Measuring Interdependence of Inflation Uncertainty

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Research question

How to measure the strength of the interdependence of inflation uncertainty?

- We estimate inflation uncertainty by *ex post* forecast errors and the interdependence of uncertainty by a probability model.
- We show a potential endogeneity bias of the probability model estimates and propose a new empirical framework exploiting heteroskedasticity in the data.

Measuring inflation uncertainty

Inflation uncertainty is measured by *ex post* forecast errors from a bivariate VAR BEKK GARCH (1,1) model using inflation of the UK and the euro area (Jan 1997-March 2016).

$$U_{t,h} = \Sigma_{t,h}^{1/2}\Sigma_{t|t-h}^{-1/2}e_{t|t-h} = \Sigma_{t,h}^{1/2}\Sigma_{t|t-h}^{-1/2}(\pi_t - \pi_{t|t-h}) \quad (1)$$

- $e_{t|t-h}$: the h-period ahead forecasts erros made at time $t-h$.
- $\Sigma_{t,h}$ and $\Sigma_{t|t-h}$: the variance-covariance matrix of e_t and $e_{t|t-h}$.

Measuring independence by a probability model: Part I

Marginal density functions

- Two Piece Normal (TPN) distribution [3]

$$f_{TPN}(x;\sigma_1,\sigma_2,\mu) = \begin{cases} Aexp\{-(x-\mu)^2/2\sigma_1^2\} & \text{if } x \le \mu \\ Aexp\{-(x-\mu)^2/2\sigma_2^2\} & \text{if } x > \mu \end{cases} \quad (2)$$

where $A = (\sqrt{2\pi}(\sigma_1+\sigma_2)/2)^{-1}$.

- Weighted Skew Normal (WSN) distribution [1]

$$U = \underbrace{X}_{\text{baseline forecast error}} + \underbrace{\alpha \cdot Y \cdot I_{Y>m} + \beta \cdot Y \cdot I_{Y<k}}_{\text{Signal part based on revised forecast error}} \quad (3)$$

where X and Y are bivariate $N(0,\sigma^2)$ with correlation coefficient, ρ.

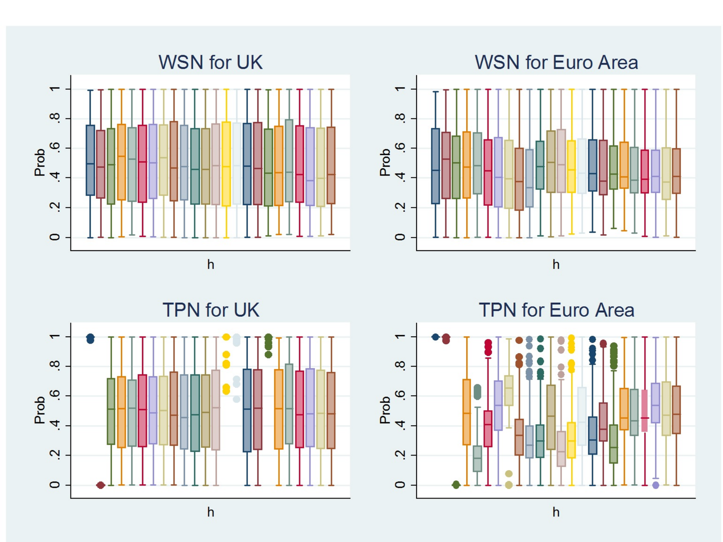

Figure 1. Box plot of probability integral transformation

- Minimum distance statistics, graphical diagnostics of *pit's*, and goodness-of-fit tests support the choice of WSN against TPN for both the UK and the euro area.

Measuring independence by a probability model: Part II

Conditional density function (copulas)

$$\hat{\gamma} = \arg\max_{\gamma} \sum_{t=1}^{T} \ln\left(c(F_1(U_1;\hat{\theta}_1), F_2(U_2;\hat{\theta}_2);\gamma)\right) \quad (4)$$

- γ: copula parameter; $\hat{\theta}_1, \hat{\theta}_2$: marginal densities estimanted by the simulated minimum distance.
- The *pdf* of Frank copula:

$$c(y_1,y_2;\gamma) = \frac{-\gamma(e^{-\gamma}-1)e^{-\gamma(y_1+y_2)}}{((e^{-\gamma y_1}-1)(e^{-\gamma y_2}-1)+(e^{\gamma}-1))^2} \quad (5)$$

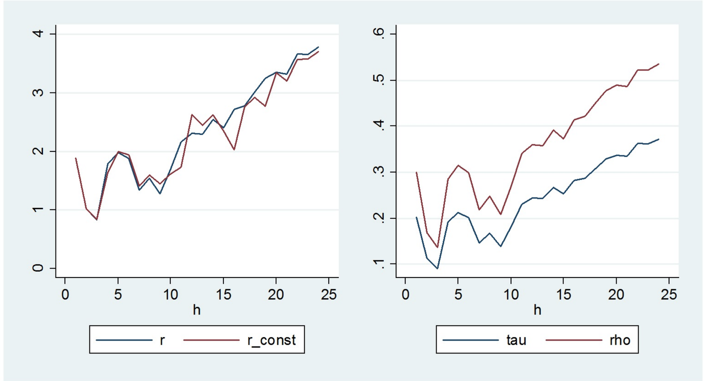

Figure 2. Copula parameters and rank correlation: same horizon

Endogenous model of interdependence

To illustrate a potential bias, an endogenous model of interdependence is assumed as in [2].

$$A\begin{bmatrix}U_1\\U_2\end{bmatrix} = \begin{bmatrix}\eta\\\varepsilon\end{bmatrix}, \text{with } A = \begin{bmatrix}1 & -\alpha\\-\beta & 1\end{bmatrix} \text{ and } \Omega = \begin{bmatrix}V_1 & C_{12}\\C_{12} & V_2\end{bmatrix} \quad (6)$$

- U_1 and U_2: inflation uncertainty of the UK and the euro area.
- η and ε: structural shocks, independent Normal distribution; Ω: var-cov matrix of $[U_1U_2]'$.
- **α and β: the coefficients capturing interdependence of uncertainty.**

Reduced form → a potential bias if endogeneity is not properly addressed in the estimation.

$$U_1 = \frac{1}{(1-\alpha\beta)}(\eta+\alpha\varepsilon),\ U_2 = \frac{1}{(1-\alpha\beta)}(\beta\eta+\varepsilon) \quad (7)$$

$$\Omega = \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix}\alpha^2\sigma_\varepsilon^2+\sigma_\eta^2 & \alpha\sigma_\varepsilon^2+\beta\sigma_\eta^2\\ \alpha\sigma_\varepsilon^2+\beta\sigma_\eta^2 & \sigma_\varepsilon^2+\beta^2\sigma_\eta^2\end{bmatrix} \quad (8)$$

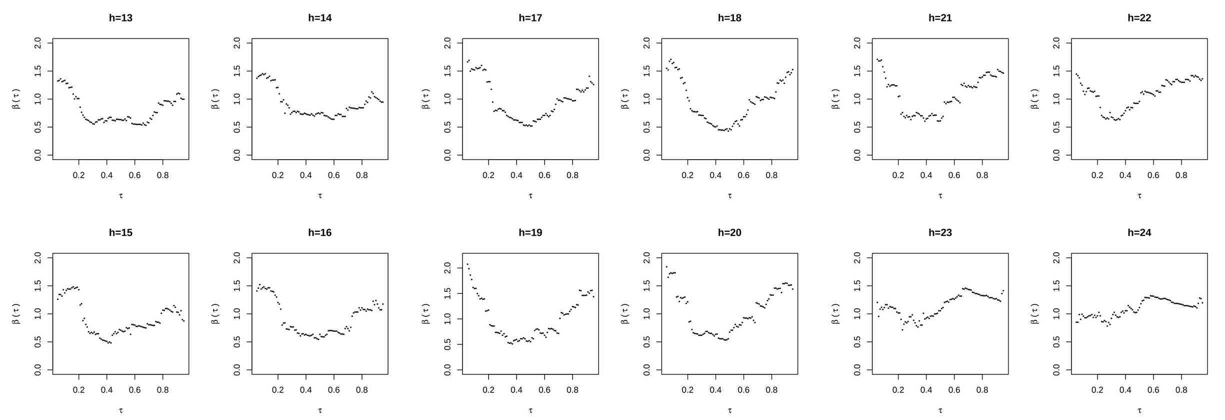

Figure 3. The estimated slope coefficients (β) of linear quantile regressions

Measuring independence by identification through heteroskedasticity

The variance-covariance matrix of the structural model:

$$A\Omega A^T == \begin{bmatrix} \cdot & -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta)\\ -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) & \cdot \end{bmatrix} \quad (9)$$

Optimization problem: the off-diagonal terms in Equation (11) need to be equal to zero.

$$\min_{\alpha,\beta} f(\alpha,\beta;V_1,V_2,C_{12}) = -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) \quad (10)$$

Impose additional assumptions: the parameters in A are stable over time; heteroskedasticity.

Define two regimes: RH if $\theta > median(\theta)$ and RL otherwise. θ is computed using different sample periods (p), forecast horizons (h), and rolling windows (rw).

$$\theta = \frac{V_1}{V_2} = m(p;h,rw) \quad (11)$$

- $p \in \{1,2,3\}$ with 1: pre-crisis period, 2: the Global Financial Crisis period, 3: post-crisis period. $h = 1,2,\ldots,24$ and $rw = 12$.
- The minimum distance estimates of α and β using V_1, V_2, and C_{12} for each regime.

Figure 4. Interdependece of inflation uncertainty: identification through heteroskedasticity

Main findings

- Crisis period:β exceeds 1 for longer term horizons → amplifying effects of the surprises in the UK inflation on the euro area inflation.
- Pre- and post-crisis period: the range of the estimates lies [-1, 1], mostly close to zero → interdependence is statistically insignificant.

Conclusions

- **Probability model** The simultaneous spillover of inflation uncertainty is stronger for uncertainty about distant future than near future.
- **Endogenous model** The strength of the propagation of inflation uncertainty intensifies during the GFS period while the interdependence dampens during the post-crisis period.

References

[1] W. Charemza, C. Díaz, and S. Makarova. Conditional term structure of inflation forecast uncertainty: The copula approach. *Romanian Journal of Economic Forecasting*, 22(1):5–18, 2019.

[2] R. Rigobon. Contagion, spillover, and interdependence. *Economia*, 19(2):69–100, 2019.

[3] K. F. Wallis. An assessment of bank of england and national institute inflation forecast uncertainties. *National Institute Economic Review*, 189(1):64–71, 2004.