Conceptual Differences Between Decision Utility and Experienced Utility: A Theory for Jevons' Wish?

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Goals

- Discuss conceptual differences between decision utility and experienced utility
- Prove existence of a unique family of experienced utilities
- Prove existence of non-experienced utilities
- Illustrate how the experienced and non-experienced utilities can be used to explain time allocation and the sequence of activities

Definitions

Definition

Decision utility is a function which represents an individual's preferences over mutually exclusive alternatives

- **Decision Utility**: neoclassical economic concept of utility

Definition

Experienced utility is a function which represents an individual's hedonic experience from an activity over time

- **Experienced Utility**: classical utilitarian concept of utility

ACTIVITY: Anything an individual spends time on
Similar to 'good' or 'alternative' in economic theory

Setting

- Conceptual differences: primal, existential and functional
- Applicable differences: requirements to use utility
- Descriptive theory: both describes and prescribes choice
- Jevons' wish: to measure the quantity of feeling

Answer two questions:

- does decision utility include experience?
- does experienced utility include or lead to a decision?

Primal Differences

- Decision utility: preference relation
- Experienced utility: hedonic experience
- Decision utility does not include experience
- Experienced utility does not include decision, leads to decision through non-experienced utilities

- non-experienced utilities: experienced utilities from activities which are not chosen
- hedonic experience more basic than preference relation

Existential Differences

DECISION UTILITY ASSUMPTIONS:

- I Finite number of goods
- II Choice of a bundle of goods
- III Preferences are rational and continuous

EXPERIENCED UTILITY ASSUMPTIONS:

- I Finite number of activities
- II Choice of a single activity
- III Rate of change of experienced utility is proportional to difference between the experienced utility and the other experienced utilities up to a positive coefficient function

Existential Differences

DECISION UTILITY ASSUMPTION:

- Finite number of goods

EXPERIENCED UTILITY ASSUMPTION:

- Finite number of activities

Similarity:

- finite number under consideration

Existential Differences

Decision Utility Assumption:

- Choice of a bundle of goods

Experienced Utility Assumption:

- Choice of a single activity

Similarity:

- single choice

Differences:

- number of goods unchanged
- one less activity for every chosen activity
- bundle represented by a value
- activity linked with a *function*

Existential Differences

Decision Utility Assumption:

- Preferences are rational and continuous

Experienced Utility Assumption:

- Rate of change of experienced utility is proportional to difference between the experienced utility and the other experienced utilities up to a positive coefficient function

Similarity:

- continuous preferences, differentiable experienced utility

Existential Differences

DECISION UTILITY ASSUMPTION:

- Preferences are rational and continuous

EXPERIENCED UTILITY ASSUMPTION:

- Rate of change is proportional to difference between experienced utility from each activity and experienced utilities from other activities up to a positive coefficient

Differences:

- decision utility theory: rational preferences
- my theory: no rationality or preferences
- non-rationality: weaker assumption, stronger theory

Existential Differences

Dynamics from experienced utility assumptions:

$$
\begin{aligned}
du_1/dt &= \beta_1(t)\,(u_1(t) - u_2(t) - \cdots - u_n(t)) \\
du_2/dt &= \beta_2(t)\,(-u_1(t) + u_2(t) - \cdots - u_n(t))\,, \\
&\cdots \\
du_n/dt &= \beta_n(t)\,(-u_1(t) - u_2(t) - \cdots + u_n(t))
\end{aligned}
$$

$$\dot{\mathbf{u}}(t) = \mathcal{B}(t)\mathbf{u}(t)$$

- Coefficients of proportionality are variable or constant
- System of pull-and-push forces
- Experienced utility from every activity is a pull force against the push forces of other experienced utilities

Lemma 1

Lemma

$\mathcal{B}(t)$ has a complete set of eigenvectors $\mathbf{k}^1, \mathbf{k}^2, \cdots, \mathbf{k}^n$ and their associated eigenvalues are given by the following linear transformation:

$$\mathbf{w}(t) = K^{-1}\mathcal{B}_d(t)K$$

Family of Experienced Utilities

Theorem (Family of Experienced Utilities)

Given positive coefficients of proportionality on an open interval, the rate of change of experienced utility from each activity is proportional to the difference between experienced utility from the activity and sum of experienced utilities from the other activities. Then there exists a unique family of experienced utilities which are:

- i *expressed explicitly:* $\mathbf{u}(t) = Ke^{\tilde{\Omega}(t)}\mathbf{c}$,
- ii *real valued and*
- iii *linearly independent.*

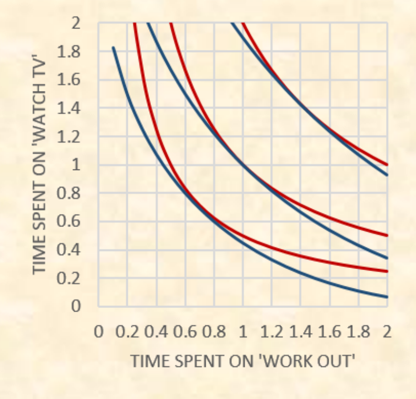
Also, given initial conditions, experienced utilities are cardinal utilities.

Functional Differences

- Decision utility: unspecified function
- Experienced utility: unique family of functions
- Decision utility: a function from $\mathbb{R}^n \to \mathbb{R}$
- Experienced utility: a function from $\mathbb{R} \to \mathbb{R}^n$

- non-uniqueness: even if a decision utility function can explain an individual's choice, there is no guarantee the function represents the same individual's preferences

Functional Differences



Functional Differences

- Decision utility: additional assumptions are needed to represent preferences with discounting and uncertainty
- Experienced utility: no additional assumptions needed, it includes both discounting and uncertainty

- decision utility represents preferences with discounting only if it is a function of a special kind

Lemma 2

Lemma

A researcher has a discounting factor η for decision utility DU and an individual has a discounting factor δ for preferences of goods $\mathbf{x} = x^1, x^2, \cdots, x^n$ over different periods. Then researcher's DU represents the individual's preferences only if it is a homogeneous function of degree $k = \ln \eta / \ln \delta$.

Lemma 3

Lemma

The family of experienced utilities represents individual hedonic experience with both discounting and uncertainty.

Remark

For constant coefficients of proportionality, the family of experienced utilities represents individual hedonic experience with constant discounting and uniformly distributed uncertainty.

Functional Differences

- Decision utility: rationality is required
- Experienced utility: rationality not required, still satisfied
- Decision utility: axiom of choice is assumed
- Experienced utility: axiom of choice not assumed; given an initial condition, still satisfied

Lemmas 4 and 5

Lemma

Experienced utilities satisfy rationality.

Lemma

Given an initial condition, experienced utilities satisfy the axiom of choice.

Corollary 1.1

Corollary

With constant coefficients of proportionality, the family of experienced utilities is expressed as:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_n(t) \end{bmatrix} = c_1 \begin{bmatrix} k_1^1 \\ k_2^1 \\ \vdots \\ k_n^1 \end{bmatrix} e^{\omega_1 t} + c_2 \begin{bmatrix} k_1^2 \\ k_2^2 \\ \vdots \\ k_n^2 \end{bmatrix} e^{\omega_2 t} + \cdots + c_n \begin{bmatrix} k_1^n \\ k_2^n \\ \vdots \\ k_n^n \end{bmatrix} e^{\omega_n t}$$

Comparisons

- Decision utility: evaluation of different specifications requires even more assumptions, especially for choices which include uncertainty
- Experienced utility: specifications are readily available by the family of experienced utilities
- Decision utility: different forms of expected and non-expected utility have not been capable to represent underlying preferences
- Experienced utility: unique functional form provided by the family of experienced utilities

Comparisons

- Artificial agents can be programmed to satisfy rationality: they maximize expected decision utility
- These artificial agents are computationally impossible
- Models with individuals behaving like artificial agents are unlikely to represent preferences
- These models impose extreme conditions on unrealistic individuals with extraordinary cognitive ability

- decision utility theory: we know neither the preferences nor the decision utility function
- my theory: knowing preferences is not required and we know the family of experienced utility functions

Comparisons

KAHNEMAN ET AL. (1997):

- Normative theory of total experienced utility extending decision utility
- Stopwatch time restarting at 0
- Experience is constant over time, e.g. with age
- Does not allow discounting

MY THEORY:

- Descriptive theory of instant experienced utility independent of decision utility
- Calendar time starting/restarting at any time
- Experience changes over time, with age or other factors
- Includes both discounting and uncertainty

Non-experienced Utilities

- Experienced utilities from the non-chosen activities while an individual is spending time on the chosen activity
- Conditioned on the chosen activity
- Help to explain the switch time from an activity to another
- Same assumptions as experienced utility
- Dynamics include experienced utility from chosen activity

Non-experienced Utilities

$$\dot{u}_{1|i} = \beta_1(t)\left(u_{1|i}(t) \cdots - u_{i-1|i}(t) - u_{i+1|i}(t) - \cdots - u_{n|i}(t)\right)$$
$$-\beta_1(t)u_i(t)$$
$$\cdots$$
$$\dot{u}_{i-1|i} = \beta_{i-1}(t)\left(-u_{1|i}(t) \cdots + u_{i-1|i}(t) - u_{i+1|i}(t) \cdots - u_{n|i}(t)\right)$$
$$-\beta_{i-1}(t)u_i(t)$$
$$\dot{u}_{i+1|i} = \beta_{i+1}(t)\left(-u_{1|i}(t) \cdots - u_{i-1|i}(t) + u_{i+1|i}(t) \cdots - u_{n|i}(t)\right)$$
$$-\beta_{i+1}(t)u_i(t)$$
$$\cdots$$
$$\dot{u}_{n|i} = \beta_n(t)\left(-u_{1|i}(t) \cdots - u_{i-1|i}(t) - u_{i+1|i}(t) \cdots + u_{n|i}(t)\right)$$
$$-\beta_n(t)u_i(t)$$

$$\dot{\mathbf{u}}_{|\mathbf{i}}(t) = \mathcal{B}_{|i}(t)\mathbf{u}_{|i}(t) + \mathbf{b}_{|i}(t)$$

Non-experienced Utilities

Theorem (Non-experienced Utilities)

Given positive coefficients of proportionality on an open interval, the rate of change of non-experienced utility from each non-chosen activity is proportional to the difference between non-experienced utility from the activity and sum of non-experienced utilities from the other non-chosen activities as well as experienced utility from the chosen activity. Then there exist unique non-experienced utilities which are:

i *expressed explicitly:* $\mathbf{u}_{|i}(t) = K_{|i} diag\left(e^{\tilde{\omega}_{1|i}(t)}, \dots, e^{\tilde{\omega}_{i-1|i}(t)}, e^{\tilde{\omega}_{i+1|i}(t)}, \dots, e^{\tilde{\omega}_{n|i}(t)}\right) \mathbf{c}_{|i} + K_{-i} diag\left(e^{\tilde{\omega}_{1}(t)}, \dots, e^{\tilde{\omega}_{i-1}(t)}, e^{\tilde{\omega}_{i}(t)}, e^{\tilde{\omega}_{i+1}(t)}, \dots, e^{\tilde{\omega}_{n}(t)}\right) \mathbf{c}$,

ii-iii *real valued and linearly independent.*

Corollary 2.1

Corollary

With constant coefficients of proportionality, non-experienced utilities are expressed as:

$$\mathbf{u}_{|i}(t) =$$
$$K_{|i}\text{diag}\left(e^{\omega_{1|i}t}, \cdots, e^{\omega_{i-1|i}t}, e^{\omega_{i+1|i}t}, \cdots, e^{\omega_{n|i}t}\right)\mathbf{c}_{|i}+$$
$$K_{-i}\text{diag}\left(e^{\omega_{1}t}, \cdots, e^{\omega_{i-1}t}, e^{\omega_{i}t}, e^{\omega_{i+1}t}, \cdots, e^{\omega_{n}t}\right)\mathbf{c}$$

Example

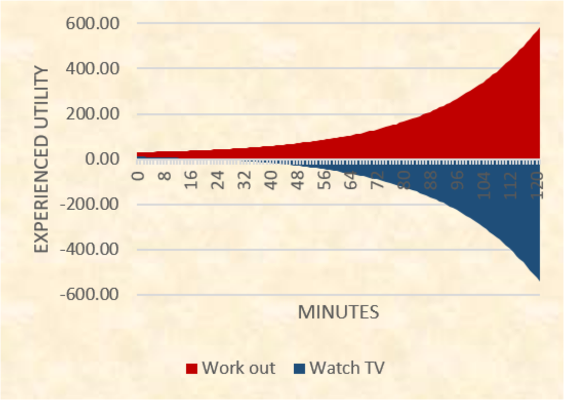
- Two activities, Work Out; Watch TV, over two hours: $u_1(t)$ from working out, $u_2(t)$ from watching TV
- Suppose $\beta_1(t) = \beta_2(t) = 1$ and at $t_0 = 0$, $\mathbf{u}(0) = (30, 10)$
- Experienced utility functions:

$$\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 20 + 10e^{2t} \\ 20 - 10e^{2t} \end{bmatrix}$$

- observed choice: the individual watches TV during the first hour and works out during the second hour.

Example

My experienced utility extends decision utility:
utility maximization, $u_1(t) > u_2(t)$ for all $t \in [0, 2]$



- the individual would only work out and not watch any TV

Example

- Non-experienced utility functions:

$$u_{1|2} = (-20e)\, e^{t} + 20 + 10e^{2t}$$
$$u_{2|1} = (20e)\, e^{t} + 20 - 10e^{2t}$$

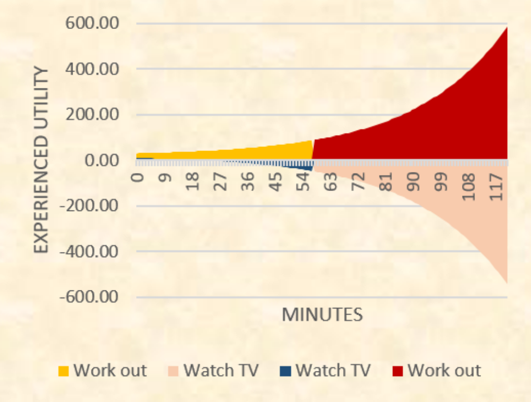
- Time allocation (A) and sequence of activities (S):

$$S_1 = \{work\ out;\ watch\ TV\}, \quad A_1 = \{1, 1\} \tag{1}$$
$$S_2 = \{watch\ TV;\ work\ out\}, \quad A_2 = \{1, 1\} \tag{2}$$

- Experience/total utility (TU): $TU_1 \approx -164$, $TU_2 \approx 244$
- decision utility theory: not possible to explain sequence
- my theory: no rationality or preferences
  possible to explain both time allocation and the sequence

Example

My experienced utility is independent of decision utility: descriptive theory, either solution (1) or (2) is plausible



Decision utility extends my experienced utility:

- normative theory, utility maximization, (2) is optimal

Concluding Remarks

- A descriptive theory of instant experienced utility independent of decision utility
- Does not require rationality or include preferences
- It can explain time allocation and the sequence of activities
- Decision utility: tool to measure the outcome, not the flow
- Experienced utility: machine to measure both the outcome and the flow