The Racial Wealth Gap and the Role of Firm Ownership

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Abstract

This paper develops an overlapping generations model that isolates the impact of the U.S. racial wealth gap in 1962 on the long-run dynamics of wealth. The model predicts that one component of the initial gap, firm ownership, coupled with the intergenerational transfer of that ownership, results in a permanent gap in overall wealth independent of other dimensions of inequality. This implies that even if all discrimination against black Americans had ceased upon the end of Jim Crow, the wealth gap would have persisted without a reparations policy addressing the fact that the initial firm ownership gap arose in the first place. As such, reparations to the descendants of slaves and Jim Crow era policies are essential to closing the racial wealth gap. The model also predicts that a reparations program will only be effective if it targets the distribution of firm ownership in the economy.

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In 1962, two years before the legal end of Jim Crow, the average black household in the U.S. possessed 16% of the wealth of the average white household.[1] Over the next 57 years the wealth gap remained virtually unchanged, peaking at just 22 percent in 1992, and falling back to 16 percent in 2016 after the Great Recession. If the "initial" 1962 wealth gap coupled with the transfer of that wealth across generations can lead to the persistent wealth gap seen in the data, then a reparations policy directly addressing the fact that the U.S. allowed the build up of the initial wealth gap in the first place will be essential to closing the gap in the future. If on the contrary, an initial wealth gap cannot generate persistent wealth inequality on its own, government intervention may still be necessary to close the wealth gap; but, it should focus solely on ending active discrimination against black Americans rather than addressing past inequities.

Darity (2005) provides the foundational argument for the necessity of reparations, maintaining that white households' overwhelming abundance of wealth relative to blacks' is due to generations of unimpeded wealth building and intergenerational wealth transfers. In contrast, previous generations of black Americans were systematically denied the opportunity to accumulate wealth throughout the history of slavery and Jim Crow. This "initial" gap in wealth then persisted after the legal end of Jim Crow because black households did not have enough built-up wealth to make intergenerational wealth transfers at comparable levels to white ones. Evidence from two studies that perform Blider-Oaxaca mean decompositions of the wealth gap - (Menchik and Jianakoplos, 1997), (Mckernan et al., 2014) - support Darity's argument by demonstrating that intergenerational transfers contribute to the mean wealth gap in a quantitatively meaningful way (10-20 percent).

The purpose of this paper is to better assess the salience of Darity's wealth gap explanation through the lens of a dynamic long run wealth accumulation model. A dynamic approach is necessary because a mean decomposition analysis can only explain differences in wealth levels at a given point in time and not differences in wealth accumulation over time (Aliprantis et al., 2019). A means coefficient analysis can explain how much of the wealth gap in a single period is due to differences in inheritances received in that period, but it cannot capture that the black households

[1]Data comes from the Survey of Consumer Finances. A detailed description of how I constructed this variable along with a plot of the wealth gaps over time can be found in Appendix A.

may receive those smaller inheritances in the first place because the previous generation of black households received smaller inheritances than the previous generation of white households.

My approach is to isolate the impact of the initial wealth gap and the intergenerational transfer of that wealth on long-run wealth inequality with an overlapping generations model with inheritance motives. The only difference between white and black households in the model is their initial stock of wealth. The model predicts that the component of the initial wealth gap comprised of the gap in firm ownership leads to a steady state wealth gap. Specifically, because white households in the model own a larger percentage of the firms in the economy than black households and can choose to pass along some fraction of those firms directly to the next generation, white households earn more profits than black households every period. This consistent extra income for white households then allows them to build up a permanent advantage in wealth. Thus, a reparations policy that addresses the initial discrepancy in firm ownership is necessary for closing the mean racial wealth gap.[2] A key consequence of my model is that a pure wealth transfer form of reparations will not sustainably close the wealth gap because the underlying wealth dynamics due to unequal firm ownership will remain and reassert themselves after the policy. A more effective form of reparations must rectify the gap in firm ownership directly.

This theory reconciles Darity's argument for the persistence of the wealth gap with more recent evidence from Aliprantis et al. (2019), who use a dynamic general equilibrium model to show that the accumulation of successive earnings gaps over time are the primary driver of the wealth gap. In the absence of these earnings gaps, the initial wealth disparity will dissipate extremely quickly. However, this argument can fit into Darity's framework if the unequal initial wealth conditions lead to unequal income generating capacity between white and black households, and white households can transfer that income generating capacity to their descendants. In this case, the initial wealth gap can explain the earnings gap, which in turn explains the wealth gap.[3] This dynamic is exactly what occurs in my model. White household's advantage in firm ownership gives them

[2]My argument only applies to the mean racial wealth gap as opposed to the similar gap at the median because firm assets are concentrated at the top of both the white and black distributions (Survey of Consumer Finances).

[3]Aliprantis et al. (2019) focus on labor income, but the logic holds for any type of income.

superior income generating capacity over black household's in the form of profits and the intergenerational transfer of firm assets functions as the intergenerational transfer of income generating capacity. In the remainder of this paper I will introduce the details of the model that underlies my arguments, show how to solve for its laws of motion and steady state, and discuss this steady state's implications.

Model Preliminaries

The overlapping generations model I use to isolate the impact of the initial wealth gap and intergenerational wealth transfers on the dynamics of the wealth gap is a variant of the two-period discrete time overlapping generations (OLG) model with impure altruism presented in Acemoglu (2009). Each of the 2 cohorts (young and old) has a constant finite population L divided evenly among finitely many H households. So, each cohort in a given household has a constant population of $l = \frac{L}{H}$. There are finitely many N identical firms owned by the households that use capital and labor in each period t to produce a single output good, which households can then consume in period t or save to rent as productive capital in period $t + 1$. The output market and the input markets are competitive and firm profits will pay out to the young cohorts each period. To model the impact of firm profits on the wealth gap, I employ decreasing returns to scale production technology in the model so there are positive economic profits at equilibrium. Households can also buy and sell shares of firm ownership in a competitive stock market. I normalize the number of shares per firm to $\frac{100}{N}$ and allow the purchase and sale of fractional shares. Because all firms are identical and will thus earn the same profits at equilibrium, each share of firm ownership entitles the owner to $\frac{1}{100/N} * \frac{1}{N}$, or one percent, of the economy's aggregate profits. So, there are a total of 100 equally valued shares of firm ownership in the economy. Each young member of the household provides one unit of labor inelastically. At the end of a given period, the young cohort in each household can buy and sell firm shares on the stock market and decide how many shares to leave to the incoming young generation.

Because the purpose of this model is to focus on the ramifications of the initial 1962 racial wealth gap, period 0 in the model represents 1962. The model has two types of households, those endowed with high levels of wealth in period 0 (type R) and those endowed with low levels of wealth in period 0 (type P). Household wealth has three components in this model, the household's stock of physical capital, its stock of marketable firm shares, and its stock of off-market firm shares. An off market firm share is a share of firm ownership that the household never sells on the stock market and simply passes down directly from generation to generation (i.e., privately owned businesses). Each household of type i with $i \in \{R, P\}$ starts with θ_i units of capital, π_0^i marketable shares of firm ownership, and ϕ_i off-market shares of firm ownership. Type R households represent white households and type P households represent black households; so, to capture the initial wealth gap, I exogenously set $\theta_R > \theta_P$, $\pi_0^R > \pi_0^P$, and $\phi_R > \phi_P$. The fraction of the H households that are type R is q^R and the fraction that are type P is q^P. By definition, $q^R H\phi_R + q^P H\phi_P = (100 - m)$, where m is the total number of marketable firm shares in the economy.

Optimization Problems and Equilibrium

Because all type R households are identical and all type P households are identical, there are only two distinct optimization problems, that of a representative type R and type P household. In each period t, each household of type i takes the interest rate, wage rate, their inheritance, and profits as given and maximizes the lifetime utility of the young cohort born in that period by solving the following problem:

$$\max_{c_{1t}^i, c_{2(t+1)}^i, b_{t+1}^i, \pi_{t+1}^i} \ln(c_{1t}^i) + \beta \ln(c_{2(t+1)}^i) + \rho\beta \ln(b_{t+1}^i) + \zeta\beta \ln(\pi_{t+1}^i)$$

$$\begin{aligned} s.t.\ \ & c_{2(t+1)}^i + b_{(t+1)}^i + v_{t+1}\pi_{t+1}^i \\ & \leq (1 + r_{t+1})(b_t^i + lW_t + (\frac{\pi_t^i + \phi_i}{100})P_t - c_{1t}^i) + v_{t+1}\pi_t^i. \end{aligned}$$

Here, $c_{1t}^i \geq 0$ is total period t consumption for the young cohort in a household of type i, $c_{2t}^i \geq 0$ is total period t consumption for the old cohort in a household of type i, $b_t^i \geq 0$ is the capital transfer made by the old cohort in the household to the young cohort in the household in period t

for a household of type i, and $\pi_t^i > 0$ and $\phi_i > 0$ are the transfer of marketable firm shares and off-market firm shares respectively made by the old cohort to the young cohort in period t. $W_t \geq 0$ is the wage rate in period t, $r_{t+1} \geq 0$ is the rate of return on productive capital in period $t+1$, $v_t \geq 0$ is the price of a firm share, and $P_t \geq 0$ is aggregate firm profits in period t. $(\frac{\pi_t^i+\phi_i}{100})P_t$ are the dividends paid out to a household of type i in period t. Each of these variables other than firm shares is expressed in real terms as units of the economy's single output good.

The instantaneous utility function is $u(c) = \ln(c)$ and $0 < \beta < 1$ is the time discount factor. I model the incentive for the old cohort in a household to leave capital and firm shares to the next generation as a warm glow. The warm glow expressions are $u(b) = \rho \ln(b)$ for capital and $u(\pi) = \zeta \ln(\pi)$ for firm shares. The factor $\rho > 0$ and $\zeta > 0$ capture the fact that cohorts may value capital transfers and firm share transfers differently from each other and their own consumption. ζ also adjusts for the difference in units between firm shares and the other goods.

The household problem can be solved with standard Lagrange multiplier techniques to produce the following optimal paths for the choice variables:

$$c_{1t}^i = \frac{b_t^i + lW_t + (\frac{\pi_t^i+\phi_i}{100})P_t + \frac{v_{t+1}}{1+r_{t+1}}\pi_t^i}{\beta + \rho\beta + \zeta\beta + 1} \tag{1}$$

$$b_{t+1}^i = \frac{\rho\beta(1+r_{t+1})(b_t^i + lW_t + (\frac{\pi_t^i+\phi_i}{100})P_t + \frac{v_{t+1}}{1+r_{t+1}}\pi_t^i)}{\beta + \rho\beta + \zeta\beta + 1} \tag{2}$$

$$\pi_{t+1}^i = \frac{\zeta\beta(1+r_{t+1})(b_t^i + lW_t + (\frac{\pi_t^i+\phi_i}{100})P_t + \frac{v_{t+1}}{1+r_{t+1}}\pi_t^i)}{v_{t+1}(\beta + \rho\beta + \zeta\beta + 1)}. \tag{3}$$

A household's optimal path of savings $\left(b_t^i + lW_t + (\frac{\pi_t^i+\phi_i}{100})P_t - c_{1t}^i\right)$ is thus

$$S_t^i = \frac{\beta + \rho\beta + \zeta\beta}{\beta + \rho\beta + \zeta\beta + 1}(b_t^i + lW_t + (\frac{\pi_t^i + \phi_i}{100})P_t) - \frac{v_{t+1}\pi_t^i}{(1+r_{t+1})(\beta + \rho\beta + \zeta\beta + 1)}. \tag{4}$$

Wealth is measured as the household's savings plus the real value of its firm shares at equilibrium.

So the equilibrium wealth ratio is

$$WR_t = \frac{\frac{\beta+\rho\beta+\zeta\beta}{\beta+\rho\beta+\zeta\beta+1}(b_t^P + lW_t + (\frac{\pi_t^P+\phi_P}{100})P_t) - \frac{v_{t+1}\pi_t^P}{(1+r_{t+1})(\beta+\rho\beta+\zeta\beta+1)} + v_t(\pi_t^P + \phi_P)}{\frac{\beta+\rho\beta+\zeta\beta}{\beta+\rho\beta+\zeta\beta+1}(b_t^R + lW_t + (\frac{\pi_t^R+\phi_R)}{100})P_t - \frac{v_{t+1}\pi_t^R}{(1+r_{t+1})(\beta+\rho\beta+\zeta\beta+1)} + v_t(\pi_t^R + \phi_R)}. \tag{5}$$

Each of the N firms in the model are identical so they each have the same production function for producing the economy's single aggregate good. In the canonical OLG model (Acemoglu, 2009), the production function is Cobb-Douglas given by $F(\mathcal{K}, \mathcal{L}) = \mathcal{K}^\alpha \mathcal{L}^\gamma$ where $0 < \alpha < 1$, $0 < \gamma < 1$, $\mathcal{K} > 0$ is physical capital, and $\mathcal{L} > 0$ is labor. The assumption of decreasing returns to scale is equivalent to $\alpha + \gamma < 1$. Each firm takes r_t and W_t as given and solves the following profit maximization problem in every period t:

$$\max_{\mathcal{K}_t, \mathcal{L}_t} \mathcal{K}_t^\alpha \mathcal{L}_t^\gamma - W_t \mathcal{L}_t - r_t \mathcal{K}_t - \delta \mathcal{K}_t$$

where $0 \leq \delta \leq 1$ is the rate of capital depreciation. The decreasing returns to scale production technology guarantees a unique solution to the firm's problem. After aggregating first order conditions across all firms, input prices and profits are given by

$$W_t = N^{1-\alpha-\gamma} \gamma K_t^\alpha L^{\gamma-1}, \tag{6}$$

$$r_t = N^{1-\alpha-\gamma} \alpha K_t^{\alpha-1} L^\gamma - \delta. \tag{7}$$

$$P_t = N^{1-\alpha-\gamma}(1-\alpha-\gamma) K_t^\alpha L^\gamma. \tag{8}$$

Finally, the market clearing conditions are

$$K_{t+1} = q^R H S_t^R + q^P H S_t^P \text{ (capital market)} \tag{9}$$

$$K_{t+1} = (1-\delta)K_t + N^{1-\alpha-\gamma} K_t^\alpha L^\gamma - q^R H(c_{1t}^R + c_{2t}^R) - q^P H(c_{1t}^P + c_{2t}^P) \;\; \forall t \text{ (output market)} \tag{10}$$

$$q^R H \pi_t^R + q^P H \pi_t^P = m \;\; \forall t \text{ (Stock Market)}. \tag{11}$$

Dynamics of the Capital Stock and Wealth Gap

We can derive alternate forms of the solution to the household problem,

$$c^i_{1t} = \frac{b^i_t + lW_t + (\frac{\pi^i_t+\phi_i}{100})P_t}{\beta+\rho\beta+1} + \frac{\frac{v_{t+1}}{1+r_{t+1}}(\pi^i_t - \pi^i_{t+1})}{\beta+\rho\beta+1} \tag{12}$$

$$b^i_{t+1} = \frac{\rho\beta(1+r_{t+1})(b^i_t + lW_t + (\frac{\pi^i_t+\phi_i}{100})P_t}{\beta+\rho\beta+1} + \frac{\rho\beta(1+r_{t+1})\frac{v_{t+1}}{1+r_{t+1}}(\pi^i_t - \pi^i_{t+1})}{\beta+\rho\beta+1}, \tag{13}$$

to get a new savings equation

$$S^i_t = \frac{\beta+\rho\beta}{\beta+\rho\beta+1}(b^i_t + lW_t + (\frac{\pi^i_t+\phi_i}{100})P_t) - \frac{\frac{v_{t+1}}{1+r_{t+1}}(\pi^i_t - \pi^i_{t+1})}{\beta+\rho\beta+1}. \tag{14}$$

Substituting equation 14 into equation 9 yields

$$K_{t+1} = \frac{\beta+\rho\beta}{\beta+\rho\beta+1}(B_t + LW_t + P_t) - \frac{\frac{v_{t+1}}{1+r_{t+1}}(m-m)}{\beta+\rho\beta+1} = \frac{\beta+\rho\beta}{\beta+\rho\beta+1}(B_t + LW_t + P_t) \tag{15}$$

where $B_t = q^R H b^R_t + q^P H b^P_t$ is the aggregate intergenerational transfer of capital in period t. Aggregating equation 13 across both types of households yields

$$B_{t+1} = \frac{\rho\beta}{\beta+\rho\beta+1}(1+r_{t+1})(B_t + LW_t + P_t) \implies B_{t+1} = \frac{\rho\beta}{\rho+\rho\beta}(1+r_t)K_{t+1}. \tag{16}$$

Combining equations 15 and 16 and substituting the equations for W_t, r_t, and P_t (6, 7, 8) shows that the law of motion for the capital stock is

$$K_{t+1} = \frac{\beta+\rho\beta}{\beta+\rho\beta+1}(\frac{\rho\beta}{\beta+\rho\beta}(1+N^{1-\alpha-\gamma}L^\gamma\alpha K_t^{\alpha-1} - \delta)K_t + N^{1-\alpha-\gamma}L^\gamma(1-\alpha)K_t^\alpha). \tag{17}$$

This law of motion shows that the capital stock converges to a unique steady state K_*, which implies that r_t, W_t, and P_t converge to r_*, W_*, and P_* respectively [4].

It is still necessary to solve for the dynamics of b^i_t, π^i_t, and v_t to understand the dynamics of

[4] A formal proof of this fact can be found in Claim 1 in Appendix B.

the wealth ratio (equation 5). Note that the optimal paths of b_t^i and π_t^i (equations 2 and 3) together imply that

$$\pi_t^i = \frac{\zeta b_t^i}{\rho v_t}. \tag{18}$$

Then, substituting 18 into equation 2 yields

$$b_{t+1}^i = \frac{\rho\beta(1+r_{t+1})}{\beta+\rho\beta+\zeta\beta+1}(1+\frac{\zeta P_t}{100\rho v_t}+\frac{v_{t+1}\zeta}{(1+r_{t+1})\rho v_t})b_t^i + \frac{\rho\beta(1+r_{t+1})}{\beta+\rho\beta+\zeta\beta+1}(lW_t+\frac{\phi_i P_t}{100}). \tag{19}$$

Next, combining the stock market clearing condition (equation 11) with the optimal path of π_t^i (equation 3) shows that

$$v_{t+1} = \frac{\zeta\beta(1+r_{t+1})(B_t+LW_t+P_t)}{m(\beta+\rho\beta+1)}, \tag{20}$$

which implies that v_t converges ot a steady state v_*.

Now, after applying steady states, the long run optimal path of b_t^i is governed by

$$b_{t+1}^i = \frac{\rho\beta(1+r_*)}{\beta+\rho\beta+\zeta\beta+1}(1+\frac{\zeta P_*}{100\rho v_*}+\frac{\zeta}{\rho(1+r_*)})b_t^i + \frac{\rho\beta(1+r_*)}{\beta+\rho\beta+\zeta\beta+1}(lW_*+\phi_i\frac{P_*}{100}). \tag{21}$$

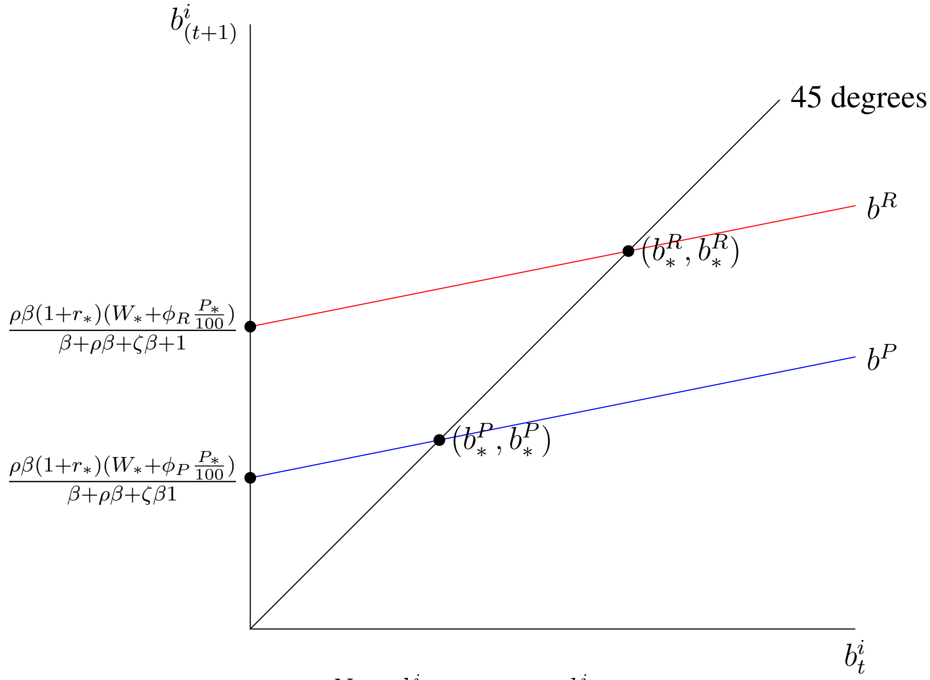
Plotting equation 21 for both types of households along with the 45 degree line reveals that b_t^i converges to a steady state b_*^i (Figure 1).[5] Based on equation 18, π_t^i converges to a steady state π_*^i. Because, $\phi_R > \phi_P$, it is clear from Figure 1 that $b_*^R > b_*^P$ and $\pi_*^R > \pi_*^P$, implying that

$$\lim_{t\to\infty} WR_t = \frac{\frac{\beta+\rho\beta+\zeta\beta}{\beta+\rho\beta+\zeta\beta+1}(b_*^P+lW_*+(\frac{\pi_*^P+\phi_P}{100})P_*)+v_*\pi_*^P(1-\frac{1}{(1+r_*)(\beta+\rho\beta+\zeta\beta+1)})+v_*\phi_P}{\frac{\beta+\rho\beta+\zeta\beta}{\beta+\rho\beta+\zeta\beta+1}(b_*^R+lW_*+(\frac{\pi_*^R+\phi_R}{100})P_*)+v_*\pi_*^R(1-\frac{1}{(1+r_*)(\beta+\rho\beta+\zeta\beta+1)})+v_*\phi_R} < 1. \tag{22}$$

The difference between ϕ_R and ϕ_P is the only factor driving the wealth gap in the long run. The initial gap in physical capital and marketable firm shares between the two types of households is not relevant to wealth inequality when the economy reaches its steady state. In fact, if all shares of firm assets were marketable and $\phi_R = \phi_P = 0$, the wealth gap would entirely erode over time. But, as long as some fraction of the firm shares in the economy are off-market, the type R

[5] A formal proof of this fact can be found in Claim 2 in Appendix B.

Figure 1: Dynamics of the Intergenerational Transfer of Capital



Note: b^i_t converges to b^i_*.

households' extra off-market shares will give them a consistent advantage in profits over the type P households each period, which causes the build up of a steady state gap in intergenerational transfers of both capital and firm shares. These gaps in turn lead to a steady state wealth gap. Effectively, the inherent rigidity in the stock market due to households' desire to keep some of their firms shares in the family prevents black households from catching up to white households in terms of both stock and capital.

Conclusion and Policy Implications

According to the findings of this paper, a reparations policy that addresses the history of racial discrimination in the U.S. is necessary to eliminate the large racial wealth gap that has persisted for over 50 years. My analysis considers a theoretical economy with no active forms of discrimination between white and black Americans. Within this framework, given the assumptions that firms earn positive profits and that there are rigidities in the stock market due to privately owned businesses, the initial gap in firm ownership between white and black households in 1962 will lead to a permanent wealth gap. Thus, a key feature of the persistent racial wealth gap is an initial conditions problem. Consequently, race blind approaches to wealth building are misguided. Even if black and white Americans had the exact same opportunities, starting with the passage of the Civil Rights Act in 1964, the descendants of slaves and the victims of Jim Crow era polices would still have had no way to catch up to whites because they started on such unequal footings. The U.S. government must take active steps to address the disparity in initial conditions before there can be any hope to close the wealth gap in this country.

My research also indicates that the most useful form of reparations will be one that targets the underlying distribution of firm ownership in the economy. Any type of lump sum wealth transfer will have limited impact if the allocation of firms in the economy stays the same. If large enough, the transfer will close the gap initially, but because it does not alter the fundamental laws of motion of the economy (equation 21), the economy would just return to the same steady state with the same wealth gap. To alter these laws of motion, a reparations program must equalize black and white control over off-market firms (ϕ_i and ϕ_r). The most straightforward way to do so would be for the federal government to simply mandate a transfer of off market firms from white households to black households. Such a policy is admittedly politically unlikely but the government could approximate this first-best policy with more moderate solutions such as entrepreneurial subsidies to black Americans (Boerma and Karabarbounis, 2021).

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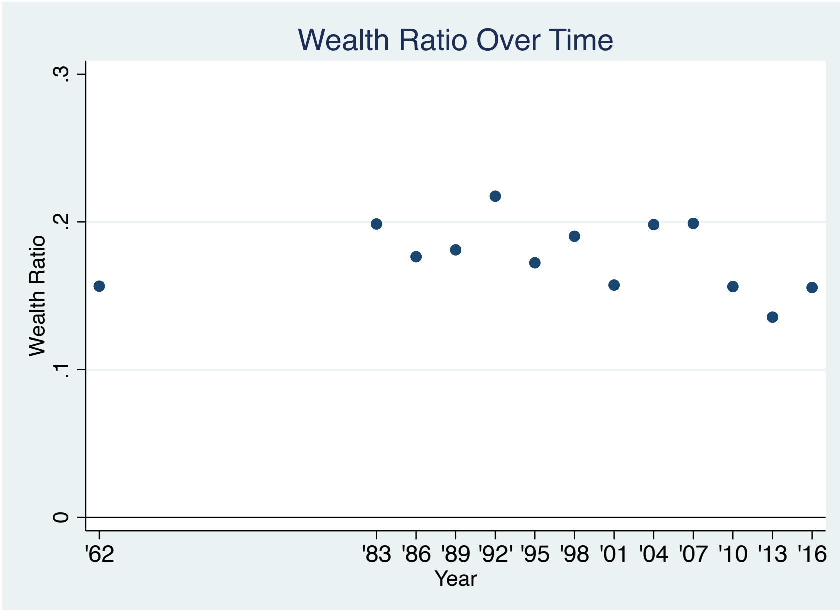
Online Appendix

A Wealth Variable Construction

Figure 2: Persistence of the Wealth Gap



Note: Each point is the estimate for the mean wealth of the black households over the mean wealth for white households in the given year.

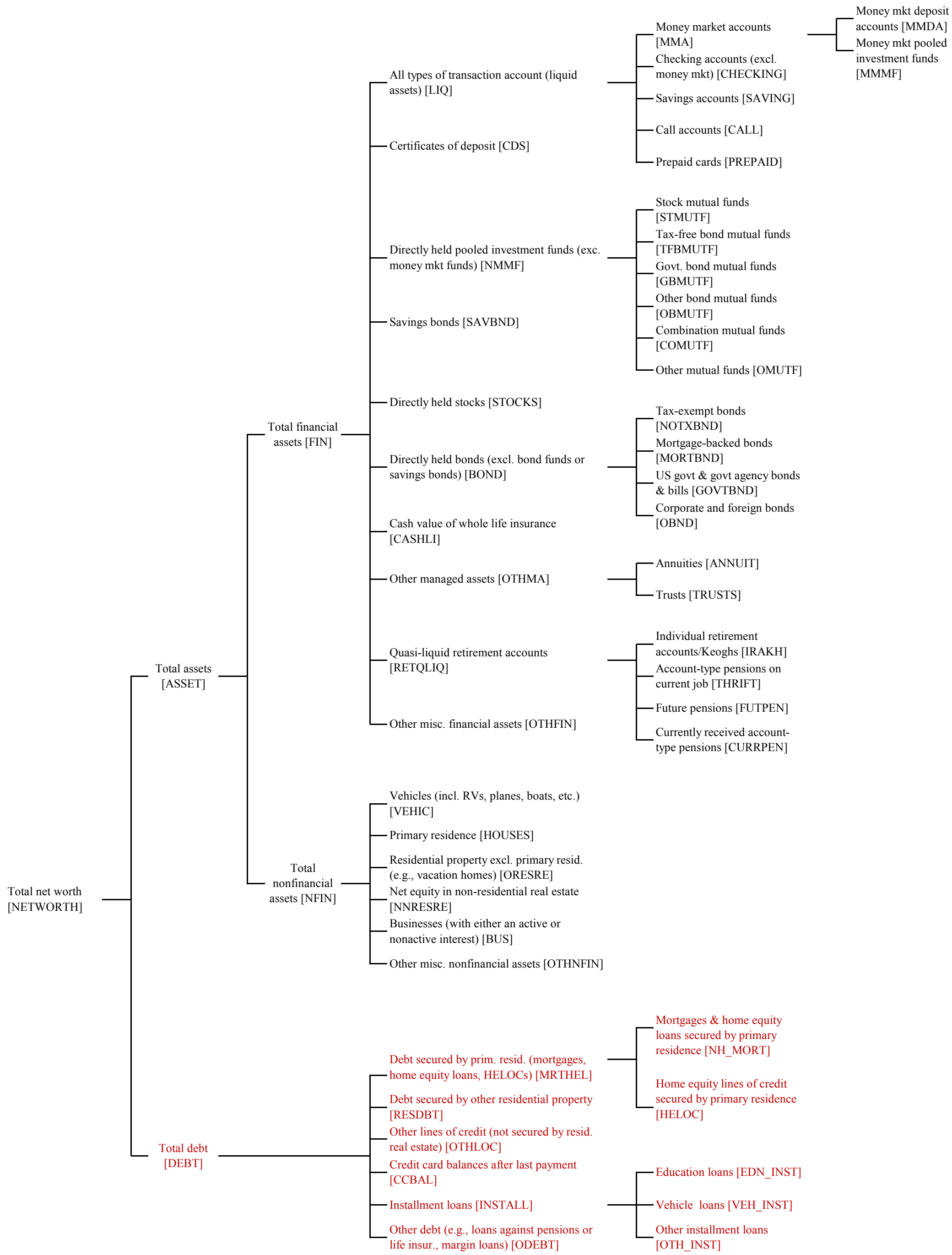
I base my wealth variable construction on Alliprantis, Carroll, and Young (2019) (ACY), who also use the Survey of Consumer Finances (SCF) to discuss the Racial Wealth Gap. The SCF surveys a nationally representative sample of U.S. households, with an over sample of households with the highest income to better capture the upper tail of the wealth distribution, every three years from 1983 to 2016. The survey contains detailed information about household wealth and income and the variable that ACY uses to reflect wealth is household net worth, the value of all a household's assets minus the value of all its debt. The SCF provides race information about the head of households and this is how ACY delineates household race. For 1962, ACY uses a precursor to the SCF, the Survey of Financial Characteristics of Consumers (SFCC). Unfortunately, there is no data available after 1962 and before 1983. ACY limits its sample to households with heads between the ages of 20 and 100 and converts wealth data to 2016 dollars using the St. Louis Fed's GDP Implicit Price Deflator. I follow ACY's methodology and reproduce their findings about the mean racial wealth gap from 1962 to 2016 (Figure 2). There are minor discrepancies between my results and the authors' due to slightly different variable constructions (they do not specify precisely how they construct their variables and I try to match them as closely as possible).

For the 1983 and 1986 SCF, the specific variables used for household net worth is b3324 and c1457 respectively. These two variables are equivalent and are comprised of total assets minus total debt. Total assets is the sum of paper assets -the sum of stocks and mutual funds, bonds, checking and savings accounts, IRA and Keogh accounts, money market accounts and CDs, profit sharing and thrift accounts, cash value of life insurance, and other financial assets - and real assets - the sum of the current market value of the home, other properties, businesses, and vehicles. Total debt is the sum of total real estate debt and total consumer debt. The weight used in 1983 is the Extended Income FRB weight (b3016), which is the recommended full sample weight. The weight used in 1986 is FRB 1986 Weight #2 (c1014), which is the recommended weight for viewing the 1986 households, who were re-interviewed from 1983, as a cross section of the 1986 population. These weights are used for all calculations (SCF 1983 Code book, SCF 1986 Code book).

For 1989-2016, the data I use specifically comes from the the SCF Bulletin extract data. Figure 3 depicts the content of the net worth variable in these data sets. I modify this variable slightly by subtracting out future pensions and currently received account type pensions to bring this variable in line with the net worth variable in 1983 and 1986 that does not include this pension information. There is only one weight variable in these data sets called wgt and it is used for all calculations.

The 1962 SCFF is the most complicated data set to deal with. First, the SFCC only distinguishes race by white, non-white, and not-ascertained. As such, I follow ACY's assumption that non-white aligns with the black delineation in the SCF (the percentage of non-white households in the SFCC is close to the percentage of black Americans in the U.S. in 1962) and that not-ascertained aligns with the white delineation in the SCF (some white ethnic groups like Jews and Italians were likely delineated as not-ascertained in 1962). The next complication is that the SCFF does not provide an aggregate wealth variable. As such, I construct it from the component parts of wealth to match the net worth variable I use for the SCF. First, I add up all the assets: checking accounts (v5-v7), U.S. savings bonds (v8-v10), U.S. government bills (v11), U.S. government notes (v12), U.S. government certificates (v13), non savings U.S. government bonds (v14-v18), state and local bonds (v19), bonds, debentures, and notes for foreign corporations or governments (v20), bonds, debentures, and notes for domestic corporations (v21), mortgage assets (v22-v24), loans to business without active interest (v25), non-mortgage loan assets to individuals (v26-v28), amount paid into annuities (v29-v31), estates in probate (v32), cash surrender value for non-term life-insurance (v39-v41), balance in savings accounts (v45-v69), market value of stock (v70-v74), market value of business with active interest (v80-v84), loans to business with active interest (v85-v88), share of undistributed profits in closely held corporation (v89-v90), market value of business without active interest (v91-v95), loans to business without active interest (v96-v100), amount that can be withdraw from profit sharing plans (v101), amount that can be withdrawn from retirement plans (v102), market value of real estate (v103,v105,v107), credit in brokerage accounts (v109), market value of automobiles (v111), oil royalties, patents, and commodity contracts (v112-v114), assets held in trusts (v176). Then, from all the assets, I subtract all the debts: loans outstanding secured by life insurance polices (v42-v44), real estate debt (v104,v106,v108), debit in brokerage accounts(v110), loans secured by stock (v115), loans secured by bonds (v116), installment debt (v117-v120), non installment debt (v121-v124). Lastly, the weight variable is v4, which is used for all calculations.

Figure 3: Construction of Net Worth in SCF 1989-2016



*Names in brackets refer to variables in the SCF Bulletin extract data
For precise variable definitions, please see the documentation and programs on the SCF website.

This flow chart is provided by the 1989-2016 SCF along with the data.
See https://www.federalreserve.gov/econres/scfindex.htm

B Formal Proofs

Claim 1: The capital stock converges to a unique steady state K_*.

Proof. I adapt a proof presented in Acemoglu (2009). Recall that the dynamics of the capital stock are governed by equation 17:

$$K_{t+1} = \frac{\beta + \rho\beta}{\beta + \rho\beta + 1}\left(\frac{\rho\beta}{\beta + \rho\beta}(1 + N^{1-\alpha-\gamma}L^{\gamma}\alpha K_t^{\alpha-1} - \delta)K_t + N^{1-\alpha-\gamma}L^{\gamma}(1-\alpha)K_t^{\alpha}\right).$$

I will first prove the existence and uniqueness of the steady state. From the above equation it is clear that $K_{t+1} = f(K_t)$ where f is a differentiable function with the properties that $\lim_{x\to\infty} f'(x) < 1$ and $\lim_{x\to 0} f'(x) = \infty$. Now, note that for positive x

$$f(x) = x \iff \frac{f(x)}{x} = 1.$$

Using L'Hopital's rule,

$$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} f'(x) = \infty$$
$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} f'(x) < 1.$$

So, there is some value $x_1 > 0$ such that $\frac{f(x_1)}{x_1} > 1$ and a value $x_2 > 0$ such that $\frac{f(x_2)}{x_2} < 1$. Now, because $\frac{f(x)}{x}$ is continuous for all $x > 0$, the intermediate value theorem guarantees there is some $x_1 < K_* < x_2$ such that $\frac{f(K_*)}{K_*} = 1$. Regarding the uniqueness of K_*, note that $\frac{f(x)}{x}$ is a decreasing function. So, for all $x > K_*$, $\frac{f(x)}{x} < 1$ and for all $x < K_*$, $\frac{f(x)}{x} > 1$. Thus K_* is the unique steady state of f that is strictly greater than 0. To see that K_t must converge to K_*, first note that $f'(x) > 0$ for all x. Then, for all $0 < K_t < K_*$,

$$K_{t+1} - K_* = f(K_t) - f(K_*) = -\int_{K_t}^{K_*} f'(x)dx < 0 \implies K_{t+1} < K_*.$$

Further,

$$\frac{K_{t+1} - K_t}{K_t} = \frac{f(K_t) - K_t}{K_t} = \frac{f(K_t)}{K_t} - 1 > 0 \implies K_{t+1} > K_t.$$

Similarly, for all $K_t > K_*$,

$$K_{t+1} - K_* = f(K_t) - f(K_*) = \int_{K_t}^{K_*} f'(x)dx > 0 \implies K_{t+1} > K_*$$

and

$$\frac{K_{t+1} - K_t}{K_t} = \frac{f(K_t) - K_t}{K_t} = \frac{f(K_t)}{K_t} - 1 < 0 \implies K_{t+1} < K_t.$$

In the baseline model, the initial capital stock is exogenously set to some positive number so $K_1 > 0$. The previous work shows that if $K_1 < K_*$, then K_t is an increasing sequence bounded above by K_* and if $K_1 > K_*$, it is a decreasing sequence bounded below by K_*. In either case K_t is a bounded monotone sequence and so must converge to some $K > 0$. Thus, applying the continuity of f,

$$f(K) = \lim_{t\to\infty} f(K_t) = \lim_{t\to\infty} K_{t+1} = K.$$

Thus K is a steady state greater than 0 so $K = K_*$. □

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Claim 2: For $i \in \{R, P\}$, b_t^i converges to a steady state

$$b_*^i = \frac{\frac{\rho\beta(1+r_*)}{\beta+\rho\beta+\zeta\beta+1}(lW_* + \phi_i \frac{P_*}{100})}{1 - \frac{\rho\beta(1+r_*)}{\beta+\rho\beta+\zeta\beta+1}(1 + \frac{\zeta P_*}{100\rho v_*} + \frac{\zeta}{\rho(1+r_*)})} > 0.$$

Proof. Recall that the dynamics of b_t^i are governed by equation 19:

$$b_{t+1}^i = \frac{\rho\beta(1 + r_{t+1})}{\beta + \rho\beta + \zeta\beta + 1}(1 + \frac{\zeta P_t}{100\rho v_t} + \frac{v_{t+1}\zeta}{(1 + r_{t+1})\rho v_t})b_t^i + \frac{\rho\beta(1 + r_{t+1})}{\beta + \rho\beta + \zeta\beta + 1}(lW_t + \frac{\phi_i P_t}{100}),$$

which has the form of a sequence $b_{t+1} = a_t b_t + c_t$ where a_t converges to some $a > 0$ and c_t converges to some $c > 0$. The proof of this claim amounts to demonstrating that b_t converges to $\frac{c}{1-a} > 0$. First note that for all $n \in \mathbb{N}$, there exists some T_n such that for all $t \geq T_n$, $a - \frac{1}{n} < a_t <$

$a + \frac{1}{n}$ and $c - \frac{1}{n} < c_t < c + \frac{1}{n}$. So, for all $t \geq T_n$,

$$(a - \frac{1}{n})b_t + (c - \frac{1}{n}) < b_{t+1} < (a + \frac{1}{n})b_t + (c + \frac{1}{n}).$$

Now, let (b_t^n) be the sequence (b_t) after truncating off the first $T_n - 1$ terms, let (U_t^n) be the sequence given by $U_0^n = b_{T_n}$ and $U_{t+1}^n = (a + \frac{1}{n})U_t^n + (c + \frac{1}{n})$, and let (L_t^n) be the sequence given by $L_0^n = b_{T_n}$ and $L_{t+1}^n = (a - \frac{1}{n})L_t^n + (c - \frac{1}{n})$. It is clear from induction that $L_t^n \leq b_t^n \leq U_t^n$ for all t. Now, suppose for contradiction that $a > 1$. Choose N such that $a - \frac{1}{N} > 1$ and $c - \frac{1}{N} > 0$. So,

$$L_{t+1}^N > (a - \frac{1}{N})L_t^N > (a - \frac{1}{N})^t L_0^N \to \infty \implies b_t^N \to \infty \implies b_t \to \infty.$$

In the specific context of the model, b_t diverging to infinity means that b_t^R and b_t^P diverge to infinity, which implies that $B_t = q^R H b_t^R + q^P H b_t^P$ diverges to infinity. However, equation 16 shows that $B_t = \frac{\rho\beta}{\rho + \rho\beta}(1 + r_t)K_t$, which converges to $\frac{\rho\beta}{\rho+\rho\beta}(1 + r_*)K_*$, which is a contradiction. Next, suppose for contradiction that $a = 1$. For all n,

$$L_t^n = (1 - \frac{1}{n})^t L_0^n + (c - \frac{1}{n})((1 - \frac{1}{n})^{t-1} + ... + (1 - \frac{1}{n})^0).$$

The first term here will converge to 0 and the second term is the geometric series

$$\sum_{t=1}^{\infty}(c - \frac{1}{n})(1 - \frac{1}{n})^{t-1} = \sum_{t=0}^{\infty}(c - \frac{1}{n})(1 - \frac{1}{n})^t = \frac{c - \frac{1}{n}}{\frac{1}{n}} = nc - 1.$$

So, $b_t^n \geq L_t^n$, which converges to $nc - 1$. Now, given some $m > 0$, there exists an N such $Nc - 1 > m + 1$. Then, there exists a T such that $t > T \implies L_t^N > m \implies b_t^N > m$. Thus, if $a = 1$, b_t diverges to infinity, which as previously shown is a contradiction. So, $a < 1$. Now choose N_0 such that $(a + \frac{1}{N_0}) < 1$ and $(a - \frac{1}{N_0}) > 0$. For all $n > N_0$,

$$U_t^n = (a + \frac{1}{n})^t U_0^n + (c + \frac{1}{n})((a + \frac{1}{n})^{t-1} + ... + (a + \frac{1}{n}^0)) \to \frac{c + \frac{1}{n}}{1 - a - \frac{1}{n}}$$

and

$$L_t^n = (a - \frac{1}{n})^t U_0^n + (c - \frac{1}{n})((a - \frac{1}{n})^{t-1} + ... + (a - \frac{1}{n}^0)) \to \frac{c - \frac{1}{n}}{1 - a + \frac{1}{n}}.$$

Finally, take $\varepsilon > 0$. Choose $N_1 > N_0$ such that

$$\frac{c + \frac{1}{N_1}}{1 - a + \frac{1}{N_1}} < \frac{c}{1-a} + \frac{\varepsilon}{2} \text{ and } \frac{c - \frac{1}{N_1}}{1 - a - \frac{1}{N_1}} > \frac{c}{1-a} - \frac{\varepsilon}{2}.$$

Then, choose T such that

$$t > T \implies U_t^{N_1} < \frac{c + \frac{1}{N_1}}{1 - a - \frac{1}{N_1}} + \frac{\varepsilon}{2} \text{ and } L_t^{N_1} > \frac{c - \frac{1}{N_1}}{1 - a + \frac{1}{N_1}} - \frac{\varepsilon}{2}.$$

Thus,

$$t > T \implies \frac{c}{1-a} - \varepsilon < \frac{c - \frac{1}{N_1}}{1 - a + \frac{1}{N_1}} - \frac{\varepsilon}{2} < L_t^{N_1} < b_t^{N_1} < U_t^{N_1} < \frac{c + \frac{1}{N_1}}{1 - a - \frac{1}{N_1}} + \frac{\varepsilon}{2} < \frac{c}{1-a} + \varepsilon.$$

So, it is clear that b_t converges to $\frac{c}{1-a} > 0$ as required. $\square$