Shaping the future: Policy shocks and the GDP growth distribution

François-Michel Boire [1] Thibaut Duprey [2] Alexander Ueberfeldt [2]

[1]University of Western Ontario [2]Bank of Canada

Introduction

We incorporate quantile regressions into a structural VAR model to empirically assess how monetary and fiscal policy influence risks around future GDP growth.

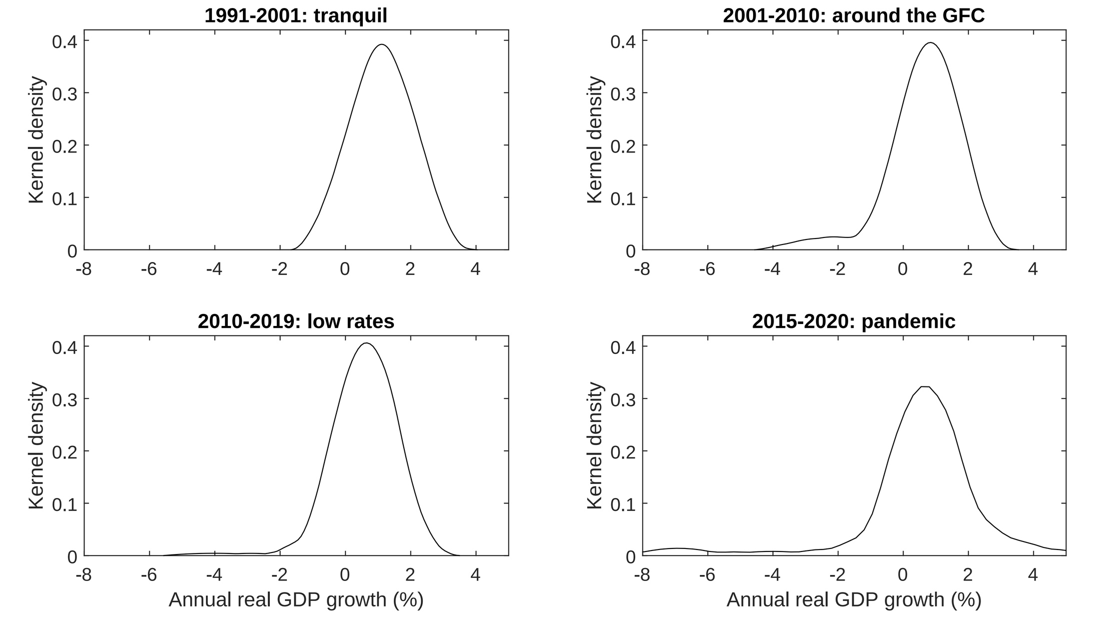


Figure 1:Unconditional distribution of GDP growth in the US

Questions

- **1** How can we assess risks around GDP growth?
  - We simulate the *entire distribution* of GDP growth with a new quantile-vector autoregression hybrid model (QR-VAR)
- **2** How can monetary and fiscal policy tools influence these risks?
  - Both policies change the location of the distribution of future GDP growth, but fiscal shocks also impact its shape.
  - When the ZLB binds, fiscal policy scales the distribution up, esp. for lower quantiles

Literature

- Larger spending multipliers in recessions / ZLB
  Auerbach and Gorodnichenko (2010); Christiano, Eichenbaum and Rebello (2011); Ramey and Zubairy (2018)
- Effectiveness of monetary policy tools
  Sims and Wu (2019); Ulate (2019)
- Asymmetric dynamics output / financial stress
  Adrian, Boyarchenko and Giannone (2019); Chavleishvili and Manganelli (2019)

A Hybrid Model

In a traditional VAR, we write a bivariate process $\{Y_1, Y_2\}$ with one lag as

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} \\ \Gamma_{2,1} & \Gamma_{2,2} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}, \quad (1)$$

where the error term $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t})'$ is such that $\mathbb{E}(\epsilon_t) = 0$ and $\mathbb{E}(\epsilon'_{t_i}\epsilon_{t_j}) = \mathbf{\Sigma}_\epsilon \mathbb{1}_{i=j}$, where $\mathbf{\Sigma}_\epsilon$ is PSD.

In the QR-VAR hybrid, we generalize the linear model of *one* variable, Y_1, to a linear quantile model. For N evenly spaced probabilities $\{p_1, \ldots, p_N\}$, the linear conditional quantiles are written as

$$\begin{bmatrix} Q_{p_1}(Y_1|y_{t-1}) \\ \vdots \\ Q_{p_N}(Y_1|y_{t-1}) \end{bmatrix} = \begin{bmatrix} \beta_1^1 & \beta_2^1 \\ \vdots & \vdots \\ \beta_1^N & \beta_2^N \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} \quad (2)$$

and independently estimated with the linear quantile regression method of Koenker and Basset (1978). Given the representation of Y_1 in Eq (1), we propose a QR-VAR hybrid model

$$\begin{bmatrix} y_{1,t} \\ \vdots \\ y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \beta_1^1 & \beta_2^1 \\ \vdots & \vdots \\ \beta_1^N & \beta_2^N \\ \Gamma_{2,1} & \Gamma_{2,2} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \eta_t^1 \\ \vdots \\ \eta_t^N \\ \epsilon_{2,t} \end{bmatrix}. \quad (3)$$

For sufficiently large N, we can approximate the conditional mean of Y_1 as $\frac{1}{N}\sum_{n=1}^N Q_{p_n}(Y_1|y_{t-1})$, from which we estimate residuals

$$\epsilon_{1,t}^* = y_{1,t} - \frac{1}{N}\sum_{n=1}^N Q_{p_n}(Y_1|y_{t-1}) = \frac{1}{N}\sum_{n=1}^N \eta_t^n. \quad (4)$$

The residuals $(\epsilon_{1,t}^*, \epsilon_{2,t})'$ are decomposed structurally as in the standard VAR.

Benefits of the QR-VAR

When Y_1 is governed by non-gaussian, conditionally heteroskedastic, and skewed dynamics, estimating the QR-VAR compared to the VAR:

- **1** Does not worsen conditional mean predictions;
- **2** Slightly over-estimates conditional variance;
- **3** Accurately estimates conditional skewness.

Simulation Approach

- **1** Fit conditional quantiles. For all $p \in \{p_1, \ldots, p_N\}$ we estimate

$$\hat{Q}_p(Y_{1,t}|\mathbf{Y}_{t-1}) = \hat{\boldsymbol{\beta}}_p \mathbf{Y}_{t-1}$$

$$\hat{\boldsymbol{\beta}}_p = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \sum_{t=1}^T \rho_p(y_{1,t} - \boldsymbol{\beta}\mathbf{y}_{t-1})$$

where $\rho_p(u) = u \cdot (p - \mathbb{1}_{\{u \leq 0\}})$.
We obtain $N = 99$ fitted quantiles $\{\hat{Q}_{0.01}(Y_{1,t}|\mathbf{y}_{t-1}) \ldots, \hat{Q}_{0.99}(Y_{1,t}|\mathbf{y}_{t-1})\}$.

- **2** Approximate conditional density $\hat{f}(y_{1,t}|\mathbf{y}_{t-1})$ with Kernel Density Approximation (KDA)

$$\frac{1}{N}\sum_{n=1}^N \frac{1}{\mathcal{H}\lambda_{p_n}} K\left(\frac{y_{1,t} - \hat{Q}_{p_n}(Y_{1,t}|\mathbf{y}_{t-1})}{\mathcal{H}\lambda_{p_n}}\right)$$

where $K(\cdot)$ is a Gaussian kernel, $\mathcal{H} > 0$ is a bandwidth, and λ_τ is a local bandwidth.

Simulate: re-sampling from $\hat{f}(\cdot \mid \mathbf{y}_{t-1})$

Application to Monetary and Spending Shocks

Panel QR-VAR (US, UK, Japan, Canada, Australia, Finland; 1964Q1–2019Q4) with 5 variables:

- **1** Annual real GDP growth per cap.
- **2** Annual real government spending growth per cap.
- **3** Change in the short term (shadow) rate
- **4** Annual CPI inflation
- **5** Log of the financial stress index (FSI)

Structural shock identification:

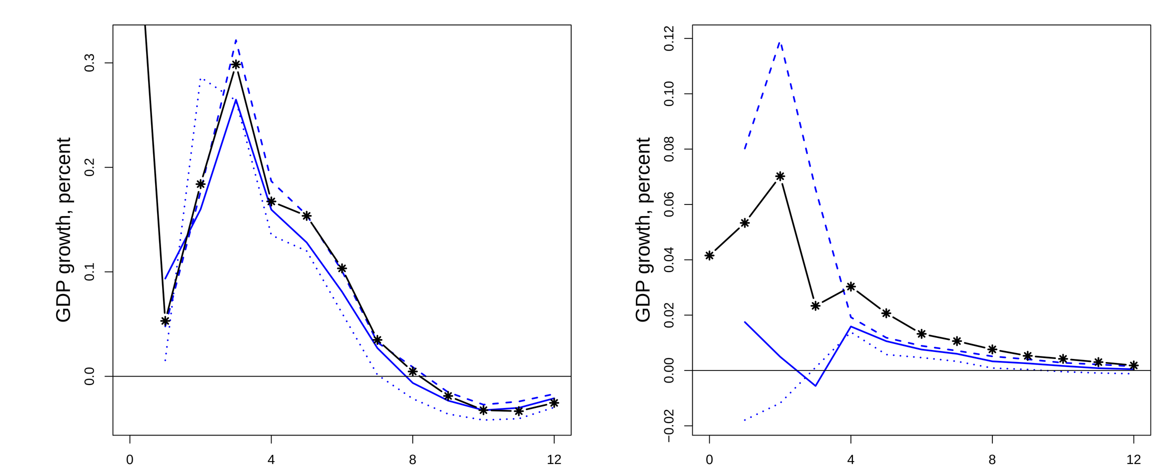
Resp. \ Shock	Spending	Supply	Monetary	Financial	Demand
Gvt. spending	+	0	0	0	0
Inflation		+	-		+
Rate		+	+		+
log-FSI				+	
GDP		-	-		+

Spending shocks identified with zero restrictions

Results

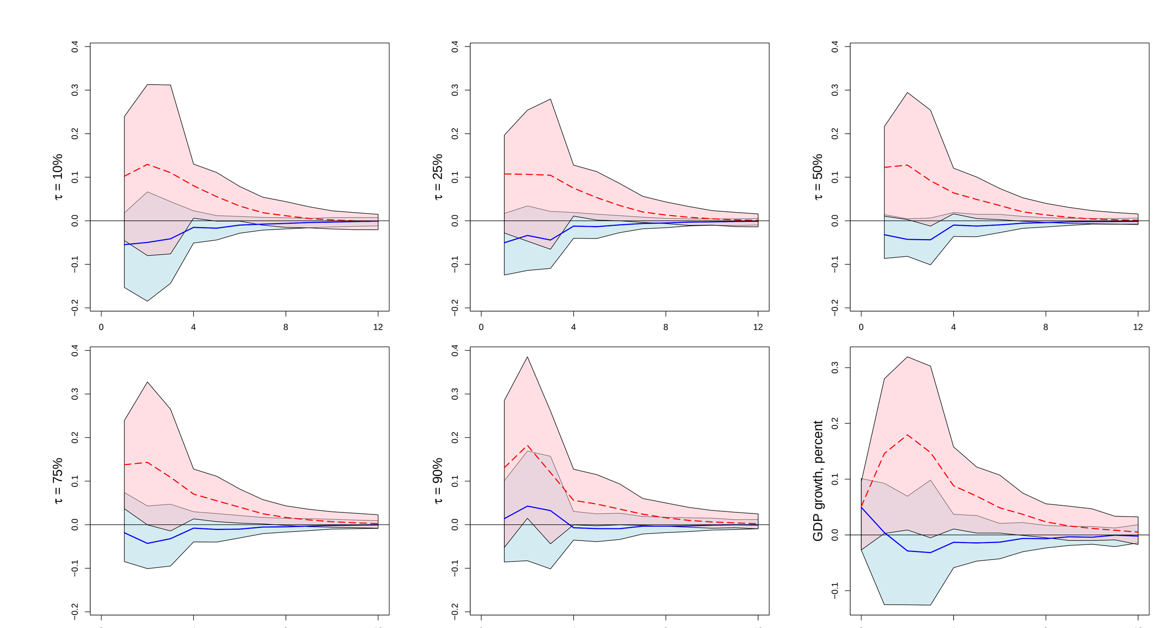
Monetary shocks move all quantiles of GDP growth evenly while spending shocks increase the odds of faster GDP growth

Impulse response of quantiles 10 (dotted), 50 (plain), 90 (dashed) and mean (starred) of GDP growth, after 50 bps monetary easing (left) or 5 ppt spending increase (right).



Spending shocks increase upside risks during ZLB events

Quantile impulse (Panels 1 to 5) or mean impulse (Panel 6)
Normal times (blue) or ZLB (red) with 90% confidence interval



Conclusion

- **1** We build a hybrid of quantile regressions and structural VAR to relax assumptions of linearity, symmetry, and uni-modality for one variable
  - QR-VAR better for skewed variable
- **2** Monetary policy does not change the shape of the distribution of future GDP growth
- **3** Fiscal policy changes the shape of the distribution
  - More so when the ZLB is binding
  - Increases lower quantiles: speeds up the recovery