimality of the choice of $\{I_s\}_{s=t}^{\infty} \geq 0$ it must be the case that (25) is weakly greater than the following objective that restricts the large shareholder to set $\{I_s\}_{s=t}^{\infty} = 0$:

$$\max_{\{\delta_{B,s}\}_{s=t}^{\infty} \in \{0,1\}} \mathbb{E}\left[\int_t^{s^{**}} \frac{m_s}{m_t} \omega d\Pi_s \middle| \mathcal{F}_{A,t} \right], \tag{27}$$

where the time of default $s^{**} \equiv \inf\{s \geq t : \delta_{B,s} dN_{C,s} = 1\}$ is now chosen optimally without access to additional signals since $\{I_s\}_{s=t}^{\infty} = 0$. Thus, if (25) is smaller than ωC_t then it must also be the case that the constrained maximum (27) is smaller than ωC_t. Dividing (27) by ω and dividing ωC_t by ω then also implies that the following inequality holds if the large shareholder refuses to co-finance and breaks the match with the firm:

$$\max_{\{\delta_{B,s}\}_{s=t}^{\infty} \in \{0,1\}} \mathbb{E}\left[\int_t^{s^{**}} \frac{m_s}{m_t} d\Pi_s \middle| \mathcal{F}_{A,t} \right] < C_t. \tag{28}$$

Inequality (28) implies that even if the shareholders other than the large shareholder received all dividends going forward, the shareholders would optimally reject to make the payment C_t (under the richer information set of the large shareholder $\mathcal{F}_{A,t}$). This result takes advantage of another fact: as large investors make take-it-or-leave-it-offers to firms, the equity absent large shareholder involvement is priced exactly as if the probability of *future* involvement was zero (in any state without involvement, $b = 0$), implying that the left-hand side of (28) indeed represents the continuation value for shareholders if the large shareholder rejects to co-finance. As a result, the firm will indeed default when the large shareholder refuses to co-finance and breaks the match with the firm. Finally, in unambiguously solvent states (when $P > C$ independent of beliefs about θ), the large shareholder could reject co-financing without causing firm default. Yet, this strategy is suboptimal since it breaks the match with the firm.

A.2. Proof of Lemma 3

In the following, I derive the representations for the log-Bayes factors (12) and (11). By definition Bayes factors associated with positive and negative innovations are given by:

$$e^{f^{+}(Z)} = \frac{\lambda_x^{+}(h,Z)}{\lambda_x^{+}(l,Z)}, \tag{29}$$

$$e^{f^{-}(Z)} = \frac{\lambda_x^{-}(h,Z)}{\lambda_x^{-}(l,Z)}. \tag{30}$$

In addition, restriction (6) implies the following equations:

$$\lambda_x(Z) = \lambda_x^{+}(h,Z) + \lambda_x^{-}(h,Z), \tag{31}$$

$$\lambda_x(Z) = \lambda_x^{+}(l,Z) + \lambda_x^{-}(l,Z). \tag{32}$$

Combining these equations with the formulas for $\mu_x(\theta, Z)$ and $\sigma_x(Z)$ provided in equations (7) and (8) yields the provided formulas for the log-Bayes factors.

A.3. Proof of Proposition 1

Lemma 1 implies that in equilibrium, the large shareholder optimally rejects co-financing only when firm default is also optimal for all equity holders, that is, optimal co-financing strategies reject only in states where these rejections also trigger firm default. Thus, for a firm that is matched with a large shareholder, the time of default is simply given by: $s_B^* \equiv \inf\{s \geq t : \delta_{B,s} dN_{C,s} = 1\}$. The Hamilton-Jacobi-Bellman equation associated with the maximization problem in (14) is given by:

$$\begin{aligned}0 = \max_{\tilde{I}\geq 0,\ \delta_B\in\{0,1\}} &\Big\{\omega(1-s)(Ye^{x} - \lambda_C(1-\delta_B(\rho,o,Z))Ye^{c}) - Y\tilde{I}(\rho,o,Z) \\ &- (r_f(Z) + \lambda_C\delta_B(\rho,o,Z))\cdot V(\rho,o,Z,Y) \\ &+ V_Y(\rho,o,Z,Y)Y\mu_Y(Z) + \frac{1}{2}V_{YY}(\rho,o,Z,Y)Y^2\sigma_Y(Z)^2 - V_Y(\rho,o,Z,Y)Y\sigma_Y(Z)\nu(Z) \\ &+ \psi\tilde{I}(\rho,o,Z)^{\eta}\cdot(\pi(o)\cdot V(\rho,\infty,Z,Y) + (1-\pi(o))\cdot V(\rho,-\infty,Z,Y) - V(\rho,o,Z,Y)) \\ &+\Lambda_{\rho,o}(\rho,o,Z)\mathbf{V}_{\rho,o}(\rho,o,Z,Y) + \Lambda_Z(Z)\mathbf{V}_Z(\rho,o,Z,Y)\}, \end{aligned} \tag{33}$$

where I define $\pi(o) \equiv \frac{e^o}{1+e^o}$, where $\mathbf{V}_{\rho,o}(\rho,o,Z,Y)$ indicates a vector that collects the values of the function $\tilde{V}(\rho,o,Z)$ evaluated at all $(\rho,o)\in\Omega_\rho\times\Omega_o$ while keeping the other arguments fixed, and where the matrix $\Lambda_{\rho,o}(\rho,o,Z)$ reflects that the states (ρ,o) move simultaneously respecting the evolution of the log-odds ratio o stated in equation (3). Given the conjecture that $V(\rho_t,o_t,Z_t,Y_t) = Y\cdot\tilde{V}(\rho,o,Z)$, it can be verified that the HJB equation scales with Y. Dividing by Y, rearranging and using the risk premium definition

$$rp_B(\rho,o,Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \bar{\Lambda}_Z(Z))\frac{\tilde{\mathbf{V}}_Z(\rho,o,Z)}{\tilde{V}(\rho,o,Z)}. \tag{34}$$

yields equation (15).

A.4. Proposition 2

The derivation of equation (37) takes advantage of the scaling property that was shown to hold in the proof of Proposition 1 in Appendix A.3. The equity risk premium rp_P is given by:

$$rp_P(b,\rho,o,Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \bar{\Lambda}_Z(Z))\frac{\tilde{\mathbf{P}}_Z(b,\rho,o,Z)}{\tilde{P}(b,\rho,o,Z)}. \tag{35}$$

A.5. Unlevered Equity Value

The unlevered firm does not face a default decision and thus, its value is not affected by the presence of a large shareholder. The unlevered equity market value is given by:

$$U(o_t, Z_t, X_t) = \mathbb{E}\left[\int_t^{s^*} \frac{m_s}{m_t}(1-\tau)X_t ds \middle| \mathcal{F}_t\right], \tag{36}$$

where the scaled unlevered value $\tilde{U}(o, Z)$ solves the following set of equations for all $(o, Z) \in \Omega_o \times \Omega_Z$:

$$\begin{aligned} 0 = \Big\{&(1-\tau) - (r_f(Z) + rp_U(o, Z) - \mu_Y(Z)) \cdot \tilde{U}(o, Z) \\ &+\Lambda_o(o, Z)\tilde{\mathbf{U}}_o(o, Z) + \Lambda_Z(Z)\tilde{\mathbf{U}}_Z(o, Z)\Big\}, \end{aligned} \tag{37}$$

and where the risk premium is given by:

$$rp_U(o, Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \bar{\Lambda}_Z(Z))\frac{\tilde{\mathbf{U}}_Z(o, Z)}{\tilde{U}\,(o, Z)}. \tag{38}$$

A.6. Value Extracted by Large Investors

Let W denote the ex ante value of the surplus extracted by the group of large investors with a given firm before a match has occurred $(b = 0)$. The scaled value $\tilde{W}$ solves the following set of equations for $(\rho, o, Z) \in \Omega_\rho \times \Omega_o \times \Omega_Z$:

$$\begin{aligned} 0 = \Big\{&\lambda_C \cdot \kappa \cdot \max\left[\tilde{V}(\rho, o, Z) + (1-\omega)\tilde{P}(1, \rho, o, Z) - \tilde{P}(0, \rho, o, Z) > \tilde{\chi}, 0\right] \\ &- \lambda_C \cdot (1-a) \cdot \kappa \cdot \mathbb{1}_{\{\tilde{V}(\rho,o,Z)+(1-\omega)\tilde{P}(1,\rho,o,Z)-\tilde{P}(0,\rho,o,Z)>\tilde{\chi}\}}\tilde{W}(\rho, o, Z) \\ &- (r_f(Z) + rp_W(\rho, o, Z) - \mu_Y(Z)) \cdot \tilde{W}(\rho, o, Z) \\ &+\Lambda_{\rho,o}(\rho, o, Z)\tilde{\mathbf{W}}_{\rho,o}(\rho, o, Z) + \Lambda_Z(Z)\tilde{\mathbf{W}}_Z(\rho, o, Z)\Big\}, \end{aligned} \tag{39}$$

where I define the risk premium:

$$rp_W(\rho, o, Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \bar{\Lambda}_Z(Z))\frac{\tilde{\mathbf{W}}_Z(\rho, o, Z)}{\tilde{W}\,(\rho, o, Z)}. \tag{40}$$

B. Optimal Leverage

In this section, I characterize firms' optimal initial leverage choices. Suppose the firm is unlevered at date $t = 0$ and its log earnings are $x_0 \in \Omega_x$. The firm chooses an interest coverage ratio ρ_0 from the set:

$$\Omega_\rho \in \{x - \log[\lambda_C] : x \in \Omega_x\}. \tag{41}$$

A choice $\rho_0 \in \Omega_\rho$ together with the initial log earnings level x_0 pins down the following choice for the log of detrended debt payments:

$$c = x_0 - \log[\lambda_C] - \rho_0. \tag{42}$$

At date $t = 0$, the firm then chooses ρ_0 to solve the following program:

$$\max_{\rho_0 \in \Omega_\rho} \left\{ \tilde{P}(b_0, \rho_0, o_0, Z_0) + \tilde{D}(b_0, \rho_0, o_0, Z_0) \right\}. \tag{43}$$

where the functions $\tilde{P}(b, \rho, o, Z)$ and $\tilde{D}(b, \rho, o, Z)$ are characterized Propositions 1 and 3.

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