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Sustainable Development Goals as a problem in the change of techniques

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Abstract

The paper explores some important theoretical issues that are relevant for the realisation of the *Social Development Goals*. The perspective is mainly "classical" and post-Keynesian, but reference is also made to other approaches. The focus is put on the dynamic problem of technological change and on the connected relative price variations, particularly the wage-profit distribution. Looking at the problem in that way should allow a better conceptual understanding of the main parameters at stake in the implementation of SDG and therefore facilitate a pragmatic evaluation of their economic implications and of the realistic choices that are open for action.

(*) European Investment Bank, opinions expressed are personal. Version dated 31.12.2021. The paper is dedicated to the memory of Eugenia Correa, who accepted and commented on a first draft in a session she organized on "Financialization processes in emerging and developing economies" at the 16th International Conference Developments in Economic Theory and Policy. Bilbao (Spain) on 27th-28th June 2019.

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Introduction

The 17 Sustainable Development Goals are part of the UN 2030 Agenda for Sustainable Development defined in Addis Ababa (UN 2015), further adopted by world leaders in 2015 and entered into force on 1st January 2016. These overarching objectives are translated into some 169 targets and 230 indicators.

Figure 1: UN Sustainable Development Goals

They cover: i) no poverty; ii) zero hunger; iii) good health and well-being; iv) quality education; v) gender equality; vi) clean water and sanitation; vii) affordable and clean energy; viii) decent work and economic growth; ix) industry innovation and infrastructure; x) reduced inequalities; xi) sustainable cities and communities; xii) responsible consumption and production; xiii) climate action; xiv) life below water; xv) Life and land; xvi) peace justice and strong institutions; xvii) partnerships for the goals.

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Reaching these goals requires a rather sizeable increase in investment in goods and services. The current estimates of the investments necessary for the realisation of the SDG vary between 20% and 80% of 2019 world GDP *at market prices* (cumulated over the 10 years of their realisation). Actual cost could be higher, of the order of 125% of 2019 world GDP (Cingolani, 2021a).

SDG are expected to be financed essentially by the private sector, despite they have largely a public good element. Hence their financial attractiveness should be discussed independently from possible public support. Looking at the investment process with post-Keynesian critical glasses, it appears unlikely that SDG could be realised on time if financed by the private sector alone without some substantial public support, even though their cost represents only about a fifth of the estimated world's private wealth (Cingolani, 2021a).

The SDG are an ambitious attempt to transform systemically the world's economic production structure from one that is harmful to environment and that is increasingly failing on economic and social cohesion to a new one that is more compatible with the goals. In this transition, which also requires a transition of the relative prices of all goods and services produced at world level and therefore present substantial challenges in terms of international cooperation and world governance, the profitability of SDG related investments for the private sector will evolve, together with the overall macro-distribution between profits and wages. SDG thus imply a technological transformation with massive distributional impacts. It is almost natural to look at this problem with post-Keynesian glasses because in this approach technology determines the structure of relative prices[2], contrary to the neoclassical approach, where the causality is reversed[3]. This paper investigates what, if any, would be the policy prescriptions for the *Sustainable Development Goals* (SDG) from a classical/post-Keynesian viewpoint. The aim is to show that applying the classical approach to the SDG issue brings fertile and interesting results, as well as interesting and important policy relevant questions that sometimes cannot be even asked in other approaches.

Section 1 below, gives some background on the neo-classical and the classical/post-Keynesian paradigms. The technology-distribution nexus is discussed in the second section, respectively against the background of the static Leontief model in the first sub-section, and against the Sraffian interpretation of the Leontief model, in the following one. Dynamics are introduced in the third section, with the help of Pasinetti's structural sectoral dynamics. The framework discussed is applied to discuss the question of the choice and changes of techniques in the fourth section.

Finally, given the public good nature of most SDG, the public sector must also be mobilised in their realisation as they would tend otherwise to be underprovided by the market. A

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[2] It is fair to read Pasinetti ([1975] 1977) as notably meaning that in a multi-sectoral economy made of single products, i.e. where each sector produces a single output, technology sets relative prices, which hence do not depend on demand. The issue is less clear-cut when joint products are considered. This is in a single period, as discussed below, in subsequent periods (continuation analysis) the model dynamics generate interactions between technology and price changes that change the overall productive structure.

[3] In a sense, if one looks at the growth theory discussions of the sixties, one could say that in the Kaldor-Pasinetti model, distribution is endogenous as it adapts to the capital-labour ratio of factor quantities, while in Solow's growth model the price of production factors is given, and the capital-labour ratio adapts to it.

practical question is thus how much public support should be provided and to what projects, which is discussed in the fifth section below, where the discussion on technology and relative prices is extended to the concept of accounting or shadow prices.

1. The classical and the neoclassical paradigms: implications for SDG

In the 1960 a controversy on capital theory developed between the "two Cambridges" in Massachusetts and in England, to which participated top economists such as Paul Samuelson, Robert Solow, Franco Modigliani, Joan Robinson, Richard Kahn, Nicholas Kaldor and, indirectly, Piero Sraffa, through his former students Pierangelo Garegnani and Luigi Pasinetti.

This was a notable instance in which the "neoclassical" mainstream honestly recognized its intellectual defeat against the alternative "classical" paradigm[4] inspired by the post-Keynesian approach and particularly by Sraffa's (1960) "classical" reconstruction of economic theory (Samuelson, 1966). However, while conceding the American side was wrong on the technical issue at stake (the so called "re-switching of techniques"), Samuelson also hinted that this issue was essentially irrelevant for practical purposes. Further on the question was essentially forgotten (see Pasinetti, 2000), if not "repressed"[5], and, perhaps in part for this reason, the mainstream approach remains largely hegemonic still today.

Nonetheless, in the last 60 years the Cambridge-UK alternative approach was substantially developed in various complementary directions. Although it has not reached the coverage and sophistication of the neo-classical paradigm, today it is certainly not anymore: "A prelude to a critique of economic theory". This can be confirmed by reading a few recent "foundational" alternative textbooks such as Petri (2021), Shaikh (2020), Lavoie (2014), Bellino (2021) and several others[6] who show a considerable degree of sophistication in effectively mixing their criticisms of the mainstream's weaknesses with the constructive development of alternatives explanatory frameworks based on different assumptions and implying different different causality chains. While the authors of these manuals do not agree on all their theories and their practical implications, there is a growing awareness that, putting all their arguments together, neoclassical policy prescriptions appear to have mostly a limited validity in a wide range of fields [7]. While there is no convergence of views on the details of alternative policy prescriptions on concrete problems, the group of hypotheses taken as postulate by these heterodox authors seem relevant for the analysis of real-world problems. If there is a suitable policy (be it "optimal", "second best" or "satisfactory in terms of limited rationality"), it should be modelled starting from such hypotheses.

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[4] In the "reproduction" sense. The word is used with reference to the distinction between the classical approach based on production (or reproduction) prices, and the neoclassical approach, based on the exchange paradigm where prices are scarcity indexes in a framework where resources are limited. See for instance Bharadwaj (1986) or Pasinetti (1986).
[5] In Freudian terms: if the psychoanalytic concepts could be applied to collective persons, one would say that the consequences of the re-switching were pushed back from the economists' "Superego" to "Das Es".
[6] Roncaglia (2005, 2019), who follows the classical approach, wrote the first two volumes of a major and very learned synthesis on the history of economic thought (see also Roncaglia & Tonveronachi, 2014). Pasinetti is about to publish a book entitled "A theory of Value" and a postumous volume of Garegnani is in preparation by some of his former students.
[7] Cingolani (2022), argues that the cases to which the neo-classical prescriptions properly apply cover presumably less than 10% of the cases possible *ex ante*.

2. Leontief's model: neo-classical and Sraffian applications to environment

The input-output model can be read both with Walrasian or classical/psot-Keynesian glasses[8]. Leontief himself kept the ambiguity on the subject. While declaring: "We are dealing here essentially with attempted application of the economic theory of general equilibrium to empirical quantitative analysis of the concrete national economy" (Leontief, [1941] 1951, p. 202) and making similar statements in Leontief (1966), later, when asked by Christian de Bresson (2004, p. 138) whether he was Walrasian or classical, he replied: "Not interesting. I have my own system".

Miller & Blair (2009), present the input-output model from a neoclassical perspective; they attribute to Leontief (1970) the merit of having first applied it the to environmental problems[9]. An example of this type of application is presented in section 2.1 below.

Pasinetti (1977) shows that the Sraffian model is intimately related to the original input-output model of Leontief. In section 2.2 below, an example is developed in which this interpretation, which puts emphasis on distribution, is used to build an original application to environment.

The common structure of Leontief's model, which applies to both examples, focuses on a single period and is therefore static. This can also be the starting point for continuation analysis and it is also relevant for the discussion of Pasinetti's structural dynamics discussed in the third and fourth sections below. This common structure can be briefly reminded here. Assuming n commodities including labour, the "closed" Leontief systems can be written, respectively for quantities and for prices, as:

$$\begin{cases} (I-A)q = 0 \\ p^t(I-A) = 0 \end{cases} \qquad (1)$$

where A is a nxn square technology matrix with elements $a_{ij,}$ i, j=1,...n-1, giving the amount of commodity i necessary to produce a unit of commodity j. In the closed model the last row of A is made of the coefficients a_{nj} j=1...n, giving the quantity of labour necessary for the production of commodity j (labour coefficients), while the last column with elements c_{in}, i=1,n, provides the final demand coefficients giving the quantity of product i demanded by each worker. The vector giving the total quantity of all commodities produced is q and its last element is the labour force (or population), while p_t is the row vector of prices, in which the last element is the wage rate.

In the open Leontief system final demand is made exogenous and therefore matrix A becomes (n-1)x(n-1). The system for quantities becomes:

[8] As emphasized by Akhabbar and Lallement (2011) there is no doubt that input-output was presented in the beginning as an empirical concrete application of Walras' model. The classical roots and implications of the input-output model have been substantially developed by Kurz and Salvadori (2000). See also the already quoted recent textbooks of Petri (2021) and Bellino (2022) as well the classic Pasinetti ([1977] 1980).

[9] Lager (1998) also drew attention to Isard (1968) as well as Ayres and Kneese (1969).

$$\begin{cases}(I-A)q = y \\ a_n q = LF\end{cases} \quad (2)$$

where q is now a 1x(n-1) row vector, giving total output produced of each commodity except labour, a_n is the vector of labour coefficients corresponding to the last row of the previous A matrix for the closed system and y is the (n-1)x1 column vector of final demand.

As noted by Pasinetti (1977, p. 61, the open system is used mainly for quantities. Its price version is the same as that of n the closed model: $p^t(I-A)=0,$ where A is the open (n-1)x(n-1) technology matrix ad p is a (n-1)x1 vector of prices excluding wages.

2.1. Neo-classical reading of Leontief quantity model (Steenge 2004)

Steenge (2004) introduces environment in the open Leontief quantity model in the following way:

$$\begin{bmatrix} I-A_{11} & -A_{12} \\ A_{21} & -I+A_{22}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}c_1\\c_2\end{bmatrix} \quad (3)$$

where: A_{11} is the Leontief technology matrix, A_{21} is a satellite matrix of emission coefficients, A_{12} is the matrix of input coefficients of the abatement industries, A_{22} is the matrix of output of pollutants per unit of eliminated pollutant, x_1 is the vector of total outputs of traditional goods, x_2 is the vector of total of pollutants being abated, c_1 if the vector of final consumption of the traditional goods, c_2 is the vector of tolerated levels of pollutants.

The corresponding price equation is:

$$[p_1' \quad p_2']\begin{bmatrix} I-A_{11} & -A_{12} \\ Q_{21} & I+Q_{22}\end{bmatrix} = [v_1' \quad v_2'] \quad (4)$$

where v_1 is the vector of direct labour input coefficients, v_2 the vector of direct labour input coefficients for the abatement activities, p_1 and p_2 represent respectively the prices of the conventional goods and the prices of the pollutants being eliminated, Q_{21} and Q_{22} are expressions for the proportion of each pollutant "eliminated at the expense of the originating industry". As noted by Stenge (2004, p. 372-373), the Leontief model provides information on the technical side (the quantity of polluting substances produced and the abated technologies that are available) and on the cost allocation mechanism. As a result, behavioural and policy aspects are introduced, and the price system (2) is no longer the dual of (1).

The quantity model implies:

$$\begin{cases}x_1 = A_{11}x_1 + A_{12}x_2 + c_1 \\ x_2 = A_{21}x_1 + A_{22}x_2 - c_2\end{cases} \quad (5)$$

If one assumes identical abatement rates and retains the polluter pays principle that polluting industries pay for the cost of abatement, the tolerated emissions can be neglected and the vector of quantities to be abated becomes:

$$x_2 = (A_{21}x_1 + A_{22}x_2)$$

so that model (1) is now given by (5):

$$\begin{bmatrix} I - A_{11} & -A_{12} \\ -\alpha A_{21} & -I + \alpha A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ 0 \end{bmatrix} \quad (6)$$

or in system form, putting A_{22}=0 and adding the labour equation (7) can be derived:

$$\begin{cases} x_1 = A_{11}x_1 + A_{12}x_2 + c_1 \\ x_2 = \alpha A_{21}x_1 \\ L = v'_1 x_1 + v'_2 x_2 \end{cases} \quad (7)$$

From which, eliminating x_2, one gets (8):

$$\begin{cases} x_1 = A_{11}x_1 + \alpha A_{12}A_{21}x_1 + c_1 \\ L = v'_1 x_1 + v'_2 \alpha A_{21}x_1 \end{cases} \quad (8)$$

which, solving for x1 gives (8bis):

$$L = (v'_1 + v'_2 \alpha A_{21})x_1$$

$$x_1 = \frac{L}{(v'_1 + v'_2 \alpha A_{21})} \text{ (8bis)}$$

hence the closed model expression becomes:

$$x_1 = A_+ x_1 \quad (9)$$

where:

$$A_+ = A_{11} + \alpha A_{12}A_{21} + \frac{c_1}{L}(v'_1 + v'_2 \alpha A_{21}) = 1 \quad (10)$$

Steenge (2004, p. 374) develops the example given originally by Leontief where:

Table 19.1 *Leontief tableau with pollution*

	Agriculture	Manufacturing	Households	Total output
Agriculture	25	20	55	100
Manufacturing	14	6	30	50
Labor	80	180	0	260
Pollution	50	10	0	60

Source: reproduced from Steenge (2004)

From which:

$$A_{11} = \begin{bmatrix} 0.25 & 0.40 \\ 0.14 & 0.12 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 0.20 \end{bmatrix} \quad c_1 = \begin{bmatrix} 55 \\ 30 \end{bmatrix} \quad A_{21} = [0.50 \quad 0.20] \quad v'_1 = [0.80 \quad 3.60]$$

with v'_2=2.

He shows that in this example the prices are always the same weather one considers the abatement activities as a new type of primary input or weather one considers them as intermediate inputs (p. 377).

2.2 Sraffian reading of the Leontief price model (Pasinetti, 1977) [10]

Amongst the "classical" interpretation of the input-output model, the Sraffian one makes particularly clear the relation between technological change and distribution. This is shown for instance in the solution of the price variant of the Leontief closed model, which shows that for a given vector of output produced, there is an arbitrage between labour and profits, which can be derived from its reduced form for prices (Pasinetti [1975] 1977, eq. V.5.18 p. 80):

$$p = a_n[I - (1 + \bar{\pi})A]^{-1}w \qquad (11)$$

where p is the vector of relative prices, a_n is the vector of labour coefficients, $\bar{\pi}$ is a rate of profit (common to all sectors) comprised between 0 and the maximum rate of profit allowed by a given technology, A is the technological matrix of input-output coefficients and w the wage rate.

Using relation (11), it is possible to illustrate the technological issues relating to the introduction of environment in a simple Sraffian reinterpretation of the static Leontief price model that does not require to abandon square matrices or introduce new scarce production factors or their remuneration.

Indeed, for any given numéraire, relation (11) above embeds the combination of wages and profits that, for a given output, are consistent with the technology defined by matrix A: it is the so-called wage-profit possibility frontier[11]. In this relation one of the two distributive variables must be set exogenously. Based on relation (1), one can obtain different wage-profit frontiers, one for each *numéraire*, but if A has the necessary stability properties, this relation between wages and profits is always negative (downward sloping) and in general nonlinear[12].

Assuming further that, in line with the classical tradition[13], salaries are paid at the beginning of the production period, relation (11) which is normally used to derive natural prices when π=0, may be used also to derive Marxian production prices, which are condidered to be part of the capital advanced by capitalists at the beginning of the production period and therefore also generate profits. In this case:

$$p[I - (1 + \pi)A] = a_n w \xrightarrow{becomes} p[I - (1 + \pi)A] = a_n w(1 + \pi)$$

Which can be re-written in the form that corresponds to V.A.16 p. 126 of Pasinetti (1977):

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[10] This section benefitted from the precious help and support of Ariel Wirkiermann, who is warmly thanked.

[11] The case considered is that of single goods (joint production is ruled out). Matrix A is assumed to have full rank.

[12] It is solution of a polynomial on degree n-1, where n is the rank of A.

[13] See Graziani (1994) for references and implications. The assumptions and notations correspond to the Appendix to chapter V of Pasinetti (1977).

$$p = (pA + a_n w)(1 + \pi) \qquad (12)$$

The vector w has as components the value of the basket of goods consumed by workers. If d is such a basket, be it subsistence or not, one has w=p*d. If the wage is the numéraire, one has w=1. Substitution into (12) gives:

$$p = p(A + da_n)(1 + \pi)$$

from which:

$$p[I - (1 + \pi)(A + da_n)] = 0 \qquad (13)$$

Where $A^{+} = (A + da_n)$ is the augmented matrix of technical coefficients which includes labour as a commodity in its technical coefficients. One can re-interpret d in relation (13) as the given subsistence wage and consider that it is already included in matrix A like in the closed system, which is now a technical matrix "augmented" for the subsistence wage because labour is included in the reproduction means through its coefficients. When this is the case, relation (2), can be read as illustrating the arbitrage between the "extra-wage", paid above the subsistence means, and profits.

The final step in the argument is to change the interpretation of the vector w=p*d in (3) from that of an extra-wage paid in addition to the subsistence wage to a vector measuring environmental abatement costs that "add" to a given wage included in the technological matrix. To avoid confusions, w in (11) is renamed as ρ, the vector of labour coefficients remains, as before, a_n, the vector d in (3) is renamed as c and represents the basket of consumption goods in the given wage expressed in real terms with pc=1 because the wage basket (or the standard commodity) is the numéraire, and a_{CO^2} is the labour absorbed by the environmental activity. The model based on (1) then becomes:

$$\begin{gathered} p = \rho a_{CO^2}[I - (1 + \pi)(A + ca_n)]^{-1} \\ pc = 1 \end{gathered} \qquad (14)$$

Relation (14) becomes now the expression of the distributional arbitrage between environmental costs and profits, where the former are expressed in terms of quantities of goods dedicated to fight environmental costs, quantities that are lost for consumption and/or for profits. From (14) one derives:

$$pc = \rho \alpha_{CO^2}[I - (1 + \pi)(A + c\alpha_n)]^{-1} c = 1$$

And the frontier is given by:

$$\rho = \frac{1}{\alpha_{CO^2}[I - (1+\pi)(A + c\alpha_n)]^{-1} c} \qquad (15)$$

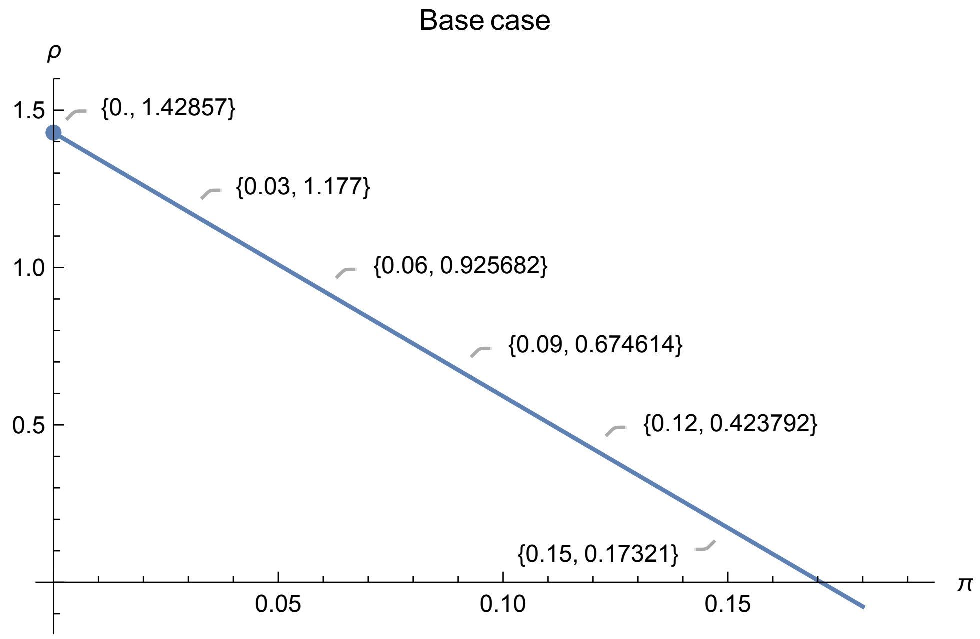
Which is an inverse relationship between ρ and π. An example in two dimensions can illustrate the point.

Let's assume the technical coefficient matrix A, the vectors a_{CO}^2, an and c to be given respectively by:

$$A = \begin{bmatrix} .2 & .5 \\ .4 & .3 \end{bmatrix}, \quad c = \begin{bmatrix} .1 \\ .4 \end{bmatrix} \quad a_{CO^2} = \begin{bmatrix} .2 \\ .2 \end{bmatrix} \quad a_n = \begin{bmatrix} .4 \\ .2 \end{bmatrix} \qquad (16)$$

Then applying relation (15) and fixing π=.15 gives ρ is .173. The Figure below, gives the possibility frontier when the profit rate is changed from 0 to .18:

Figure 2: wage-profit possibility frontier in the base case



With the given technology, a rate of profit of 15% is associated to a maximum sustainable environmental cost of .17, whereas at a rate of 12% the sustainable cost is .42. At the maximum rate of profit the sustainable environmental cost is zero and with zero profits society can devote 1.42 to fight environment.

Now if this example is taken as the base case, one can examine what happens when some of the exogenous parameters change. For example, let's examine first the case when a polluting but labour-saving new technology is introduced. In this case what changes with respect to (16) are the vectors a_{CO}^2 and a_n, which become:

$$a_{CO^2} = \begin{bmatrix} .2 \\ \mathbf{.6} \end{bmatrix} \quad a_n = \begin{bmatrix} .4 \\ \mathbf{.1} \end{bmatrix} \quad (17)$$

One can see in the Figure 3.a below that in this case the frontier rotates to the left, with the result that the maximum wage is reduced, but the maximum profit is increased. The two frontiers intersect at a level of profit around 13-14%.

The second case is that of an increase in the real wage, so that the only change with respect to the base case is in the c vector, which becomes:

$$c = \begin{bmatrix} .3 \\ .4 \end{bmatrix} \quad (18)$$

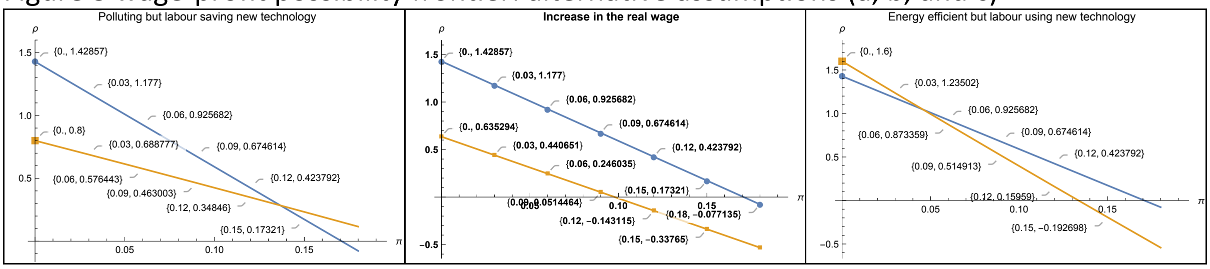
In that case one can see in the Figure 3.b below that in this case the frontier is translated downwards with a reduction of both the maximum profit rate and the maximum level of environmental costs sustainable.

The third case is that of the introduction of an energy efficient but labour using new technology. This means that the vectors that would be changes would be the vectors $a_{CO}{}^2$ and a_n , which become:

$$a_{CO^2} = \begin{bmatrix} .2 \\ .1 \end{bmatrix} \qquad a_n = \begin{bmatrix} .4 \\ .3 \end{bmatrix} \quad (19)$$

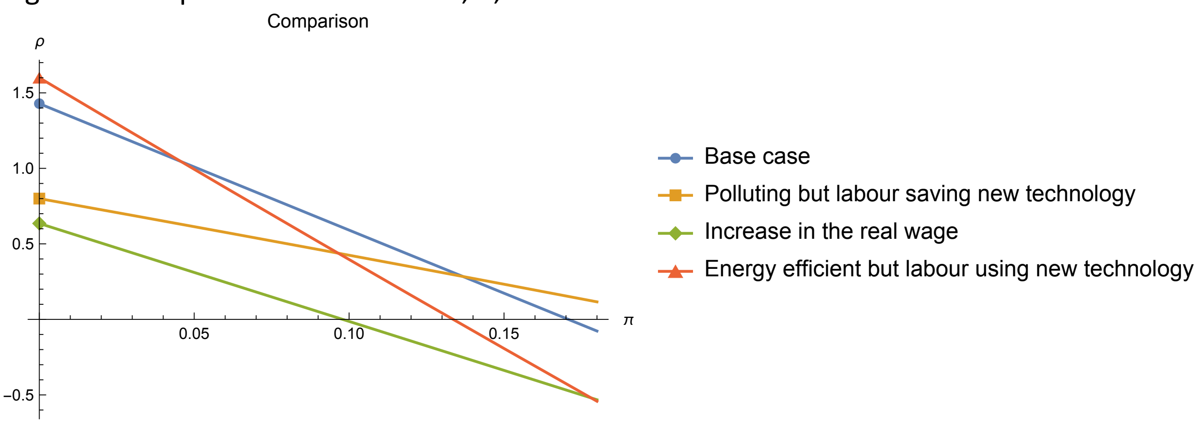
Figure 3.c then shows that in this case the possibility frontier is rotated to the left, so that the maximum sustainable environmental cost in increased to 1.6, but the maximum profit rate is reduced to some 13%. The intersection of the new frontier with the base case is achieved at a profit rate around 5%

Figure 3 wage-profit possibility frontier: alternative assumptions (a, b, and c)



The three scenarios are compared together with the base case in Figure 4 below. They can be seen as illustrating different possible solutions for financing an increased environmental cost within a static single period model where technology is given.

Figure 4: comparison of scenarios a, b, and c

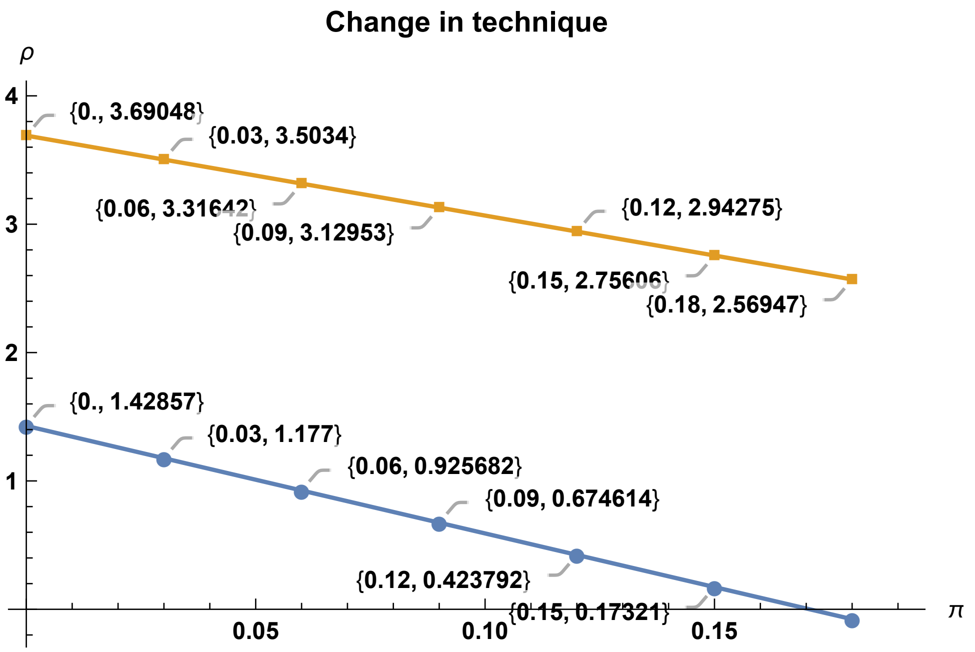


Finally, as an introduction to the arguments developed in sections 3 and 4 below, the case of technological change is considered, in which the coefficients of the matrix A are also changed. In that case we could have:

$$A = \begin{bmatrix} .1 & .4 \\ .3 & .2 \end{bmatrix}, \quad c = \begin{bmatrix} .1 \\ .4 \end{bmatrix} \quad a_{CO^2} = \begin{bmatrix} .2 \\ .2 \end{bmatrix} \quad a_n = \begin{bmatrix} .4 \\ .1 \end{bmatrix}$$

where the full possibility frontier is moved outwards, allowing to generate a higher maximum rate of profit and a higher sustainable environmental cost.

Figure 4: comparison of scenarios a, b, and c



As argued in the next sections, the problem of realising SDG is essentially to find a way to have a sequence of technological and price changes that allows to reach the SDG investment objectives while maintaining the economy on a stable growth path.

3. Pasinetti's structural dynamics (1973, 1981)

Leontief model is a static one and Pasinetti's (1977) wage-profit possibility frontier illustrates an arbitrage at one point in time in the case of a given technology. This can be seen as a single period or as a sequential model in a stationary condition. Pasinetti ([1973] 1980) introduced time explicitly in a model that remains static but which, treating investment as separate from consumption, fits better to the "single period" of a sequential dynamic model. This model is used by Pasinetti to introduce vertical integration, which is a necessary preliminary step for discussing structural dynamics (Pasinetti, 1981). After the transformation of vertical integration, beyond the single period, structural dynamics provide the appropriate framework for discussing the change of techniques.

In this section, Pasinetti's structural dynamics model of 1981 is presented, after a definition of vertical integration. For the latter Annex I develops a worked example with 4 sectors.

3.1 Vertical integration (open Leontief sequential model with depreciation and investment)

The starting point is the original Leontief open system with depreciation and investment. Depreciation is assumed constant in each sector as a percentage of the capital coefficient. It is included in a "diminished" technology matrix A^- which is explained below. The Leontief quantity system becomes:

$$\begin{cases} (I - A^{\ominus})x_t = y_t & (20a) \\ l_t x_t = L_t & (20b) \\ s_t = A x_t & (20c) \end{cases}$$

where the following notations are used:
- x_t = vector of the physical quantities for the m commodities produced in period t.
- y_t = vector of the physical net product of the economic system. y_t, represents what is available for consumption and investment, after replacement of x_t.
- s_t = vector of the stock physical capital required at the start of each period
- $p_t = [p_1(t) \quad \cdots \quad p_m(t)]$ vector of the prices of the m commodities
- e = unit column vector of dimension m
- $\hat{x}$ = diagonal matrix obtained from x_t
- X^c = Matrix of circulating capital flows
- X^f = Matrix of fixed capital flows
- $\hat{\delta}$ = diagonal matrix of fixed capital consumption coefficients
- $X^f\hat{\delta}$ = matrix of capital consumption flows
- A^c = Matrix of circulating capital coefficients
- A^f = Matrix of fixed capital coefficients

One can define the matrix which gives the sum of the circulating capital and depreciation as $A^{\ominus} = A^c + A^f\hat{\delta}$ whose elements represent that part of the beginning of year stock of capital goods that are used up each year in the production process.

$$A^{\ominus} = A^c + A^f\hat{\delta} \qquad (21)$$

By definition the full matrix A gives the amount of capital goods (circulating and fixed) required each year for the production of one physical unit of the commodity i by industry j is given by :

$$A = A^{\ominus} + (1 - \hat{\delta})A^f \qquad (22)$$ [14].

The vertical integrated system is obtained by solving 20a for x_t:

[14] The matrix A is given by the sum of A^c and A^f: $A = A^c + A^f$. The particular case in which all capital coefficients are zeros is represented by A^f=0, and in this case: $A^{\ominus} = A$.

$$x_t = \left(I - A^{\ominus}\right)^{-1} y_t \qquad (23)$$

where: $\left(I - A^{\ominus}\right)^{-1}$ is the Leontief matrix. Replacing x_t in (10b), one obtains:

$$L_t = l_t x_t = \underbrace{l_t\left(I - A^{\ominus}\right)^{-1}}_{v_t} y_t = v_t y_t \quad (24)$$

where v_t is the vector of vertically integrated coefficients. Similarly replacing x_t in (20c) provides the vector of vertically integrated capital coefficients:

$$s_t = A x_t = \underbrace{A\left(I - A^{\ominus}\right)^{-1}}_{H} y_t = H y_t \quad (25)$$

The system can thus be written as in version 20B below:

$$\begin{cases} x_t = \left(I - A^{\ominus}\right)^{-1} y_t = L y_t & (20B\ a) \\ L_t = l_t\left(I - A^{\ominus}\right)^{-1} y_t = v_t y_t & (20B\ b) \\ s_t = A\left(I - A^{\ominus}\right)^{-1} y_t = \ H y_t & (20B\ c) \end{cases}$$

If each element of the vector of final demand is considered separately (by taking the vector made of zeros plus the element y_i for output i), one can derive Sraffa's sub-systems for each commodity (see Annex)[15].

If, in addition, matrix A is measured in units of productive capacity, which correspond to the columns of matrix H and indicate the productive capacity necessary in the system to produce one unit of commodity i, the system can be written in the vertically integrated form 20C, where the only production coefficients appearing in the technology matrix are the vertical integrated coefficients for labour and for the n-1 capital goods (for depreciation and investment):

$$\begin{bmatrix} -1 & & & & & & & c_{1n} \\ & -1 & & & & & & c_{2n} \\ & & -1 & & & & & \vdots \\ & & & -1 & & & & c_{n-1,n} \\ a_{k_1 1} & & \cdots & & -1 & & \cdots & c_{k_1 n} \\ & \ddots & & & & -1 & \cdots & \vdots \\ & & a_{k_{n-1} n-1} & & & & -1 & c_{k_{n-1} 1n} \\ a_{n1} & & a_{n,n-1} & a_{nk_1} & \cdots & \cdots & a_{nk_{n-1}} & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_{k_1} \\ \vdots \\ x_{k_{n-1}} \\ LF \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (20C)$$

where, to simplify, it is assumed that capital goods enter only in consumption goods:

- $(c_{1n}, \ldots c_{n-1,n})^t$ is the vector of n-1 consumption coefficients;
- $(ck_{1n}, \ldots ck_{n-1n})^t$ is the vector of capital coefficients
- $(ak_1 n, \ldots ak_{n-1} n)$ is the depreciation rate for capital goods
- $(a_{n1}, \ldots a_{n,n-1})$ is the capital coefficient vector for consumption goods
- $(a_{nk1}, \ldots a_{nk,n-1})$ is the capital coefficient vector for capital goods

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[15] It is assumed that all commodities are basic, i.e. enter into the production of all other commodities. The more general case is developed in Pasinetti ([1977] 1980).

- $(x_1, \ldots x_n)^t$ is the vector of n-1 outputs of consumption goods
- $(xk_1, \ldots xk_n)^t$ is the vector of n-1 outputs of capital goods
- Lf is the labour force (output of the product "labour")

Taking T_i as the average lifetime of the i[th] capital good, one has: $a_{k,i} = 1/T_i$; (20C) can thus be re-written as:

$$\begin{bmatrix} -1 & & & & & & & c_{1n} \\ & -1 & & & & & & c_{2n} \\ & & -1 & & & & & \vdots \\ & & & -1 & & & & c_{n-1,n} \\ \frac{1}{T_1} & & \cdots & & -1 & & \cdots & c_{k_1 n} \\ & \ddots & & & & -1 & \cdots & \vdots \\ & & \frac{1}{T_{n-1}} & & & & -1 & c_{k_{n-1} n} \\ a_{n1} & & a_{n,n-1} & a_{nk_1} & \cdots & \cdots & a_{nk_{n-1}} & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_{k_1} \\ \vdots \\ x_{k_{n-1}} \\ LF \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (20D)$$

Let's call A_{int} the matrix appearing in system 20C[16]. Setting the determinant of this matrix to zero is a condition for its singularity, which, for homogenous systems such as 20c, implies that there exist solutions for x different from 0. It also provides Pasinetti's macroeconomic condition for full employment, which can be written in this case as:

$$\sum_{i=1}^{n-1} a_{ni} c_{in} + \sum_{i=1}^{n-1} a_{nk_i} c_{k_i} + \sum_{i=1}^{n-1} \frac{1}{T_i} a_{nk_i} c_{in} = 1 \quad (21)$$

Solving (20C) for the case of n-1 final commodities, each with its own capital good, gives:

$$\begin{cases} x_1 = c_{1n} LF \\ x_2 = c_{2n} LF \\ \vdots \\ x_{n-1} = c_{n-1n} LF \\ x_{k_1} = \left[c_{k_{1n}} + \frac{c_{1n}}{T_1} \right] LF \\ x_{k_2} = \left[c_{k_{2n}} + \frac{c_{2n}}{T_1} \right] LF \\ \vdots \\ x_{k_{n-1}} = \left[c_{k_{n-1,n}} + \frac{c_{n-1n}}{T_1} \right] LF \end{cases} \quad (22)$$

The system for prices can be written:

[16] "Int." stands for intermediary. This makes reference to the fact that Pasinetti's (1973) vertical integration model in (10C) can be interpreted also in terms of vertical hyper-integration and that, therefore, Pasinetti's (1981) dynamics presented below, which starts from 10C at time zero (or its more elaborated version in which capital is also used for producing capital under the assumption that this is done in the same sector, which is not presented here), should also be read in terms of vertical hyper-integration (see Garbellini, 2009).

$$\begin{bmatrix} -1 & & & \frac{1}{T_1}+\pi_1 & & & & a_{n1} \\ & -1 & & & \frac{1}{T_2}+\pi_2 & & & a_{n2} \\ & & -1 & & & \ddots & & \vdots \\ & & & -1 & & & \frac{1}{T_{n-1}}+\pi_{n-1} & a_{n,n-1} \\ \frac{1}{T_1} & & \cdots & & -1 & & \cdots & a_{nk_1} \\ & \ddots & & & & -1 & \cdots & \vdots \\ & & \frac{1}{T_{n-1}} & & & & -1 & a_{nk_{n-1}} \\ c_{1n} & \cdots & c_{n-1,n} & c_{k_1 n} & \cdots & \cdots & c_{k_{n-1},n} & -1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_{k_1} \\ \vdots \\ p_{k_{n-1}} \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (23)$$

The matrix in (2) is different from (20D) but has the same determinant (and full employment condition). The solution of the system for prices is:

$$\begin{cases} p_1 = \left[a_{n1}w + a_{nk_1}\left(\frac{1}{T_1} + \pi_1\right)\right]w \\ \vdots \\ p_{n-1} = \left[a_{n,n-1} + a_{nk_{n-1}}\left(\frac{1}{T_{n-1}} + \pi_{n-1}\right)\right]w \\ p_{k_1} = a_{nk_1}w \\ \vdots \\ p_{kn-_1} = a_{nk_{n-1}}w \end{cases} \quad (14)$$

Relations (23) and (24) are the solution of the model for the single period meeting simple assumptions on the exogenous technological change, demand, and distributional parameters and one can examine how this single period evolves dynamically taking account technological and other exogenous changes.

3.2 Structural dynamics

Beyond the short-term of the single period, in which the dynamics of (22) and (24) are driven by w and LF and, in the long-term each technical coefficient in the matrixes of systems (20D) and (23) is changing under the impact of exogenous technological progress and demand growth. To see this, let's note that population (or labour force) grows at an exogenous rate g:

$$x_n(t) = x_n(0)e^{gt} \quad (25)$$

and assume that productivity changes at a particular rate ρ_j in every sector:

$$a_{nk_j} = a_{nk_0}e^{-\rho_j t} \qquad j = 1,2 \ldots . n-1. \quad (26)$$

Per capita demand grows at a rate r_i during a sub-period of length θ (θ=t-ηz):

$$a_{in}(t) = a_{in}(t-\theta)e^{r_i\theta} \quad i = 1,2, \ldots n-1 \quad (27)$$

The dynamic capital accumulation equilibrium conditions are:

$$a_{k_i n}(t) = (g + r_i)a_{in}(t) = (g + r_i)a_{in}(t-\theta)e^{r_i\theta} \quad i = 1,2, \ldots n-1 \quad (28)$$

If μ is the proportion of active to total population and ν is the proportion of working hours worked during the period considered, the effective demand condition (condition for non-singularity of the matrix) becomes:

$$1 = \frac{1}{\mu\nu}\sum a_{ni}(t-\theta)a_{in}(t-\theta)\, e^{(r_i-\rho_i)\theta} + \frac{1}{\mu\nu}\sum \left(g + r_i + \tfrac{1}{T_i}\right) a_{nk_i}(t-\theta)a_{in}(t-\theta)e^{(r_i-\rho_i)\theta} \qquad (29)$$

As commented by Pasinetti (1981, p. 87) the condition shows that:

> *"The conclusion is straightforward. Even if we start from an equilibrium position (i.e. even if full employment of the labour force and full productive capacity utilisation are realised at a given point of time) the structural dynamics of the economic system cause that position to change and therefore make it impossible in general to automatically maintain full employment through time."*

The dynamic movements of quantities and relative prices are given by:

$$\begin{cases} x_i(t) = Ae^{(g+r_i)\theta} \\ x_{k_i}(t) = \left(g + r_i + \tfrac{1}{T_i}\right) Ae^{(g+r_i)\theta} \end{cases} \qquad i = 1, \ldots [n(t)-1] \quad (30)$$

$$\begin{cases} p_i(t) = Be^{-\rho_i\theta} + \left(\pi + \dfrac{1}{T}\right) Ce^{-\rho_{k_i}\theta} \\ p_{k_i}(t) = Ce^{-\rho_{k_i}\theta} \end{cases} \qquad i = 1, \ldots [n(t)-1] \quad (31)$$

where:

$$A = a_{in}(t-\theta)x_n(t-\theta)$$
$$B = a_{ni}(t-\theta)\overline{w}$$
$$= a_{nk_i}(t-\theta)\overline{w}$$

Therefore, each physical quantity grows at the rate g+r$_i$, and the structure of production evolves during time. Each relative price is also changing over time at a rate that depends inversely on the sectoral rate of technical progress. Taking $\overline{w}$ as the basis for the price system over time, each price is decreasing at a rate which is a weighted average of the pace at which productivity changes are taking place in the sector to which it refers, and in the sector which produces capital goods for it. The structure of prices therefore also changes over time, adapting to technical change.

Sectoral employment can be noted: $E_i(t) = a_{ni}(t)x_i(t)$ and $E_{ki}(t) = a_{nk_i}(t)x_{k_i}(t)$. Its dynamic is governed by:

$$\begin{cases} E_i(t) = Me^{(g+r_i-\rho_i)\theta} \\ E_{ki}(t) = \left(g + r_i + \tfrac{1}{T}\right) Ne^{(g+r_i-\rho_i)\theta} \end{cases} \qquad i = 1, \ldots . n(t)-1 \quad (32)$$

with:

$$M = a_{ni}(t-\theta)a_{in}(t-\theta)\nu^{-1}x_n(t-\theta) \quad (33)$$

and:

$$N = a_{nk_i}(t-\theta)a_{in}(t-\theta)\nu^{-1}x_n(t-\theta) \quad (34)$$

The natural rate of profit is given by:

$$\pi_i^* = g + r_i \qquad i = 1, \dots n-1 \quad (35)$$

The natural rate of profit is thus given by the sum of the rate of population growth and the sectoral rate of increase of per capita demand (both exogenous). The profit rate is uniform only if population grows at constant technical coefficients.

The natural economy is "pre-institutional". There is no single rate of profit as there would be in an economy in which the institution of the competitive markets would prevail. Profits just cover the natural investment needs which depend on population and demand growth, and these are different in growing or declining sectors. If one requires the uniformity of profit rates, as it would be necessary under free competition, equilibrium profits π_e, that would respect the other equilibrium conditions for investment and full-employment, would be given by:

$$\pi_e = \frac{1}{s_c}(g + r^*) \qquad (36)$$

where s_c is the rate of savings of capitalists and r* is a weighted average of the sectoral rate of growth of per capita demand. Relation (26) is the sectoral equivalent of the "Cambridge growth equation".

Contrary to the natural profit rate, which is not unique, in the natural economy there is a single natural rate of interest i*, which under fairly general conditions is equal to the rate of growth of wages σ_w. This is the rate of interest that guarantees that , whatever teh numéraire, prices of commodities are such that labour commanded is equal to labour embodied in value terms.

In absolute terms investment noted $x_{ki}^{"}$, which excludes depreciation, will grow proportionally to sectoral output at a variable rate equal to the sum of population growth and sectoral per capita demand according to:

$$x_{k_i}^{"} = (g + r_i)x_i(t) \quad i = 1, \dots, n-1 \quad (37)$$

The relations (37) specify the amounts of additional productive capacity that must be built in each single sector to achieve full employment. As noted by Pasinetti:

> *Since, in each sector, the rate of profit is determined exogenously by the same factors that enter the (V.4.3) ... [here, 37 above]..., the whole problem of choice of technique can be solved independently of the (V.4.3), and has a unique solution, except in the very particular case in which, with a discrete technology, the rate of profit happens to settle precisely at a*

switch point between two techniques. We shall assume, for simplicity, that this is not the case. This means that the capital accumulation equilibrium conditions (V.4.3) will normally determine investments in each particular sector, not only in terms of productive capacity, but also in terms of actual machines. As a straightforward consequence, the total amount of new investment in the economic system as a whole, which we may call I, can be derived by a simple sum:" [...]

$$I = \sum (g + r_i) p_{k_i} x_i \qquad (38)$$

[...] "The importance of this expression is that, by determining total equilibrium investment, it also ipso facto determines the equilibrium amount of total new savings for the economic system as a whole. By dividing both sides of (VII.10.1) [here (38) above] by aggregate final national income, we also obtain the equilibrium proportion of final national income that must be
saved and devoted to new investment, if full employment is to be maintained. It is important to realise that this equilibrium ratio (the over-all savings ratio) emerges as being determined directly by the exogenous factors of our analysis. As may be seen immediately by expanding its components, it is determined by consumers' consumption coefficients, population, technology, and their movements through time." (Pasinetti, 1981, p. 145)

[Section to be possibly expanded with a numerical example]

4. Choice versus change of techniques

Against the background of his natural economy, Pasinetti (1981) emphasized the difference existing between "the choice of techniques", which concerns that part of the capital stock that is renewed or expanded every year, from the "change in techniques", which concerns the substitution of the full input-output matrix of the economy and that is progressively realised in several years.

"In traditional capital theory, choice of technique and changes of technique have always been presented as two facets of the same problem. But again the association has been a consequence of dealing exclusively with stationary economic systems, which represent the special case in which the techniques which are relevant for the problem of choice happen to coincide with the techniques which are relevant in the process of change. As soon as we let time and technical progress into the picture, this coincidence disappears and two different sets of techniques become relevant for the two problems." (Pasinetti, 1981, p. 189)

Distribution impacts on and is conditioned by both the choice and the change in techniques, as Pasinetti already noted in his *Lectures on the Theory of Production*:

"The choice of the technique which minimizes the costs of production depends not only on the technology of the economic system but also on the distribution of income between profits and wages." (Pasinetti ([1975] 1977), p. 152).

"Thus in the case of basic commodities, since they are themselves used in the production of all commodities, changing a method of production in a single industry

has effects throughout the whole economic system [...] In analytical terms this means that [...] the solution to the problem of choice of technique can only be sought with reference to the economic system as a whole." (Pasinetti ([1975] 1977), p. 156).

Indeed, it is worthwhile stressing that, assuming linear sectoral growth in a vertically integrated economy implies that it is the full Sraffian subsystem whose coefficients decrease at a proportional rate, which is on the one hand reasonable at the "macro level" of vertically integrated sectors and on the other hand obviously has impacts on the whole economy.

4.1 Choice of technique: The choice of technique refers to each single production unit at a given point in time. Suppose that Ω alternative techniques f_j^{α}, ... f_j^{ω}, are known for producing the same quantity of commodity $\overline{x_j}$. $\alpha,\beta...\omega$ represent the alternative technical methods and K_j^k(k= α, β, ...ω) is the vector of machines and intermediate commodities necessary for the vertically integrated sector j to produce one unit of x_j. $x_{nj}^{x}$is the physical quantity of labour requied in the same vertically integrated sector, where w is the wage rate and π_i are the sectoral profit rates. At micro level all these prices are known, therefore the problem od technical choice is one of cost minimisation. The "choice of technique function" is:

$$Min\begin{cases} p_{k_j}^{\alpha}K_j^{\alpha}+x_{nj}^{\alpha}w, \\ \vdots \\ \vdots \\ p_{k_j}^{\omega}K_j^{\omega}+x_{nj}^{\omega}w \end{cases} \qquad j=1,\dots n-1 \qquad (39)$$

and the choice will depend notably on the prices of capital goods appearing in (40), which in the more complex formulation corresponding to (143, where capital goods enter also in capital goods, reads:

$$p_{k_i}=\left[\left(\frac{T_{k_i}}{T_{k_j}-\gamma_j-\pi_{k_i}\gamma_j T_{k_j}}\right)Ta_{nk_j}\right]w \quad j=1,\dots.n-1 \quad (40)$$

(40) shows that any change in the wage rate will change in the same manner the cost of all available technical methods in the same proportions, hence the choice of technique depends on the technical parameters and is independent of the wage rate. The choice of technique depends on the technical coefficients but not on the wage rate. On the contrary profits appear inside the square brackets and therefore have a differential impact on the choice of technique.

This last result seems in line with traditional theory, but (40) does not imply that the effect of a change in the profit rate should be positive or negative, as Pasinetti shows by derivating (30) with respect to profit. A technique that is profitable at a high rate of profit, may cease to be so if the rate of profit is decreased, but may become profitable again at an even lower rate[17].

[17] This is the essence of the result of the switching of tecniques debate, which implies *in fine* that there is no negative *a priori* relation between the price of a factor and its quantity, which is a major assumption of the traditional theory, notably essential for demonstrating the stability of its equilibrium.

The fact that the choice of techniques does not depend on wages in on the contrary in stark contradiction with traditional theory, which is based on the single good case, where an increase in wage automatically implies an opposite movement of the (necessarily single) profit rate, which, as discussed above, influences in turn the choice of techniques.

If the rate of profit is the natural rate given by (35), one can insert (40) in the choice of technique function, replacing π_i with $(g+r_i)$. In this case the terms in square brackets in (40) are a sum of physical quantities of labour (direct, indirect, and hyper-direct) multiplied by the wage rate, which has now been factored out. Hence, the principle of cost minimisation at micro level minimizes the quantity of labour used at macro level and "the 'natural' price system is indeed an efficient price system" (Pasinetti, 1981, p. 198).

4.2 Change of technique: Over time, the choice of technique in the sequence of single periods brings to the change of techniques, which is a macro-dynamic process involving changes in all the macro and micro variables of the system (interest rate and wages, but also sectoral outputs and prices). This is an important difference with the mainstream approach, which only considers the static micro-choice of techniques and ignores the subsequent process. It is also the true reason why Pasinetti rejects the neo-classical production function[18]. The reason is that, while the "fixed-coefficient" production function, accepted by post-Keynesians, describes "ex post" the reality of the complementarity and therefore the non-substitutability of labour and capital in the single period once new capacity is installed, the infinite substitutability of capital and labour implicit in the neo-classical production function concerns the "ex ante" choice of techniques of relation (39), which cannot be envisaged within the single period.

> *"This means introducing precisely those particular assumptions - i.e. stationary technical knowledge - that make the techniques which are relevant for the problem of choice coincide with the techniques which are relevant for a process of change. As has been pointed out already, this means frustrating any possibility of a dynamic analysis. Note moreover that - in spite of the superficially more elegant formulation - to assume coincidence of phenomena that are in general not coincidental is precisely the opposite of what is meant by 'generalisation'." (p. 203)*

This also implies the rejection of marginal productivity, which comes out to be essentially an irrelevant concept (p. 205).

The dynamic process of technical change results from the increase in the technical choices available in (39) brought by innovation, which, when actually adopted in the fixed-coefficients production function, progressively change the vertically integrated production parameters for the investment realised during the year, for which the hypothesis retained in the natural economy is that they evolve proportionally over time.

[18] In addition to the better known complementary one of the re-switching of techniques. The latter infirms the neo-classical regularity properties of the production function on the sign of its first and second derivatives, because it implies no negative relation between prices of factors and their quantity.

This framework allows a precise definition of technical progress. Starting from the simplified price relation when capital goods enter only consumption goods:

$$p_i(t) = a_{ni}(t) + \left(\pi_i + \tfrac{1}{T}\right) a_{nk_i}(t) \qquad i = 1, .. n-1 \qquad (41)$$

there is technical progress in the production of commodity i when, "for any given rate of profit" prices decrease through time $\frac{dp_i(t)}{dt} < 0$. When the profit rate is at its natural level given by (35), relation (41) becomes a sum of physical quantities of labour: direct, indirect, and hyper-direct. One can thus write:

$$l_i(t) = a_{ni}(t) + \tfrac{1}{T} a_{nk_i}(t) + (g + r_i) a_{nk_i}(t) \qquad i = 1, .. n-1 \qquad (42)$$

where $l_i(t)$ is the vertically integrated labour coefficient for commodity i. When this coefficient decreases in time ($\frac{dl_i(t)}{dt} < 0$), which happens when p(t) decreases, there has been a a saving of labour in the system. This allows to define technical progress as implying a saving of labour. With this definition, technical progress can be classified as:

- **neutral on capital intensity** (Harrod neutral), when technical progress takes place at the same rate in the production of commodity i and in the production of the capital good for commodity i (ρi=ρk). In such case, the capital-output ratio remains constant, and the capital-labour coefficient increases at the average productivity rate.
- **capital-intensity increasing**, when (ρi>ρk) and both the capital-output and the capital-labour ratios increase.
- **capital-intensity decreasing**, when (ρi<ρk). In this case the capital-output decreases and there are 3 possible sub-cases:
  i) ***physical capital neutral***: when the capital-labour ratio remains constant ("Hicks neutrality").
  ii) ***physical capital using***: when the capital-labour ratio increases.
  iii) ***physical capital saving***: when the capital-labour ratio decreases. In this case there are again three possible sub-cases:
    a) **direct labour neutral** when the ratio of output to labour remains constant (Solow neutrality)
    b) **direct labour saving** when the ratio of output to labour increases
    c) **direct labour using** when the ratio of output to labour increases.

If, as it appears to be the case, in reality technical progress is mainly Harrod-neutral, the fact that one observes that the capital labour ratio continuously increases can not be interpreted as a process of substitution of capital for labour. There is in fact no "movement along the production function" due to an increase in wages. The causality goes from technical progress to wages and not the other way round. This implies that neither substitution of capital for labour nor a change in capital intensity are related to the rate of profit.

This important last result does not mean that the rate of interest has no influence on the rate of investment in the natural economy. Keynes' marginal efficiency of capital remains relevant, in the sense that all projects can be ordered at microeconomic level as a function of their expected profitability, which depends on present costs (relative price of machines, of intermediate commodities, the wage rate and the interest rate) and on the future expected demand, independently from the degree of capital intensity of any single project. It makes therefore sense to select for investments those projects that have a return above the rate of interest. But this process has nothing to do with a substitution of capital for labour. The marginal project made profitable by a decrease in interest rates can be the more capital intensive.

5. Prices and technology: cost-benefits considerations

Leontief's input-output model seems an unescapable reference for looking at SDG investment and particularly at their climate change dimension. It offers the advantage of being interpretable both on neo-classical and Classical/post-Keynesian grounds. In particular, the natural system developed by Pasinetti in his structural dynamics, is attractive for its normative properties: it is efficient, achieves full employment, and keeps investment and savings at the level compatible with these requirements and keeps prices at the level at which labour embodied and labour commanded are equal.

Being normative in nature (Roncaglia, 1991), Pasinetti's structural dynamics almost naturally raises the question of what criteria should be taken as a reference in valuing the net socio-economic benefits of public policy actions aiming at attaining the SDG, notably for what concerns investment. This is a crucial step in determining the amount of necessary public support (Cingolani, 2021a).

But the discussion carried out until now on the natural economy has been quite theoretical. This section discusses to what extent it is relevant for the investment choice in an actual economy. Coming back to the starting question of the paper, if the market takes investment decisions based on observed market prices, what criterion should be used by the public sector to support these investments (respectively not to support); or to promote additional pure public sector SDG related investments financed 100% by public funds?[19] It is not clear what these prices should be exactly. In Cingolani (2021b) a small model is developed illustrating the problem of setting and agreeing on an incentive to realise an environmental

[19] It is not unfrequent that sizable public funds are budgeted to achieve certain policy objectives, to be spent for projects to be determined at a later stage, even if often based on partially pre-defined eligibility requirements. An example is the *Western Balkans Investment Framework*, established to support six countries of the Western Balkans in their preparation for accession to the European Union. In the last 11 years the European Commission and several associated donors (18 bilateral countries and 3 founding International Financial Institutions) have provided more than 1.4 billion Euros of grants to finance a pipeline of projects of total cost of some EUR 22bn, currently under implementation. In addition, loans for EUR 8 bn have been signed by the original founders IFIs and others that joined later, and another EUR 7 bn has been committed for the same pipeline. To commit the grant funds and identify the loans provided by the respective IFIs, the committee in charge for the allocation of the grant budget requests (and often finances directly) several feasibility studies as part of the programme, but in the end the decision is taken based on a mix of criteria, partly political. One of the reasons, is also that, as argued above, there is no way to integrate all the parameters of relevance in a single "automatic algorithm" that would offer a methodology rigorous enough to be accepted by all donors. The degree of common public support provided by this instrument thus results ex post from the decisions of this committee, which are partly political (see Cingolani, 2021a).

project of a given socio-economic rate of return, but this assumes that accounting prices are known .

Should the reference be to "observed market prices", which set the incentives for the private investors? The answer seems to be unambiguously no, because today prevailing market prices causes massive harm on environment and deep economic and social problems, therefore they are unlikely to be compatible as such with the realisation of the SDG, otherwise the latter would not have had to be developed. Based on the discussion above on the links between technology and prices, it can be expected that these market prices will have to evolve substantially in the process of realisation of the SDGs. But what woud be the role of an "entrepreneurial State" à la Mazzucato in promoting this evolution?

In the following, reference prices to be used in cost benefit calculations will be called accounting prices[20]. The establishment of accounting or shadow prices is one of the main purposes of cost-benefit analysis (Weimer, 2008), on which one must admit that it has been much more developed by mainstream authors than by post-Keynesians[21]. However, the the bases for the development of an alternative post-Keynesan cost-benefit approach are fully there in the form of a different value paradigm (Pasinetti, 1986). In addition, valuable analyses have already been developed by heterodox authors in the field of public economics, which incidentally bring sometimes to conclusions convergent with the results obtained by neo-classical authors[22].

To clarify ideas, one could start by following the approach of Allais ([1981] 1989) and of the best neo-classical authors in concentrating attention on the comparison of two alternative states of the economy, which could correspnd to the terms of a choice between doing an investment or not. To simplify to the extreme, if one assumes that a project will realise a change in the quantities supplied by Δq, the general idea of cost-benefit analysis is that the social impact of the project should be evaluated at $p_S\Delta q$, where p_S are the "accounting prices" (or another relevant index of social utility). On the contrary, the market will evaluate the same project at $p_M\Delta q$, where pM are observed market prices. The general idea, originating in Pigou (1932), is that to the extent that $p_S>p_M$ there is *prima facie* evidence for providing public support to the project, in principle up to an amounts p_S-p_M.

But are the notions of "accounting price" and "market price" sufficiently clear for practical application? Below some indications are given that let expect that the answer is negative.

First let's briefly discuss accounting prices. It is frequent to take as reference for social welfare judgments normative (or "equilibrium") prices. For example, in mainstream analysis, these would be the walrasian market-clearing prices. For many authors, in the absence of

[20] The term is preferred to that of shadow prices, more often used in the literature, as it suggests an analogy with the duality between quantity and prices existing in linear programming and therefore seem to indicate that these prices result from a optimisation programme that is necessarily static an thus far away from the very uncertain reality to which these prices should be applied (and even farther away from how this reality is interpreted on post-Keynesian grounds). Tinbergen (1958), who pioneered the accounting prices concept as a correction to be given to factor prices to consider several distortions to free competition existing in developing economies, introduced it with the term "accounting prices". Only later the term shadow prices prevailed, which alludes to a vector of prices coming out of an optimisation process with respect to a specific social welfare function.

[21] For a criticism of cost-benefit analysis, see Hansson (2010).

[22] Such as for instance on the necessary link between accounting prices and policy objectives (Cingolani, 2021a)

"distortions", these prices are assumed to coincide with observed market prices. There is for instance an influential tradition, associated for instance with the manual of Little-Mirrlees (1968), which suggests taking as much as possible international market prices as "shadow prices" for cost-benefit analysis.

Transposing this logic in the alternative classical value paradigm, what would be the equivalent normative prices? In Pasinetti's structural dynamics, these would clearly be the natural prices. But there is at least one additional normative price concept in the classical-post-Keynesian literature, which is that of long-period prices. One should thus clarify the relations existing between the notions of market price, neo-classical equilibrium prices, natural prices, and long-period prices, as it is done below in Annex II.

The brief historical digression of Annex II allows to conclude that, if a normative concept of prices is to be retained for the social evaluation of investments, this could be based on one of the following concepts, for which the profit distribution assumptions are reminded:

- market prices, if these are assumed to coincide at every moment with equilibrium prices, such as it is the case for market fundamentalists (for instance the followers of the market efficiency hypothesis). In this case (extra-)profits would logically cancel, as continuously prevailing equilibrium prices would also imply maximum efficiency.
- neo-classical equilibrium prices when the latter are not necessarily supposed to coincide with market prices.
- "Smithian" natural prices, in the sense of prices based on labour costs and the equalisation of sectoral rates of return. Here the extra-profits would be included in the difference between market and natural prices, but a "natural remuneration" of capital would be kept included in natural prices.
- Marx's production prices, based on the equalisation of sectoral profit rates to a level comprised between 0 and the maximum profit rate, which, to make it simple, is the rate coming out of relation (1) when the wage is minimal at zero (see Pasinetti, 1977, Appendix V).
- Long-term prices in the tradition of Garegnani, which to a large extent are a variant of Marx's reproduction prices.
- Pasinetti's natural prices, discussed above, where profits finance investments and not more.

If normative prices should be taken as reference for defining accounting prices, the most reasonable choice seems to be that of opting for Pasinetti's natural prices. This is because:

- the option of market price and of neo-classical prices should be discarded, first because they require completely unrealistic assumptions (full convexity of consumption and production sets, stability of equilibrium, no uncertainty, etc.) and second because it is irrelevant for policy purposes: if this solution is retained, there would be no difference between the social and the private evaluation of projects.
- As originally defined, the traditional classical approach was abandoned due to the problems with the labour theory of value (transformation of values into prices, standard of value etc). Sraffa's analysis seems to offer a solution to these problems, particularly in the interpretation offered by Pasinetti. Being based on a consistent definition of costs, both are preferable to the straight classical approach; which is therefore also discarded.

- In Marx's production prices, the actual profit rate remains indeterminate on economic grounds, it depends on the result of an "exogenous" class struggle between labour and capital, and therefore accounting prices are difficult to be agreed upon at international level and applied to a wide range of cases.
- Long-period prices are dependent on the practical relevance of the long-period equilibrium concept and on its "stability". Although favourable results have been reached in thiss case (Baggio, 1987, Petri 2021), to the knowledge of the author the case has not been made in a framework of "fundamental uncertainty" as that retained by Keynes, which brought several influential post-Keynesians, such as Kalecki, Robinson, or Davidson and others either to dismiss completely the equilibrium notion or privilege only a short-term equilibrium.

Pasinetti's natural economy framework, focussing on long-term structural adjustment, seems the preferable solution amongst the above, because of its simplicity (despite the subtle sophistication) and because of the normative properties of natural prices such as full employment, dynamic equilibrium of investment, efficiency, and cost minimisation, etc.

However, the idea that normative conditions should be used in the real-world has been criticized, mainly within the neo-classical literature, and at least two of the arguments raised seem quite solid.

The second-best approach has shown that if there is a more than one departure from optimal conditions, it is not sure that removing this "distortion" would improve general welfare (Lipsey & Lancaster, 1956-57). This indicates that if one uses normal price values in a non-optimal world (or natural prices in an institutionalised context) the result would not be unambiguous. The question needs to be explored further.

In addition, one can interpret Allais ([1981] 1989) as implying that, close to a situation of maximum efficiency, it is possible to use neo-classical equilibrium prices to calculate the impact of a change in quantities brought by the project (in other words using $p_M\Delta q$). But on the contrary, far from the maximum efficiency equilibrium, the so called "loss" has a formal definition, but cannot be based on neoclassical equilibrium prices. Therefore, the question of what accounting prices to use remains open. Close to equilibrium, traditional cost - benefit analysis is of relevance, but what is the meaning of being close to maximum efficiency in a dynamic context characterized by technical change? In particular, what is this meaning of traditional CBA in the context of the systemic change as the one supposed to be brought by the SDG?

What the heterodox analyses allow to add to these criticisms of traditional CBA, is that distribution, which is normally taken as given in standard cost-benefit calculations (CBA), should be an integral part of the evaluation of investment involving technical change, as clearly demonstrated in the context of the natural economy by Pasinetti. CBA normally assumes technology as given and it is generally applied to projects that are small with respect to the size of the economy, so that the change brought about by the project cannot affect the structure of relative prices and impact on distribution. In the case of SDG, the target is instead to promote a substantial transformation of the technology of the economy, which requires important additional investments. Therefore, the assumption that distribution considerations can be neglected must be dropped. The question for the SDGs therefore remains open: what prices should be taken as reference for the process of choice of techniques?

In front of such theoretical indeterminacy, a pragmatic solution could be to agree at political level on the accounting prices as suggested in Cingolani (2021a).

Conclusions

The analysis carried out remains preliminary and should be completed, probably by developing one further numerical example concerning structural dynamics. Nonetheless the models and the examples presented allow to draw some conclusions relevant for the SDG.

First, looking at climate change, the single period model presented in section 2.1 shows that it is possible to use the standard static neo-classical input-output model adding a satellite environmental account to establish comparative statics exercises concerning the quantitative impact of different investment scenarios concerning the suitable level of pollution. This model can be interpreted on neoclassical grounds and therefore in a context of scarce resources where investment expenditure for social and environmental purposes not already provided by the market is a net cost to society. The model of section 2.2 shows that if technology changes, additional environmental and social expenditures do not necessarily imply a reduction of the expenses for the consumption of other goods. Already in this comparative statics framework if there is a technological change that expands the wage-profit possibility frontier, it is possible to increase environmental expenditure while also increasing income. The model already introduces the main theme of the paper, which concerns the link between technology and the price structure which implies the need to look at these two aspects together from a planning viewpoint. The market price structure should evolve as developed in the 3rd and 4th sections, in such a way as to allow their realisation. Suitable accounting prices should be developed to establish the necessary public sector support for the SDG.

The discussion of the structural dynamics model developed in section 3 illustrates further the need for adequate public programming of the investment necessary for the realisation of the SDG. The natural dynamics of a capitalist market economy is likely to evolve towards increased unemployment due to the combined of different sectoral rates of technical progress. For that reason, aggregate demand must be managed, potentially by the creation ex novo of new markets by the public sector with a view to absorb the manpower released from other sectors. The structural dynamics will be governed by the interaction between technical progress and the production technology installed, described by a fixed coefficient production function. In this process of change the causality chain doesn't go from salaries as the prime element of costs to labour savings technical progress but it is a dynamic process in which the salary does not impact on sectoral rates of development of technical progress. This confirms that a "just transition" characterized by an increase in the levels of standard of leaving and the attainment of environmental and social objectives is perfectly possible, but must ne programmed in order to avoid to accumulate imbalances.

In general, the models presented show that the realisation of the SDG require strong international coordination around policy objectives at macro-economic level and cannot rely only on market automatism.

For this cooperation to be effective, due account should be taken of the main points where the causality chains of classical analysis depart from those of the mainstream. In particular, as noted:

- (40) shows that any change in the wage rate will change in the same manner the cost of all available technical methods in the same proportions, hence the choice of technique depends on the technical parameters and is independent of the wage rate. The choice of technique depends on the technical coefficients but not on the wage rate.

- (40) does not imply that the effect of a change in the profit rate should be positive or negative. A technique that is profitable at a high rate of profit, may cease to be so if the rate of profit is decreased, but may become profitable again at an even lower rate, which shows the relevance of the capital controversies for practical decisions. This also implies the rejection of marginal productivity, which turns out to be "essentially irrelevant". It also implies that neither substitution of capital for labour nor a change in capital intensity are related to the rate of profit.

Beyond the central question of what is the price structure that could incentivise the private sector to attain the SDG, there is the equally important question of what accounting prices should be taken into consideration by the public sector, in particular for setting the incentives for the private sector to realize the SDG investments needed. The point raised in section 5, already and noted in Cingolani, 2021a and 2021b is that these prices are not known. It is thus proposed to agree politically on them, as there seem to be consensus, that whatever they should be, they should be defined based on the existing policy objectives. One should also pragmatically recognize that accounting prices should be agreed in line with existing policies and therefore the relevant national, regional, and local jurisdictions should be involved in their establishment.

Given their importance, these questions deserve to be further studied.

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Annex I: Vertically integrated input-output structure: a worked example[23]

Pasinetti's (1977) model can be seen as a static one illustrating the wage-profit frontier in the case of a given technology at a specific point in time, or for a stationary economy. Pasinetti ([1973] 1980) introduces time explicitly, which allows to also treat investment as separate from consumption.

The explicit appearance of the time variable t introduces the dynamics in the model. To introduce investment, it is useful to derive the equivalent of relation (11) from the quantity system, in which circulating capital is distinguished from fixed capital. The initial assumption is that capital is consumed at a constant rate in time, different for each sector. The Leontief matrix A of section 2.2 becomes the sum of two matrices, of which one represents the coefficient of circulating capital and the other one the fixed capital coefficients. If a fraction δ_i of the stock of fixed capital of every sector is consumed during the production period, sectoral depreciation rates can be put along the principal diagonal of a matrix δ.

The following notations are retained:

$x_t = \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix}$ vector of the physical quantities of the m commodities produced in period t.

$y_t = \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix}$ vector of the physical net product of the economic system. y_t, represents what is available for consumption and investment, after replacement of Xt.

[23] The author thanks Nadia Garbellini for her kind and very substantial help in putting together this Annex.

$s_t = \begin{bmatrix} s_1(t) \\ \vdots \\ s_m(t) \end{bmatrix}$ vector of the stock physical capital required at the start of the period

$p_t = [p_1(t) \quad \cdots \quad p_m(t)]$ vector of the prices of the m commodities

$e = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ unit column vector of dimension m

$\hat{x} = \begin{bmatrix} x_1(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_m(t) \end{bmatrix}$ diagonal matrix obtained from x_t

$X^c = \begin{bmatrix} x^c_{1,1} & \cdots & x^c_{1,m} \\ \vdots & \ddots & \vdots \\ x^c_{m,1} & \cdots & x^c_{m,m} \end{bmatrix}$ Matrix of circulating capital flows

$X^f = \begin{bmatrix} x^f_{1,1} & \cdots & x^f_{1,m} \\ \vdots & \ddots & \vdots \\ x^f_{m,1} & \cdots & x^f_{m,m} \end{bmatrix}$ Matrix of fixed capital flows

$\hat{\delta} = \begin{bmatrix} \delta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_m \end{bmatrix}$ diagonal matrix of fixed capital consumption coefficients

$X^f \hat{\delta} = \begin{bmatrix} x^f_{1,1} & \cdots & x^f_{1,m} \\ \vdots & \ddots & \vdots \\ x^f_{m,1} & \cdots & x^f_{m,m} \end{bmatrix} \begin{bmatrix} \delta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_m \end{bmatrix}$ matrix of capital consumption flows

The production identity for the period is written:

$$X^C e + X^f \hat{\delta} e + y_t = x_t \qquad (2)$$

or:

$$\begin{bmatrix} x^c_{1,1} & \cdots & x^c_{1,m} \\ \vdots & \ddots & \vdots \\ x^c_{m,1} & \cdots & x^c_{m,m} \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} x^f_{1,1} & \cdots & x^f_{1,m} \\ \vdots & \ddots & \vdots \\ x^f_{m,1} & \cdots & x^f_{m,m} \end{bmatrix} \begin{bmatrix} \delta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_m \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix}$$

Let's assume that X^c , X^f_d and y_t are known to be:

$$X^c = \begin{bmatrix} 20 & 10 & 12 & 8 \\ 7 & 15 & 9 & 11 \\ 10 & 8 & 18 & 13 \\ 10 & 11 & 7 & 21 \end{bmatrix}, \quad X^f \delta = \begin{bmatrix} 10 & 4 & 2 & 5 \\ 3 & 12 & 4 & 8 \\ 7 & 6 & 11 & 5 \\ 8 & 3 & 4 & 10 \end{bmatrix} \text{ and } y_t = \begin{bmatrix} 10 \\ 15 \\ 8 \\ 20 \end{bmatrix}$$

Applying (2) one gets:

$$\begin{bmatrix}20 & 10 & 12 & 8\\ 7 & 15 & 9 & 11\\ 10 & 8 & 18 & 13\\ 10 & 11 & 7 & 21\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}+\begin{bmatrix}10 & 4 & 2 & 5\\ 3 & 12 & 4 & 8\\ 7 & 6 & 11 & 5\\ 8 & 3 & 4 & 10\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}+\begin{bmatrix}10\\15\\8\\20\end{bmatrix}=\begin{bmatrix}81\\84\\86\\94\end{bmatrix}$$

The matrices of technical coefficients can be derived as follows:

$$A^c = X^c\hat{x}^{-1} = \begin{bmatrix}20 & 10 & 12 & 8\\ 7 & 15 & 9 & 11\\ 10 & 8 & 18 & 13\\ 10 & 11 & 7 & 21\end{bmatrix}\begin{bmatrix}81 & 0 & 0 & 0\\ 0 & 84 & 0 & 0\\ 0 & 0 & 86 & 0\\ 0 & 0 & 0 & 94\end{bmatrix}^{-1} = \begin{bmatrix}0.25 & 0.12 & 0.14 & 0.09\\ 0.09 & 0.18 & 0.10 & 0.12\\ 0.12 & 0.10 & 0.21 & 0.14\\ 0.12 & 0.13 & 0.08 & 0.22\end{bmatrix}$$

$$A^f\hat{\delta} = X^f\hat{x}^{-1}\hat{\delta} = X^f\hat{\delta}\hat{x}^{-1} = \begin{bmatrix}10 & 4 & 2 & 5\\ 3 & 12 & 4 & 8\\ 7 & 6 & 11 & 5\\ 8 & 3 & 4 & 10\end{bmatrix}\begin{bmatrix}0.1 & 0 & 0 & 0\\ 0 & 0.1 & 0 & 0\\ 0 & 0 & 0.1 & 0\\ 0 & 0 & 0 & 0.1\end{bmatrix}^{-1}\begin{bmatrix}81 & 0 & 0 & 0\\ 0 & 84 & 0 & 0\\ 0 & 0 & 86 & 0\\ 0 & 0 & 0 & 94\end{bmatrix}^{-1} = \begin{bmatrix}0.25 & 0.12 & 0.14 & 0.09\\ 0.09 & 0.18 & 0.10 & 0.12\\ 0.12 & 0.10 & 0.21 & 0.14\\ 0.12 & 0.13 & 0.08 & 0.22\end{bmatrix}$$

where it is assumed that the coefficient of depreciation δ is equal to 10% for all sectors. One can also define the matrix which gives the sum of the circulating capital and depreciation as $A^{\ominus} = A^c + A^f\hat{\delta}$ and whose elements represent that part of the beginning of year capita stock of capital goods that are used up each year in the production process. By definition the matrix A of the capital coefficients giving the amount of capital goods (circulating and fixed) required each year for the production of aone physical unit of the commodity i by industry j is given by : $A = A^{\ominus} + (1-\hat{\delta})A^f$. The matrix A of relation (1) is thus given by the sum of A^c and A^f: $A = A^c + A^f$.

Knowing $A^f\hat{\delta}, A^f$ can be derived from:

$$A^f = A^f\hat{\delta}\widehat{\delta^{-1}} = \begin{bmatrix}0.25 & 0.12 & 0.14 & 0.09\\ 0.09 & 0.18 & 0.10 & 0.12\\ 0.12 & 0.10 & 0.21 & 0.14\\ 0.12 & 0.13 & 0.08 & 0.22\end{bmatrix}\begin{bmatrix}10 & 0 & 0 & 0\\ 0 & 10 & 0 & 0\\ 0 & 0 & 10 & 0\\ 0 & 0 & 0 & 10\end{bmatrix} = \begin{bmatrix}1.23 & 0.48 & 0.23 & 0.53\\ 0.37 & 1.43 & 0.47 & 0.85\\ 0.86 & 0.71 & 1.28 & 0.53\\ 0.99 & 0.36 & 0.47 & 1.06\end{bmatrix}$$

One can define the matrix which gives the sum of the circulating capital and depreciation as $A^{\ominus} = A^c + A^f\hat{\delta}$ and whose elements represent that part of the beginning of year stock of capital goods that are used up during the year in the production process.

$$A^{\ominus} = A^c + A^f\hat{\delta} = \begin{bmatrix}0.25 & 0.12 & 0.14 & 0.09\\ 0.09 & 0.18 & 0.10 & 0.12\\ 0.12 & 0.10 & 0.21 & 0.14\\ 0.12 & 0.13 & 0.08 & 0.22\end{bmatrix}+\begin{bmatrix}0.25 & 0.12 & 0.14 & 0.09\\ 0.09 & 0.18 & 0.10 & 0.12\\ 0.12 & 0.10 & 0.21 & 0.14\\ 0.12 & 0.13 & 0.08 & 0.22\end{bmatrix}=\begin{bmatrix}0.37 & 0.17 & 0.16 & 0.14\\ 0.12 & 0.32 & 0.15 & 0.20\\ 0.21 & 0.17 & 0.34 & 0.19\\ 0.22 & 0.17 & 0.13 & 0.33\end{bmatrix}$$

By definition the matrix A of the capital coefficients giving the amount of capital goods (circulating and fixed) required each year for the production of one physical unit of the commodity i by industry j is given by : $A = A^{\ominus} + (1-\hat{\delta})A^f$. The matrix A of relation (11) is thus given by the sum of A^c and A^f: $A = A^c + A^f$. The particular case in which all capital coefficients are zeros is represented by A^f=0, and in this case: $A^{\ominus} = A$.

$$A = A^c + A^f = \begin{bmatrix} 0.25 & 0.12 & 0.14 & 0.09 \\ 0.09 & 0.18 & 0.10 & 0.12 \\ 0.12 & 0.10 & 0.21 & 0.14 \\ 0.12 & 0.13 & 0.08 & 0.22 \end{bmatrix} + \begin{bmatrix} 1.23 & 0.48 & 0.23 & 0.53 \\ 0.37 & 1.43 & 0.47 & 0.85 \\ 0.86 & 0.71 & 1.28 & 0.53 \\ 0.99 & 0.36 & 0.47 & 1.06 \end{bmatrix} = \begin{bmatrix} 1.48 & 0.60 & 0.37 & 0.62 \\ 0.46 & 1.61 & 0.57 & 0.97 \\ 0.99 & 0.81 & 1.49 & 0.67 \\ 1.11 & 0.49 & 0.55 & 1.29 \end{bmatrix}$$

In terms of technical coefficients, the identity (2) becomes:

$$x = A^c x + A^f \delta x + y = A^{\ominus} x + y \quad (3)$$

from which one gets Leontief inverse L solving for x: $(I - A^{\ominus})x = y$; and,

$$x = (I - A^{\ominus})^{-1} y = Ly \qquad (4)$$

$$L = (I - A^{\ominus})^{-1} = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.37 & 0.17 & 0.16 & 0.14 \\ 0.12 & 0.32 & 0.15 & 0.20 \\ 0.21 & 0.17 & 0.34 & 0.19 \\ 0.22 & 0.17 & 0.13 & 0.33 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 2.69 & 1.27 & 1.20 & 1.28 \\ 1.30 & 2.39 & 1.12 & 1.31 \\ 1.62 & 1.37 & 2.50 & 1.46 \\ 1.52 & 1.28 & 1.15 & 2.52 \end{bmatrix} \quad (4)$$

The Leontief matrix gives the amount of heterogeneous goods that are directly and indirectly needed in the whole economic system to obtain one physical unit of commodity i as final good.

If we define the row vector l_t giving the labur force required by the economic system in year t, measured in man-years and assume:

$$l_t = [200 \quad 250 \quad 400 \quad 300]$$

the correspondent vector of labour coefficients is defined as before as $a_n^t = l_t \hat{x}$

$$a_n^t = [200 \quad 250 \quad 400 \quad 300] \begin{bmatrix} 81 & 0 & 0 & 0 \\ 0 & 84 & 0 & 0 \\ 0 & 0 & 86 & 0 \\ 0 & 0 & 0 & 94 \end{bmatrix}^{-1} = [2.47 \quad 2.98 \quad 4.65 \quad 3.19]$$

Let's consider the economic system from the viewpoint of the requirements necessary for producing each commodity only. This is the viewpoint of "vertically integrated sector", which is complementary to that of "industries" of the input-output table. For the first commodity the subsystem is defined using as vector of final demand:

$$y_1 = \begin{bmatrix} y_{t1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The total output necessary to produce 10 units of final demand for commodity 1 y_{t1} is:

$$x_1 = Ly_1 = \begin{bmatrix} 2.69 & 1.27 & 1.20 & 1.28 \\ 1.30 & 2.39 & 1.12 & 1.31 \\ 1.62 & 1.37 & 2.50 & 1.46 \\ 1.52 & 1.28 & 1.15 & 2.52 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 26.86 \\ 13.03 \\ 16.19 \\ 15.24 \end{bmatrix}$$

Defining the vector v^T:

$$v^T = a_n^t\left(I - A^{\ominus}\right)^{-1} = [2.47 \quad 2.98 \quad 4.65 \quad 3.19]\begin{bmatrix} 2.69 & 1.27 & 1.20 & 1.28 \\ 1.30 & 2.39 & 1.12 & 1.31 \\ 1.62 & 1.37 & 2.50 & 1.46 \\ 1.52 & 1.28 & 1.15 & 2.52 \end{bmatrix} = [22.9 \quad 20.7 \quad 21.6 \quad 21.9]$$

the labour total direct and indirect labour requirements to produce y1 are then given by L_1:

$$L_1 = a_n^t x_1 = a_n^t L y_1 = \underbrace{a_n^t\left(I - A^{\ominus}\right)^{-1}}_{v^T} y_1 = v^T y_1 = [22.9 \quad 20.7 \quad 21.6 \quad 21.9]\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 229.02$$

which is the vertically integrated labour coefficient for commodity 1.

Similarly, one can define a matrix H of vertically integrated productive capacity for commodity i as the matrix whose column vectors h_i give the the bundle of physical quantitities which are directly and indirectly required as stocks, in the whole economic system, to obtain one physical unit of commodity i as final demand.

$$H = A\left(I - A^{\ominus}\right)^{-1} = \begin{bmatrix} 1.48 & 0.60 & 0.37 & 0.62 \\ 0.46 & 1.61 & 0.57 & 0.97 \\ 0.99 & 0.81 & 1.49 & 0.67 \\ 1.11 & 0.49 & 0.55 & 1.29 \end{bmatrix}\begin{bmatrix} 2.69 & 1.27 & 1.20 & 1.28 \\ 1.30 & 2.39 & 1.12 & 1.31 \\ 1.62 & 1.37 & 2.50 & 1.46 \\ 1.52 & 1.28 & 1.15 & 2.52 \end{bmatrix} = \begin{bmatrix} 6.29 & 4,59 & 4.07 & 4.77 \\ 5.71 & 6.43 & 4.88 & 5.96 \\ 7.13 & 6.08 & 6.58 & 6.18 \\ 6.46 & 4,96 & 4,72 & 6,10 \end{bmatrix}$$

from which the vector S_1 giving the direct and indirect stock of capital necessary to produce ten units of y_1 s final demand becomes:

$$S_1 = Ax_1 = A\,Ly_1 = \underbrace{A\left(I - A^{\ominus}\right)^{-1}}_{H} y_1 = Hy_1 = \begin{bmatrix} 6.29 & 4,59 & 4.07 & 4.77 \\ 5.71 & 6.43 & 4.88 & 5.96 \\ 7.13 & 6.08 & 6.58 & 6.18 \\ 6.46 & 4,96 & 4,72 & 6,10 \end{bmatrix}\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 62.98 \\ 57.19 \\ 71.38 \\ 64.67 \end{bmatrix}$$

One can also look at the consumption versus depreciation amounts that are needed directly or indirectly to provide either a unit or the total amount of final demand for product i. For this one must multiply the relevant matrices by the diagonalised vector x_1:

$$X_1^c = A^c\widehat{x_1} = \begin{bmatrix} 0.25 & 0.12 & 0.14 & 0.09 \\ 0.09 & 0.18 & 0.10 & 0.12 \\ 0.12 & 0.10 & 0.21 & 0.14 \\ 0.12 & 0.13 & 0.08 & 0.22 \end{bmatrix}\begin{bmatrix} 26.86 & 0 & 0 & 0 \\ 0 & 13.03 & 0 & 0 \\ 0 & 0 & 16.19 & 0 \\ 0 & 0 & 0 & 15.24 \end{bmatrix} = \begin{bmatrix} 6.63 & 1.55 & 2.26 & 1.30 \\ 2.32 & 2.33 & 1.69 & 1.78 \\ 3.32 & 1.24 & 3.39 & 2.11 \\ 3.32 & 1.71 & 1.32 & 3.40 \end{bmatrix}$$

$$X_{(1)}^f\delta = A^f\hat{x}_1\hat{\delta} = \begin{bmatrix} 1.23 & 0.48 & 0.23 & 0.53 \\ 0.37 & 1.43 & 0.47 & 0.85 \\ 0.86 & 0.71 & 1.28 & 0.53 \\ 0.99 & 0.36 & 0.47 & 1.06 \end{bmatrix}\begin{bmatrix} 26.86 & 0 & 0 & 0 \\ 0 & 13.03 & 0 & 0 \\ 0 & 0 & 16.19 & 0 \\ 0 & 0 & 0 & 15.24 \end{bmatrix}\begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 3.32 & 0.62 & 0.38 & 0.81 \\ 0.99 & 1.86 & 0.75 & 1.30 \\ 2.32 & 0.93 & 2.07 & 0.81 \\ 2.65 & 0.47 & 0.75 & 1.62 \end{bmatrix}$$

If one sums the requirements for circulation and depreciation, one gets a matrix of intermediate exchanges X_1:

$$X_1^c + X_{(1)}^f \delta = X_1 = \begin{bmatrix} 6.63 & 1.55 & 2.26 & 1.30 \\ 2.32 & 2.33 & 1.69 & 1.78 \\ 3.32 & 1.24 & 3.39 & 2.11 \\ 3.32 & 1.71 & 1.32 & 3.40 \end{bmatrix} + \begin{bmatrix} 3.32 & 0.62 & 0.38 & 0.81 \\ 0.99 & 1.86 & 0.75 & 1.30 \\ 2.32 & 0.93 & 2.07 & 0.81 \\ 2.65 & 0.47 & 0.75 & 1.62 \end{bmatrix} = \begin{bmatrix} 9.95 & 2.17 & 2.63 & 2.11 \\ 3.32 & 4.19 & 2.45 & 3.08 \\ 5.64 & 2.17 & 5.46 & 2.92 \\ 5.97 & 2.17 & 2.07 & 5.02 \end{bmatrix}$$

If this matrix is summed over rows, it gives the quantity of every commodity that is produced to produce another commodity:

$$X_1 e = \begin{bmatrix} 9.95 & 2.17 & 2.63 & 2.11 \\ 3.32 & 4.19 & 2.45 & 3.08 \\ 5.64 & 2.17 & 5.46 & 2.92 \\ 5.97 & 2.17 & 2.07 & 5.02 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 16.86 \\ 13.03 \\ 16.19 \\ 15.24 \end{bmatrix}$$

the difference with total output necessary for subsystem 1 is given by final demand:

$$x_t - X_1 e = \begin{bmatrix} 26.86 \\ 13.03 \\ 16.19 \\ 15.24 \end{bmatrix} - \begin{bmatrix} 16.86 \\ 13.03 \\ 16.19 \\ 15.24 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Let us also define:

π = uniform rate of profit
w(t) = uniform wage rate

The price system is then given by:

$$p^T = w a_n^T + p^T A^{\ominus} + \pi p^T A$$

from which on gets for p^T:

$$p^T - p^T A^{\ominus} = w a_n^T + \pi p^T A$$
$$p^T \left(I - A^{\ominus}\right) = w a_n^T + \pi p^T A$$

from which:

$$p^T = w \underbrace{a_n^T \left(I - A^{\ominus}\right)^{-1}}_{v^T} + \pi p^T \underbrace{A \left(I - A^{\ominus}\right)^{-1}}_{H}$$

The price system can thus be rewritten in terms of vertically integrated labour and production capacity coefficients as:

$$p^T = w v^t + \pi p^T H$$

again, isolating p^T one gets finally:

$$p^T - \pi p^T H = w v^t$$
$$p^T (I - \pi H) = w v^t$$

and:

$$p^T = wv^t(I - \pi H)^{-1}$$

Assuming w=1 and π=4%:

$$p^T = v^t(I - 0.04H)^{-1} = [22.9 \quad 20.7 \quad 21.6 \quad 21.9]\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 0.04\begin{bmatrix} 6.29 & 4{,}59 & 4.07 & 4.77 \\ 5.71 & 6.43 & 4.88 & 5.96 \\ 7.13 & 6.08 & 6.58 & 6.18 \\ 6.46 & 4{,}96 & 4{,}72 & 6{,}10 \end{bmatrix}\right)^{-1}$$

Therefore, to get the price vector, one needs the Leontief matrix and A, or alternatively A^- and A.

$$p^T = wv^t(I - \pi H)^{-1} = wa_n^t L(I - \pi AL)^{-1} = wa_n^t\left(I - A^{\ominus}\right)^{-1}\left(I - \pi A\left(I - A^{\ominus}\right)^{-1}\right)^{-1}$$

Note that when one knows L, one gets A^-:

$$L = \left(I - A^{\ominus}\right)^{-1} -> L^{-1} = I - A^{\ominus} -> A^{\ominus} = I - L^{-1}$$

Annex II: A digression on normative prices

Classical authors tended to neglect market prices i.e., the price actually paid on the market for a particular commodity, because they thought that these were influenced by contingent factors outside the scope of economic theory. In particular Smith defined what he called "natural price" of a commodity as being essentially its factor costs, i.e. the sum of labour, capital and rent.

> *" [...] when the price of any commodity is neither more nor less than what is sufficient to pay the rent of the land, the wages of labour, and the profits of the stock employed in the raising, preparing and bringing it to market, according to their natural rates, the commodity is then sold for what may be called its natural price. (Smith 1776, p. 72, quoted by Viaggi, 2008).*

Following Smith, classical authors neglected market prices and considered that they would gravitate around natural prices, which represented the actual value of commodities. For Smith, the actual process of gravitation would turn precisely around observed market prices. If market prices were higher than natural prices (based on labour content) and therefore the quantity supplied would be below effectual demand (defined as that demand that would prevail at natural prices), competition between producers would increase supply up to the point where profit rates would be equalised between commodities, thus bringing down market prices of the commodity in question to a level closer to its natural price. The same mechanism would work with contrary effects in the opposite case of "excess supply".

Later classical authors developed the ideas of Smith. Building on Ricardo, Marx introduced the concept of production prices which was that level of prices that would result from the equalisation of profits between different commodities. Like Ricardo he was faced with the problem that production prices would potentially not be consistent with the "exchange value" of commodities, expected to reflect labour content. Marshall developed the notion of "normal price" which remains based on costs, but does not reflect necessarily an idea of equalisation of the sectoral rates of return, and thus goes beyond "perfect competition".

The classical approach was abandoned with the marginal revolution, where, together with scarcity (borrowed form Ricardo's theory of rent), utility became the main determinant of value. One can note however that modern classical authors such as Petri (2016), argue that the early neo-classical authors (Walras, Jevons and Menger) reasoned in term of "long-period" prices and kept a reference to the idea that market prices would gravitate around them, being defined essentially in line with Marx's production prices (Petri, 2021, chapter 2).

These authors argue that the neoclassical stream actually abandoned the reference to long-term prices only after the definition of inter-temporal general and temporary equilibrium by Hayek, Lindahl and Hicks in the ealry 1930s (Garegnani, 1976). Other authors, starting from Hicks in Value and Capital, think that Walras should be read as modelling short-term equilibrium (see also the 1960s "Italian" debate on capital theory[24] in *Il Giornale degli Economisti* discussed in Cingolani, 2016). Whether interpreted as short or long-term equilibrium, the capital theory of Walras is self-contradictory, as it cannot be assumed that the initial capital endowment are equilibrium ones (Garegnani, 1960).

[24] As opposed to the Cambridge controversy, in which some of the Italian authors, such as Garegnani, were also very active.