Liquidity Allocation and Endogenous Uncertainty

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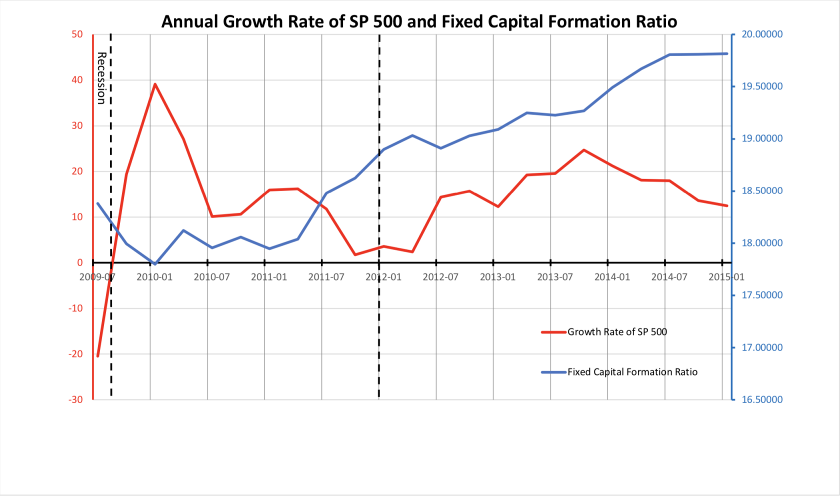
Motivation: Phenomenon

For most of advanced economies, during the slow economic recovery from the Great Recession,

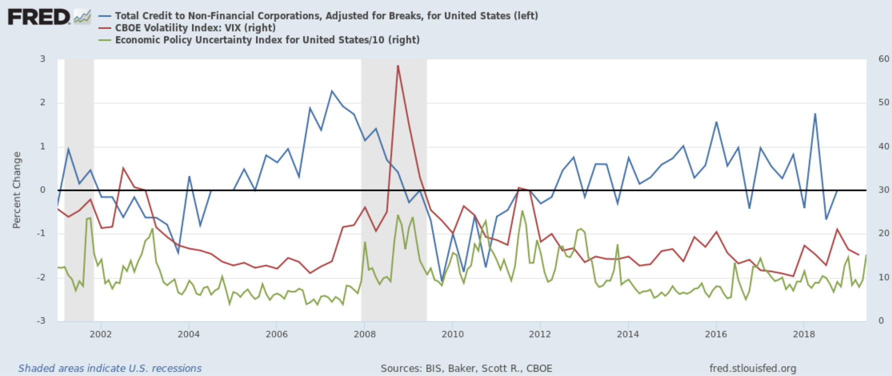
- **Weak Investment** but **Soar of Stock Market**
- **Saving Glut of Non-Financial Corporation**: net funds inflow from real economy to financial system
- high level of **Uncertainty**

Motivation: Phenomenon

Growth Rate of Equity Price and Investment Ratio



Fund Flows and Uncertainty



Empirical Evidences

- Evidences for Slow Economic Recovery:
  - Becker, Davis and Murphy (2010)
  - Fernald, Hall, Stock, and Watson (2017)
  - Ball (2014)
  - Hall (2015)
- Evidences for Saving Glut of non-Financial Corporate and Weak Investment:
  - Chen, Karabarbounis and Neiman (2017)
  - Gruber and Kamin (2016)

What I Do

- A Tractable Theory to Rationalize Above Phenomenon
  - An **Endogenous Liquidity Allocation Mechanism** between Real Economy and Financial System
  - The Interaction between Endogenous Liquidity Allocation and Endogenous Aggregate Uncertainty
- Numerical Analysis

Main Intuitions

- Assumptions:
  - Physical capital has less liquidity than its corresponding equity
  - Entrepreneurs have to take partial risk of their own investment
- Holding of risky physical capital depends on the capital structure of corporation, so does the investment
  - costly adjustment of physical capital position
  - incomplete risk-sharing
  - Net worth works as risk buffer

Main Intuitions

Endogenous Liquidity Allocation Mechanism:

- Recovering:
  - Entrepreneurs with low net worth level, prefer high liquidity financial assets, and even disinvest
  - More funds from real economy flow into financial system, push up equity price and amplify financial risks
  - Higher financial risks retard investment and leads to an adverse liquidity loop
- Booming:
  - high net worth level of entrepreneurs stimulates high investment demand and high equity price

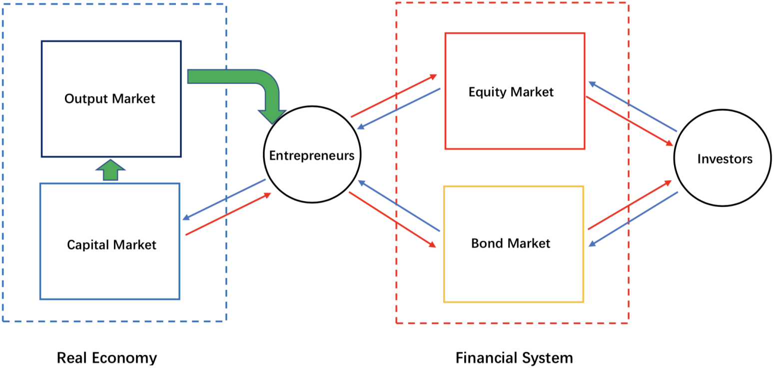
Related Literature

- DSGE of Endogenous Risks
  - Brunnermeier and Sannikov (2014, 2017)
  - He and Krishnamurthy (2012, 2013, 2019)
  - Di Tella (2017)
- Dynamic Corporate Investment
  - Bolton, Chen, Wang (2011), Bolton, Wang, Yang (2019)
  - Isohatala, Milne, and Robertson (2014)
  - He and Kondor (2016)
- Liquidity Difference between Assets
  - Kiyotaky and Moore (2012)

Economic Environment

- Infinite Identical Risk-Averse Entrepreneurs Whose Total Mass is 1
- Infinite Identical Risk-Neutral Financial Investors Whose Total Mass is 1
- Only Entrepreneurs Can Run Physical Capital
- Entrepreneurs Can Raise Funds from Financial Markets by Issuing Equity and Bonds
- Financial Investors ONLY Participate in Transactions in Financial Market

Economic Structure



The Economic Structure

Key Assumptions

1. Slower Adjustment of Macroeconomy than Financial Market
   - Physical Capital Has Less Liquidity than Its Corresponding Equity
   - Transaction Cost for Purchasing Capital: $\underbrace{\Psi(\kappa)}_{\text{Capital Formation}} < \kappa$
   - Capital and Equity are Imperfect Substitutive
2. Financial Frictions: "Skin in the Game"
   - Entrepreneurs Have to Take Partial Risk of Their Own Investment
   - Equity Issuance Constraint: $\chi \geq \overline{\chi}$

Other Assumptions

- The Evolving Process of Physical Capital:

$$dK = [\Phi(\iota) + \Psi(\kappa) - \delta]Kdt + \sigma KdZ$$

  - dZ : Aggregate Productivity Shock

- Guessed Process of Equity Price:

$$dq = \mu^q qdt + \sigma^q qdZ$$

- Return Rate of Equity:

$$dR = (\frac{A - \iota - p\kappa}{q})dt + [\Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q]dt + (\sigma + \sigma^q)dZ$$

Dynamic Optimization Question of Entrepreneurs

$$\max_{\{C,\nu,\iota,\kappa,\chi\}} E_0 \left[\int_0^\infty e^{-\rho t} \frac{C^{1-\gamma}}{1-\gamma} dt \right]$$

s.t.

$$0 \le h \le 1,$$

$$(1-h)\nu W = \chi q K \ge \bar{\chi} q K,$$

$$dW = (1-h)\nu W dR + h\nu W d\tilde{R} + (1-\nu)\, W r dt - C dt,$$

$$\frac{d(qK)}{qK} = \left[\underbrace{\Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q}_{\mu^V} \right] dt + \left(\underbrace{\sigma + \sigma^q}_{\sigma^V} \right) dZ$$

Optimal Choices of Entrepreneurs

- Optimal Investment Ratio

$$[\underbrace{\frac{(1-\gamma)\varphi(w)-w\varphi'(w)}{(1-h)\nu w\varphi'(w)}+1}_{\text{Relative Price of Capital Denominated by Equity}}]\Phi'(\iota)=\frac{1}{q}$$

  - $w \equiv \frac{W}{qK}$: the capital ratio
  - $\varphi(w) \equiv J(W,qK)/(qK)^{1-\gamma}$

- Relative Price of Capital

$$\frac{(1-\gamma)\varphi(w)-w\varphi'(w)}{(1-h)\nu w\varphi'(w)}+1=\frac{qKJ'_{qK}+(1-h)\nu WJ'_{W}}{(1-h)\nu WJ'_{W}}<1$$

because $J'_{qK}<0$

Optimal Choices of Entrepreneurs

- Trade-Off between Producing Capital and Purchasing Capital

$$\underbrace{\Phi'(\iota)K}_{\text{Capital Formation by Producing}} = \underbrace{\Psi'(\kappa)K/p}_{\text{Capital Formation by Purchasing}}$$

- Comparison between Capital Price and Equity Price: At Equilibrium, $p < q$

$$[\frac{(1-\gamma)\varphi(w) - w\varphi'(w)}{(1-h)\nu w\varphi'(w)} + 1]\Psi'(\kappa) = \frac{p}{q}$$

- Different from BS (2014, 2017) Who Assume Physical Capital $\Longleftrightarrow$ Equity:

$$\Phi'(\iota)K = K/q$$

Optimal Choices of Entrepreneurs

- Asset Pricing of Equity

$$
\begin{aligned}
& (1-h)E(dR)/dt + h\mu^{\tilde{R}} + \underbrace{\frac{\lambda(1-h)}{(qK)^{-\gamma}\varphi'(w)}}_{\text{Liquidity Premium}} \\
= \quad & r + \underbrace{\pi^{e}[(1-h)(\sigma+\sigma^{q}) + h\sigma^{\tilde{R}}]}_{\text{Risk Premium}}
\end{aligned}
$$

  - λ : the Lagrangian Multiplier of Equity Issuance Constraint $\chi \geq \overline{\chi}$
  - Liquidity Premium Comes from "Skin of Game"
  - Risk Pricing by Entrepreneurs:

$$
\pi^{e} \equiv \gamma(\sigma+\sigma^{q}) - \frac{\{\nu[(1-h)(\sigma+\sigma^{q}) + h\sigma^{\tilde{R}}] - (\sigma+\sigma^{q})\}w\varphi''(w)}{\varphi'(w)}
$$

Optimal Choices of Investors

- Dynamic Optimization Question

$$\max_{\{\underline{C},\underline{\nu}\}} E_0\left[\int e^{-rt}\underline{C}dt\right]$$

s.t.

$$d\underline{W} = \underline{\nu W}dR + (1-\underline{\nu})\underline{W}rdt - \underline{C}dt$$

- Asset Pricing of (Outside) Equity

$$\underbrace{\frac{A-\iota-p\kappa}{q} + \Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma\sigma^q}_{E(dR)/dt} = r$$

Markov Equilibrium

- The Markov Equilibrium Has A Single State Variable

$$\eta \equiv \frac{\int_0^1 W(i)di}{\int_0^1 qK(i)di}$$

- No One Purchases Physical Capital

$$\kappa = 0 \Rightarrow \Psi(\kappa) = 0$$

- The Net Liquidity Flow to Financial System

$$\int_0^1 dF(i)di \equiv \underbrace{\int_0^1 d[(1-\nu(i))W(i)]di}_{\text{inflow}} - \underbrace{\int_0^1 d[(1-\chi(i))qK(i))]di}_{\text{outflow}}$$

- The Ratio of Net Liquidity Flow:

$$\int_0^1 dF(i)di / \int_0^1 qK(i)di$$

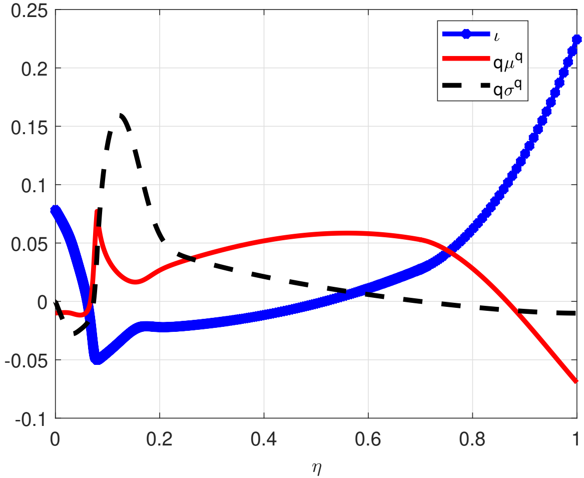
Parameterization

- Similar Parameter Values as BS (2014)

Parameter	Meaning	Value
ρ	time discount rate of entrepreneurs	6%
r	time discount rate of investors	5%
γ	risk aversion of entrepreneurs	2
A	productivity	12%
δ	depreciation rate	3%
σ	capital quality shock	2%
$\overline{\chi}$	equity issuance constraint	70%
ϕ	capital formation function by investment	10
ψ	capital formation function by purchasing	10

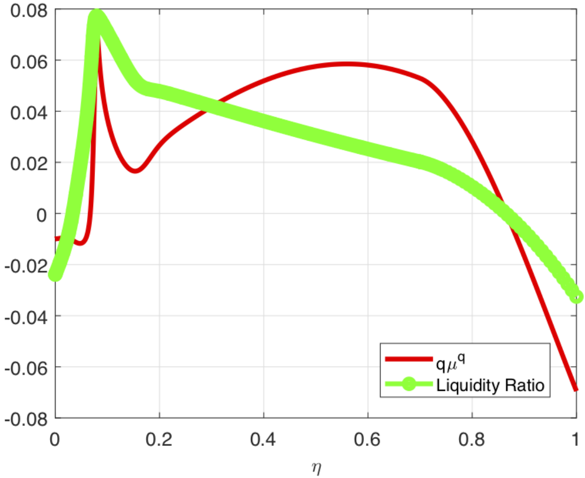
Global Dynamics

Investment Ratio, Equity Price Growth and Equity Market Risk

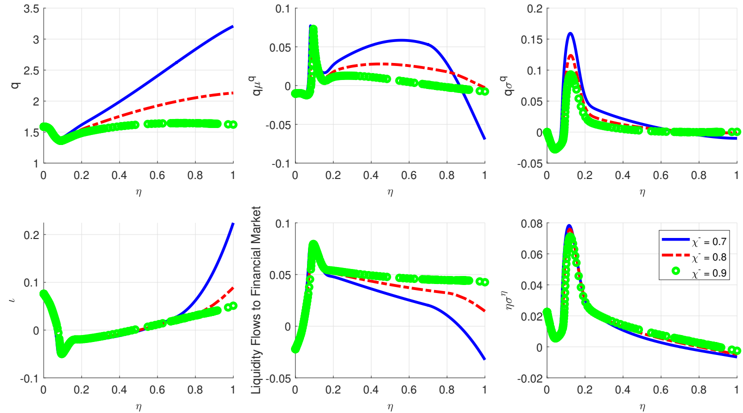


Global Dynamics

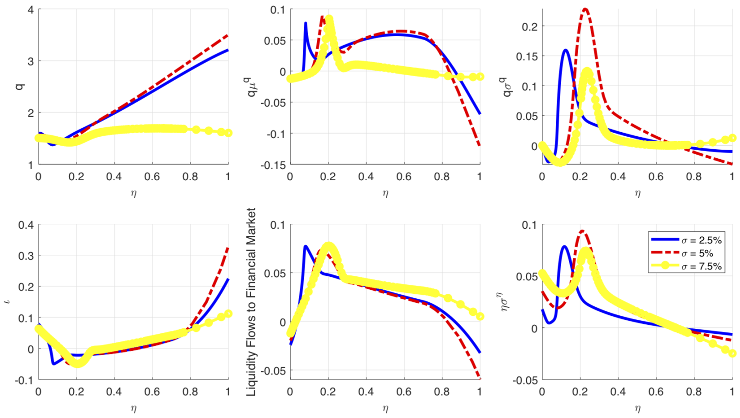
Equity Price Growth and Net Liquidity to Financial System



Global Dynamics with Different Extents of Equity Issuance Constraint



Global Dynamics with Different Extents of Exogenous Risk



Conclusion

- A Tractable DSGE Model of Endogenous Liquidity Allocation Mechanism between Real Economy and Financial System
- The Interactions between Endogenous Risks and Liquidity Allocation Help Us to Understand the Inconsistency between Business Cycle and Financial Cycles.
- Policy Implications:
  - QE policy is not perfect
  - QE policy has a potential to lead to the weak investment and saving glut of non-financial corporation during the economic recovery by allocating more liquidity into financial markets rather than into the real economy

Ext1: Capital Requirement for Financial Investors

- Financial investors face capital requirement constraint: $\underline{\nu} \leq \bar{m}$
- Financial investors can be risk-averse

$$\underline{\rho} = \underline{\nu}[E(dR/dt) - r] + r \geq r$$

$$E(dR/dt) - r \quad = \quad \underbrace{\frac{\zeta}{\underline{W}}}_{RPi}$$

$$\underbrace{\frac{\zeta}{\underline{W}}}_{RPi} + \underbrace{\frac{\lambda_2}{(qK)^{-\gamma}\varphi'}}_{LPe} = \underbrace{\left[\gamma + (1-\nu)\frac{\eta\varphi''}{\varphi'}\right](\sigma + \sigma^q)^2}_{RPe} > 0$$

Ext1: Capital Requirement for Financial Investors

Proposition: When $0 \leq \eta < 1 - \frac{1-\overline{\chi}}{\overline{m}}$,entrepreneurs face a binding equity issuance constraint and hold no outside equity, and financial investors' capital requirement constraint are not binding, i.e., $\nu = \overline{\chi}/\eta$, $\underline{\nu}<\overline{m}$, and $qH = 0$; When $1 - \frac{1-\overline{\chi}}{\overline{m}} < \eta \leq 1$,entrepreneurs' equity issuance constraint is not binding, and financial investors face a binding capital requirement constraint, i.e., $\nu > \overline{\chi}/\eta$, $\chi > \overline{\chi}$, and $\underline{\nu}=\overline{m}$; When $\eta = 1 - \frac{1-\overline{\chi}}{\overline{m}}$, both entrepreneurs' equity issuance constraint and investors's capital requirement constraint are binding, entrepreneurs hold no outside equity, i.e., $\nu = \overline{\chi}/\eta$, $\underline{\nu}=\overline{m}$, and $qH = 0$.

Ext2: Capital Misallocations

- Entrepreneurs can rent out $1-\psi$ $(0 \leq \psi \leq 1)$ fraction of physical capital to investors whose productivity is $\underline{A} << A$ to hedge against partial labor productivity shock dZ:

$$dR = \left[\underbrace{\frac{\psi A + (1-\psi)\underline{A} - \iota - p\kappa}{q} + \Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \psi\sigma\sigma}_{E(dR/dt)}\right. \\ +(\psi\sigma + \sigma^q)dZ$$

$$LPe = RPe = [\gamma + (1-\nu)\frac{w\varphi''(w)}{\varphi'(w)}](\psi\sigma + \sigma^q)^2 = \pi^e(\psi\sigma + \sigma^q)$$

- Or, Entrepreneurs can sell out physical capital to investors and pay some transaction cost

Ext3: Heterogenous Productivity Agents

Agent i's choice will affect his return rate of physical capital, $dr^K(i)$

$$\max_{\{C^i,\chi^i,h^i,\nu^i,\iota^i,\kappa^i\}} E_0 \int_0^\infty e^{-\rho t} \frac{(C^i)^{1-\gamma}}{1-\gamma} dt$$

s.t.

$$\chi^i \geq \bar{\chi},$$

$$\left(1-h^i\right) \nu^i W^i = \chi^i q K^i,$$

$$1 \geq h^i \geq 0,$$

$$dW^i = \left(1-h^i\right) \nu^i W^i dr^K(i) + h^i \nu^i W^i dR + \left(1-\nu^i\right) W^i r dt - C^i dt,$$

$$\frac{d\left(qK^i\right)}{qK^i} = \left[\Phi\left(\iota^i\right) + \Psi\left(\kappa^i\right) - \delta + \mu^q + \sigma\sigma^q\right] dt + \left(\sigma + \sigma^q\right) dZ.$$