Endogenous Abatement Technology *

Preliminary version

Ghassane Benmir† Josselin Roman‡

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Abstract

In this paper, we first explore empirical evidence on i) the role the ETS carbon pricing system plays in emissions reduction at the Euro Zone (EZ) level, and ii) fiscal and macro-financial drivers of green innovation. We use macro time series data from EZ and US as well as panel data from the EZ, receptively. We find that the ETS price plays a significant role in emissions reduction, as well as in steering green innovation. However, above a certain level, it negatively impacts green research and development (R&D), whereas long-term loans help boost green R&D. Second, to investigate the role financial policy could play in stimulating green R&D, we build a general equilibrium model where we show how green innovation could help achieve the net-zero target at lower output costs compared to fiscal carbon policies. We then expand the model to account for both financial intermediaries and endogenous green growth, the latter of which implies increasingly efficient abatement. Using Bayesian techniques, we first estimate the model and then construct counterfactual policy implementation scenarios, where we show that financial subsidies, macroprudential policies, and monetary policy differently affect the path of the trend growth in green innovation, and that they all have the same pro-cyclical dynamics. Finally, we investigate the net-zero emissions target under the three above-mentioned policies in order to assess their efficacy.

Keywords: Climate Mitigation Pathways, Endogenous Green R&D, Subsidy, Macroprudential Policy, Green QE.

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†London School of Economics and Political Science; email: g.benmir@lse.ac.uk

‡University Paris Dauphine – PSL Research University. email: josselin.roman@dauphine.psl.eu. The authors are extremely grateful to Anna Creti, Simon Dietz, Ivan Jaccard, Tierra McMahon, Luca Taschini, Gauthier Vermandel, and Bertrand Villeneuve, for useful discussions and for providing comments and advice. All errors and omissions are our own. The usual disclaimer applies.

1 Introduction

In recent years, monetary policy makers have become increasingly concerned by the challenges posed by climate change. As a step toward more actions, the ECB decided, as part of its monetary policy strategy review, to monitor more closely climate risk and the consequences it could have on financial stability and monetary policy transmission. For the time being, however, fiscal policy has been the main instrument to mitigate present and future damages from climate change.

While carbon pricing is the major tool used in climate mitigation policies nowadays, this policy is not a free lunch as it induces unintended effects. In Europe, Canada, and California (US), as well as elsewhere, governments have opted for a market cap and trade system instead of a targeted price, where carbon permits are traded, which facilitates the attainment of desired emission level reductions. As, this market design is not set optimally from a welfare perspective and is subject to market volatility and business cycle fluctuations, a number of inefficiencies arise (e.g. welfare losses and risk premium distortions as highlighted in Benmir and Roman [2020]). In order to address the inefficiencies induced by such a carbon market design, finding ways to steer green innovation without solely relying on increasing carbon pricing becomes a major priority.

The goal of this paper is twofold. First, we seek to empirically investigate the different linkages between carbon emissions, fiscal and financial carbon policies, and green innovation. Second, we want to shed light on how fiscal and financial policies could help steer some of the main drivers that contribute to the next zero carbon emissions transition. To do so, we build a quantitative model to address the evidence and provide a framework that allows for analyzing the role of various green innovation policies in the transition to a low carbon economy.

With respect to the first goal, we rely on empirical data on the Euro Zone (EZ), the US, and a panel of the 19 EZ countries. We finds that a fraction of emissions reduction is accounted for by carbon pricing policies (e.g. ETS system), and shows that carbon pricing might not always steer green innovation, which in turn is a major contributor to emissions reduction. Furthermore, macro-financial factors (e.g. long-term loans) are found to play a significant, positive role in boosting green innovation.

Regarding the second goal of the paper, the model introduces two modifications to the standard real business cycle economy: i) it explicitly accounts for the process of endogenous green innovation by lowering the cost of abatement; ii) it includes an agency friction in financial markets that may disrupt the financing of investments in innovation à la Queralto [2020]). Endogenous green innovation financed by the banking sector allow for the substantial emissions reduction by triggering higher levels of abatement, and without subsequently relying on increasingly higher levels of carbon pricing.

In the spirit of Romer [1990], Acemoglu et al. [2012], and Anzoategui et al. [2019], we introduce sustained growth in green R&D arising from an endogenously expanding variety of green technologies. Green entrepreneurs invest in projects that could lead to an improvement of the green technology, but lacks the funds to finance the necessary expenditures. When it is successful, the green technology allows firms to abate at a cheaper cost, which in turn lower emissions. To obtain funds, our green firms borrow from banks. The outcome from green innovation efforts consists of novel varieties of intermediate goods, which are then used in final abatement efforts.

The main quantitative application of our model is to explore the EZ net-zero transition pathways, as well as business cycle fluctuations, under the presence of green innovation boosting policies (i.e. fiscal and macroprudential). Three main reasons are behind the focus on the EZ. First, the ETS (European Trading System) carbon pricing market is the most advanced environmental fiscal policy in the world. Second, the European Union (EU) global strategy in emissions reduction is moving toward finding ways in which green innovation could be steered more efficiently. Finally, the availability of data allows for running both empirical exercises and counterfactual scenarios.

Using a DSGE framework as a foundation, the present paper builds on Heutel [2012], Fischer and Springborn [2011], and Golosov et al. [2014], among others, to account for the effect of the environmental externality on the economy, while also following Gertler and Karadi [2011] to model financial intermediaries. The novelty of the model is that we introduce green innovators in the spirit of Romer [1990], Comin and Gertler [2006], and Acemoglu et al. [2012]. The main divergences of our paper with this literature are that: i) endogenous growth in green R&D directly impacts the abatement technology by making it cheaper, thus triggering higher abatement levels, ii) green innovators need to obtain funds from financial intermediaries to set up projects as in Anzoategui et al. [2019] and Queralto [2020], and iii) we estimate the model trends and endogenous growth structural parameters using data on R&D and green innovation patents expenditure.

The paper is divided into three main sections: i) an empirical analysis on the linkages between carbon pricing, green R&D, and macro-financial factors; ii) a transition pathway analysis using a reduced form model; and iii) an analysis of output and green innovation trends as well as net-zero pathways, using a full fledged estimated model with both financial intermediaries and an endogenously-determined abatement technology.

2 Motivational Evidence: Emission, Carbon Pricing, and Green Innovation

2.1 Data

Data used[1] in this section were obtained from the ECB Statistical Data Warehouse, Eurostat database, the University of Oxford ourworldindata.org database, FRED database, OECD database, European Patent Office (EPO) database, and the European Environment Agency.[2]

The data set includes series from all 19 EZ countries, the EZ area aggregate, as well as the US, with data spanning from the first quarter (Q1) of year 2000 to the last quarter (Q4) of year 2019.

[1] All data were either extracted directly on a quarterly basis or transformed from a monthly frequency to a quarterly frequency.

[2] For a detailed list of data used and treatment, please refer to the appendix, subsection A.1.

Table IV presents the descriptive statistics for the data set we use in our first analysis (i.e. the difference-in-difference between the EZ and the US). First, we ensure that all macro data are end of the date quarterly, and in millions of currency. We transform the emissions and population data to per million. After operating these harmonizations, we compute the deflated growth rate for all data. Finally, we add 4 and 8 lags[3] to the green patents, as this represents the time for the green innovation to be adopted by firms.

Table V presents the descriptive statistics for the data set we use in our second analysis (i.e. the Panel OLS on the EZ 19 countries). We use the same macro variables, however, this time we focus on the 19 EZ countries. We also add green patents data, the ETS price data, and the long-term loans granted by the financial sector to domestic non-financial corporations. As in the first case, we add lags (4, 8, and 12) to the ETS carbon price and to the long-term loans, as this represents the time for both the fiscal and financial policies to impact green innovation.

2.2 Carbon Pricing and Emission Reduction: EZ–US Difference-in-Difference Analysis

The empirical evidence on the role of fiscal carbon policies on emissions reduction is found to be significantly different depending on the market structure and design of the fiscal policy. As highlighted by (Sumner et al. [2011], Meckling et al. [2017], Haites [2018], and Best et al. [2020]), this isn't an easy task as it is challenging to disentangle the effects of carbon pricing from those of other climate and energy policies (Somanathan et al. [2014] , Narassimhan et al. [2018]). Yet, to date, there isn't a clear consensus on the effectiveness of carbon pricing, where, on one hand, case studies in North America (both British Colombia and California) show that carbon pricing had a significant impact on emissions reduction (Murray and Rivers [2015] and Martin and Saikawa [2017]), while on the other hand, Bel and Joseph [2015] as well as Haites [2018], when looking at the EU, don't find that ETS carbon pricing has contributed as much as it did in the US in terms of emissions reduction.

[3]Where 4 lags is 1 year and 8 lags are 2 years.

The emission carbon pricing is one difference among many between the socio-economic policies of the EZ and the US. The two major economic areas are among the three biggest contributors to the world CO_2 emissions. Although both pledged to significantly reduce their emissions levels, the carbon policies and market design of the two economic areas are significantly different. First, we conduct an empirical analysis to assess the efficiency of the ETS carbon market price levels and the market design for mitigating and reducing emissions. To do so, we compare the situation between the EZ and the US using a difference-in-difference technique. We chose to focus on the third phase of the ETS (2013–2020), which was first announced in 2009 and later amended in 2010 and 2011, as this face saw the introduction of new rules governing the free allocations of emissions allowances given to energy-intensive industries, whereas in the first and second phases, allocations were not based on historical production multiplied by best available emissions technology benchmarks.

The nature of our data set and research question—which explores the impacts of a public policy (in this case the introduction of ETS carbon policy) on emissions reduction—suggests a comparison between the pre and post policy implementation of phase three in order to assess the effectiveness of the policy. Thus, if a control could be found that would allow us to capture other policies that could also affect emissions reduction that are not directly related to the policy we are analyzing, then difference-in-difference would be an accurate method. Our first choice was the US, as there is no major carbon policy system in place and comparable socio-economic and demographic attributes, as well as technological advancements, are observed in both economic areas, meaning that we would able to capture other aspects of public policy that could interfere with our analysis. Looking at EZ and US, we first check the socio-economic and demographic data summarized in Table IV. It shows that both economic areas are highly similar for the selected attributes. Then, we test for the trends of the mean of emissions for both areas in order to assess the assumption of parallel trends before the policy (ETS 3rd phase (2013)) announcement date and to determine if the difference in the trends after the policy holds. We consider the 2010 amendments news date instead of the initial 2009 announcement date of the third phase ETS implementation, as in 2009 the announcement didn't include many monitoring and reporting guideline, which are essential for the efficient conduct of the policy, while the 2010 announcement included new stringent monitoring and reporting guidelines for greenhouse gas emissions from the capture, transport and geological storage of carbon dioxide.[4] Figure II displays clearly the validity of a Diff-in-Diff approach for our control variable and treatment with respect to the announcement date.

To estimate the impact of the ETS price on emissions reduction, we use a regression model where we compare the average changes in emissions between two economic areas. Furthermore, we use the Newley-West estimator for robust standard errors to avoid the auto-correlation stemming from the spline we operated on the emissions when transforming the frequency of the data to quarterly:

$$ln(E_i) = \alpha + \beta_1 Policy_i + \beta_2 Treatment_i + \beta_3 (Treatment_i \times Policy_i) + \sum_i \beta_i X_i + error_i \tag{1}$$

As shown in Table I, we first find that the carbon ETS system played a significant role in emissions reduction in EZ as compared to the US. The results are also quite consistent when adding, changing, and/or substituting controls. We find that the coefficient of interest (the diff-in-diff estimator) falls between -.07 and -.19, thus suggesting that the ETS pricing model contributed to between 7 to 19 percent of emissions reduction in the EZ.

We also, confirm that green innovations achieved through an increase in green patents contribute to decreasing emission levels. The results are also significantly consistent whether we consider a 1-year lag or a 2-year lag for green patents to materialize.

However, we don't find any significant impacts of oil prices between the two areas nor do we conclude on a significant role of government spending or investment on emissions reduction.

We find that the trade balance for goods plays a significant role in reducing emissions, thus

[4] https://ec.europa.eu/clima/eu-action/eu-emissions-trading-system-eu-ets/development-eu-ets-2005-2020_en

suggesting that the ETS carbon pricing didn't have any significant leakage outcomes during the studied period. As for services, we cannot conclude that they play a significant role in emissions reduction nor in emissions increase. These findings are in line with Dechezleprêtre et al. [2019] where they find no evidence that the EU ETS has led to carbon leakages. It is also supported by Venmans et al. [2020] who show that carbon pricing didn't have linkages that impact trade.

TABLE I

ETS Price Impact on Emissions: EZ-US Difference-in-Difference Regression

ln(Emissions per capita) (quarterly)	(1)	(2)	(3)	(4)	(5)	(6)
Policy (Q1 2013)	-0.0614**	-0.0111	0.0186	0.0649***	0.0496**	-0.0170
	(0.0309)	(0.0261)	(0.0276)	(0.0166)	(0.0198)	(0.0350)
Treatment (EZ)	-1.369***	-1.230***	-1.269***	-1.300***	-1.160***	-1.727***
	(0.0861)	(0.0986)	(0.0947)	(0.0741)	(0.0673)	(0.253)
Diff-in-diff Estimator	-0.0730***	-0.112***	-0.121***	-0.191***	-0.137***	-0.0932**
	(0.0276)	(0.0225)	(0.0229)	(0.0255)	(0.0266)	(0.0420)
ln(GDP per capita)	-1.032***	-0.534***	-0.581***	-1.150***	-0.895***	
	(0.168)	(0.202)	(0.187)	(0.184)	(0.152)	
ln(R&D Green) 4 lags		-0.178***				
		(0.0366)				
ln(R&D Green) 8 lags			-0.205***		-0.194***	-0.0957***
			(0.0371)		(0.0377)	(0.0336)
Trade Balance (Goods)				-0.105***	-0.120***	-0.0757***
				(0.0165)	(0.0233)	(0.0276)
Trade Balance (Services)				-0.277***	0.0430	0.168
				(0.0468)	(0.0727)	(0.103)
ln(Oil Price)					-0.00104	0.00745
					(0.0114)	(0.0112)
ln(Consumption per capita)						-1.009***
						(0.335)
ln(Gov Spending per capita)						-0.322
						(0.212)
ln(Investment per capita)						0.127
						(0.111)
Constant	9.159***	10.00***	10.03***	8.947***	9.520***	6.908***
	(0.129)	(0.208)	(0.184)	(0.166)	(0.200)	(0.560)
Observations	160	152	144	160	144	144

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2.3 Green Innovation: EZ Panel OLS Analysis

Turning now to the assessment of the impacts of both fiscal and macro-financial variables on green innovation (i.e. green patents), we use a pool of panel data from the 19 EZ countries. The focus of our analysis is on the fiscal (ETS carbon pricing) and financial (long-term credits to non-financial firms) impacts on green innovation. Unfortunately, due to scarcity of data on green subsidies for the EZ, we are unable to clearly show the impact of such policies on green patenting. However, different studies (e.g. Bai et al. [2019]) show the positive and significant impact of such fiscal tools in facilitating green innovation.

Previous papers—such as Acemoglu et al. [2012] and Aghion et al. [2016], which (using panel data) assess the impacts of carbon policies (via subsidies or taxes) on fuel prices and clean innovation, or such as Acemoglu et al. [2019], which rely on diff-in-diff between the US and the EU to assess shell gas discovery and its impact on patents and green innovation—fail to capture the impacts of macro-financial variables on R&D. As such, we focus on investigating both fiscal and macro-financial drivers of green innovation.

Understanding the role macro-financial variables could play in steering green innovation could become a major tool in mitigating climate change and efficiently reducing emissions. In this second part of our empirical assessment, we conduct a panel regression analysis to investigate the role financial loans could play in boosting green patents. We start our analysis from Q1 of 2008 to Q4 of 2019 in order to have a balanced panel sample for all the EZ 19 countries, as data on the ETS carbon pricing are only available from 2008. Then to assess the drivers of green innovations, which we proxy through green patents, we regress the green patents for each of the EZ countries on both the ETS prices and macro-financial indicators, namely the long-term loans, as well as on a number of macro controls, time fixed and country fixed effects:

$$GreenPatent_{i,t} = \beta_1 ETS_{i,t} + \beta_2 FI_{i,t} + \sum_i \beta_i X_{i,t} + T_t + State_i + error_{i,t} \tag{2}$$

Table II results suggest both a significant and positive role of the ETS price system as well as the long term credit trends for boosting green innovation. The results are consistently significant as we run robustness checks with different timing lags for both the ETS carbon price and for long-term loans, and they underscore the importance of conducting more R&D.

Output is found to play an important role, suggesting that the stronger the economic growth, the higher the levels of green innovation. This is inline with the finding of Song et al. [2015], where green innovation benefits from the positive spillovers of economic growth.

TABLE II

Green Innovation Drivers: Panel OLS Regression

Green R&D	(1)	(2)	(3)
ETS Price Level (1 year lag)	22.65*		
	(12.92)		
Long-term Loan (1 year lag)	0.0801***		
	(0.0149)		
ETS Price Level (2 years lag)		7.882*	
		(4.167)	
Long-term Loan (2 years lag)		0.0990***	
		(0.0140)	
ETS Price Level (3 years lag)			7.761**
			(3.724)
Long-term Loan (3 years lag)			0.112***
			(0.0140)
GDP per capita	1.502***	1.474***	1.442***
	(0.290)	(0.350)	(0.422)
Constant	-772.8**	-392.9***	-389.4***
	(339.0)	(119.8)	(119.9)
Observations	772	700	628
R-squared	0.969	0.970	0.968
Time fixed effect	Y	Y	Y
Country fixed effect	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: The regression features both time and countries fixed effects that are not reported for simplicity.

Table III shows that, although carbon pricing is found to have played a significant role in steering green innovation over the 11-year period in the EZ, it might have negative effects on green innovation above a certain threshold (for prices higher than 15 euros). This result is confirmed with higher pricing cutoffs (i.e. prices higher than 20 euros and 25 euros). The robustness checks in Table VI and Table VII confirm that the above results remain largely significant and unchanged when considering different lags for both the carbon pricing and long-term loans to non-financial firms.

TABLE III

Green Innovation Drivers: Panel OLS Regression - Thresholds Effects

Green R&D	(1)	(2)	(3)	(4)	(5)
ETS Price > 5	9.351				
	(27.77)				
ETS Price > 10		13.84			
		(30.19)			
ETS Price > 15			-142.7*		
			(82.42)		
ETS Price > 20				-142.7*	
				(82.42)	
ETS Price > 25					-105.0*
					(58.73)
Long-term Loan (1 year lag)	0.0781***	0.0781***	0.0781***	0.0781***	0.0781***
	(0.0146)	(0.0146)	(0.0146)	(0.0146)	(0.0146)
GDP per capita	1.566***	1.566***	1.566***	1.566***	1.566***
	(0.292)	(0.292)	(0.292)	(0.292)	(0.292)
Constant	-172.2***	-176.7***	-162.8***	-162.8***	-162.8***
	(38.05)	(41.19)	(46.63)	(46.63)	(46.63)
Observations	790	790	790	790	790
R-squared	0.968	0.968	0.968	0.968	0.968
Time fixed effect	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: The regression features both time and countries fixed effects that are not reported for simplicity.

3 General Framework

In this section we present a standard endogenous growth model enhanced with an environmental externality à la Heutel [2012] and the possibility of emission abatement for firms. We assume that this abatement technology can be improved exogenously. The goal is to check whether the model is able to replicate the empirical finding presented above, and perform a forecast simulation for the EZ. In the next section, we will show how it is possible to endogenize the cost and efficiency of the abatement technology.

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \dots \infty)$. The production sector produces two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms. Public authorities decide on the fiscal and environmental policy.

3.1 The Household

The economy is populated by a continuum of measure one of households, and each household has a unit measure of members. Households make decisions on consumption, labor supply, investment in physical capital and saving through a risk-free one-period international bond. A fraction of the workers are specialized workers L_s who supply labor inelastically to entrepreneurs. Regular workers in a household are monopolistic suppliers of a differentiated specific labor type, used to produce intermediate goods. Both types of labor return wages to the family. Profits made by firms are paid to entrepreneurs for their innovative ideas.

We note that, as it is highlighted by **?**, **?**, and Queralto [2020], the business cycle literature typically features preferences with $\Gamma_t = 1$ for all t. These business cycle frameworks assume no long-run growth. However, as we are also interested in the transition pathways, Γ_t cannot be considered as constant for the long-run simulations. Thus, it is important to consider trend growth in hours worked. In addition, the presence of $\Gamma_t^{1-\sigma}$[5] ensures a balanced growth path with constant hours. Furthermore, as long as the volatility of the growth rate $(\gamma^Y)^{1-\sigma}$ is small, fluctuations in $\Gamma_t^{1-\sigma}$ will have a small impact on labor supply at medium frequencies,

[5]We adjust the growth rate with $1 - \sigma$ as we consider a separable dis-utility of labour.

consistent with the usual formulation of GHH preferences.

The household maximization problem reads:

$$\max_{\{C_t, I_t, K_{t+1}, L_t, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \Gamma_t^{1-\sigma} L_{t+i}^{1+\varphi} \right], \tag{3}$$

s.t.

$$C_t + B_{t+1} + I_t + f(K_t, I_t) = W_t L_t + Ws_{t,s} \bar{Ls}^s + T_t + R_t B_t + R_t^K K_t \tag{4}$$

$$K_{t+1} = (1-\delta) K_t + I_t \tag{5}$$

where $\beta \in (0, 1)$ is the discount factor, parameters σ, $\varphi > 0$ shape the utility function of the representative household associated with risk consumption C_t, and labor L_t. The consumption index C_t is subject to external habits with degree $h \in [0; 1)$ while $\chi > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked. Labor supply L_t is remunerated at real wage W_t. As we assume that government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired. Households also choose the level of investment I_t and lend capital K_t at a return rate R_t^K. Adjustment costs $f(K_t, I_t) = \frac{\gamma_I}{2}(\frac{I_t}{K_t} - \delta)^2 I_t$ allow for capital building time, as in Christiano et al. [2005]. $\bar{Ls}^s$ is the inelastic labor supply to the R&D sector remunerated at real wage $Ws_{t,s}$. Note that firms do not reverse profits back to households. These profits will instead be revenues for entrepreneurs, as shown in the next section.

The first order conditions read[6]:

[6] We note ϱ_t^C and ϱ_t^K the Lagrange multipliers associated with budget and capital constraints, respectively.

$$\varrho_t^C = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{6}$$

$$\varrho_t^C = \chi \frac{\Gamma_t^{1-\sigma} L_t^{\varphi}}{W_t}, \tag{7}$$

$$1 = \beta E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\}, \tag{8}$$

$$\varrho_t^C = \frac{\varrho_t^K}{1 + f_I(.)}, \tag{9}$$

$$\varrho_t^K = \beta E_t \{ (1-\delta) \varrho_{t+1}^K + \varrho_{t+1}^C (R_{t+1}^K - f_K(.)) \}, \tag{10}$$

where the stochastic discount factor (i.e. the expected variation in marginal utility of consumption) reads as follows $\Lambda_{t-1,t} = \frac{\varrho_t^C}{\varrho_{t-1}^C}$.

3.2 R&D Entrepreneurs

As in Comin and Gertler [2006] entrepreneurs are an unbounded mass of prospective innovators with the ability to introduce new varieties of intermediates in each period. Each entrepreneur use resources to create a new project $RD_{t,s}$. Both new projects $RD_{t,s}$ and existing varieties $A_{t,s}$ face the risk of an exogenous exit shock $(1 - \phi_{RD,s})$. This process is meant to capture in a simple way the life-cycle dynamics of firms. Note that we also consider that entrepreneurs are not using energy heavy output, thus emitting zero CO_2 emissions. The evolution of the aggregate stock of innovations $A_{t,s}$ reads as follows:

$$A_{t+1,s} = \phi_{RD,s}(A_{t,s} + RD_{t,s}), \tag{11}$$

Entrepreneurs are able to produce new varieties by employing materials and skilled workers as inputs, according to the following production function:

$$RD_{t,s} = N_{t,s}^{\eta_s} (A_{t,s} Ls_{t,s})^{1-\eta}, \ \eta_s \in (0,1), \tag{12}$$

where $N_{t,s}$ is the amount of materials used (in units of final output) and $Ls_{t,s}$ is the number of skilled workers hired. Once the variety created, entrepreneurs lend it to monopolist firms in exchange for patent exclusivity. The monopolists then manufacture the new good and reverse profits Π_t (as shown in Equation 29) back to the entrepreneurs. Furthermore, as in Romer [1990], in order to generate endogenous growth, the entrepreneurs production function captures the externality of the aggregate level of knowledge $A_{t,s}$.

The entrepreneurs problem will read as follows:

$$\max_{\{RD_{t,s},N_{t,s},Ls_{t,s}\}} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left[\Pi_t RD_{t+i,s} - (N_{t+i,s} + Ws_{t+i,s} Ls_{t+i,s}))\right] \tag{13}$$

s.t.

$$RD_{t+i,s} = N_{t+i,s}^{\eta_s} (A_{t+i,s} Ls_{t+i,s})^{1-\eta}. \tag{14}$$

The first order conditions read:

$$1 = MC_t^{RD,s} \eta_s N_{t,s}^{\eta_s - 1} (A_{t,s} Ls_{t,s})^{1-\eta_s}, \tag{15}$$

$$Ws_{t,s} = MC_t^{RD,s} (1-\eta_s) A_{t,s} N_{t,s}^{\eta_s} (A_{t,s} Ls_{t,s})^{-\eta_s}, \tag{16}$$

$$\Pi_t = MC_t^{RD,s}, \tag{17}$$

where $MC_t^{RD,s}$ the Lagrange multiplier associated to the production constraint. Entrepreneurs equalize their marginal cost to the profit they receive form the the firms and are subject to the inelastic supply of skilled labor $Ls_{t,s} = \bar{Ls}^s$).

3.3 The Firms

3.3.1 The Final Firms

The final good is produced by a competitive sector, which uses the different varieties of intermediates produced by entrepreneurs as inputs, yielding the following production function:

$$Y_t = \int_0^{A_{t,s}} \left(Y_{jt}^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}. \tag{18}$$

Final firms are looking for profit maximization at a given price P_t, subject to the intermediate goods j with prices P_{jt}:

$$P_t = \left(\int_0^{A_{t,s}} P_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{19}$$

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \tag{20}$$

3.3.2 The Intermediate Firms

Contrary to the standard RBC framework, representative firms (indexed by j) of the modeled economy seek face a trade-off between the desired level of abatement level and the environmental policy level, in addition to the usual capital and labor trade-off.

As the environmental externality is a global phenomena, firms do not internalize its impacts, thus, they incur the externality costs as the social planner or government imposes an environmental policy in order to fix the market failure. Setting an environmental policy then pushes firms to optimally choose a level of abatement to maximize their profit. Following Heutel [2012], the environmental externality enters the Cobb-Douglas production function of the firms, through a damage function linked to the level of temperature à la Nordhaus and Moffat [2017] as follows:

$$Y_{jt} = \varepsilon_t^A d(T_t^o) K_{jt}^{\alpha} L_{jt}^{1-\alpha},\ \alpha \in (0,1), \tag{21}$$

where $d(T_t^o)$ is a convex polynomial function of order 2 displaying the temperature level ($d(T_t^o) = ae^{-(\frac{b}{{\Gamma_t}^2} {T_t^o}^2)}$), with $(a,b) \in \mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017]. As in the case of the disutility of labour, we introduce ${\Gamma_t}^2$ to the damage sensitivity parameter b, such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} {T_t^o}^2}$. The goal is to allows for the existence of the balance growth path without a loss of generality, as over the business cycle or for a period of less than 30 years $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} {T_t^o}^2} \approx ae^{-b{T_t^o}^2}$[7]. ε_t^A is an exogenous technology shock that follows an AR(1) shock process: $\log(\varepsilon_t^A) = \rho_A \log(\varepsilon_{t-1}^A) + \sigma_A \eta_t^A$, with $\eta_t^A \sim \mathcal{N}(0,1)$.

[7]This point is further discussed in the Balanced Growth Path section.

As argued by Dietz and Venmans [2019], global temperature $d(T_t^o)$ is assumed to be linearly proportional to the level of cumulative emissions:

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o. \tag{22}$$

Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E_t^*, \tag{23}$$

where E_t is the aggregate flow of emissions at time t ($\int_0^1 E_{jt}dj$) and γ_d is the decay rate. E_t^* represents the rest of the world emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere.[8]

The emissions level is modeled by a nonlinear technology (i.e. abatement technology μ) that allows for reducing the inflow of emissions:

$$E_{jt} = (1-\mu_{jt})\vartheta Y_{jt}. \tag{24}$$

The emissions E_{jt} at firm level are proportional to the production Y_{jt} with ϑ the carbon intensity parameter. Contrary to Lontzek et al. [2015], we consider $\vartheta_t = \vartheta$ constant overtime and calibrate it using Euro Area emission to GDP levels, as in our model, we capture the effects of green R&D directly through the abatement cost.

Furthermore, we allow for emissions reduction at the firm level through an abatement effort μ_{jt}. When firms decide on abatement efforts, they incur a technology cost:

$$Z_{jt} = f(\mu_{jt})Y_{jt}, \tag{25}$$

where

$$f(\mu_{jt}) = g(\theta_t^1)\mu_{jt}^{\theta_2},\ \theta_2 > 1, \tag{26}$$

and

$$g(\theta_t^1) = \frac{\theta_1}{\Gamma_t^{\theta_1}\epsilon_t^{\theta_1}},\ \theta_1 > 0, \tag{27}$$

[8]For simplicity we assume that the rest of the world emissions follow the same growth rate of our closed economy: $E_t^* = E^*\Gamma_t$.

with θ_1 and θ_2 representing the cost efficiency of abatement parameters. In this section, we assume that the cost function of abatement $g(\theta_t^1)$ follows an exogenous trend $\Gamma_t^{\theta_1}$ and can be hit by a random shock $\epsilon_t^{\theta_1}$.[9] The goal is to capture exogenously the impact of improvments in green technology that we will concretely model in the next section. This will result in a decrease in abatement costs that will allow for substantially higher levels of abatement μ_{jt}.

A decrease in $g(\theta_t^1)$ triggers a drop in the marginal cost of abatement, which we define as:

$$MC_\mu = \frac{f(\mu_{jt})'}{\mu_{jt}} \tag{28}$$

Thus, the profits of our representative intermediate firms Π_{jt} will be affected by the presence of the environmental externality. The revenues are the real value of intermediate goods Y_{jt}, while the costs arise from wages W_t (paid to the labor force L_{jt}), investment in capital K_{jt} (with returns R_t^K), abatement μ_{jt} (the firms are facing), and the price of emissions E_{jt} associated with the environmental policy.

$$\Pi_{jt} = \frac{P_{jt}}{P_t} Y_{jt} - W_t L_{jt} - R_t^K K_{jt} - g(\theta_t^1)\mu_{jt}^{\theta_2} Y_{jt} - \tau_{et} E_{jt} \tag{29}$$

The cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital, labor, as well as the abatement, respectively:

$$R_t^K = \alpha \Psi_{jt,k} \frac{Y_{jt}}{K_{jt}}, \tag{30}$$

$$W_t = (1-\alpha)\Psi_{jt,k} \frac{Y_{jt}}{L_{jt}}, \tag{31}$$

$$\tau_{et} = \frac{g(\theta_t^1)\theta_2}{\upsilon} \mu_{jt}^{\theta_2 - 1}. \tag{32}$$

The first two equation equation Equation (30) and (31) are the standart optimal choice of capital and labor, with $\Psi_{jt} = \Psi_t$ the marginal cost component related to the same capital-labor ratio all firms choose. This marginal cost component is common to all intermediate firms. When capturing the CO_2 externalityn firms face an additional tradde-off (equation

[9] $\epsilon_t^{\theta_1}$ follows an $AR(1)$ shock process: $\log(\varepsilon_t^{\theta_1}) = \rho_{\theta_1} \log(\varepsilon_{t-1}^{\theta_1}) + \sigma_{\theta_1} \eta_t^{\theta_1}$, with $\eta_t^{\theta_1} \sim \mathcal{N}(0,1)$.

(32)) between paying the environmental policy τ_t or incurring abatement cost related to the abatement levels they chose μ_t.[10] This last optimality condition highlights the key role of the carbon price dynamics in shaping the abatement level of firms.

We can now rewrite the firm problem as following:

$$\Pi_{jt} = \left(\frac{P_{jt}}{P_t} - MC_t^f\right) Y_{jt}, \tag{33}$$

where,

$$MC_{jt}^f = MC_t^f = \Psi_t + g(\theta_t^1)\mu_{jt}^{\theta_2} + \tau_{et}(1-\mu_t)\varphi, \tag{34}$$

The total marginal cost captures both abatement and emissions costs. Note that in the case of the laissez-faire scenario, $MC_t^f = \Psi_t$ as the firms are not subject to emissions and abatement constraints.

The aggregate production function of the intermediate firms will now features the measure A_t. Using both the Cobb-Douglas production form (21) and the final firms production equation (18), we can rewrite the production function as following:

$$Y_t = A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^{\alpha} L_t^{1-\alpha}. \tag{35}$$

The firm profit maximization with respect to output and prices, yields the following pricing rule:[11]

$$MC_t^f = \frac{P_{jt}}{P_t}\frac{\theta-1}{\theta} \tag{36}$$

Each intermediate producer sets its price equals to a constant markup over the marginal cost. Finally, the profits equation will also capture the measure $A_{t,s}$ and can be presented as following:[12]

$$\Pi_t = \frac{1}{\theta}\frac{Y_t}{A_{t,s}}. \tag{37}$$

[10] In addition, both the environmental policy τ_t and abatement effort μ_t are common to all firms, as the environmental cost, which firms are subject to, is constant.

[11] With $\frac{P_{jt}}{P_t} = 1$, as we abstract from price stickiness.

[12] For the full mathematical derivations please refer to the appendix.

3.4 Government

Government levies a lump sum tax and sets an environmental policy to finance its spending as following:

$$T_t + \tau_{et} E_t = G_t, \tag{38}$$

with the public expenditure G_t, taxes T_t, and revenue from emissions tax $\tau_{et} E_t$. The government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \tag{39}$$

3.5 The environmental policy

Competitive Equilibrium

To pin down the optimal policy,[13] we solve for the Competitive Equilibrium ("CE"). The CE in this economy is defined as follows:

Definition 1 *A competitive equilibrium consists of an allocation* $\{C_t, L_t, K_{t+1}, E_t, X_t, T_t^o\}$*, a set of prices* $\{P_t, R_t, R_t^K, W_t\}$ *and a set of policies* $\{\tau_t, T_t, B_{t+1}\}$ *such that*

- *the allocations solve the consumers', firms' problems given prices and policies,*
- *the government budget constraint is satisfied in every period,*
- *temperature change satisfies the carbon cycle constraint in every period, and*
- *markets clear.*

Definition 2 *The optimal solution sets the carbon price* τ_t *as an optimal policy* τ_t^**, which maximizes the total welfare in equation (3):*

$$\tau_t^* = SCC_t. \tag{40}$$

with SCC_t *the social cost of carbon:*

$$SCC_t = \eta\beta \frac{\lambda_{t+1}}{\lambda_t} SCC_{t+1} + (\upsilon_1^o \upsilon_2^o)\beta \frac{\lambda_{t+1}}{\lambda_t} \S_{t+1}^T, \tag{41}$$

[13] As we consider a closed economy, we assume that cooperation takes place in such a way to avoid free-riding and potential carbon leakages. This is achieved by setting E^* to a constant.

and with,

$$\S_t^T = (1-\upsilon_1^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T - \sum_k \Psi_t \varepsilon_t^A \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{42}$$

Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 3 *The public authorities, however, do not always optimally set the carbon policy. For instance, in the EU area, public authorities target an emissions level that is consistent with their objective of a 55% emissions reduction by 2030. As in Benmir and Roman [2020] we model this situation by assuming that the cap on emissions implies a specific carbon price that can be hit by exogenous shocks and which also incorporates an endogenous trend:*

$$\tau_t = \Gamma_t^{\tau}\, Carbon\ Price.^{14} \tag{43}$$

where $\Gamma_t^{\tau} = \gamma^{\tau}\varepsilon_t^{\tau}\Gamma_{t-1}^{\tau}$ is the stochastic growth rate of the tax which allows to reduce emissions to be aligned with the cap policy, and where ε_t^{τ} the stochastic AR(1) shock on tax that represents the market volatility of the ETS system.

This stylized representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives. That said, the targeted CO_2 level/price is assumed to be constant at the business cycle frequency.

3.6 Normalization and Aggregation

In equilibrium, factors and goods markets clear as shown below. First, the market-clearing conditions for aggregate capital, investment, labor, and wages, read as: $A_t K_t = \int_0^1 K_{jt} dj$, $I_t = \int_0^1 I_{jt} dj$, $A_t L_t = \int_0^1 L_{jt} dj$, and $W_t = \int_0^1 W_{jt} dj$. Similarly, global aggregate emissions and aggregate emissions cost reads as: $E_t = \int_0^1 E_{jt} dj$, and emissions cost $Z_t = \int_0^1 Z_{jt} dj$, respectively. Finally, the resource constraint of the economy reads as follows:

$$Y_t = C_t + G_t + I_t + N_{t,s} + f(.)I_t + Z_t. \tag{44}$$

[14] Although the policy used in the EU is E_t = Cap Policy, it is analogous to set τ_t = Carbon Price that would allow for decreasing emissions to match the cap.

3.7 Transition Pathways with Exogenous Abatement Technology

3.7.1 Calibration

Calibrated parameters for the standard endogenous growth model are reported in Table VIII and Table IX. For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio at 40 percent,[15] the share of hours worked per day at 0.33 for firms and 0.15 for entrepreneurs, and the capital share in the production function α at 0.3. The inverse elasticity of net investment to the price of capital γ_I is set at 1.728 as in Gertler and Karadi [2011] and the coefficient of relative risk aversion σ in the utility function at 2, as argued by Stern [2008] and Weitzman [2007]. We set the discount factor at 0.9975 to get a steady state real interest rate of 1 percent. This choice is motivated by the low interest rate environment witnessed in recent years.

Regarding the environmental part, we calibrate the damage function according to Dietz and Stern [2015]. The global temperature parameters υ_1^o and υ_2^o are set following Dietz and Venmans [2019] to pin down the 'initial pulse-adjustment timescale' of the climate system. The level of the remainder of the world's emissions E^* is set at 1.59 in order to replicate the global level of carbon in the atmosphere of 840 gigatons. We use the carbon intensity parameter ϑ to match the observed ratio of emissions to output for the Euro Area (EA) at 21%.[16] The abatement parameters θ_1 and θ_2 are taken from Heutel [2012]. The decay rate of emissions δ_x is set at 0.21 percent. Finally, the firms' marginal cost parameter θ is set to 11.

3.7.2 Transition Pathways Simulations

In order to solve for the medium/long-run pathways scenarios, we use the extended path algorithm (Adjemian and Juillard [2013]), which allows for both integrating deterministic trends and stochastic shocks, as it is shown in Benmir and Roman [2020].

[15] We match the level of the Euro Area.

[16] We compute the emissions to output ratio as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from FRED.

The goal of this section is to find and analyze a theoretical pathway consistent with the objective of the EU for 2030 under the presence of i) a targeted carbon price policy, ii) an exogenously growing green technology, and iii) an optimal policy coupled with an exogenously growing green technology.

We thus find the trajectory of the output, the marginal cost of abatement, and the carbon price, that leads to a desired reduction in emissions (55 percent relative to the level of 1990). We then highlight the main differences between relying solely on a carbon policy or solely on an abatement technology, versus using an optimal policy which maximizes the welfare (but would alone fails to attain the 55 percent emissions reduction desired) coupled with an abatement technology that is increasing over time.

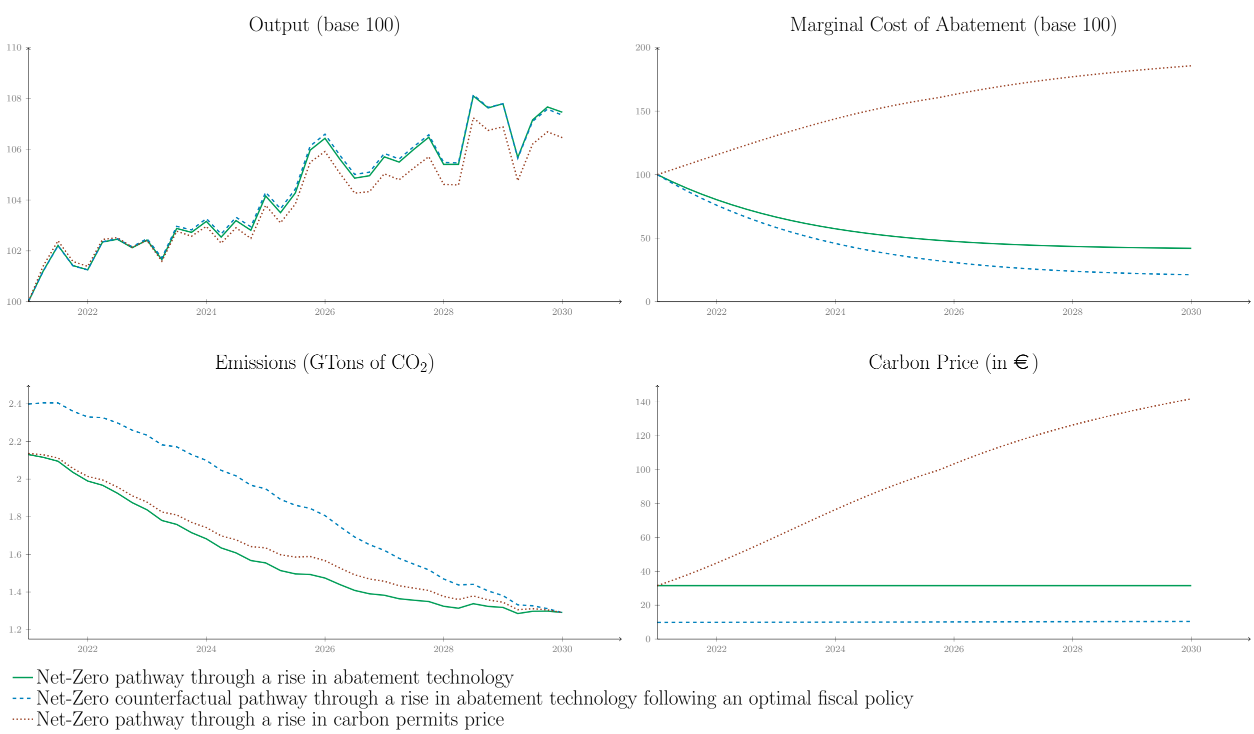
Figure I shows what carbon price and/or reduction in abatement costs trajectories would be needed to be on track for achieving the net-zero target in the EZ, assuming a growth trend of 0.8 percent.[17] We also add a stochastic shock process to TFP, that we calibrate according to the estimation in Smets and Wouters [2003]. This allows us to simulate a realistic transition scenario, where the trend in growth is anticipated, but shocks can distort this deterministic process in the short run. The blue dashed line is a scenario where we build a counterfactual highlighting the pathway if an optimal policy is set and coupled with decreasing marginal abatement costs. The green solid line is a scenario where green technology—coupled with a fixed tax rate—is the only long-run driver of emissions reduction. Finally the dotted red line corresponds to the scenario where the targeted environmental policy (e.g. EU ETS cap system) is the only instrument used to mitigate the climate externality and keeps the economy on track for achieving the desired level of emissions reduction. Relying on a targeted tax alone, requires high levels of carbon price to be on target for net-zero by 2050, and induces a higher output loss than both other scenarios where green innovation is boosted to allow for lower marginal cost of abatement, which in turn triggers higher abatement levels. We find that either fixing the environmental policy at a targeted level and allowing for green innovation to boost abatement levels, or using an

[17] The average real growth rate per capita in the EZ area from 2000 to 2020

optimal fiscal policy coupled with green innovation are more efficient in keeping higher levels of output than just relying on a carbon fiscal policy alone. It suggests that an optimal policy with green innovation boosting is the optimal choice from a welfare perspective.

This results comforts our empirical finding where both a fiscal environmental policy and green innovation growth which we characterized in the empirical section via increasing numbers of green patents (i.e. lower costs of abatement as new technology are efficient and thus allow for abating at lower costs) are both major contributors to significant emissions reduction. In addition, higher fiscal carbon prices are also shown to negatively impacts the costs of abatement where green patents tends to decrease following spikes in carbon prices.

FIGURE I. Net-Zero Transition Pathways - 2030



4 Introducing Endogenous Green Technology

In this section, we introduce green entrepreneurs who produce innovations in the abatement technology. An improvement in green technologies will, in turn, reduce the cost of abatement for firms. However, green innovators will need to rely on loans from banks to start new projects. Thus, we also show how financial intermediaries are modeled. Finally, we propose a set of policies that could help fostering green innovations. The goal is to show how public policies could ultimately impact the abatement efficiency.

4.1 Household

In this new setup, households are populated of both workers and bankers, with measures $1-f$ and f respectively. In addition, a new specialized green R&D worker force Ls_g join the global R&D skilled labour Ls_s as well as the unspecialized labour force L_t are now either specialized in who supply labor inelastically to entrepreneurs. Bankers manage a financial intermediary that uses borrowed funds to make loans to green innovators. There is perfect consumption insurance among family members.

There is random turnover between bankers and workers: a banker becomes a worker with probability $1-\theta_B$ at which time he or she transfers accumulated earnings to the family. Workers become bankers with probability $(1-\theta_B)\frac{f}{1-f}$ so there is a measure $(1-\theta_B)f$ of new bankers each period. This allows for offsetting the number that exit. The household transfers a small amount of resources to new bankers so they are able to start operations. Banker exit is introduced as a device to ensure that the financial imperfection will remain relevant—otherwise banks might reach a point where internal resources are enough to finance all desired lending.

The budget constraint of households is modified to display wages for skilled labor employed by green entrepreneurs, as well as profits from the ownership of financial intermediaries.

$$C_t + B_{t+1} + I_t + f(K_t, I_t) = W_t L_t + W s_{t,g} \bar{Ls}^g + W s_{t,s} \bar{Ls}^s + \Pi_t^{FI} + T_t + R_t^K K_t + R_t B_t \quad (45)$$

where $\bar{Ls}^g$ is the inelastic labor supply to green entrepreneurs associated with wage $Ws_{t,g}$, and Π_t^{FI} are the profit from the financial intermediaries.[18]

[18] These changes to the household budget constrain do not have any impact on the first order conditions presented in subsection 3.1.

4.2 Green Innovators

Similarly to the R&D entrepreneurs presented in subsection 3.2, we follow Comin and Gertler [2006] and introduce an unbounded mass of prospective green innovators with the ability to improve the abatement technology. However, we differ from their set up insofar as we consider that the innovators are green R&D creators that allows for improving the abatement efficiency via a reduction in abatement costs ($g(\theta_t^1)$). Each green innovator use resources to create a new project $RD_{t,g}$. Both new projects $RD_{t,g}$ and existing technologies $A_{t,g}$ face the risk of an exogenous exit shock $(1-\phi_{RD,g})$. Similarly to the R&D entrepreneurs, we assume that green innovators do not emit CO_2 while developing new technologies.

Our innovators or research and development centers need to obtain funding from banks to finance entry. Here the idea is that financial intermediaries are the economic entities with the expertise and knowledge when evaluating and monitoring green entrepreneurial projects.

The total number of green technologies in operation at any given time t is denoted by $A_{t,g}$, while the green projects $RD_{t,g}$ are the number of new technologies in process in period t. Accordingly, the evolution of the aggregate stock of green innovations, $A_{t,g}$, is given by:

$$A_{t+1,g} = \phi_{RD,g}(A_{t,g} + RD_{t,g}), \tag{46}$$

To be more specific, each green innovator can produce a new potential technology by employing materials and skilled workers as inputs, according to the following production function:

$$RD_{t,g} = N_{t,g}^{\eta_g}(A_{t,g}Ls_{t,g})^{1-\eta},\ \eta_g \in (0,1), \tag{47}$$

where $N_{t,g}$ is the amount of materials used (in units of final output) and $Ls_{t,g}$ is the number of skilled workers hired. $A_{t,g}$ denotes the aggregate green technological level of the economy, which as explained below is equal to the total number of technologies in operation. Similarly to the R&D entrepreneurs, the innovators production function captures the externality of the aggregate level of knowledge $A_{t,g}$, which allows for generating endogenous growth.[19]

[19] For simplicity, we consider that spillovers on the green innovation only originate from the green technological level $A_{t,g}$.

Once the technology created, entrepreneurs lend it to monopolist firms in exchange for patent exclusivity. The monopolists then use these technologies to lower their abatement cost and pay a rent Z_t corresponding to abatement costs to the green innovators.

As in Queralto [2020], we assume that green entrepreneurs can borrow to face the entry cost without any friction. More specifically, when seeking funding, our innovators can emit a financial intermediaries security which is perfectly contingent on the success of the green project. However, as in Gertler and Karadi [2011], banks do face frictions relative to their leverage ratio, as we will show in the next section. As long as the innovation does not become obsolete, the underlying securities pay in each future period. If the innovation becomes obsolete, then the payoff is zero. We denote the price of one unit of these securities $Q_{t,e}$.

The green innovators optimize over the revenues from selling securities subject to the inherent costs of developing the innovation by using materials N_t and paying wages Ws_t to the skilled labor Ls_t. The maximization problem reads as follows:

$$\max_{\{RD_{t,g}, N_{t,g}, Ls_{t,g}\}} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left[Q_{t+i,e} RD_{t+i,g} - (N_{t+i,g} + Ws_{t+i,g} Ls_{t+i,g}) \right] \tag{48}$$

s.t.

$$RD_{t+i,g} = N_{t+i,g}^{\eta_g} (A_{t+i,g} Ls_{t+i,g})^{1-\eta_g} \tag{49}$$

The first order condition reads (denoting $MC_t^{RD,g}$ the Lagrange multiplier associated to the production constraint):

$$1 = MC_t^{RD,g} \eta_g N_{t,g}^{\eta_g - 1} (A_{t,g} Ls_{t,g})^{1-\eta_g}, \tag{50}$$

$$Ws_{t,g} = MC_t^{RD,g} (1-\eta_g) A_{t,g} N_{t,g}^{\eta_g} (A_{t,g} Ls_{t,g})^{-\eta_g}, \tag{51}$$

$$Q_{t,e} = MC_t^{RD,g}. \tag{52}$$

Using these first order conditions[20] and equation (47), we can rewrite the price of the inherent

[20] With $Ws_{t,g} = \frac{1-\eta_g}{\eta_g} \frac{N_{t,g}}{A_{t,g} Ls_{t,g}}$

security $Q_{t,e}$ in terms of the marginal cost components as following:[21]

$$Q_{t,e} = MC_t^{RD,g} = \frac{1}{\eta_g} \left(\frac{1}{\bar{Ls}^g} \right)^{\frac{1-\eta_g}{\eta_g}} \left(\frac{RD_{t,g}^N}{A_{t,g}} \right)^{\frac{1-\eta_g}{\eta_g}}, \tag{53}$$

Contrary to the previous section, where the cost of abatement was driven by an exogenous process, the cost function of abatement is now steered by endogenous green technological changes. Thus, green innovators projects will ultimately lead to higher abatement and lower emissions. The equation (27) now reads:

$$f(\mu_t) = \left(\int_0^{A_{t,g}} f(\mu_{jt})^{\frac{1}{\theta_3}} dj \right)^{\theta_3} \tag{54}$$

Thus,

$$g(\theta_t^1) = \theta_1 A_{t,g}^{-\theta_3}, \ \theta_1 > 0 \text{ and } \theta_3 > 0, \tag{55}$$

where θ_3 is now the elasticity of the cost of abatement with respect to the green technology.

4.3 Financial Intermediaries

A representative financial intermediary make use of deposits from households as well as its own net worth to leverage and invest in green entrepreneurs. We model this part following Gertler and Karadi [2011]. We can write the representative bank's balance sheet as:

$$Q_{t,e} S_{t,e} = N_t + B_t, \tag{56}$$

where $S_{t,e}$ are financial claims on green innovators and $Q_{t,e}$ their relative price. Note that market clearing implies that $S_{t,e} = A_{t,g} + RD_{t,g}$, as assets held by banks must match the total number of existing green technologies. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, banks' retained earnings evolve as follows:

$$N_t = R_{t,e} Q_{t-1,e} S_{t-1,e} - R_t B_{t-1}, \tag{57}$$

[21] We also use the market clearing condition for skilled labor: $Ls_{t,g} = \bar{Ls}^g$.

$$N_t = (R_{t,e} - R_t)Q_{t-1,e}S_{t-1,e} + R_t N_{t-1}, \tag{58}$$

where $R_{t,e}$ denotes the gross rate of return on a unit of the bank's claims on green innovators:

$$R_{e,t} = \frac{\phi_{RD_g}(Z_t + Q_{t,e})}{Q_{t-1,e}}. \tag{59}$$

Financial intermediaries will maximize equity on an infinite horizon, yielding the following objective function:

$$V_t^B = E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1-\theta_B)\theta_B^{i-1} N_{t+1+i} \right\}, \tag{60}$$

where θ_B is the probability of a bank exiting the market. The constraint on banks arise from the existence of a supervisory regulator. Drawing on Pietrunti [2017], we assume that this regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t^B \geq \lambda Q_{t,e} S_{t,e}. \tag{61}$$

In this simplified setup, banks only hold one asset, so the regulator will set a value for λ in order to target a specific capital ratio for banks. By modifying this parameter, the macroprudential authority will be able to tighten or relax the constraint on banks, which will impact the number of entrepreneurial projects the financial sector can fund. In our baseline model, we will calibrate λ to match the capital ratio of European banks at the steady state. We guess that the value function is linear of the form $V_t = \Gamma_t^B N_t$ so we can rewrite V_t^B as:

$$V_t^B = \max_{S_{t,e}} E_t \left\{ \beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \right\}, \tag{62}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t^B$. Maximization subject to constraint (61) yields the following first order and slackness conditions:

$$\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,e} - R_{t+1}) \right\} = \nu_t \lambda, \tag{63}$$

$$\nu_t \left[\Gamma_t^B N_t - \lambda Q_{t,e} S_{t,e} \right] = 0, \tag{64}$$

where ν_t is the multiplier for constraint (61). We can thus write the capital ratio as $\Xi_t = \lambda/\Gamma_t^B$. Finally, we rewrite the value function to find Γ_t^B:

$$\begin{aligned} V_t^B &= \nu_t \lambda Q_{t,e} S_{t,e} + \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \right\} \\ \Gamma_t^B N_t &= \nu_t \Gamma_t^B N_t + \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \right\} \\ \Gamma_t^B &= \frac{1}{1-\nu_t} \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \right\}. \end{aligned} \tag{65}$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B (R_{t,e} - R_t) Q_{t-1,e} S_{t-1,e} + (\theta_B R_t + \omega) N_{t-1}, \tag{66}$$

with $\omega \in [0;1)$ the proportion of funds transferred to entering bankers.

4.4 Carbon Policy and Green Innovation

As argued in the section above on the model equilibrium, many economies rely on a permit-market-based instrument instead of an optimal carbon price (e.g. the ETS in the EU and the carbon permit markets in Canada in California (US)). Thus, in order to reach the Paris Agreement objective of the net-zero emissions by 2050, such carbon pricing strategy requires carbon prices to constantly increase, which in turn incentivizes firms to engage in continuously higher abatement efforts. However, investing in abatement technologies is costly and has a number of consequences such as welfare losses as shown in Benmir and Roman [2020]. Steering green innovation via other tools besides carbon pricing would be less welfare distortionary. Incentivizing green innovation that lowers the cost of abatement, however, might prove difficult if the price of carbon increases substantially and in places where no green abatement technology is yet available.

Definition 4 *A government, when relying on a carbon permit market solely to tackle the climate externality, sets a carbon cap:*

$$E_t = Cap_t \tag{67}$$

which inherently determines a carbon price level τ_{et}:

$$\tau_{et} = Carbon\ Price_t. \tag{68}$$

where Cap_t is the path of the cap on emissions consistent with the net-zero objective, and $Carbon\ Price_t$ the inherent carbon price associated with this objective. To reach the net-zero target, the price is expected to steadily increase in order to match the expected decrease in the cap.

However, under the presence of endogenous green innovation that contributes to lowering the cost of abatement, the social planner is not limited anymore in terms of tools it could use, and is able to rely on both a carbon price τ_{et} and the green technologies $A_{t,g}$:

Definition 5 *To decrease emissions, firms engage in higher abatement efforts:*

$$\mu_t = 1 - \frac{Cap_t}{\upsilon Y_t} \tag{69}$$

with $\Delta\left(\frac{Cap_t}{\upsilon Y_t}\right) < 0$. Otherwise, the optimal social cost of carbon presented in the initial exogenous framework would be able to achieve the target. Therefore, the carbon price, as defined in equation (32), is driven by two instruments, namely, i) the environmental cap Cap_t and ii) the green technologies $A_{t,g}$:

$$Carbon\ Price_t = \theta_1 \theta_2 \frac{\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1}}{\upsilon} A_{t,g}^{-\theta_3} \tag{70}$$

Effectively, when $Cap_t = \upsilon Y_t = \bar{E}_t$[22] (i.e. a laissez-faire economy)

$$\min(Carbon\ Price_t) = 0 \tag{71}$$

And when $Cap_t = 0 \Rightarrow \mu_t = 1$ (i.e. a net-zero objective)

$$\max(Carbon\ Price_t) = \theta_1 \theta_2 \frac{1}{\upsilon} A_{t,g}^{-\theta_3} \tag{72}$$

Definition 6 *When it is impossible to implement an optimal policy[23] $\tau_{et} > SCC_t$, public authorities insure a specific carbon price by setting a cap on emissions. However, the design and trajectory of the cap policy Cap_t could also have indirect consequences on green innovation. Depending on the cap policy implemented, this could have the opposite effect. That is, instead of increasing the total cost of abatement for firms $Z_t = f(\mu_t)Y_t$, the loss in output could translate to a lower total cost of abatement Z_t. This decrease in Z_t would reduce banks' investments in green projects. Ultimately, it would lead to slower green innovation and a*

[22] $\bar{E}_t$ the steady state level of emissions at each period t

[23] Implementing an optimal policy requires major institutional constraints and carbon pricing monitoring, which cannot be achieved with the current public institutions (Delpla and Gollier [2019]).

lower growth rate of $A_{t,g}$.[24]

$$\frac{\partial Z_t}{\partial Cap_t} = \frac{\partial f(\mu_t)}{\partial Cap_t} Y_t + \frac{\partial Y_t}{\partial Cap_t} f(\mu_t) \tag{73}$$

$$= \left(\frac{\partial f(\mu_t)}{\partial Cap_t} + \frac{1}{d(T_t^o)} \frac{\partial d(T_t^o)}{\partial Cap_t} f(\mu_t) \right) Y_t \tag{74}$$

Thus, there exists a Cap_t^* for a level of $A_{t,g}$[25]*, such that $\frac{\partial f(\mu_t)}{\partial Cap_t} + \frac{1}{d(T_t^o)} \frac{\partial d(T_t^o)}{\partial Cap_t} f(\mu_t) = 0$.*

Corollary 1 $\Delta Cap_t < 0 \Rightarrow \Delta Z_t > 0 \Rightarrow \Delta R_{t,e} > 0 \Rightarrow \Delta RD_{t,g} > 0$
An increase in the carbon price (i.e. a decrease in the cap), triggers more abatement, which in turn increases the cost of abatement $Z_t = f(\mu_t)Y_t$, as firms would equate their marginal benefit from investing in abatement to the carbon price. This increase in Z_t imply a higher rate of return on entrepreneurs equity $R_{e,t}$ as entrepreneurs' profits are reversed to banks. The higher the profatibility of entrepreneurs, the more banks would direct investment toward green projects, which would spur green innovation $A_{t,g}$.

Corollary 2 $\Delta Cap_t << 0 \Rightarrow \Delta Z_t < 0 \Rightarrow \Delta R_{t,e} < 0 \Rightarrow \Delta RD_{t,g} < 0$
A significant change in the cap policy design might not always result in an increase of the cost of abatement $Z_t = f(\mu_t)Y_t$. Although a rise in the carbon price increases $f(\mu_t)$ on one hand, it also decreases profits Π_t and output Y_t on the other hand. There exist a point where the decease in output Y_t is superior to the increase in $f(\mu_t)$, which results in a decrease in Z_t. This would in turn lower the rate of return $R_{t,e}$, which would contribute negatively to green innovation.

Proposition 1 *To ensure we meet the net-zero target with a deacreasing cap on emissions, while trying to mitigate the effect on welfare of a rising carbon price, we investigate three macro-financial tools that could foster green innovation: i) the fiscal authority uses revenues from carbon pricing policy to subsidize green innovators; ii) the macroprudential authority adapt its capital requirement to give an incentive to financial intermediaries to invest in green entrepreneurs' equity, thus generating a greater number of successful green technologies; and iii) the central bank engages in an asset purchase program aiming to ease funding conditions for the green innovation sector.*

i) Fiscal Policy

As presented in the model section, the government finances its government spending as follows:

$$T_t + (1 - \bar{s})\tau_t E_t = G_t, \tag{75}$$

[24] As shown in Appendix section B.5, note that, as we will get closer to the end of transition to net-zero, the high level of $A_{t,g}$ will imply a decreasing cost Z_t. Through a feedback loop, this will make investment in green projects less interesting for banks and the growth rate of green technologies will be lower.

[25] Note that Z_t is concave in our case.

with the public expenditure G_t finding its source from taxes T_t and revenues from the carbon tax $\tau_t E_t$.

In this setting we will consider the possibility for the government to divert part $\bar{s}$ or all of the environmental policy revenues back to the green innovators (if $\bar{s} = 0$ no subsidy is diverted to the green innovators). In this case, subsidies would raise profits of green entrepreneurs and ultimately be reversed to banks as interest:

$$R_{e,t} = \frac{\phi_{RD}(Z_t + Q_{t,e} + \bar{s}\tau_t E_t)}{Q_{t-1,e}}. \tag{76}$$

ii) Macroprudential Authority

As detailed in section 4.3, the macroprudential authority imposes a capital constraint on banks modeled through the parameter λ that pins down the steady state capital ratio. In a more sophisticated model, claims on green entrepreneurs could be one of several assets held by banks. In this case, different weights could be applied to different assets, and the regulator could favor a specific sector.[26] Our setup is without loss of generality, since modifying λ in our model is similar to modifying the weight on loans to entrepreneurs in a model with several assets, keeping all other weights constant.

Furthermore, we also allow the macroprudential authority to react to changes in the stock of emissions. By doing so, the macroprudential authority is able to steer credit to the green entrepreneurs when emissions flow of CO_2 in the atmosphere is going far away from its steady state. The macroprudential rule in this setting will read as follows:

$$\lambda_t = 1 - \lambda(E_t - \bar{E}) \tag{77}$$

iii) Quantitative Easing

In the previous sections, we introduced the link between the financial sectors and the development of green technologies. We also laid the ground for policy intervention through the existence of a macroprudential authority. As recently put forward in the monetary policy

[26] See Benmir and Roman [2020].

strategy review of the ECB, central banks have a role to play in the fight against climate change. Whether on inflation stabilization or financial stability grounds, there is a growing understanding of risks arising from global warming and potential room for central bank intervention. We now introduce a central bank that can substitute for financial intermediaries in financing green entrepreneurs. Thus, total claims on entrepreneurs are split between government and private holdings:

$$Q_{t,e}S_{t,e} = Q_{pt,e}S_{pt,e} + Q_{gt,e}S_{gt,e}, \tag{78}$$

with $Q_{gt,e}S_{gt,e}$ the total real value of loans to entrepreneurs held by the central bank. $Q_{pt,e}S_{pt,e}$ is the total real value of loans to firms of type k held by financial intermediaries as defined in section 4.3. The central bank decides in every period to hold a portion $\psi_{t,e}$ of total loans to green entrepreneurs:

$$Q_{gt,e}S_{gt,e} = \psi_{t,e}Q_{t,e}S_{t,e}. \tag{79}$$

We assume that the central bank reacts to deviations of carbon emissions from their targeted level (i.e. steady state at the business cycle frequency) in order to decide the share of assets $\psi_{t,e}$ it holds. This rule reads as follows:

$$\psi_{t,e} = \phi^s(E_t - \bar{E}), \tag{80}$$

where the reaction parameter ϕ^s is set at 10[27]. Note that in our baseline model $\bar{\psi}_{t,e} > 0$ in order to account for the fact that the ECB keeps a substantial share of private assets in its portfolio.

4.5 Normalization and Aggregation

When introducing green innovators, the resource constraint of the economy is modified as follows:

$$Y_t = C_t + G_t + I_t + N_{t,g} + N_{t,s} + f(.)I_t. \tag{81}$$

[27] This corresponds to a maximum 12% of asset purshases at over sample period, and is aligned with Gertler and Karadi [2011].

5 The Balanced Growth Path

From the empirical data on global patents, green patents, and output, both green investment $N_{t,g}$ and global R&D investments $N_{t,s}$ are found to have higher trend growth than output. This empirical finding requires us to balance the growth rates of the green and global R&D investments on the supply side of the resource constraint of our economy to ensure balanced growth. Thus, to allow for a balanced growth path, we introduce investment-specific trends à la Greenwood et al. [1997] that we denote as $V_{t,g} = \gamma_g^V V_{t-1,g}$ and $V_{t,s} = \gamma_s^V V_{t-1,s}$, where γ_g^V and γ_s^V are constant growth rates. These investment goods $N_{t,g}$ and $N_{t,s}$ are produced from final goods by means of a linear technology, whereby $\frac{1}{V_{t,g}}$ and $\frac{1}{V_{t,s}}$ units of final goods yield one unit of investment goods, respectively.[28]

Furthermore, the non-linear climate damages within the production function does not allow for a balanced growth path when considered as the following: $d(T_t^o) = ae^{-bT_t^{o2}}$. To allow for a balanced growth path trajectory, we show that over the period horizon we consider for our estimation (2000-2020), the low growth rate Γ_t had a small to no effect on the damage function dynamics $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}} \approx ae^{-bT_t^{o2}}$. Capturing the growth rate of the economy within the damage function allows for simplifying the de-trended form of the damage function without a loss of generality, given that over the period sample of our estimation, climate damages that are corrected for the economy growth rate Γ_t are not significantly different from climate damages that are not corrected for the economy growth rate. In addition, given that both climate is defined as the average change over the past 30 years, and that the stock of emissions is a slow moving variable, our 20 year sample period allows us to consider the damage function as a de-trended equation, which allows for reconciling the balanced growth path.

Our economy presents three sources of permanent growth: i) an endogenous source of growth $A_{t,s}$, ii) two exogenous sources of growth $V_{t,s}$ and $V_{t,g}$, and iii) a fourth endogenous source of green innovation growth $A_{t,g}$ which impacts the efficiency of abatement. Having

[28]The slope of this investment-specific trend crucially appears in the measurement equation of the model and is estimated.

these different sources of growth requires that we de-trend our model as a number of variables (e.g. output, emissions, investment, ...) will not be stationary. In the online appendix we present the de-trended economy. The aggregate variables of our economy,[29] include: output per capita Y_t, investment per capita I_t, consumption per capita C_t, government spending G_t, lump sum taxes T_t, capital per capita K_{t-1}, emissions E_t, abatement costs $Z_t/V_{t,g}$, green investment expenditures $N_{t,g}/V_{t,g}$, global R&D investment expenditures $N_{t,s}/V_{t,s}$, stock of emissions X_t, Temperature T_t^o, R&D varieties per capita $RD_{t,s}$, and green innovation varieties per capita $RD_{t,g}$, wages W_t, skilled labour wages $W_{t,s}$, relative price of financial claims $Q_{t,e}$, debt to households B_t, net worth N_t, and the banks value function V_t^B, and all grow at the same rate Γ_t, which reads as the following:

$$\Gamma_t = A_{t,s}^{\overline{\frac{1}{(\theta-1)(1-\alpha)}}} \tag{82}$$

where $\Gamma_t = \gamma_t^Y \Gamma_{t-1}$, the stock growth of R&D $A_{t,s}$ is $\gamma_t^{A_s} = \frac{A_{t,s}}{A_{t-1,s}}$, and the stock growth of green innovation $A_{t,g}$ is $\gamma_t^{A_g} = \frac{A_{t,g}}{A_{t-1,g}}$.

6 Quantitative Analysis

6.1 Calibration and Estimation

6.1.1 Data and Measurement Equations

The model is estimated using Bayesian methods and EZ quarterly data over the sample time period 2000Q1 to 2019Q4. Data are taken from both Eurostat and the European Patent Office. We focus on the period between 2000 and 2019, as the decoupling between emissions and output started to be more significant in the 2000s. Furthermore, empirical data also support this strategy, since investment in decarbonized technologies started to exhibit a trend at the same time.

In order to estimate the key shocks and parameters of our model, we start by making our four series (output, emissions, R&D and green innovation expenditures, which we proxy via

[29]Along the balanced growth path.

patents numbers) stationnary. We first divide the sample by the working age population. Second, data are taken in logs and we then use a first difference filtering to obtain growth rates. Finally, we use the GDP price index to deflate all nominal variables.

To measure the empirical contribution of endogenous growth in green and standard technologies, we follow Vermandel [2019] and use a cost-based approach. As there is no data available for quarterly investment in both green technologies and global R&D, we use the number of patents filed to proxy expenditures.

Measurement equations are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \\ \text{Real Per Capita R\&D Expenditure Growth} \\ \text{Real Per Capita Green Innovation Expenditure Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_t^Y + \Delta \log (\tilde{y}_t) \\ \log \gamma_t^Y + \Delta \log (\tilde{e}_t) \\ \log(\gamma_t^Y / \gamma_s^V) + \Delta \log (\tilde{n}_{t,s}) \\ \log(\gamma_t^Y / \gamma_g^V) + \Delta \log (\tilde{n}_{t,g}) \end{bmatrix}, \tag{83}$$

where tilde denote de-trended variables.[30]

6.1.2 Calibration and Prior Distribution

As the main objective of our paper is to assess trends in R&D and green innovation growth, all standard macro-finance and environmental parameters are calibrated from the literature. The calibration values for the standard macro block and the environmental components are reported in table VIII and table IX. Table X reports the calibration of financial parameters related to the full model. We set the probability of remaining a banker θ_B at 0.972 as in Gertler and Karadi [2011]. We find the values of the proportional transfer to the entering banker ω and the regulatory parameter λ to approximately match both the debt to equity ratio[31] and the capital ratio in the EA. Because we only model loans to entrepreneurs, that are seen to carry a high level of risk, we assume that the regulator applies

[30] The balanced growth path of the model can be found in the appendix.

[31] We compute the debt to equity ratio by taking the sum of the debt to equity ratios of the 19 EZ countries, weighted by their relative shares in total banks assets, using data from Eurostat and the ECB.

a 150% weight[32] to such assets before multiplying it by the theoretical capital requirement for banks of 10.5%. This yields an effective capital ratio of 15.75% in our baseline model.

For the remaining set of parameters and shocks, we rely on Bayesian methods. In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method (MCMC). In the following fit exercise, we solve the model using a linear approximation to the policy function, and employ the Kalman filter to form the likelihood function. Table XI summarizes the prior—as well as the posterior—distributions of the structural parameters for the U.S. economy. As in Smets and Wouters [2003] the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while the standard deviation of shocks follow an inverse gamma distribution with mean 0.001 and standard deviation of 0.005.

The output growth rate γ_y and green innovation growth rate γ_{A_g} are estimated using a prior standard deviation of a gamma distribution with mean 0.05 and 0.01, respectively, while we use a beta distribution with mean 0.125 and 0.15 for the investment share in R&D η_s and green innovations η_g. Finally, the exogenous R&D and green innovation investment growth rates γ_{V_s} and γ_{V_g} are estimated using a normal distribution with means 1 and standard deviations of 0.2.

6.1.3 Posterior Distribution

In addition to prior distributions, table XI reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel MCMC chains of 20,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor so as to match an average acceptance rate close to 25-30 percent for each chain.

Results of the posterior distributions for each estimated parameter are listed in table XI.

[32] Corresponding to the highest weight possible for corporate loans according to Basel III regulation.

It is clear from table XI that the data were informative, as the shape of the posterior distributions differs from the priors. Results for structural shocks parameters that are common with Smets and Wouters [2003] are in line with the values they find. Regarding investment elasticities η_k with $k \in \{s, g\}$, our values are close to Queralto [2020]. As for the endogenous and exogenous trends, our estimates are consistent with the observed empirical output and green innovation investment growth rates.

6.2 Endogenous Trends

In this section, we first discuss the results of our estimation of endogenous growth trends in output and green innovation. We then perform a counterfactual exercise to assess the relevance of policies aiming at boosting the growth trend in green innovation.

6.2.1 Estimated Trends

Figure III and figure IV display the estimated trends in output and green technology, respectively. Those two trends are highly correlated,[33] but the trend on green innovation is approximately twice as high as the trend on output. This can explain the decoupling between emissions and output witnessed over the studied period. The trend on green innovation also exhibits more volatility at the business cycle frequency, which is consistent with the fact that the green technology sector is less mature than standard R&D.

6.2.2 Incentive Policies for Green Innovation

Now that we have retrieved the time path of the two endogenous trends, we perform counterfactual exercises by retrospectively implementing public policies designed to affect the behavior of green entrepreneurs and trigger a higher growth in green innovation.

Tax, Subsidies and Green Innovation

Our first counterfactual exercise is to implement a subsidy scheme as defined in subsection 4.4. By reversing revenues from the carbon tax to green entrepreneurs, the goal is to

[33] This is not surprising, since the model features a spillover effect from the global technology to the green technology.

foster investment in green technologies. Figure V shows the time path of the trend on green innovation when the tax levied through the carbon permit market is turned into subsidies for green entrepreneurs compared to the baseline model where the revenues from the tax simply finance government spending. The subsidy policy would have worked very well from 2004 to 2011, by raising the trend growth on green innovation by 0.1% to 0.3%. The effect is much more diffuse, however, after the year 2012. This can be explained by the fact that the ratio of emissions to output started to decline around this time, implying lower revenues from the carbon tax and, hence, lower subsidies for green innovators.

Macruprudential Policy and Green Innovation

In our second counterfactual exercise, we implement a macroprudential policy rule that reacts to deviations of the emissions level of carbon from its steady state. Figure VI displays the time path of the trend on green innovation when the macroprudential policy is active compared to the baseline model. The idea here is to give an incentive to banks to lend more freely to the green entrepreneurs when the emissions flow of CO_2 is too high. To do so, the macroprudential authority lowers λ_t, following the macroprudential rule specified in subsection 4.4. This implies a decrease in the capital ratio of banks, but also more funds available to green entrepreneurs to start new projects. The policy is effective in steering new projects and the green technology from 2004 to 2011. The reason is that, emissions increased in the first 7 years due to the inefficiencies of the ETS system phase 1 and 2 which were still experimental. The macroprudential authority reacting to deviations from the detrended steady state got increasingly worried about emissions dynamics and progressively loosened the capital requirements on banks, leading to the launch of more new projects by green entrepreneurs.

Interestingly, the two public policies studied here seem to be achieving similar results but with different magnitudes. In the counterfactual research we conduct, we don't consider an optimal design of the macroprudential rule which could react differently to changes in emissions in order to maximize the growth in green technologies. One could imagine a rule where the financial authority only reacts to an increase in emissions ($E_t - \bar{E} > 0$) for instance. We leave this work on optimal macroprudential rules for future research.

QE Policy and Green Innovation

In our third counterfactual exercise, we implement a QE policy rule that reacts to deviations of the emissions level of carbon from its steady state, similarly to the macroprudential rule. Figure VII displays the time path of the trend on green innovation when the QE policy is active compared to the baseline model. In this scenario, the aim is to allow the central bank to directly fund green entrepreneurs, which would boost the number of green projects and ultimately lead to a higher growth in green technologies. Just like the macroprudential policy, the QE rule is not set optimally, with respect to growth in green technologies. We find that this policy is very similar to the macroprudential policy and acts pro-cyclically as the subsidy policy. The explanation for this pro-cyclicality feature lies in the way we model innovation in green technologies. As shown in section 4.4, a higher total cost of abatement leads to higher profits for entrepreneurs and triggers more growth in green technologies. Periods of high emissions (compared to steady state) also imply a higher abatement cost for firms, as we consider the carbon price constant. Thus, the policies we consider will only reinforce this effect, by incentivizing banks to lend more to green entrepreneurs when profits in this sector are already rising.

6.3 Transition Pathways with Endogenous Abatement Technology

In this section, we characterize the dynamics of the economy when considering the net-zero pathway consistent with the objective of the EU for 2050 ($E_t = Cap_t$) under the presence of i) a fiscal subsidy scheme where 70% of the environmental revenues are reversed to the financial intermediaries to incentive higher investments in green technologies, ii) a permanent macroprudential policy, which lowers the capital constraint on financial intermediaries by 30%, thus allowing them to increase investments in green entrepreneurs, and iii) an asset purchase program where the central bank buys around 1% of total claims on green entrepreneurs per year. We use the estimated values of the structural parameters to replicate the growth rates in productivity and green technologies of the EZ economy. Furthermore, as we are unable to estimate the elasticity θ_3 of abatement costs $f(\mu_t)$ to green technology $A_{t,g}$ due to data unavailability, we consider three different cases that corresponds to three different values of $\theta_3 \in (0, 1)$.

Figure VIII shows the dynamics of our key variables (output, emissions, carbon price, marginal abatement cost, green technology, and global R&D) under a net-zero scenario. The carbon price is significantly driven by the elasticity θ_3. The scenario where $\theta_3 = 1$ (the blue line) is the most optimal in terms of welfare, as the price of carbon is constantly decreasing, which is not the case when $\theta_3 = .7$ and $\theta_3 = .3$. With a higher theta, the output growth rate is also higher as profits are less impacted negatively by the carbon price. This impact on profits in turn lowers the global R&D investments and level. Turning to innovation in green technologies, a higher elasticity lowers the marginal cost of abatement, which leads to a lower carbon price to meet the emissions reduction goal. We note that a scenario where $\theta_3 = 1$ is highly unlikely as carbon prices are increasing nowadays, suggesting that $\theta_3 < 1$.[34]

Figure IX, figure X, and figure XI display the counterfactual exercises where the public authorities implement either a fiscal, macroprudential, or monetary policy. Since the level of θ_3 is highly uncertain, we show the transition paths for the 3 values considered above. Focusing, however, on the the case where $\theta_3 = .3$ (the most conservative case), a financial fiscal subsidy, which reverses 70% of the carbon policy revenues to green innovators, is found to be the most effective in steering both growth in green technologies as well as global R&D. The macroprudential policy and QE policy both act as carbon price stabilizers (a lower increase in the first half of the 30 years than the subsidy case). In all scenarios, the carbon price increases in the first 15 years, until the technology is mature enough to trigger higher abatement without having to raise the price on carbon as explained in section 4.4.

[34] Further research could be done to investigate the elasticity of θ_3 of abatement cost to green technologies to better characterize the economy dynamics.

7 Conclusion

In this article, we first conduct an empirical analysis on the role of the ETS in emissions reduction within the EZ using a diff-in-diff analysis, with the US as the control area. We find that the cap and trade EU system contributed significantly to emissions reduction. We then rely on a panel data set on the EZ to assess the impacts of fiscal environmental policies and long-term bank lending on green innovation. We find that both the environmental policy and the availability of funds play an important and significant role in boosting green innovation. However, we also find that above a certain threshold, the carbon price has a negative effect on green innovation.

Second, we develop a dynamic general equilibrium model based on the empirical evidence to assess the role fiscal and macro-financial policies can play both in the long-run and in the short-run.

We use a reduced form model to get the long-run transition pathways toward the net-zero transition and find that making abatement technology available and cheap coupled with an optimal environmental policy is the most efficient tool (from a welfare perspective) in achieving climate goals. Relying solely on a carbon price could reach the same target, but comes with higher welfare costs.

Finally, we use a full fledged model incorporating both endogenous green innovation growth and financial intermediaries to quantitatively estimate trends on output and green innovation. We then assess the role subsidies, macroprudential policies, and QE, could play in boosting green innovation. We show that these three policies differently affect the path of the trend growth in green innovation, but that they have the same pro-cyclical dynamics. In addition, we show that financial subsidies are more effective than macroprudential and QE rules in reaching the net-zero while ensuring a lower carbon price over time. This leads us to conclude that policy makers could optimally foster growth in projects that enable cheaper and more effective abatement by giving incentives to financial intermediaries and entrepreneurs. In the context of the fight against climate change, and keeping in mind the ambitious goals that it requires, these findings represent both a glimmer of hope and a call for more action.

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A Appendix: Empirical Part

A.1 Data Sources

The data used[35] in this section were obtained from following sources:

- "Long-term loans granted by the financial sector to domestic non-financial corporation" were extracted from the ECB Statistical Data Warehouse.
- All EZ macro data (output, consumption, government spending, investment, export of goods, export of services, import of goods, import of services, taxes on goods, subsidies on goods) were obtained from the Eurostat database.
- EZ and US Emission data were obtained from the University of Oxford ourworldindata.org database.[36]
- All US macro data are obtained through Fred database.
- Both US and EZ area and countires deflators, as well as crude oil price are extracted from Fred database.
- Quarterly population for all samples are obtained from the OECD database.
- 'Green Patent' data are extracted from the European Patent Office (EPO) database.[37]
- ETS carbon price data are obtained from the European Environment Agency.

[35] All data used were either extracted directly on a quarterly basis or transformed from a monthly frequency to a quarterly frequency.

[36] The only data available for the EZ countries are yearly aggregates. We use a spline to transform yearly emission data to quarterly frequency in order to have a balanced dataset.

[37] Data on green patents are selected through the new search filter introduced by the EPO: "cpc = y02", which allows for identifying patents with green applicability.

A.2 Empirical Results

FIGURE II. Parallel Trends Hypothesis

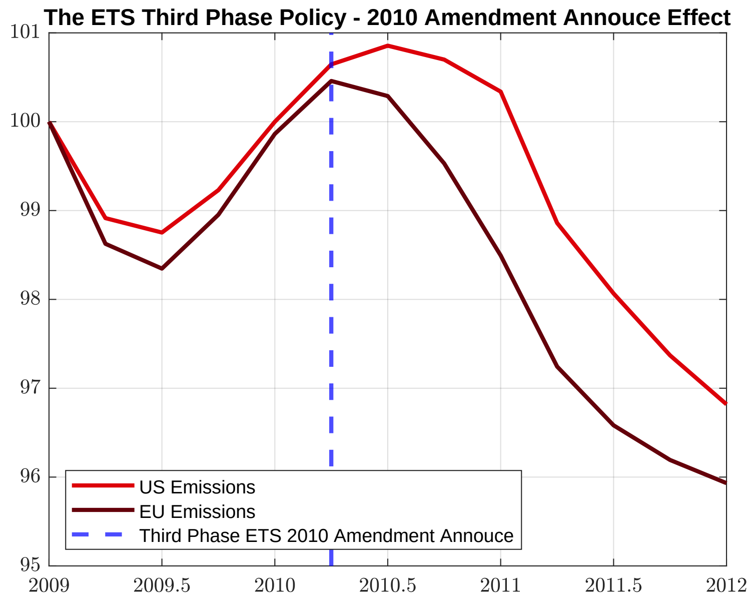


TABLE IV

Descriptive Statistics EZ and US.

Variable	Obs	Mean	Std. Dev.	Min	Max
The EZ					
GDP in (Million of Currency)	80	2361.266	344.8842	1725.153	3013.108
Emissions in $GTCO_2$	80	2646960	252602.9	2172255	2985500
Population	80	333.7475	6.509189	319.8963	342.2888
Deflator	80	92.7957	7.953556	77.50166	105.901
Green Patents	80	3111.275	679.2269	1139	4309
Oil Price	80	51.71275	19.41105	20.9	95.61
Gov Spending	80	484.9802	79.18786	336.0314	621.4932
Household Consumption	80	1270.093	167.5075	948.4206	1565.283
Gross capital formation	80	511.4903	70.16437	400.4561	690.9054
Exports of good	80	729.8737	173.5684	450.8578	1028.471
Exports of services	80	245.521	84.31034	127.6664	423.7959
Imports of good	80	681.3173	150.7942	433.557	941.4877
Imports of services	80	230.6274	77.03146	126.3731	414.0665
The US					
GDP in (Million of Currency)	80	3819.033	821.707	2500.714	5436.849
Emissions in $GTCO_2$	80	5668626	340328.9	4634741	6139822
Population	80	306.8736	14.38066	280.4759	327.2556
Deflator	80	95.2847	10.36493	77.396	112.95
Green Patents	80	6782.775	2355.937	2496	10575
Oil Price	80	62.41812	26.59745	19.96	139.96
Gov Spending	80	830.0017	215.3109	466.8755	1204.646
Household Consumption	80	2581.976	568.6365	1653.4	3689.8
Gross capital formation	80	198.5823	38.74323	145.228	281.3773
Exports of good	80	257.3163	73.25903	140.6633	356.7225
Exports of services	80	118.5654	42.93334	54.34583	184.0133
Imports of good	80	402.8038	96.94235	225.3292	538.12
Imports of services	80	84.67429	24.1856	43.94917	124.3208

TABLE V
Descriptive Statistics EZ aggregate.

Variable	Obs	Mean	Std. Dev.	Min	Max
The aggregate EZ					
GDP in (Million of Currency)	864	140894.1	207878.3	1509.7	872335
Population	856	19233.6	24911.43	486	83145
ETS price	846	11.63263	6.868454	3.8696	27.13354
Deflator	864	98.48927	5.351341	80.69107	115.0133
Green Patents	864	202.1134	485.9407	0	2672
Oil Price	864	61.73833	16.73311	30.26	95.61
Gross capital formation	864	29631.05	43870.17	80	189979
Long-term loans	862	196488.1	254660.6	2612.26	920094

TABLE VI

Green Innovation Drivers: Panel OLS Regression - Robustness A

Green R&D	(1)	(2)	(3)	(4)	(5)
ETS Price > 5	16.87				
	(23.13)				
ETS Price > 10		20.79			
		(26.25)			
ETS Price > 15			-150.1*		
			(82.27)		
ETS Price > 20				-150.1*	
				(82.27)	
ETS Price > 25					-111.4*
					(56.91)
Long-term Loan (2 year lag)	0.0972***	0.0972***	0.0972***	0.0972***	0.0972***
	(0.0141)	(0.0141)	(0.0141)	(0.0141)	(0.0141)
GDP per capita	1.539***	1.539***	1.539***	1.539***	1.539***
	(0.343)	(0.343)	(0.343)	(0.343)	(0.343)
Constant	-203.2***	-207.2***	-186.4***	-186.4***	-186.4***
	(40.35)	(43.74)	(43.33)	(43.33)	(43.33)
Observations	718	718	718	718	718
R-squared	0.969	0.969	0.969	0.969	0.969
Time fixed effect	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: Although we do not present the time and countries fixed effects (for simpliciy), the regression capture both time and countries fixed effects.

TABLE VII

Green Innovation Drivers: Panel OLS Regression - Robustness B

Green R&D	(1)	(2)	(3)	(4)	(5)
ETS Price > 5	16.84				
	(19.71)				
ETS Price > 10		14.38			
		(23.02)			
ETS Price > 15			11.77		
			(26.14)		
ETS Price > 20				-146.1*	
				(80.79)	
ETS Price > 25					-108.5*
					(55.39)
Long-term Loan (3 year lag)	0.109***	0.109***	0.109***	0.109***	0.109***
	(0.0143)	(0.0143)	(0.0143)	(0.0143)	(0.0143)
GDP per capita	1.455***	1.455***	1.455***	1.455***	1.455***
	(0.409)	(0.409)	(0.409)	(0.409)	(0.409)
Constant	-214.4***	-212.0***	-209.3***	-197.6***	-197.6***
	(47.27)	(48.76)	(51.85)	(46.72)	(46.72)
Observations	646	646	646	646	646
R-squared	0.968	0.968	0.968	0.968	0.968
Time fixed effect	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: Although we do not present the time and countries fixed effects (for simpliciy), the regression capture both time and countries fixed effects.

B Appendix: Model Part

B.1 Calibration

TABLE VIII
Standard parameter values (quarterly basis)

	Calibrated parameters	Values
β	Discount factor	0.9975
α	Capital share	0.33
δ	Depreciation rate of capital	0.025
h	Habits formation parameter	0.8
σ	Risk aversion	2
φ	Disutility of labor	1
θ	Price elasticity	11
$\bar{L}$	Labor supply	0.33
$\bar{L}_s$	Labor supply	0.15
$\bar{g}/\bar{y}$	Public spending share in output	0.4

TABLE IX

Environmental and Entrepreneurs parameter values (quarterly basis)

	Calibrated parameters	Values
η	Material share	.125
a	Damage function parameter	1.004
b	Damage function parameter	0.02
υ_1^o	Temperature parameter	0.5
υ_2^o	Temperature parameter	0.00125
E^*	Emissions from the rest of the world	1.59
ϑ	Carbon intensity	0.287
δ_x	CO_2 natural abatement	0.0021
θ_1	Abatement cost parameter	0.05
θ_2	Abatement cost parameter	2.7
θ_3	Abatement cost parameter	-0.6

TABLE X

Financial parameter values (quarterly basis)

	Calibrated parameters	Values
γ_I	Capital adjustment cost	1.728
ω	Proportional transfer to the entering bankers	0.008
λ	Steady state risk weight on loans	0.43
θ_B	Probability of staying a banker	0.98

TABLE XI

Prior and Posterior distributions of structural parameters

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes:					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.001	0.005	0.0061 *[0.0050 ; 0.0071]*
Std. emission	σ_E	$\mathcal{IG}_1$	0.001	0.005	0.0082 *[0.0070 ; 0.0093]*
Std. R&D	σ_{A_s}	$\mathcal{IG}_1$	0.001	0.005	0.0352 *[0.0307 ; 0.0401]*
Std. green innovation	σ_{A_g}	$\mathcal{IG}_1$	0.001	0.005	0.0451 *0.0392 ; 0.0512]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.9641 *[0.9349 ; 0.9934]*
AR(1) emission	ρ_E	$\mathcal{B}$	0.50	0.20	0.9796*[0.9636 ; 0.9983]*
AR(1) R&D	ρ_{A_s}	$\mathcal{B}$	0.50	0.20	0.5456 *[0.3704 ; 0.7129]*
AR(1) green innovation	ρ_{A_g}	$\mathcal{B}$	0.50	0.20	0.9237 *[0.8509 ; 0.9832]*
Endogenous growth parameters:					
Trend slope	$\gamma_y - 1$	$\mathcal{G}$	0.005	0.001	0.0043*[0.0029 ; 0.0058]*
Green innovation trend slope	$\gamma_{A_g} - 1$	$\mathcal{G}$	0.01	0.002	0.0100 *[0.0067 ; 0.0132]*
R&D investment exogenous trend	γ_{V_s}	$\mathcal{N}$	1	0.20	1.0020 *[1.0011 ; 1.0027]*
Green investment exogenous trend	γ_{V_g}	$\mathcal{N}$	1	0.20	1.0097 *[0.9951 ; 1.0276]*
R&D investment elasticity	η_g	$\mathcal{B}$	0.15	0.20	0.0721 *[0.0001 ; 0.1501]*
Green investment elasticity	η_s	$\mathcal{B}$	0.125	0.20	0.1088 *[0.0001 ; 0.2170]*
Log-marginal data density					666.668864

Notes: $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal, and $\mathcal{G}$ the Gamma distribution.

TABLE XII
Steady state values

	Baseline	Macroprudential	Subsidies	QE
Output	0.8318	0.8330	0.8401	0.8318
Consumption	0.3776	0.3781	0.3813	0.3776
Emissions	0.1749	0.1749	0.1750	0.1749
Emissions to Output	0.2102	0.2100	0.2083	0.2102
Overall Technology	1	1.0102	1.0720	1
Green Projects	0.1055	0.1065	0.1130	0.1055
Abatement Cost	0.0536	0.0535	0.0523	0.0536
Abatement Share	0.2675	0.2685	0.2742	0.2675
Tax in Euros	28.50	28.50	28.50	28.50
Entrepreneurs' Profits	0.0756	0.0750	0.0756	0.0756
Entrepreneurs' Risk Premium	0.0029	0.0020	0.0029	0.0029
Banks' Capital Ratio	0.1581	0.1107	0.1581	0.1581

B.2 Trend Figures

FIGURE III. The Economy Trend Growth Rate (in %).

FIGURE IV. The Green Innovation Trend Growth Rate (in %).

FIGURE V. Counterfactual Subsidy Exercise.

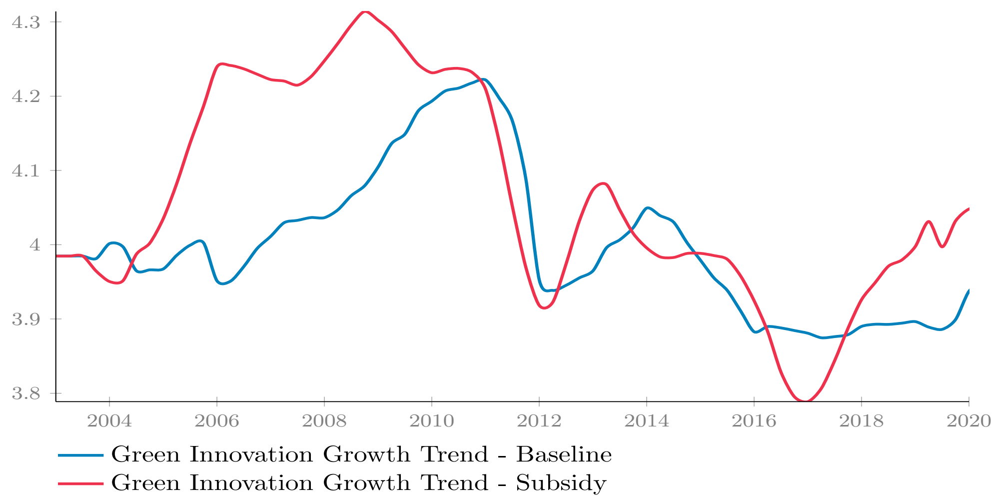


FIGURE VI. Counterfactual Macroprudential Exercise.

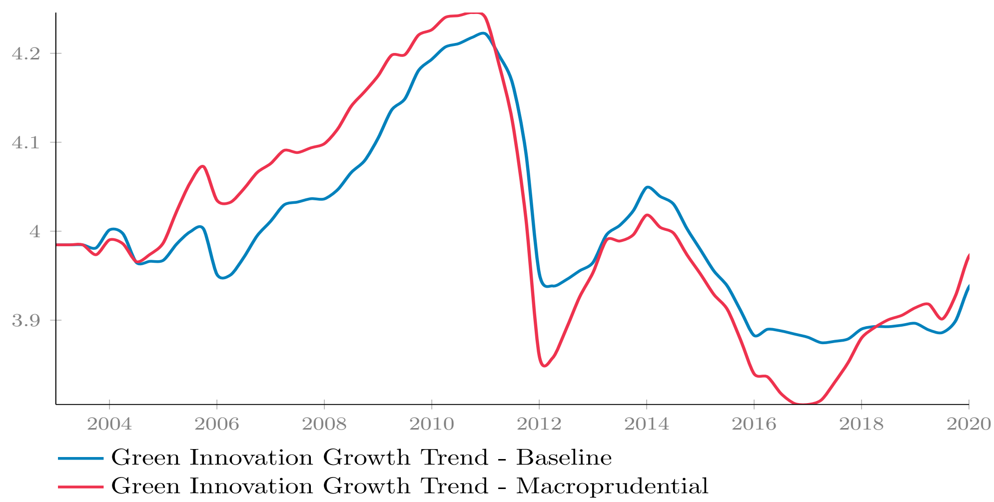
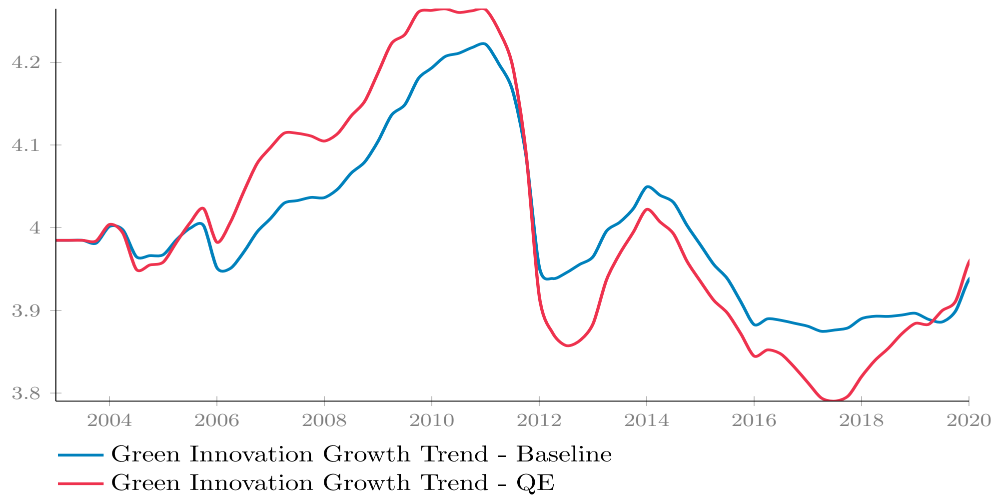


FIGURE VII. Counterfactual QE Exercise.



B.3 Transition Pathways Figures

FIGURE VIII. The Net-Zero Transition Pathway Under Different Abatement to Green Technology Elasticities θ_3.

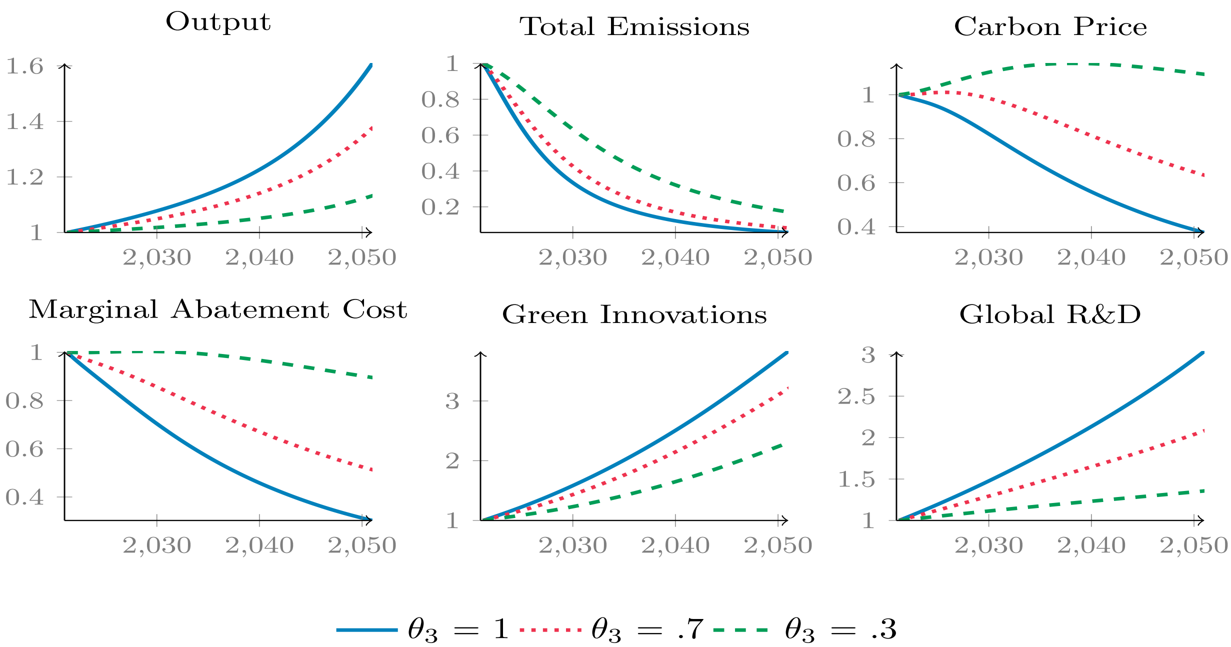


FIGURE IX. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = 1$).

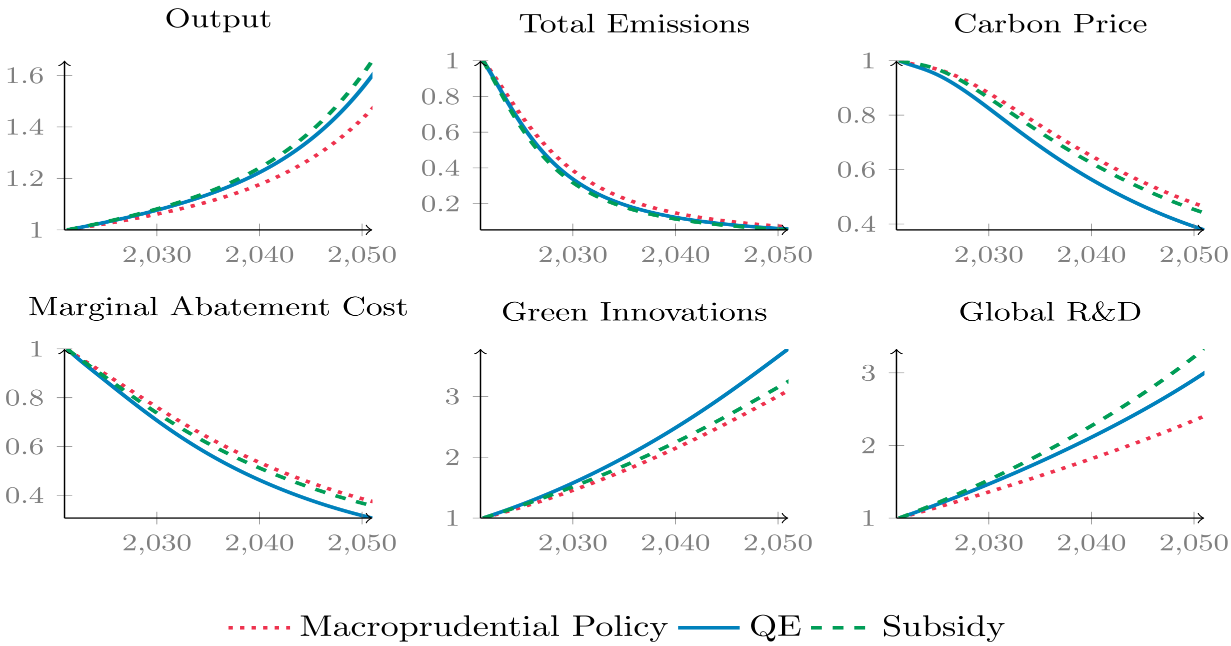


FIGURE X. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = .7$).

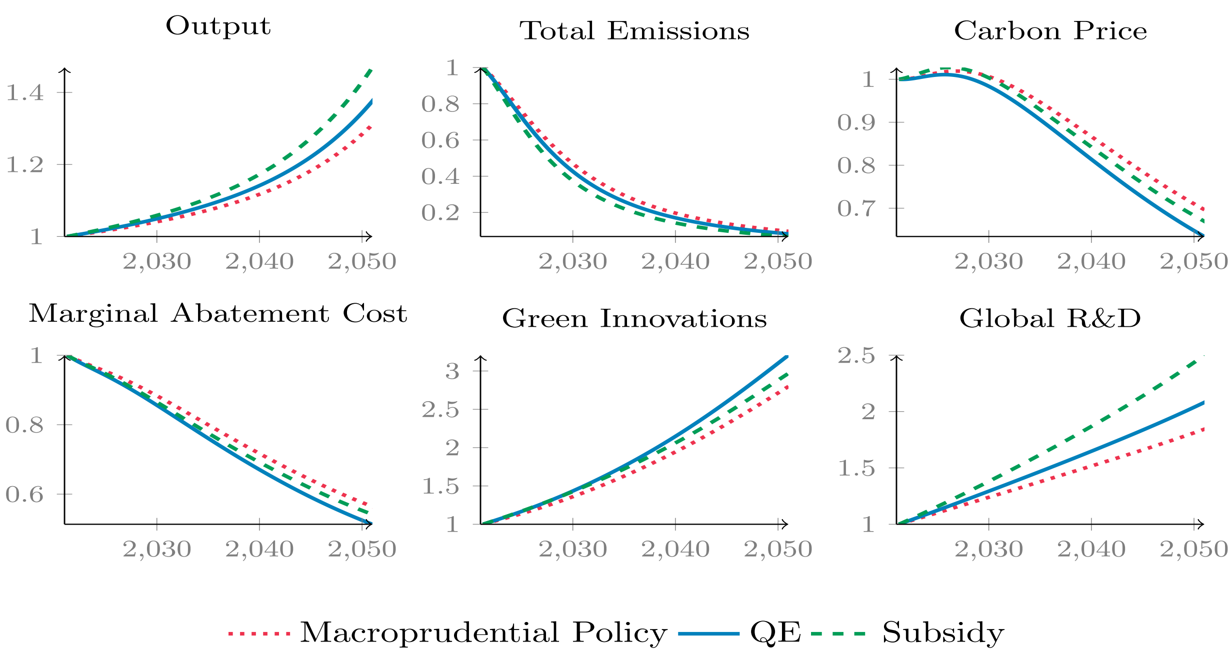
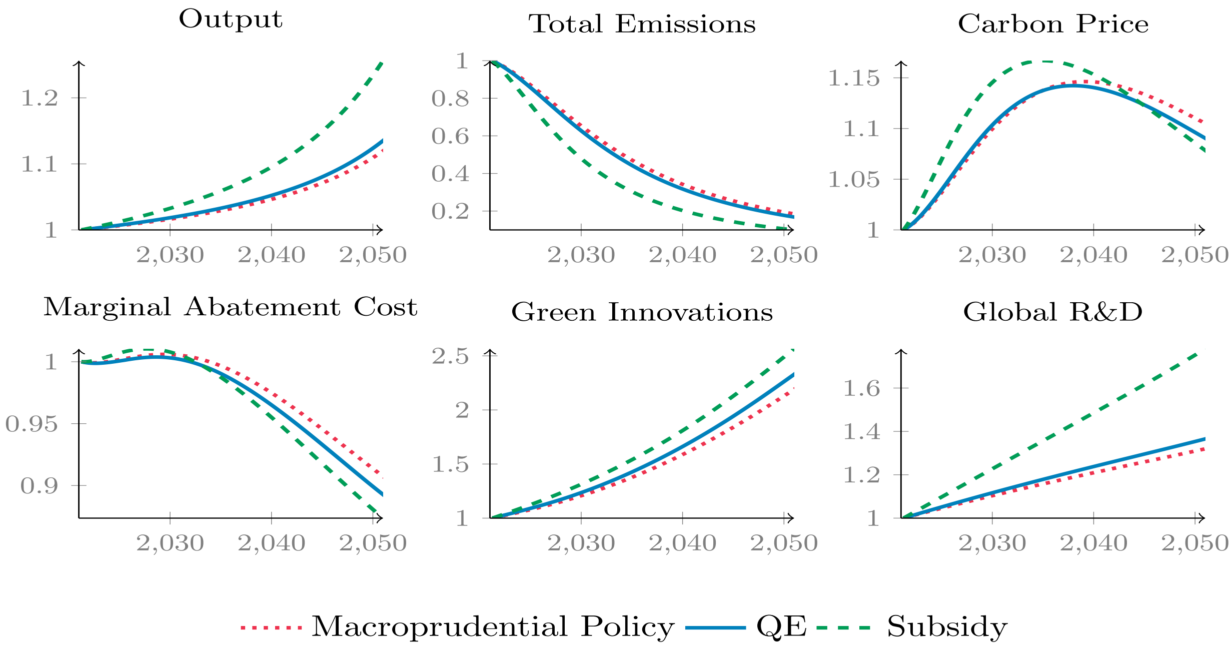


FIGURE XI. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = .3$).



B.4 Model Equilibrium

B.4.1 The Social Planner Solution

The planners social problem for the households reads as following[38]:

$$
\begin{aligned}
&\max E_t \sum_{i=0}^{\infty} \beta^i \Bigg(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \Gamma_t^{1-\sigma} L_{t+i}^{1+\varphi} \\
&+ \lambda_t (W_t L_t + W_t^s \bar{Ls}^s + W_t^g \bar{Ls}^g + R_t^K K_t + \Pi_t + T_t + R_t B_t - C_t - I_t - B_{t+1}) \\
&+ \lambda_t \varrho_t^C ((1-\delta) K_t + I_t - K_{t+1}) \\
&+ \lambda_t q_t (Y_t - W_t L_t - R_t^K K_t - f(\mu_t) Y_t - \Pi_t) \\
&+ \lambda_t \Psi_t (d(T_t^o) K_t^\alpha L_t^{1-\alpha} - Y_t) \\
&+ \lambda_t \S_t^X (X_t - \eta X_{t-1} - E_t - E^*) \\
&+ \lambda_t \S_t^T (T_t^o - \upsilon_1^o (\upsilon_2^o X_{t-1} - T_{t-1}^o) - T_{t-1}^o) \\
&+ \lambda_t \S_t^E (E_t - (1-\mu_t) \varphi_t Y_t) \Bigg),
\end{aligned}
$$

where the Social Cost of Carbon SCC_t is $\S_t^X$, and Ψ_t the marginal cost component related to the firms problem.

The first order conditions determining the SCC_t are the ones with respect to T_t^o, X_t, E_t, μ_t and Π_t:

$$\lambda_t \S_t^T = \beta(1-\upsilon_1^o)\lambda_{t+1}\S_{t+1}^T - \lambda_t \Psi_t \varepsilon_t^A \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^\alpha L_t^{1-\alpha} \tag{84}$$

$$\lambda_t \S_t^X = \beta(\upsilon_1^o \upsilon_2^o)\lambda_{t+1}\S_{t+1}^T + \beta\eta\lambda_{t+1}\S_{t+1}^X \tag{85}$$

$$\lambda_t \S_{t,k}^E = g(\varkappa)\lambda_t \S_t^X \tag{86}$$

$$\lambda_t q_{t,k} f'(\mu_{t,k}) = \varphi_{t,k}\lambda_t \S_{t,k}^E \tag{87}$$

$$\lambda_t = \lambda_t q_{t,k}. \tag{88}$$

[38]Please note that the social planner problem is not impacted by the financial intermediaries nor by the R&D entrepreneurs or the green innovators.

Rearranging these FOCs we obtain the following SCC_t:

$$\S_t^T = (1 - \upsilon_1^o)\Lambda_{t,t+1}\S_{t+1}^T - \sum_k \Psi_{t,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K_t^\alpha L_t^{1-\alpha} \tag{89}$$

$$\S_t^X = (\upsilon_1^o \upsilon_2^o)\Lambda_{t,t+1}\S_{t+1}^T + \eta\Lambda_{t,t+1}\S_{t+1}^X \tag{90}$$

$$\S_t^E = g(\varkappa)\S_t^X \tag{91}$$

$$f'(\mu_t) = \varphi_t \S_t^E \tag{92}$$

The competitive equilibrium problem for the firms reads as following:

$$\begin{aligned} &\max E_t \sum_{i=0}^{\infty}\Bigg(\left(\frac{P_{jt}}{P_t}Y_t - W_t L_t - R_t^K K_t - f(\mu_t)Y_t - \tau_t E_t - \Pi_t\right) \\ &+ \lambda_t\Psi_t(d(T_t^o)K_{t-1}^\alpha L_t^{1-\alpha} - Y_t) \\ &+ \lambda_t \S_t^F(E_t - (1-\mu_t)\varphi_t Y_t)\Bigg) \end{aligned}$$

The first order conditions determining the tax rate τ_t are the ones with respect to E_t and μ_t:

$$\S_t^F = \tau_t \tag{93}$$

$$f'(\mu_t) = \S_t^F \varphi_t \tag{94}$$

Thus, from both the household and firm FOCs, we get:

$$\S_t^F = \tau_t \tag{95}$$

$$\S_t^F = \S_t^E \tag{96}$$

$$f'(\mu_t) = \S_t^E \varphi_t \tag{97}$$

$$\S_t^T = (1 - \upsilon_1^o)\Lambda_{t,t+1}\S_{t+1}^T - \Psi_t\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t-1}^\alpha L_t^{1-\alpha} \tag{98}$$

$$\S_t^X = (\upsilon_1^o \upsilon_2^o)\Lambda_{t,t+1}\S_{t+1}^T + \eta\Lambda_{t,t+1}\S_{t+1}^X \tag{99}$$

$$\S_t^E = \S_t^X \tag{100}$$

B.4.2 The Firms

The firm maximization of profits reads:

$$\Pi_{jt} = \max_{P_{jt}, Y_{jt}} \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}, \tag{101}$$

s.t.

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \tag{102}$$

The first order condition yields:

$$\frac{P_{jt}}{P_t} = \frac{\theta}{\theta - 1} MC_t^f \tag{103}$$

Now using the pricing equation $P_t = (\int_0^{A_{t,s}} P_{jt}^{1-\theta} dj)^{\frac{1}{1-\theta}}$ we get:

$$\frac{P_{jt}}{P_t} = A_{t,s}^{\frac{1}{\theta-1}} \tag{104}$$

Thus, we can rewrite the first order condition as:

$$\frac{\theta}{\theta - 1} MC_t^f = A_{t,s}^{\frac{1}{\theta-1}}. \tag{105}$$

Therefore,

$$\Pi_{jt} = \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}, \tag{106}$$

$$= \frac{1}{\theta} \frac{Y_t}{A_{t,s}} \tag{107}$$

Turning now to the Cobb-Douglas production function, we use the inputs market-clearing conditions $\int_0^{A_{t,s}} L_{jt} dj = A_{t,s} L_t$ and $\int_0^{A_{t,s}} K_{jt} dj = A_{t,s} K_t$ to retrieve the final form of the production function:

$$Y_t = A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^\alpha L_t^{1-\alpha}. \tag{108}$$

The rest of the first order condition remains similar to the ones presented in the reduced form model.

B.4.3 The Households, Innovators, and Financial intermediaries

For the household, the entrepreneurs, and the banking sector, all equilibrium equations are presented in the core text.

B.5 Carbon Cap and Green Innovation

By substituting the environmental cap policy equation ($E_t = Cap_t$) into the emissions flow equation ($E_t = (1 - \mu_t)\upsilon Y_t$), we get:

$$\mu_t = 1 - \frac{Cap_t}{\upsilon Y_t} \tag{109}$$

Using the FOC on abatement Equation 32:

$$\text{Carbon Price}_t = \theta_1\theta_2 \frac{\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1}}{\upsilon} A_{t,g}^{-\theta_3} \tag{110}$$

We see that the carbon price could be steered by either Cap_t and/or $A_{t,g}$[39]. It is then clear that when:

$\Delta A_{t,g}^{\theta_3} > \Delta \left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1} \Rightarrow$ Carbon Price$_t$ decrease.

While when:

$\Delta A_{t,g}^{\theta_3} < \Delta \left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1} \Rightarrow$ Carbon Price$_t$ increase.

Turning now to the abatement cost, we have:

$$f(\mu_t) = \theta_1 \left(1 - \frac{\text{Cap}_t}{\upsilon Y_t}\right)^{\theta_2} A_{t,g}^{-\theta_3} \tag{111}$$

Likewise, when:

$\Delta A_{t,g}^{\theta_3} > \Delta \left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2} \Rightarrow$ the per unit abatement cost decrease.

While when:

$\Delta A_{t,g}^{\theta_3} < \Delta \left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2} \Rightarrow$ the per unit abatement cost increase.

As the total abatement cost $Z_t = f(\mu_t)Y_t$ enters the banks returns equation $R_{t,e} = \frac{\phi_{RD_g}(Z_t + Q_{t,e})}{Q_{t-1,e}}$, a drop in Z_t would reduce the returns $R_{e,t}$. In turn, the decrease in $R_{e,t}$ gives

[39] The changes on Y_t being very small over the business cycle with respect to climate damages, we don't focus on their effects on carbon prices.

less incentives to financial intermediaries to finance green equity innovators, which end up decreasing their overall number of innovations $A_{t,g}$.