Volatility Uncertainty and Jumps

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Motivation

The 1987 stock market crash showed that option pricing models **fail** to price options with **short TTM** and **deep-OTM puts**

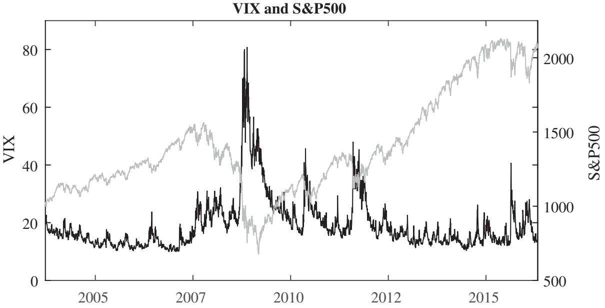
→ Solution: **state-dependent jump intensity** that is linked to **volatility** (Bates, 2000)

$$\lambda_t = \alpha_0 + \lambda^V V_t + ...$$

→ Implications:

- **Strong linear** link between jump intensity and volatility
- Only **source of time-variation** in jump risks is volatility

Motivation



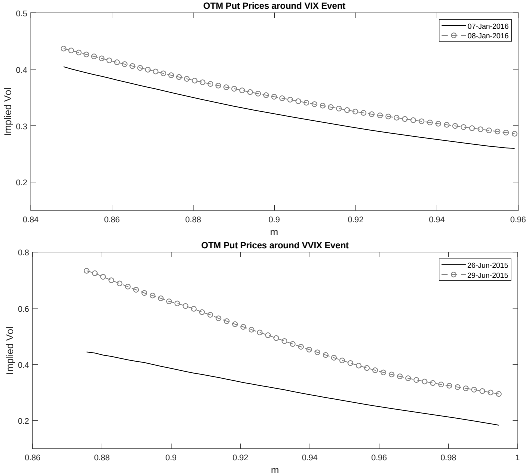
- Linking jump risks to volatility seems reasonable
  - → Negative jumps in stock market occur when volatility is high
- Andersen, Fusari, Todorov (2015, 2019): After turbulent times, left tail **stays elevated** long after volatility mean-reverts
  - → Disconnect between time-series dynamics?

This Paper

In an almost **non-parametric** setting, we ask:

- Are expected jump risks and volatility linearly tied?
  - Very weak relationship at best
  - Significance completely gone once higher moments are included
- Which moment is related to jump risks? **Volatility Uncertainty**
  - Main driver of evolution of jump risks
  - Higher volatility uncertainty increases downside risk and decreases upside potential
  - Predicts realized price jumps
- How can option pricing models account for our findings?
  - Decoupling jump risk evolution from volatility is crucial
  - Separately modeling left and right tail necessary

Event Study - Large VIX and VVIX Shocks



$\rightarrow$ Changes in volatility uncertainty have an isolated effect on tails

Higher Moments and Tail Measure

- Main analysis based on option-implied information (under risk-neutral measure)
- We extract higher moments in standard-fashion with portfolios of weighted option prices
  - Vol^2 and SKEW using S&P500 options
  - VolVol^2 using VIX options
- For tail measure, we follow Bollerslev, Todorov, and Xu (2015)
  - Use (deep) out-of-the-money options
  - Fit them to jump intensity

$$\nu_t(dy) = \left(\phi_t^+ \times e^{-\alpha_t^+ y}\mathbf{1}_{\{y>0\}} + \phi_t^- \times e^{\alpha_t^- y}\mathbf{1}_{\{y<0\}}\right)$$

  - independent left (LJV) and right (RJV) tail
  - time-variations in shape of tail possible

Data

- Time-span: January 3, 2007 until April 29, 2016
- Option Metrics: monthly and weekly S&P500 options, monthly VIX options
- Basic filters; Time-to-maturity of options: $1 < \text{TTM} < 45$
- Calculate our measures on a **weekly** basis, then
  1. orthogonalize them due to correlations
  2. take first differences due to autocorrelation
  3. standardize measures

Evolution of Left Tail

$$\Delta \mathsf{LJV}_t = \alpha + \beta \Delta X_t + \epsilon_t$$

	(1)	(2)	(3)	(4)	(5)
ΔVol^2	0.2578		0.1954		0.2241
	(1.67)		(1.41)		(1.63)
ΔVolVol^2		0.2943			
		(3.44)			
$\Delta \text{VolVol}^{2,\perp}$			0.2025	0.3156	0.2303
			(3.01)	(4.22)	(3.30)
ΔSKEW				-0.2061	-0.2652
				(-4.66)	(-4.75)
adj. R^2	0.0644	0.0845	0.0996	0.1153	0.1345

Evolution of Right Tail

$$\Delta \mathsf{RJV}_t = \alpha + \beta \Delta X_t + \epsilon_t$$

	(1)	(2)	(3)	(4)	(5)
ΔVol^2	-0.0515		-0.0097		-0.0090
	(-1.74)		(-0.46)		(-0.41)
ΔVolVol^2		-0.1220			
		(-3.17)			
ΔVolVol$^{2,\perp}$			-0.1356	-0.1297	-0.1331
			(-3.09)	(-3.28)	(-3.05)
ΔSKEW				-0.1006	-0.0696
				(-1.88)	(-1.38)
adj. R^2	0.0006	0.0129	0.0153	0.0276	0.0248

Predicting Realized Risks

Analysis so far under risk-neutral measure. Can volatility uncertainty also explain realized risks?

- Determine realized variance and tripower variation
- Difference isolates realized price jumps

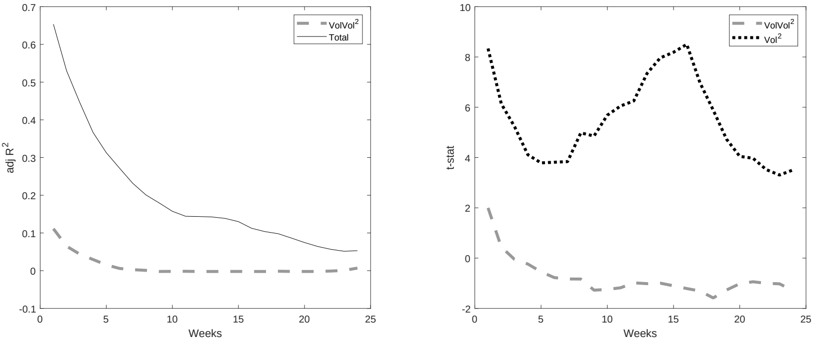
Run predictive regressions of form

$$\text{Realized Risk}_{[t+h-1,t+h]} = \gamma + \beta_{Vol}\,\text{Vol}_t^2 + \beta_{VolVol}\,\text{VolVol}_t^2 + \epsilon_t,$$
$$h = 2, \ldots, 25.$$

and compare the R^2 of multiple regression to R^2 of simple regression.

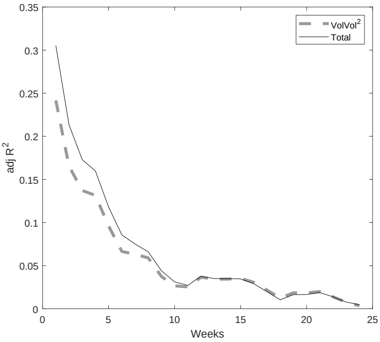
Note: Non-overlapping regressions, we predict the weekly avg. in $t+h$. Standard errors are HAC-estimators that correct for autocorrelation.

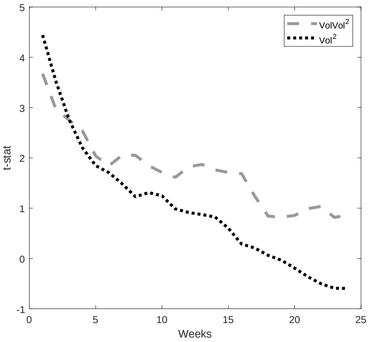
Realized Variance



- Almost no predictive power of volatility uncertainty on total risk

Realized Price Jumps





- Price jumps can be well predicted by volatility uncertainty
- Vol uncertainty not only explains expected jump risks ($\mathbb{Q}$) but also realized jump risks ($\mathbb{P}$)

Testing Option Pricing Models

- What happens if jump intensity is only tied to volatility?
- Test model of Eraker (2004)

$$\frac{dS_t}{S_t} = (r-\mu)dt + \sqrt{V_t}dW_t^{S,\mathbb{Q}} + dJ_t^{S,\mathbb{Q}}$$
$$dV_t = \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t)dt + \sigma_V\sqrt{V_t}dW_t^{V,\mathbb{Q}} + dJ_t^{V,\mathbb{Q}}$$
$$\lambda_t = \lambda_0 + \lambda_1 V_t$$

- How do we test? For **each week**
  - Extract state variables by minimizing distance between model's variance expectations and model-free IV
  - Simulate model 50,000 times
  - Determine model-implied option prices and risk measures

Eraker Model - Results

	ΔLJV			ΔRJV		
	(1)	(2)	(3)	(4)	(5)	(6)
ΔVol^2	0.8102		0.8115	0.1248		0.1254
	(3.49)		(3.61)	(1.92)		(1.99)
ΔVolVol^2		-0.3104			-0.0835	
		(-4.77)			(-1.99)	
$\Delta\text{VolVol}^{2,\perp}$			-0.1015			-0.0525
			(-2.45)			(-1.40)
adj. R^2	0.6557	0.0945	0.6654	0.0136	0.0049	0.0143

- Volatility is clearly the main driver
- Counterfactual negative link between left tail and volatility uncertainty
- VolVol^2 irrelevant for right tail

→ Overall, OTM option price dynamics are not in line with data

Summary

- Paper analyzes the interdependencies between expected tail risks and higher moments of return distribution
- We show that volatility uncertainty has a distinct impact on both tails of the risk-neutral distribution
- Expected volatility uncertainty predicts realized price jumps but not realized volatility
- Findings present a challenge for many modern option pricing models
  - model tests suggest that decoupling the intensity from volatility is necessary
  - separately model left and right jump intensity

Backup – Liquidity of SPX Options

$m \in$	$(-\infty, -4]$	$(-4, -2.5]$	$(-2.5, -1]$	$(-1, 1]$	$(1, 2.5]$	$(2.5, 4]$	$(4, \infty)$
Vol[#]	0.10	0.02	0.02	0.05	0.02	0.01	0.04
Vol[$]	0.96	0.21	0.40	1.18	0.32	0.13	0.32
$\widehat{\text{Vol}[\%]}$	0.36	0.07	0.09	0.20	0.09	0.06	0.13
$\widetilde{\text{Vol}[\%]}$	0.39	0.06	0.08	0.20	0.08	0.06	0.13
OI[#]	1.25	0.15	0.18	0.25	0.16	0.13	0.41
OI[$]	43.55	3.80	4.69	6.97	4.34	3.32	21.58
$\widehat{\text{OI}[\%]}$	0.47	0.06	0.08	0.11	0.07	0.06	0.16
$\widetilde{\text{OI}[\%]}$	0.52	0.06	0.07	0.10	0.06	0.05	0.14
$\widehat{\text{Bid-Ask Spread}}$	0.21	0.07	0.06	0.06	0.08	0.15	0.25
$\widetilde{\text{Bid-Ask Spread}}$	0.04	0.04	0.04	0.05	0.06	0.07	0.05

Backup – Self-Exciting Jump Model

- Kaeck (2018) uses a rich specification:

$$\frac{dS_t}{S_t} = (r - q - \mu)dt + \sqrt{V_t}dW_t^{S,\mathbb{Q}} + dJ_t^{\lambda,\mathbb{Q}}$$
$$dV_t = \kappa_V^{\mathbb{Q}}(m_t - V_t)dt + \sigma_V\sqrt{V_t}(\rho dW_t^{S,\mathbb{Q}} + \sqrt{1-\rho^2}dW_t^{V,\mathbb{Q}}) + dJ_t^{\lambda,\mathbb{Q}}$$
$$dm_t = \kappa_m^{\mathbb{Q}}(\theta_m^{\mathbb{Q}} - m_t)dt + \sigma_m\sqrt{m_t}dW_t^{m,\mathbb{Q}}$$
$$d\lambda_t = \kappa_l^{\mathbb{Q}}(\theta_l^{\mathbb{Q}} - \lambda_t)dt + \sigma_l\sqrt{\lambda_t}dW_t^{l,\mathbb{Q}} + dJ_t^{\lambda,\mathbb{Q}}$$

- λ_t is the jump intensity for all jumps
  - follows independent process
  - can jump itself (self-exciting)

Backup – Kaeck Model Results

	ΔLJV			ΔRJV		
	(1)	(2)	(3)	(4)	(5)	(6)
ΔVol^2	-0.0094		-0.0449	0.0635		0.0268
	(-1.38)		(-2.50)	(1.22)		(0.51)
ΔVolVol^2		0.1634			0.1818	
		(2.10)			(2.06)	
$\Delta\text{VolVol}^{2,\perp}$			0.1670			0.1906
			(2.08)			(2.10)
adj. R^2	-0.0017	0.1231	0.1213	0.0045	0.0728	0.0811

- Results for left tail close to empirics
- Counterfactual positive link between right tail and volatility uncertainty

$\rightarrow$ Need to model left and right tail separately