The Perils of Tracking r-Star*

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Abstract

The natural rate of interest (r-star) has been a critical benchmark for monetary policymaking recently. We show that there exist fiscal limits to a monetary policy rule that targets r-star. When monetary policy is constrained by fiscal sustainability concerns, an interest rate rule that tracks r-star generates large and persistent movements in inflation and output gap, thereby producing macroeconomic instability and welfare losses. The mechanism operates through the government budget constraint, as interest rate changes affect the value of government debt and, thereby, inflation. This leads to perils of tracking r-star, in a model widely used for monetary policy analysis.

Keywords: r-star; monetary and fiscal policy; government debt; inflation; policy regimes; government budget constraint

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1 Introduction

The end of the Great Recession and the ongoing normalization of monetary policy by the Federal Reserve have renewed interest in the natural rate of interest. The usefulness of the natural rate, often dubbed "r-star," as a benchmark for monetary policymakers can be understood most clearly in the canonical New Keynesian (NK) model. Setting the nominal interest rate to target the natural rate closes the output gap, as given by the household optimality condition in that model. In the absence of shocks that lead to a trade-off for monetary policy, closing the output gap in turn completely stabilizes inflation, as given by the firms' optimality condition in that model. This then ensures that the first-best allocation is achieved under such a policy.

As several empirical studies have shown that the natural rate has changed non-trivially over time (for example, Laubach and Williams 2003, Justiniano and Primiceri 2012, Lubik and Matthes 2015, and Holston et al. 2017), it has raised the possibility and desirability of allowing a time-varying intercept, as given by "r-star," in standard Taylor rule formulations.[1] In fact, monetary policy making has already started to take the variation in the natural rate into account while making interest rate decisions. For instance, then-Chair Janet Yellen in speeches often explained FOMC decisions in terms of tracking r-star and argued for allowing time-varying intercept in monetary policy rules.[2]

While these insights are economically sound from a theoretical perspective and have also influenced actual policy making recently, whether the central bank should actually track r-star is still being debated. The existing debates focus mostly on issues related to implementation. For example, Taylor and Wieland (2016) argue against adopting a Taylor rule that tracks r-star over the business

[1] These studies additionally also find a downward trend in r-star over time. Empirically, studies also distinguish between long-run and short-run/cyclical r-star. The focus here is on the cyclical r-star.

[2] As one example, in a speech on March 2015 titled "The New Normal Monetary Policy," then-Chair Janet Yellen mentioned "*...the economy's underlying strength has been gradually improving, and the equilibrium real federal funds rate has been gradually rising. ... and as the equilibrium real funds rate continues to rise, it will accordingly be appropriate to raise the actual level of the real federal funds rate in tandem, all else being equal.*"

As another example, in a speech on January 2017 titled "The Economic Outlook and the Conduct of Monetary Policy," then-Chair Janet Yellen mentioned "*The Taylor rule is often implemented by assuming that the real, or inflation-adjusted, value of the longer-run neutral interest rate–which I will call R* for convenience–is equal to 2 percent, roughly the average historical value of the real federal funds rate prior to the financial crisis... this version of the Taylor rule prescribes a much higher path for the federal funds rate than the median of participants' assessments of appropriate policy. One important factor explaining this divergence is the FOMC's growing recognition that the longer-run neutral level of the real federal funds rate has likely declined below 2 percent, contrary to what is often assumed in implementations of the Taylor rule ...Such revisions would imply shifts in the level of the Taylor rule's prescriptions by as much as 1-1/4 percentage points, holding other factors constant. Clearly, sensible implementation of policy rules requires adjustments to take such changes into account, as a failure to do so would result in poor monetary policy decisions and poor economic outcomes.*"

cycle, because the welfare-relevant r-star is necessarily both model and shock specific. Moreover, even with perfect knowledge of the structure of the economy, real time estimates of latent variables (and thus r-star) are potentially imprecise, as argued by Beyer and Wieland (2017).[3] As following the *wrong* r-star not only leads to macroeconomic instability, but also generates undesirable interest rate volatility, this concern provides some caution against adopting such a policy proposal.

We provide a new perspective on this debate by presenting a *full-information* environment in which tracking, even the *correct*, r-star generates macroeconomic instability and lower welfare. While it is well-understood that prescriptions for monetary policy depend crucially on the prevailing fiscal policy (and vice versa,) the literature on r-star has paid little attention to the role of fiscal policy. The potential role for fiscal policy is precluded by the implicit assumption that inflation is completely insulated from fiscal conditions – in particular, the state of government indebtedness. We find that tracking r-star is undesirable when there exists a channel through which an accumulation of nominal public debt leads to inflation. We call this the *fiscal* channel.

Our main result thus is that there exists a *fiscal limit* to a monetary policy regime that incorporates r-star targeting. What is the intuition for this result? As usual, increasing the policy rate by the central bank in response to a rise in r-star stabilizes inflation through the conventional *aggregate demand* channel, as we outlined above at the beginning. This however, also increases interest payments on public debt, thereby raising its outstanding value. Conditional on the existence of this alternative *fiscal* channel, it in turn leads to a rise in inflation and hence the output gap. This channel thus operates in the opposite direction and can easily dominate the conventional channel.

Is our result and mechanism practically relevant? The current and expected level of the U.S. government debt coupled with ever-increasing interest payments suggest that overlooking the fiscal channel may be a mistake. To highlight this, we present several actual and projected components of government budget and debt from the CBO. Figure 1 shows that net interest payments will grow significantly and play a progressively greater role in deficits and debt dynamics. Specifically, panel (a) shows that while net interest on debt is around \$ 300 billion currently, by 2029, it is projected to be over \$ 900 billion. As a result, net interest will start to take a non-trivial share of U.S. government outlays. Panel (b) shows that while net interest on debt currently stands around 2% of GDP, it is projected to rise to 6% of GDP by 2049, which would be larger than total discretionary spending

[3]Hamilton et al. (2016) also show substantial uncertainty in estimates of r-star.

and comparable to social security outlays.

Figure 2 makes a related point. Panel (a) shows that while net interest payments currently are smaller than primary deficits, they are projected to be bigger in the near future. Lastly, but perhaps most importantly, panel (b) shows that different interest rate paths on government debt can have a non-trivial effect on debt-GDP ratios. For instance, if interest rates are 1% point higher relative to baseline, CBO projects that by 2049, the debt-GDP ratio will be around 200%, as opposed to the baseline projection of 140%.

Besides the increasing role of interest payments and the impacts of interest rate changes on public debt, another important observation from the CBO projections is that the government deficit and debt are projected to increase and not return to "normal" in the foreseeable future. These forecasts imply that the government is not expected to substantially adjust the primary surplus in response to changes in the debt level. This can be seen clearly already in panel (b) of Figure 2, which plots historical and projected path of the debt-GDP ratio. In its baseline projection, the CBO projects debt-GDP to continuously increase and reach over 140% by 2049. Overall, the situation showcased in Figures 1-2 is precisely the environment in which the *fiscal* channel tends to operate.

We consider such an environment in a standard monetary model – the same set-up in which tracking r-star would be desired in the absence of the fiscal channel. The lack of proper fiscal policy adjustments prevents monetary policy from actively pursuing inflation stabilization.[4] Instead, the monetary authority, constrained by government debt sustainabillity concerns, responds insufficiently to changes in the rate of inflation. Such accommodation by monetary policy allows inflation to adjust to stabilize public debt dynamics and achieve fiscal sustainabillity. Under this policy regime, variations in interest rates triggered by movements in r-star will change the level of public debt and thus the rate of inflation – fluctuations that would be avoided if the central bank did not track r-star. We then show that not tracking r-star leads to higher macroeconomic stability and welfare. This leads to perils of tracking r-star.

Our results thus constitute a cautionary note on the policy recommendation that the Federal Reserve should track r-star going forward. This is because the recent tax policy changes and high level of government debt and deficits are likely to make the fiscal channel relevant as we discussed above. This coupled with unprecedented and overt White House pressures on monetary policy

[4]Aggressive inflation targeting, in this case, would generate explosive inflation and output dynamics in the model.

recently suggest that central bank accommodation of fiscal concerns in future might be plausible.[5] This is the second ingredient needed for our mechanism to be in operation. Thus, the policy regime that underlies our theoretical results could actually be in operation (currently or) in the near future in the U.S.[6] Overall, our results highlight the general point that policy recommendations on the nature of the monetary policy rule should not be made without considering the fiscal policy in place and the implications of interest rate policy on the government budget constraint.

In Section 2, we first provide an analytical characterization of the main channels in a prototypical NK model. In Section 3, we consider a general version of the model that has richer propagation mechanisms, such as habits in consumption decision and partial indexation in price setting. We also introduce interest rate and tax smoothing in the model to capture the observed inertia in the policy instruments.[7]

In this richer model, while all the main results emphasized in the simple model hold, an interesting new insight emerges. In particular, including a r-star intercept in monetary policy rules with interest smoothing may be destabilizing – even in the absence of the fiscal channel. Under such a policy rule, an initial change in the interest rate – triggered by even a transitory shock to r-star – generates persistent movements of the interest rate in subsequent periods. Such future movements of the interest rate, however, are in principle totally unrelated to realizations of the future r-star and in fact, can be far from them. In such a case, a monetary policy rule featuring r-star, coupled with interest rate smoothing, generates macroeconomic instability even without the fiscal channel. While this result is not the primary focus of our paper, it does provide another warning on policy recommendations that argue for a time-varying intercept in monetary policy rules – independently from fiscal conditions. It thus highlights that details of the monetary policy rule followed by the central bank are important, even in widely used model environments.[8]

[5] As one example, consider the July 5, 2019 tweet from the President: "*Our most difficult problem is not our competitors, it is the Federal Reserve." The Fed "raised rates too soon, too often" and "doesn't have a clue*!"

[6] Previous work has argued that such a regime existed in U.S. history. For instance, Woodford (2001) discusses the bond-price support regimes of the 1940s as one example while Sims (2011) and Jacrobson et al. (2019) argue for such a regime to explain the rise of inflation in the 1970s and the recovery of 1933 respectively. Bianchi and Ilut (2017) find support for the argument in Sims (2011) using an estimated NK model in which monetary and fiscal policies switch over time. Leeper (2010) argues that such a regime is likely to operate in future in the U.S. given growing old-age promised benefits. Sims (2016) uses such a regime to analyze several current monetary policy issues.

[7] Policymakers' gradualist behaviors are well documented. For example, Coibion and Gorodnichenko (2012) show that the Federal Reserve has been smoothing the interest rate *beyond* what is required by the fundamental inertia in economic conditions.

[8] This result is also practically relevant as Curdia et al. (2015) show that an interest rate rule including both r-star intercept *and* interest rate smoothing term fits the U.S. data well.

Related literature Our paper is related to several strands of the literature. In addition to the work discussed above, Edge et al. (2008) provide estimates of cyclical variation in r-star using a quantitative model. Also using quantitative estimated models, Curdia et al. (2015) and Barsky et al. (2014) show the plausibility and desirability of monetary policy tracking r-star. These policy recommendations apply however, only when the fiscal channel we identify is not in operation.

We are building on by now a large literature that specifies monetary and fiscal policy regimes jointly in business cycle monetary models. Leeper (1991), Sims (1994, 2004), Woodford (1994), Loyo (2000), and Cochrane (2001) presented the theoretical underpinnings. The "fiscal theory," since then, has been developed further and applied to explore a wide range of issues.[9] Among others, Sims (2011), Cochrane (2018) and Bhattarai et al. (2014) use a model with a rich set of frictions and show that the central bank's attempt to stabilize inflation ends up raising the volatility of inflation in the absence of fiscal backing.[10] While this paper shares a similar insight at the general level, the main novelty here is an analytical characterization of the relationship between the responsiveness to the natural rate in Taylor rules and the dynamics of inflation, output and public debt; we thereby bridge the gap between the fiscal theory and the empirical literature on r-star that finds evidence for r-star-tracking interest rate rules in the U.S. data. The analytical result in turn allows us to develop a clear intuition for normative implications of following such a rule under different policy regimes.[11] Another distinction from the existing studies comes from our finding that tracking r-star can be destabilizing even in the absence of the fiscal channel, as mentioned above.

Our paper is also related to Benhabib et al. (2001) who show perils of following standard Taylor rules. The key issue emphasized in Benhabib et al. (2001) is that even when monetary policy responds sufficiently strongly to inflation, there is global indeterminacy in standard monetary models once the zero lower bound on nominal interest rate is taken into account. Our focus here however is different as we emphasize that the nature of fiscal policy matters in whether one reaches the recommendation that the central bank should follow a Taylor rule that includes the natural rate of interest.

[9]For example, see other papers of the aforementioned authors such as Cochrane (2011, 2014), Leeper and Walker (2013), Leeper et al. (2017), and Sims (2011, 2013) as well as the papers discussed in footnote 6. Canzoneri et al. (2011) and Leeper and Leith (2016) provide an excellent survey of the literature.

[10]The first two also include a richer maturity structure of government debt which is absent in our analysis.

[11]This paper focuses on the optimal response to r-star by the central bank that follws a standard interest rate rule. We refer the interested readers to Leeper and Zhou (2013) and section 4 and 5 of Leeper and Leith (2016) for (general) optimal monetary and fiscal policy mix in the presence of the fiscal channel.

2 Inspecting the mechanisms

We discuss the mechanisms using a prototypical NK model. We first present the model and analytical results. We then supplement them with numerical illustrations.

2.1 Model

Since the basic NK model is well known, a complete description is relegated to the appendix. The private sector equilibrium conditions as well as policy rules and government budget constraints in the model, after log-linearization, are given by:

$$\tilde{Y}_t = \mathbb{E}_t\tilde{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) + \hat{r}^*_t, \tag{1}$$

$$\hat{\pi}_t = \kappa\tilde{Y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{2}$$

$$\hat{R}_t = \gamma\hat{r}^*_t + \phi\hat{\pi}_t, \tag{3}$$

$$\hat{\tau}_t = \psi\hat{b}_{t-1}, \tag{4}$$

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\hat{\pi}_t - \beta^{-1}\hat{\tau}_t + \bar{b}\hat{R}_t, \tag{5}$$

where $\tilde{Y}_t$ is the output gap, which is the difference between actual output and the flexible-price output, $\hat{R}_t$ is the nominal interest rate, $\hat{\pi}_t$ is inflation, $\hat{\tau}_t$ is the tax revenues, $\hat{b}_t$ is the real maturity value of outstanding government debt, and $\hat{r}^*_t$ is the natural rate of interest or r-star.[12]

Equations (1) and (2) represent the Euler equation and the Phillips curve in the model, respectively, and arise from private sector optimization. Parameter β is the discount factor of the household and κ represents the slope of the Phillips curve, which is a composite of the structural parameters.

Monetary policy is modelled using a rule (3) that features a systematic response of the nominal interest rate $\hat{R}_t$ to inflation $\hat{\pi}_t$ with feedback parameter ϕ and to the natural rate $\hat{r}^*_t$. Parameter γ measures how closely the central bank tracks $\hat{r}^*_t$. Previous work on r-star has considered mostly two special cases ($\gamma = 0$ and $\gamma = 1$), but here we consider a more general specification where γ can take on different values in $[0, 1]$.

Fiscal policy is modelled using a rule (4) that features a systematic response of the tax revenues

[12] $\hat{X}_t$ denotes the log deviation of a variable X_t from its steady state $\bar{X}$, except for two fiscal variables, $\hat{\tau}_t$ and $\hat{b}_t$. The latter two variables represent respectively the deviation of government tax revenues (net of transfers) and of the maturity value of government debt from their steady-state levels, measured as a fraction of steady-state output ($\bar{Y}$): $\hat{b}_t = \left(b_t - \bar{b}\right)/\bar{Y}$ and $\hat{\tau}_t = \left(\tau_t - \bar{\tau}\right)/\bar{Y}$. We write $\hat{\tilde{Y}}_t$ as $\tilde{Y}_t$ to avoid cluttered notations.

$\hat{\tau}_t$ to the real maturity value of outstanding government debt $\hat{b}_{t-1}$ with feedback parameter ψ. For simplicity, it is assumed that the government issues one-period nominal bonds and levies lump-sum taxes. Then we can write the flow budget constraint of the government as equation (5).

The natural rate $\hat{r}_t^*$ is a composite of the structural shocks. To study the fiscal channel in isolation from other sources that potentially render tracking r-star suboptimal, the model features $\hat{r}_t^*$ as the only driving force and abstracts from other disturbances such as cost-push shocks. In addition, we assume that $\hat{r}_t^*$ is a white noise in this section, as it both keeps the notation simple and also helps us see clearly how the fiscal channel generates an endogenous propagation mechanism.[13]

2.2 Main results

To obtain intuition for our results, it is helpful to substitute out the two policy instruments $\left\{\hat{R}_t, \hat{\tau}_t\right\}$ using the policy rules (3) and (4), thereby reducing the model to a dynamic system of $\left\{\tilde{Y}_t, \hat{\pi}_t, \hat{b}_t\right\}$:

$$\tilde{Y}_t = \mathbb{E}_t\tilde{Y}_{t+1} - \phi\hat{\pi}_t + \mathbb{E}_t\hat{\pi}_{t+1} + \underbrace{(1-\gamma)\,\hat{r}_t^*}_{\text{aggregate demand channel}}, \tag{6}$$

$$\hat{\pi}_t = \kappa\tilde{Y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{7}$$

$$\hat{b}_t = \beta^{-1}(1-\psi)\hat{b}_{t-1} - \bar{b}\left(\beta^{-1} - \phi\right)\hat{\pi}_t + \underbrace{\bar{b}\gamma\hat{r}_t^*}_{\text{fiscal channel}}. \tag{8}$$

Increasing γ from 0 to 1 potentially affects model dynamics through two opposing channels. On the one hand, it has a stabilizing effect on inflation and the output gap through the conventional aggregate demand channel: increasing γ from 0 to 1 lowers the influence of shocks to $\hat{r}_t^*$ on the output gap in the aggregate demand equation (6), which in turn stabilizes inflation through the Phillips curve (7). On the other hand, increasing γ may amplify inflation and output gap fluctuations through the new fiscal channel. For a given shock to $\hat{r}_t^*$, a higher γ leads to a bigger increase in the real value of outstanding public debt in the government budget constraint (8).[14] If the fiscal feedback parameter ψ is small so that $\beta^{-1}(1-\psi) > 1$ and the coefficient on $\hat{b}_{t-1}$ is thus explosive, inflation – the second term on the right-hand-side of (8) – needs to rise to decrease the real value of debt. This inflationary devaluation of government debt should be accommodated by a weak response of monetary policy to

[13] This assumption is not critical for our results – other than the exact form of the solutions. The appendix provides the solution in the case where $\hat{r}_t^*$ follows an AR(1) process, $\hat{r}_t^* = \rho_r\hat{r}_{t-1}^* + \varepsilon_t$.

[14] This channel is stronger when steady-state debt-GDP ratio, $\bar{b}$, is larger.

inflation ($\phi < 1$) for non-explosive dynamics. However, this leads to destabilization of the output gap.

We thus need a non-responsive fiscal authority ($\beta^{-1}(1-\psi) > 1$ or $\psi < \bar{\psi} \equiv 1-\beta$) for the fiscal channel to be operative in this simple model. Otherwise, and if it is combined with a sufficiently large monetary policy feedback parameter ($\phi > 1$), it is well known that the first two equations that constitute the *monetary* bloc, (6)-(7), and the last equation that represents the *fiscal* bloc, (8), are decoupled and the time path of $\left\{\hat{\pi}_t, \tilde{Y}_t\right\}$ is determined separately from fiscal conditions.[15] To see such a decoupling formally, we provide the solution of the model under this standard '*monetary* regime':

$$\hat{\pi}_t = \Gamma_M(\gamma)\,\hat{r}_t^*, \quad \tilde{Y}_t = \Lambda_M(\gamma)\,\hat{r}_t^*, \tag{9}$$

where $\Gamma_M(\gamma) \equiv \frac{\kappa(1-\gamma)}{\kappa\phi+1}$ and $\Lambda_M(\gamma) \equiv \frac{(1-\gamma)}{\kappa\phi+1}$ are non-negative and decreasing in γ on $[0,1]$. Equation (9) clearly shows that setting $\gamma = 1$ unambiguously leads to a full stabilization of inflation and the output gap.

Given (9), (8) gives the solution of debt in this standard monetary regime:

$$\hat{b}_t = \beta^{-1}(1-\psi)\hat{b}_{t-1} + \Theta_M(\gamma)\,\hat{r}_t^*,$$

where $\Theta_M(\gamma) \equiv \bar{b}\left[\left(\frac{\kappa\beta^{-1}+1}{\kappa\phi+1}\right)\gamma + \frac{\kappa\left(\phi-\beta^{-1}\right)}{\kappa\phi+1}\right]$, which is increasing in γ and positive almost everywhere.[16] An increase in $\hat{r}_t^*$, therefore, typically raises the level of debt – more so when the central bank tracks r-star. Such changes in the fiscal condition, however, have no implications for inflation and the output gap. The fiscal channel is thus non-operative for inflation and output dynamics in the monetary regime.

In contrast, when the aforementioned conditions on the policy parameters are not met, there is no such decoupling. Conditional on a non-responsive fiscal policy ($\psi < \bar{\psi}$), non-explosive/stable inflation and output dynamics arise only if the central bank sets $\phi < 1$. The following proposition summarizes several properties of the solution under this alternate, '*fiscal* regime'.[17]

Proposition 1 *When the fiscal channel is operative (i.e.* $\psi \in \left(-\infty, \bar{\psi}\right)$ *and* $\phi \in [0,1)$*), the solution*

[15] If fiscal authority is sufficiently responsive but the central bank is weakly responsive ($\phi < 1$), the model is subject to sunspot shocks and provides no restrictions on the effects of r-star.

[16] A (very weak) sufficient condition is $\phi > \beta^{-1}$.

[17] All proofs are provided separately in section D of the appendix for smooth exposition.

for debt, inflation, and the output gap is given by

$$\hat{b}_t = \Theta(\gamma)\hat{r}_t^* + \Omega_b \hat{b}_{t-1} = \Theta(\gamma)\sum_{k=0}^{\infty}\Omega_b^k \hat{r}_{t-k}^*, \tag{10}$$

$$\hat{\pi}_t = \Gamma(\gamma)\hat{r}_t^* + \Omega_\pi \hat{b}_{t-1} = \Gamma(\gamma)\hat{r}_t^* + \Omega_\pi \Theta(\gamma)\sum_{k=1}^{\infty}\Omega_b^{k-1}\hat{r}_{t-k}^*, \tag{11}$$

$$\tilde{Y}_t = \Lambda(\gamma)\hat{r}_t^* + \Omega_Y \hat{b}_{t-1} = \Lambda(\gamma)\hat{r}_t^* + \Omega_Y \Theta(\gamma)\sum_{k=1}^{\infty}\Omega_b^{k-1}\hat{r}_{t-k}^*, \tag{12}$$

where the coefficients are composites of the structural parameters. Moreover,

1. Ω_b, Ω_π *and* Ω_Y *are independent of how the central bank responds to the natural rate (*γ*); they are all positive.*

2. Θ*,* Γ*, and* Λ *are linear functions of* γ*, conditional on other structural parameters;* Γ *and* Λ *are positive* $\forall\gamma \geq 0$*.*

The proposition provides a simple representation of the model that helps us see how tracking the natural rate affects inflation (and output) dynamics via the fiscal channel. As in the case of the monetary regime, an increase in r-star is expansionary, leading to an increase in inflation and the output gap on impact, as $\Gamma(\gamma), \Lambda(\gamma) > 0\ \forall\gamma \geq 0$. In addition, the outstanding value of government debt is now a state variable, which allows even transitory shocks to have persistent effects on the endogenous variables. Importantly, the fiscal channel is now operative: an increase in outstanding public debt requires inflation and the output gap to rise as discussed above (i.e. Ω_π and Ω_Y are positive).

As in the conventional monetary regime, the coefficients on $\hat{r}_t^*$ depends on parameter γ. The way the parameter affects these coefficients however, is quite different in the fiscal regime. We summarize the results in the following proposition and elaborate on it in the ensuing paragraphs.

Proposition 2 *When the fiscal channel is operative, at* $\gamma = 0$, $\Theta(\gamma) < 0$, $\Gamma(\gamma) > 0$*, and* $\Lambda(\gamma) > 0$*. Moreover,*

$$\Theta'(\gamma) > 0 \quad \textit{for } \psi \in \left(-\infty, \bar{\psi}\right) \quad \textit{and} \quad \phi \in [0,1),$$

$$\Gamma'(\gamma) > 0 \quad \textit{for } \psi \in \left(-\infty, \bar{\psi}^*\right) \quad \textit{and} \quad \phi \in [0,1),$$

where $0 < \bar{\psi}^* \leq \bar{\psi}$ *is a reduced-form parameter.*[18]

It is well understood that inflation volatility is dominant for welfare in the model (e.g. Woodford 2003). Our primary interest, therefore, is on inflation dynamics represented by (11), which has two components, $\Gamma(\gamma)\,\hat{r}_t^*$ and $\Omega^\pi \hat{b}_{t-1}$. These two capture respectively the effects on current inflation $\hat{\pi}_t$ of current and past innovations to r-star.

Let us first consider the former (contemporaneous) effect, which is measured by $\Gamma(\gamma)$. The proposition states that $\Gamma'(\gamma) > 0$. Intuitively, the central bank following (3) raises the nominal rate in response to increases in $\hat{r}_t^*$, thereby increasing interest payments and thus $\hat{b}_t$, which in turn requires $\hat{\pi}_t$ to respond more. This *fiscal channel* is stronger when the central bank tracks r-star more closely (i.e. γ is greater.)

We now turn to the latter (lagged) effects. Because of the endogenous state variable, even a transitory innovation to r-star has long-lasting effects on inflation; equivalently, current inflation $\hat{\pi}_t$ depends *also* on $\hat{r}_{t-k}^*$ for all $k \geq 1$. One can study such persistent effects by inspecting debt dynamics given by (10). First, consider the benchmark case where $\gamma = 0$. In this case, $\Theta(\gamma)$ turns out to be negative, which enables debt to work as a stabilizing force: an increase in $\hat{r}_t^*$, which is inflationary, lowers the real value of debt. A lower debt level in turn puts downward pressure on inflation when the fiscal channel is operative. As we progressively increase γ, however, such a stabilizing effect is weakened because a positive shock to $\hat{r}_t^*$ now also increases interest payments, and thus debt, as the central bank responds to the shock by increasing the nominal rate. This is reflected in (10) by the increase of the coefficient on $\hat{r}_t^*$: $\Theta(\gamma)$ increases and eventually becomes positive when γ gets sufficiently large (e.g. $\gamma = 1$.)[19] In that case, the presence of $\hat{b}_{t-1}$ in (11) has a multiplier, rather than stabilizing, effect on inflation.

Because of the aforementioned two mechanisms, a shock has *larger* and *more persistent* effects on inflation when the central bank tracks the natural rate more closely. This finding is illustrated in Figure 3. The figure presents the impulse responses of the variables to a one percentage point transitory shock to $\hat{r}^*$ under the monetary regime (in row (a)) and under the fiscal regime (in row (b)).[20] As mentioned above, debt responds more to shocks as γ moves from 0 to 1 in either regime

[18] Note that the result, $\Gamma'(\gamma) > 0$, requires a 'sufficiently' unresponsive fiscal policy – although $\bar{\psi}^*$ and $\bar{\psi}$ are almost identical numerically.

[19] Numerically, $\Theta(\gamma)$ is in fact positive except for very small values of γ.

[20] The parameterization is described in Table 1, which is standard. We note that for the policy coefficients, we use

(shown in the third column). Under the monetary regime, where the fiscal channel is absent, such behavior of debt has no implications for inflation (shown in the first panel). By contrast, when the fiscal channel is operative in the fiscal regime, it generates larger inflation fluctuations (shown in the fifth panel). Importantly, the model shows persistent responses of inflation even to transitory shocks because the fiscal channel provides an endogenous propagation mechanism based on debt dynamics.

Let us now turn to the dynamics of the output gap given by (12). The equation reveals that the output gap $\tilde{Y}_t$, just like inflation, is a function of the current and past realizations of r-star. However, we find that $\Lambda(\gamma)$, which measures the contemporaneous effect of shocks, can be either increasing or decreasing in γ, depending on other structural parameters. One way to see this ambiguity is to notice that the output gap is determined by the difference between current inflation and (discounted) expected future inflation as shown in the Phillips curve (2). If the central bank tracks the natural rate more closely (i.e. a higher γ), current inflation responds more to $\hat{r}_t^*$, as discussed above. Future inflation, however, is also expected to respond more because more debt is carried over to the next period. While the (net) contemporaneous effect on the output gap of tracking r-star is ambiguous, the lagged effect, captured by $\Omega_Y \hat{b}_{t-1}$, is unambiguously amplified when γ is larger for the same reason laid out above. In that sense, tracking r-star also destabilizes the output gap, not just inflation. These results are illustrated in the sixth panel of Figure 3.

Figure 4 presents the variance of inflation, the output gap and the nominal rate under two regimes; they are scaled to one at $\gamma = 0$.[21] The variances are quadratic and convex in γ as the coefficients in (10)-(12) are linear in γ. It can be analytically shown that inflation volatility (measured by the variance) is minimized at γ less than one under the fiscal regime, which is expected from the propositions and Figure 3. Numerically, we find that the value of γ that minimizes inflation volatility is typically close to zero in all reasonable parameterizations.

In the example shown in Figure 4, when the central bank fully tracks r-star (i.e. $\gamma = 1$), inflation volatility is more than 400 times bigger than that under $\gamma = 0$ (shown in the fifth panel). We also find that the output gap volatility is minimized at γ less than one, yet slightly larger than the value of γ that minimizes inflation volatility. The reason is that $\Lambda(\gamma)$, the coefficient on $\hat{r}_t^*$ in (12) is often decreasing in γ, thereby producing smaller initial responses when γ is greater, as shown in the sixth

$\phi = 1.5$ and $\psi = 0.1$ under the monetary regime and $\phi = 0.5$ and $\psi = 0$ under the fiscal regime. We show sensitivity analysis on the policy coefficients later.

[21] For example, we report $\frac{\mathbb{V}_\gamma(\pi_t)}{\mathbb{V}_{\gamma=0}(\pi_t)}$, where $\mathbb{V}_\gamma(\pi_t)$ in the unconditional variance of inflation at γ.

panel of Figure 3. As stated above, however, shocks have longer-lasting effects through $\hat{b}_{t-1}$ under larger γ, thereby forcing output to deviate more persistently from its natural level. This lagged effect yields the variance of the output gap minimized at a small value of γ (as shown in the sixth panel.)

Lastly, we also consider the effect of tracking r-star on welfare. In particular, following the literature, we calculate the unconditional expectation of the welfare losses (relative to the efficient allocation) associated with given policies. It has been shown that the second order approximation for a given γ, $\mathbb{L}_\gamma$, can be expressed as a weighted sum of inflation and the output gap volatility:

$$\mathbb{L}_\gamma = \vartheta \left[\mathbb{V}_\gamma \left(\pi_t \right) + \lambda \mathbb{V}_\gamma \left(\tilde{Y}_t \right) \right], \tag{13}$$

where ϑ and λ are positive reduced-form parameters (Woodford 2003). We then report the ratio, $\mathbb{L}_\gamma / \mathbb{L}_{\gamma=0}$, in the last column of Figure 4: the welfare losses in general cases relative to the reference point where the central bank does not track r-star ($\gamma = 0$). The shape of the curves in that column resembles those of the first column because inflation volatility is dominant: λ is very small.[22] As stated above, under the conventional monetary regime, the central bank can completely stabilize inflation and the output gap simultaneously by fully tracking r-star ($\gamma = 1$) and thereby achieving the efficient allocation (shown in the fourth panel). In sharp contrast, under the fiscal regime, when the central bank fully tracks r-star (i.e. $\gamma = 1$), our measure of the welfare losses is more than 50 times bigger than that under $\gamma = 0$ (shown in the last panel).[23] In addition, in this case, there exists no γ that leads to $\hat{\pi}_t = 0 \; \forall t$, and it is infeasible to stabilize both inflation and the output gap simultaneously. "Divine coincidence" therefore does not exist in this prototype NK model – even in the absence of cost-push shocks.

The results are qualitatively the same under alternative parameterization, as implied by the propositions. For the interested readers, the appendix presents the cases in which i) the policy rules have different coefficient values (shown in Figures A.1 and A.2) and ii) r-star follows a persistent autoregressive process (shown in Figures A.3 and A.4).

In summary, our analysis in this section reinforces the general idea that any prescription for monetary policy has to take into account the prevailing fiscal policy regime. Given nonresponsive

[22] $\lambda = \frac{\kappa}{\theta}$, where θ is the elasticity of substitution between variety of goods.

[23] Numerically, we find that the value of γ that minimizes welfare losses is typically close to zero in all reasonable parameterization.

fiscal policy, the central bank will not be able to respond aggressively to inflation ($\phi > 1$); that would generate explosive inflation and output dynamics. When the central bank has to accommodate fiscal policy ($\phi < 1$), a fiscal channel is in operation and tracking r-star leads to macroeconomic instability and lower welfare. Our analysis showcases the perils of tracking r-star in the framework where tracking it is normally shown to produce the first best outcome.

3 The model with endogenous propagation mechanisms

We now consider an extension to the basic NK model we presented above. The model features some endogenous propagation mechanisms, as there is inertia in both private sector and government behavior. It is essentially the same as the ones presented in Woodford (2003) and Curdia et al. (2015) with respect to private sector behavior – a NK model featuring habits in consumption decision and partial indexation in price setting. We also introduce interest rate and tax smoothing in the model.[24]

This model, in our view, strikes a balance between empirical realism and analytical tractability, and thus serves our purpose well. On the one hand, although the model is simpler than the medium-scale dynamic equilibrium models currently used in policy making institutions, it is known to fit the time series on inflation, output and interest rate reasonably well and has been used in quantitative studies such as Curdia et al. (2015) and Bhattarai et al. (2016). On the other hand, its relative simplicity, compared to the medium-scale models, enables us to derive analytically a welfare-based loss function that is intuitive and easy to understand and conduct a normative analysis in a manner very similar to the one in the previous section. Furthermore, it also helps us to see a new insight most clearly.

The private sector equilibrium conditions as well as policy rules and government budget constraints in the model, after log-linearization, are given by:

$$x_t = \mathbb{E}_t x_{t+1} - (1-\beta\eta)\left(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}_t^*\right) \tag{14}$$

$$\hat{\pi}_t - \chi\hat{\pi}_{t-1} = \kappa_H\left[\left(\tilde{Y}_t - \delta\tilde{Y}_{t-1}\right) - \beta\delta\mathbb{E}_t\left(\tilde{Y}_{t+1} - \delta\tilde{Y}_t\right)\right] + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1} - \chi\hat{\pi}_t\right] \tag{15}$$

$$\hat{R}_t = \rho_R R_{t-1} + (1-\rho_R)\left(\gamma\hat{r}_t^* + \phi_\pi\hat{\pi}_t + \phi_Y\tilde{Y}_t\right), \tag{16}$$

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1-\rho_\tau)\left(\psi_b\hat{b}_{t-1} + \psi_Y\tilde{Y}_t\right), \tag{17}$$

[24] Since the private sector component of the model is well known, we refer the interested readers to Woodford (2003) for further details.

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\hat{\pi}_t - \beta^{-1}\hat{\tau}_t + \bar{b}\hat{R}_t.$$

where $x_t \equiv \left(\tilde{Y}_t - \eta\tilde{Y}_{t-1}\right) - \beta\eta\mathbb{E}_t\left(\tilde{Y}_{t+1} - \eta\tilde{Y}_t\right)$. The new parameters, η, χ, ρ_R and ρ_τ measure the degree of consumption habit, price indexation, interest rate and tax smoothing respectively. Moreover, $\delta \in [0, \eta]$ and κ_H are a composite of the structural parameters.[25]

Compared to the Euler equation (1) in the simple model, (14) here accounts for habit formation while compared to the Phillips curve (2) in the simple model, (15) here accounts for partial indexation of prices. In terms of policy rules, in addition to smooth changes in the instruments, we also allow the policymakers to respond to the output gap with coefficients ϕ_Y and ψ_Y, as given in (16) and (17) respectively. The government budget constraint remains unchanged from before.

In this model, the welfare relevant loss function (13) is now generalized to

$$\mathbb{L}_\gamma = \vartheta_H\left[\mathbb{V}_\gamma\left(\hat{\pi}_t - \chi\hat{\pi}_{t-1}\right) + \frac{\kappa_H}{\theta}\mathbb{V}_\gamma\left(\tilde{Y}_t - \delta\tilde{Y}_{t-1}\right)\right],$$

where ϑ_H is a reduced-form parameter. Notice the simple NK model in the previous section is a nested case of this model.

Overall, the main results on the fiscal channel shown in the previous section holds in this richer model. An interesting new insight, however, emerges due to monetary policy inertia. We consider several cases below to highlight the role of interest rate smoothing as well as the robustness of our main results on the fiscal channel.

3.1 Case I

To develop the intuition, we first set $\rho_R = 0$. This case is in the spirit of our main question, "Would tracking r-star be desirable going forward?", and moreover, also consistent with the counterfactual analysis conducted in Barsky et al (2014).

In this case, the results are the same as in the simple NK model. The degree of habit, indexation, tax smoothing, and the extent of persistence in r-star do not change the results qualitatively. For illustration, we present the results in Figures 5 and 6 where we set the new "inertia" parameters

[25] The slope of the Phillips curve is given as $\kappa_H \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\varphi\theta)}\frac{\eta}{\delta}\frac{1}{(1-\beta\eta)} = \kappa\left[\frac{\eta}{\delta}\frac{1}{(1-\beta\eta)}\frac{1}{(1+\varphi)}\right]$, and $\delta \in [0, \eta]$ is the smaller root of the quadratic equation, $\frac{\eta}{1-\beta\eta}\left(1+\beta\delta^2\right) = \left[\varphi + \frac{1+\beta\eta^2}{1-\beta\eta}\right]\delta$.

to 0.7 (i.e. $\eta = \chi = \rho_\tau = 0.7$), except for ρ_R which we fix at zero. The other parameters remain the same as in Figure 3 and 4. We also report the variance of (semi-) first difference in inflation and in the output gap that enter the welfare relevant loss function above ($\Delta_\pi \equiv \hat{\pi}_t - \chi \hat{\pi}_{t-1}$ and $\Delta_Y \equiv \tilde{Y}_t - \delta \tilde{Y}_{t-1}$).

Figures 5 and 6 reveal that fully tracking r-star (i.e. without interest smoothing) under the monetary regime achieves complete stabilization of inflation and the output gap, which leads to $\Delta_\pi = \Delta_Y = 0$. This generates the first best outcome. For the fiscal regime, where the fiscal channel is in operation, like before for the simple NK model, not tracking r-star at all is optimal.

Once again, the intuition can be obtained by substituting out the nominal rate, $\hat{R}_t$, from the system using the Taylor rule. In particular, the Euler equation (14) becomes

$$x_t = \mathbb{E}_t x_{t+1} - (1-\beta\eta)\left(\phi_\pi \hat{\pi}_t + \phi_Y \tilde{Y}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) + (1-\beta\eta)(1-\gamma)\,\hat{r}_t^*.$$

The last term above implies that fully tracking r-star (i.e. $\gamma = 1$) removes exogenous disturbances completely from the monetary bloc, which necessarily leads to full stabilization of inflation and the output gap under the monetary regime. Under the fiscal regime, the fiscal channel works in the same fashion as before, through (8).

3.2 Case II

We here allow for interest rate smoothing. In this case, strongly tracking r-star can lead to macroeconomic instability and lower welfare even under the monetary regime. It is easier to understand the reason in the case of an i.i.d r-star, which is illustrated in Figure 7. The parameters are the same as in the previous subsection, except for the interest rate smoothing parameter ρ_R; it is now set to 0.7 along with the other parameters that govern inertial private sector and fiscal behavior.

Consider the monetary regime first. With a transitory increase in r-star, the nominal rate increases less (than what is required by an increase in r-star due to the smoothing) on impact, but in the following periods, the nominal rate is persistently higher than normal, although the r-star shock is completely transitory. Because of such overshooting of the nominal rate in the subsequent periods, households expect persistently high interest rate. Then, inflation and output gap can actually go down with a large value of γ, which causes macroeconomic instability.[26] The optimal value of γ can be significantly less than one, as illustrated clearly in Figure 8. The results under the fiscal regime are overall still the same as in the simple NK model.

Another way to see the potential disadvantage of tracking r-star even in the monetary regime is by analyzing the Euler equation (14), which, after substituting out $\hat{R}_t$, can be written as:

$$x_t = \mathbb{E}_t x_{t+1} + (1-\beta\eta)\,\mathbb{E}_t\hat{\pi}_{t+1} - (1-\beta\eta)\left\{\rho_R^t R_0 + (1-\rho_R)\sum_{k=0}^{t-1}\rho_R^k\left(\phi_\pi\hat{\pi}_{t-k} + \phi_Y\tilde{Y}_{t-k}\right)\right\}$$
$$+ (1-\beta\eta)\left\{(1-(1-\rho_R)\gamma)\,\hat{r}_t^* - \rho_R(1-\rho_R)\,\gamma\sum_{k=1}^{t-1}\rho_R^k\hat{r}_{t-k}^*\right\}. \tag{18}$$

In the monetary regime, the driving forces are given by the second row of (18). As before, raising γ decreases the coefficient on the current r-star, which is stabilizing. When the central bank does interest rate smoothing however, raising γ increases the (absolute) size of the coefficients on lagged r-star. This latter effect, when γ is sufficiently large, is destabilizing through inflation and output gap expectations – the first two terms on the right-hand-side of (18).

3.3 Case III

The third case uses the same parameterization as the second case above, except that r-star now follows a persistent autoregressive process. The results are in Figures 9 and 10 where we set ρ_r to 0.8. While the results are qualitatively the same as in the previous case, the overshooting problem identified in the previous subsection is not as significant because r-star itself is persistent. The optimal value of γ is still less than one in the monetary regime, yet bigger than what we obtain under i.i.d r-star case. Importantly, once again, tracking r-star is not desired in the fiscal regime for the same reason laid out in the case of the simple NK model.

4 Conclusion

There exists a *fiscal limit* to a monetary policy regime that incorporates r-star targeting. In particular, our analysis suggests that tracking (even the *correct*) r-star will be desirable only if the public

[26] Perhaps, "tracking r-star" in this case is not a correct terminology, because the nominal rate does not track r-star period-by-period and moreover actually deviates more from r-star in later periods as γ increases.

expects a sufficient tax increase in response to government debt increases – a condition uncertain to hold in future in the U.S., based on CBO projections. Our paper constitutes a cautionary note on the policy recommendation that the Federal Reserve should track r-star going forward.

Table and Figures

Table 1: Parameter values used in the numerical analysis

Parameter	Description	Value	Note
β	Discount factor	0.99	Long-run interest rate
κ	Phillips curve slope	0.0245	Based on underlying parameter values
$\bar{b}$	Steady-state debt-GDP	0.4	U.S. data
ϕ	Inflation coefficient in monetary rule	1.5	Monetary regime
		0.5	Fiscal regime
ψ	Debt coefficient in fiscal rule	0.1	Monetary regime
		0	Fiscal regime

Notes: The table presents parameter values used in the baseline and sensitivity analysis. The source or targeted moment is described as well. The slope of the Phillips curve, κ, is implied by the relation, $\kappa = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\varphi\theta)}(1+\varphi)$, where $\alpha = 0.75$, $\varphi = 1$ and $\theta = 6$ are respectively the infrequency of price adjustment, the elasticity of labor supply, and the elasticity of substitution between varieties in the model.

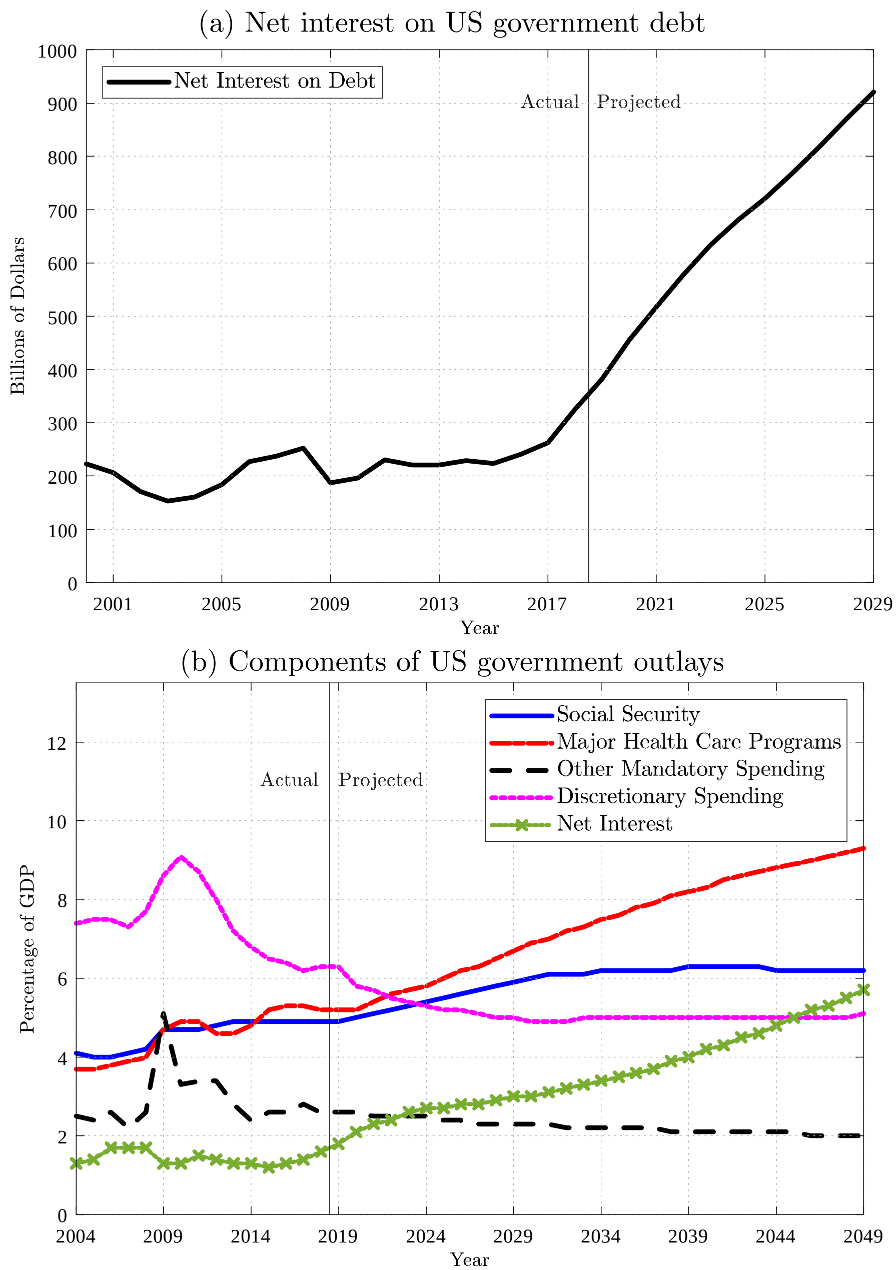

Figure 1: Net interest payments and other outlays

Notes: Panel (a) presents actual and projected net interest payments on US government debt. Panel (b) presents actual and projected components of US government outlays as % of GDP. The data source is CBO.

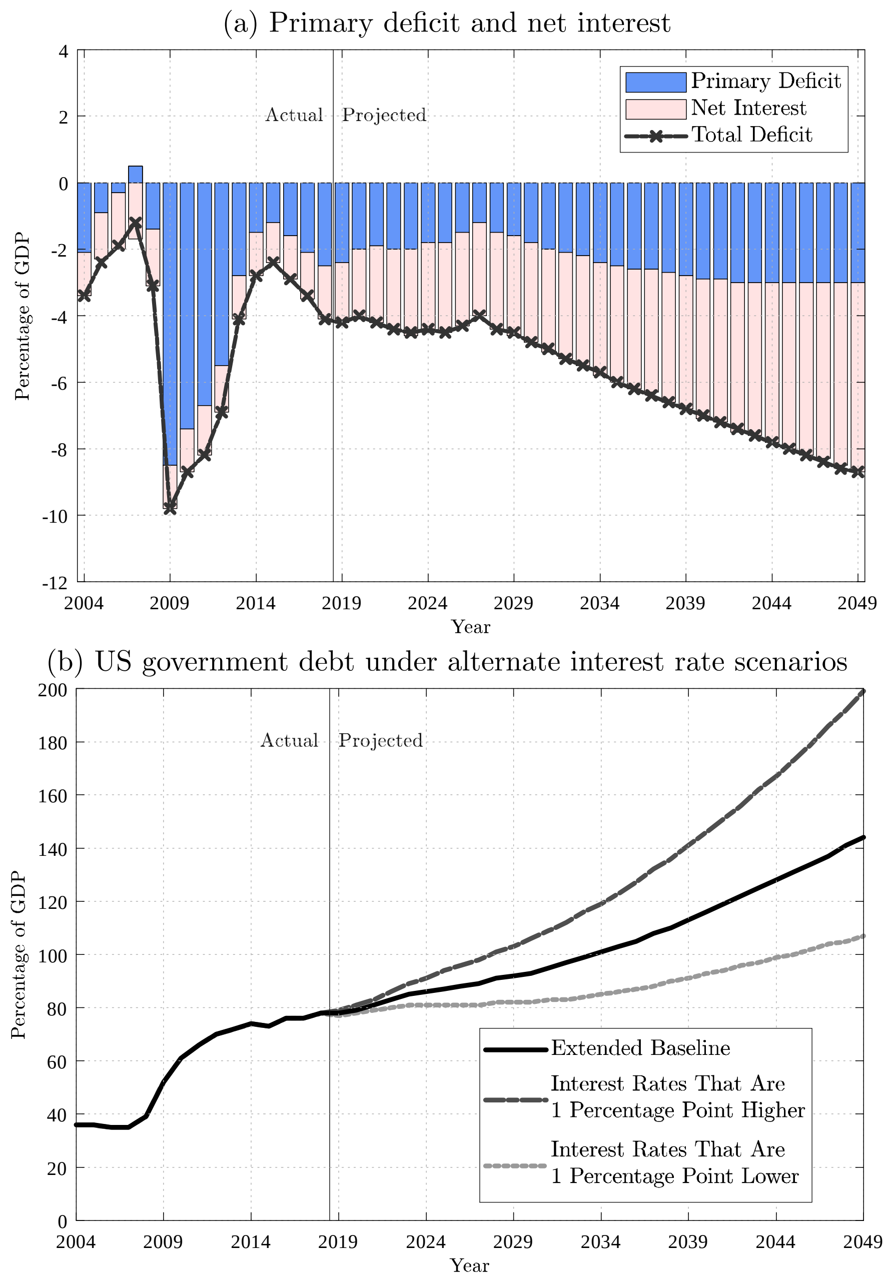

Figure 2: Effects of net interest on deficits and debt

Notes: Panel (a) presents actual and projected path of primary deficits and net interest payments as % of GDP. Panel (b) presents actual and projected US government debt to GDP ratio under different scenarios for interest rate path. The data source is CBO.

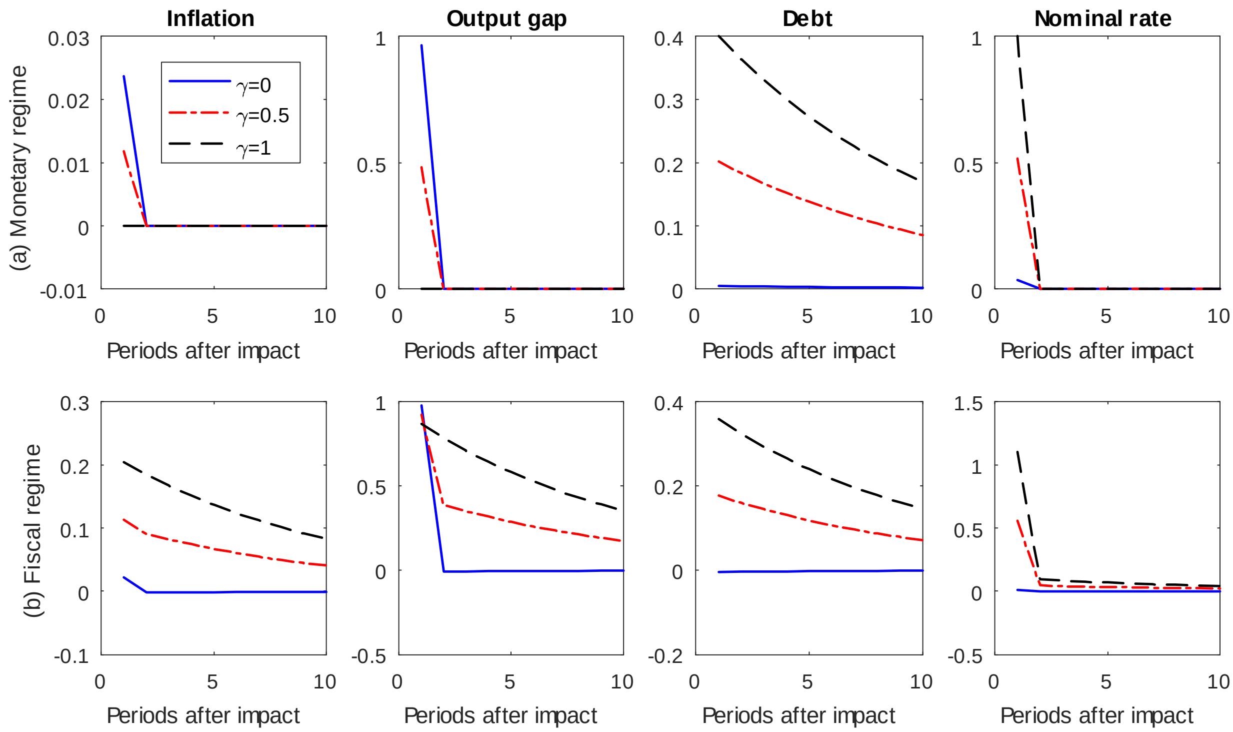

Figure 3: Impulse responses to an innovation to r-star.

Notes: The figure shows responses of inflation, output gap, debt, and nominal interest rate to a $\hat{r}^*$ shock. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The parameterization of the model is given in detail in Table 1. We use policy coefficients of $\phi = 1.5$ and $\psi = 0.1$ for the monetary regime and $\phi = 0.5$ and $\psi = 0$ for the fiscal regime. The shock size is one percentage point.

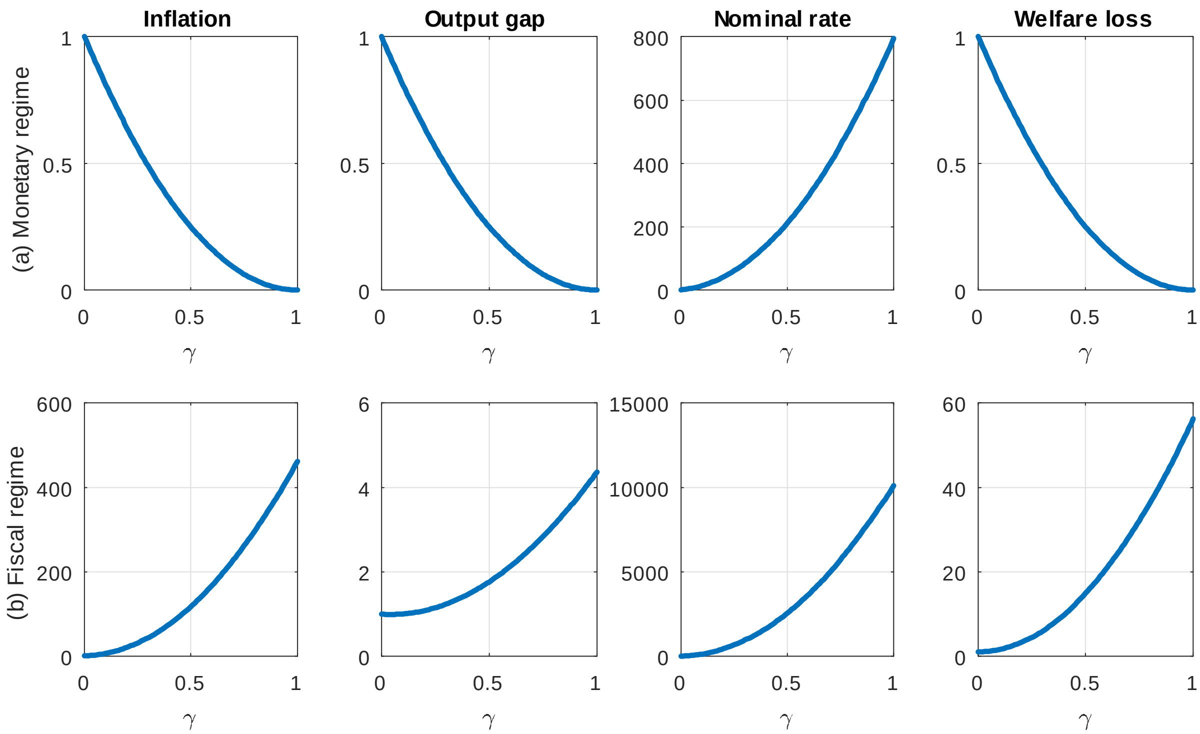

Figure 4: Relative variance and welfare loss.

Notes: The figure shows variances of inflation, output gap, and nominal interest rate as well as welfare loss. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The variances are in relative terms, where the variances at $\gamma = 0$ have been normalized to 1. The welfare loss is compared to the case where $\gamma = 0$. The parameterization of the model is given in detail in Table 1. We use policy coefficients of $\phi = 1.5$ and $\psi = 0.1$ for the monetary regime and $\phi = 0.5$ and $\psi = 0$ for the fiscal regime.

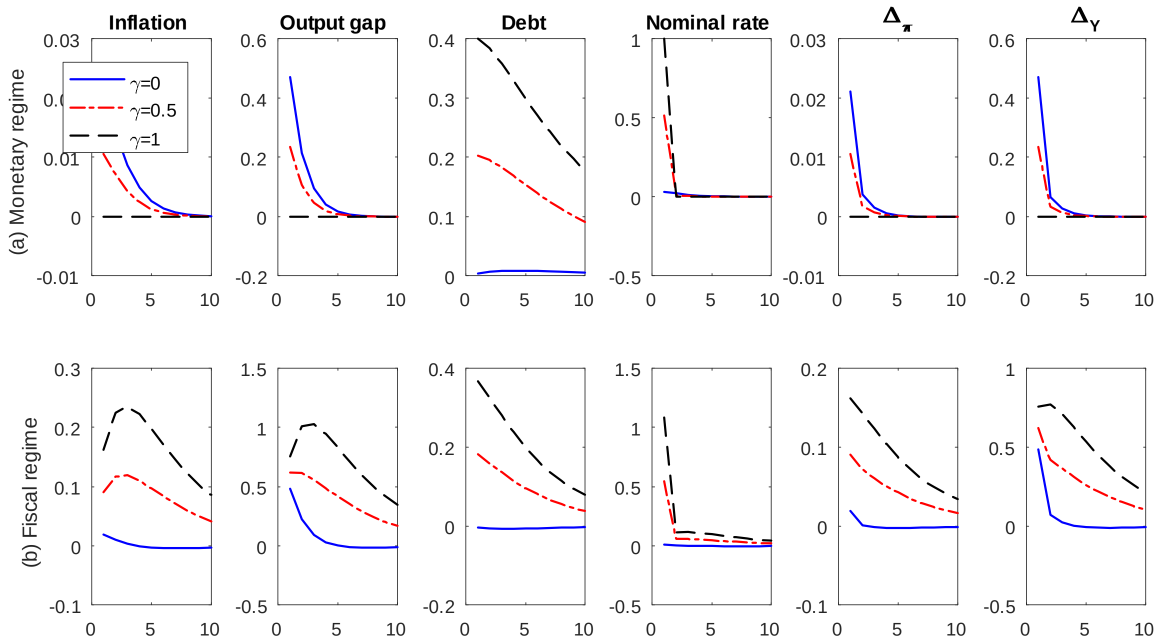

Figure 5: Impulse responses to an innovation to r-star in the model with inertia but no interest rate smoothing.

Notes: The figure shows responses to a $\hat{r}^*$ shock in the model with inertia but no interest rate smoothing in the monetary policy rule. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The parameterization of the model is described in the text.

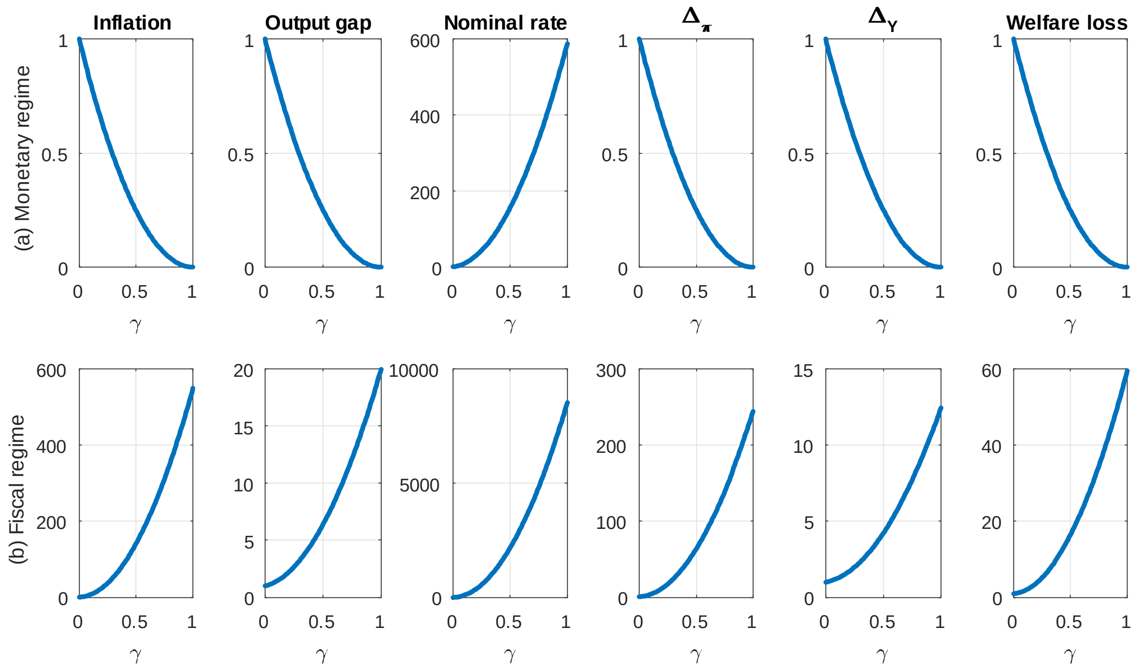

Figure 6: Relative variance and welfare loss in the model with inertia but no interest rate smoothing.

Notes: The figure shows variances as well as welfare loss in the model with inertia but no interest rate smoothing. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The variances are in relative terms, where the variances at $\gamma = 0$ have been normalized to 1. The welfare loss is compared to the case where $\gamma = 0$. The parameterization of the model is described in the text.

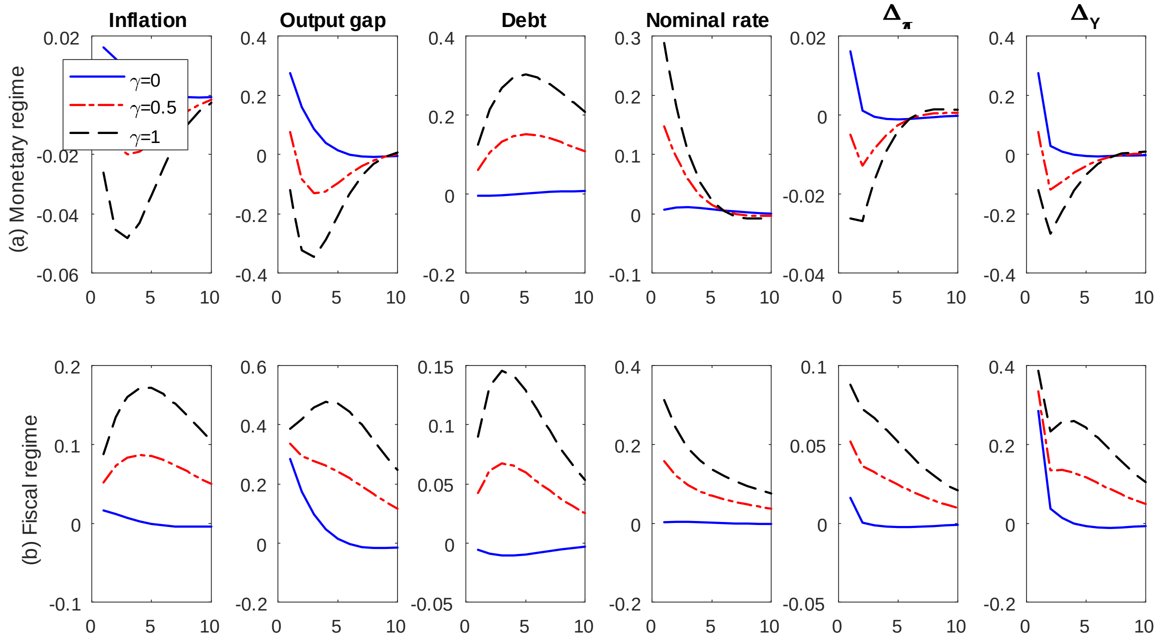

Figure 7: Impulse responses to an innovation to r-star in the model with inertia and interest rate smoothing where r-star is iid.

Notes: The figure shows responses to a $\hat{r}^*$ shock in the model with inertia and interest rate smoothing where $\hat{r}^*$ is iid. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The parameterization of the model is described in the text.

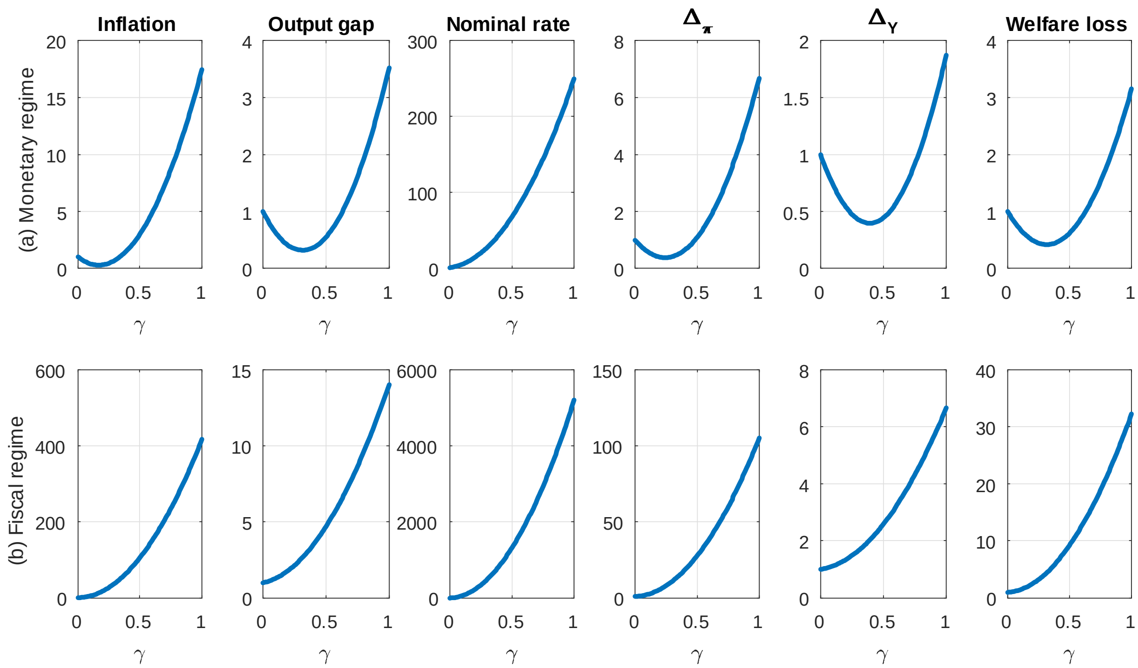

Figure 8: Relative variance and welfare loss in the model with inertia and interest rate smoothing where r-star is iid.

Notes: The figure shows variances as well as welfare loss in the model with inertia and interest rate smoothing where $\hat{r}^*$ is iid. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The variances are in relative terms, where the variances at $\gamma = 0$ have been normalized to 1. The welfare loss is compared to the case where $\gamma = 0$. The parameterization of the model is described in the text.

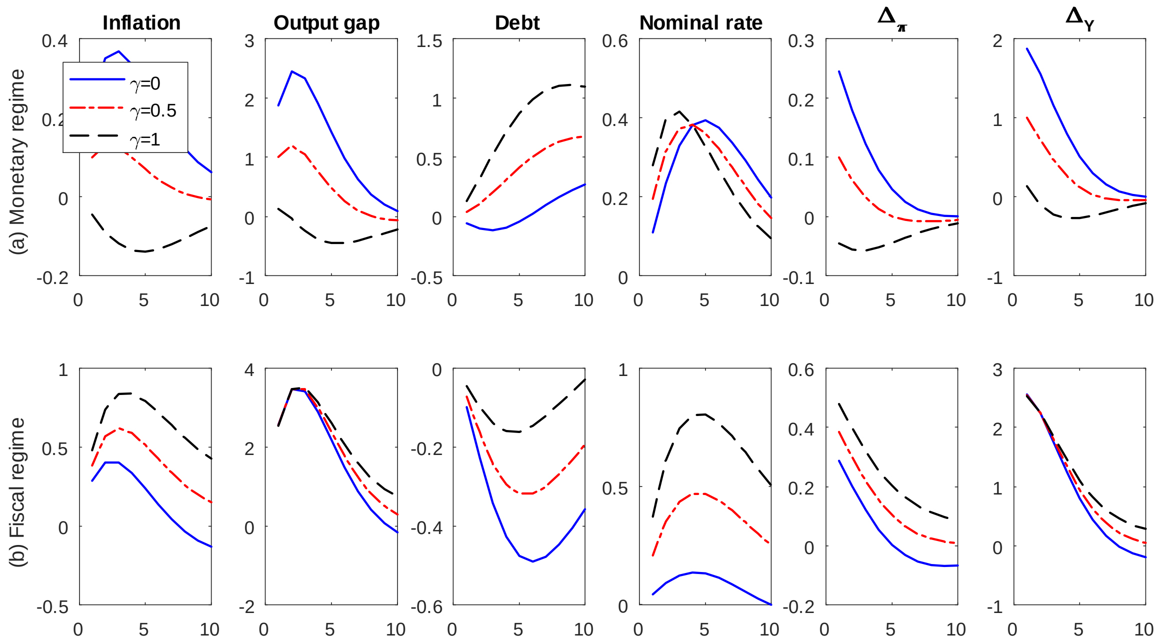

Figure 9: Impulse responses to an innovation to r-star in the model with inertia and interest rate smoothing where r-star is persistent.

Notes: The figure shows responses to a $\hat{r}^*$ shock in the model with inertia and interest rate smoothing where $\hat{r}^*$ is persistent. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The parameterization of the model is described in the text.

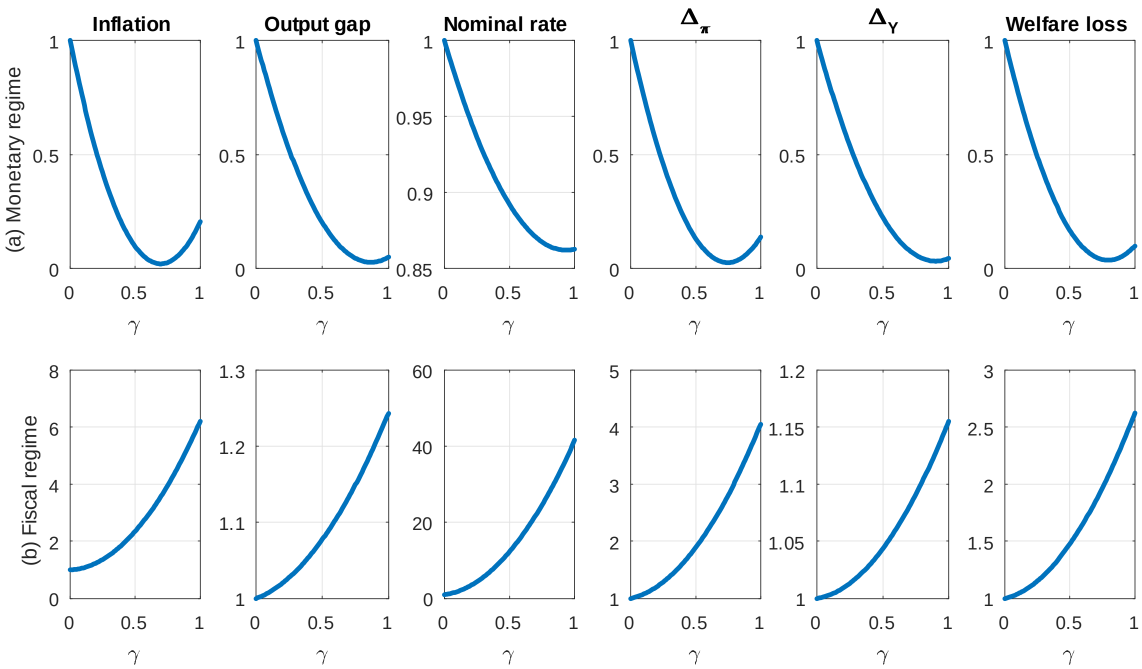

Figure 10: Relative variance and welfare loss in the model with inertia and interest rate smoothing where r-star is persistent.

Notes: The figure shows variances as well as welfare loss in the model with inertia and interest rate smoothing where $\hat{r}^*$ is persistent. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The variances are in relative terms, where the variances at $\gamma = 0$ have been normalized to 1. The welfare loss is compared to the case where $\gamma = 0$. The parameterization of the model is described in the text.

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Appendix

A Additional figures

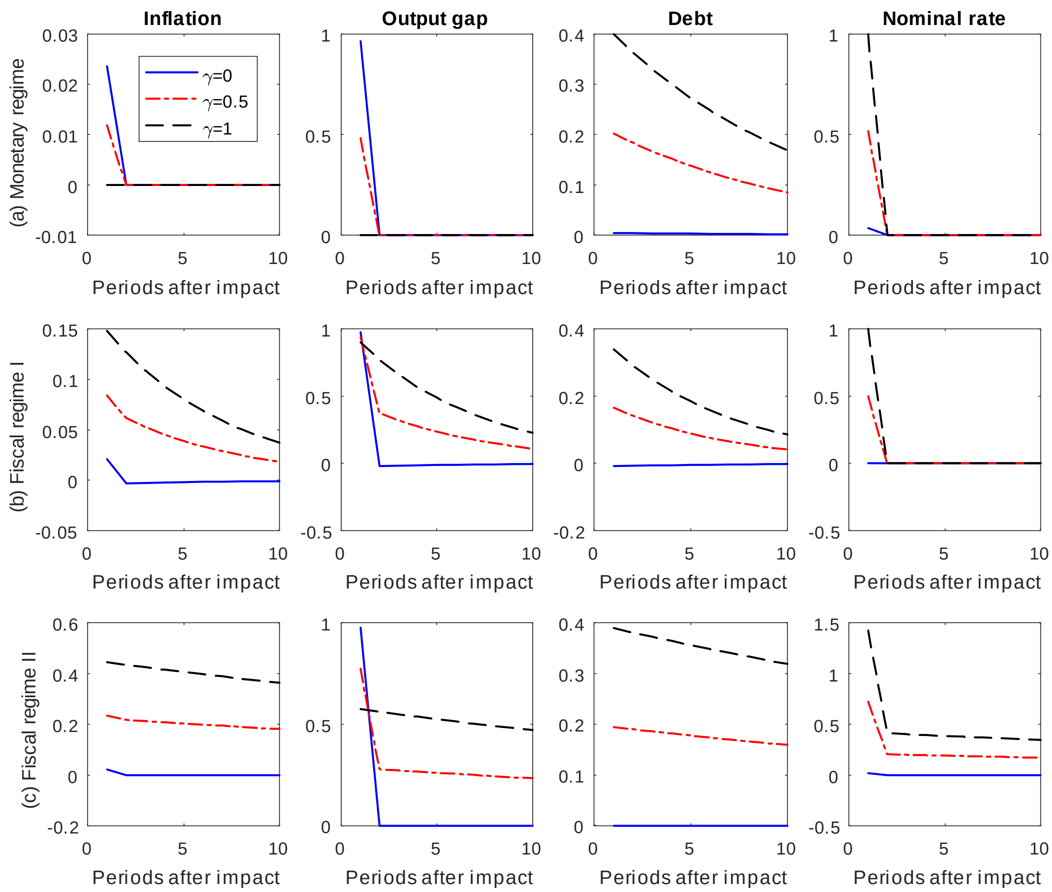

Figure A.1: Impulse responses to an innovation to r-star (different policy rule parameterizations).

Notes: The figure shows responses of inflation, output gap, debt, and nominal interest rate to a $\hat{r}^*$shock. Row (a) shows results for the monetary regime. Rows (b) and (c) show results for alternate parameterizations of the fiscal regime. We use policy coefficients of $\phi = 1.5$ and $\psi = 0.1$ for the monetary regime. Row (b) shows results for the fiscal regime in which $\phi = 0$ and $\psi = 0$. Row (c) shows results for the fiscal regime in which $\phi = 0.95$ and $\psi = 0.005$. The parameterization of the rest of the model is given in detail in Table 1. The shock size is one percentage point.

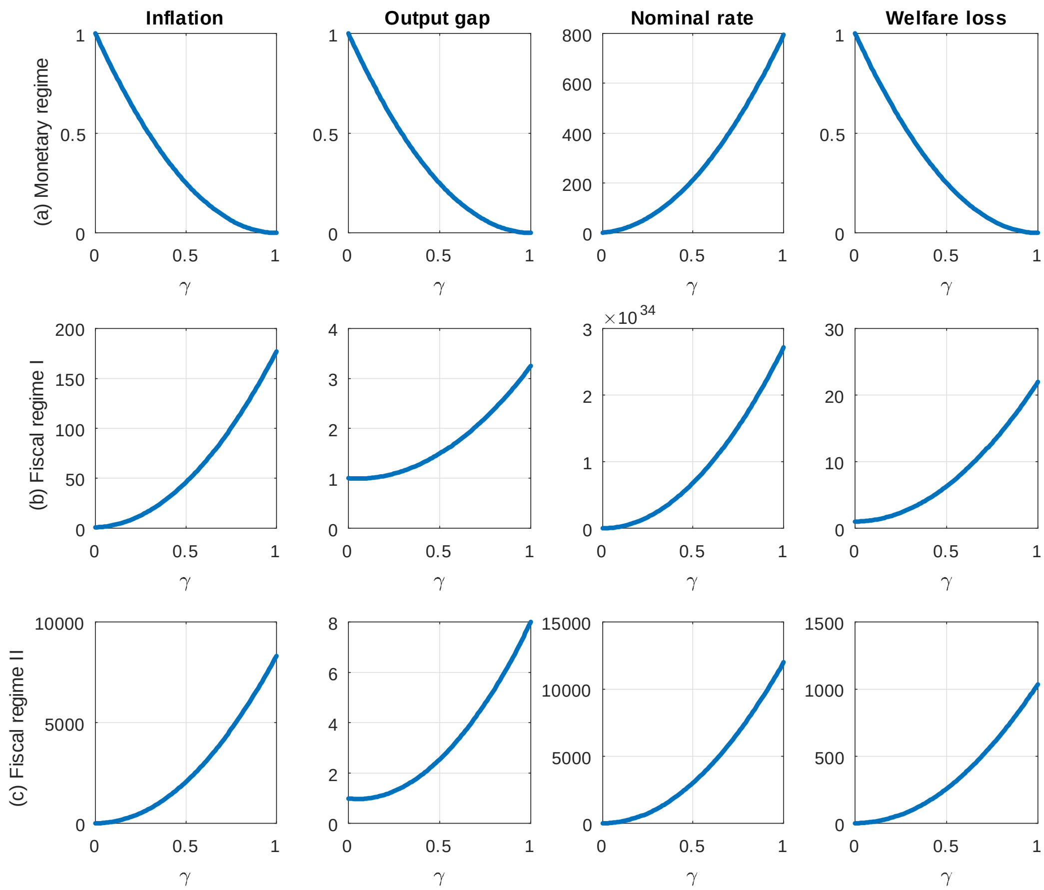

Figure A.2: Relative variance and welfare loss (different policy rule parameterizations).

Notes: The figure shows variances of inflation, output gap, and nominal interest rate as well as welfare loss. Row (a) shows results for the monetary regime. Rows (b) and (c) show results for alternate parameterizations of the fiscal regime. We use policy coefficients of $\phi = 1.5$ and $\psi = 0.1$ for the monetary regime. Row (b) shows results for the fiscal regime in which $\phi = 0$ and $\psi = 0$. Row (c) shows results for the fiscal regime in which $\phi = 0.95$ and $\psi = 0.005$. The variances are in relative terms, where the variances at $\gamma = 0$ have been normalized to 1. The welfare loss is compared to the case where $\gamma = 0$. The parameterization of the rest of the model is given in detail in Table 1.

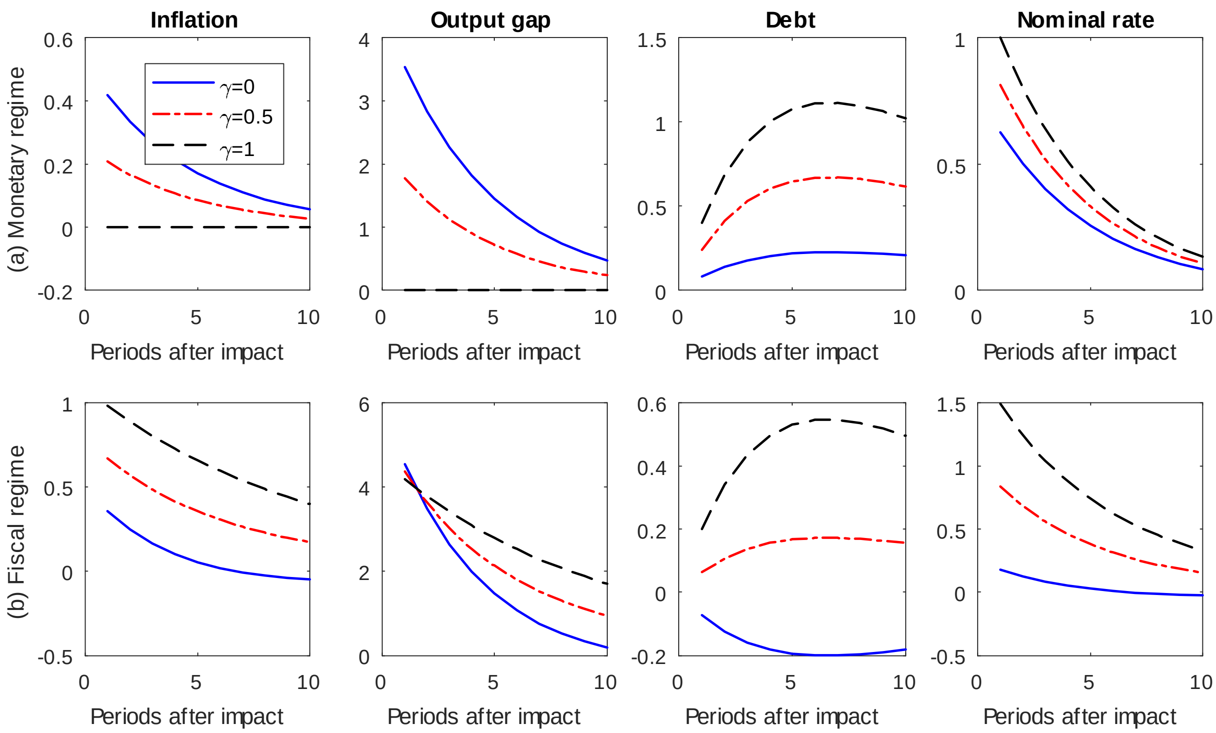

Figure A.3: Impulse responses to an innovation to r-star when r-star is a persistent process.

Notes: The figure shows responses of inflation, output gap, debt, and nominal interest rate to a $\hat{r}^*$ shock . Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. We use policy coefficients of $\phi = 1.5$ and $\psi = 0.1$for the monetary regime and $\phi = 0.5$ and $\psi = 0$ for the fiscal regime. $\hat{r}^*_t$ follows an AR(1) process, $\hat{r}^*_t = \rho_r \hat{r}^*_{t-1} + \varepsilon_t$ with $\rho_r = 0.8$. The shock size is one percentage point. The parameterization of the rest of the model is given in Table 1.

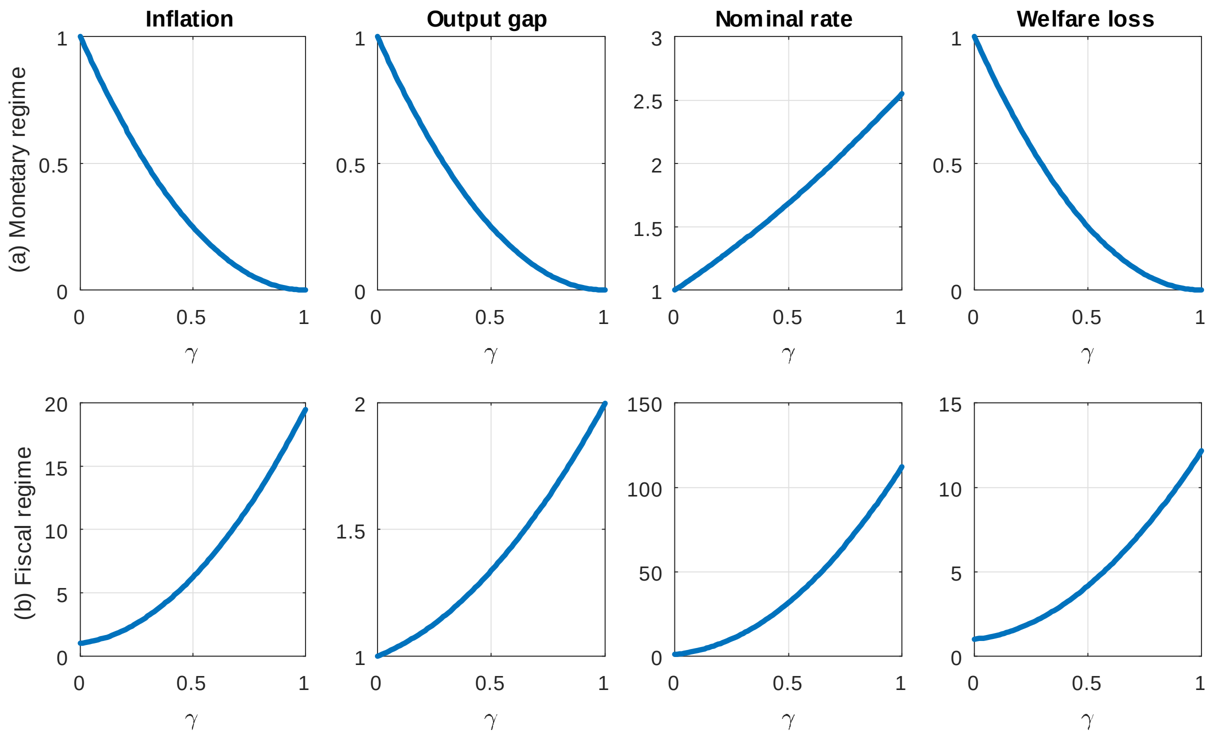

Figure A.4: Relative variance and welfare loss when r-star is a persistent process.

Notes: The figure shows variances of inflation, output gap, and nominal interest rate as well as welfare loss. Row (a) shows results for the monetary regime. Row (b) shows results for the fiscal regime. The variances are in relative terms, where the variances at $\gamma = 0$ have been normalized to 1. The welfare loss is compared to the case where $\gamma = 0$. We use policy coefficients of $\phi = 1.5$ and $\psi = 0.1$ for the monetary regime and $\phi = 0.5$ and $\psi = 0$ for the fiscal regime. $\hat{r}_t^*$ follows an AR(1) process, $\hat{r}_t^* = \rho_r \hat{r}_{t-1}^* + \varepsilon_t$ with $\rho_r = 0.8$. The parameterization of the rest of the model is given in detail in Table 1.

B The model

B.1 Households

Identical households choose sequences of $\{C_t, B_t, N_t, D_{t+1}\}$ to solve:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left[\log C_t - \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di\right]$$

subject to

$$P_t C_t + B_t + \mathbb{E}_t \left[Q_{t,t+1} D_{t+1}\right] = R_{t-1} B_{t-1} + D_t + \int_0^1 W_t(i) N_t(i) di + \Pi_t - P_t \tau_t,$$

where C_t is consumption, $N_t(i)$ is labor hours supplied to firm i, P_t is the price level, B_t is the amount of one-period risk-less nominal government bond, R_t is the gross nominal interest rate, W_t is the nominal wage rate, Π_t is profits of intermediate firms, and τ_t is government taxes net of transfers. The parameter, $\varphi \geq 0$, denotes the inverse of the Frisch elasticity of labor supply, while d_t represents an intertemporal preference shock. In addition to the government bond, households trade at time t one-period state-contingent nominal securities D_{t+1} at price $Q_{t,t+1}$.

B.2 Firms

Perfectly competitive firms produce the final good, Y_t, by assembling intermediate goods, $Y_t(i)$, through a constant-elasticity-of-substitution (CES) technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ denotes the elasticity of substitution between intermediate goods. The corresponding price index for the final consumption good is

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}},$$

where $P_t(i)$ is the price of the intermediate good i. The optimal demand for $Y_t(i)$ is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t.$$

Monopolistically competitive firms produce intermediate goods using the production function, $Y_t(i) = a_t N_t(i)$, where $N_t(i)$ denotes the labor hours employed by firm i and a_t represents exogenous economy-wide productivity. Prices are sticky. A firm adjusts its price, $P_t(i)$, with probability $1-\alpha$ each period, to maximize the present discounted value of future profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left[P_t(i) - \frac{W_{t+k}}{A_{t+k}}\right] Y_{t+k}(i).$$

B.3 Government

Each period, the government collects lump-sum tax revenues τ_t and issues one-period nominal bonds B_t to finance its consumption G_t, and interest payments. Accordingly, the flow budget constraint is given by:

$$\frac{B_t}{P_t} = R_{t-1} \frac{B_{t-1}}{P_t} + G_t - \tau_t.$$

For simplicity, we assume $G_t = 0$. The flow budget constraint can be rewritten as:

$$R_t^{-1} b_t = b_{t-1} \frac{1}{\pi_t} - \tau_t,$$

where $b_t \equiv R_t \frac{B_t}{P_t}$ denotes the real maturity value of government debt.

Monetary and fiscal policies are described by simple rules. The central bank sets the nominal interest rate according to:

$$\frac{R_t}{\bar{R}} = \left(r_t^*\right)^{\gamma} \pi_t^{\phi},$$

where $\bar{R}$ is the steady-state value of R_t. Similarly, the fiscal authority sets the tax revenues according to:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\psi},$$

where $\bar{\tau}$ and $\bar{b}$ are respectively the steady state value of τ_t and b_t.

C Approximate model

C.1 Log-linear approximation

We log-linearize the equilibrium conditions around the non-stochastic steady state with values $\left\{\bar{\pi},\ \bar{Y},\ \bar{R},\ \bar{b},\ \bar{\tau}\right\}$. In particular, we assume that inflation is zero in the steady state: $\bar{\pi} = 0$. Since the log-linearized model is standard, we omit a detailed derivation. The approximate model is characterized by the following equations:

$$\begin{aligned}
\hat{Y}_t &= \mathbb{E}_t \hat{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) - \mathbb{E}_t[\Delta \hat{d}_{t+1}], \\
\hat{\pi}_t &= \kappa \left(\hat{Y}_t - \hat{Y}_t^n\right) + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \\
\hat{R}_t &= \gamma \hat{r}_t^* + \phi \hat{\pi}_t, \\
\hat{\tau}_t &= \psi \hat{b}_{t-1}, \\
\hat{b}_t &= \beta^{-1} \hat{b}_{t-1} - \beta^{-1} \bar{b} \hat{\pi}_t - \beta^{-1} \hat{\tau}_t + \bar{b} \hat{R}_t.
\end{aligned}$$

In the equations above, we use $\hat{X}_t$ to denote the log deviation of a variable X_t from its steady state $\bar{X}$ ($\hat{X}_t = \ln X_t - \ln \bar{X}$), except for two fiscal variables, $\hat{b}_t$ and $\hat{\tau}_t$. Following Woodford (2003), we let them represent respectively the deviation of the maturity value of government debt and of government tax revenues (net of transfers) from their steady-state levels, measured as a percentage of steady-state output: $\hat{b}_t = \left(b_t - \bar{b}\right)/\bar{Y}$ and $\hat{\tau}_t = \left(\tau_t - \bar{\tau}\right)/\bar{Y}$. In our simple model, (the log-deviation of) the natural level of output and the slope of the Phillips curve are respectively given as $\hat{Y}_t^* = \hat{a}_t$ and $\kappa = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\varphi\theta)}(1+\varphi)$. The coefficient on the real rate in the first equation is unity because, we assume log-utility on consumption.

The model can be reduced to a dynamic system of $\left\{\hat{\pi}_t, \hat{b}_t, \tilde{Y}_t\right\}$:

$$\begin{aligned}
\tilde{Y}_t &= \mathbb{E}_t \tilde{Y}_{t+1} - \phi \hat{\pi}_t + \mathbb{E}_t \hat{\pi}_{t+1} + (1-\gamma)\, \hat{r}_t^*, \\
\hat{\pi}_t &= \kappa \tilde{Y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \\
\hat{b}_t &= \beta^{-1}(1-\psi) \hat{b}_{t-1} - \bar{b}\left(\beta^{-1} - \phi\right) \hat{\pi}_t + \bar{b} \gamma \hat{r}_t^*,
\end{aligned} \tag{C.1}$$

where $\tilde{Y}_t \equiv \hat{Y}_t - \hat{Y}_t^*$ represents the output gap and $\hat{r}_t^*$ is given as:

$$\hat{r}_t^* = \mathbb{E}_t\left[\Delta \hat{a}_{t+1}\right] - \mathbb{E}_t[\Delta \hat{d}_{t+1}].$$

Note that demand-type shocks raise $\hat{r}_t^*$, while supply-type shocks lower $\hat{r}_t^*$. The natural rate follows an AR(1) process:

$$\hat{r}_t^* = \rho_r \hat{r}_{t-1}^* + \varepsilon_{r,t}.$$

C.2 Diagonalization

The solution of the system (C.1) can be obtained through standard methods. First, we write (C.1) in a matrix form:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbb{E}_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} 1 & \phi & 0 \\ -\kappa & 1 & 0 \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) & \beta^{-1}(1-\psi) \end{pmatrix} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -(1-\gamma) \\ 0 \\ \bar{b}\gamma \end{pmatrix} r_t^*.$$

Pre-multiplying $\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$ yields:

$$\mathbb{E}_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \underbrace{\begin{pmatrix} \kappa\beta^{-1}+1 & \phi-\beta^{-1} & 0 \\ -\kappa\beta^{-1} & \beta^{-1} & 0 \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) & \beta^{-1}(1-\psi) \end{pmatrix}}_{=G} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -(1-\gamma) \\ 0 \\ \bar{b}\gamma \end{pmatrix} r_t^*.$$

We then diagonalize the system using an eigenvalue decomposition of the coefficient matrix, G, and rewrite the system as

$$V^{-1}\mathbb{E}_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} V^{-1} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + V^{-1} \begin{pmatrix} -(1-\gamma) \\ 0 \\ \bar{b}\gamma \end{pmatrix} r_t^*, \tag{C.2}$$

where the columns of V are the eigenvectors, and

$$\begin{aligned} e_1 &= \frac{1}{2\beta}\left(\beta+\kappa+1+\sqrt{(\beta+\kappa+1)^2-4\beta(1+\kappa\phi)}\right), \\ e_2 &= \beta^{-1}(1-\psi), \\ e_3 &= \frac{1}{2\beta}\left(\beta+\kappa+1-\sqrt{(\beta+\kappa+1)^2-4\beta(1+\kappa\phi)}\right), \end{aligned}$$

are the eigenvalues of G. One can show that V and V^{-1} have the form of:

$$\begin{aligned} V &= \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ 1 & 1 & 1 \end{pmatrix}, \\ V^{-1} &= \begin{pmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix}, \end{aligned}$$

where the elements are nonlinear functions of the model parameters. Finally, we define $X_t \equiv \begin{pmatrix} x_{1,t} & x_{2,t} & x_{3,t} \end{pmatrix}^T \equiv V^{-1}\begin{pmatrix} \tilde{Y}_t & \hat{\pi}_t & \hat{b}_{t-1} \end{pmatrix}^T$, and rewrite the system as:

$$\mathbb{E}_t X_{t+1} = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} X_t + \begin{pmatrix} -q_{11}(1-\gamma) \\ -q_{21}(1-\gamma)+\bar{b}\gamma \\ -q_{31}(1-\gamma) \end{pmatrix} \hat{r}_t^*. \tag{C.3}$$

The elements of X_t are given by:

$$\begin{aligned} x_{1,t} &= q_{11}\tilde{Y}_t + q_{12}\hat{\pi}_t, \\ x_{2,t} &= q_{21}\tilde{Y}_t + q_{22}\hat{\pi}_t + \hat{b}_{t-1}, \end{aligned}$$

$$x_{3,t} = q_{31}\tilde{Y}_t + q_{32}\hat{\pi}_t.$$

D Proofs

We now provide (terse) proofs of the propositions in the paper. The next section provides a more detailed proof and discussion of the mechanisms for a case where r-star follows an autoregressive process.

D.1 Proof of Proposition 1

When $\phi \in [0,1)$ and $\psi \in (-\infty, \bar{\psi})$ (i.e. under the fiscal regime), $e_1 > 1$, $e_2 > 1$ and $e_3 \in (0,1)$, and thus the first two rows in (C.3) provide linear restrictions. We use these restrictions to solve for $\hat{\pi}_t$:

$$\hat{\pi}_t = \underbrace{\frac{\beta(e_2 - e_3)}{\bar{b}(1-\beta\phi)}}_{\Omega_\pi}\hat{b}_{t-1} + \underbrace{\left[\frac{\kappa\beta^{-1}}{e_1 e_2} + \gamma\left(\frac{\Omega_\pi \bar{b} e_1 - \kappa\beta^{-1}}{e_1 e_2}\right)\right]}_{\Gamma(\gamma)}\hat{r}_t^*. \tag{D.1}$$

Next, we plug (D.1) into (8) to obtain the law of motion for $\hat{b}_t$:

$$\hat{b}_t = e_3\hat{b}_{t-1} + \underbrace{\bar{b}\left(\gamma\frac{e_1 e_3 + \kappa\beta^{-1}(\beta^{-1} - \phi)}{e_1 e_2} - \frac{\kappa\beta^{-1}(\beta^{-1}-\phi)}{e_1 e_2}\right)}_{\Theta(\gamma)}\hat{r}_t^*. \tag{D.2}$$

Finally, we use (7) to solve for $\tilde{Y}_t$:

$$\tilde{Y}_t = \underbrace{\kappa^{-1}\Omega_\pi(1-\beta e_3)}_{\Omega_Y}\hat{b}_{t-1} + \underbrace{\kappa^{-1}\left[\Gamma(\gamma) - \beta\Omega_\pi\Theta(\gamma)\right]}_{\Lambda(\gamma)}\hat{r}_t^*. \tag{D.3}$$

Given $\phi \in [0,1)$, $\psi \in (-\infty, \bar{\psi})$, $e_1 > 1$, $e_2 > 1$ and $e_3 \in (0,1)$, $\Omega_b = e_3$, $\Omega_\pi = \frac{\beta(e_2-e_3)}{\bar{b}(1-\beta\phi)}$ and $\Omega_Y = \kappa^{-1}\frac{\beta(e_2-e_3)}{\bar{b}(1-\beta\phi)}(1-\beta e_3)$ are positive and independent of γ, whereas $\Theta(\gamma)$, $\Gamma(\gamma)$ and $\Lambda(\gamma)$ are linear functions of γ. Let us consider $\Lambda(\gamma)$:

$$\begin{aligned}\Lambda(\gamma) &= \kappa^{-1}\left[\Gamma(\gamma) - \beta\Omega_\pi\Theta(\gamma)\right] \\ &= \frac{\beta^{-1}}{e_1 e_2} + \beta\Omega_\pi\bar{b}\frac{\beta^{-1}(\beta^{-1}-\phi)}{e_1 e_2} + \gamma\left[\left(\frac{\Omega_\pi\bar{b}e_1\kappa^{-1} - \beta^{-1}}{e_1 e_2}\right) - \beta\Omega_\pi\bar{b}\frac{e_1 e_3\kappa^{-1} + \beta^{-1}(\beta^{-1}-\phi)}{e_1 e_2}\right].\end{aligned}$$

Notice that $\Lambda(0) = \frac{\kappa\beta^{-1}}{e_1 e_2} + \beta\Omega_\pi\bar{b}\frac{\kappa\beta^{-1}(\beta^{-1}-\phi)}{e_1 e_2} > 0$ and $\Lambda(1) = \Omega_\pi\bar{b}\kappa^{-1}\left(\frac{1-\beta e_3}{e_2}\right) > 0$. Since $\Lambda(\gamma)$ is linear in γ, we must have $\Lambda(\gamma) > 0$ on $[0,1]$. Similarly, $\Gamma(\gamma) > 0$ on $\gamma \in [0,1]$.

D.2 Proof of Proposition 2

From (D.1)-(D.3), it is easy to see that $\Theta(0) < 0$, $\Gamma(0) > 0$, and $\Lambda(0) > 0$. Moreover, $\Theta'(\gamma) = \bar{b}\frac{e_1 e_3 + \kappa\beta^{-1}(\beta^{-1}-\phi)}{e_1 e_2} > 0$. We now turn to $\Gamma'(\gamma) = \frac{\beta(e_2-e_3)e_1 - \kappa\beta^{-1}(1-\beta\phi)}{e_1 e_2(1-\beta\phi)}$. Rearranging terms yields

$$\Gamma'(\gamma) = \frac{\left[\beta^2(1-e_3) + \beta + \beta\kappa\right]e_2 - (\beta+\kappa)}{e_1 e_2(1-\beta\phi)}.$$

Since the denominator is unambiguously positive for all parameter values under the fiscal regime, $\Gamma'(\gamma) > 0$ if and only if the numerator is also positive. The numerator is a linear and increasing function of e_2 because the slope, $\left[\beta^2(1-e_3) + \beta + \beta\kappa\right]$, is positive. This implies $\Gamma'(\gamma) > 0$ for sufficiently large e_2 – or for sufficiently small ψ; that is, when $\psi < \bar{\psi}^*$ where $\bar{\psi}^* \equiv \frac{\beta e_1 - (\beta+\kappa)}{\beta e_1}$. It remains to show that $\bar{\psi}^*$ is positive. The denominator of $\bar{\psi}^*$ is positive. Consider the numerator, $g(\phi) \equiv \beta e_1 - (\beta + \kappa)$. Given other parameters, $g(\phi)$ has the smallest value at $\phi = 1$ because $g'(\phi) < 0$. Evaluate $g(\phi)$ at $\phi = 1$:

$$g(1) = \beta e_1 - (\beta + \kappa) = \beta \left(\frac{\kappa + 1}{\beta} \right) - (\beta + \kappa) = 1 - \beta > 0,$$

which implies $\bar{\psi}^* > 0$. Finally, redefining $\bar{\psi}^*$ as $\bar{\psi}^* \equiv \min\left\{\bar{\psi}^*, \bar{\psi}\right\}$, we establish that:

$$\Gamma'(\gamma) > 0 \text{ for } 0 < \bar{\psi}^* \leq \bar{\psi} \equiv 1 - \beta.$$

E Solution and discussions in the case of persistent shocks

This section provides a more detailed proof and discussion of the mechanisms for a case where r-star follows an autoregressive process.

E.1 Monetary regime

Under monetary regime, e_1 and e_3 are outside the unit circle, while e_2 is inside the circle. The system thus has a unique stable solution. We use the method of undetermined coefficients and obtain:

$$\hat{\pi}_t = \Gamma_M(\gamma)\, \hat{r}_t^* \quad \text{and} \quad \tilde{Y}_t = \Lambda_M(\gamma)\, \hat{r}_t^*$$

where

$$\Gamma_M(\gamma) \equiv \frac{\kappa(1-\gamma)}{\kappa(\phi - \rho_r) + (1-\rho_r)(1-\beta\rho_r)},$$
$$\Lambda_M(\gamma) \equiv \frac{(1-\gamma)(1-\beta\rho_r)}{\kappa(\phi - \rho_r) + (1-\rho_r)(1-\beta\rho_r)}.$$

The solution given in the main text is a particular case in which $\rho_r = 0$. The central bank can stabilize simultaneously inflation and the output gap (i.e. $\hat{\pi}_t = \tilde{Y}_t = 0$) by fully tracking r-star (i.e. $\gamma = 1$).

E.2 Fiscal regime

We now turn to the case in which $\phi \in [0, 1)$ and $\psi \in \left(-\infty, \bar{\psi}\right)$ where $\bar{\psi} \equiv 1 - \beta$ is the upper bound for the regime. In this case, it can be shown that $e_1 > 1$, $e_2 > 1$ and $e_3 \in (0, 1)$. The first two rows in (C.3) thus provide linear restrictions:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1}\right)^k \mathbb{E}_t z_{1,t+k}^*, \tag{E.1}$$

$$x_{2,t} = \frac{1}{e_2} \sum_{k=0}^{\infty} \left(\frac{1}{e_2}\right)^k \mathbb{E}_t z_{2,t+k}^*, \tag{E.2}$$

where

$$z_{1,t}^* = q_{11}(1-\gamma)\, \hat{r}_t^*,$$
$$z_{2,t}^* = \left[q_{21}(1-\gamma) - \bar{b}\gamma\right] \hat{r}_t^*.$$

The equations above imply:

$$\mathbb{E}_t z_{1,t+k}^* = q_{11}(1-\gamma)\, \rho_r^k \hat{r}_t^*$$
$$\mathbb{E}_t z_{2,t+k}^* = \left[q_{21}(1-\gamma) - \bar{b}\gamma\right] \rho_r^k \hat{r}_t^*$$

Plugging these equations into (E.1) and (E.2), we obtain:

$$x_{1,t} = \frac{1}{e_1}\sum_{k=0}^{\infty}\left(\frac{1}{e_1}\right)^k \mathbb{E}_t z^*_{1,t+k} = q_{11}(1-\gamma)\frac{1}{e_1-\rho_r}\hat{r}^*_t \tag{E.3}$$

$$x_{2,t} = \frac{1}{e_2}\sum_{k=0}^{\infty}\left(\frac{1}{e_2}\right)^k \mathbb{E}_t z^*_{2,t+k} = \left[q_{21}(1-\gamma)-\bar{b}\gamma\right]\frac{1}{e_2-\rho_r}\hat{r}^*_t \tag{E.4}$$

Equation (E.3) implies:

$$\tilde{Y}_t = -\frac{q_{12}}{q_{11}}\hat{\pi}_t + \frac{1-\gamma}{e_1-\rho_r}\hat{r}^*_t \tag{E.5}$$

We plug (E.5) into (E.4) to get:

$$q_{21}\left[-\frac{q_{12}}{q_{11}}\hat{\pi}_t + \frac{1-\gamma}{e_1-\rho_r}\hat{r}^*_t\right] + q_{22}\hat{\pi}_t + \hat{b}_{t-1} = \left[q_{21}(1-\gamma)-\bar{b}\gamma\right]\frac{1}{e_2-\rho_r}\hat{r}^*_t$$

Solving for $\hat{\pi}_t$ as a function of the state variables, $\left\{\hat{b}_{t-1}, r^*_t\right\}$, we obtain

$$\hat{\pi}_t = \Omega_\pi \hat{b}_{t-1} + \Omega_\pi\left\{(1-\gamma)\frac{q_{21}(e_1-e_2)}{(e_1-\rho_r)(e_2-\rho_r)} + \gamma\left(\frac{\bar{b}}{e_2-\rho_r}\right)\right\}\hat{r}^*_t \tag{E.6}$$

where

$$\begin{aligned}\Omega_\pi &= \frac{q_{11}}{q_{12}q_{21}-q_{11}q_{22}}\\ q_{21} &= \frac{\bar{b}\kappa(1-\beta\phi)}{\psi^2+(\beta+\kappa-1)\psi-\kappa(1-\beta\phi)}.\end{aligned}$$

To express the coefficients on the state variables in terms of model parameters, we use the results in the following lemmas; the proofs are provided at the end of this appendix.

- **Lemma 1** $\Omega_\pi = \frac{\beta(e_2-e_3)}{\bar{b}(1-\beta\phi)} > 0$.
- **Lemma 2** $\Omega_\pi q_{21}(e_1-e_2) = -\kappa\beta^{-1}$.

We then simplify (E.6) as:

$$\hat{\pi}_t = \frac{\beta(e_2-e_3)}{\bar{b}(1-\beta\phi)}\hat{b}_{t-1} + \left\{\underbrace{(1-\gamma)\left[\frac{\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)}\right]}_{\text{conventional channel}} + \underbrace{\gamma\frac{\bar{b}\Omega_\pi}{e_2-\rho_r}}_{\text{fiscal channel}}\right\}\hat{r}^*_t$$

The coefficient on $\hat{b}_{t-1}$ does not depend on γ. However, γ has two opposing effects on the coefficient on $\hat{r}^*_t$. On the one hand, an increase in γ lowers the response of inflation through the conventional channel. On the other hand, an increase in γ raises the response of $\hat{\pi}_t$ through the fiscal channel: When $\hat{r}^*_t$ is included in the Taylor rule, an increase in $\hat{r}^*_t$ *additionally* raises interest payments and public debt, which in turn requires an additional increase in the rate of inflation. Combining the two channels together, we write the above as:

$$\hat{\pi}_t = \frac{\beta(e_2-e_3)}{\bar{b}(1-\beta\phi)}\hat{b}_{t-1} + \left\{\frac{\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)} + \gamma\left[\frac{\Omega_\pi\bar{b}(e_1-\rho_r)-\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)}\right]\right\}\hat{r}^*_t$$

or

$$\hat{\pi}_t = \Omega_\pi\hat{b}_{t-1} + \Gamma(\gamma)\hat{r}^*_t \tag{E.7}$$

where

$$\Omega_\pi \equiv \frac{\beta\left(e_2-e_3\right)}{\bar{b}\left(1-\beta\phi\right)},$$
$$\Gamma\left(\gamma\right) \equiv \frac{\kappa\beta^{-1}}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)}+\gamma\left[\frac{\Omega_\pi\bar{b}\left(e_1-\rho_r\right)-\kappa\beta^{-1}}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)}\right].$$

We later prove that $\Gamma\left(\gamma\right)$ is positive and increasing in γ; that is, the fiscal channel dominates the conventional channel in the model. Notice that the solution given in the main text is a particular case of (E.7).

Next we plug (E.7) into the government budget constraint to obtain the law of motion for $\hat{b}_t$:

$$\begin{aligned}\hat{b}_t &= \beta^{-1}(1-\psi)\hat{b}_{t-1}-\bar{b}\left(\beta^{-1}-\phi\right)\left[\Omega_\pi\hat{b}_{t-1}+\Gamma\left(\gamma\right)\hat{r}_t^*\right]+\bar{b}\gamma\hat{r}_t^*,\\ &= e_3\hat{b}_{t-1}+\bar{b}\left\{\underbrace{\gamma+\phi\Gamma\left(\gamma\right)}_{\text{interest payments}}-\underbrace{\beta^{-1}\Gamma\left(\gamma\right)}_{\text{inflation}}\right\}\hat{r}_t^*.\end{aligned}$$

Similarly to inflation, an increase in γ has two opposing effects on the coefficient on $\hat{r}_t^*$: i) via inflation and ii) via interest payments. On the one hand, inflation responds more on impact when γ is larger (as discussed earlier), which stabilizes debt. Hence, the coefficient on $\hat{r}_t^*$ is smaller with a bigger γ. On the other hand, the interest rate also increases by more when γ is greater – because not only inflation rises more (captured by $\phi\Gamma\left(\gamma\right)\hat{r}_t^*$) but also the central bank tracks r-star more closely (captured by $\gamma\hat{r}_t^*$). This latter effect destabilizes debt and produces a larger coefficient on $\hat{r}_t^*$.

To develop intuition further, we plug $\Gamma\left(\gamma\right)$ into the equation above and obtain

$$\hat{b}_t = \Omega_b\hat{b}_{t-1}+\Theta\left(\gamma\right)\hat{r}_t^*, \tag{E.8}$$

where

$$\Omega_b \equiv e_3 = \frac{1}{2\beta}\left(\beta+\kappa+1-\sqrt{\left(\beta+\kappa+1\right)^2-4\beta\left(1+\kappa\phi\right)}\right)$$
$$\Theta\left(\gamma\right) \equiv \bar{b}\left(\gamma\frac{\left(e_1-\rho_r\right)\left(e_3-\rho_r\right)+\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)}-\frac{\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)}\right).$$

The solution given in the main text is a particular case of (E.8). The coefficient $\Theta\left(\gamma\right)$ is increasing in γ (verified in the next section.) When $\gamma=0$ and Θ is negative, $-\frac{\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)}$. In this case, debt works as a *stabilizing* force: an increase in $\hat{r}_t^*$ leads to an increase in inflation, which lowers debt level; a reduced debt level in turn puts a downward pressure on inflation. Therefore inflation overall does not increase as much. Now consider the case in which $\gamma>0$. An increase in γ raises $\Theta\left(\gamma\right)$, as the aforementioned interest payment effect kicks in. This weakens the stabilizing role of government debt. Moreover, $\Theta\left(\gamma\right)$ eventually becomes positive when γ gets sufficiently large (e.g. $\gamma=1$.) In such cases, the presence of $\hat{b}_{t-1}$ in (E.7) has a multiplier, rather than stabilizing, effect on inflation.

Finally, we use the Phillips curve to solve for the output gap:

$$\kappa\tilde{Y}_t = \underbrace{\Omega_\pi\hat{b}_{t-1}+\Gamma\left(\gamma\right)\hat{r}_t^*}_{\hat{\pi}_t}\underbrace{-\beta\Omega_\pi\left(e_3\hat{b}_{t-1}+\Theta\left(\gamma\right)\hat{r}_t^*\right)-\Gamma\left(\gamma\right)\beta\rho_r\hat{r}_t^*}_{-\beta\mathbb{E}_t\hat{\pi}_{t+1}},$$

which implies

$$\tilde{Y}_t = \Omega_Y\hat{b}_{t-1}+\Lambda\left(\gamma\right)\hat{r}_t^*, \tag{E.9}$$

where

$$\Omega_Y \equiv \kappa^{-1}\Omega_\pi\left(1-\beta e_3\right),$$
$$\Lambda\left(\gamma\right) \equiv \kappa^{-1}\left[\left(1-\beta\rho_r\right)\Gamma\left(\gamma\right)-\beta\Omega_\pi\Theta\left(\gamma\right)\right].$$

As for inflation, the coefficient on $\hat{b}_{t-1}$ does not depend on γ while the coefficient on $\hat{r}_t^*$ depends on γ. An increase in γ leads to an increase in $\Gamma(\gamma)$ and $\Theta(\gamma)$ as discussed above. It can be shown whether $\Lambda(\gamma)$ is increasing or decreasing depends on parameter values – especially on the degree of nominal rigidities; when prices are extremely sticky, $\Lambda'(\gamma) > 0$; otherwise, $\Lambda'(\gamma) < 0$.

E.3 Detailed proofs of the propositions

The results in Proposition 1 are already shown in the previous subsection. We therefore focus on Proposition 2 that shows the results on the relationship between the coefficients on r-star and γ, the measure of r-star tracking by the central bank.

E.3.1 $\Theta(\gamma)$

We first prove that $\Theta(\gamma)$ is increasing in γ.

$$\Theta(\gamma) = \bar{b}\left(\gamma\frac{(e_1-\rho_r)(e_3-\rho_r)+\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{(e_1-\rho_r)(e_2-\rho_r)} - \frac{\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{(e_1-\rho_r)(e_2-\rho_r)}\right),$$

which is linear in γ. At $\gamma = 0$, $\Theta(\gamma) = -\frac{\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{(e_1-\rho_r)(e_2-\rho_r)}$ is negative. Now consider the slope:

$$\begin{aligned}
\Theta'(\gamma) &= \bar{b}\frac{(e_1-\rho_r)(e_3-\rho_r)+\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{(e_1-\rho_r)(e_2-\rho_r)}\\
&= \bar{b}\frac{e_1e_3-(e_1+e_3)\rho_r+\rho_r^2+\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{(e_1-\rho_r)(e_2-\rho_r)}\\
&= \bar{b}\frac{\beta^{-1}(1+\kappa\phi)-\beta^{-1}(\beta+\kappa+1)\rho_r+\rho_r^2+\kappa\beta^{-1}\left(\beta^{-1}-\phi\right)}{(e_1-\rho_r)(e_2-\rho_r)}\\
&= \bar{b}\frac{(1+\kappa\phi)-(\beta+\kappa+1)\rho_r+\beta\rho_r^2+\kappa\left(\beta^{-1}-\phi\right)}{\beta(e_1-\rho_r)(e_2-\rho_r)}\\
&= \bar{b}\frac{1-(\beta+\kappa+1)\rho_r+\beta\rho_r^2+\kappa\beta^{-1}}{\beta(e_1-\rho_r)(e_2-\rho_r)}\\
&= \bar{b}\frac{(1-\rho_r)+\kappa\left(\beta^{-1}-\rho_r\right)-\beta\rho_r(1-\rho_r)}{\beta(e_1-\rho_r)(e_2-\rho_r)}\\
&= \bar{b}\frac{(1-\rho_r)(1-\beta\rho_r)+\kappa\left(\beta^{-1}-\rho_r\right)}{\beta(e_1-\rho_r)(e_2-\rho_r)} > 0.
\end{aligned}$$

Therefore, $\Theta(\gamma)$ is increasing in γ.

E.3.2 $\Gamma(\gamma)$

We now turn to $\Gamma(\gamma)$ which is given as

$$\begin{aligned}
\Gamma(\gamma) &\equiv \frac{\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)} + \gamma\left[\frac{\Omega_\pi\bar{b}(e_1-\rho_r)-\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)}\right]\\
&= \frac{\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)} + \gamma\left[\frac{\beta(e_1-\rho_r)\frac{(e_2-e_3)}{(1-\beta\phi)}-\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)}\right].
\end{aligned}$$

The slope of the linear function $\Gamma(\gamma)$ is given by

$$\Gamma'(\gamma) = \frac{\beta(e_2-e_3)(e_1-\rho_r)-\kappa\beta^{-1}(1-\beta\phi)}{(e_1-\rho_r)(e_2-\rho_r)(1-\beta\phi)}$$

$$= \frac{\beta\left(e_2-e_3\right)\left(e_1-e_2+e_2-\rho_r\right)-\kappa\beta^{-1}\left(1-\beta\phi\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{\beta\left(e_2-e_3\right)\left(e_1-e_2\right)-\kappa\beta^{-1}\left(1-\beta\phi\right)+\beta\left(e_2-e_3\right)\left(e_2-\rho_r\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{-\frac{1}{\beta}\left[\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)\right]-\frac{\kappa}{\beta}\left(1-\beta\phi\right)+\beta\left(e_2-e_3\right)\left(e_2-\rho_r\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{-\frac{1}{\beta}\left[\psi^2+\left(\beta+\kappa-1\right)\psi\right]+\beta\left(e_2-e_3\right)\left(e_2-\rho_r\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

Substitute ψ out using $\psi = 1-\beta e_2$:

$$\Gamma'\left(\gamma\right) = \frac{-\frac{1}{\beta}\left[\left(1-\beta e_2\right)^2+\left(\beta+\kappa-1\right)\left(1-\beta e_2\right)\right]+\beta\left(e_2-e_3\right)\left(e_2-\rho_r\right)}{\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{\beta^2\left(e_2-e_3\right)\left(e_2-\rho_r\right)-\left(1-\beta e_2\right)^2-\left(\beta+\kappa-1\right)\left(1-\beta e_2\right)}{\beta\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{\beta^2\left(e_2^2-(e_3+\rho_r)e_2+\rho_r e_3\right)-\left(\beta^2 e_2^2-2\beta e_2+1\right)-\left(\beta+\kappa-1\right)+\left(\beta+\kappa-1\right)\beta e_2}{\beta\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{\beta^2\left(\rho_r e_3-(e_3+\rho_r)e_2\right)+2\beta e_2-\left(\beta+\kappa\right)+\left(\beta+\kappa-1\right)\beta e_2}{\beta\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}$$

$$= \frac{\left[\beta^2\left(1-e_3\right)+\beta\left(1-\beta\rho_r\right)+\beta\kappa\right]e_2+\beta^2\rho_r e_3-\left(\beta+\kappa\right)}{\beta\left(e_1-\rho_r\right)\left(e_2-\rho_r\right)\left(1-\beta\phi\right)}.$$

Since the denominator is unambiguously positive for all parameter values under fiscal regime, $\Gamma'\left(\gamma\right) > 0$ if and only if the numerator is also positive. The numerator is a linear and increasing function of e_2 because the slope, $\left[\beta^2\left(1-e_3\right)+\beta\left(1-\beta\rho_r\right)+\beta\kappa\right]$, is positive. This implies $\Gamma'\left(\gamma\right) > 0$ for sufficiently large e_2 – or for sufficiently small ψ; that is, when $\psi < \bar{\psi}^*$ where

$$\bar{\psi}^* \equiv 1-\frac{\left(\beta+\kappa\right)-\beta^2\rho_r e_3}{\beta\left(1-e_3\right)+\left(1-\beta\rho_r\right)+\kappa} = 1-\frac{\beta+\kappa-\beta^2\rho_r e_3}{\beta\left(e_1-\rho_r\right)}$$

It remains to show that $\bar{\psi}^*$ is positive. The denominator of $\bar{\psi}^*$ is positive. Consider the numerator, $g\left(\phi\right) \equiv \beta\left(e_1-\rho_r\right)-\left(\beta+\kappa-\beta^2\rho_r e_3\right)$. Given other parameters, $g\left(\phi\right)$ has the smallest value at $\phi = 1$ because $g'\left(\phi\right) < 0$. Evaluate $g\left(\phi\right)$ at $\phi = 1$:

$$g\left(1\right) = \beta\left(e_1-\rho_r\right)-\left(\beta+\kappa-\beta^2\rho_r e_3\right) = \beta\left(\frac{\kappa+1}{\beta}-\rho_r\right)-\left(\beta+\kappa-\beta^2\rho_r\right)$$
$$= \left(1-\beta\right)\left(1-\beta\rho_r\right) > 0,$$

which implies $\bar{\psi}^* > 0$. Finally, redefining $\bar{\psi}^*$ as $\bar{\psi}^* \equiv \min\left\{\bar{\psi}^*, \bar{\psi}\right\}$, we establish that:

$$\Gamma'\left(\gamma\right) > 0 \text{ for } 0 < \bar{\psi}^* \leq \bar{\psi} \equiv 1-\beta.$$

In addition, we can show that $\bar{\psi}^*$ depends on the slope of the Phillips curve and satisfies:

$$\lim_{\kappa\to\infty}\bar{\psi}^* = 0$$
$$\lim_{\kappa\to 0}\bar{\psi}^* = 1-\beta.$$

E.3.3 $\Lambda(\gamma)$

As stated in the main text, the sign of $\Lambda'(\gamma)$ can either be positive or negative depending on other parameters. In this subsection, we illustrate this point, considering a special case; in particular, we assume r_t^* is i.i.d and $\phi = \psi = 0$ under the fiscal regime.

From (E.9), we have

$$\begin{aligned}\kappa\Lambda(\gamma) &\equiv \Gamma(\gamma) - \beta\Omega_\pi \bar{b}\left[\gamma - \left(\beta^{-1} - \phi\right)\Gamma(\gamma)\right] \\ &= \left[1 + \beta\Omega_\pi \bar{b}\left(\beta^{-1} - \phi\right)\right]\left\{\frac{\kappa\beta^{-1}}{e_1 e_2} + \gamma\left[\frac{\Omega_\pi \bar{b} e_1 - \kappa\beta^{-1}}{e_1 e_2}\right]\right\} - \beta\Omega_\pi \bar{b}\gamma.\end{aligned}$$

Differentiate $\Lambda(\gamma)$ with respect to γ:

$$\kappa\Lambda'(\gamma) = \left[1 + \beta\left(e_2 - e_3\right)\right]\left[\frac{\beta e_1 \frac{(e_2 - e_3)}{(1-\beta\phi)} - \kappa\beta^{-1}}{e_1 e_2}\right] - \frac{\beta\left(e_2 - e_3\right)}{\left(\beta^{-1} - \phi\right)}.$$

Using $\phi = \psi = 0$ and rearranging terms, we get

$$\kappa\Lambda'(\gamma) = \left[2 - \beta e_3\right]\left[\frac{\beta e_1\left(\beta^{-1} - e_3\right) - \kappa\beta^{-1}}{\beta^{-1} e_1}\right] - \frac{\beta e_1\left(\beta^{-1} - e_3\right)}{\beta^{-1} e_1}.$$

We therefore have

$$\Lambda'(\gamma) < 0 \quad \Longleftrightarrow \quad \beta e_1\left(\beta^{-1} - e_3\right) - \left[2 - \beta e_3\right]\left[\beta e_1\left(\beta^{-1} - e_3\right) - \kappa\beta^{-1}\right] > 0$$

After rearranging terms, we can show

$$\begin{aligned}&\beta e_1\left(\beta^{-1} - e_3\right) - \left[2 - \beta e_3\right]\left[\beta e_1\left(\beta^{-1} - e_3\right) - \kappa\beta^{-1}\right] \\ &= \left(\beta e_3 + \kappa - 1\right)\left(\beta^{-1} - 1\right) + \kappa\left(1 - e_3\right).\end{aligned}$$

We can see that $\left(\beta e_3 + \kappa - 1\right)\left(\beta^{-1} - 1\right) + \kappa\left(1 - e_3\right)$ can be either positive or negative. For example, it is positive (i.e. $\Lambda'(\gamma) < 0$) when κ is sufficiently large.

F Proofs of Lemma 1 and 2

F.1 Lemma 1

Note that

$$\begin{aligned}\frac{q_{13}q_{21} - q_{11}q_{23}}{\det\left(V^{-1}\right)} &= v_{23}, \\ \frac{q_{11}q_{22} - q_{12}q_{21}}{\det\left(V^{-1}\right)} &= v_{33},\end{aligned}$$

where

$$v_{23} = \frac{2\left(1 - \psi\right) - \left(\beta + \kappa + 1 - \sqrt{\left(\beta + \kappa + 1\right)^2 - 4\beta\left(1 + \kappa\phi\right)}\right)}{2\bar{b}\left(1 - \beta\phi\right)}.$$

Therefore,

$$\begin{aligned}0 \times q_{21} - q_{11} = -q_{11} &= \det\left(V^{-1}\right) \times v_{23}, \\ q_{11}q_{22} - q_{12}q_{21} &= \det\left(V^{-1}\right) \times v_{33} = \det\left(V^{-1}\right) \times 1.\end{aligned}$$

It follows that

$$\begin{aligned}
\Omega_\pi &= \frac{q_{11}}{q_{12}q_{21}-q_{11}q_{22}} = v_{23} \\
&= \frac{2\left(1-\psi\right)-\left(\beta+\kappa+1-\sqrt{\left(\beta+\kappa+1\right)^2-4\beta\left(1+\kappa\phi\right)}\right)}{2\bar{b}\left(1-\beta\phi\right)} \\
&= \frac{2\left(1-\psi\right)-2\beta e_3}{2\bar{b}\left(1-\beta\phi\right)} = \frac{\beta\left(e_2-e_3\right)}{\bar{b}\left(1-\beta\phi\right)} > 0.
\end{aligned}$$

F.2 Lemma 2

$$\Omega_\pi q_{21}\left(e_1-e_2\right) = \frac{\beta\left(e_2-e_3\right)}{\bar{b}\left(1-\beta\phi\right)}\frac{\bar{b}\kappa\left(1-\beta\phi\right)}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)}\left(e_1-e_2\right)$$

$$= \kappa\beta\frac{\left(e_2-e_3\right)\left(e_1-e_2\right)}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)} = \kappa\beta\frac{e_1e_2-e_2^2-e_1e_3+e_2e_3}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)}$$

$$= \kappa\beta\frac{e_2\left(e_1+e_3-e_2\right)-e_1e_3}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)} = \kappa\beta\frac{\frac{\left(1-\psi\right)\left(\beta+\kappa+\psi\right)}{\beta^2}-e_1e_3}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)}$$

$$= \kappa\beta\frac{\frac{\left(1-\psi\right)\left(\beta+\kappa+\psi\right)}{\beta^2}-\frac{1+\kappa\phi}{\beta}}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)} = \kappa\beta\frac{\frac{1}{\beta^2}\left[\left(1-\psi\right)\left(\beta+\kappa+\psi\right)-\beta-\kappa\beta\phi\right]}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)}$$

$$= \kappa\beta^{-1}\frac{-\left[\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)\right]}{\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)} = -\kappa\beta^{-1}.$$