Dynamic Equilibrium with Costly Short-Selling and Lending Market

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Motivation

- Significant fraction of trades occur in short-selling and lending market
- Key imperfection in this market: short-selling is costly
- Vast empirical literature investigates costly short-selling effects on
  - shorting fee
  - stock price, risk premium, volatility
  - short interest and its predictive power
  - short-selling risk and short-selling activity
- Existing theoretical studies primarily in stylized static settings
  - not suitable to address many empirical regularities

This Paper

- Provide a comprehensive analysis of costly stock short-selling
- Model generates
  - rich implications supporting extensive empirical evidence
  - new insights and predictions
  - simple and straightforward intuitions
- We also provide an analysis of
  - economic determinants of benefits of being a stock lender
  - endogenously determining optimal size of lenders
- In the model
  - belief disagreement (constant or time-varying) generates demand for short-selling and supply of lendable shares
  - (pessimistic) short-sellers borrow shares from (optimistic) lenders by paying a **shorting fee** (determined endogenously)
  - investors' subjective stock risk premia are all different

Main Results

- Shorting fee increases in belief disagreement, but decreases in lenders' size
- Stock price increases in shorting fee, but decreases in lenders' size
- Stock risk premium decreases in shorting fee, while its volatility is increased with costly short-selling
- Short interest increases in shorting fee, and negatively predicts future stock returns
- Short-selling risk matters: a higher short-selling risk can lead to lower stock returns and less short-selling activity

Related Theoretical Literature

- Short-Sale constraints and restrictions
  - static: Miller (1977), Diamond and Verrecchia (1987), Hong and Stein (2003), Nezafat, Schroder, and Wang (2017)
  - dynamic: Harrison and Kreps (1978), Detemple and Murthy (1997), Scheinkman and Xiong (2003), Gallmeyer and Hollifield (2008), Chabakauri (2015)
- Models with shorting fee
  - static: Blocher, Reed, and Van Wesep (2013), Banerjee and Graveline (2014)
  - dynamic: Duffie, Gârleanu, and Pedersen (2002), Daniel, Klos, and Rottke (2018), Evgeniou, Hugonnier, and Prieto (2019), Nutz and Scheinkman (2019)

though none has our key results (e.g. short interest and its predictive power, short-selling risk and activity)

Securities Market

- Continuous-time economy with infinite horizon
- Two securities: a risky stock and a riskless asset
- Stock
  - in fixed supply Q
  - claim to dividend flow D

$$dD_t = \mu dt + \sigma d\omega_t$$

  - stock price S is determined endogenously and posited to follow

$$dS_t + D_t dt = \mu_{St} dt + \sigma_{St} d\omega_t$$

- Riskless asset
  - in perfectly elastic supply
  - pays a constant interest rate r

Investors' Beliefs and Short-Selling

- Economy is populated by optimistic and pessimistic investors
- All investors commonly observe the dividend process
  - have different beliefs about its mean
- Optimistic investors, with mass 0.5, perceive the mean $\mu + \theta$
- Pessimistic investors, with mass 0.5, perceive the mean $\mu - \theta$
- Due to identical masses and belief symmetry
  - average belief is unbiased
- **Disagreement** θ is first constant then time-varying

Stock Short-Selling and Lending Market

- Stock short-selling costs follow standard market practices
  - short-sellers pay a shorting fee to borrow shares from lenders, and sell those borrowed shares to other willing buyers
- Further categorize optimists and pessimists into two groups
  - optimistic investors
    - **Lenders**, ℓ, with mass λ, lend a fraction α of their long position (**partial lending**)
    - **Holders**, h, with mass $0.5 - \lambda$, do not lend

    size of lenders λ is taken as given first, but endogenized later
  - pessimistic investors
    - **Short-Sellers**, s, with mass λ_s, pay **shorting fee** ϕ_t for each share they borrow from lenders
    - **Non-Participants**, n, with mass $0.5 - \lambda_s$

    shorting fee ϕ_t is determined endogenously. w.l.o.g. $\lambda_s = 0.5$

Investors' Preferences and Optimization

- Each type of investor's, $i = \ell, h, s$, optimization problem

$$\max_{c_i,\psi_i} \mathrm{E}_i \left[\int_0^\infty e^{-\rho t} \frac{e^{-\gamma c_{it}}}{-\gamma} dt \right]$$

subject to her dynamic budget constraint

$$dW_{it} = W_{it} r dt + \psi_{it} \pi_{it} dt + \psi_{it} \sigma_{St} d\omega_{it} - c_{it} dt$$

- Distinct type-specific subjective risk premia

$$\pi_{it} = \begin{cases} \pi_{St} + \frac{\sigma_{St}}{\sigma}\theta + \alpha\phi_t & \text{for } i = \ell \\ \pi_{St} + \frac{\sigma_{St}}{\sigma}\theta & \text{for } i = h \\ \pi_{St} - \frac{\sigma_{St}}{\sigma}\theta + \phi_t & \text{for } i = s \end{cases}$$

$\pi_{St} = \mu_{St} - rS_t$: common objective risk premium

Equilibrium with Riskless Shorting Fee

- Equilibrium market clearing conditions
  - stock: $\lambda\psi_{\ell t} + (\frac{1}{2} - \lambda)\psi_{ht} + \frac{1}{2}\psi_{st} = Q$
  - short-selling and lending: $\lambda\alpha\psi_{\ell t}\phi_t + \frac{1}{2}\psi_{st}\phi_t = 0$
- Shorting fee

$$\phi = -\frac{\gamma\sigma^2 Q}{(1/2-\lambda\alpha)-\lambda\alpha(1-\alpha)/(1/2+\lambda\alpha)}\frac{1}{r} + \frac{\theta}{(1/2+\lambda\alpha)-\lambda\alpha(1-\alpha)/(1/2-\lambda\alpha)}\frac{1}{r}$$

  - shorting fee increases in disagreement θ
  - shorting fee decreases in lenders' size λ and partial lending α
- Stock price

$$S_t = \bar{S}_t + \left(\frac{1}{2} + \lambda\alpha\right)\frac{1}{r}\phi$$

$\bar{S}_t = \frac{1}{r}D_t + \frac{\mu}{r^2} - \frac{\gamma\sigma^2 Q}{r^2}$ benchmark (costless short-selling) stock price

  - stock price increases in shorting fee ϕ
  - stock price decreases in lenders' size λ and partial lending α

Economy with Time-Varying Disagreement

- Retain all primitives of previous economy
  - except for investors' beliefs, now implying time-varying disagreement
- Optimistic and pessimistic investors perceive the mean $\mu \pm \theta_t$

$$d\theta_t = \kappa\left(\mu_\theta - \theta_t\right)dt + \sigma_\theta d\omega_{\theta t}$$

  - microfounded by different interpretations of signals in Bayesian learning environment (e.g., Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010))
- Stock price now posited to follow

$$dS_t + D_t dt = \mu_{St} dt + \sigma_{1t} d\omega_t + \sigma_{2t} d\omega_{\theta t}$$

Equilibrium with Risky Shorting Fee

- Shorting fee

$$\phi_t = \phi_0 + \underbrace{\phi_1}_{>0} \theta_t$$

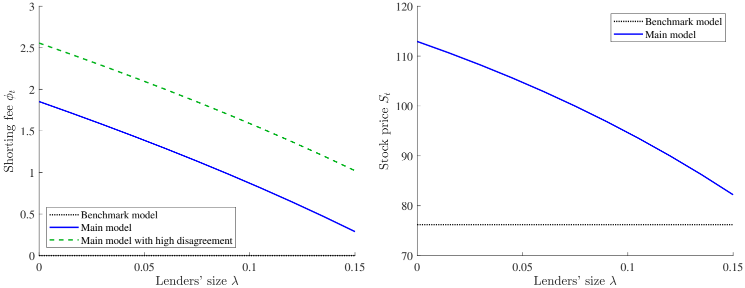
  - shorting fee increases in disagreement θ_t (D'Avolio (2002))

- Stock price

$$S_t = \bar{S}_t - \frac{\mu}{r^2} + \frac{\gamma\sigma^2 Q}{r^2} + A + \underbrace{B}_{>0} \phi_t$$

  - stock price increases in shorting fee ϕ_t (Jones and Lamont (2002), Ofek, Richardson, and Whitelaw (2004), Blocher, Reed, and Van Wesep (2013), Prado (2015))

Effect of Lenders' Size



- Shorting fee and stock price decrease in lenders' size λ as before (Prado, Saffi, and Sturgess (2016))

Stock Risk Premium and Volatility

- Stock risk premium

$$\pi_{St} = \bar{\pi}_S + \text{ constant } - (\kappa + r)\, B\phi_t$$

$\bar{\pi}_S = \gamma\sigma^2 Q/r$ benchmark (costless short-selling) risk premium

  - stock risk premium decreases in shorting fee ϕ_t (Jones and Lamont (2002), Cohen, Diether, and Malloy (2007), Blocher, Reed, and Van Wesep (2013), Drechsler and Drechsler (2016), Duong, Huszár, Tan, and Zhang (2017))

- Stock volatility

$$\sigma_S = \sqrt{\bar{\sigma}_S^2 + \sigma_\theta^2 \phi_1^2 B^2}$$

$\bar{\sigma}_S = \sigma/r$ benchmark (costless short-selling) volatility

  - stock volatility higher than that with costless short-selling (Drechsler and Drechsler (2016))

Short Interest

- Short interest (fraction of outstanding shares shorted)

$$\mathcal{SI}_t = \text{constant} + \underbrace{\frac{1}{2r}\Big(\frac{(\kappa + r - 2K_{s2}\sigma_\theta^2)\phi_1 B + 1/r - \phi_1}{\gamma\sigma_S^2 Q} - \frac{r}{\gamma\sigma^2 Q}\Big)\frac{1}{\phi_1}}_{>0}\phi_t$$

  - short interest increases in shorting fee ϕ_t (D'Avolio (2002), Beneish, Lee, and Nichols (2015), Drechsler and Drechsler (2016))

Short Interest and Predictability

- In predictive regression

$$S_{t+h} - S_t = \alpha_{\mathcal{SI}} + \beta_{\mathcal{SI}} \mathcal{SI}_t + \epsilon_{t+h}$$

slope coefficient

$$\beta_{\mathcal{SI}} = -2r\phi_1 B \frac{\gamma\sigma_S^2 Q}{\left(\kappa + r - 2K_{s2}\sigma_\theta^2\right)\phi_1 B + 1/r - \phi_1} \left(1 - e^{-\kappa h}\right) < 0$$

  - short interest negatively predicts future stock returns (Seneca (1967), Figlewski (1981), Senchack and Starks (1993), Desai, Ramesh, Thiagarajan, and Balachandran (2002), Asquith, Pathak, and Ritter (2005), Boehmer, Huszar, and Jordan (2010), Beneish, Lee, and Nichols (2015), Rapach, Ringgenberg, and Zhou (2016))
  - change in short interest negatively predicts future returns (Boehmer, Jones, and Zhang (2008), Diether, Lee, and Werner (2009))

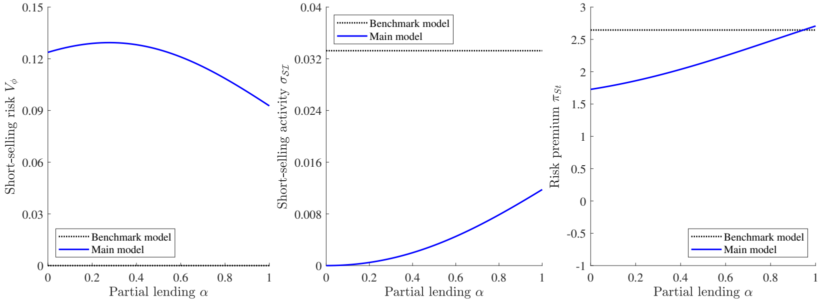
Short-Selling Risk and Short-Selling Activity

- Short-selling risk

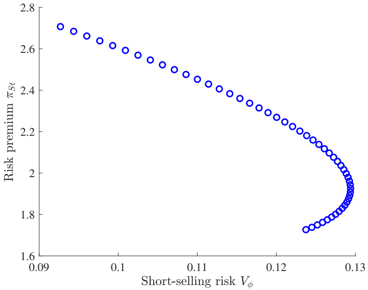
$$V_\phi = \lim_{t\to\infty} \mathrm{Var}\left[\phi_t\right] = \phi_1^2 \frac{\sigma_\theta^2}{2\kappa}$$

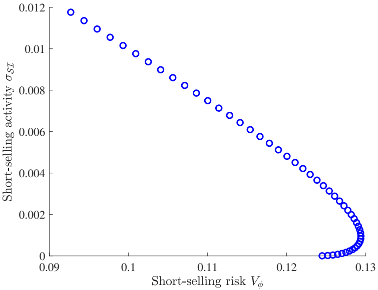
- Short-selling activity

$$\sigma_{\mathcal{SI}} = \sqrt{\mathrm{Var}_t\left[d\mathcal{SI}_t\right]/dt} = \frac{\sigma_\theta}{2r}\frac{\left(\kappa + r - 2K_{s2}\sigma_\theta^2\right)\phi_1 B + 1/r - \phi_1}{\gamma\sigma_S^2 Q}$$



Effects of Short-Selling Risk





- Differences in partial lending can explain findings of Engelberg, Reed, and Ringgenberg (2018) that stocks with higher short-selling risk typically have lower returns and less short-selling activity

Benefits of Lending

- Indirect utility

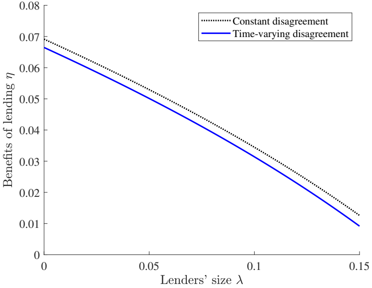
$$J_i\left(W_{it},\theta_t,t\right)=\max_{(c_i,\psi_i)}\mathrm{E}_{it}\left[\int_t^\infty e^{-\rho u}\frac{e^{-\gamma c_{iu}}}{-\gamma}du\right]$$

  - lenders have higher indirect utility than non-lending holders

- Benefit of lending η solves

$$J_\ell\left(\left(1-\eta\right)W_0,\theta_0,0\right)=J_h\left(W_0,\theta_0,0\right)$$

Benefits of Lending (cont'd)



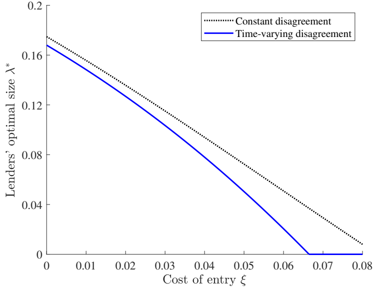
- Benefits of lending decreases in lenders' size λ
- Benefits of lending is non-monotonic, first increases then decreases, in partial lending α

Lenders' Optimal Size

- Endogenously determine lenders' optimal size given entry cost ξ
- Lenders' optimal size λ^* solves

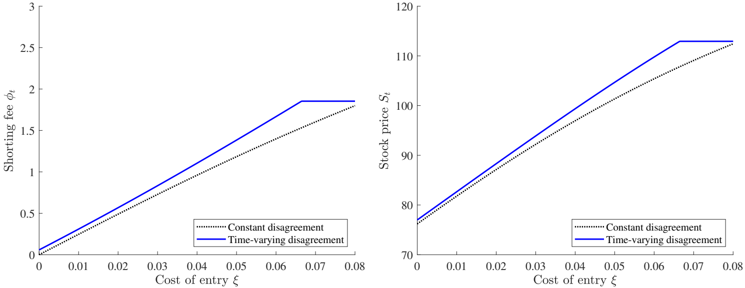
$$J_\ell\left(\left(1-\xi\right)W_0, \theta_0, 0; \lambda^*\right) = J_h\left(W_0, \theta_0, 0; \lambda^*\right)$$

Lenders' Optimal Size (cont'd)



- Lenders' optimal size decreases in cost of entry ξ
- Lenders' optimal size is non-monotonic, first increases then decreases, in partial lending α

Effects of Cost of Entry



- Shorting fee and stock price increase in cost of entry (through endogenously determined lenders' optimal size)

Conclusion

- Provide a comprehensive analysis of costly stock short-selling and lending market within a familiar dynamic asset pricing framework
- Model generates rich implications supporting extensive evidence
- Main results
  - stock price increases in shorting fee, while stock price and shorting fee decrease in lenders' size
  - stock risk premium decreases in shorting fee, while stock volatility is increased with costly short-selling
  - short interest increases in shorting fee and negatively predicts stock returns
  - higher short-selling risk can be associated with lower stock returns and less short-selling activity