Capital Heterogeneity and Investment Prices: How much are Investment Prices Declining?*

Preliminary

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Abstract

Not as much as you may think. Investment-specific technological change (ISTC), reflected in the declining price of new investment goods, has been recognized as an important potential driver of economic growth, business cycles, the labor share, and the equilibrium real rate. However, the declines in investment prices are heavily concentrated in a few capital categories, most notably computers, while most categories exhibit little change. How one aggregates these price changes is hence critical to evaluating the aggregate importance of ISTC. We demonstrate theoretically the correct aggregation approach using a simple standard neoclassical model with multiple capital goods. Importantly, the correct aggregation depends on the question at stake. Second, empirically, we evaluate the quantitative impact of using the correct aggregation procedure. We find that the contribution of ISTC to long-run growth, to business cycles, and to the labor share is smaller than if one ignores aggregation issues.

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1 Introduction

One of the key stylized facts of modern macroeconomics is that investment prices are falling relative to consumption prices. This price decline is often used as a measure of investment-specific technological change, which has been argued to explain the considerable majority of output per worker growth in the U.S. economy (Greenwood et al. [1997]), and a significant share of macroeconomic fluctuations ((Greenwood et al. [2000], Fisher [2006])). But it is has also been used as an explanation for a wide range of phenomena—including some of the most prominent macro trends, like the falling labor share (Karabarbounis and Neiman [2014]) and the secular decline in real interest rates (Sajedi and Thwaites [2016]).

Underlying this investment price, however, is an enormous amount of heterogeneity across different types of capital goods. Relative to consumption, the quality-adjusted price of computers has fallen by a factor of about 3000 since 1970, according to the Bureau of Economic Analysis (BEA), while the price of structures—residential and non-residential alike—has actually risen.

Much research abstracts from this heterogeneity and simply relies on one simple, easily available measure: the gross investment price index in the national accounts. By construction, this index aggregates the prices of different types of capital goods with weights proportional to their gross investment flows. Computers, for instance, receive a high weight, because they turn over quickly, with a high gross flow. Houses, despite making up nearly half of the private capital stock, receive a low weight, as they depreciate slowly, with a low gross flow.

But is this the correct measure of investment-specific technical change? For instance, an alternative measure produced by the BEA weights investment prices using the stock, rather than the flow, of each capital goods. Because computers are a small share of the stock, they receive a low weight, while houses receive a high weight. According to that measure, there has been nearly no change in the price of capital. Figure 1 depicts these two price series.

Our contribution in this paper is to study this question and provide several simple theoretical results outlining which measure is relevant for which question. We then take a first stab at evaluating the quantitative importance of using the relevant price index for each question. Overall, we find that using the right index makes a substantial difference.

We start with perhaps the single most prominent implication of falling investment prices: the role of investment-specific technological change in growth. We adopt the standard framework for this question, but generalize it to allow for many capital goods, each

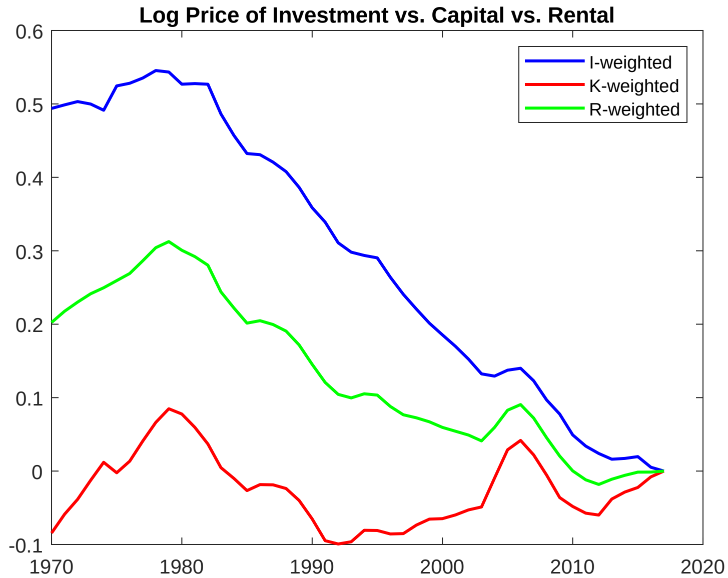

Figure 1: Log of flow-, stock-, and rental- weighted relative investment prices. Both prices normalized to 1 in 2018.

with its own price trend. The theory implies that the correct aggregate measure uses neither investment flows nor capital stocks as weights; instead, it uses the rental costs of each type of capital. Such weights are already used in traditional quantity-based growth accounting —based on the work of Jorgenson and others—but, to our knowledge, have not previously been used to study the equilibrium effecs of changes in investment prices themselves. As Figure 1 illustrates, this rental-weighted price index falls in-between the other stock- and flow- weighted indices.

We show that along a balanced growth path, the rental weights lie in between the flow- and stock- weights, with a simple formula connecting the three measures. When the formula is applied, the long-term importance of investment-specific technological change turns out to be roughly half what we would conclude from the usual, flow-weighted price index.

We then illustrate the role of investment prices for the "big ratios". Our theoretical analysis demonstrates that stock-weighted indices are the relevant ones to understands changes in these ratios. We apply this to the decline in net investment (e.g., Gutierrez and Philippon [2017]), and the increasing gap between real interest rates and the marginal product of capital (e.g., Marx et al. [2019], Farhi and Gourio [2019]). We show how not taking this into account would lead to misleading conclusions that investment prices are driving some changes.

We next study the role of ISTC for business cycles (Greenwood et al. [2000], Fisher

[2006]). We construct the correct measure of investment price shocks in a standard RBC model, which is close to—but not exactly—the stock-weighted deflator. We find that this change affects the results of the VAR analysis.

Finally, we look at the possible implications of changing investment prices for other pressing long-run questions: the long-term decline in r^*, and the labor share. This requires that we go beyond Cobb-Douglas production, generalizing to a CES functional form. We show that for both questions, the rental-share weighted index is the appropriate one.

The paper is organized as follows. The remainder of the introduction discusses the related literature. We then discuss our theoretical framework and obtain some results for balanced growth, for business cycle shocks, and for some extensions. Turning to the empirical analysis, we first review some basic features of the data, before providing some calculations about the effect of ISTC on economic growth for the US, for business cycles, and for the long-run macroeconomic Kaldorian ratios.

Literature Review To be added.

2 Theory

We start with a simple model that allows for clear analytical results - indeed, the simplest possible extension of the standard neoclassical model, with a Cobb-Douglas production function over multiple capital goods. We first present how capital aggregation affects the equilibrium growth rate, and discuss the alternative price indices implied by the model, as well as the "big ratios". We then discuss transitional dynamics and in particular present a result about the effect of ISTC shocks, which is relevant for business cycle analysis. Finally, we extend our simple model in various directions to (1) allow a more flexible empirical analysis and (2) to illustrate the effect of ISTC on the labor share and the equilibrium interest rate r^*, which requires relaxing the assumption of unit elasticity of substitution between capital and labor.

2.1 Model

Model Setup We work in continuous time and without uncertainty. For now we assume inelastic labor supply, and that the labor force L_t grows exogenously at rate g_L.

Households have standard power preferences (which are required for balanced growth):

$$\int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt. \tag{1}$$

The consumption good, which price is normalized to unity, is produced by a competitive representative firm with a constant-returns-to-scale Cobb-Douglas production technology, using labor L and n types $K_1, \ldots, K_n$ of capital as inputs:

$$Y_t = A_t L_t^{\alpha_L} K_{1t}^{\alpha_{K_1}} \cdots K_{nt}^{\alpha_{K_n}}. \tag{2}$$

Here A_t denotes total factor productivity (TFP), which is assumed to grow exogenously at rate g_A. We assume there is a linear technology to convert p_{it} units of the consumption good into one unit of investment in capital type i. This pins down the relative price of type i investment at p_{it}. Each p_{it} grows at an exogenous rate g_{p_i} that captures the investment-specific technical change (ISTC) for that capital type.[1] Capital depreciate at type-specific rates δ_i:

$$\dot{K}_{it} = I_{it} - \delta_i K_{it}, \tag{3}$$

where $\dot{K}_{it}$ denotes the time derivative of capital K_{it}. The goods market clears:

$$Y_t = C_t + \sum_{i=1}^{n} p_{it} I_{it}, \tag{4}$$

To economize on notation, we will sometimes write α_i for α_{K_i} i.e. omit the K subindex, and similar g_i means g_{p_i}. We will also denote $\alpha_K = \sum_{i=1}^n \alpha_{K_i}$ the total capital share.

Equilibrium The equilibrium is characterized by optimization of households and firms. On the household side, we have a standard consumption Euler equation:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma}, \tag{5}$$

where r_t denote the real interest rate. The representative firm's first-order conditions for capital demand are, for $i = 1...N$:

$$\alpha_{K_i} \frac{Y_t}{K_{it}} = R_{it}, \tag{6}$$

[1] Note that, following much of the literature, we do not model explicitly the production of investment goods. It is well known that a multisector model where the sectors have the same share parameters $\alpha_L, \alpha_{K_1}, ..., \alpha_{K_n}$ is isomorphic to this linear technology.

where R_{it} denotes the rental rate of capital type i. The usual no-arbitrage condition relates this rental rate to the capital price p_{it} through the user cost formula:

$$R_{it} = p_{it} \left(r_t + \delta_i - \frac{\dot{p}_{it}}{p_{it}} \right). \tag{7}$$

An equilibrium consists of paths for $\{Y_t, C_t, r_t, (K_{it}, R_{it}, I_{it})_{i=1,\ldots,n}\}_{t\geq 0}$ that satisfy equations (2)-(7), where $\{A_t, L_t, (p_{it})_{i=1,\ldots,n}\}_{t\geq 0}$ are the exogenous forcing processes.

2.2 Balanced Growth Analysis

In this section, we first characterize the balanced growth path (BGP) of this economy, then the relations between several investment price indices, and finally the "big ratios".

Equilibrium Growth Rate Suppose that the growth rate of the exogenous variables TFP A, labor L, and capital prices are constant:

$$g_A = \dot{A}/A, \tag{8}$$

$$g_L = \dot{L}/L, \tag{9}$$

$$g_{p_i} = \dot{p}_i/p_i, \tag{10}$$

The economy then has a balanced growth path, i.e. an equilibrium with a constant growth rate of output, a constant interest rate r_t, and constant consumption-output C_t/Y_t and investment-output ratio (for each investment type) $p_{it}I_{it}/Y_t$ for all i.[2]

To solve for the growth rate of output, note first that the ratio of real capital i to real output falls inversely with the price of capital i. This comes from combining (6) and (7):

$$\frac{K_{it}}{Y_t} = \frac{\alpha_{K_i}}{r_t + \delta_i - g_{p_i}} \frac{1}{p_{it}}. \tag{11}$$

It follows that along a BGP the growth rate of capital i equals output growth less the growth rate of the price p_i:

$$g_{K_i} = g_Y - g_{p_i}. \tag{12}$$

We can then use the production function to solve for the output growth rate g_Y. Log-

[2] Note that the numeraire is consumption, and output is measured in consumption units, so the consumption-output ratio is both a nominal and real ratio, while the investment-output ratios are nominal ratios (i.e. in consumption units).

differentiating (2), we obtain:

$$g_Y = g_A + \alpha_L g_L + \sum_{i=1}^{n} \alpha_{K_i} g_{K_i}, \tag{13}$$

and substituting in (12), we obtain after simplification:

$$g_Y - g_L = \frac{g_A}{\alpha_L} - \frac{\sum_{i=1}^{n} \alpha_{K_i} g_{p_i}}{\alpha_L}, \tag{14}$$

i.e. the growth rate of output per worker is the sum of the growth rate of TFP g_A, amplified by "capital deepening" $1/\alpha_L$, and a weighted average of the growth rates of each investment type's technical progress, again amplified by capital deepening.

We define the *rental-weighted* index of investment prices as:

$$g_{p_R} \equiv \sum_{i=1}^{n} \frac{\alpha_{K_i}}{\alpha_K} g_{p_i}. \tag{15}$$

Technically, this is a divisia index that weights each capital type' prices according to its share in the production function, α_{K_i}. Because these shares are also equal to rental shares of income, we call this a rental-weighted index. In the next subsections, we compare this index to the other indices that are commonly used. Using this definition, we can restate equation (14) as the following:

Proposition 1 *Along the balanced growth path, the growth rate of output per worker depends on TFP growth and on the rental-weighted index of investment prices according to:*

$$g_Y - g_L = \frac{g_A - \alpha_K g_{p_R}}{\alpha_L}. \tag{16}$$

This formula generalizes the formula commonly used in the ISTC literature, which applies this formula with (typically) only one capital type and hence one investment price. Of course, with one capital type, there is no aggregation problem. In reality however, aggregation is an important issue, as we illustrated in the introduction. This result shows that the correct aggregation of individual prices to measure the effect of ISTC on the aggregate growth rate of the economy is the rental-weighted one. This leads to several natural questions: how can we measure the rental-cost weighted index? And how do it compare to the other price indices used by researchers or produced by statistical agencies?

Price Indices We can define Divisia price indices using the formula:

$$\frac{\dot{p}_t^s}{p_t^s} = \sum_{i=1}^{n} s_{it} \frac{\dot{p}_{it}}{p_{it}}$$

where the s_{it} are weights (that sum to one). The key question is which weights to use.[3] A first natural choice is to use the investment shares, leading to what we call the flow-weighted, or investment-weighted index:

$$s_{it}^I \propto p_{it} I_{it},$$

where by "proportional to" we simply mean that the shares are normalized to sum to unity, i.e.

$$s_{it}^I = \frac{p_{it} I_{it}}{\sum_{j=1}^{n} p_{jt} I_{jt}}.$$

This index measures the change in cost of the investment bundle (in consumption units). It is the deflator of investment found in the BEA national income and product accounts (NIPA) and used in much of ISTC research.

An alternative choice is the capital share, leading to what we call the stock-weighted, or capital-weighted, price index:

$$s_{it} \propto p_{it} K_{it}.$$

This index measures the change in the cost of an investment bundle with the composition of the *current* capital stock (again in consumption units). It is the deflator of capital stocks found in the BEA fixed asset tables (FAT).

Yet another choice is the rental weighted index we discussed above:

$$s_{it} \propto R_{it} K_{it}.$$

This index can be interpreted as the rate of change in the cost of purchasing capital that delivers the same bundle of capital services. It is the dual to a quantity index aggregated using user costs, which correctly measures the change in aggregate input to the production function.Note, however, that the price index is distinct from an index measuring the aggregate rental cost itself, which requires knowing how r_t and $\dot{p}_{it}$ change. This quantity index is produced by the BLS as part of its annual multifactor productivity program (and by John Fernald on a quarterly basis). There are two potential approaches to measuring

[3]We assume that the underlying individual prices p_{it} are correctly measured and focus on the aggregation problem. Much research has already discussed

this index - one is to make assumptions to impute unobserved rental rates (as the BLS does) - another, as we will show, is to rely on a balanced growth relationship.

Rental-cost, Capital and Investment shares on the BGP We can calculate the values of the investment, capital and rental shares along the BGP. Define the current-cost value of all capital,

$$K_t^W \equiv \sum_{i=1}^{n} p_{it} K_{it}, \tag{17}$$

and define aggregate investment,

$$I_t \equiv \sum_{i=1}^{n} p_{it} I_{it}, \tag{18}$$

and aggregate profits:

$$\Pi_t \equiv \sum_{i=1}^{n} R_{it} K_{it}, \tag{19}$$

then the capital share of type i is:

$$s_{it}^K = \frac{p_{it} K_{it}}{K_t^W} = \frac{p_{it} K_{it}}{\sum_{j=1}^{n} p_{jt} K_{jt}}, \tag{20}$$

and using equation (11), we see that, along the BGP, the capital share of type i is constant and equal to

$$s_i^K = \frac{\frac{\alpha_i}{r+\delta_i-g_{p_i}}}{\sum_{j=1}^{n} \frac{\alpha_j}{r+\delta_j-g_{p_i}}}, \tag{21}$$

or in shorthand

$$s_i^K \propto \frac{\alpha_i}{r+\delta_i-g_{p_i}}. \tag{22}$$

Similarly, we can find the investment shares and rental shares, and collect all these result in:

Proposition 2 *Along the BGP, the capital, investment and rental shares are:*[4]

$$s_i^K \propto \frac{\alpha_i}{r+\delta_i-g_{p_i}}, \tag{23}$$

$$s_i^I \propto \alpha_i \frac{g_Y+\delta_i-g_{p_i}}{r+\delta_i-g_{p_i}}, \tag{24}$$

$$s_i^R \propto \alpha_{K_i}. \tag{25}$$

[4] As a side note, the formulas for the rental and capital shares also hold off-BGP.

These three formulas show clearly that the shares are different, because (as we will review in section 3) the depreciation rates and price trends are extremely heterogenous across capital types.

Rental cost weighted index as an average of Stock and Flow-weighted indices. Of course, one key issue is that while we can measure investment flows or capital stocks directly, rental costs (or α_{K_i}) are harder to measure. We now show how, along the BGP, the rental-cost share is a combination of the investment share and the capital share; as a result, the rental-cost index is a combination of the investment index and the capital index.

Proposition 3 *Along the BGP, we have:*

$$s_i^R = \frac{s_I}{\alpha_K} s_i^I + \left(1 - \frac{s_I}{\alpha_K}\right) s_i^K,$$

and consequently

$$g_{p_R} = \frac{s_I}{\alpha_K} g_{p_I} + \left(1 - \frac{s_I}{\alpha_K}\right) g_{p_K}$$

where s_I is the aggregate investment share and α_K is the aggregate capital share.

Proof. Start from

$$\begin{aligned} R_i K_i &= \left(r + \delta_i - g_{p_i}\right) P_i K_i \\ &= \left(r - g_Y\right) P_i K_i + \left(g_Y + \delta_i - g_{p_i}\right) P_i K_i \\ &= \left(r - g_Y\right) P_i K_i + P_i I_i \end{aligned}$$

Hence,

$$\sum_{i=1}^{n} R_i K_i = \alpha_K Y = \left(r - g_Y\right) K + I$$

It follows that

$$\begin{aligned} s_i^R &= \frac{R_i K_i}{\sum_{j=1}^{n} R_j K_j} \\ &= \frac{\left(r - g_Y\right) P_i K_i + P_i I_i}{\left(r - g_Y\right) K + I} \\ &= \frac{P_i K_i}{K}\left(1 - \frac{s_I}{\alpha_K}\right) + \frac{P_i I_i}{I}\frac{s_I}{\alpha_K}. \end{aligned}$$

■

This result reveals that the rental cost-weighted index is a hybrid of two conceptually simpler, and readily available indices, the flow and stock-weighted index.

The Big Ratios and Calibration Macroeconomists routinely use aggregate "big ratios" to calibrate[5] or evaluate[6] their models. But which capital aggregation is adequate for this?

Proposition 4 *Along the BGP,*

$$\frac{I}{K} = \frac{\sum_{i=1}^{n} p_i I_i}{\sum_{i=1}^{n} p_i K_i} = g_Y + \delta^K - g_{p_K} \tag{26}$$

$$\frac{\Pi}{K} = \frac{\sum_{i=1}^{n} R_i K_i}{\sum_{i=1}^{n} p_i K_i} = r + \delta^K - g_{p_K} \tag{27}$$

$$\frac{K}{Y} = \frac{\sum_{i=1}^{n} p_i K_i}{Y} = \frac{\alpha_K}{r + \delta^K - g_{p_K}} \tag{28}$$

In this result (which follows from simple algebra), I is the total current investment, while $K = K^W$ is the current cost stock of capital, both measured in consumption good units. Overall, to calibrate one-capital model, one should use the $stock - weighted$ depreciation rate and investment price growth. Many authors instead use the flow-weighted investment price growth.[7] In section **??** below, we illustrate how taking this simple fact into account changes the view of some secular trends in these big ratios.

2.3 Transitional Dynamics

We now turn to the analysis of transitional dynamics of this same model. The main goal is to study the effect of ISTC shocks. In a first step, we show how to characterize the transitional dynamics of this model. In a second step, we use this characterization to demonstrate the "correct" approach to aggregating capital for ISTC shocks.

[5] For a thorough discussion of the calibration approach, see for instance Cooley [1995]

[6] For some recent work discussing the big ratios, see for instance Eggertsson et al. [2018], and many others.

[7] For instance, one of us is guilty of this (Farhi and Gourio [2019])

Characterizing Transitional Dynamics For expositional simplicity, we start from the social planner problem:

$$\max_{C_t, L_t, (I_{it}, K_{it})_{i=1..N}} U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} v(L_t) dt \tag{29}$$

$$s.t. :$$

$$\dot{K}_{it} = I_{it} - \delta_i K_{it} \tag{30}$$

$$Y_t = C_t + \sum_{i=1}^{N} P_{it} I_{it} \tag{31}$$

$$Y_t = A_t L_t^{\alpha_L} \prod_{i=1}^{N} K_{it}^{\alpha_i} \tag{32}$$

where the exogenous forcing processes are $(A_t)_{t\geq 0}$, $\left((P_{it})_{i=1}^{N}\right)_{t\geq 0}$ and we have an initial condition $(K_{i0})_{i=1}^{N}$. Note that we now allow for elastic labor supply.[8] To simplify the exposition we assume no population growth, but this can be easily relaxed. The equilibrium is an object $\{Y_t, L_t, C_t, \lambda_t, r_t, w_t, (I_{it}, K_{it})_{i=1...N}\}_{t\geq 0}$, characterized by the constraints above (30)-(32) plus the usual first-order conditions:

$$C_t^{-\sigma} v(L_t) = \lambda_t, \tag{33}$$

$$\frac{C_t v'(L_t)}{v(L_t)} = w_t, \tag{34}$$

$$w_t = \alpha_L \frac{Y_t}{L_t}, \tag{35}$$

$$r_t = \rho + \sigma \frac{\dot{C}_t}{C_t} - \frac{v'(L_t) L_t}{v(L_t)} \frac{\dot{L}_t}{L_t}, \tag{36}$$

$$\frac{P_{it} K_{it}}{Y_t} = \frac{\alpha_i}{r_t + \delta_i - g_{it}}. \tag{37}$$

Our first step is to show that this problem can be simplified by using a single state variable, the wealth of the economy:

$$K_t^W = \sum_{i=1}^{N} P_{it} K_{it} \tag{38}$$

[8] We use the functional form above for the utility function because it is required for balanced growth.

We have from equation (37):

$$\frac{K_t^W}{Y_t} = \sum_{i=1}^{N} \frac{\alpha_i}{r_t + \delta_i - g_{it}} \tag{39}$$

And by differentiation of equation (38) we obtain after some algebra (derivation in appendix):

$$\dot{K}_t^W = \alpha_L Y_t - C_t + r_t K_t^W, \tag{40}$$

which is simply the "budget constraint": the growth of financial wealth equals financial income plus labor income less consumption.

Moreover, we can substitute the capital demand in the production function to obtain

$$Y_t = A_t^{\frac{1}{\alpha_L}} L_t \prod_{i=1}^{N} \left(\frac{\alpha_i}{P_{it}} \frac{1}{r_t + \delta_i - g_{it}} \right)^{\frac{\alpha_i}{\alpha_L}}. \tag{41}$$

Overall, the equilibrium is characterized by: $\{C_t, L_t, w_t, K_t^W, Y_t, r_t\}_{t \geq 0}$ that satisfy equations (39)-(41) plus the Euler equation, the labor supply and labor demand conditions (equations (34)-(36)). The economic intuition is straightforward - given the linear technology that transforms all types of investment into consumption instantaneously, and in the absence of irreversibility constraints, there is no need to keep track of all capital stocks - it is sufficient to know the aggregate value-weighted stock of capital.[9] These equations allow to solve for the evolution of the aggregates given an initial $K_0^W = \sum_{i=1}^{N} P_{i0} K_{i0}$ and exogenous $\{A_t, P_{it}\}_{t \geq 0}$.[10]

The effect of ISTC shocks We now use the characterization of transitional dynamics to study the role of investment price shocks for "business cycles". In particular, to understand the correct weighting of investment prices, we consider the following standard "MIT shock" experiment. Suppose that prior to $t = 0$, the economy is on its the balanced growth path with $\{A_t, (P_{it})_{i=1..N}\}$ following exponential trends that are expected to continue forever. At $t = 0$, unexpectedly, there is a one-time, permanent shock to the *level* of all the prices P_i, after which the price trends continue as before. That is, prior to $t = 0$,

[9] We should note that this result does not show that capital can be "aggregated"; the stock of capital K^W is a sufficient state variable but the equilibrium still requires knowledge of the underlying capital-specific parameters e.g. prices, price trends, shares and depreciation rates.

[10] Note that from these variables one can infer the evolution of the other variables of the model, e.g. the individual capital stocks K_{it} from equation (37).

people expected that for $t \geq 0$ and $i = 1...N$,

$$P_{it} = P_{i0} e^{g_i t},$$

but after $t = 0$ they now expect

$$P_{it} = P'_{i0} e^{g_i t}.$$

How does this shock affect the equilibrium $\{C_t, L_t, w_t, K_t^W, Y_t, r_t\}_{t \geq 0}$?

Clearly, the long-run growth rate of the economy is unaffected, and in the long-run we have simply a "parallel shift" in the (log) BGP. On top of that, there are transitional dynamics to reach this new balanced growth path since the initial capital stock of the economy is not in general on the new BGP. These dynamics depend on the structural parameters of the model (for instance, the elasticity of intertemporal substitution) and on the vector of investment price shock $\frac{P'_{i0}}{P_{i0}}$. The following result shows that effect of this shock vector on the full path of responses can be represented by a scalar: a combination of these investment prices changes is a "sufficient statistic" for the shock. This combination is an average of the investment-weighted and stock-weighted shocks.

Proposition 5 *The effect of a small, one-time, unexpected permanent change to the vector of investment prices P_{i0} is governed by*

$$\omega = s_I g_{p_I} + (1 - s_I) g_{p_K}, \tag{42}$$

where s_I is the aggregate investment share of GDP, and g_{p_I} and g_{p_K} are respectively the flow-weighted and stock-weighted changes in prices:

$$g_{p_I} = \sum_{i=1}^{n} s_i^I \frac{P'_{i0}}{P_{i0}}, \tag{43}$$

$$g_{p_K} = \sum_{i=1}^{n} s_i^K \frac{P'_{i0}}{P_{i0}}. \tag{44}$$

One way to understand this result is to consider two potential vectors of changes in prices, e.g. P'_{i0} and P''_{i0}. The result states that these two changes will generate the exact same *path* of macroeconomic aggregates, if and only if they have the same ω.

Proof. The proof is not complicated; it involves rewriting the equilibrium conditions around the new balanced growth path and noting that these conditions are then independent of the prices P_{i0}, so that the only effect works through the initial conditions of the state variable. It only remains then to note that the effect of the price changes on that state variable work through ω.

First, we denote the *new* BGP (after the change in prices)$as(\widetilde{C}_t, \widetilde{K}_t^W, \widetilde{Y}_t, \widetilde{L}_t, \widetilde{w}_t, \widetilde{r}_t)$ - this is the solution to the model if the capital stock at $t = 0$ were on the BGP (which of course it is not, generically). Clearly, $\widetilde{r}_t = r^*$ is constant (equal to $\rho + \sigma g_Y$, and equal to the previous BGP), and so is $\widetilde{L}_t = L^*$. The rest of this BGP is characterized by the equations

$$\widetilde{Y}_t = (A_t)^{1/\alpha_L} L_* \prod_{i=1}^{N} \left(\frac{\alpha_i}{P_{it}} \frac{1}{r^* + \delta_i - g_i} \right)^{\alpha_i/\alpha_L},$$

$$\frac{\widetilde{K}_t^W}{\widetilde{Y}_t} = \sum_{i=1}^{N} \frac{\alpha_i}{r^* + \delta_i - g_i},$$

$$\frac{\widetilde{C}_t v'(L^*)}{v(L^*)} = \widetilde{w}_t,$$

$$\widetilde{w}_t = \alpha_L \frac{\widetilde{Y}_t}{\widetilde{L}^*},$$

$$\widetilde{C}_t \quad = \quad \left(\alpha_L + (r^* - g_Y) \sum_{i=1}^{N} \frac{\alpha_i}{r^* + \delta_i - g_i} \right) \widetilde{Y}_t$$

To study the deviations from the *new* BGP, define for each variable Z_t the "detrended" version $\widehat{Z}_t$:

$$\widehat{Z}_t = \frac{Z_t}{\widetilde{Z}_t},$$

and rewrite the system of equation as $\left(\widehat{C}_t, \widehat{K}_t^W, \widehat{Y}_t, r_t, \widehat{w}_t, \widehat{L}_t\right)$:

$$\widehat{w}_t = \alpha_L \frac{\widehat{Y}_t}{\widehat{L}_t},$$

$$r_t - r^* = \sigma \frac{\dot{\widehat{C}}_t}{\widehat{C}_t} - \frac{v'(\widehat{L}_t L^*) \widehat{L}_t}{v(\widehat{L}_t L^*)} \frac{\dot{\widehat{L}}_t}{\widehat{L}_t},$$

$$\widehat{Y}_t = \widehat{L}_t^{\frac{1}{\alpha_L}} \prod_{i=1}^{N} \left(\frac{r^* + \delta_i - g_i}{r_t + \delta_i - g_i} \right)^{\frac{\alpha_i}{\alpha_L}},$$

$$\widehat{K}_t^W = \widehat{Y}_t \frac{\sum_{i=1}^{N} \frac{\alpha_i}{r_t + \delta_i - g_i}}{\sum_{i=1}^{N} \frac{\alpha_i}{r^* + \delta_i - g_i}},$$

And (derived in appendix):

$$\dot{\widehat{K}}_t^W = \frac{Y^*}{K^*}\widehat{Y}_t - \widehat{C}_t\frac{C^*}{K^*} + (r_t - g_Y)\,\widehat{K}_t^W.$$

This system of equations does not involve the level of P_i.[11] Hence, the full dynamics of the system are not affected by P_i - except through its effect on the initial condition of this system, i.e. the state variable $\widehat{K}_0^W$.

It only remains to measure how much this state variable is affected at time 0. There are two effects, both on the numerator K_0^W and the denominator $\widetilde{K}_0$. First, on the denominator, at time 0 the capital is revaluated using the new prices - this is a stock-weighted change. Second, the balanced growth path shifts - according to the formula for output, which involves a rental-weighted change, multiplied by the capital share over the labor share according to equations 2.3 and 2.3, (and similar to our proposition 1). In math, the overall effect is

$$\begin{aligned}\omega &= p_K + (\alpha_K/\alpha_L)p_R \\ &= p_K + (\alpha_K/\alpha_L)\left(\frac{s_I}{\alpha_K}\times p_I + \left(1-\frac{s_I}{\alpha_K}\right)\times p_K\right) \\ &= \frac{1}{\alpha_L}\left((1-s_I)p_K + s_I p_I\right).\end{aligned}$$

■

It is noteworthy that the proof does not actually require that we start on the BGP. It is also clear that while this model is quite simple, the logic would work with models with more frictions (e.g., sticky prices). On the other hand, we rely heavily on our strong assumption that capital allocation is frictionless (no irreversibility or capital adjustment costs).

As a numerical illustration of this result, we solved using a shooting method a model with 3 capital goods and compared the responses to one price shock and to alternative price shocks that have either the same investment-weighted, same capital-weighted, same rental-weighted, or same "shock-weighted" (i.e., same ω). Figure 2 depicts the responses and shows how the first and last one are superimposed (The o and + are almost overlaid) while the others are quite different.

[11] It is easy to check that the steady-state consumption-output C/Y and wealth-output K^W/Y ratios are independent of the level of p_{it}. These prices scale up and down the entire economy but their levels does not affect these ratios. (The *growth rate* of prices, on the other hand, does affect these ratios in general.)

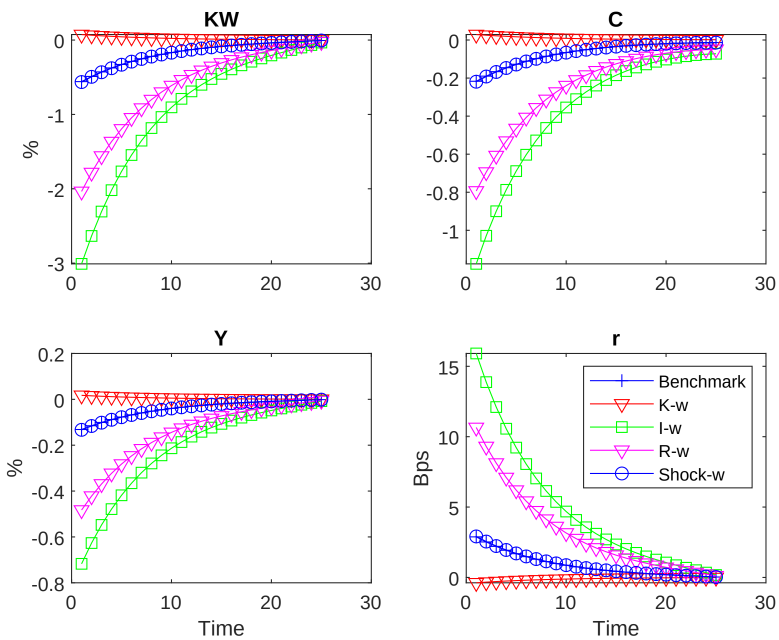

Figure 2: Numerical illustration of Proposition 5: Response to a permanent change in investment prices at time 0 for different price vector changes that have either the same I-w, K-w, R-w, or Shock-w as the benchmark one. KW denotes the wealth, C consumption, Y output and r the interest rate.

2.4 Model Extensions

We first discuss an extension of the baseline model that allows for time-varying parameters and does not assume the BGP. We then discuss two extensions to non-unitary elasticity of substitution, which are used to study the effect of ISTC on the labor share or the rate of return r^*.

2.4.1 Extension with time-varying parameters (Incomplete - Skip)

The previous section demonstrates in a simple framework the differences between various investment price indices and their relation to long-run growth. That section makes strong assumptions to obtain a balanced growth path. Here we relax some of these assumptions, which allows to quantify the role of ISTC in a more flexible framework.

Specifically, we drop two assumptions (1) that we have the balanced growth path and (2) that some of the exogenous parameters are constant. We now allow arbitrary exogenous $\{\rho_t, \alpha_{it}, \delta_{it}, g_{it}, g_{At}, P_{i0}, L_t, A_0\}$. Given an initial condition $\{K_{i0}\}$, the problem is

$$
\begin{aligned}
\max_{C_t, I_{it}, K_{it}} U &= \int_0^\infty e^{-\int_0^t \rho_s ds} \frac{C_t^{1-\sigma}}{1-\sigma} dt \\
s.t. &: \\
\dot{K}_{it} &= I_{it} - \delta_{it} K_{it} \\
Y_t &= C_t + \sum_{i=1}^N P_{it} I_{it} \\
Y_t &= A_t L_t^{\alpha_{Lt}} \prod_{i=1}^N K_{it}^{\alpha_{it}}
\end{aligned}
$$

where

$$
\begin{aligned}
P_{it} &= P_{i0} e^{\int_0^t g_{is} ds}, \\
A_t &= A_0 e^{\int_0^t g_{As} ds}.
\end{aligned}
$$

The capital demand are given by

$$\frac{P_{it} K_{it}}{Y_t} = \frac{\alpha_{it}}{r_t + \delta_{it} - g_{it}}, \tag{45}$$

and the Euler equation reads

$$r_t = \rho_t + \sigma \frac{\dot{C}_t}{C_t}. \tag{46}$$

This generalized model aggregates in the exact same was as our benchmark model. Define

$$K_t^W = \sum_{i=1}^N P_{it} K_{it}, \tag{47}$$

then it is straightforward to show that

$$\dot{K}_t^W = r_t K_t^W + \alpha_{Lt} Y_t - C_t \tag{48}$$

and the production function implies

$$Y_t = A_t^{\frac{1}{\alpha_{Lt}}} L_t \prod_{i=1}^N \left(\frac{1}{P_{it}} \frac{\alpha_{it}}{r_t + \delta_{it} - g_{it}} \right)^{\frac{\alpha_{it}}{\alpha_{Lt}}}, \tag{49}$$

and we have

$$\frac{K_t^W}{Y_t} = \sum_{i=1}^{N} \frac{\alpha_{it}}{r_t + \delta_{it} - g_{it}}. \tag{50}$$

Overall, we still have a system of four equations in four unknowns $\{K_t^W, Y_t, C_t, r_t\}$. Once we have solved four these variables, we can deduce K_{it}, I_{it}, etc. We can solve this system using a shooting method given the exogenous processes.

2.4.2 Labor Share

We know consider the effect of ISTC on the labor share. We allow for a non-unitary elasticity of substitution between capital and labor:

$$Y = (b_K K^{\frac{\sigma-1}{\sigma}} + b_L L^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}},$$

where the capital aggregate is still Cobb-Douglas:

$$K = K_1^{\gamma_{K_1}} ... K_n^{\gamma_{K_n}}.$$

Of course, with permanent trend in ISTC, the shares are non-stationary. We hence assume now that the prices are trendless but consider a shock to the level of investment prices. Our simple result is the following:

Proposition 6 *Consider a one-time permanent small change to the vector p_i. Then change in gross labor share is:*

$$(\sigma - 1)\alpha_K \widehat{p}^R.$$

That is, the relevant price index here is the rental-weighted one. This is intuitive since the rental-weighted price is the one that determines the level of output.

To illustrate the effect that using the rental-weighted rather than investment-weighted index can have on the implied change in the labor share, we construct for various elasticities of substitution, the implied change in labor share since 1970 given observed prices changes. The results are shown in Table 1. For instance, if one assumes the elasticity is larger than 1 (e.g. as in Karabarbounis and Neiman [2014]) say 1.25, then the labor share should have declined by 9 points given the ISTC measured using the investment-weighted prices. But if one - correctly - uses the rental-weighted prices instead, the implied change is only 3 points. On the other hand, if the elasticity is less than unity, then ISTC would have pushed the labor share up. Our point, obviously, is not to argue in favor of either, but simply to show that (1) there is a correct investment price deflator for this question, and (2) the results are quite sensitive to which investment price is used.

	Iw	Rw
$\sigma = 1.5$	-0.17	-0.07
$\sigma = 1.25$	-0.09	-0.03
$\sigma = 0.75$	0.09	0.03
$\sigma = 0.5$	0.17	0.07

Table 1: Contribution of ISTC to change of labor share, for different EOS and using I-w or R-w. 1970-2017.

2.4.3 ISTC and the decline of r^* (Incomplete)

We now generalize slightly more our model to allow for an upward-sloping (rather than horizontal) supply of capital. The goal is to consider the role of ISTC in the determination of the equilibrium rate of return. Some authors (e.g. Summers, Sajedi and Thwaites [2016]) have argued that ISTC, by reducing the consumption sacrifice needed to reach a certain investment, has depressed the real interest rate.

We now assume that the supply of savings is given by $W_t L_t S(r_t)$, where S is an increasing function of the interest rate. This savings function can be rationalized by many models, such as the Aiyagari precautionary savings model, or OLG models. Equilibrium in asset markets now require

$$\sum_{i=1}^{n} p_{it} K_{it} = W_t L_t S(r_t).$$

Our main result is the following:

Proposition 7 *Consider a one-time unexpected permanent small shock to the vector p_{i0}. Then the change in r^* is $\zeta \widehat{p}^R$, where ζ is a function of model parameters.*

This result shows that, for this purpose, the correct aggregation is again rental-weighted.

3 Basic facts

In this section, we present some basic descriptive facts about the behavior of capital prices. We first discuss data sources, then discuss the investment and capital shares, the depreciation and price trends by capital type, and finally the key differences between investment-weighted (or flow-weighted) investment prices, and capital-weighted (or stock-weighted) investment prices.

3.1 Data

We use public, readily available spanning the period 1970-2017.[12] Our main source is the Fixed Asset Tables of the Bureau of Economic Analysis (B.E.A.).[13] Specifically we use Tables 2.1 (current cost capital), 2.2 (quantity index for capital), 2.4 (current-cost depreciation), 2.7 (investment) and 2.8 (quantity index for investment). These tables cover all of private fixed assets and provide a detailed breakdown by capital type. In the appendix (section 8.2), we list all the types of capital available in our data, together with some summary statistics.

We also use standard NIPA[14] data to construct a non-durables and services consumption deflator.[15] Finally, for our growth decomposition, we use real output and hours data from the Bureau of Labor Statistics (BLS) multifactor productivity program.[16][17]

The flow-weighted investment deflator is constructed as the ratio of the investment deflator (i.e., table 2.7 divided by table 2.8) to our consumption deflator. We can construct this deflator for any capital type, by choosing the appropriate line of tables 2.7 and 2.8. The stock-weighted investment deflator similarly obtains by diving the capital deflator (the ratio of table 2.1 to 2.2) to the consumption deflator.[18] In appendix, we show that, at a finely disaggregated level, the investment and capital prices nearly coincide. The differences between these series appear at an aggregated level only, owing precisely to the different weighting that is our focus. (See figure 24 in appendix and the discussion there.)

Throughout our paper, we will use the BEA price indices, instead of some other data source. We follow this approach in large part because our point is orthogonal to the question of the measurement of investment prices (i.e., the appropriate adjustment for quality and for new goods). Early work by Gordon showed that the BEA deflators likely underestimated the pace of progress for investment and durable goods. Gordon proposed an alternative measure, which has been used in much of the literature, and which was subsequently updated by Cummins and Violante. Since then, however, the BEA has considerably refined its measurement of investment prices. At this point, it is unclear if the mismeasurement of investment prices is more serious than that of consumption prices. Moreover, the BEA has expanded the set of measured investment substantially by includ-

[12] The start date is driven by the availability of detailed data by capital type.

[13] See https://apps.bea.gov/iTable/index_FA.cfm

[14] See https://apps.bea.gov/iTable/index_nipa.cfm.

[15] We use a Fisher index of nondurables goods and services.

[16] See https://www.bls.gov/mfp/

[17] Add caveats/adjustments about comparability of NFBS to GDP.

[18] For some of the graphs, we normalize these relative prices to have a value of one in the last year.

ing software, R&D, and entertainment products, for which no alternative price index is readily available.

3.2 Shares

As a reminder, figure 3 illustrates the shares of investment (top panel) and the capital stock (bottom panel) in the four broad categories of investment currently measured by the BEA: non-residential equipment, residential and non-residential structures, and intellectual property products (IPP). This latest category was created by the BEA in 2012 by combining software (which was measured since 1999 but previously included with equipment), together with research and development (R&D) and entertainment products. IPP has risen significantly from about 10% of investment spending in the 1980s to about 25% today.

Currently, the two largest categories in terms of flow spending are equipment and IPP, with about 35% and 25% of total private investment, respectively. Yet, these are the two smallest categories in terms of capital stocks, as shown in the bottom panel of figure 3: residential and non-residential structures account for around 45% and 30%, respectively, of the stock.

Of course, this difference between investment and capital shares largely reflect differences in depreciation rates and growth rate of capital prices. Table 2 reports these statistics for the major components of investment that we study, and figure 4 illustrates these graphically.

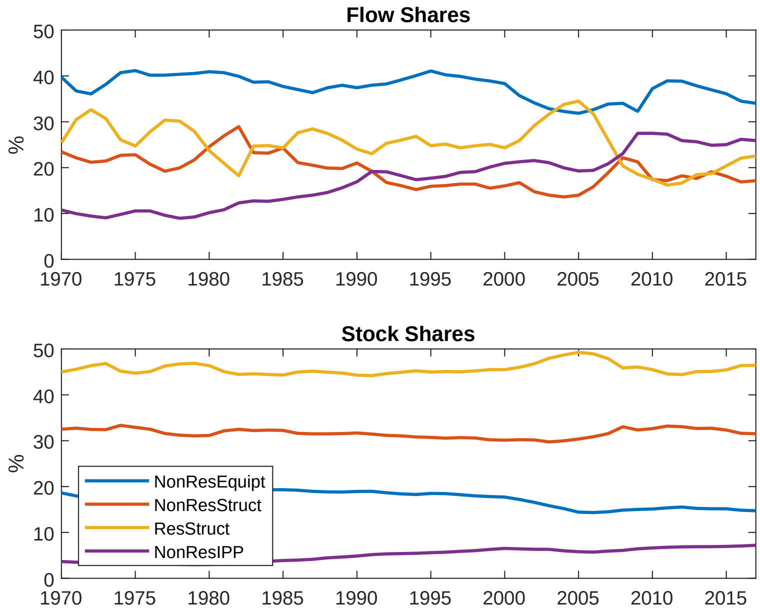

Figure 3: Investment (flow) shares (top panel) and capital (sto ck) shares (bottom panel) for equipment, non-residential and residential structures, and IPP.

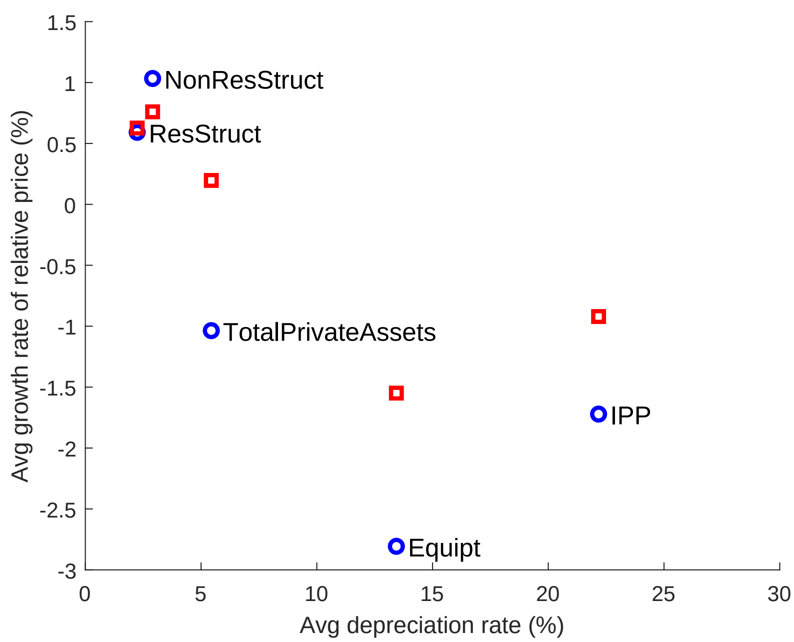

Figure 4: For each component of investment, we depict the depreciation rate (time-series average) against the average growth rate of investment prices (blue circles) or capital price (red squares).

	Share I	Share K	Deprec. rate	dPI/PI	dPK/PK	dI/I
Private fixed assets	100.00	100.00	5.44	-1.04	0.20	3.83
Equipment	38.11	17.63	13.44	-2.81	-1.55	5.49
Nonresidential structures	19.28	31.63	2.91	1.03	0.76	1.22
Residential structures	25.21	45.68	2.24	0.59	0.63	2.58
Intellectual property products	17.41	5.06	22.17	-1.72	-0.92	6.37

Table 2: For each component of investment, we report the time-series average of the share of investment, the share of capital, the depreciation rate, the growth rate of the price of investment and the price of capital, and the growth rate of real investment.

3.3 Falling and Rising Investment Prices

It is well known that the fall of investment prices is driven by equipment prices. In this section we draw attention to two somewhat less well known facts. First, even within equipment, the decline of prices is concentrated in a very limited number of categories. Second, the price of structures has been rising, in particular since 1990.

Figure 5 displays the relative price of each equipment category and illustrates the extraordinary decline in the cost of computers, which swamps any other price action during this period: computer prices have, according to the BEA, fallen by a factor of over 3000 relative to consumption prices. Figure 6 uses a log scale for readability. Communication equipment prices also decline, though at a slightly slower pace. The other equipment goods exhibit fairly moderate price declines, and many are not far from trend-less.

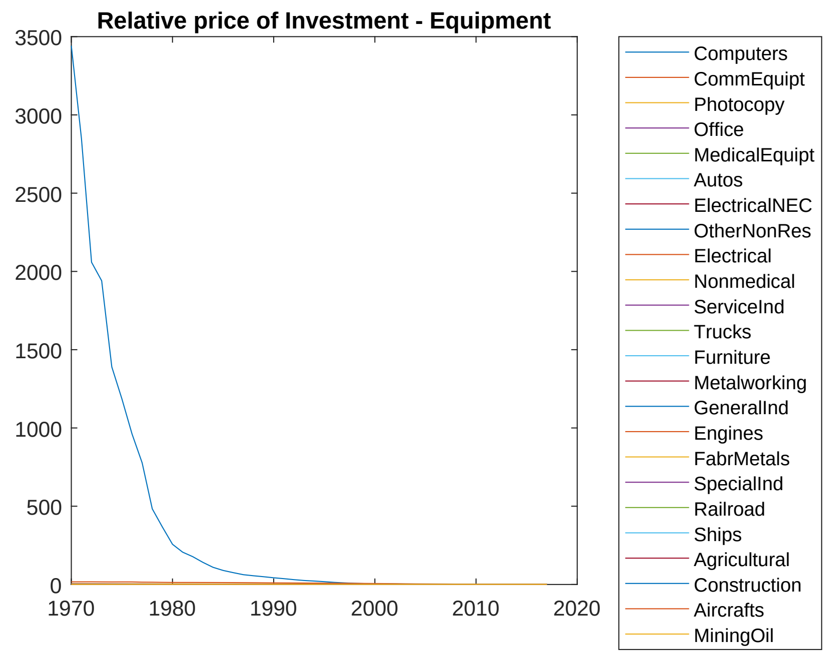

Figure 5: Relative price of each capital type (level 4) in equipment to consumption.

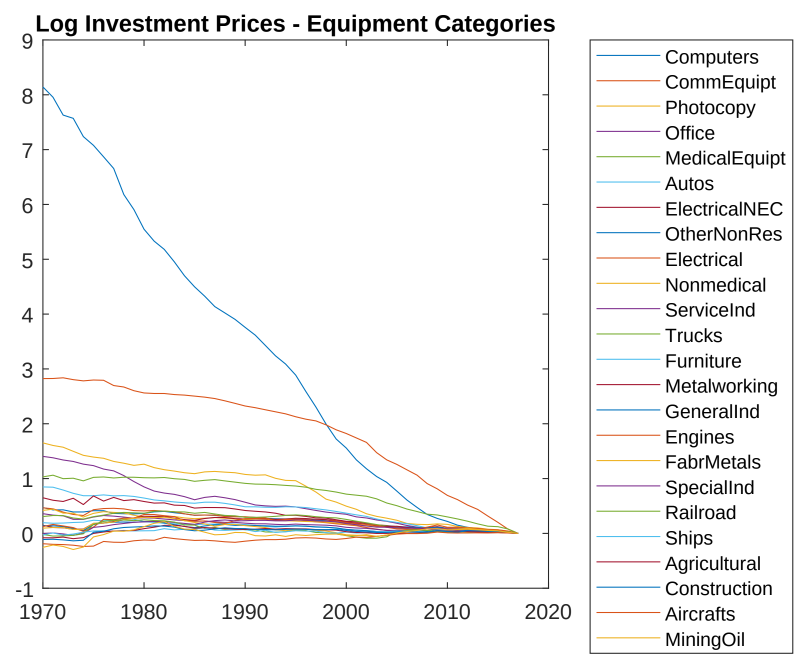

Figure 6: Log Relative price of each capital type (level 4) in equipment to consumption.

The natural question is: how much of the decline in the overall price of investment (or of equipment), is accounted for by these few categories? Figure 7 answers this by showing the average contribution to the growth rate of the overall (flow-weighted) relative price of investment, and figure 8 does the same for the price of equipment and IPP.

Overall, in our data, the price of investment declines by 1.04% per year on average. The contribution of computers to this decline is around 0.59% per year on average.[19] In other words, the single category of computers - which amounts to 3.5% on average of flow spending (and 0.5% of the stock of capital) - is responsible for over half of the decline of the investment price. Two other categories are major contributors: communication equipment (a contribution of 0.26%), and software (prepackaged, customer and own-account), for 0.24% in total. If we strip out these three categories, there is no investment price decline. This, of course, implies that how we *weight* these few categories is key.

On the other hand, figure 7 shows that several structures have rising relative prices, in particular permanent residential housing, and energy structures (power and oil and gas drilling). Overall, as was seen in figure 4, the price of structures relative to consumption is rising slightly.[20]

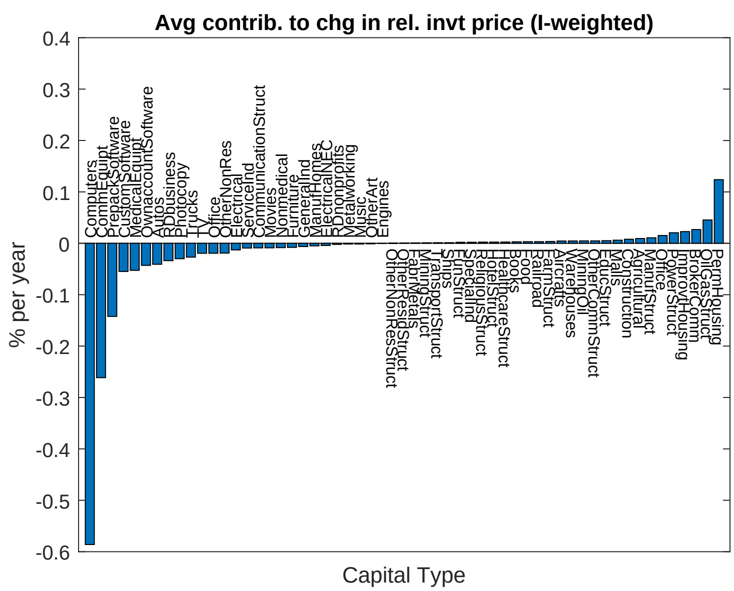

Figure 7: Average contribution of each capital type to the change in the relative price of investment (using investment shares as weights).

[19]This contribution is defined as the change in the investment price that would occur if all other prices were stable (relative to consumption) and only computer prices were changing.

[20]There are of course a number of measurement issues, including the quality adjustment of structures, and how to tease away the value of land from the value of structures. See Rupert et al.

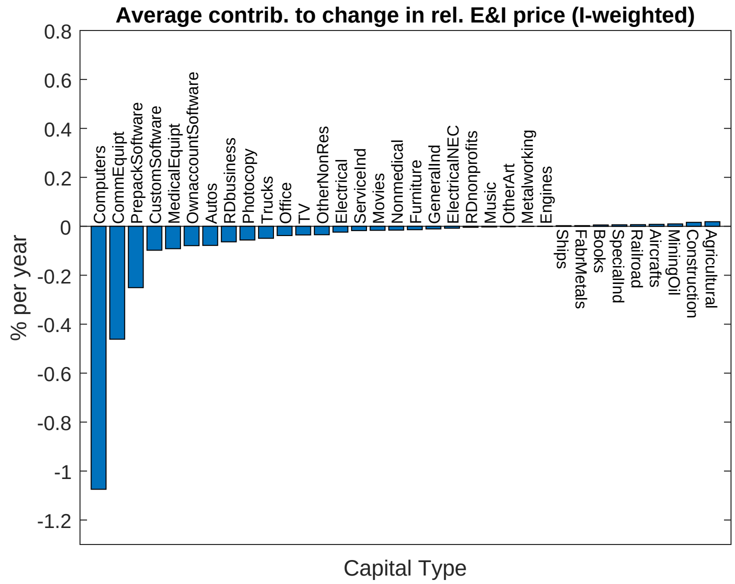

Figure 8: Average contribution of each capital type to the change in the relative price of equipment and intangibles investment (using investment shares as weights).

3.4 Difference between Stock-Weighted and Flow-Weighted deflators

We now illustrate the difference between the stock-weighted and flow-weighted deflators. Figure 1 illustrates the sharp difference between these two series. The blue line is the flow-weighted deflator of investment prices, and falls continuously over time starting around 1980. The decline of over 0.5 log point since 1980 reflects that investment goods, relative to consumption goods, are now about 40% cheaper than they used to be. The red line is the stock-weighted deflator of investment prices. While this price fluctuates, in particular rises during the housing boom, it appears nearly trendless over this period. Clearly, there are stark differences as far as trends are concerned.

The main driver of the difference is that, as we just saw, many investment prices, in particular structures, do not fall. These prices get a much larger weight in the stock-weighted deflator, based on figure 3, and a much lower on in the flow-weighted deflator. On the other hand, high-tech equipment such as computers gets a much smaller weight in the stock-weighted deflator (by a factor of about 7), reducing dramatically its contribution. To illustrate this, in figure 10 we depict the contribution of each type to the stock-weighted and to the flow-weighted investment deflators.

At the heart of our point is the fact that capital types differ substantially in depreciation rates and in investment price growth. Broadly speaking, these features are correlated, so that high depreciation types, which have a high investment share relative to their capital share, also see a large price decline: this pattern is illustrated in figure 11 (and was also present in figure 4 above).[21] Overall, figure 9 shows that the differences between I-weighted and K-weighted deflators remain important if we disaggregate between equipment, residential and non-residential structures, and IPP.

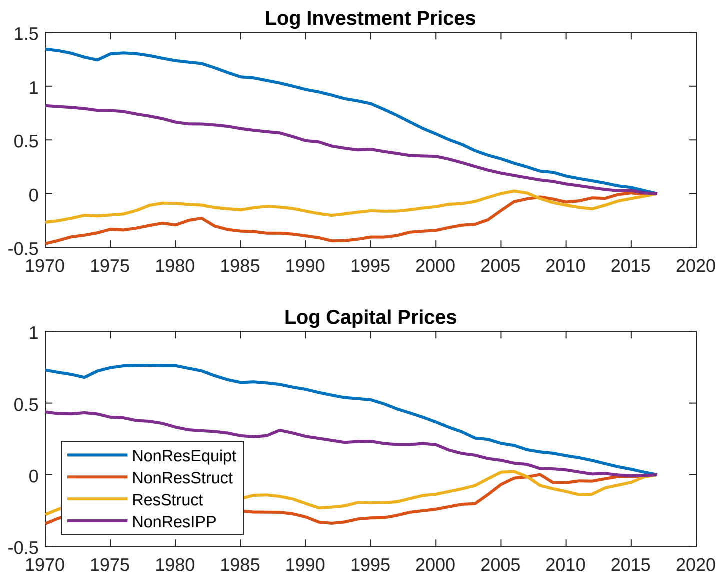

Figure 9: Log of flow- and stock-weighted relative investment prices.

[21] This correlation is a statistical regularity - it does not have to be true either from an accounting point of view or from an economic point of view.

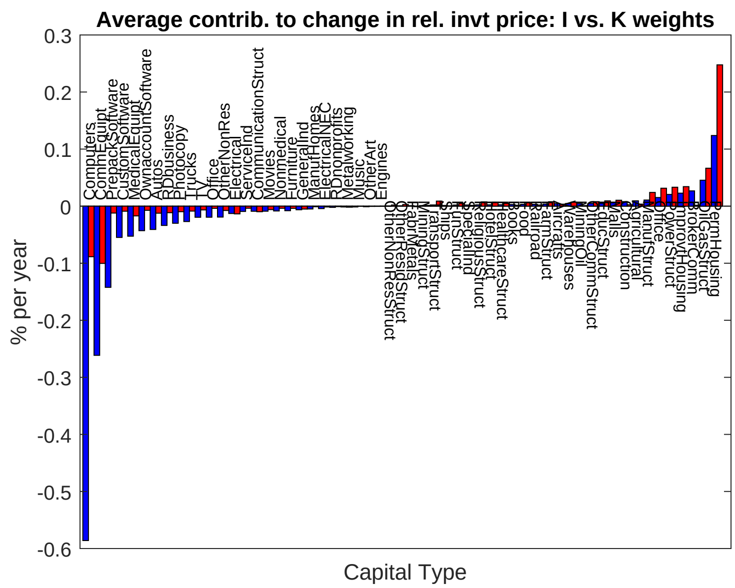

Figure 10: Average contribution of each capital type to the change in the relative price of investment: investment weights (blue) vs. capital weights (red).

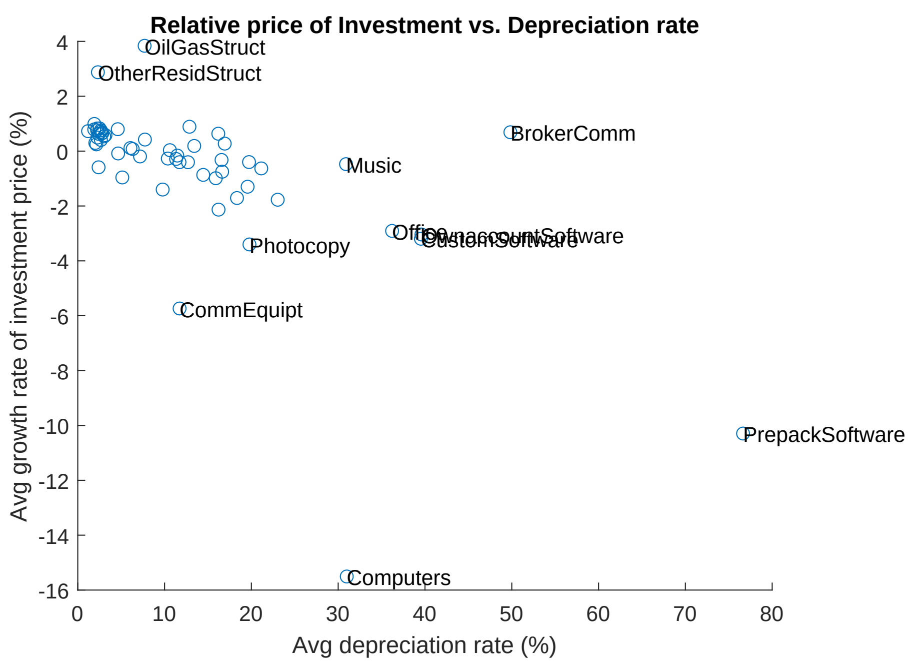

Figure 11: The figure plots, for each capital type, the time-series average of the depreciation rate against the time-series average growth rate of the price of investment.

4 Long Run Growth

One key implication of our model is that the correct investment price index which determines the long-run growth of the economy is a rental share-weighted average of the underlying prices. All previous authors, to our knowledge, use an investment-weighted index. In this section, we illustrate how doing the correct measurement affects the conclusion that investment-specific technical change is responsible for a majority of productivity growth. We first present a very simple calculation using the balanced growth model, and we then generalize it.

4.1 BGP analysis

Our calculation closely follows the model of section 2. According to Proposition 1, along the BGP path, the growth rate of output per hour (i.e., labor productivity) is a combination of the growth rate of total factor productivity and of the rental-share weighted price index for investment goods g_{p_R}:

$$g_Y - g_L = \frac{g_A}{1-\alpha_K} - \frac{\alpha_K}{1-\alpha_K} g_{p_R}, \tag{51}$$

where g_A is the growth rate of total factor productivity and α_K the aggregate capital share. Moreover, the rental-share weighted price of investment goods is, according to Proposition 3, a convex combination of the flow-weighted and stock-weighted investment prices (all in growth rates), and the weight is the ratio of investment to profits.

Our approach, then, is to obtain from the data the stock- and flow-weighted price indices, and the investment-profits ratio, and hence obtain the rental share-weighted prices g_{p_R}. We furthermore measure α_K as the capital share. We can then infer the TFP growth rate g_A needed to match the growth rate of output per hour from equation (51). We hence know how much of growth is due to TFP, namely $\frac{g_A}{1-\alpha_K}$, and how much is due to ICT change, $-\frac{\alpha_K}{1-\alpha_K} g_{p_R}$.

We produce this calculation, both for the correct price index g_{p_R} and in the case where the researcher mistakenly uses g_{IW} instead. Tables 3 and 4 report the results. The first table reports the data moments used for the calculation. In the full sample (1970-2017), the growth rate of labor productivity is 1.2% per year,[22]. The growth rate of the I-weighted price index is around -1% per year, while the K-weighted price index actually increases

[22] This figure is obtained as the growth rate of the nonfarm business sector output per hours from the BLS MFP program, with two adjustments: first, we reduce this number by 0.3% per year to make it cover the entire economy (to reflect the mean difference in growth rate of NFBO and GDP), and second, we also adjust for the relative price of NDS PCE and GDP.

slightly (by 0.2%). Given that the investment-profit ratio is around 0.5, the R-weighted price index is almost an exactly equal average of the two, i.e. around -0.4% per year. The table then goes through this calculation for various subsamples. The decline in investment prices accelerates after 1985. The gap between the I-w and R-w price indices shrinks slightly after 2006.

Table 4 presents the key results from the exercise. In the full sample, our procedure ascribes 0.98% of the 1.19% growth rate in output per worker to total factor productivity, and 0.21% to ISTC progress. In percentage terms, over 80% of the increase in output per worker is due to TFP. In contrast, a researcher who uses he I-w index would find that ISTC contributes 0.52% to growth, and TFP only 0.66%, leading to a much more equal 55%-45% decomposition. Hence, using the correct price index leads to a substantially different conclusion on the sources of economic growth for the United States over the past 50 years.[23] Interestingly, over the three subsamples that we consider (pre-1985, 1985-2005, and post-2005), the importance of ISTC rises steadily. But the difference between the I-weighted and R-weighted indices remain very significant. For instance, in the last subsample, our procedure gives a role of 60% to ISTC, while the I-weighted procedure attributes a role of over 100% (meaning that TFP is actually falling).[24]

	DlogY/H	**Inv/Prof**	**Price IW**	**Price KW**	**Price RW**
1970-2017	1.19	0.51	-1.02	0.23	-0.41
1970-1984	1.17	0.55	-0.23	0.65	0.16
1985-2005	1.49	0.52	-1.49	0.09	-0.73
2006-2017	0.68	0.45	-1.12	-0.01	-0.51

Table 3: The table reports, for various samples, the average growth rate of output per hour, the ratio of investment to profits, and the growth rate of three investment price indices: the investment (flow) weighted, the capital (stock) weighted, and the rental share weighted.

[23]In their seminal paper Greenwood et al. [1997], find an even somewhat greater role for ISTC. There are a few technical differences between their calculations and ours (different sample, different calibration procedure, etc.), and two major ones: they use the Gordon price index while we use the BEA one; and importantly, they use a two-capital model, which reduces the aggregation bias. See appendix TBA for a detailed comparison.

[24]Our calculations rely on the capital share representing the fair compensation of capital). One potential extension is to allow for market power.

	Data	Iw:ITC	Iw:TFP	Rw:ITC	Rw:TFP
1970-2017	1.19	0.52	0.66	0.21	0.98
(%)	100.00	43.80	55.91	17.46	82.37
1970-1984	1.17	0.11	1.06	-0.08	1.25
(%)	100.00	9.30	90.60	-6.52	106.60
1985-2005	1.49	0.75	0.73	0.37	1.11
(%)	100.00	50.79	48.84	24.75	74.97
2006-2017	0.68	0.64	0.04	0.29	0.39
(%)	100.00	93.64	6.32	42.35	57.49

Table 4: The table reports, for various samples, the average growth rate of output per hour, and the derived growth rate of output due to ISTC and to TFP, both for the case in which the calibration uses investment-weighted prices (Iw, columns 2-3) and for the case where the calibration uses rental share-weighted prices (Rw, columns 4-5). The % values indicate the share of growth in output per hour accounted for by each source (ISTC vs. TFP).

4.2 Comparison with BLS indices

Our focus on rental-weighted of course brings to mind the capital services indices produced by the BLS. Figure 12 compares the capital services index of the BLS (for the nonfarm business sector) with the stock-weighted capital stock of the Fixed Asset tables and with our own rental-weighted index (both calculated for the nonresidential sector so as to more closely align the coverage with the BLS). Remarkably, our simple rental-weighted calculation comes out close to the BLS - hence, the balanced growth assumption may not be a bad one. By contrast, the stock of capital in the fixed asset tables has grown more slowly - largely reflecting that capital growth has been higher in capital types that have high rental shares relative to stock shares (e.g., computers, ICT, and software).

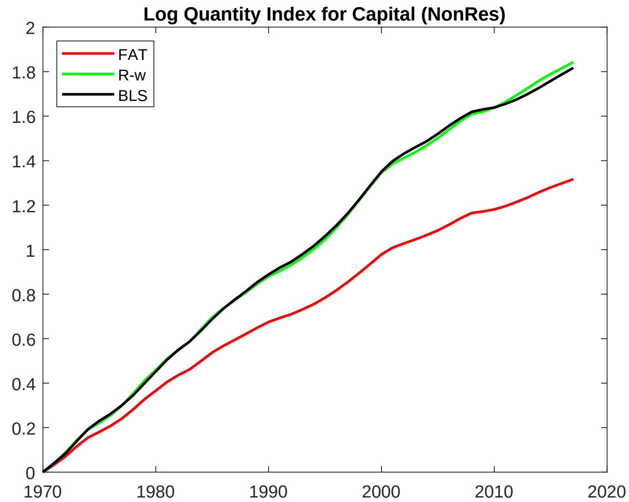

Figure 12: Comparison of quantity indices for nonresidential sector: stock-weighted, rental-weighted (our version), and BLS nonfarm business sector index of capital services quantity.

More relevant for us is the price index of capital services; here, note that our index is somewhat different conceptually from the BLS since we do not try to account for variation in the user cost itself - only from the prices. Figure 13 depicts the evolution of the BLS capital service price index, our rental-weighted index, the flow-weighted deflator, and the stock-weighted (fixed assets) deflator, again for the nonresidential sector only. We find that our index again follows fairly closely the BLS one with regards to the trends, though the BLS one fluctuates more (likely because it attempts to capture the variation in the user cost).[25]

[25]The BLS measure of capital services works by constructing, for each industry, an estimate of return on capital and hence rental rates, which are then attributed to the capital goods based on the use of capital by each industry.

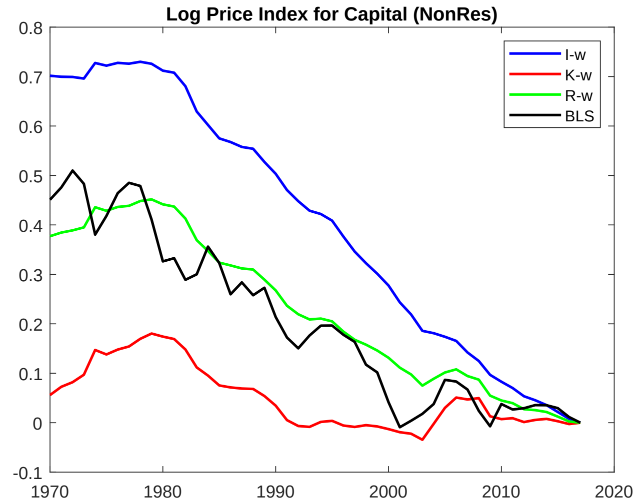

Figure 13: Comparison of price indices for nonresidential sector: flow-weighted, stock-weighted, rental-weighted (our version), and BLS nonfarm business sector index of capital services price.

4.3 Extension with Time-Varying Parameters, Off BGP

TBA

5 Investment prices and economic fluctuations (Preliminary)

In this section we revisit the contribution of investment specific technical change (ISTC) to economic fluctuations. Specifically, we run VARs in the style of Fisher [2006] that use long-run restrictions to infer the role of ISTC shocks and other productivity shocks. Our VARs have three variables: an investment price index (in growth rate), labor productivity growth, and log hours. We compare the VARs that use the investment deflator vs. the ones that use the measure suggested by proposition 5 - a combination of the investment deflator and the stock deflator, though with a much larger weight on the stock deflator. Like Fisher, we use quarterly data with and estimate over 1982:IV to 2019:IV with 4 lags. Because we use quarterly data, we have 14 categories of capital goods.

Figure 14 compares the growth rate of the flow-weighted investment price used in much of this literature with the series of relevant price change (i.e. ω). (Both series are deflated with the non-durables and services consumption deflator.) The correlation between the two series is high but not perfect. In particular, during the 1990s, and again in the mid-2000s, our index declines less than the flow-weighted index.

Figure 15 compares the impulse response to a one-standard-deviation identified ISTC shock using our index or the flow-index. This shock is recessionary in both cases; however, the magnitudes are different, with the decline of output and hours much smaller for our index (despite a larger shock).

Figure 16 depicts the variance decomposition of hours. Unsurprisingly in light of the diminished impulse response, the ISTC shocks now account for a significantly smaller share of variance of hours. In contrast, the role of TFP is now greater.

We should note that while these VARs do show some significant differences, the differences are much smaller if one focuses solely on equipment prices as a source of shocks. This is because there is less heterogeneity in weights within equipment.

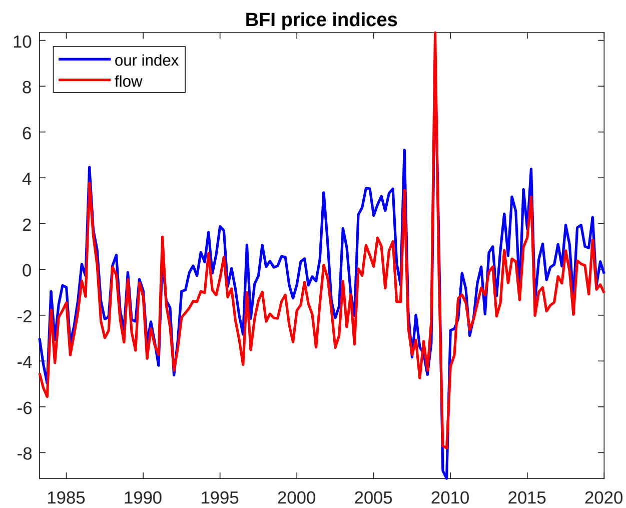

Figure 14: Growth rate of flow-weighted investment price and of the relevant price index ω. Total investment - both residential and nonresidential.

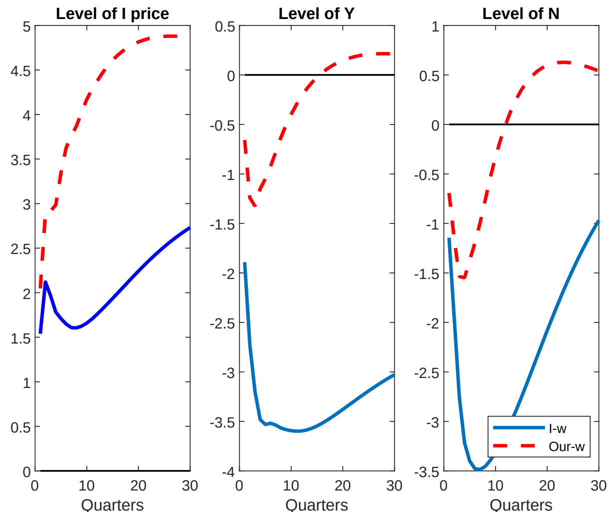

Figure 15: Impulse responses to an identified positive shock to investment prices in the VAR using either the flow-index or our index.

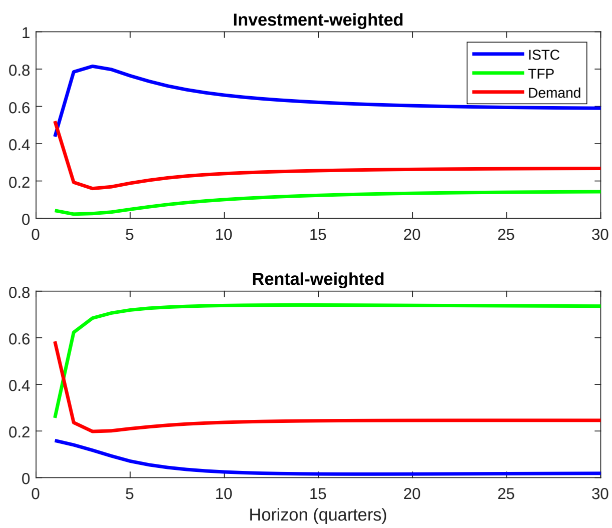

Figure 16: Variance decomposition of Hours across the three shocks, for the flow-weighted deflator (top panel) and our deflator (bottom panel).

6 Capital Heterogeneity and The Big Ratios

Recently, the behavior of the Kaldorian "big ratios" has again attracted attention among macroeconomists, who have reconsidered the stability of the capital share, capital-output, investment-output, or profitability (profit-capital) ratios.[26] In this section, we discuss how taking into account capital heterogeneity affects the measurement of these ratios, and the changes that have been documented.

6.1 The decline of investment

The puzzle A number of researchers have highlighted the weakness of investment over the past 20 years. Figure 17 illustrates this pattern by depicting gross and net investment-to-capital ratios. Researchers have studied a variety of potential explanations, including the rising importance of intangibles, rising market power, the offshoring of capital-intensive manufacturing production, and the rising influence of shareholder who may constrain CEOs'desires of expansion.[27]

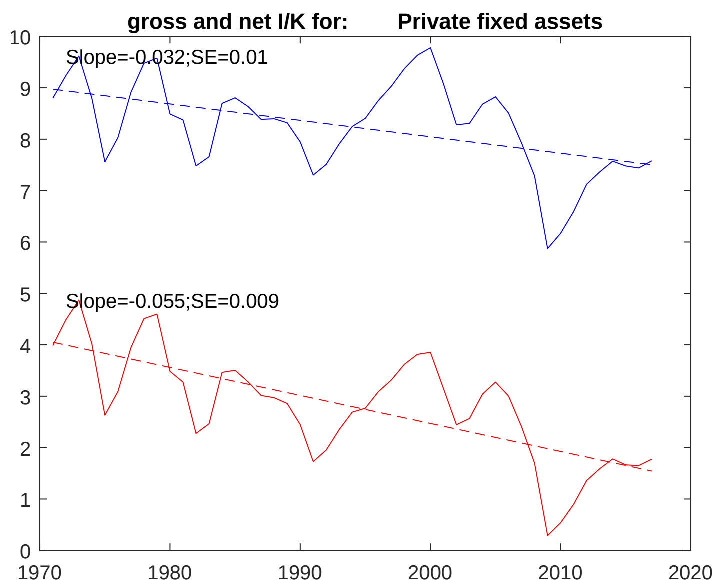

Figure 17: Gross (top line, blue) and net (bottom line, red) investment to capital ratio, for overall private fixed assets, with linear trends superimposed. In %.

Our balanced growth perspective

[26]See, among many others, (Karabarbounis and Neiman [2014], Eggertsson et al. [2018], **?**, **?**, **?**, etc.

[27]Add citation/discussion to Philippon, Crouzet and Eberly, Alexander and Eberly, Hall, etc.

In our balanced growth model, the gross aggregate investment to capital ratio is stable and equals

$$\frac{I_t}{K_t} = \delta^K + g_Y - \frac{\dot{p}_t^K}{p_t^K}, \tag{52}$$

where g_Y is the growth rate of output, δ^K is the *stock-weighted* depreciation rate, and $\frac{\dot{p}_t^K}{p_t^K}$ is the *stock-weighted* investment price growth. The stock-weighted depreciation rate is

$$\delta = \sum_{i=1}^{n} \delta_i s_i^K, \tag{53}$$

where s_i^K is the share of capital of type i:

$$s_{it}^K = \frac{P_{it} K_{it}}{\sum_{j=1}^{n} P_{jt} K_{jt}}, \tag{54}$$

which along the balanced growth path equals

$$s_{it}^K = s_i^K = \frac{\frac{\alpha_i}{uc_i}}{\sum_{j=1}^{n} \frac{\alpha_j}{uc_j}}. \tag{55}$$

The stock-weighted investment price growth rate follows a similar formula, where we replace the type-specific depreciation rate with type-specific investment price growth. Most researchers correctly use the stock-weighted depreciation rate, but many substitute the *flow-weighted investment* price deflator for the capital-weighted deflator. This is an important consideration in the calibration of growth models.

The role of investment prices and growth

We can use equation 52 to understand the evolution of the investment-capital ratio. The net investment-capital ratio can only decline for two reasons: the growth rate of the economy is declining, or the (stock-weighted) growth rate of investment prices is rising (i.e. less technical progress). (A third possible reason, of course, is that the economy is not in balanced growth or, worst still, that our model is misspecified.)

Figure 6.1 shows that the role of growth in driving the decline of the net investment-gdp is critical. This figure depicts 11-year centered moving average of net I/K and GDP growth. Clearly, the vast majority of the decline is accounted for by the decline in growth - a simple but possibly underappreciated point - any causal story for the decline of investment has to be one that explains lower growth.

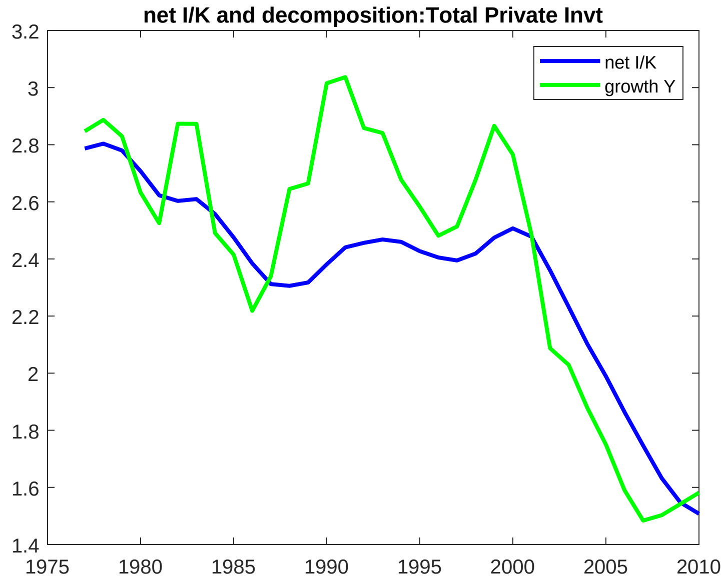

Figure 18 does the same exercise for non-residential equipment since 1990, and adds the role of prices to the figure, as well as a residual (red line). Clearly, the role of prices is important for that series, together with growth, but the residual plays only a limited role, suggesting again that explanations for the decline of investment ought to focus on explaining the behavior of GDP and investment prices.

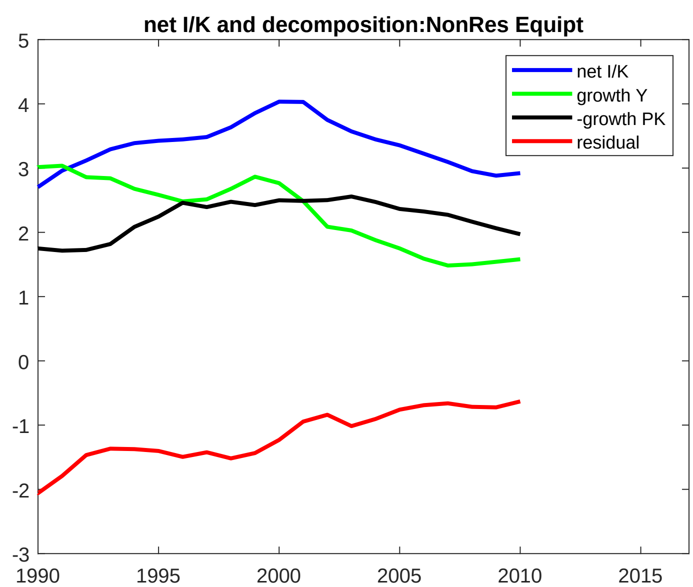

Figure 18: net I/K for non residential equipment and role of GDP growth (green), price of investment (black) and residual (red).

Finally, note that equation 52 also applies at the capital type level:

$$\frac{I_{it}}{K_{it}} = \delta_i + g_Y - \frac{\dot{p}_{it}}{p_{it}}. \tag{56}$$

This equation can be applied across types - high investment-capital ratios reflect high depreciation or large price declines - and it can also be applied over time - a permanent decline in the investment-capital ratio must be due to lower depreciation, lower growth, or higher rate of increase (lower rate of decrease) of the price of investment. Figure 19 illustrates this equation for the case of computers.

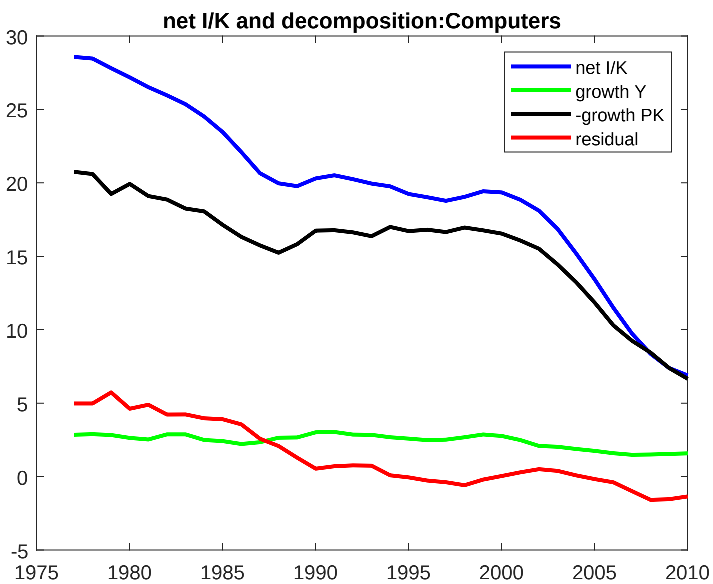

Figure 19: net I/K for computers and role of GDP growth (green), price of investment (black) and residual (red).

6.2 The stability of profitability

The puzzle Rates of return on safe assets such as U.S. Treasuries have declined substantially over the past 25 years. Yet, measured rates of return on private capital appear stable. Gomme, Ravikumar and Rupert in an influential contribution use National Income data together with capital stocks from the Fixed Assets Tables to calculate a series for the aggregate return on private capital, or profitability ratio, i.e. payouts to capital per unit of capital, and show that it is trend-less.[28] We construct in figure 20 below a similar measure. Previous research has highlighted three main factors for the evolution of this ratio: rents

[28]See also Mulligan for earlier related work.

owing to market power (e.g. Barkai), intangible capital (e.g. Crouzet and Eberly), and risk premia (e.g Farhi and Gourio).

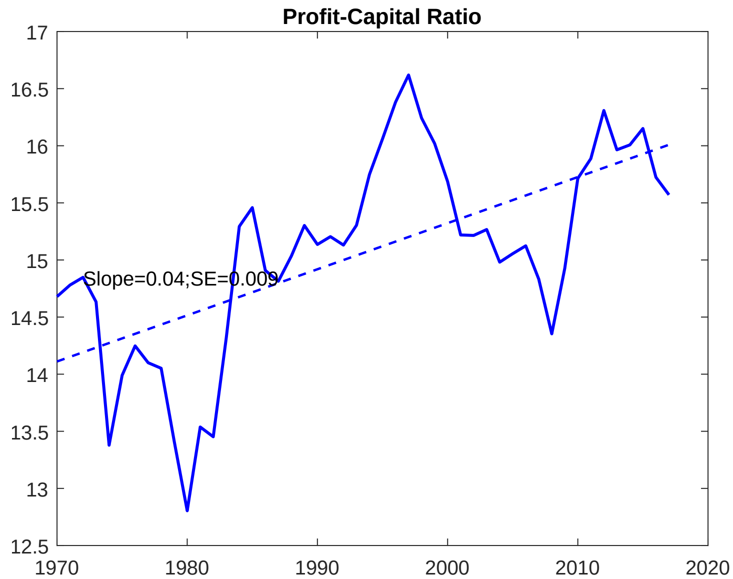

Figure 20: Profitability of private capital, profits are from table 1.10 (with the usual adjustments for proprietor income), and private capital is current cost total private fixed assets.

Our balanced growth perspective In our balanced growth model, the aggregate profit to capital ratio equals

$$\frac{\Pi_t}{K_t} = \frac{\sum_{i=1}^{n} \Pi_{it}}{\sum_{i=1}^{n} P_{it} K_{it}} = r + \delta^K - \frac{\dot{p}_t^K}{p_t^K}, \tag{57}$$

Here too, the appropriate index is the stock-weighted investment price growth. The question is now, can the lack of decline of profitability be due to an increase in $\frac{\dot{p}_t^K}{p_t^K}$, in particular relative to the flow-weighted index which has been used by most researchers instead?

The role of investment prices and depreciation

Figure depicts the growth rate of the stock-weighted price of investment , together with a linear trend, and figure does the same for the flow-weighted one. If anything, these growth rates are declining, meaning that ISTC is accelerating during this period (though, to be sure, there are important fluctuations). This means that the required profitability is becoming higher over time due to this decline, thus explaining partly the trend. However, replacing the flow-weighted index with the stock-weighted index reduces this positive effect on profitability. Moreover, it also reduces the increase in the effect during this period. We conclude that taking this into account hardens the puzzle.

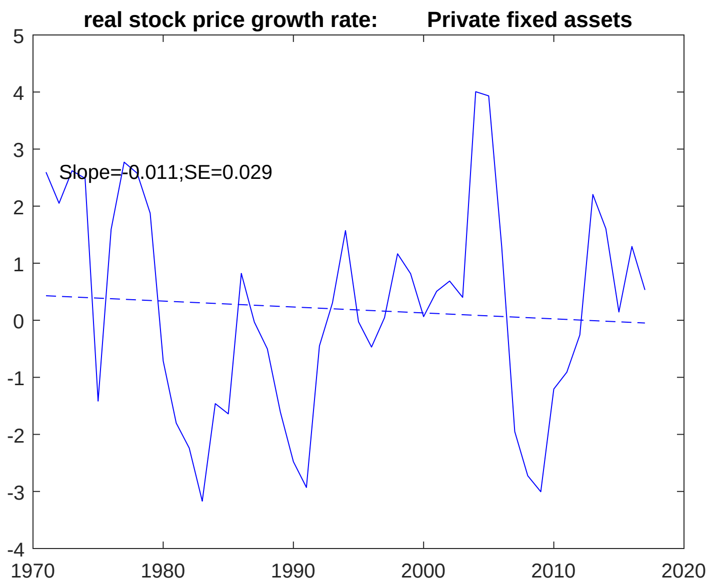

Figure 21: Growth rate of (stock-weighted) price of investment for total private fixed assets, with linear trend.

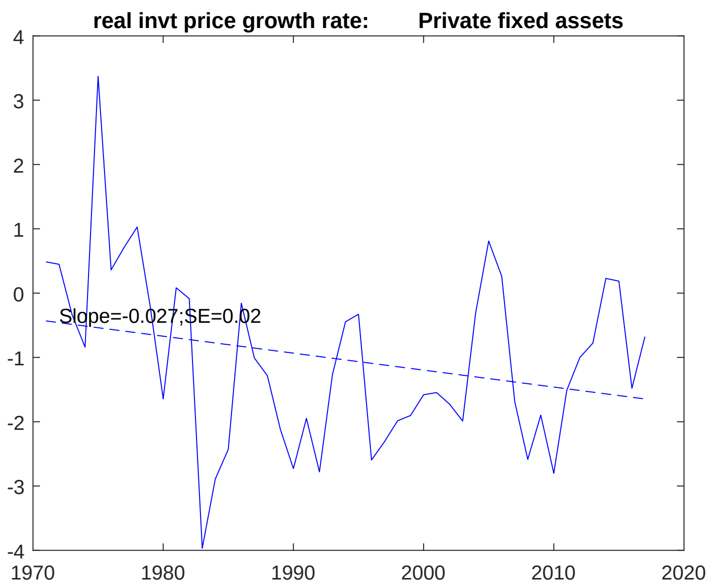

Figure 22: Growth rate of (flow-weighted) price of investment for total private fixed assets, with linear trend.

6.3 The rise of the capital-output ratio

The puzzle Following in particular the work of Piketty, there has been some interest in the evolution of the capital-output ratio. Here we focus on the standard measure of capital:

reproducible private fixed assets as measured by the B.E.A. fixed asset tables.[29] Figure 23 depicts the evolution of this capital (at current cost) to GDP, which has increased by about 15% since 1970, though with some fluctuations. This is a deviation from the stylized "stability" and in need of an explanation - changing technology, lower interest rates, or rising market power are potential drivers that have been considered in the literature.[30]

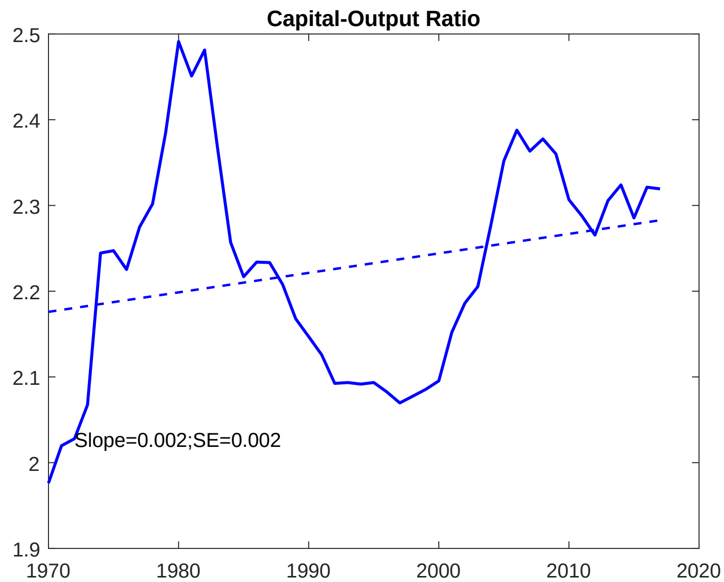

Figure 23: Capital-output ratio. Capital is total private fixed assets at current cost, and output is total nominal GDP.

Our balanced growth perspective In our balanced growth model, the (current cost) capital to output ratio is:

$$\frac{K_t}{Y_t} = \frac{\sum_{i=1}^{n} P_{it} K_{it}}{Y_t} = \frac{\sum_{i=1}^{n} \alpha_i}{r + \delta^K - \frac{\dot{p}_t^K}{p_t^K}} \tag{58}$$

The role of investment prices and growth

TBA

[29] Piketty includes a broader set of assets, notably land and financial assets.

[30] See, for instance, Farhi and Gourio for a decomposition.

7 Conclusion

To be added.

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8 Appendix

8.1 Proofs

Derivation of equation 40

$$\begin{aligned}
\dot{K}_t^W &= \sum_{i=1}^{N} g_{it} P_{it} K_{it} + \sum_{i=1}^{N} P_{it} \dot{K}_{it} \\
&= \sum_{i=1}^{N} (g_{it} - \delta_i) P_{it} K_{it} + Y_t - C_t \\
&= \sum_{i=1}^{N} (g_{it} - \delta_i) \frac{\alpha_i}{r_t + \delta_i - g_{it}} Y_t + Y_t - C_t \\
&= \sum_{i=1}^{N} \alpha_i \left(-1 + \frac{r_t}{r_t + \delta_i - g_{it}} \right) Y_t + Y_t - C_t
\end{aligned}$$

Derivation of equation 2.3

$$\begin{aligned}
\frac{\dot{\widehat{K}}_t^W}{\widehat{K}_t^W} &= \frac{\dot{K}_t}{K_t} - \frac{\dot{\widetilde{K}}_t}{\widetilde{K}_t} \\
\dot{\widehat{K}}_t^W &= \frac{\dot{K}_t}{\widetilde{K}_t} - \widehat{K}_t^W g_Y \\
&= \frac{\alpha_L Y_t - C_t + r_t K_t^W}{\widetilde{K}_t} - \widehat{K}_t^W g_Y \\
&= \frac{\alpha_L \widehat{Y}_t \widetilde{Y}_t - \widehat{C}_t \widetilde{C}_t + r_t \widehat{K}_t^W \widetilde{K}_t}{\widetilde{K}_t} - \widehat{K}_t^W g_Y \\
&= \frac{\alpha_L \widehat{Y}_t \widetilde{Y}_t - \widehat{C}_t \widetilde{C}_t}{\widetilde{K}_t} + (r_t - g_Y) \widehat{K}_t^W \\
&= \alpha_L \frac{Y^*}{K^*} \widetilde{Y}_t - \widehat{C}_t \frac{C^*}{K^*} + (r_t - g_Y) \widehat{K}_t^W
\end{aligned}$$

8.2 Summary Statistics

Table 5 lists the types of capital available in our data, together with four numbers. The first is the level of aggregation (ranging from 0 for total private fixed assets, to 1 for the big categories (E, S & I), to 2 with the additional non-residential vs. residential distinction, to 3 for the broader subcategories (e.g., Information Processing Equipment), to 4, 5, or 6 for more detailed categories. The second is the sector to which the category belongs (1 for the entire economy, 2 for equipment, 3 for non-residential equipment, 4 for residential equipment, 5 for structures, 6 for non-residential structures, 7 for residential structures, 8 for intellectual property products (IPP), 9 for non-res. IPP.[31] The third and fourth are the time-series average of the investment (flow) share (the ratio of nominal investment of the type over total private fixed investment) and of the capital (stock) share (the ratio of the nominal current-cost capital of the type over the total private fixed capital stock (also at current cost))).

Tables 6, 7 and 8 present some additional summary statistics, grouped by equipment, structures, and intellectual property. Here for brevity, we do not use the long name but a shorter version. We report for each type the time-series average of the depreciation rate, of real investment growth and real capital growth, and of investment and capital price growth. As explained in section 3.1, the prices here are calculated as the ratio of (i) the deflator implied by the fixed asset table (i.e. the ratio of the current cost to the real stock estimates) and (ii) the deflator for nondurable goods and services personal consumption expenditures (constructed using NIPA data as the Fisher aggregator of the nondurable goods and services deflator).

Figure 24 shows the average growth rate of investment and capital price for disaggregated capital types (i.e. level 4, in the language of table 5). These mean growth rates align very closely, with a R2 of 0.99 and a slope of 0.96.[32]

[31] Note that the residential equipment category is very small (around 1% of investment), and we will hence omit it from our analysis. There is currently no residential IPP category.

[32] If we look at this relation pointwise, i.e. year-by-year rather then averaging, the relation remains very tight, with a slope of XXX and a R2 above YYY. (Note - there's an outlier here, need to check data - 'Other Resid Structures' is behaving weirdly.)

	Sector	Aggreg.	Avg share I	Avg share K
Private fixed assets	1.00	0.00	100.00	100.00
Equipment	2.00	1.00	38.11	17.63
Nonresidential equipment	3.00	2.00	37.57	17.43
Information processing equipment	3.00	3.00	12.10	3.71
Computers and peripheral equipment	3.00	4.00	3.51	0.54
Communication equipment	3.00	4.00	4.33	1.74
Medical equipment and instruments	3.00	4.00	1.87	0.62
Nonmedical instruments	3.00	4.00	1.02	0.42
Photocopy and related equipment	3.00	4.00	0.83	0.27
Office and accounting equipment	3.00	4.00	0.54	0.11
Industrial equipment	3.00	3.00	9.01	6.20
Fabricated metal products	3.00	4.00	1.03	0.61
Engines and turbines	3.00	4.00	0.43	0.36
Metalworking machinery	3.00	4.00	1.61	0.99
Special industry machinery, n.e.c.	3.00	4.00	1.81	1.19
General industrial, including materials handling, equipment	3.00	4.00	2.76	1.70
Electrical transmission, distribution, and industrial apparatus	3.00	4.00	1.37	1.34
Transportation equipment	3.00	3.00	7.98	3.73
Trucks, buses, and truck trailers	3.00	4.00	3.75	1.21
Light trucks (including utility vehicles)	3.00	5.00	1.90	0.57
Other trucks, buses, and truck trailers	3.00	5.00	1.85	0.65
Autos	3.00	4.00	2.29	0.69
Aircraft	3.00	4.00	1.15	0.88
Ships and boats	3.00	4.00	0.33	0.34
Railroad equipment	3.00	4.00	0.47	0.61
Other equipment	3.00	3.00	8.47	3.80
Furniture and fixtures	3.00	4.00	1.74	0.89
Agricultural machinery	3.00	4.00	1.40	0.77
Construction machinery	3.00	4.00	1.44	0.61
Mining and oilfield machinery	3.00	4.00	0.55	0.22
Service industry machinery	3.00	4.00	1.16	0.47
Electrical equipment, n.e.c.	3.00	4.00	0.30	0.10
Other nonresidential equipment	3.00	4.00	1.90	0.75
Residential equipment	4.00	5.00	0.53	0.20
Structures	5.00	1.00	44.49	77.31
Nonresidential structures	6.00	2.00	19.28	31.63
Commercial and health care	6.00	3.00	6.88	10.72
Office 1	6.00	4.00	2.52	3.97
Health care	6.00	4.00	1.49	2.26
Hospitals and special care	6.00	5.00	1.16	1.79
Hospitals	6.00	6.00	0.95	1.44
Special care	6.00	6.00	0.22	0.35
Medical buildings	6.00	5.00	0.33	0.47
Multimerchandise shopping	6.00	4.00	1.02	1.58
Food and beverage establishments	6.00	4.00	0.50	0.83
Warehouses	6.00	4.00	0.57	0.87
Other commercial 2	6.00	4.00	0.78	1.21
Manufacturing	6.00	3.00	2.62	4.33
Power and communication	6.00	3.00	3.41	6.14
Power	6.00	4.00	2.37	4.64
Electric	6.00	5.00	1.72	3.20
Other power	6.00	5.00	0.65	1.44
Communication	6.00	4.00	1.04	1.50
Mining exploration, shafts, and wells	6.00	3.00	2.98	2.66
Petroleum and natural gas	6.00	4.00	2.76	2.44
Mining	6.00	4.00	0.22	0.22
Other structures	6.00	3.00	3.39	7.78
Religious	6.00	4.00	0.33	0.93
Educational and vocational	6.00	4.00	0.50	0.93
Lodging	6.00	4.00	0.81	1.12
Amusement and recreation	6.00	4.00	0.56	0.95
Transportion	6.00	4.00	0.45	2.05
Air	6.00	5.00	0.06	0.10
Land 3	6.00	5.00	0.39	1.95
Farm	6.00	4.00	0.50	1.33
Other 4	6.00	4.00	0.25	0.47
Residential structures	7.00	2.00	25.21	45.68
Housing units	7.00	3.00	14.55	35.89
Permanent site	7.00	4.00	13.84	35.13
1 to 4 unit	7.00	5.00	11.79	30.18
5-or more-unit	7.00	5.00	2.05	4.95
Manufactured homes	7.00	4.00	0.71	0.75
Brokers' commissions and other ownership transfer costs 5	7.00	3.00	4.42	0.69
Improvements	7.00	3.00	6.17	8.88
Other residential 6	7.00	3.00	0.07	0.22
Intellectual property products	8.00	1.00	17.41	5.06
Nonresidential intellectual property products	9.00	2.00	17.41	5.06
Software	9.00	3.00	5.54	0.77
Prepackaged 7	9.00	4.00	1.91	0.17
Custom	9.00	4.00	2.15	0.35
Own account	9.00	4.00	1.48	0.25
Research and development 8,9	9.00	3.00	9.22	3.12
Business	9.00	4.00	8.66	2.91
Manufacturing	9.00	5.00	6.64	2.27
Pharmaceutical and medicine manufacturing	9.00	6.00	1.17	0.56
Chemical manufacturing, excluding pharmaceutical and medicine	9.00	6.00	0.53	0.23
Semiconductor and other electronic component manufacturing	9.00	6.00	0.58	0.14
Other computer and electronic product manufacturing	9.00	6.00	1.20	0.26
Motor vehicles, bodies and trailers, and parts manufacturing	9.00	6.00	0.83	0.18
Aerospace products and parts manufacturing	9.00	6.00	0.57	0.18
Other manufacturing	9.00	6.00	1.77	0.73
Nonmanufacturing	9.00	5.00	2.02	0.64
Scientific research and development services	9.00	6.00	0.17	0.06
All other nonmanufacturing	9.00	6.00	1.85	0.58
Nonprofit institutions serving households (NPISHs)	9.00	4.00	0.56	0.21
Universities and colleges 10	9.00	5.00	0.07	0.02
Other nonprofit institutions	9.00	5.00	0.49	0.19
Entertainment, literary, and artistic originals	9.00	3.00	2.65	1.17
Theatrical movies	9.00	4.00	0.62	0.39
Long-lived television programs	9.00	4.00	1.13	0.39
Books	9.00	4.00	0.38	0.21
Music	9.00	4.00	0.36	0.09
Other	9.00	4.00	0.15	0.09

Table 5: Full name of each category; Sector and Aggregation level; time-series average (over 1970-2017) of the share of nominal investment and current cost capital.

	Agg	Avg of: Dep	real dI/I	real dK/K	dPI/PI	dPK/PK
Equipt	1.00	13.44	5.49	3.83	1.05	2.36
NonResEquipt	2.00	13.41	5.50	3.82	1.05	2.38
InfoProces	3.00	17.12	11.89	9.29	-3.87	-1.87
Computers	4.00	30.99	25.61	23.22	-12.27	-11.30
CommEquipt	4.00	11.73	9.36	8.65	-1.97	-2.04
MedicalEquipt	4.00	16.22	8.06	8.16	1.75	1.88
Nonmedical	4.00	14.45	5.73	5.34	3.04	3.01
Photocopy	4.00	19.78	4.04	3.96	0.41	0.42
Office	4.00	36.21	4.11	1.96	0.88	0.77
Industrial	3.00	9.35	1.96	2.12	3.68	3.58
FabrMetals	4.00	11.47	1.05	1.63	3.81	3.81
Engines	4.00	7.17	3.77	2.25	3.77	3.72
Metalworking	4.00	11.33	1.90	1.48	3.68	3.67
SpecialInd	4.00	10.61	1.55	1.33	4.01	3.98
GeneralInd	4.00	10.37	2.71	2.52	3.67	3.65
Electrical	4.00	5.13	2.68	2.97	2.96	2.94
Transportation	3.00	14.53	4.79	2.60	3.20	3.54
Trucks	4.00	21.14	7.66	3.75	3.30	3.40
LightTrucks	5.00	21.54	32.84	5.48	3.05	3.12
OtherTrukcs	5.00	20.69	3.25	1.89	3.79	3.86
Autos	4.00	23.03	5.92	3.40	2.10	2.00
Aircrafts	4.00	7.74	4.80	2.70	4.39	4.34
Ships	4.00	6.32	2.65	1.10	4.04	3.99
Railroad	4.00	6.07	3.15	0.35	4.09	4.13
OtherEquipt	3.00	15.02	2.66	2.66	3.67	3.65
Furniture	4.00	12.70	3.13	3.02	3.53	3.50
Agricultural	4.00	13.42	1.52	1.00	4.17	4.13
Construction	4.00	16.93	3.64	1.96	4.26	4.21
MiningOil	4.00	16.19	5.96	3.36	4.66	4.52
ServiceInd	4.00	16.63	2.85	2.76	3.17	3.18
ElectricalNEC	4.00	19.57	4.10	3.65	2.60	2.53
OtherNonRes	4.00	15.90	4.25	4.58	2.93	2.94
ResEquipment	5.00	16.20	4.78	4.86	0.71	0.61

Table 6: Equipment: short names, aggreg. level, and time-series average (over 1970-2017) of the depreciation rate, the growth rate of real investment and real capital, and the growth rate of the relative investment and capital prices (both relative to the (PCE NDS) consumption deflator).

	Agg	Avg of: Dep	real dI/I	real dK/K	dPI/PI	dPK/PK
Structures	1.00	2.51	1.67	2.08	4.73	4.64
NonResStruct	2.00	2.91	1.22	1.91	5.03	4.74
CommercialHealth	3.00	2.44	1.87	2.91	4.62	4.63
Office	4.00	2.50	2.56	2.91	4.81	4.76
HealthcareStruct	4.00	2.02	1.34	3.39	4.26	4.27
HospitalsPlus	5.00	1.89	1.10	3.40	4.34	4.28
Hospitals	6.00	1.89	1.25	3.43	4.34	4.28
Specialcare	6.00	1.88	1.14	3.25	4.40	4.30
MedicalStruct	5.00	2.48	3.13	3.36	4.38	4.23
Malls	4.00	2.66	2.14	2.62	4.68	4.69
Food	4.00	2.65	0.89	2.31	4.72	4.70
Warehouses	4.00	2.24	4.96	3.14	4.81	4.66
OtherCommStruct	4.00	2.74	1.60	2.74	4.61	4.67
ManufStruct	3.00	3.19	1.73	1.49	4.55	4.54
PowerCommStruct	3.00	2.27	1.86	2.01	4.26	4.34
PowerStruct	4.00	2.23	2.27	1.67	4.75	4.68
ElectricStruct	5.00	2.15	3.01	1.99	4.70	4.63
OtherPowerStruct	5.00	2.41	2.69	0.99	4.91	4.85
CommunicationStruct	4.00	2.40	2.04	3.11	3.33	3.32
MiningOilStruct	3.00	7.44	2.79	1.02	7.64	7.80
OilGasStruct	4.00	7.70	2.74	0.83	7.98	8.10
MiningStruct	4.00	4.61	5.26	3.53	4.77	4.59
OtherStruct	3.00	2.35	1.45	1.12	4.57	4.54
ReligiousStruct	4.00	1.89	-1.25	0.96	4.76	4.60
EducStruct	4.00	1.91	2.58	2.43	4.97	4.82
HotelStruct	4.00	2.85	5.92	2.99	4.64	4.56
FunStruct	4.00	3.05	1.96	1.65	4.50	4.55
TransportStruct	4.00	2.14	2.15	-0.27	4.22	4.19
AirStruct	5.00	2.51	6.00	2.21	5.38	4.50
LandTransStruct	5.00	2.13	1.93	-0.40	4.13	4.15
FarmStruct	4.00	2.42	-0.10	0.29	4.60	4.61
OtherNonResStruct	4.00	2.26	-1.82	2.03	4.48	4.43
ResStruct	2.00	2.24	2.58	2.18	4.56	4.59
Housing	3.00	1.26	2.55	1.97	4.64	4.66
PermHousing	4.00	1.18	2.67	1.96	4.70	4.67
units1to4	5.00	1.15	3.31	1.99	4.59	4.61
units5more	5.00	1.41	2.26	1.74	5.12	5.14
ManufHomes	4.00	4.65	0.98	2.73	3.84	3.90
BrokerComm	3.00	49.84	3.84	4.49	4.66	4.75
ImprovtHousing	3.00	2.67	2.97	2.94	4.38	4.36
OtherResidStruct	3.00	2.32	7.66	0.52	7.02	4.60

Table 7: Same as Table 6, but for structures.

	Agg	Avg of: Dep	real dI/I	real dK/K	dPI/PI	dPK/PK
IPP	1.00	22.17	6.37	5.19	2.16	2.98
NonResIPP	2.00	22.17	6.37	5.19	2.16	2.98
Software	3.00	45.75	12.78	11.96	-1.11	-0.35
PrepackSoftware	4.00	76.65	24.95	25.73	-6.85	-7.27
CustomSoftware	4.00	39.53	11.60	12.71	0.60	0.81
OwnaccountSoftware	4.00	39.59	8.14	8.12	0.77	0.71
RD	3.00	19.51	4.39	4.79	3.53	3.73
RDbusiness	4.00	19.73	4.38	4.78	3.53	3.73
RDmanufacturint	5.00	19.79	3.45	4.01	3.53	3.73
RDdrugs	6.00	10.00	7.61	8.32	3.55	3.71
RDchemical	6.00	16.54	0.98	1.40	3.59	3.72
RDsemiconductor	6.00	26.79	9.14	9.09	5.21	3.87
RDelectronics	6.00	34.23	3.40	3.39	3.55	3.75
RDcars	6.00	34.89	3.03	2.64	3.56	3.71
RDair	6.00	23.43	2.36	2.09	3.51	3.73
RDother	6.00	16.53	2.54	2.72	3.54	3.73
RDnonmanuf	5.00	18.04	11.90	11.43	3.71	3.65
RDscientific	6.00	15.48	Inf	Inf	2.02	2.91
RDnonmanufelse	6.00	18.22	11.76	11.19	3.71	3.65
RDnonprofits	4.00	16.56	4.74	4.92	3.61	3.65
RDuniv	5.00	17.60	6.38	6.16	3.29	3.61
RDothernonprofits	5.00	16.61	4.47	4.73	3.38	3.68
Art	3.00	14.93	3.84	3.20	2.91	2.92
Movies	4.00	9.78	5.84	3.18	2.47	2.50
TV	4.00	18.34	5.69	4.91	2.15	2.13
Books	4.00	12.87	0.67	1.28	4.86	4.86
Music	4.00	30.90	1.94	1.61	3.43	3.21
OtherArt	4.00	11.72	2.05	2.29	3.49	3.43

Table 8: Same as Table 6, but for intellectual property products (IPP).

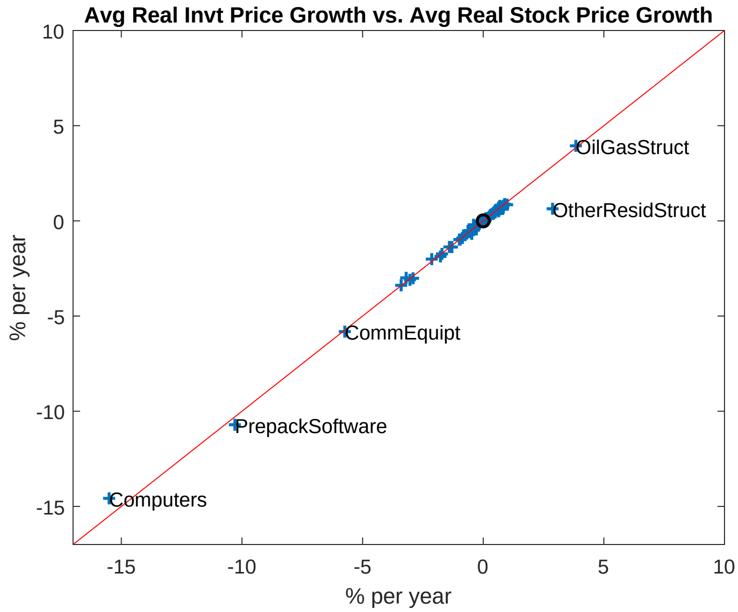

Figure 24: For each capital type, we plot the time-series average growth rate of capital price against the average growth rate of investment prices, and we add the 45 degree line.