Exit Expectations, Time Inconsistency, and the Optimal Design of a Currency Union

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Abstract

In a currency union, monetary policies are determined collectively by the member countries. For countries' lacking commitment, one benefit of belonging to a currency union is that a country's monetary policy gains credibility by loosing their monetary autonomy. Focusing on this benefit of belonging to a currency union, this paper analyses the effect of an exogenous rise in the expectation of members' exit on the currency union's optimal design. Our two main results are as follows. First, higher expectations of a country's exit enable the public to anticipate a higher probability of domestic discretionary policymaking in the future. Since the public knows that the authority has an incentive to generate surprise inflation, we have a higher expected inflation rate accompanied by a lower growth rate at the rational expectation equilibrium. Second, a higher exit expectations of advanced countries increases the optimal share of developing countries in the currency union from the advanced countries' perspective. In other words, unlike the traditional theory of optimal currency areas, a higher exit probability requires a greater portion of member countries with dissimilar economic backgrounds to be in the currency area. (JEL: E5; E4)

Keywords: Optimal currency area; monetary union; inflationary bias; commitment

1 Introduction

This paper examines the effect of exit expectations on the optimal design of a currency union. We consider a currency union in which member countries belong to the currency

union to improve its credibility. Incorporating exit expectations is useful to better understand the workings of currency unions for the following reasons.

First, inflation expectations are well-known to play a central role in determining the effectiveness of monetary policy, although few studies have investigated the effects of exit expectations.[1] In a currency union, a high exit probability implies that the public sector expects a high probability of domestic monetary policymaking (rather than collective policymaking in the union) in the future. Thus, the possibility of members' exit affects economic outcomes through changing public's expectations.

Second, theoretical, empirical, and experimental studies on collective decision-making have demonstrated that the outcomes of collective decisions depend on institutional design including exit rules.[2] In a currency union, the exit of a developing country implies the fewer members prefer a low inflation rate in the future. Therefore, the possibility of exits affects economic outcomes by changing the bargaining power balance among the members.

Also, exits from the European Monetary Union (EMU) have been precipitated an enormous public debate over the last decade. Some politicians in European countries have advocated withdrawal from the EMU, and the price of a bet (Figure 1) is positive and changes depending on the political-economic circumstance at the time. Regardless of its actual political feasibility, those changes in public expectations influence monetary policy and future economic outcomes.

Using these discussions as a motivation, this paper studies the effect of a positive exit possibility on the monetary policy in a currency union and its optimal design. We consider a currency union consists of countries that have an incentive to implement surprise inflation, and belonging to a currency union only eliminates member countries' ability to control monetary policies for their own sakes. This inflexibility can improve welfare by improving the credibility of monetary policy. Thus, similar to Dixit (2000); Chari et al. (2020); Alesina and Barro (2002), credibility is the main source of benefit from belonging to a currency union. By exiting a currency union, on the other hand, a country obtains the control over its monetary policy. Thus, in this setting, the exit option is appealing to a country facing a high ex-post temptation to implement surprising inflation.

We build a model consists of countries in advanced and developing region, and consider the advanced countries' optimal design of a currency union.[3] Those regions are only

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[1]For instance, the canonical works of Barro and Gordon (1983a,b) show that a time-inconsistency problem arises when a monetary authority has the flexibility to alternate policies but does not have commitment power.

[2]As Olson (1965) famously wrote "the movement in and out of the group must no longer be ignored".

[3]In Section 4, we examine the opposite scenario in which developing countries design a currency union
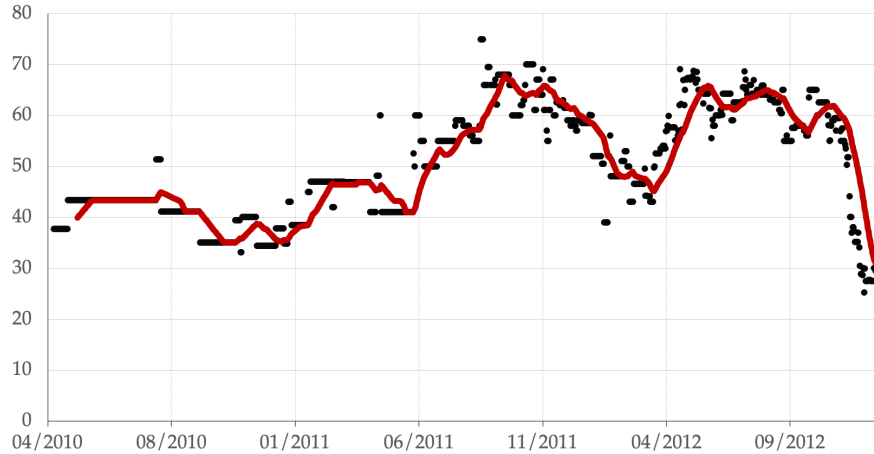
Figure 1: The prices of a bet "any country currently using the Euro to announce intention to drop it before midnight ET 31 Dec 2014" on the trading exchange website *Intrade.com*. The red-line plots a 30 days moving average of the prices. Data is available at `http://intrade-archive.appspot.com/event.jsp?event=79890`.

heterogeneous in the expected value of output shock, which becomes a source of conflicts in the common interest rate choice. With this model, we investigate the optimal share of developing countries—the share of developing countries in a currency union—from advanced countries' perspectives. Our focus is to find out the effect of increasing exit probability on the optimal share of developing countries.

Our main results are twofold. First, we show that the optimal share of developing countries is positive meaning that advanced countries' welfare improves by allowing some advanced countries to belong to the currency union. This result is straightforward. Adding new developing countries to the currency union increases the ex-post share of output-growing members that prefer a lower level of inflation, so the committee selects a lower common interest rate. The public sector anticipates this mechanism, which decreases the expected inflation.

Second, we show that a higher exit probability increases the optimal share of developing countries. By exiting a currency union, a country obtains its own monetary authority; thus, they may implement discretionary monetary policy. Therefore, inflationary bias arises under autarky. Under a public sector's rationally expectations, expected inflation rises as the exit probability increases. As a result, a higher exit probability requires a larger share of developing countries to decrease expected inflation to the optimal level.

Our findings contribute contribute to the theory of optimal currency areas by adding a new dimension for discussions—adding exit probability induces the optimal currency

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that advanced countries enter.

area to include more dissimilar member countries. Classical theory in the tradition of Mundell (1961) has considered that governments face a trade-off determining whether to join a currency union—they must weigh the benefits of facilitating trade and financial transactions against the costs of losing their independent monetary authority. Under this assumption, greater wage flexibility (or labor mobility) increases the benefits of belonging to the currency union, while more similar economic conditions between member countries reduce the cost of losing monetary policy independence. In other words, without non-monetary benefits, there are no incentives for a country to join a currency union.

The Mundell's criterion assumes the governments' commitment; thus, they do not suffer from the time-inconsistency of monetary policymaking studied by Kydland and Prescott (1977); Barro and Gordon (1983a). Assuming limited commitment power, many papers have pointed out that belonging to a currency union can mitigate the time-inconsistency problem. Alesina and Barro (2002) showed that countries suffering commitment issues gain more benefit from belonging to a currency union by obtaining a more credible common currency. Chari et al. (2020) demonstrated that countries with dissimilar temptation shocks, which exacerbate time inconsistency problems as in Barro and Gordon (1983a), should form a currency union. They showed that relatively credible countries, such as Germany, can raise credibility by allowing less credible countries, such as Southern European countries, to belong the currency union.

Similar to Chari et al. (2020), we consider countries without commitment and show that the optimal currency union consists of countries with dissimilar economic environments. Unlike Chari et al. (2020), our focus is to study the effect of exit probability on the optimal currency union. We demonstrate that an increase in exit expectations increases the share of less credible member countries in the optimal currency union.

This paper is related to the large literature on discretionary monetary policymaking without commitment. The early literature focuses on incentive design by a single central banker (Barro and Gordon, 1983b; Chari and Kehoe, 1990; Rogoff, 1985; Stokey, 1989; Walsh, 1995). In particular, Rogoff (1985) showed that an inflationary bias can be reduced by delegating a conservative central banker who cares relatively little about unemployment or output stimulation. Recent literature has investigated institutional design of monetary policy committees to improve a monetary authority's credibility (Bullard and Waller, 2004; Sibert, 2003; Riboni and Ruge-Murcia, 2010; Riboni, 2010; Eslava, 2010; Dal Bo, 2006; Mihov and Sibert, 2006). Another branch of the literature has examined institutional design in single-country New-Keynesian models (Riboni and Ruge-Murcia, 2008b; Hahn, 2016; Hefeker and Zimmer, 2015) and currency union models (Von Hagen and Süppel, 1994; Aaron-Cureau and Kempf, 2006; Saito, 2018; Farvaque and Matsueda,

2009).

Finally, our paper is related to studies on regime changes in a currency union. Eijffinger et al. (2018); Kriwoluzky et al. (2019); Na et al. (2018); Schmitt-Grohé and Uribe (2016) studied the sovereign debd default and exit from a currency union in a small open New Keynesian economy. Unlike these papers, this paper focuses on the monetary aspects of a currency union and investigates the effect of exogenous exit probability on its optimal design. Similar to Kriwoluzky et al. (2019), we suppose that exit probability is exogenously given and abstract exit contamination among members.[4]

The remainder of this paper is organized as follows. Section 2 presents our currency union model and solves policy outcomes within the currency union and under autarky. Section 3 derives the main results describing the effect of exit possibility on outcomes and the optimal currency union. Section 4 discusses several extensions to the model. Finally, Section 5 concludes the paper and describes promising future research directions.

2   The model

This section presents our currency union model under exit expectations. A currency union consists of member countries with heterogeneous output shocks. Time is discrete and indexed by t. There are two regions—advanced region A and developing region B. The currency union is consists of continuum of measure ω^A countries in region A and continuum of measure ω^B countries region B, where $\omega^A + \omega^B = 1$.

Among the regions, countries are ex-ante heterogeneous in terms of their expectations of region-specific output shocks (v^j, γ^j); where $v^j \in \mathcal{U}$ is an i.i.d. region-specific supply shock, and $\gamma^j \in \mathcal{G}$ is the i.i.d. region specific demand shock. Within each region, countries are ex-ante identical but ex-post heterogeneous in terms of country-specific shocks (η, ϵ); here, $\eta \in \mathcal{H}$ is the i.i.d. country-specific supply shock, and $\epsilon \in \mathcal{E}$ is the i.i.d. country-specific demand shock. Let E be the expectation operator. To simplify the analysis, we assume that shocks satisfy the following condition.

Assumption 1.

$$E[v^B] > E[v^A] = 0,$$

$$E[\gamma^A] = E[\gamma^B] = 0,$$

$$E[\eta] = E[\epsilon] = 0.$$

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[4]Eijffinger et al. (2018) considered the contagion of exits between members. Drazen and Masson (1994); Obstfeld (1996) studied models with self-fulfilling exits.

Under Assumption 1, the developing region is expected to draw a higher supply shock, while the expected level of the demand shock is the same among all nations. For all shocks $x \in \{v^A, \gamma^A, v^B, \gamma^B, \eta, \epsilon\}$, we denote their probability as $f_x(x)$ and their variance as $Var_x(x)$.

Countries lack commitment ability and therefore have an incentive to implement surprise inflate economy, a la Barro and Gordon (1983a), if they fo not belong to the currency union. In the currency union, members collectively choose a common interest rate; thus, each country does not have the flexibility of implementing monetary policy for its own sake. This inflexibility can improve a country's credibility by influencing public's expectations. We focus on this aspect of a currency union—the benefits (and costs) of belonging a currency union are only come from not having an independent monetary authority. We suppose that the interest rate in the currency union is determined by bargaining among all members; therefore, the policy outcome in the currency union depends on the balance of the bargaining power among the member countries,

After observing the common monetary policy, member countries can exit from the currency union. For the sake of tractability, we assume that countries' regimes are exogenously determined by regime shock s; each country's regime, $s \in \{Exit, Union\}$, is determined with probability p^j and $1 - p^j$, respectively. We also suppose that p^j is independent of any output shock. In this setting, an increase in the exit probability can be interpreted as the spread of exit rumors, which does not necessarily reflect actual economic incentive.

We examine the advanced countries' optimal design of the share of developing countries within the currency union, ω^B. With an optimally designed institution, advanced countries can achieve the commitment outcome without having commitment powers. Our focus is to investigate the effect of rising exit expectations on the optimal share of ω^B.

The timing of events is summarized as follows.

Stage 0. Given (p^A, p^B), advanced countries determine the share of developing countries in the currency union ω^B.

Stage 1. The public rationally forms an expectation regarding the inflation rate.

Stage 2. Output shocks are determined.

Stage 3. The member countries collectively determine the currency union's interest rate, i^{cu}.

Stage 4. Regimes are determined.

Stage 5. Policy implementation:

1. If $s = Exit$, the country discretionarily choose an interest rate, i_{exit}^{j}.

2. If $s = Union$, the country implements the currency union's interest rate, i^{cu}.

In Section 2.1, we describe the economic structure of the model; in Section 2.2, we define the objective of monetary authorities; in Section 2.3, we examine the policy outcomes; and in Section 3, we study the outcomes at the rational expectations equilibrium.

2.1 Economy

The output demand of a country in region j is as follows:

$$y^{j} = -\alpha \left(i - \pi^{j} \right) + \gamma + \epsilon^{j}, \tag{1}$$

where i is the nominal interest rate, π^{j} is the inflation rate, and $\alpha > 0$ is the interest rate elasticity of output demand. The output supply of a country in region j is as follows:

$$y^{j} = \beta \left(\pi - \pi_{e}^{j} \right) + \eta + v^{j}, \tag{2}$$

where π_{e}^{j} is the expected inflation rate, and $\beta > 0$ is the inflation rate elasticity of the output supply. For simplicity, we assume that α and β are the same across countries, and to rule out unrealistic behavior of inflation and output, we suppose that both α and β are positive and satisfy the condition $\beta > \alpha$. Then, we have the following:

$$y^{j} = y^{j}(i; \pi_{e}^{j}, \gamma, \eta) = \frac{\beta \left(-\alpha i + \alpha \pi_{e}^{j} + \left(\gamma + \epsilon^{j} \right) \right) - \alpha \left(\eta + v^{j} \right)}{\beta - \alpha}, \tag{3}$$

$$\pi^{j} = \pi^{j}(i; \pi_{e}, \gamma, \eta) = \frac{-\alpha i + \beta \pi_{e}^{j} + \gamma + \epsilon^{j} - \left(\eta + v^{j} \right)}{\beta - \alpha}. \tag{4}$$

2.2 Monetary policy objective

Monetary policy in the currency union is conducted by a committee consisting of each country's representatives. The per-period social loss function of a member in region j is

given by the following:

$$L^j(i; \pi_e^j, \gamma, \eta) = \frac{\lambda}{2}\left(y^j(i; \pi_e^j, \gamma, \eta) - k\right)^2 + \frac{1}{2}\pi^j(i; \pi_e^j, \gamma, \eta)^2, \tag{5}$$

where $k \geq 0$ is the target output rate and $\lambda \geq 0$ is the weight on the output rate. We suppose that all of the shocks are publicly observable, meaning that there is no information uncertainty about preferences and economic conditions.

Given the expected inflation rate π_e and the realized economics shocks, a country's ex-post optimal interest rate, $i_*^j(\pi_e^j, \gamma, \eta)$, minimizes Eq.(5) subject to Eqs. (3)-(4). In an interior solution, it holds that:

$$i_*^j(\pi_e^j, \gamma, \eta) = \frac{\beta(1 + \alpha\beta\lambda)\pi_e^j - \beta(\beta - \alpha)\lambda k - (1 + \alpha\beta\lambda)\left(\eta + v^j\right)}{\alpha\left(1 + \beta^2\lambda\right)} + \frac{\gamma + \epsilon^j}{\alpha}. \tag{6}$$

Each country's ex-post bliss interest rate depends on the expected inflation rate, output target, and supply and demand shocks. Since the shocks are heterogeneous among countries, the optimal interest rates are heterogeneous among members' representatives as well. In our model, this heterogeneity is the only source of conflicts in the currency union in which each member adopts a common interest rate.

2.3 Policy outcomes

A country under autonomy discretionarily chooses its monetary policy; therefore, its optimal interest rate (6) is implemented. Thus, it holds that:

$$i_{exit}^j(\pi_e^j, \gamma, \eta) = i_*^j(\pi_e^j, \gamma, \eta). \tag{7}$$

Substituting the bliss interest rate (6) into Eq. (4), the country's bliss inflation rate under autonomy is given by the following:

$$\pi^j(\pi_e^j, \eta) = \frac{\beta^2\lambda\pi_e^j + \beta\lambda k - \beta\lambda\left(\eta + v^j\right)}{1 + \beta^2\lambda}. \tag{8}$$

The union's policy objective does not reflect domestic interests. It is straightforward that an interior solution to the bargaining problem, i_{cu}, coincides with the weighted sum of the ex-post optimal interest rates among members. The interest rate i_*^{cu} minimizes:

$$\sum_{j\in\{A,B\}} \omega^j \int_{\gamma\in\mathcal{G}} \int_{\eta\in\mathcal{H}} L^j\left(i_{cu}; \pi_e^j, \gamma, \eta\right) f_\eta(\eta)f_\gamma(\gamma)d\gamma d\eta, \tag{9}$$

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where $\omega^A + \omega^B = 1$. To avoid unnecessary complication, we impose the following assumption.

Assumption 2. *The currency union is large enough so that country-specific shocks offset each other within a region:* $\int \gamma f(\gamma) d\gamma = E[\gamma]$ *and* $\int \eta f(\eta) d\eta = E[\eta]$.

Under Assumption 2, Eq. (9) is written as follows:

$$\sum_{j \in \{A,B\}} \omega^j L^j \left(i_{cu}; \pi_e^j, E[\gamma], E[\eta] \right). \tag{10}$$

Therefore, the union's policy objective does not reflect domestic interests. It is straightforward that an interior solution to the bargaining problem, i_{cu} , coincides with the weighted sum of the ex-post optimal interest rate among members. Therefore,

$$i_{cu} = \sum_{j \in \{A,B\}} \omega^j i_*^j (\pi_e^j, E[\gamma], E[\eta]). \tag{11}$$

The policy outcome i_{cu} reflects the views of all committee members. This rule is similar to monetary policymaking process in the EMU in which monetary policy responds not only to the larger countries, but also to the economic environment of the entire monetary union.[5]

By plugging (6) into (11), we obtain the following:

$$i_{cu} = i_*^m(\pi_e^m; E[\gamma], E[\eta]) = \frac{\beta(1 + \alpha\beta\lambda)\pi_e^m - \beta(\beta - \alpha)\lambda k - (1 + \alpha\beta\lambda)(E[\eta] + v^m)}{\alpha(1 + \beta^2\lambda)} + \frac{E[\gamma] + \epsilon^m}{\alpha}, \tag{12}$$

where $v^m = (1 - \omega)(v^A + E[\gamma]) + \omega(v^B + E[\gamma])$ and $\epsilon^m = (1 - \omega)(\epsilon^A + E[\eta]) + \omega(\epsilon^B + E[\eta])$. Therefore, the interest rate in the currency union coincides with the optimal interest rate of weighted average member m. Substituting the interest rate (12) in the union into (4) yields a country's inflation rate as follows:

$$\pi_{cu}^j = \pi_{cu}(i_{cu}, \pi_e^j; \gamma, \eta) = \frac{1}{\beta - \alpha}\left(-\alpha i_{cu} + \beta\pi_e^j + (\gamma + \epsilon^j) - (\eta + v^j)\right), \tag{13}$$

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[5]Empirical studies on the ECB' collective decisions, such as Mihov (2001) and Sturm and Wollmershäuser (2008), support the consensus rule in policymaking that advanced countries do not have extensive weights on policymaking; hence the policies do not only reflect the economic conditions of those countries. Riboni and Ruge-Murcia (2010) empirically show that policymaking in the Bank of Canada, the Bank of England, the European Central Bank, the Swedish Riksbank, and the U.S. Federal Reserve are best described by a consensus rule.

$$y_{cu}^j = y_{cu}(i_{cu}, \pi_e^j; \gamma, \eta) = \frac{\beta}{\beta - \alpha} \left(-\alpha i_{cu} + \alpha \pi_e^j + \left(\gamma + \epsilon^j \right) - \frac{\alpha}{\beta} \left(\eta + v^j \right) \right). \quad (14)$$

In comparison to the inflation rate under autonomy (8), the inflation rate in the union (13) depends on the entire union's economic shocks through the common interest rate, i_{cu}.

3 Main results

This section presents (i) the outcomes under rational expectations and (ii) the optimal design of the currency union. In Section (3.1), we consider a currency union in which the member countries do not have exit options; therefore, the exit probability is zero for all members. In Section (3.2), we examine the effect of increasing exit probability on the outcomes and structure of the optimal currency union.

3.1 No-exit benchmark

We first study a benchmark at which members cannot exit from the currency union. By imposing public sectors' rational expectations into the m's optimal inflation (13), we obtain the average member's expected inflation rate: $\pi_e^m = \beta \lambda \left(k - E\left[v^m \right] \right)$. Substituting it into (12) yields the currency union's interest rate at the rational expectation equilibrium as follows:

$$i_*^{cu} = \beta \lambda k - \frac{(1 + \lambda \alpha \beta) v^m + \epsilon^m}{\alpha}. \quad (15)$$

With (15), an arbitrary member in region j's inflation rate is written as follows:

$$\pi_{cu}^j(\pi_e^j; \gamma, \eta) = \frac{\beta \left(\pi_e^j + \alpha \lambda k \right) + \left(\frac{(1 - \beta^2 \lambda)(1 + \alpha \beta \lambda)}{1 + \beta^2 \lambda} \right) v^m - \epsilon^m - \left(\eta + v^j - (\gamma + \epsilon^j) \right)}{\beta - \alpha}. \quad (16)$$

Without the possibility of exit, the public sectors' rational expectation implies that $\pi_e^A = E[\pi^A]$. Arranging Eq. (16) yields the following:

$$\pi_e^A = \beta \lambda k - \frac{(1 + \alpha \beta \lambda) \omega^B E[v^B]}{\alpha}. \quad (17)$$

Note that under Assumption 1, we have $E[v^m] = \omega E\left[v^B \right]$, $\epsilon^m = 0$ and $v^m = \omega v^B$. Eq. (17) implies that if the currency union consists of only advanced countries ($\omega^B = 0$), then

the expected inflation rate $(\beta \lambda k > 0)$ surpasses the optimal inflation rate (0);. Therefore, the member countries suffer from the time-inconsistency problem.

Proposition 1. *In a currency union without exit options, the expected inflation rate (17) equal to its optimal level (0) if:*

$$\omega_*^B = \frac{\alpha \beta \lambda k}{(1 + \alpha \beta \lambda) E[v^B]} > 0. \tag{18}$$

If $\omega^B = \omega_*^B$, we have:

$$E[\pi_{cu}^A] = 0, \ E[y_{cu}^A] = 0. \tag{19}$$

These results say that if $\omega^B = \omega_*^B$, the advanced countries' expected outcome coincides with the outcome that occurs when they can commit to the optimal inflation rate. In such a case, the advanced countries' expected inflation rate and output rate are equal to the optimal levels, zero. In other words, advanced countries' expected social loss is minimized if their expected inflation rate (17) is zero.

Proposition 1 establishes an important benchmark for the results that follow and entails the following economic implications. First, the positivity of the optimal measure (18) means that advanced countries are better off forming a currency union with developing countries. In other words, in contrast to the traditional optimal currency area theories, it shows the positive impact of having members with heterogeneous economic backgrounds. Allowing developing countries to join a currency union can reduce the inflation bias and raise the credibility of the common monetary policy as well as the advanced countries' social welfare.

Second, the magnitude of the optimal measure ω_*^B positively depends on the output target k. A greater temptation of surprise inflation increases the advanced countries' optimal inflation. Consequently, a higher share of developing countries is required to achieve the advanced countries' optimal inflation level. At the same time, the value of ω_*^B negatively depends on the developing region's expected supply shock $E[v^B]$. A higher expected shocks decreases their bliss inflation, resulting in less measure on them to achieve the advanced countries' optimal outcome.

3.2 Exit expectations

We now depart from the no-exit assumption and consider the possibility of exit from the currency union. Specifically, we investigate the effect of a marginal increase in the advanced countries' exit possibility on the optimal measure of the developing countries,

ω_*^B. As in Section (3.1), with the optimal measure, advanced countries can achieve their bliss expected inflation rate; thus, it is optimal from the advanced countries' perspective.

Let $p^j \in [0,1)$ be countries in region j's ex-ante probability of exiting from the currency union. Note that countries are identical within each region before realizing country-specific shocks. With the possibility of exit, the public sector's rational expectation implies the following:

$$\pi_e^j = p^j E[\pi^j(\pi_e^j, \eta)] + (1 - p^j) E[\pi_{cu}^j(\pi_e^j; \gamma, \eta)]. \tag{20}$$

Here, the expectation operators integrate all possible country-specific supply shocks. By plugging in the inflation rate under autonomy (8) and in the currency union (13) into Eq. (20), we have the expected inflation rate under rational expectations as follows:

$$\pi_e^A = \beta \lambda k - \frac{(1 - p^A)(1 + \lambda \beta^2)(1 + \alpha \beta \lambda)\omega^B E[v^B]}{\beta(1 + \alpha \beta \lambda)p^A + \alpha(1 + \beta^2 \lambda)}. \tag{21}$$

Proposition 2. *A higher probability of exit increases the inflation rate and decreases the output gap.*

$$\frac{d\pi_e^A}{dp^A} = \frac{(\alpha + \beta + 2\alpha\beta^2\lambda)(1 + \lambda\beta^2)(1 + \alpha\beta\lambda)\omega E[v^B]}{(\alpha(1 + \beta^2\lambda) + \beta(1 + \alpha\beta\lambda)p^A)^2} > 0, \tag{22}$$

$$\frac{\partial \pi_{cu}^A(i_{cu*}; \gamma, \eta)}{\partial p^A} = \left(\frac{\beta^2\lambda}{1 + \beta^2\lambda}\right)\frac{\partial E[\pi^A]}{\partial p^A} > 0, \tag{23}$$

$$\frac{\partial y_{cu}^A(i_{cu}, \gamma, \eta)}{\partial p^A} = -\left(\frac{\beta}{(1 + \beta^2\lambda)}\right)\frac{dE[\pi^A]}{dp} < 0. \tag{24}$$

This finding shows that the exit probability is an additional source of inflation bias associated with a negative output gap (stagflation). Intuitively, country j's higher exit probability implies a higher probability of country j implementing its own optimal policy i_t^j under autonomy. As a result, since $E[i_t^{CU}] < E[i_t^A]$ and $E[\pi_t^A(i_t^{CU})] < E[\pi_t^A(i_t^A)]$ under Assumption 1, the public sector expects a higher expected inflation rate as the exit probability increases.

Similar to the results in Section 3-1, if the expected inflation rate (21) equals zero, the expected output gap equals zero as well. Therefore, the expected social loss is minimized. Rearranging the expected inflation rate (21) leads to the following result.

Proposition 3. *In a currency union with the possibility of exit, the expected inflation rate (21) equals its optimal level (which is zero) if:*

$$\omega_{**}^B(p^A) = \frac{(\alpha(1 + \beta^2\lambda) + \beta(1 + \alpha\beta\lambda)p^A)\beta\lambda k}{(1 - p^A)(1 + \beta^2\lambda)(1 + \alpha\beta\lambda)E[v^B]} > 0, \tag{25}$$
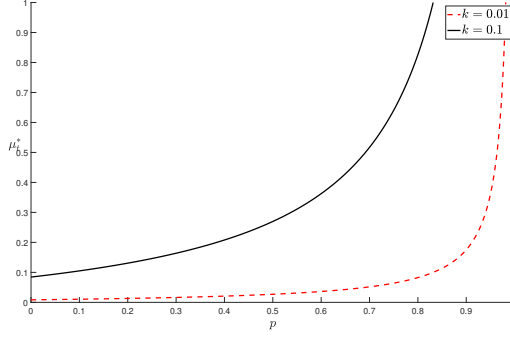
Figure 2: Optimal share and exit probability

If $\omega^B = \omega^B_{**}$, we have:

$$E[\pi^A_{cu}] = 0, \ E[y^A_{cu}] = 0, \tag{26}$$

Proposition 3 states that by setting $\omega^B = \omega^B_{**}(p^A)$, the advanced countries' expected outcomes coincide with those when they can commit to the optimal inflation rate. The next result illustrates the effect of exit probability on $\omega^B_{**}(p^A)$.

Corollary 1.

$$\frac{d\omega^B_{**}(p^A)}{dp^A} > 0, \ \frac{d^2\omega^B_{**}(p^A)}{d\left(p^A\right)^2} > 0. \tag{27}$$

Figure (2) illustrates $\omega^B_{**}(p^A)$. An increase in p^A raises the expected inflation rate (21). Thus, advanced countries require a greater share of developing countries in the union to decrease the increased inflation expectations. This result also indicates that $\omega^B_{**}(p^A)$ exponentially increases as p^A increases, meaning that the marginal impact of exit rumor is high when the public sector believes that the exit decision is likely to happen. The optimal share $\omega^B_{**}(p^A)$ is minimized with $p^A = 0$, then, it holds that $\omega^B_{**}(p^A) = \omega^B_*$.

Intuitively, when the threat of exit increases, the optimal currency union requires more developing countries in the currency union. Departing from the traditional Mundellian criterion, which encourages homogeneity among members, this indicates that having heterogeneous members is beneficial especially when the public expects countries' to exit from the currency union.

Corollary 2. The advanced countries cannot achieve the commitment outcome if:

$$\frac{\left(\alpha(1 + \beta^2\lambda) + \beta\left(1 + \alpha\beta\lambda\right)p^A\right)\beta\lambda k}{(1 - p^A)(1 + \beta^2\lambda)\left(1 + \alpha\beta\lambda\right)} > E[v^B] \tag{28}$$

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If the economy satisfies Condition (28), ω_{**}^B is greater than 1, which is not feasible according to its definition. If $E\left[v^B\right]$ is too low or (and) p^A is too high, the expected inflation rate is positive no matter how many developing countries are in the currency union.

Corollary 3. (Output target and developing regions' expected output)

$$\frac{d\omega_{**}^B(p^A)}{dk} = \frac{\omega_{**}^B(p^A)}{k} > 0, \quad \frac{d^2\omega_{**}^B(p^A)}{dk^2} = 0. \tag{29}$$

$$\frac{d\omega_{**}^B(p^A)}{dE[v^B]} = -\frac{\omega_{**}^B(p^A)}{E[v^B]} < 0, \quad \frac{d^2\omega_{**}^B(p^A)}{dE[v^B]^2} = \frac{2\omega_{**}^B(p^A)}{E[v^B]^2} > 0. \tag{30}$$

Corollary 3 characterizes the impact of the target rate of output, k, on the optimal share of developing countries in the currency union. The higher output target rate k causes a greater inflationary bias and decreases the interest rate that is implemented by the currency union (12). To control expected inflation, a greater share of developing nations that have a low bliss inflation rate is required. The effect of increasing developing countries' supply shock $E[v^B]$ is similar but in the opposite direction. Since it decreases the developing countries' optimal inflation rate, a greater share of developing countries is required.

Corollary 4.

$$\frac{d\omega_{**}^B(p^A)}{d\alpha} > 0, \quad \frac{d^2\omega_{**}^B(p^A)}{d\alpha^2} < 0. \tag{31}$$

$$\frac{d\omega_{**}^B(p^A)}{d\beta} > 0, \quad \frac{d^2\omega_{**}^B(p^A)}{d\beta^2} \leq 0 \, if \, \beta^2\lambda > 1. \tag{32}$$

$$\frac{d\omega_{**}^B(p^A)}{d\lambda} > 0, \quad \frac{d^2\omega_{**}^B(p^A)}{d\lambda^2} < 0. \tag{33}$$

Corollary 4 and Figure 5 show the effects of increasing the preference parameters—namely, the interest rate elasticity of output, α, the inflation rate elasticity of output, β, and the preferences of monetary delegates, λ—on the currency union's optimal share of developing countries, $\omega_{**}^B(p^A)$. A change in each parameter value has the same effect—a high parameter value increases the expected inflation rate (21). As a consequence, similar to a reduction in $E\left[v^B\right]$, the currency union requires a greater share of developing countries to reduce the expected inflation rate.

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(a) Sensitivity of $E[v^B]$

(b) Sensitivity of α

(c) Sensitivity of β
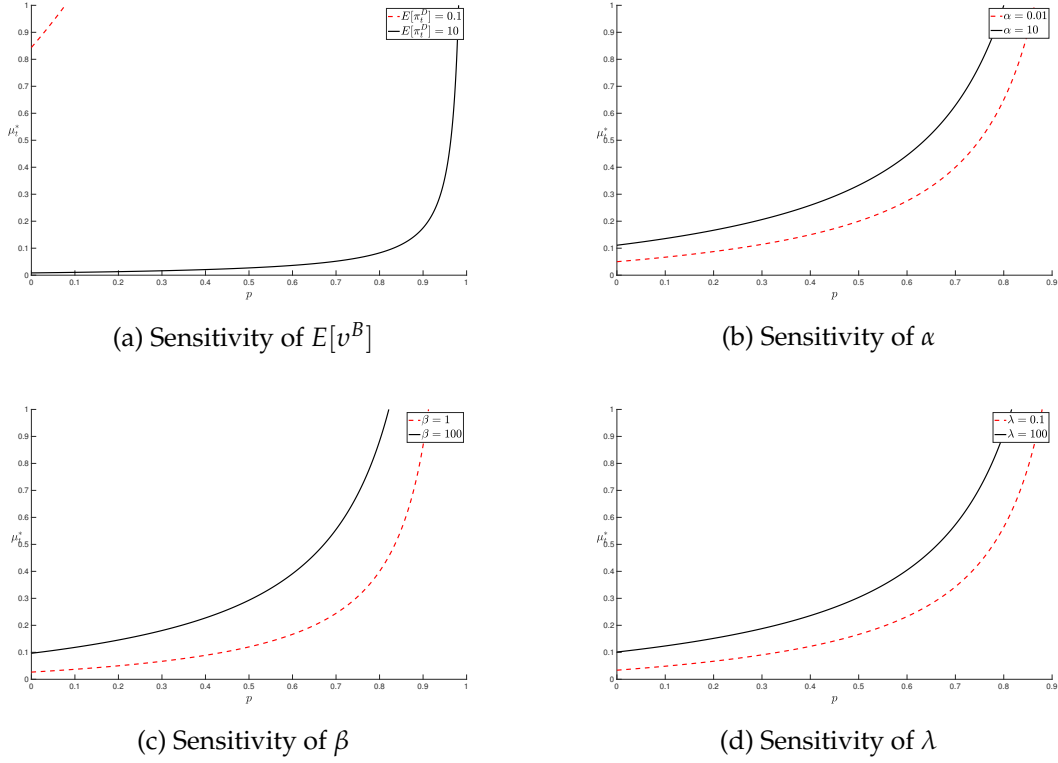
(d) Sensitivity of λ

Figure 3: Sensitivity Analysis

4 Extensions and Discussions

4.1 Optimal currency area from developing countries' perspectives

We focused on the advanced countries' optimal design of a currency union. This setting can be interpreted as a realistic situation in which advanced countries form a currency union and decide the number of developing countries to allow. Now, we study the opposite scenario in which developing countries form a currency union and determine the number of advanced countries allowed to join.

It is straightforward that a similar calculation to the derivation of Eq. (21) yields the developing countries' expected inflation rate under rational expectations as follows:

$$
\pi_e^B = \beta\lambda k - \frac{(1 - p^B)(1 + \lambda\beta^2)\left(1 + \alpha\beta\lambda\right)\omega^B E\left[v^B\right]}{\beta\left(1 + \alpha\beta\lambda\right)p^B + \alpha(1 + \beta^2\lambda)}. \tag{34}
$$

A greater share of advanced countries (ω^A) decreases the relative share of developing countries in the currency union (ω^B) and therefore increases the expected inflation rate by decreasing the second-term on the right-hand side of Eq. (35). Intuitively, a larger

share of advanced countries means more bargaining power for countries that prefer a higher inflation rate. As a result, the currency union chooses a lower common interest rate.

The developing countries' expected inflation rate is equal to its optimal level if $\omega^B = \omega^B_{***}(p^B)$ such that:

$$\omega^B_{***}(p^B) = \frac{\beta\lambda k \left(\beta\left(1 + \alpha\beta\lambda\right)p^B + \alpha(1 + \beta^2\lambda)\right)}{(1 - p^B)(1 + \lambda\beta^2)\left(1 + \alpha\beta\lambda\right)E\left[v^B\right]} > 0. \tag{35}$$

Developing countries cannot achieve the commitment outcome if the following hods:

$$\frac{\beta\lambda k \left(\beta\left(1 + \alpha\beta\lambda\right)p^B + \alpha(1 + \beta^2\lambda)\right)}{(1 - p)(1 + \lambda\beta^2)\left(1 + \alpha\beta\lambda\right)} > E\left[v^B\right]. \tag{36}$$

Note that $\omega^B_{**} \leq 1$ by its definition. Intuitively, when the value of $E\left[v^B\right]$ is sufficiently low so that the developing countries' expected inflation rate (35) is positive, allowing an advanced country to belong to the currency union increases the expected inflation even higher. Therefore, in such a case, developing countries choose $\omega^A = 1$, meaning that they do not allow any advanced countries belong to the currency union.

4.2 Entrance condition

We assumed that advanced countries can choose the share of developing countries in the currency union. This assumption is valid only if developing countries prefer to belong to the currency union. It is natural to suppose that countries prefer being in a currency union if the expected welfare of belonging to the currency union is greater than that under autonomy. In this section, we ask under what conditions countries in both regions are better off by joining the optimal currency union.

Autonomy.

We first deliver the expected social loss under autonomy in which countries deliberately choose monetary policy for their own sakes. Imposing the rational expectation $\pi^j_e = E\left[\pi^j(\pi^j_e, \eta)\right]$ into Eq. (8) yields the following:

$$E[\pi^j] = \beta\lambda\left(k + E[\eta + v^j]\right). \tag{37}$$

Therefore, the outcomes in discretionary equilibrium, π_d^j, y_d^j, are given as follows:

$$\pi_d^j = \frac{\beta\lambda\left(k - E[\eta + v^j]\right)}{1 + \beta^2\lambda}, \ y_d^j = \frac{\beta^2\lambda\left(E[\eta + v^j] - s\right)}{1 + \beta^2\lambda}. \tag{38}$$

As is well known by Barro and Gordon (1983) and Kydland and Prescott (1977), the inflation rate is increasing in the positive output target $k > 0$, which reflects policymakers' desire to increase output. Since the inflation bias does not influence output, welfare under autonomy decreases as the inflation bias increases.

Entry.

The decisions on whether or not to join a currency union depend on the currency union's institution. Again, we assume that the preferences and economic conditions of every country are perfectly observable. Then, countries anticipate the advanced countries' design of (ω^A, ω^B) and the currency union's interest rate i^{cu}. Therefore, a country in region j is better off by belonging a currency union if:

$$E\left[L^j(y_{cu}^j, \pi_{cu}^j)\right] + \delta < E\left[L^j(y_d^j, \pi_d^j)\right]. \tag{39}$$

Under Assumption 1, we obtain the following advanced countries' entry condition:

$$Var(v^A) < \Phi k^2, \tag{40}$$

where $\Phi := \frac{\left(1 - \lambda(1+\beta^2\lambda)\right)\left(1+\beta^2\lambda\right)^2(\beta-\alpha)^2}{(\beta-\alpha)^2 - 1 - 2\beta^2\lambda - (\beta^2\lambda)^2} > 0$. Belonging the currency union is preferable when the output target (or inflation bias) k is relatively greater than the variance in the regional supply shock $Var(v^A)$.[6] This result follows as a corollary to the classical discussion on "rule versus commitment." If a country suffers from inflation bias, the benefits of committing to an inflexible monetary policy surpass those of having the ability to discretionarily make monetary policy. If the economic condition is unstable, a country prefers to have its own domestic monetary authority to prepare for a severe recession or boom.

In our model, countries do not have commitment power, so the only way to commit to a policy is to join the currency union. Therefore, countries join the currency union if the economic environment is relatively calm, which we implicitly assumed throughout this paper. This finding suggests that in turbulent economic conditions, members of the currency union may have more incentive to relinquish the common currency and deliberately implement monetary policy independently.

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[6]We cannot analytically solve the developing countries' entering condition. Howeve,r it is straightforward that the intuition must be same for those of advanced countries—a higher inflation bias, a higher welfare gain of joining the currency union.

5 Conclusion

We have examined a simple currency union model that yields several useful findings regarding the impact of exit expectations on economic outcome as well as the optimal structure of a currency union. A higher exit probability induces the public sector to put a higher weight on the future probability of autonomous discretionary monetary policymaking; countries experience a higher expected inflation rate at the rational expectation equilibrium, resulting in a higher actual inflation. We studied the optimal share of developing countries—the expected output of which is higher than that of advanced countries— in the currency union. We showed that as the probability of exit increases, advanced countries require a greater fraction of developing countries in the currency union to decrease expected inflation to its optimal level.

In terms of future research, it would be interesting to endogenize entry and exit decisions, and investigate the rules on exit and entrance on the outcome. Specifically, exit and entrance can be (i) free or (ii) restricted. With a free exit (entrance) rule, countries can freely make decisions by itself. With a restricted exit (entrance) rule, a country's decision is subject to the existing members' approval (e.g., the decision is accepted if a majority of the member accept it). This direction for future research can shed light on the impact of a currency union's exit (entrance) rules on outcome as well as its optimal design.

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