Duration structure of unemployment hazards and the trend unemployment rate

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New method to estimate the trend unemployment rate

Develop a new method to estimate the trend unemployment rate taking into account the time-varying duration profile of unemployment hazards

- characterized by three time-varying factors—level, slope and curvature → Duration structure of unemployment hazards

Duration-structure trend unemployment rate (DS-TUR)

- The unemployment rate composed of the trend components of time-varying parameters constituting the duration structure of unemployment hazards and the trend inflows to unemployment
- The identification of trend unemployment rate is achieved not only from the trends in labor-force flows but also from the low-frequency changes in the distribution of unemployment duration.

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Key findings

1. The *DS-TUR* exhibits a secular decline between 1980 and 2000, a slow uptrend during the 2000s, and a decline between 2011 and 2019.

- The slow uptrend during the 2000s reflects the secular rise in long-term unemployment.

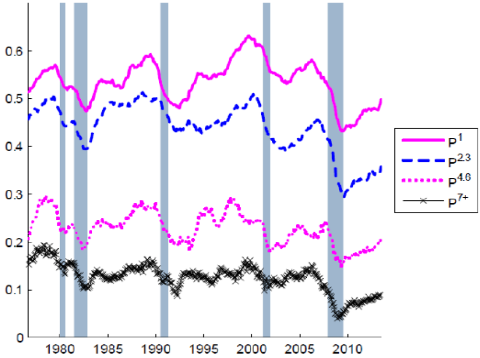
2. Without mismatch or the extension of UI benefits considered, the *DS-TUR* exhibits a rise and fall during 2007-2011.

3. The short-term component has trended down since 1980, while the long-term component shows an uptrend between 2000-2011 $\rightarrow$ falling frictional unemployment, rising structural unemployment.

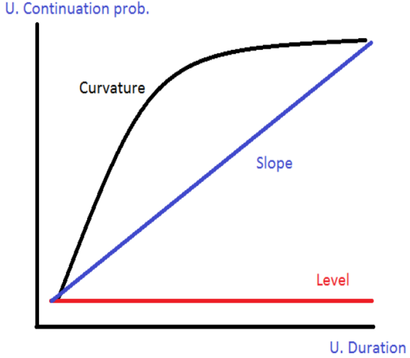
4. The short-term unemployment-rate gap has a strong Phillips correlation with the PCE inflation.

Model: Duration structure of unemployment hazards

Unemployment-exit probability



Duration profile of U-continuation probability



Laguerre function is used to model the nonlinear duration profile of unemployment continuation probabilities.

Model: Duration structure of unemployment hazards

1. Term structure of interest rates

$$f(\tau) = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} 1 \\ e^{\tau/\lambda} \\ (\tau/\lambda)e^{\tau/\lambda} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}$$

2. Duration structure of unemployment hazards

$$\begin{aligned} x_t(\tau) &= \underbrace{\beta_{0t}}_{Level} + \underbrace{\beta_{1t} e^{-(12-\tau)/\lambda_t}}_{Slope} + \underbrace{\beta_{2t}((12-\tau)/\lambda_t)e^{-(12-\tau)/\lambda_t}}_{Curvature}, \text{ for } \tau < 12 \\ &= \underbrace{\beta_{0t}}_{Level} + \underbrace{\beta_{1t}}_{Slope}, \text{ for } \tau \geq 12 \end{aligned}$$

Assumption: an individual's unemployment exit-probability does not change once unemployed longer than one year (Kroft, Lange, and Notowidigdo, 2013).

Data used in the analysis

U_t^1 = number of people newly unemployed in month t (S.A.)

$U_t^{2.3}$ = number of people unemployed for 2-3 months

$U_t^{4.6}$ = 4-6 months

$U_t^{7.12}$ = 7-12 months

$U_t^{13.+}$ = more than 1 year

$y_t = (U_t^1, U_t^{2.3}, U_t^{4.6}, U_t^{7.12}, U_t^{13.+})'$ for t = 1976:M1 - 2019:M12

Dynamic accounting identity: Ahn and Hamilton (2020)

$$U^1 = w.$$

$$p(\tau) = \exp[-\exp(x(\tau))] \quad \text{for} \quad \tau = 1, 2, 3, ...,$$

probability to stay unemployed next month of those unemployed for τ month

$$P(k) = p(1)p(2)...p(k),$$

probability to stay unemployed for k consecutive months

$$U^{2.3} = [wP(1) + wP(2)], \quad U^{4.6} = \sum_{k=3}^{5} [wP(k)]$$

$$U^{7.12} = \sum_{k=6}^{11} [wP(k)], \quad U^{13.+} = \sum_{k=12}^{47} [wP(k)]$$

We have five unknown parameters, w, β_0, β_1, β_2, and λ, which allows us to fit the five data points, $U^1, U^{2.3}, U^{4.6}, U^{7.12}, U^{13.+}$, exactly.

State Space Model: Measurement equation

$$
\begin{aligned}
U_t^1 &= w_t + r_t^1 \\
U_t^{2.3} &= [w_{t-1}P_t(1) + w_{t-2}P_t(2)] + r_t^{2.3} \\
U_t^{4.6} &= \sum_{k=3}^{5} [w_{t-k}P_t(k)] + r_t^{4.6} \\
U_t^{7.12} &= \sum_{k=6}^{11} [w_{t-k}P_t(k)] + r_t^{7.12} \\
U_t^{13.+} &= \sum_{k=12}^{47} [w_{t-k}P_t(k)] + r_t^{13.+}
\end{aligned}
$$

State Space Model: Measurement equation

$$P_t(j) = p_{t-j+1}(1)p_{t-j+2}(2)...p_t(j).$$

$$r_t \sim N(0, R)$$

$$\underbrace{R}_{5\times 5} = \begin{bmatrix} R_1^2 & 0 & 0 & 0 & 0 \\ 0 & R_{2.3}^2 & 0 & 0 & 0 \\ 0 & 0 & R_{4.6}^2 & 0 & 0 \\ 0 & 0 & 0 & R_{7.12}^2 & 0 \\ 0 & 0 & 0 & 0 & R_{13.+}^2 \end{bmatrix}.$$

State Space model: State equation

Assume driving variables evolve smoothly over time

- $w_t = w_{t-1} + \epsilon_t^w$
- $\lambda_t = \lambda_{H,t-1} + \epsilon_t^\lambda$
- $\beta_{0t} = \beta_{0,t-1} + \epsilon_t^{\beta_0}$
- $\beta_{1t} = \beta_{1,t-1} + \epsilon_t^{\beta_1}$
- $\beta_{2t} = \beta_{2,t-1} + \epsilon_t^{\beta_2}$

State Space model: State equation

Let ξ_t be the vector $(w_t, \lambda_t, \beta_{0t}, \beta_{1t}, \beta_{2t})'$ and $\varepsilon_t = (\varepsilon_t^w, \varepsilon_t^\lambda, \varepsilon_t^{\beta_0}, \varepsilon_t^{\beta_1}, \varepsilon_t^{\beta_2})'$.

$$\underbrace{\xi_t}_{5\times 1} = \xi_{t-1} + \underbrace{\varepsilon_t}_{5\times 1}$$

$$\underbrace{\varepsilon_t}_{5\times 1} \sim N(\underbrace{0}_{5\times 1}, \underbrace{\Sigma}_{5\times 5})$$

$$\underbrace{\Sigma}_{5\times 5} = \begin{bmatrix} (\sigma^w)^2 & 0 & 0 & 0 & 0 \\ 0 & (\sigma^\lambda)^2 & 0 & 0 & 0 \\ 0 & 0 & (\sigma^{\beta_0})^2 & 0 & 0 \\ 0 & 0 & 0 & (\sigma^{\beta_1})^2 & 0 \\ 0 & 0 & 0 & 0 & (\sigma^{\beta_2})^2 \end{bmatrix}.$$

State Space Model

$$\underbrace{\begin{bmatrix} \xi_t \\ \xi_{t-1} \\ \xi_{t-2} \\ \vdots \\ \xi_{t-46} \\ \xi_{t-47} \end{bmatrix}}_{240\times 1} = \underbrace{\begin{bmatrix} \underbrace{I}_{5\times 5} & \underbrace{0}_{5\times 5} & 0 & 0 & \dots & 0 & 0 & 0 \\ I & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & I & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & I & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & I & 0 \end{bmatrix}}_{240\times 240} \underbrace{\begin{bmatrix} \xi_{t-1} \\ \xi_{t-2} \\ \xi_{t-3} \\ \vdots \\ \xi_{t-46} \\ \xi_{t-48} \end{bmatrix}}_{240\times 1} + \underbrace{\begin{bmatrix} \underbrace{\varepsilon_t}_{5\times 1} \\ \underbrace{0}_{5\times 1} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{240\times 1}.$$

Nonlinear state space model $\rightarrow$ **Extended Kalman filter**

Estimates: level, slope and curvature

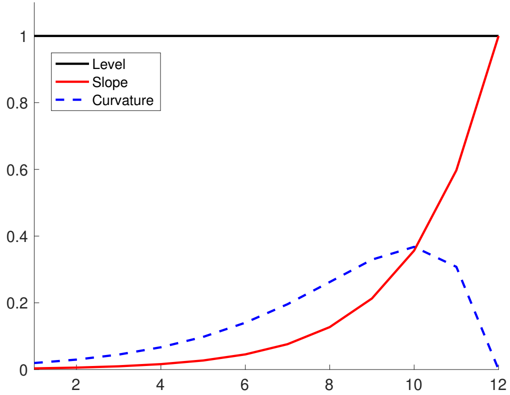
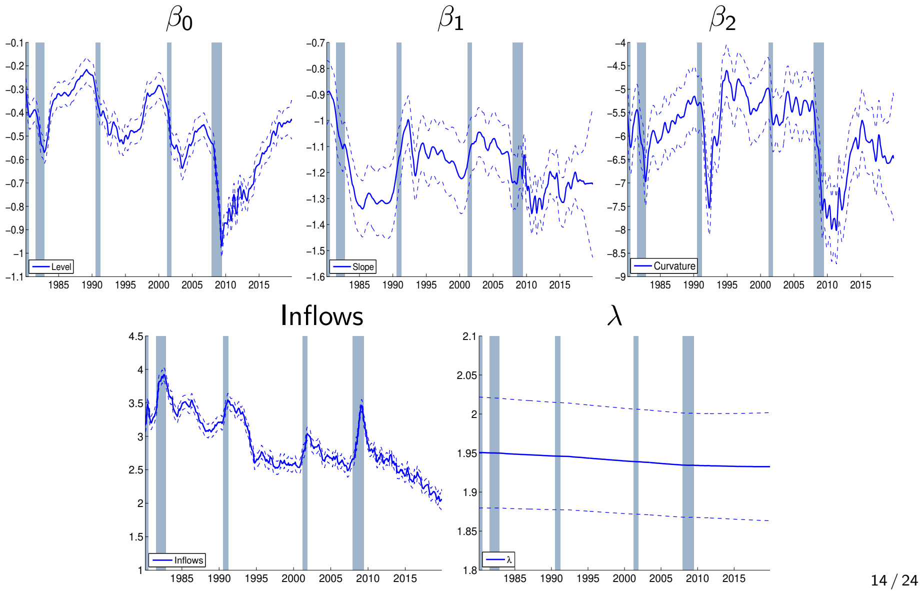


Figure: Estimates of level, slope and curvature with the value of $\lambda_t = 1.94$

Estimates: factor loadings, inflows, and λ



Monthly unemployment-continuation probabilities

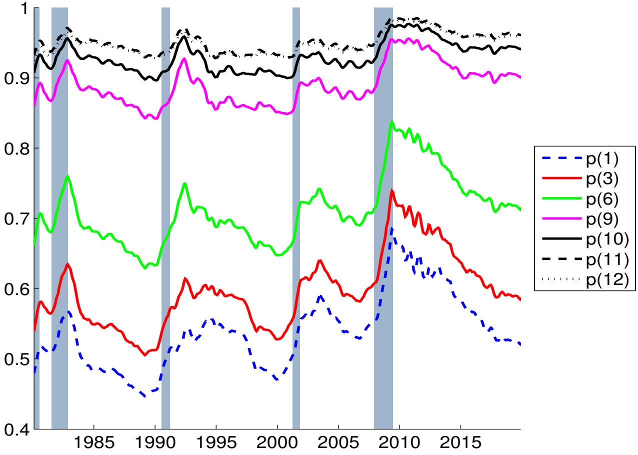


Figure: The model-implied monthly unemployment continuation probabilities by duration of unemployment

Duration-structure trend unemployment rate (DS-TUR)

Definition of DS-TUR The unemployment rate consisted only of the trend components of $(\hat{w}_{t|T}, \hat{\beta}_{0,t|T}, \hat{\beta}_{1,t|T}, \hat{\beta}_{2,t|T})'$ and the level of $\hat{\lambda}_{t|T}$.

1. Extract the trend components of the four parameters, and feed the trends back into the accounting identity model to recover the trend unemployment
2. Divide the trend unemployment by the trend labor force

$$DS\text{-}TUR_t = 100 \times \frac{\mathbf{1}' h(\hat{\psi}_{t|T}, \hat{\psi}_{t-1|T}, ..., \hat{\psi}_{t-47|T}, \hat{\lambda}_{t|T}, \hat{\lambda}_{t-1|T}, ..., \hat{\lambda}_{t-47|T})}{\hat{\psi}^{LF}_{t|T}}$$

where

$\hat{\psi}_{t|T} = [\hat{\psi}^{w}_{t|T}, \hat{\psi}^{\beta 0}_{t|T}, \hat{\psi}^{\beta 1}_{t|T}, \hat{\psi}^{\beta 2}_{t|T}]'$
$\mathbf{1}$: a (5×1) vector of ones
$h(\cdot)$: the measurement equations without measurement errors.

Trend-cycle decomposition

Estimate the standard model for trend-cycle decomposition (specified in a quarterly frequency) with a Bayesian method (Chan et al.(2019))

Assume that $\hat{w}_{q|T}$ follows an $I(2)$ process. The trend component ψ_q^w has a time-varying growth rate, μ_q^w:

$$\hat{w}_{q|T} = \underbrace{\psi_q^w}_{trend} + \underbrace{c_q^w}_{cycle}$$

Trend

$$\begin{aligned} \psi_q^w &= \mu_q^w + \psi_{q-1}^w \\ \mu_q^w &= \mu_{q-1}^w + \epsilon_{\psi q}^w, \qquad \epsilon_{\psi q}^w \sim N(0, (\sigma_\psi^w)^2) \\ \Rightarrow \triangle\psi_q^w &= \triangle\psi_{q-1}^w + \epsilon_{\psi q}^w. \end{aligned}$$

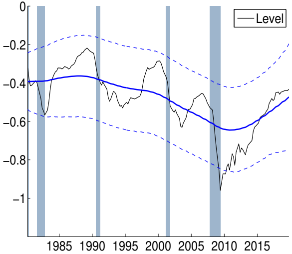
Cycle

$$c_q^w = \phi_1^w c_{q-1}^w + \phi_2^w c_{q-2}^w + \epsilon_{cq}^w, \quad \epsilon_{cq}^w \sim N(0, (\sigma_c^w)^2)$$

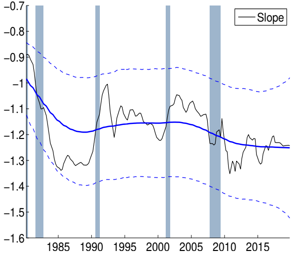
The same model is used to estimate the trends of other parameters.

Trend estimates

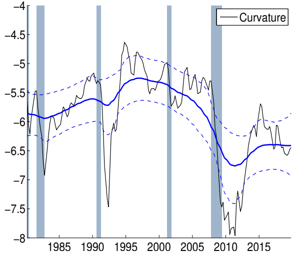
β_0



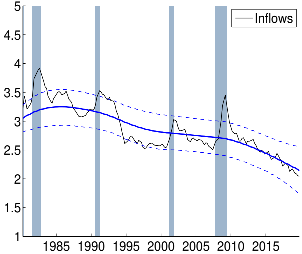
β_1



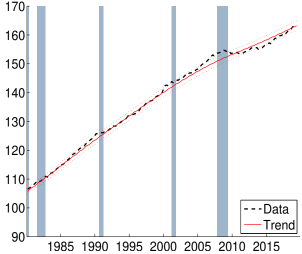
β_2



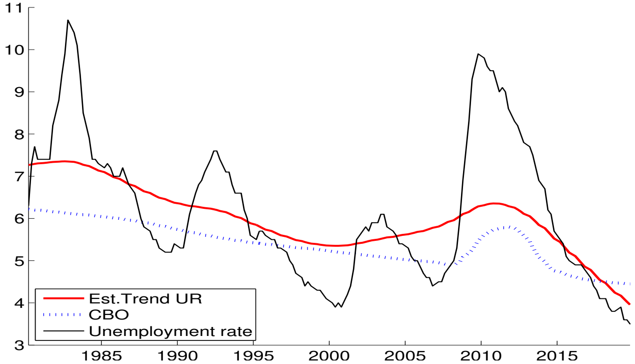
Inflows



Labor force



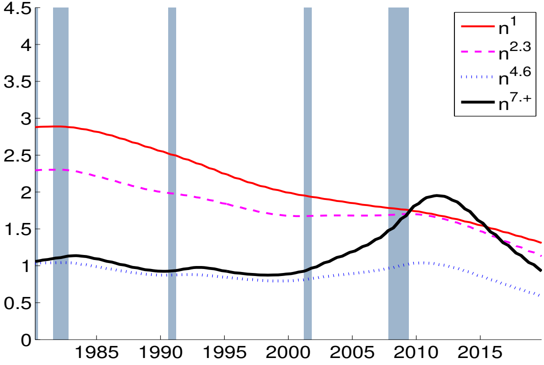
DS-TUR (1980-2019)



The low-frequency variation in the distribution of unemployment duration is used to identify the trend unemployment rate.

- The persistent effects of structural changes in the labor markets (e.g., mismatch and extended UI benefits) are captured by the long-term trend unemployment.

Duration components of DS-TUR



Decline in short-term U. since 1980s → decreased frictional unemployment
Increase in long-term U. during 2000s → increased structural unemployment

Implications for the Phillips curve

Consider a simple Phillips curve model

$$\pi_q = c_0 + c_1 gap_q + c_2 \pi_{q-1} + c_3 \pi_{q-2} + c_4 \pi_q^e + c_5 \pi_{q-1}^e + e_q$$

where π_q^e denotes the average 1-year-ahead inflation expectations from the Michigan survey in quarter q.

Alternatively, also replace gap_q with gap_q^j for $j = 1$, 2.3, 4.6, 7.+, where

$$gap_q^j = ru_q^j - \text{DS-TUR}_q^j$$

Key results

- The short-term unemployment rate gap has a strong Phillips correlation with PCE inflation, while the Phillips correlation with the aggregate gap is small.
- Replacing the unemployment-rate gaps with the unemployment rates by the duration, none of the Phillips correlation coefficients are statistically significant.

Implications for the Phillips curve

Table: Estimation results (2000:Q1-2019:Q4)

	gap_q	gap_q^1	$gap_q^{2.3}$	$gap_q^{1.3}$	$gap_q^{1.6}$	$gap_q^{7.+}$	gap_q^{CBO}
c_0	-0.30	-0.22	-0.31	-0.28	-0.25	-0.28	-0.33
S.E.	(0.33)	(0.32)	(0.32)	(0.31)	(0.32)	(0.33)	(0.33)
Gap	-0.070*	-1.32**	-0.60**	-0.46**	-0.24**	-0.071	-0.071*
S.E.	(0.037)	(0.46)	(0.20)	(0.15)	(0.088)	(0.059)	(0.037)
π_{q-1}	0.99**	0.92**	0.90**	0.89**	0.91**	1.026**	0.98**
1 S.E.	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.10)	(0.11)
π_{q-2}	-0.37**	-0.29**	-0.32**	-0.31**	-0.33**	-0.38**	-0.37**
S.E.	(0.090)	(0.091)	(0.089)	(0.088)	(0.089)	(0.092)	(0.091)
π_q^e	0.63**	0.54**	0.61**	0.58**	0.60**	0.64**	0.64**
S.E.	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)
π_{q-1}^e	-0.28**	-0.23*	-0.23**	-0.22**	-0.25**	-0.31**	-0.27**
S.E.	(0.14)	(0.14)	(0.14)	(0.13)	(0.13)	(0.14)	(0.14)
Adj. R^2	0.77	0.78	0.78	0.79	0.78	0.77	0.77

Implications for the Phillips curve

Table: Estimation results (2000:Q1-2019:Q4)

	ru_q	ru_q^1	$ru_q^{2.3}$	$ru_q^{1.3}$	$ru_q^{1.6}$	$ru_q^{7.+}$
c_0	-0.087	-0.24	0.075	0.058	0.056	-0.25
S.E.	(0.34)	(0.51)	(0.38)	(0.43)	(0.30)	(0.33)
UR	-0.048	0.0042	-0.26	-0.10	-0.086	-0.076
S.E.	(0.030)	(0.23)	(0.20)	(0.16)	(0.063)	(0.050)
π_{q-1}	1.00**	1.049**	0.99**	1.021**	1.00**	1.010**
S.E.	(0.11)	(0.19)	(0.11)	(0.11)	(0.11)	(0.10)
π_{q-2}	-0.36**	-0.37**	-0.34**	-0.35**	-0.35**	-0.38**
S.E.	(0.090)	(0.094)	(0.092)	(0.093)	(0.092)	(0.091)
π_q^e	0.64**	0.62**	0.63**	0.62**	0.63**	0.65**
S.E.	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)
π_{q-1}^e	-0.29**	-0.34**	-0.29**	-0.32**	-0.31**	-0.29**
S.E.	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)
Adj. R^2	0.76	0.76	0.77	0.76	0.76	0.77

Conclusion

This paper

1. introduces the duration structure of unemployment hazards
2. develops a novel method to estimate the trend unemployment rate

Main takeaways:

1. The identification of trend unemployment rate is achieved not only from the trends in labor-force flows but also from the low-frequency changes in the distribution of unemployment duration.
2. Secular decline in short-term trend unemployment rate $\rightarrow$ decline in frictional unemployment rate; Secular rise in long-term trend unemployment rate $\rightarrow$ rise in structural unemployment rate.
3. The short-term unemployment-rate gap has a strong Phillips correlation with PCE inflation.