Nonlinear Search and Matching Explained*

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Abstract

Prevailing wisdom suggests the matching function choice is innocuous in search and matching models. We show this is not the case when accounting for nonlinearities. Using a closed-form global solution to a textbook model, we show the Den Haan et al. (2000) matching function features gross complementarity between vacancies and unemployed workers that creates procyclical variation in the matching elasticity and nonlinear job finding rate dynamics. These effects are absent from the Cobb-Douglas matching function. A quantitative assessment shows the latter specification provides a better account of key nonlinearities in the U.S. labor market.

Keywords: Matching Function; Complementarity; Nonlinearity; Finding Rate; Unemployment
JEL Classifications: E24; E32; E37; J63; J64

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1 INTRODUCTION

The matching function—the mapping from unemployed workers and vacancies into matches—is a core component of search and matching models. While a growing literature uses versions of this model to interpret nonlinearities in the data (e.g., Abbritti and Fahr, 2013; Den Haan et al., 2020; Dupraz et al., 2019; Ferraro, 2018; Hairault et al., 2010; Petrosky-Nadeau et al., 2018; Pizzinelli et al., 2020), the matching function choice has received little discussion, with some papers using the Cobb-Douglas (CD) specification and others using the Den Haan et al. (2000, DRW) specification. Prevailing wisdom suggests this choice is innocuous.[1] We show that is not true because the matching function impacts the extent of the nonlinearity in the model. A quantitative assessment shows the CD matching function provides a better account of key nonlinearities in the U.S. labor market.

The DRW matching function generates stronger nonlinearities because it exhibits gross complementarity between vacancies and unemployed workers and a procyclical matching elasticity—the elasticity of matches with respect to the number of unemployed workers. Gross complementarity implies that the matching elasticity is increasing in labor market tightness, so an increase in the stock of unemployed workers generates more matches when vacancy creation is high. In contrast, the CD matching function has a constant matching elasticity, which shuts down this mechanism.

The matching elasticity matters because it determines the slope of the mapping from productivity shocks to the job finding rate. Under the CD specification, the constant elasticity implies that the job finding rate is approximately linear in productivity, with the slope declining in the size of the elasticity. Intuitively, when the matching elasticity is higher, unemployed workers are relatively more important than vacancies in the matching process. Since only vacancies respond on impact to productivity shocks, a higher matching elasticity weakens the transmission of productivity shocks to the job finding rate. In contrast, the procyclical variation in the matching elasticity generated by the DRW matching function implies that the job finding rate is a concave function of productivity. When productivity is high, the matching elasticity is high and vacancies are relatively less important in the creation of new matches. As a result, the transmission of productivity shocks to the job finding rate is weaker than when productivity is low and vacancies are relatively more important.

Unemployment is inherently nonlinear due to its interactions with the job finding rate (Hairault et al., 2010; Jung and Kuester, 2011). This generates state-dependent responses to a productivity shock. We show the DRW matching function creates nonlinear job finding rate dynamics that amplify the responses of unemployment. Intuitively, a productivity shock causes a stronger response of the job finding rate. If unemployment is high, that leads to a larger response of unemployment inflows or outflows. This does not happen for plausible calibrations of the CD matching function.

[1] For example, Petrosky-Nadeau and Wasmer (2017) say the "business cycle moments of the model using either [the CD or DRW] functional form are similar." The stated justification for the DRW matching function is that it restricts the job filling and finding rates to the unit interval without requiring truncation in the solution and simulation of the model.

We find little evidence for a key prediction of the DRW matching function: concavity of the job finding rate. An inspection of the raw data shows there is no concavity before the Great Recession, even though there were several recessions during that period, including the highest unemployment rate in our sample. While the deepest part of the Great Recession features low job finding rates and appears to offer some support for concavity, the recovery shows the data is much better explained by a non-transitory downward shift in finding rates, consistent with the well-documented decline in matching efficiency since 2009 (Barnichon and Figura, 2015; Lubik, 2009; Sedláček, 2014; Veracierto, 2011). We corroborate this story using regression analysis. When we account for the secular decline in matching efficiency, the concavity estimates are small and statistically insignificant. This shows the CD matching function provides a more convincing account of the data.

To quantitatively assess whether the matching function choice matters, we estimate our theoretical model with the CD and DRW specifications. While both models produce similar first-order dynamics, the DRW matching function generates skewness in the job finding rate as a generic feature of business cycles, unlike in the data where the skewness is an artifact of the non-transitory decline in matching efficiency. As a consequence, it significantly over-predicts the skewness and kurtosis of the unemployment rate. In contrast, the weaker nonlinearities of the CD matching function produce empirically consistent skewness and kurtosis in the labor market, indicating its superior empirical fit. We show these differences explain why a model with a DRW matching function can generate deep recessions and the disaster dynamics emphasized by Petrosky-Nadeau et al. (2018), while a CD matching function cannot. In addition, the welfare costs of business cycles are about 50% larger under the DRW matching function. Finally, we confirm that our results are robust to changing the wage bargaining protocol and to including job separation rate shocks in the model.

Related Literature Our closed-form solution offers a clean way to analyze the sources of nonlinearity in the textbook search and matching model. In important earlier work, Petrosky-Nadeau et al. (2018) numerically analyze the nonlinearities and disaster dynamics in a similar model. They argue that downward rigidity in the marginal cost of hiring is the main driver of the nonlinearities. Our analytical results show the nonlinearities primarily stem from procyclical variation in the matching elasticity created by the DRW matching function and the law of motion for unemployment. While a recent literature uses the DRW specification, these papers do not analyze how this choice influences outcomes other than noting that it restricts the job finding and job filling rates to the unit interval (Ferraro, 2018; Hagedorn and Manovskii, 2008; Hashimzade and Ortigueira, 2005; Petrosky-Nadeau et al., 2018).[2] Our analysis emphasizes that the matching function choice is not innocuous.

Earlier work examined nonlinearities in the search and matching model using the CD matching function. For example, Hairault et al. (2010) argue the impact of productivity shocks on the job

[2]Others include Petrosky-Nadeau and Zhang (2017, 2021), Ferraro and Fiori (2020), and Bernstein et al. (2021).

finding rate is ambiguous because the job finding rate is a concave function of labor market tightness, while labor market tightness is a convex function of productivity. The lower the matching elasticity, the lower the average job finding rate and the higher the welfare costs of business cycles. Similarly, Jung and Kuester (2011) show that search frictions amplify the welfare costs of business cycles because of the nonlinear interaction between the unemployment and job finding rates in the law of motion for unemployment. They also find these interactions create state-dependent effects of productivity shocks.[3] Our analysis of nonlinear dynamics focuses on the role of the matching function choice. We show the DRW specification generates quantitatively significant nonlinearities that do not exist under the CD specification. Furthermore, we use the skewness and kurtosis of the unemployment and job finding rates to provide new empirical support for the CD matching function.

Our identification strategy for the structural labor market parameters synthesizes the results of earlier papers. To generate realistic volatilities of unemployment and vacancies, we combine the matching elasticity with the "fundamental surplus," defined as the marginal product of labor minus any resources not allocated to vacancy creation (Ljungqvist and Sargent, 2017). The fundamental surplus sets the overall level of labor market volatility, while the matching elasticity determines how the volatility is split between vacancies and unemployment (Mortensen and Nagypal, 2007). Additionally, we follow Hagedorn and Manovskii (2008) and target the wage elasticity. Relative to this earlier work, we show our strategy allows the model to perfectly match these empirical targets.

The paper proceeds as follows. Section 2 lays out our model. Section 3 derives a closed-form solution and discusses the sources of nonlinearity. Section 4 provides reduced-form empirical support for the CD matching function. Section 5 describes our estimation methods. Section 6 quantitatively assesses our model and provides additional evidence in favor of the CD matching function. Section 7 shows our insights are robust to alternative modeling assumptions. Section 8 concludes.

2 ENVIRONMENT

We use a textbook search and matching model similar to Hagedorn and Manovskii (2008). Time is discrete, and the population size (equal to the labor force) is normalized to unity. We consider risk neutral and risk averse households. Under risk aversion, we assume perfect insurance so all households choose the same consumption path (Andolfatto, 1996; Den Haan et al., 2000; Merz, 1995).

Search and Matching Entering period t, there are n_{t-1} employed workers and $u_{t-1} = 1 - n_{t-1}$ unemployed workers. Within the period, a fraction $\bar{s}$ of the employed workers exogenously lose their jobs. The newly separated workers are able to search for new jobs within the same period as their job loss, but they have less time to search for new jobs in period t than those who became unemployed in a previous period. Therefore, let $\chi \in [0, 1]$ denote the fraction of a period that newly

[3] Lepetit (2020) computes welfare costs in a New Keynesian model with labor search and a CD matching function.

separated workers are able to spend searching for work in the same period as their job loss.[4] Given these labor market flows, the total number of unemployed searching workers in period t is given by

$$u_t^s = u_{t-1} + \chi \bar{s} n_{t-1}. \tag{1}$$

The matching process is described by a constant returns to scale matching function $\mathcal{M}(u_t^s, v_t)$, where v_t is vacancy postings.[5] We consider the CD and DRW specifications used in the literature:

$$\mathcal{M}(u_t^s, v_t) = \begin{cases} \xi (u_t^s)^\alpha v_t^{1-\alpha}, & \text{CD}, \\ u_t^s v_t / ((u_t^s)^\iota + v_t^\iota)^{1/\iota}, & \text{DRW}, \end{cases} \tag{2}$$

where $\xi > 0$ denotes matching efficiency and $\alpha \in (0,1)$ and $\iota > 0$ govern the relative importance of unemployed searching workers to vacancies in the matching process. Section 3 shows how these parameters map into key elasticities. The number of matches in period t, m_t, is then defined by

$$m_t = \min\{\mathcal{M}(u_t^s, v_t), u_t^s, v_t\}. \tag{3}$$

We can use the matching process to define the job finding and job filling rates,

$$f_t = m_t / u_t^s, \quad q_t = m_t / v_t, \tag{4}$$

where the feasibility condition, (3), ensures $f_t, q_t \in [0,1]$. The DRW matching function guarantees $m_t = \mathcal{M}(u_t^s, v_t)$, whereas (3) could bind under the CD matching function. When $m_t = \mathcal{M}(u_t^s, v_t)$, we can express the job finding and job filling rates in terms of labor market tightness $\theta_t \equiv v_t / u_t^s$,

$$f_t = \begin{cases} \xi \theta_t^{1-\alpha}, & \text{CD}, \\ 1/(1+\theta_t^{-\iota})^{1/\iota}, & \text{DRW}, \end{cases} \qquad q_t = \begin{cases} \xi \theta_t^{-\alpha}, & \text{CD}, \\ 1/(1+\theta_t^{\iota})^{1/\iota}, & \text{DRW}. \end{cases}$$

Since each period lasts one month, we assume newly matched workers begin employment in the same period they are matched with a firm (Blanchard and Galí, 2010). Hence, employment follows

$$n_t = (1-\bar{s}) n_{t-1} + f_t u_t^s. \tag{5}$$

The unemployment rate u_t includes anyone who is not employed in period t, so it is given by

$$u_t \equiv u_t^s - m_t = 1 - n_t. \tag{6}$$

[4]Shimer (2005) sets $\chi = 0.5$ when constructing a measure of the monthly job finding rate in the data. Our theoretical insights do not depend on the value of χ. However, including this additional parameter allows us to match the mean unemployment and job finding rates, which is important to accurately quantify the nonlinearity in the model.

[5]Pissarides (2000) formalizes the axioms of the matching function and its role in a search and matching model. Petrongolo and Pissarides (2001) survey the empirical evidence for constant returns to scale in the matching function.

Firms A firm chooses vacancies and employment $\{v_t, n_t\}$ to maximize the present value of dividends, $V_t = a_t n_t - w_t n_t - \kappa v_t + E_t[x_{t+1} V_{t+1}]$, subject to $n_t = (1-\bar{s})n_{t-1} + q_t v_t$ and $v_t \geq 0$, where $\kappa > 0$ is the vacancy posting cost, w_t is the wage rate, and E_t is an expectation operator conditional on time-t information. The pricing kernel, $x_{t+1} = \beta(c_t/c_{t+1})^\gamma$, where c_t is consumption, $\beta \in (0,1)$ is the discount factor, and $\gamma \geq 0$ is the coefficient of relative risk aversion. Productivity, a_t, follows

$$a_{t+1} = \bar{a} + \rho_a(a_t - \bar{a}) + \sigma_a \varepsilon_{a,t+1},\ 0 \leq \rho_a < 1,\ \varepsilon_a \sim \mathbb{N}(0,1). \tag{7}$$

The optimality conditions imply

$$\tfrac{\kappa - \lambda_{v,t}}{q_t} = a_t - w_t + (1-\bar{s})E_t\left[x_{t+1}\tfrac{\kappa - \lambda_{v,t+1}}{q_{t+1}}\right], \tag{8}$$

$$\lambda_{v,t} v_t = 0, \quad \lambda_{v,t} \geq 0, \tag{9}$$

where $\lambda_{v,t}$ is the multiplier on the non-negativity constraint $v_t \geq 0$. Condition (8) sets the marginal cost of hiring, $(\kappa - \lambda_{v,t})/q_t$, equal to the marginal benefit of hiring, which consists of the flow profits from the match, $a_t - w_t$, plus the savings from not having to post the vacancy in the future.

Wages To solve the model in closed-form, we specify a wage rule given by

$$w_t = \eta a_t + (1-\eta)b, \tag{10}$$

where $\eta \in (0,1)$ and $b > 0$. Following Hall and Milgrom (2008) and Freund and Rendahl (2020), this rule is motivated by an alternating-offers bargaining game in which workers discount future payoffs at rate η and receive flow payoff b before a wage agreement is reached. Following Jung and Kuester (2011), the wage rule can alternatively be derived by maximizing the Nash product $(a_t - w_t)^{1-\eta}(w_t - b)^\eta$, where η is the worker's bargaining power and b is the outside option. Our quantitative results are robust to Nash bargaining that maximizes the total surplus of a new match.

Equilibrium The aggregate resource constraint is given by

$$c_t + \kappa v_t = a_t n_t. \tag{11}$$

An equilibrium includes sequences of quantities $\{c_t, n_t, u_t, u_t^s, v_t, m_t, q_t, \lambda_{v,t}\}_{t=0}^\infty$, prices $\{w_t\}_{t=0}^\infty$, and productivity $\{a_t\}_{t=1}^\infty$ that satisfy (1)-(11) given the initial state $\{n_{-1}, a_{-1}\}$ and shocks $\{\varepsilon_{a,t}\}_{t=0}^\infty$.

3 ANALYTICAL RESULTS

3.1 SOLUTION

For tractability, assume risk neutrality. Then combine (8) and (10) to obtain

$$\tfrac{\kappa - \lambda_{v,t}}{q_t} = (1-\eta)(a_t - b) + \beta(1-\bar{s})E_t[\tfrac{\kappa - \lambda_{v,t+1}}{q_{t+1}}]. \tag{12}$$

We guess and verify that (12) has a solution of the form,

$$(\kappa - \lambda_{v,t})/q_t = \delta_0 + \delta_1(a_t - \bar{a}), \tag{13}$$

where

$$\delta_0 = \frac{(1-\eta)(\bar{a}-b)}{1-\beta(1-\bar{s})}, \qquad \delta_1 = \frac{1-\eta}{1-\beta(1-\bar{s})\rho_a},$$

and $\lambda_{v,t} > 0$ only when $q_t = 1$.[6] The remaining variables can then be solved for recursively as functions of (a_t, n_{t-1}) with (1)-(11). δ_0 is the steady-state marginal cost of hiring, while δ_1 measures the response of marginal cost to changes in the marginal product of labor. Intuitively, δ_1 is increasing in the profit share of a match, $1-\eta$, and the persistence of the productivity shock, ρ_a.

We use our solution to highlight the matching function and its interaction with the law of motion for unemployment as a new source of nonlinearity.[7] In addition, we can immediately clarify the mechanism described in Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018), in which the real marginal cost of hiring, κ/q_t, runs into a "downward rigidity". As (13) makes clear, κ/q_t only faces such a rigidity when productivity falls so low that it causes vacancies to hit the nonnegativity constraint. Since $v_t > 0$ in the data and almost all simulations, this cannot be the primary source of nonlinearity, which instead stems from the matching function and law of motion.

3.2 NONLINEAR JOB FINDING RATE DYNAMICS

We use (13) and the matching function to derive the supply curve for vacancy creation. Assuming $v_t > 0$ and $\lambda_{v,t} = 0$ for tractability, we obtain

$$\kappa/q_t = \begin{cases} (\kappa/\xi)\theta_t^{\alpha}, & \text{CD}, \\ \kappa\left(1+\theta_t^{\iota}\right)^{1/\iota}, & \text{DRW}, \end{cases} \tag{14}$$

where $v_t = \theta_t u_t^s$ implies that vacancies inherit the properties of labor market tightness, θ_t, since u_t^s is pre-determined. This expression captures the positive relationship between the marginal cost of hiring and vacancies. Since $\delta_1 > 0$, vacancies are increasing in productivity. In equilibrium, higher productivity raises the marginal benefit of hiring, causing firms to post more vacancies. A rise in vacancies increases θ_t, which raises the marginal cost of hiring until it equals the marginal benefit.

Inverting (14) yields labor market tightness in terms of marginal cost. Combining this with (13) and the matching function implies the finding rate is an increasing function of productivity, where

$$f_t = \begin{cases} \xi^{\frac{1}{\alpha}}\left(\frac{\delta_0+\delta_1(a_t-\bar{a})}{\kappa}\right)^{\frac{1-\alpha}{\alpha}}, & \text{CD}, \\ \left(1-\left(\frac{\kappa}{\delta_0+\delta_1(a_t-\bar{a})}\right)^{\iota}\right)^{1/\iota}, & \text{DRW}. \end{cases} \tag{15}$$

[6] Note that for v_t arbitrarily close to 0, $m_t = v_t$ by (3). By continuity, $\lambda_{v,t} > 0$ implies $q_t = 1$. Therefore, if productivity is low enough that $\delta_0 + \delta_1(a_t - \bar{a}) < \kappa/q_t$ for all $q_t \in [0,1]$, then $q_t = 1$ and $\lambda_{v,t} = \kappa - \delta_0 - \delta_1(a_t - \bar{a})$.

[7] Den Haan et al. (2020) independently developed a similar solution to shed light on the effects of volatility shocks.

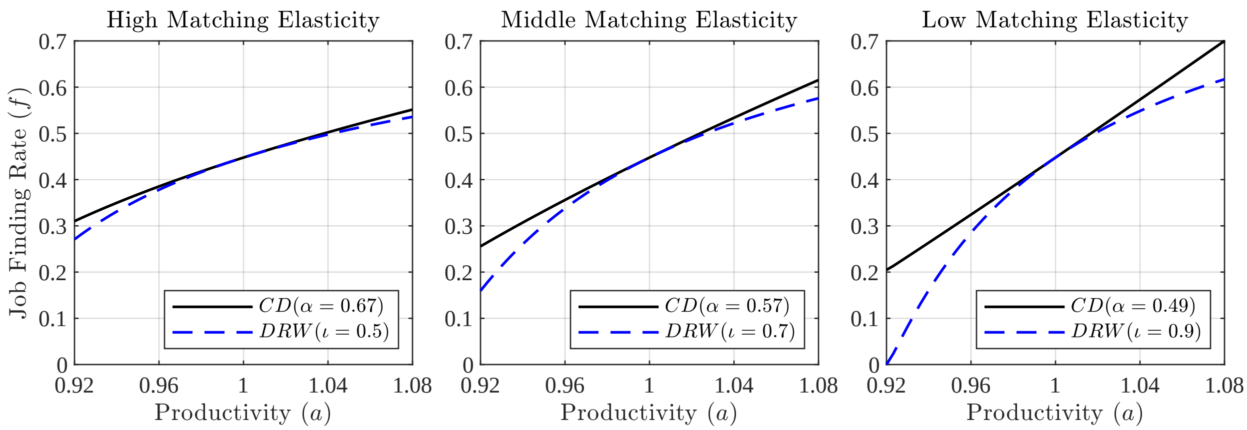

Figure 1: Sensitivity of the job finding rate to changes in productivity.

Intuitively, more vacancies generate more matches given a mass of unemployed searching workers.

To observe the nonlinearities in (15), Figure 1 plots f_t for different values of the curvature parameter, ι, in the DRW matching function. In each case, the CD parameter, α, is set so that both matching functions have the same steady-state matching elasticity $\bar{\epsilon}_{m,u^s}$, defined as the elasticity of matches with respect to unemployed searching ($\bar{\epsilon}_{m,u^s} = \bar{f}^\iota = \alpha$). The other parameters are set to their estimated values in Section 6.1. The values of $\bar{\epsilon}_{m,u^s}$ are all in the range of matching elasticities in the data (Mortensen and Nagypal, 2007; Petrongolo and Pissarides, 2001). To facilitate interpretation, we compute the slope of the job finding rate function in terms of the matching elasticity,

$$\frac{df_t}{da_t} = \begin{cases} \frac{1-\alpha}{\alpha}\phi_t^{CD}, & \text{CD}, \\ \frac{1-\epsilon_{m_t,u_t^s}}{\epsilon_{m_t,u_t^s}}\phi_t^{DRW}, & \text{DRW}, \end{cases} \tag{16}$$

where ϕ_t^{CD} and ϕ_t^{DRW} combine terms that do not impact the key properties of the job finding rate.[8]

Figure 1 shows the DRW matching function is a source of nonlinearity in the job finding rate. Increasing ι makes the job finding rate more concave by increasing the steepness of the policy function at low productivities relative to high productivities. More concavity drives greater deviations between the CD and DRW job finding rates, especially at low productivities. We can see the source of concavity analytically in (16) by noting that the DRW slope term, $(1 - \epsilon_{m_t,u_t^s})/\epsilon_{m_t,u_t^s} = (1-f_t^\iota)/f_t^\iota$ is time-varying. Since the matching elasticity is increasing in f_t, the slope coefficient is decreasing in f_t. Therefore, the job finding rate is more sensitive to productivity shocks when f_t is low. Intuitively, when f_t is low, $\epsilon_{m_t,v_t} = 1 - \epsilon_{m_t,u_t^s}$ is high and vacancies are important in the production of matches. This amplifies the transmission of productivity shocks to the job finding rate.

To understand why the matching elasticity is procyclical under the DRW matching function,

[8]Specifically, $\phi_t^{CD} = (\xi f_t^{1-2\alpha})^{1/(1-\alpha)}\delta_1/\kappa$ and $\phi_t^{DRW} = f_t(1-f_t^\iota)^{1/\iota}\delta_1/\kappa$. Given a plausible range for the job finding rate, $f_t \in (0.2, 0.6)$, setting a realistic value of $\alpha = \bar{f}^\iota \approx 0.5$ implies that ϕ_t^{CD} and ϕ_t^{DRW} are nearly constant.

note that the elasticity of substitution between unemployed searching workers and vacancies is $1/(1+\iota) < 1$, so that the matching function features gross complementarity between inputs. Therefore, a larger stock of unemployed searching workers generates more matches when vacancy creation is high and the labor market is tight. This is captured by the matching elasticity $\epsilon_{m_t,u_t^s} = f_t^\iota$, which is increasing in the job finding rate and hence in labor market tightness. It is also useful to note that relatively little time-variation in the matching elasticity is necessary to generate significant nonlinearities in the job finding rate. For example, when $\bar{\epsilon}_{m,u^s} = 0.5$, a $\pm 5\%$ change in ϵ_{m_t,u_t^s} causes a $\pm 20\%$ change in the slope of the job finding rate in the neighborhood of steady state.

In contrast, the CD matching function is not a significant source of nonlinearity. For all values of α, the job finding rate policy function is close to linear. The slope of the job finding rate in (16) is decreasing in α but is not time-varying due to the lack of gross complementarity in the matching function (i.e., the elasticity of substitution is equal to 1). Instead, increasing α makes unemployed searching workers more important and vacancies less important in the production of new matches. Since only vacancies are able to respond on impact to changes in productivity, a higher α weakens the transmission of productivity shocks to the job finding rate, resulting in a flatter policy function.

3.3 Nonlinear Unemployment Dynamics

The matching function choice also has important consequences for unemployment dynamics. The law of motion for unemployment is given by

$$u_t = u_{t-1} + \bar{s}(1 - u_{t-1}) - f_t u_t^s. \tag{17}$$

Under both matching functions, the $f_t u_t^s$ term creates state-dependence and positively skewed unemployment dynamics (Hairault et al., 2010; Jung and Kuester, 2011). Intuitively, a change in the job finding rate has a larger effect on unemployment when it is applied to a larger pool of workers.

Since u_t^s is pre-determined, $du_t/da_t = -u_t^s df_t/da_t$, so the unemployment response to productivity shocks inherits the properties of the job finding rate response. Under the DRW matching function, unemployment responds more to shocks during recessionary periods when the job finding rate is already low, creating additional state-dependence that is absent under the CD specification.

4 Empirical Job Finding Rate Dynamics

A testable difference between the DRW and CD matching functions is the concavity of the job finding rate. We inspect the raw data and then run regressions to show there is little evidence of concavity, suggesting the CD matching function provides a better account of labor market dynamics.

4.1 Data Sources and Construction

We use monthly data from 1955-2019. Following Shimer (2005), the monthly job finding rate is $f_t \equiv 1 - (U_{t+1} - U_{t+1}^s)/U_t$, where U_t is total unemployed and U_t^s is the subset who are unemployed one month or less in the Current Population

Survey.[9] The unemployment rate is $u_t = U_t/LF_t$, where LF_t is the labor force. The vacancy rate v_t is based on the series in Barnichon (2010) until 2000, after which it is equal to job openings as a share of the labor force in the Job Openings and Labor Turnover Survey. These series correct for trends in the print and online help-wanted indexes published by the Conference Board. To account for potential aggregation bias, we construct a measure of unemployed searching workers, $u_t^s = \chi s_t + (1-\chi s_t)u_{t-1}$, where $\chi = 0.5$ is consistent with Shimer (2005). Following Shimer (2012), the job separation rate is $s_t \equiv 1-\exp(-\tilde{s}_t)$, where $\tilde{s}_t$ is the solution to $U_{t+1} = (1-e^{-\tilde{f}_t-\tilde{s}_t})\tilde{s}_t LF_t/(\tilde{f}_t+\tilde{s}_t) + e^{-\tilde{f}_t-\tilde{s}_t}U_t$ and $\tilde{f}_t \equiv -\log(1-f_t)$. Labor market tightness, $\theta_t = v_t/u_t^s$.

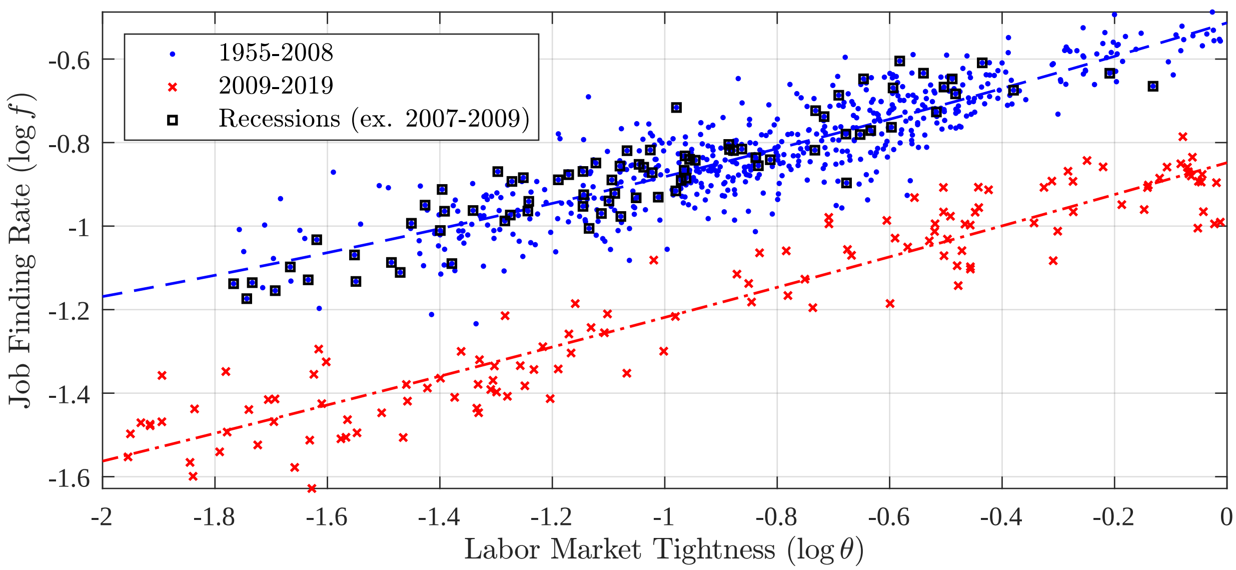

Figure 2: Job finding rate schedule before and after the Great Recession. Dashed lines are quadratic trends.

4.2 RAW DATA ANALYSIS

Our analytical results show that only the DRW matching function generates concave job finding rate dynamics. To look for this concavity empirically, we work with the matching functions directly, expressing the job finding rate in terms of labor market tightness,

$$\log f_t = \begin{cases} \log\xi + (1-\alpha)\log\theta_t, & \text{CD}, \\ \log\theta_t - \frac{1}{\iota}\log(1+\theta_t^\iota), & \text{DRW}. \end{cases} \tag{18}$$

The log-finding rate is linear under the CD specification and concave under the DRW specification.

Figure 2 plots $\log f_t$ against $\log\theta_t$. The blue dots show data before the Great Recession (1955-2008), with recessions highlighted in black squares. The red crosses show data since the Great Recession (2009-2019). The raw data clearly show that the log-finding rate was linear in labor market tightness before the Great Recession. This is in spite of the fact that there were multiple recessions

[9]We follow Elsby et al. (2009) to correct for the 1994 redesign of the Current Population Survey that lowered U_t^s.

before 2009, including the 1982 downturn which generated the highest unemployment rate in our sample (10.8% vs. a peak of 10.1% in the Great Recession). To generate any evidence of a concave relationship, we need to combine the pre-2009 sample with the deepest part of the Great Recession, captured by the bottom-left corner of red-cross data points. This specific reliance on the Great Recession casts doubt on the notion that concavity is a general feature of job finding rate dynamics and suggests that the Great Recession was unlike previous downturns. This possibility gains further weight when we consider the subsequent economic recovery, moving north-east along the red-cross data sample. Rather than returning to the pre-2009 cluster of data points, the job finding rate remains depressed as labor market tightness increases. Through the lens of the matching function, this pattern is driven by a non-transitory decline in matching efficiency during the Great Recession, consistent with empirical estimates (Barnichon and Figura, 2015; Lubik, 2009; Sedláček, 2014; Veracierto, 2011).[10,11] When we view the data from the Great Recession onwards as a whole, the relationship between the job finding rate and tightness is again log-linear, just with a lower intercept.

4.3 Regression Analysis

To formally investigate what combination of concavity and matching efficiency best explains the job finding rate, we estimate a linear regression model of the form:

$$\log f_t = \alpha_0 + \alpha_1 t + \alpha_2 \mathbf{1}\{t \geq t^*\} + \beta_1 \log\theta_t + \beta_2(\log\theta_t)^2 + \varepsilon_t. \tag{19}$$

The matching elasticity implied by this regression is $\epsilon_{m_t,u_t^s} = 1 - \beta_1 - 2\beta_2\log\theta_t$, which nests the CD matching function when $\beta_2 = 0$. Empirical support for the DRW matching function rests on whether β_2 is significantly negative, so $\log f_t$ is concave in $\log\theta_t$. We allow for non-transitory changes in matching efficiency using $\alpha_2\mathbf{1}\{t \geq t^*\}$, which alters the regression intercept for data in periods $t \geq t^*$ for some break date t^*. $\alpha_1 t$ is a linear time trend that applies to the whole sample.[12]

Table 1 reports estimates for the change in matching efficiency, α_2, and concavity, β_2, for a range of break points, t^*, over the Great Recession. The estimates for the decline in matching efficiency are stable and statistically significant across all values of t^*, in line with the stark change in intercept shown in Figure 2. In contrast, the magnitudes and t-statistics of the concavity estimates are sensitive to the break point choice. Even when the estimate of β_2 is marginally statistically significant, the extent of the concavity is very weak. Comparing the R^2 values across the different break points shows the model with the best fit ($t^* =$ 2008M11) actually features zero concavity.

Our results are further supported by the Bai and Perron (1998) test for the timing and number of intercept breaks. The test reveals that the data prefer intercept breaks in 2008M11 and 1989M1,

[10] For example, Barnichon and Figura (2015) estimate that matching efficiency declined during the Great Recession because of sharp increases in the fraction of long-term unemployed and the fraction of workers on permanent layoff.

[11] The decline in matching efficiency does not necessarily contradict the DRW matching function. Adding a matching efficiency parameter would not affect its concavity properties but would bound f_t and q_t above by the $\min\{\xi, 1\}$.

[12] We obtain similar results when using a break in time trend rather than intercept (Hall and Schulhofer-Wohl, 2018).

		Break ($1000 \times \alpha_2$)		Concavity (β_2)	
Break Point (t^*)	R^2	Estimate	t-stat	Estimate	t-stat
2007M12	0.906	-0.252	-22.26	-0.021	-1.70
2008M6	0.912	-0.262	-25.10	-0.009	-0.76
2008M11	0.916	-0.270	-27.91	0.000	0.03
2009M6	0.913	-0.259	-25.30	-0.029	-2.25

Table 1: Matching function parameter estimates. The estimates are based on monthly data from 1955-2019. The t-statistics are computed using robust standard errors. 2007M12 and 2009M6 are the first and last months of the Great Recession, 2008M6 is a reference point for 2008, and 2008M11 fits the data the best.

rather than one break during the Great Recession. The 2008M11 estimate is consistent with the downward shift in Figure 2 and the results of the regression with the best fit in Table 1. The break in 1989M1 likely captures an effect from the Great Moderation. When accounting for both breaks, we find concavity remains weak and statistically insignificant. These results corroborate our raw data analysis and show that a CD matching function in conjunction with a non-transitory decline in matching efficiency provides a more convincing account of the empirical relationship between the job finding rate and labor market tightness than the concavity from the DRW matching function.

5 MODEL IDENTIFICATION AND ESTIMATION

To quantitatively assess the matching function choice, we estimate our model under each specification. This section describes the identification scheme that permits an apples-to-apples comparison.

5.1 IDENTIFICATION For simplicity, we explain our identification scheme using the deterministic steady state, but the intuition also holds in the dynamic equilibrium system. The disagreement payoff b governs the economy's "fundamental surplus fraction" (Ljungqvist and Sargent, 2017), defined as the upper bound on the fraction of a worker's output that can be allocated to vacancy creation. A small fundamental surplus fraction is crucial to deliver realistic volatilities of unemployment and vacancies (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017). To see this, combine the vacancy creation condition, (8), with the wage rule, (10), in steady state to obtain

$$\kappa/\bar{q} = (1-\eta)(\bar{a}-b)/(1-\beta(1-\bar{s})).$$

Differentiating then yields an expression for the elasticity of tightness with respect to productivity,

$$\bar{\epsilon}_{\theta,a} = \frac{\bar{a}}{\bar{a}-b} \times \frac{1}{\bar{\epsilon}_{m,u^s}},$$

where $(\bar{a}-b)/\bar{a}$ is the fundamental surplus fraction and $\bar{\epsilon}_{m,u^s}$ is the steady-state elasticity of matches with respect to unemployed searching. Given estimates for $\bar{\epsilon}_{m,u^s}$ typically range from

0.3-0.7 (Mortensen and Nagypal, 2007; Petrongolo and Pissarides, 2001), a large response of labor market tightness to changes in labor productivity requires a small fundamental surplus fraction, which requires that b is close to the marginal product of labor, $\bar{a}$. A small fundamental surplus fraction makes labor market tightness, and hence unemployment and vacancies, sensitive to changes in labor productivity. Therefore, we identify b by targeting the two standard deviations in the data.

While b changes the volatilities of unemployment *and* vacancies, $\bar{\epsilon}_{m,u^s}$ affects their *relative* volatilities. To see this, differentiate the steady-state conditions $\bar{u} = \bar{s}(1-\chi\bar{f})/(\bar{s}(1-\chi\bar{f})+\bar{f})$ and $\bar{v} = \bar{\theta}\bar{u}^s$ to determine the elasticities of unemployment and vacancies with respect to tightness:

$$\bar{\epsilon}_{u,\theta} = -(1-\bar{u})(1-\bar{\epsilon}_{m,u^s})/(1-\chi\bar{f}),$$
$$\bar{\epsilon}_{v,\theta} = 1-(1-\chi\bar{s}/\bar{u}^s)(1-\bar{u})(1-\bar{\epsilon}_{m,u^s})/(1-\chi\bar{f}).$$

As $\bar{\epsilon}_{m,u^s}$ increases, the responsiveness of unemployment to changes in labor market tightness shrinks relative to the responsiveness of vacancies. Intuitively, when $\bar{\epsilon}_{m,u^s}$ is higher, an increase in matches requires a smaller increase in unemployed searching, and hence in unemployment. Therefore, when matches fluctuate, unemployment fluctuates less relative to vacancies. Hence, we identify $\bar{\epsilon}_{m,u^s}$ by targeting the *relative* standard deviations of unemployment and vacancies in the data.

To ensure an apples-to-apples comparison of the matching functions, we use the definitions $\bar{\epsilon}_{m,u^s} = \alpha = \bar{f}^{\iota}$. Identifying $\bar{\epsilon}_{m,u^s}$ pins down α in the CD matching function. Since we set $\bar{f}$ to target the average job finding rate in the data, we can also pin down ι in the DRW matching function. Setting $\bar{f}$ also pins down the steady-state DRW job filling rate, $\bar{q} = (1-\bar{f}^{\iota})^{1/\iota}$. We then set the CD matching efficiency so $\bar{q}$ is consistent across the two matching functions, which implies

$$\xi = \bar{q}^{1-\alpha}\bar{f}^{\alpha}.$$

Recall from (10) that η governs the responsiveness of the wage rate to changes in the marginal product of labor, which is driven by labor productivity. Hence, we follow Hagedorn and Manovskii (2008) and identify η by targeting the elasticity of the wage rate with respect to labor productivity.

We set $\bar{u}$ and $\bar{s}$ to target the average unemployment and job separation rates. We then solve for the vacancy posting cost, κ, and intra-period search duration, χ, using the steady-state conditions:

$$\kappa = \bar{q}(1-\eta)(\bar{a}-b)/(1-\beta(1-\bar{s})),$$
$$\chi = ((1-\bar{u})\bar{s}-\bar{f}\bar{u})/((1-\bar{u})\bar{s}\bar{f}).$$

Steady-state productivity, $\bar{a}$, is normalized to 1. We identify the process parameters, $\{\rho_a, \sigma_a\}$, by targeting the quarterly standard deviation and autocorrelation of labor productivity in the data.

5.2 SOLUTION We solve the nonlinear model with the policy function iteration algorithm in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). The algorithm minimizes the Euler equation errors on each node in the discretized state space. It then computes the maximum distance between the policy functions on any node and iterates until it falls below the tolerance criterion. We approximate the productivity process with Gauss-Hermite quadrature and use piecewise linear interpolation to calculate the updated policy functions. Following Garcia and Zangwill (1981), we ensure that $v_t \geq 0$ by introducing an auxiliary variable, μ_t, that satisfies $v_t = \max\{0, \mu_t\}^2$ and $\lambda_{v,t} = \max\{0, -\mu_t\}^2$. μ_t maps into vacancies when $v_t > 0$ and the Lagrange multiplier, $\lambda_{v,t}$, when $v_t = 0$. See Appendix B for more information.

5.3 ESTIMATION The discount factor, β, is set to 0.9983, consistent with an annual real interest rate of 2%, and the coefficient of relative risk aversion, γ, is set to 1, consistent with log utility. The empirical targets are stored in $\hat{\Psi}_T^D$ and estimated with a two-step Generalized Method of Moments (GMM) estimator, where $T = 780$ months. Given these values, the parameters are estimated with a Simulated Method of Moments. For parameterization $\mathcal{P}$ and shocks $\mathcal{E} = \{\varepsilon_a\}$, we solve the nonlinear model and simulate it $R = 1{,}000$ times for T months, the same length as our data. The model analogues of the Ψ_T^D empirical targets are the mean moments across the R simulations, $\bar{\Psi}_{R,T}^M(\mathcal{P}, \mathcal{E})$.

The parameter estimates are obtained by minimizing the following quadratic loss function:

$$J(\mathcal{P}, \mathcal{E}) = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\mathcal{P}, \mathcal{E})]'[\hat{\Sigma}_T^D(1 + 1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\mathcal{P}, \mathcal{E})],$$

where $\hat{\Sigma}_T^D$ is the diagonal of the GMM estimate of the variance-covariance matrix of the empirical targets. The targets are based on quarterly data, in percent deviations from a Hamilton (2018) filtered trend.[13] Each period in the model is 1 month, so we aggregate the simulated time series to a quarterly frequency. We then detrend the simulated data by computing percent deviations from the time average, so the units of each moment are directly comparable to their counterpart in the data.

6 QUANTITATIVE ASSESSMENT

We estimate our model with data from 1955-2019. Using a sample that excludes the decline in matching efficiency (1955-2008) yields similar estimates, so our results do not depend on the break discussed in Section 4. The estimates based on the shorter sample are shown in Appendix C.

6.1 ESTIMATED PARAMETERS Table 2a reports the parameter estimates under each matching function. Both models perfectly match the empirical targets, indicating the strength of our identification scheme. In the CD model, the disagreement payoff, b, is 0.934, just below the estimate of

[13] Specifically, we regress each time series on its most recent 4 lags following an 8 quarter window. Hodrick (2020) shows this approach is more accurate than an HP filter when time series, such as ours, are first-difference stationary.

Model Parameter		CD	DRW	Empirical Target	Data
Intra-Period Search Duration	χ	0.5399	0.5295	Average Unemployment Rate	5.89
Vacancy Posting Cost	κ	0.3550	0.2137	Average Job Finding Rate	42.14
Job Separation Rate	$\bar{s}$	0.0327	0.0327	Average Job Separation Rate	3.27
Disagreement Payoff	b	0.9336	0.9333	Unemployment Standard Deviation	23.66
Matching Elasticity	$\bar{\epsilon}_{m,u^s}$	0.5040	0.5904	Vacancy Standard Deviation	21.69
Bargaining Weight	η	0.5723	0.5722	Wage-Labor Productivity Elasticity	0.59
Productivity Persistence	ρ_a	0.9537	0.9537	Labor Prod. Autocorrelation	0.89
Productivity Standard Deviation	σ_a	0.0083	0.0083	Labor Prod. Standard Deviation	2.61

(a) Estimated parameter values. Both models perfectly match the empirical targets ($J = 0$).

	Full Sample		Pre-2009		CD		DRW	
Moment	Mean	SE	Mean	SE	1	2	1	2
$SD(f)$	14.96	1.47	12.42	0.92	18.56	18.42	13.24	13.22
$Corr(u,v)$	−0.74	0.04	−0.65	0.05	−0.89	−0.92	−0.85	−0.95
$Skew(u)$	1.10	0.35	0.88	0.37	0.91	−0.03	1.90	0.70
$Skew(f)$	−0.60	0.45	0.03	0.36	0.01	0.03	−0.78	−0.73
$Kurt(u)$	1.05	0.57	0.38	0.47	1.30	−0.16	5.92	0.68
$Kurt(f)$	0.29	0.54	−0.63	0.25	−0.15	−0.15	0.93	0.76

(b) Untargeted moments. Models: (1) All nonlinear; (2) All nonlinear except linear law of motion.

Table 2: Baseline model.

0.955 in Hagedorn and Manovskii (2008). Workers' bargaining power, η, is 0.572, similar to Jung and Kuester (2011) who also use a linear wage rule. The steady-state matching elasticity, $\alpha = \bar{\epsilon}_{m,u^s}$, is 0.50, in the middle of the $(0.3, 0.7)$ range suggested by Petrongolo and Pissarides (2001). The steady-state job filling rate is $\bar{q} = (1 - \bar{f}^{\iota})^{1/\iota} = 0.429$, where $\iota = \log\alpha/\log\bar{f}$ and $\bar{f} = 0.437$. This ensures that $\bar{q}$ is consistent across models. Matching efficiency is then $\xi = \bar{q}^{1-\alpha}\bar{f}^{\alpha} = 0.433$.

The estimated DRW parameters are generally similar to their CD counterparts. One exception is the matching elasticity, $\bar{\epsilon}_{m,u^s} = \bar{f}^{\iota}$, which is higher at 0.59, to compensate for its time variation. Given $\bar{\epsilon}_{m,u^s} = \alpha$, the matching parameter, $\iota = \log\alpha/\log\bar{f} = 0.656$. The implied elasticity of substitution is $1/(1+\iota) = 0.604$, which is well below the unitary elasticity implied by a CD matching function. Furthermore, estimating the second-order approximation of the DRW matching function in (19) using simulated data yields an estimate for $\beta_2 = 0.08$, which is almost triple the largest empirical estimates and indicates more pronounced concavity than in the data. Our elasticity of substitution is also much higher than the implied elasticity of 0.47 in Petrosky-Nadeau et al. (2018), which would generate much larger nonlinearities from the DRW matching function.

Our estimation shows the matching functions have similar first-order properties. In addition to matching the volatilities of unemployment and vacancies, Table 2b shows both models generate realistic job finding rate volatilities and correlations between unemployment and vacancies. These similarities likely explain the conventional wisdom that the matching function choice is innocuous.

A frequently mentioned advantage of the DRW matching function is that it guarantees job filling and job finding rates between 0 and 1, whereas the CD matching function requires truncation. We find this advantage is small. In simulations of our estimated model with a CD matching function, f_t is always inside this interval, and q_t requires truncation in only 0.196% of the simulations.

6.2 IMPLICATIONS OF THE MATCHING FUNCTION CHOICE

Nonlinearities The bottom of Table 2b shows the two matching functions produce vastly different results when we examine higher-order moments. Consider first the model with a CD matching function (column CD-1). While the model generates substantial skewness in the unemployment rate (0.91), it does not generate any skewness in the job finding rate. The fixed matching elasticity shuts down the gross complementarity mechanism, leaving the law of motion as the only source of nonlinearity. The nonlinear unemployment dynamics also generate modest excess kurtosis (1.30).

The extent of the nonlinearity is greatly amplified when the law of motion for unemployment interacts with the time-variation in the matching elasticity that occurs in the DRW model (column DRW-1). Relative to the CD model, skewness in unemployment more than doubles to 1.90, while job finding rate skewness is significant at -0.78. The excess kurtosis of unemployment increases fivefold to 5.92, and there is pronounced excess kurtosis in the finding rate of 0.93, indicating very non-Gaussian dynamics. These nonlinearities only require small movements in the matching elasticity, which has a standard deviation of 0.06. Thus, the fluctuations in the matching elasticity rarely leave the $(0.3, 0.7)$ range in the literature (Mortensen and Nagypal, 2007; Petrongolo and Pissarides, 2001). This illustrates that the DRW mechanism is a significant source of nonlinearity.

We can isolate the strength of the procyclical matching elasticity by re-solving the model with a linearized law of motion of unemployment, thus shutting down the other source of nonlinearity. The results are reported in column DRW-2. By itself, procyclical variation in the matching elasticity generates 37% of the skewness in unemployment and 94% of the skewness in the job finding rate, corroborating the importance of a time-varying matching elasticity for nonlinear finding rate dynamics. Notably, the excess kurtosis of unemployment drops to 0.68, indicating the strength of the interaction between a procyclical matching elasticity and the law of motion for unemployment. The same comparison with the CD matching function confirms there are no other sources of nonlinearity (column CD-2). Linearizing the law of motion essentially removes all traces of nonlinearity.

Empirical Fit To account for the structural break in the data discussed in Section 4, we report the empirical skewness and kurtosis of unemployment and the job finding rate over the full sample and up to the Great Recession (column Pre-2009). While the skewness of unemployment is mostly stable over time (full sample: 1.10; pre-2009 sample: 0.88), the skewness in the job finding rate is sample-dependent (full sample: -0.60; pre-2009 sample 0.03), and neither estimate is significantly different from zero. This instability is an artifact of the underlying job finding rate dynamics plotted in Figure 2. The downward shift in the job finding rate schedule after 2009 creates artificial negative skewness when estimated over the whole sample. The excess kurtosis of unemployment and the job finding rate are also sample-dependent, as both are much smaller in the pre-2009 sample.

With these empirical features in mind, the CD matching function clearly provides a stronger account of the data. The skewness of unemployment is in line with the data, and the near zero skewness of the job finding rate accords well with the notion that the change in finding rate dynamics after 2009 is better captured by a decline in matching efficiency rather than concavity. In contrast, our simulations show that the DRW matching function generates skewness in the job finding rate as a generic feature of business cycles, unlike in the data where the skewness hinges on the decline in matching efficiency that occurred only during the Great Recession. As a consequence of these counterfactual nonlinearities, it also grossly overstates the skewness and kurtosis of unemployment, and generates counterfactually high kurtosis in the finding rate, even over the full sample.[14]

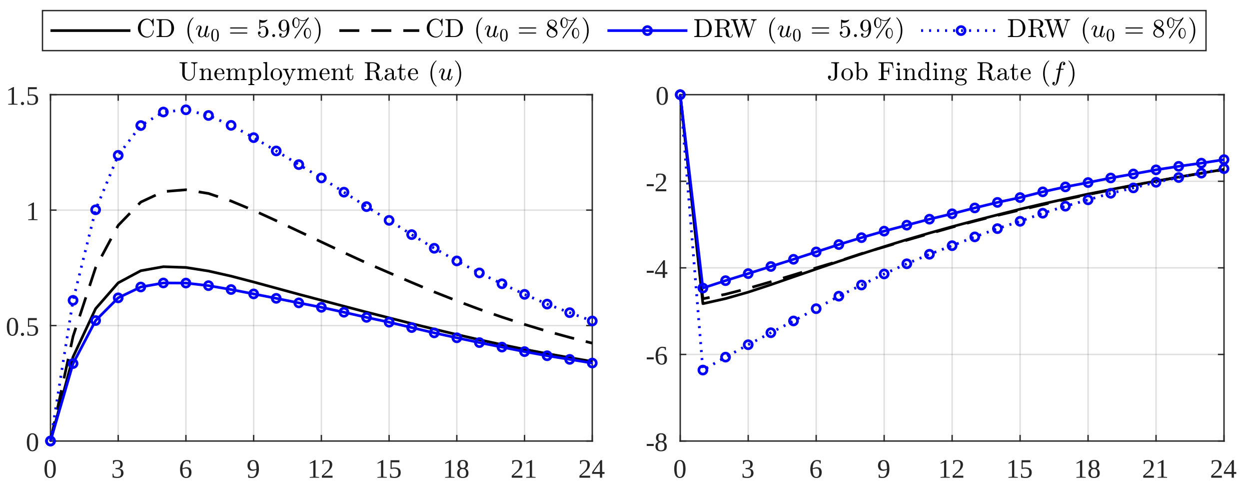

Figure 3: Percentage point responses to a -2 standard deviation productivity shock.

Deep Recessions Recent work has argued that the textbook search and matching model is capable of producing deep recessions and endogenous disaster dynamics (Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau et al., 2018). Our analysis shows this finding rests sensitively on the matching function choice. While the kurtosis moments capture some of this effect, Figure 3 provides further context by plotting generalized impulse responses of unemployment and the job finding rate to a 2 standard deviation negative labor productivity shock.[15] We illustrate the state-dependence of the responses by initializing each of the simulations at steady state ($u_0 = 5.9\%$) and a recession ($u_0 =$

[14]In addition to skewness in levels, recent papers have also considered the skewness of changes in unemployment, known as steepness asymmetry (Bernstein et al., 2021; Dupraz et al., 2019; Ferraro, 2017, 2018). These papers show additional model features are required to generate this feature of the data, regardless of the matching function choice.

[15]Following Koop et al. (1996), the response of x_{t+h} over horizon h is given by $\mathcal{G}_t(x_{t+h}|\varepsilon_{a,t+1} = -2, \mathbf{z}_t) = E_t[x_{t+h}|\varepsilon_{a,t+1} = -2, \mathbf{z}_t] - E_t[x_{t+h}|\mathbf{z}_t]$, where $\mathbf{z}_t$ is a vector of initial states and -2 is the shock size in period $t+1$.

8%). When we initialize the simulations at steady state, the choice of matching function is innocuous. This follows from the fact that both matching functions generate similar first-order dynamics.

Large differences in the responses emerge when the shock hits in a recessionary state. While both unemployment responses are larger due to the state-dependence in the law of motion, the DRW matching function generates almost a 50% larger peak response compared to the CD specification. The larger response is driven by a larger decline in the job finding rate, which is absent in the CD model and follows from the decline in the matching elasticity caused by the gross complementarity embedded in the matching function. This shows the DRW matching function is essential for the textbook labor search model to generate deep recessions and endogenous disaster dynamics.

Welfare Cost of Business Cycles To further examine the implications of the matching function, we compute the welfare cost of business cycles by implementing the experiment in Lucas (1987, 2003). We first compute the representative household's lifetime utility in an economy where consumption always equals its stochastic steady state, $\tilde{c}$. Then we compute expected welfare in the stochastic economy and solve for the percentage of stochastic consumption λ households would require to make them indifferent between the two consumption paths. Formally, $\lambda = 100 \times (\exp(\ln \tilde{c} - \frac{1-\beta}{1-\beta^{T-1}} \frac{1}{N_E} \sum_{j=1}^{N_E} E_0[\sum_{t=0}^{T} \beta^t \ln c_{j,t} | \mathbf{z}_{j,0}]) - 1)$, where $T = 3000$, $\mathbf{z}_{j,0}$ is the jth draw from the ergodic distribution with path $\{c_{j,t}\}$ and $N_E = 20{,}000$ is the number of draws.

With the CD matching function, we find $\lambda = 0.29\%$, so households require an additional 0.29% of consumption in each period to accept the fluctuations from business cycles. This number is over 5 times larger than the linear model (0.05%), which is equal to the welfare cost reported in Lucas (2003) when consumption is a Gaussian process. Under the DRW matching function, the cost of business cycles is 0.43%, almost 50% higher than the cost with the CD matching function. These results further emphasize the importance of the matching function for characterizing nonlinearities.

7 MODEL EXTENSIONS

Our key insights are robust to Nash bargaining and job separation rate shocks. In both models, the CD matching function continues to provide a better account of higher-order moments in the data.[16]

7.1 NASH BARGAINING

Under Nash bargaining, the wage rate is given by

$$w_t = \eta(a_t + \kappa(1 - \chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta)b. \tag{20}$$

Combine (20) with the vacancy posting condition and impose risk neutrality to obtain

$$\kappa/q_t = (1 - \eta)(a_t - b) + \beta E_t[x_{t+1}((1 - \bar{s})\kappa/q_{t+1} - \eta\kappa(1 - \chi\bar{s})\theta_{t+1})]. \tag{21}$$

[16] Pissarides (2009) and Petrosky-Nadeau et al. (2018) add a fixed hiring cost so the marginal cost of vacancy creation is $\kappa_0/q_t + \kappa_1$, rather than κ/q_t. Our analytical and numerical results are robust to this alternative specification.

Model Parameter		CD	DRW	Empirical Target	Data
Intra-Period Search Duration	χ	0.5372	0.5289	Average Unemployment Rate	5.89
Vacancy Posting Cost	κ	0.3214	0.2051	Average Job Finding Rate	42.14
Job Separation Rate	$\bar{s}$	0.0327	0.0327	Average Job Separation Rate	3.27
Disagreement Payoff	b	0.9280	0.9247	Unemployment Standard Deviation	23.66
Matching Elasticity	$\bar{\epsilon}_{m,u^s}$	0.5157	0.5860	Vacancy Standard Deviation	21.69
Bargaining Weight	η	0.1074	0.1139	Wage-Labor Productivity Elasticity	0.59
Productivity Persistence	ρ_a	0.9537	0.9537	Labor Prod. Autocorrelation	0.89
Productivity Standard Deviation	σ_a	0.0083	0.0083	Labor Prod. Standard Deviation	2.61

(a) Estimated parameter values. Both models perfectly match the empirical targets ($J = 0$).

	Full Sample		Pre-2009		CD		DRW	
Moment	Mean	SE	Mean	SE	1	2	1	2
$SD(f)$	14.96	1.47	12.42	0.92	17.93	17.88	14.05	14.01
$Corr(u,v)$	−0.74	0.04	−0.65	0.05	−0.89	−0.92	−0.85	−0.95
$Skew(u)$	1.10	0.35	0.88	0.37	1.13	0.17	1.98	0.80
$Skew(f)$	−0.60	0.45	0.03	0.36	−0.19	−0.18	−0.84	−0.83
$Kurt(u)$	1.05	0.57	0.38	0.47	2.08	−0.10	6.40	0.94
$Kurt(f)$	0.29	0.54	−0.63	0.25	−0.09	−0.09	1.09	1.05

(b) Untargeted moments. Models: (1) All nonlinear; (2) All nonlinear except linear law of motion.

Table 3: Model with Nash bargaining.

Since both the CD and DRW matching functions imply that θ_{t+1} is generally a nonlinear function of κ/q_{t+1}, (21) shows that the key effect of Nash bargaining is to make the marginal cost of hiring a nonlinear function of productivity. Importantly, the extent to which this nonlinearity matters is governed by the size of the Nash bargaining parameter η. To solve (21) in closed form, consider the DRW matching function with $\iota = 1$, so $q_t = 1/(1+\theta_t)$ and $\theta_t = 1/q_t - 1$. Then (21) becomes

$$\kappa/q_t = (1-\eta)(a_t - b) + \beta E_t[(1-\bar{s}-\eta(1-\chi\bar{s}))\kappa/q_{t+1} + \eta\kappa(1-\chi\bar{s})]. \tag{22}$$

We can once again guess and verify that (22) has a solution of the form,

$$\kappa/q_t = \delta_0 + \delta_1(a_t - \bar{a}),$$

where

$$\delta_0 = \frac{(1-\eta)(\bar{a}-b) + \beta\eta\kappa(1-\chi\bar{s})}{1-\beta(1-\bar{s}-\eta(1-\chi\bar{s}))}, \qquad \delta_1 = \frac{1-\eta}{1-\beta(1-\bar{s}-\eta(1-\chi\bar{s}))\rho_a}.$$

In this case, the marginal cost of hiring is a linear function of productivity. Consistent with the general case where $\iota \neq 1$, the extent to which Nash bargaining matters depends on the size of η.

We estimate the model under each matching function. Table 3a shows the parameter estimates and empirical targets, which both models perfectly match ($J = 0$). Importantly, the estimates of the matching elasticity, $\bar{\epsilon}_{m,u^s}$, are close to estimates in our baseline model, and the estimate of η are relatively small, suggesting that Nash bargaining will have a minor effect on our quantitative results.

We quantify this intuition by comparing Table 3b with Table 2b. Focusing on the higher-order moments, Nash bargaining has a relatively small effect compared to the matching function choice. For example, in the estimated DRW model (column DRW-1), Nash bargaining increases the skewness of unemployment by 4%, from 1.9 to 1.98. It is more powerful under the CD matching function (column CD-1), increasing the skewness of unemployment by 24% and generating modest negative skewness in the job finding rate. However, these effects are still much smaller than switching to the DRW matching function (comparing columns CD-1 and DRW-1), which increases the skewness of unemployment by 75% and more than quadruples the skewness of the job finding rate.

7.2 JOB SEPARATION RATE SHOCKS

We introduce job separation rate shocks, given their emphasis in the literature. Following Coles and Kelishomi (2018) and Mercan et al. (2021), the productivity and job separation rate processes are correlated. The two processes evolve according to

$$a_t = \bar{a} + \rho_a(a_{t-1} - \bar{a}) + \rho_{as}\sigma_s\varepsilon_{s,t} + \sigma_a\varepsilon_{a,t},\ 0 \le \rho_a < 1,\ \varepsilon_a \sim \mathbb{N}(0,1),$$
$$s_t = \bar{s} + \rho_s(s_{t-1} - \bar{s}) + \rho_{as}\sigma_a\varepsilon_{a,t} + \sigma_s\varepsilon_{s,t},\ 0 \le \rho_s < 1,\ \varepsilon_s \sim \mathbb{N}(0,1),$$

where ρ_{as} governs the correlation between the shocks. The vacancy creation condition becomes

$$\tfrac{\kappa-\lambda_{v,t}}{q_t} = a_t - w_t + E_t[x_{t+1}(1-s_{t+1})\tfrac{\kappa-\lambda_{v,t+1}}{q_{t+1}}]. \tag{23}$$

Unfortunately, this model does not have a closed-form solution, even under risk-neutrality. However, it is clear from (23) that the marginal cost of hiring becomes a nonlinear function of both shocks, whereas it was a *linear* function of only labor productivity in our baseline model. Thus, job separation rate shocks add a potentially important source of asymmetry to the labor market, in addition to the asymmetry from the matching function and the law of motion for unemployment.

Table 4a shows the parameter estimates for both models. Once again, they are perfectly identified. The estimated cross-correlation between the two shocks is negative, consistent with the literature. Most of the labor market parameters are similar to the model without job separation shocks. Notable exceptions are the steady-state matching elasticities, $\bar{\epsilon}_{m,u^s}$, which are slightly higher than the baseline estimates but still within the conventional range. This decreases ι, which suggests the DRW matching function will generate slightly weaker nonlinearities than in our baseline model.

Table 4b shows that adding job separation rate shocks improves both models' abilities to match the volatility of the job finding rate and the Beveridge curve. This further demonstrates that both models have similar first-order properties. However, even with a lower estimate of ι, the DRW model still overstates the skewness and kurtosis of unemployment and the job finding rate, while the CD model continues to provide a more realistic account of higher-order properties of the data.

Model Parameter		CD	DRW	Empirical Target	Data
Intra-Period Search Duration	χ	0.5214	0.5159	Average Unemployment Rate	5.89
Vacancy Posting Cost	κ	0.1401	0.0731	Average Job Finding Rate	42.14
Job Separation Rate	$\bar{s}$	0.0327	0.0327	Average Job Separation Rate	3.27
Disagreement Payoff	b	0.9418	0.9419	Unemployment Standard Deviation	23.66
Matching Elasticity	$\bar{\epsilon}_{m,u^s}$	0.6158	0.6716	Vacancy Standard Deviation	21.69
Bargaining Weight	η	0.5744	0.5744	Wage-Labor Productivity Elasticity	0.59
Productivity Persistence	ρ_a	0.9537	0.9537	Labor Prod. Autocorrelation	0.89
Productivity Standard Deviation	σ_a	0.0083	0.0083	Labor Prod. Standard Deviation	2.61
Separation Rate Persistence	ρ_s	0.8985	0.8985	Separation Rate Autocorrelation	0.79
Separation Rate Standard Deviation	σ_s	0.0012	0.0012	Separation Rate Standard Deviation	8.97
Shock Cross-Correlation	ρ_{as}	-0.0814	-0.0814	Prod. and Sep. Rate Correlation	-0.47

(a) Estimated parameter values. Both models perfectly match the empirical targets ($J = 0$).

	Full Sample		Pre-2009		CD		DRW	
Moment	Mean	SE	Mean	SE	1	2	1	2
$SD(f)$	14.96	1.47	12.42	0.92	14.41	14.29	13.04	12.85
$Corr(u,v)$	-0.74	0.04	-0.65	0.05	-0.75	-0.74	-0.73	-0.75
$Skew(u)$	1.10	0.35	0.88	0.37	0.88	0.11	1.41	0.46
$Skew(f)$	-0.60	0.45	0.03	0.36	-0.19	-0.17	-0.80	-0.75
$Kurt(u)$	1.05	0.57	0.38	0.47	1.35	-0.07	3.80	0.38
$Kurt(f)$	0.29	0.54	-0.63	0.25	0.00	-0.03	1.05	0.90

(b) Untargeted moments. Models: (1) All nonlinear; (2) All nonlinear except linear law of motion.

Table 4: Model with productivity and separation rate shocks.

8 CONCLUSION

Analyzing macroeconomic nonlinearities through the lens of search and matching models is an exciting and growing area of research. We contribute to this enterprise by analyzing the implications of a key model ingredient: the matching function. Using closed-form analytics and an estimated model, we show the matching function choice greatly affects the model's nonlinearities, in contrast to the prevailing view that it is innocuous. The matching function is a powerful source of nonlinearity when it features a procyclical matching elasticity. This is the case for the DRW matching function, but not the CD matching function. A quantitative assessment of the CD and DRW matching functions shows the former provides a better account of nonlinearities in the U.S. labor market.

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A Data Sources and Transformations

We use the following time-series from 1955-2019 provided by Haver Analytics:

1. **Labor Productivity**, Non-farm Business Sector, All Persons,
   Seasonally Adjusted, Quarterly, 2012=100 (LXNFS@USECON)
2. **Labor Share**, Non-farm Business Sector, All Persons,
   Seasonally Adjusted, Quarterly, Percent (LXNFBL@USECON)
3. **Job Openings**, Job Openings and Labor Turnover Survey,
   Seasonally Adjusted, Monthly, Thousands (LJJTLA@USECON)
4. **Unemployed, 16 Years & Over**
   Seasonally Adjusted, Monthly, Thousands (LTU@USECON)
5. **Civilian Labor Force: 16 yr & Over**
   Seasonally Adjusted, Monthly, Thousands (LF@USECON)
6. **Civilians Unemployed for Less Than 5 Weeks**
   Seasonally Adjusted, Monthly, Thousands (LU0@USECON)

We also use the **Help Wanted Advertising Index (HWI)** from Barnichon (2010), which is in units of the labor force. The series corrects for online advertising and is available on the author's website.

We applied the following transformations to the above data sources:

1. **Unemployment Rate**: $u_t = 100(LTU_t/LF_t)$.
2. **Vacancy Rate**: HWI from 1954M1-2000M12 and $LJJTLA/LF$ from 2001M1-2019M12.
3. **Short-term Unemployed** (U^s): The redesign of the Current Population Survey (CPS) in 1994 reduced U_t^s. To correct for this bias, we follow Elsby et al. (2009) and scale $LU0$ by the time average of the ratio of $LU0/LTU$ for the first and fifth rotations groups to $LU0/LTU$ across all rotation groups. Using IPUMS-CPS data, we extract EMPSTAT ("Employment Status"), DURUNEMP ("Continuous weeks unemployed") and MISH ("Month in sample, household level"). Unemployed persons have EMPSTAT equal to 20, 21, or 22. Short-term unemployed are persons who are unemployed and have DURUNEMP equal to 4 or less. Incoming rotation groups have MISH equal to 1 or 5. Using the final weights, WTFINL, we calculate the $LU0/LTU$ for the first and fifth rotations groups by conditioning on the appropriate values of MISH and DURUNEMP. We then apply the X-12 seasonal adjustment function in STATA. Finally, we take an average of the seasonally adjusted series from 1994-2019 and divide by the average of $LU0/LTU$ across all rotation groups. This process yields a scale factor of 1.1725. Therefore, U^s equals $LU0$ prior to 1994 and $1.1725 \times LU0$ after 1994.

4. **Job Finding Rate**: $f_t = 1 - (LTU_t - U_t^s)/LTU_{t-1}$.

5. **Real Wage**: $w_t = LXNFBL_t \times LXNFS_t$.

6. **Wage Elasticity**: Slope coefficient from regressing w_t on an intercept and $LXNFS_t$.

7. **Unemployed Searching**: $u_t^s = \chi s_t + (1 - \chi s_t)u_{t-1}$, where $\chi = 0.5$ (Shimer, 2005).

8. **Labor Market Tightness**: $\theta_t =$ Vacancy Rate$/u_t^s$.

9. **Job Separation Rate**: $s_t = 1 - \exp(-\tilde{s}_t)$, where $\tilde{s}_t$ satisfies

$$LTU_{t+1} = \frac{(1 - \exp(-\tilde{f}_t - \tilde{s}_t))\tilde{s}_t LF_t}{\tilde{f}_t + \tilde{s}_t} + \exp(-\tilde{f}_t - \tilde{s}_t)LTU_t, \quad \tilde{f}_t = -\log(1 - f_t).$$

B SOLUTION METHOD

Our baseline model only includes productivity shocks. This section describes the more general problem that also includes job separation rate shocks. The nonlinear equilibrium system is given by

$$E_t[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \mathcal{E}_{t+1})|\mathbf{z}_t, \mathcal{P}] = 0,$$

where g is a vector-valued function, $\mathbf{x}_t$ is a vector of variables, $\mathcal{E}_t = \{\varepsilon_{a,t}, \varepsilon_{s,t}\}$ is a vector of shocks, $\mathbf{z}_t$ is a vector of endogenous and exogenous state variables, and $\mathcal{P}$ is a vector of model parameters.

The bounds on the state variables, a_t, s_t, and n_{t-1} are set to $[0.925, 1.075]$, $[0.024, 0.041]$, and $[0.85, 0.9999]$, which contain at least 99% of the ergodic distribution. We discretize each state into 11 evenly-spaced points. The product of the points in each dimension, D, is the total nodes in the state space ($D = 1{,}331$). The realization of $\mathbf{z}_t$ on node d is denoted $\mathbf{z}_t(d)$. We discretize the exogenous states separately from the shocks, $\varepsilon_{a,t+1}$ and $\varepsilon_{s,t+1}$, which are discretized according to Gauss-Hermite quadrature for standard-normal i.i.d. random variables using 7 points (i.e., $\pm 2.65SD$). The Gauss-Hermite method provides integration weights, $\phi(m)$, for $m \in \{1, \ldots, M\}$.

Since vacancies $v_t \geq 0$, we introduce an auxiliary variable, μ_t, such that $v_t = \max\{0, \mu_t\}^2$ and $\lambda_{0,t} = \max\{0, -\mu_t\}^2$, where $\lambda_{v,t}$ is the Lagrange multiplier on the non-negativity constraint. If $\mu_t \geq 0$, then $v_t = \mu_t^2$ and $\lambda_{v,t} = 0$. When $\mu_t < 0$, the constraint is binding, $v_t = 0$, and $\lambda_{v,t} = \mu_t^2$. Therefore, the constraint on v_t is transformed into a pair of equalities (Garcia and Zangwill, 1981).

The following steps outline our nonlinear policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the linearized model. Then map the solution for the policy functions to the discretized state space. This provides an initial conjecture.

2. On iteration $j \in \{1, 2, \ldots\}$ and each node $d \in \{1, \ldots, D\}$, use Chris Sims's `csolve` to find $\mu_t(d)$ to satisfy $E[g(\cdot)|\mathbf{z}_t(d), \mathcal{P}] \approx 0$. Guess $\mu_t(d) = \mu_{j-1}(d)$. Then apply the following:

   (a) Solve for all variables dated at time t, given $\mu_t(d)$ and $\mathbf{z}_t(d)$.

(b) Linearly interpolate the policy function, μ_{j-1}, at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mu_{t+1}(m)$ on every integration node, $m \in \{1,\ldots,M\}$.

(c) Given $\{\mu_{t+1}(m)\}_{m=1}^{M}$, solve for the other elements of $\mathbf{x}_{t+1}(m)$ and compute

$$E[g(\mathbf{x}_{t+1},\mathbf{x}_t(d),\mathcal{E}_{t+1})|\mathbf{z}_t(d),\mathcal{P}] \approx \textstyle\sum_{m=1}^{M} \phi(m) g(\mathbf{x}_{t+1}(m),\mathbf{x}_t(d),\mathcal{E}_{t+1}(m)).$$

Set $\mu_j(d) = \mu_t(d)$ when `csolve` converges.

3. Repeat step 2 until $\text{maxdist}_j < 10^{-6}$, where $\text{maxdist}_j \equiv \max\{|\mu_j - \mu_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate nonlinear solution.

The algorithm is programmed in Fortran with Open MPI and run on the BigTex supercomputer.

C ADDITIONAL RESULTS

Model Parameter		CD	DRW	Empirical Target	Data
Intra-Period Search Duration	χ	0.5463	0.5405	Average Unemployment Rate	5.77
Vacancy Posting Cost	κ	0.3219	0.2022	Average Job Finding Rate	44.32
Job Separation Rate	$\bar{s}$	0.0345	0.0345	Average Job Separation Rate	3.45
Disagreement Payoff	b	0.9278	0.9280	Unemployment Standard Deviation	21.53
Matching Elasticity	$\bar{\epsilon}_{m,u^s}$	0.5330	0.6050	Vacancy Standard Deviation	21.25
Bargaining Weight	η	0.5861	0.5862	Wage-Labor Productivity Elasticity	0.60
Productivity Persistence	ρ_a	0.9517	0.9517	Labor Prod. Autocorrelation	0.89
Productivity Standard Deviation	σ_a	0.0090	0.0090	Labor Prod. Standard Deviation	2.78

(a) Estimated parameter values. Both models perfectly match the empirical targets ($J = 0$).

	Full Sample		Pre-2009		CD		DRW	
Moment	Mean	SE	Mean	SE	1	2	1	2
$SD(f)$	14.96	1.47	12.42	0.92	16.56	16.47	12.45	12.41
$Corr(u,v)$	−0.74	0.04	−0.65	0.05	−0.91	−0.93	−0.87	−0.95
$Skew(u)$	1.10	0.35	0.88	0.37	0.87	0.03	1.67	0.68
$Skew(f)$	−0.60	0.45	0.03	0.36	−0.07	−0.04	−0.76	−0.71
$Kurt(u)$	1.05	0.57	0.38	0.47	1.15	−0.16	4.49	0.56
$Kurt(f)$	0.29	0.54	−0.63	0.25	−0.16	−0.14	0.80	0.64

(b) Untargeted moments. Models: (1) All nonlinear; (2) All nonlinear except linear law of motion.

Table 5: Baseline model, pre-Great Recession sample (1955-2008).