Identification and Estimation of Demand Models with Endogenous Product Entry and Multiple Equilibria

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Motivation

- Estimation of demand of differentiated product when **firms do not offer some products in some markets**.
- Product entry if the **firm's expected profit > 0**.
- Firms' information about demand may include variables that are unobservable to the researcher. **Endogenous selection in demand estination**.
- This **selection problem is not standard:**
  - Unobsersables enter non-additively in selection condition.
  - High dimension of demand unobservables.
  - Multiple equilibria in entry game.

This Paper

- We consider a **structural model** that combines:
  - BLP demand of differentiated products.
  - Price competition between active firms/products.
  - Entry game with flexible information structure.

- In the entry game, the **specification of firms' information about demand unobservables** is crucial for the robustness of a method that tries to control for selection.

- In our model, specification of unobservables is such that:
  - Nonparametric distribution of all the unobservables.
  - Flexible: firms may know from all to nothing of demand unobs.
  - Both private and common info. unobservables.
  - Multiple equilibria unobservables.

Main contribution

- We prove the **identification of demand parameters in this model.**
- Our identification approach is **contructive and sequential** and implies a two-step estimation procedure.
- **[Step 1]** Nonparametric identification of entry probabilities conditional on the information that firms have about demand observables and unobservables.
- **[Step 2]** GMM semiparametric estimator of demand that controls for endogenous prices and selection.

Related Literature

- Recent literature on the **problem of zeroes in market shares.**

- There are potentially multiple **sources of zeroes** in market shares: small sample of consumers; consumer demand; stockouts; product not offered **product not oferred**.

- Closest papers to ours:

  Ciliberto et al. (2018); Li et al. (2019); Dube et al. (2020).

- In contrast to Ciliberto et al. (2018) and Li et al. (2019):

  - Nonparametric specification of unobservables;

  - Not joint estimation of demand and entry game but much simpler sequential estimation of demand.

Outline

[1] Model

[2] Identification Results

... *[3] Estimation method*

[4] Monte Carlo experiments

... *[5] Empirical application - airlines*

1. MODEL

Model - Demand

- BLP demand model. J single-product firms indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$.
- $a_{jm} \in \{0, 1\}$: indicator that product/firm j is available in market m.
- Market shares:

$$s_{jm} = \int \frac{a_{jm} \ \exp\{\delta_{jm} + v(p_{jm}, \mathbf{x}_{jm}, \upsilon)\}}{1 + a_{im} \ \exp\{\delta_{im} + v(p_{im}, \mathbf{x}_{im}, \upsilon)\}} dF_{\upsilon}(\upsilon|\boldsymbol{\sigma})$$

Lemma 1. *If outside alternative $j = 0$ is available, then Berry (1994) invertibility applies to the subsystem of available products such that:*

$$d_j^{-1}(\mathbf{s}_m, \sigma) = \alpha \ p_{jm} + \mathbf{x}'_{jm} \ \boldsymbol{\beta} + \xi_{jm} \ \ \textbf{if and only } a_{jm} = 1$$

Model - Price competition

- Bertrand competition as in BLP model.

Assumption 1. *Suppose that:*

- *(i) No random coefficients in* p_{jm}.
  (ii) Marginal cost mc_{jm} *is constant and* $mc_{jm} = \omega_{jm} + \widetilde{mc}_j(\mathbf{x}_{jm})$.
  (iii) The equilibrium selection mechanism does not depend on (ξ_{jm}, ω_{jm}). ■

Lemma 2. *Under Assumption 1, the equilibrium variable profit function* V_j *has the following structure:*

$$
\begin{array}{lcl}
V_{jm} & = & V_j(\mathbf{a}_m, \mathbf{x}_m, \boldsymbol{\xi}^*_m) \;\; \textit{where } \boldsymbol{\xi}^*_m \equiv (\xi^*_{jm} : j \in \mathcal{J}) \\
\textit{with} & & \xi^*_{jm} = \xi_{jm} + \alpha\ \omega_{jm}. \qquad \blacksquare
\end{array}
$$

Model - Entry game and information structure

Assumption 2. *The information set of firm j at the moment of its entry decision in market m consists of the triple $(\mathbf{x}_m, \boldsymbol{\kappa}_m, \eta_{jm})$.*
(a) $\boldsymbol{\kappa}_m \equiv (\kappa_{jm} : j \in \mathcal{J})$ is a vector of noisy signals for the demand-cost variables ξ_j^* such that, for every product j:

$$\xi_j^* = \kappa_{jm} + e_{jm}$$

where e_{jm} represents the error or noise in signal κ_{jm} and it is independent of $(\mathbf{x}_m, \boldsymbol{\kappa}_m)$.
(b) Variable η_{jm} in the fixed cost function $f_j(\mathbf{x}_{jm}, \eta_{jm})$ is private information of firm j and independently distributed over firms with CDF F_η, and is additive:

$$f_j(\mathbf{x}_{jm}, \eta_{jm}) = \overline{f}_j(\widetilde{\mathbf{x}}_{jm}) + \eta_{jm}. \qquad \blacksquare$$

Model - Bayesian Nash Equilibrium

- $\pi_j(\mathbf{a}, \mathbf{x}_m, \boldsymbol{\kappa}_m)$ is firm j's expected variable profit given its information about demand and costs, $(\mathbf{x}_m, \boldsymbol{\kappa}_m)$, and conditional on the hypothetical entry profile $\mathbf{a}$.

$$\pi_j(\mathbf{a}, \mathbf{x}_m, \boldsymbol{\kappa}_m) \quad = \quad \int V_j(\mathbf{a}, \mathbf{x}_m, \boldsymbol{\kappa}_m + \mathbf{e}_m)\ p\left(\mathbf{e}_m\right)\ d\mathbf{e}_m - \overline{f}_j(\widetilde{\mathbf{x}}_{jm})$$

- Given $(\mathbf{x}_m, \boldsymbol{\kappa}_m)$, a **Bayesian Nash Equilibrium (BNE)** is J-tuple of entry probabilities, $(P_{jm} : j \in \mathcal{J})$, that satisfies:

$$P_{jm} = F_\eta\left(\pi_{jm}^P\right) \text{ for every } j \in \mathcal{J},$$

where π_{jm}^P is firm j's expected profit – up to η_{jm}. That is,

$$\pi_{jm}^P = \sum_{\mathbf{a}_{-j} \in \{0,1\}^{J-1}} \left(\prod_{i \neq j} \left[P_{im}\right]^{a_i} \left[1 - P_{im}\right]^{1-a_i} \right) \pi_j(\mathbf{a}_{-j}, \mathbf{x}_m, \boldsymbol{\kappa}_m)$$

2. IDENTIFICATION RESULTS

Selection problem & Identification of demand

- Regression equation for demand is:

$$d_j^{-1}(\mathbf{s}_m, \sigma) = \alpha \; p_{jm} + \mathbf{x}'_{jm} \; \boldsymbol{\beta} + \lambda_j(\mathbf{x}_m) + \widetilde{\xi}_{jm},$$

where $\lambda_j(\mathbf{x}_m)$ is the selection control function:

$$\lambda_j(\mathbf{x}_m) \equiv \mathbb{E}\left[\xi_{jm} \mid \mathbf{x}_{jm}, \; a_{jm} = 1\right]$$

- Our model implies that:

$$\begin{aligned} \lambda_j(\mathbf{x}_m) &= \sum_{\kappa_m=1}^{L} f_\kappa(\kappa_m) \; \widetilde{\lambda}_j \left(P_j(\mathbf{x}_m, \kappa_m), \kappa_m\right) \\ &= \psi_j(\mathbf{P}_{jm}) \end{aligned}$$

with:

$$\widetilde{\lambda}_j \left(P_j(\mathbf{x}_m, \kappa_m), \kappa_m\right) = \mathbb{E}\left[\xi_{jm} \mid \mathbf{x}_{jm}, \; \kappa_m, \; a_{jm} = 1\right].$$

$$\mathbf{P}_{jm} \equiv \left(P_j(\mathbf{x}_m, \kappa) : \kappa = 1, 2, ..., L\right).$$

Sequential Identification

[Step 1]. Nonparametric identification of $\mathbf{f}_\kappa \equiv (f_\kappa(\kappa) : \kappa = 1, 2, ..., L)$ and $\mathbf{P}_{jm} \equiv (P_j(\mathbf{x}_m, \kappa) : \kappa = 1, 2, ..., L)$.

[Step 2]. Given $\mathbf{P}_{jm}$, identification of $(\sigma, \alpha, \boldsymbol{\beta})$ in the semiparametric regression model:

$$d_j^{-1}(\mathbf{s}_m, \sigma) = \alpha \; p_{jm} + \mathbf{x}_{jm}' \; \boldsymbol{\beta} + \psi_j(\mathbf{P}_{jm}) + \widetilde{\xi}_{jm}.$$

- We have two results on Step 1 identification. So far, we have focused on the result based on the restriction that $\boldsymbol{\kappa}_m$ has finite support (**nonparametric finite mixture**): Xiao (2018); Aguirregabiria and Mira (2019).

- Step 2 identification is based on previous results in the semiparametrics literature: Powell (2001), and Aradillas-Lopez, Honore, and Powell (2007).

3. MONTE CARLO EXPERIMENTS

Monte Carlo Experiments: DGP

- **Demand:** Nested logit; $J = 3$; one observable x_{jm}; ξ_{jm} is a mixture of two normals.
- $\kappa_m \in \{\ell, h\}$ two market types ($L = 2$).
- There is also observable z_m that affects entry cost but not demand.

Monte Carlo Experiments: DGP

Table 2. Summary Statistics from DGP

	Percentage of Zeros	Avg market Share	Average p-c/p
Firm 1	81.3%	0.16	79.9 %
Firm 2	81.4%	0.16	79.9%
Firm 3	81.4%	0.16	80%

Estimators: Bias and Variance

Monte Experiment with $M = 1000$

		True	OLS	2SLS	Our	Our(True P)
β	Mean	2	1.2161	1.6352	1.7754	1.9835
	Std. Dev.		(0.1483)	(0.1974)	(0.2573)	(0.0203)
α	Mean	-2	-1.9046	-1.9254	-1.9657	-1.9967
	Std. Dev.		(0.0134)	(0.0136)	(0.0115)	(0.0017)
σ	Mean	0.6	0.5892	0.6677	0.6277	0.6042
	Std. Dev.		(0.0113)	(0.0287)	(0.0146)	(0.0022)

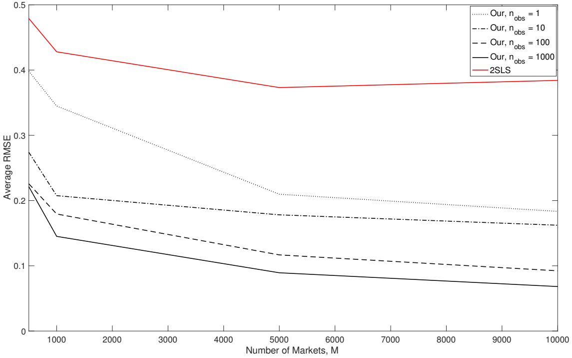
Estimators: Ratio or RMSE

Monte Experiments with for different values of M
Ratios Between RMSEs of Different Estimators

	$M = 500$		$M = 1,000$	
	Our/2SLS	Our/Our-True	Our/2SLS	Our/Our-True
β	0.8435	10.9144	0.8236	13.0788
α	0.5138	7.9534	0.4772	9.8504
σ	0.5743	7.0440	0.4262	6.6343
	$M = 5,000$		$M = 10,000$	
	Our/2SLS	Our/Our-True	Our/2SLS	Our/Our-True
β	0.5835	17.7773	0.4947	18.0781
α	0.1838	7.4314	0.1232	6.4403
σ	0.1509	4.3426	0.0958	3.2940

Estimators: Ratio or RMSE

Figure 1: Average RMSE of "Our" and 2SLS estimators



Conclusions

- New results on the identification of demand of differentiated products when firms' decision to offer a product is endogenous.

- The model and method emphasizes:
  - Nonparametric specification of the unobservables.
  - Flexible information structure for the unobservables.
  - Computationally simple sequential approach.