Firm Revenue Elasticity and Business Cycle Behaviour

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Motivation

Aggregate measures of price markups are informative for various macroeconomic topics. However, recent literature (e.g., Syverson, 2019; Bond et al., 2020) on market power in macroeconomics notes the limitations of using revenue elasticities to proxy output elasticities when estimating markups. In other words, revenue elasticities may not unlock markups. Given this, we take a step back and ask: ***What can revenue elasticities tell us about firm business cycle behaviour?*** This paper investigates the role of firms' revenue elasticities in propagating macroeconomic shocks.

Framework

A firm's production function is given by $Q = \mathcal{F}(AX)$, where Q, X, and A denotes its output, variable input bundle, and factor-augmenting productivity. The demand and inverse demand functions are $Q = \mathcal{D}(P)$ and $P = \mathcal{P}(Q)$, where P is the price. Then, the output and price elasticities are

$$\gamma = \frac{\partial \mathcal{F}}{\partial X}\frac{X}{Q} = \mathcal{F}'(AX)\frac{AX}{Q} \quad \text{and} \quad -\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q} = -\left[\frac{\partial \mathcal{P}}{\partial Q}\frac{Q}{P}\right]^{-1}.$$

From the output and inverse demand functions, the revenue function is given by $PQ = \mathcal{P}(Q)Q = \mathcal{P}(\mathcal{F}(AX))\mathcal{F}(AX) = \mathcal{R}(AX)$. Revenue elasticity is given by

$$\zeta = \frac{\partial \mathcal{R}}{\partial X}\frac{X}{PQ} = \left[\frac{\partial \mathcal{P}}{\partial Q}\frac{\partial \mathcal{F}}{\partial X} + P\frac{\partial \mathcal{F}}{\partial X}\right]\frac{X}{PQ} = \left[-\left(\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q}\right)^{-1} + 1\right]\frac{\partial \mathcal{F}}{\partial X}\frac{X}{Q}.$$

Furthermore, proift maximization yields the familiar expression for the markup as a function of price elasticity of demand:

$$\mu = \left(-\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q}\right)\left(-\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q} - 1\right)^{-1}.$$

Revenue elasticity

We can express revenue elasticity as follows

- **[Elasticity estimation approach:]** Revenue elasticity is the ratio of output elasticity to the markup: i.e., $\zeta = \gamma/\mu$.
- **[Cost-share approach:]** Revenue elasticity is equal to the ratio of variable costs to revenue: i.e., $\zeta = WX/PQ$.

Propagation mechanism

Higher revenue elasticity firms generate greater business cycle amplification in reacting to supply shocks.

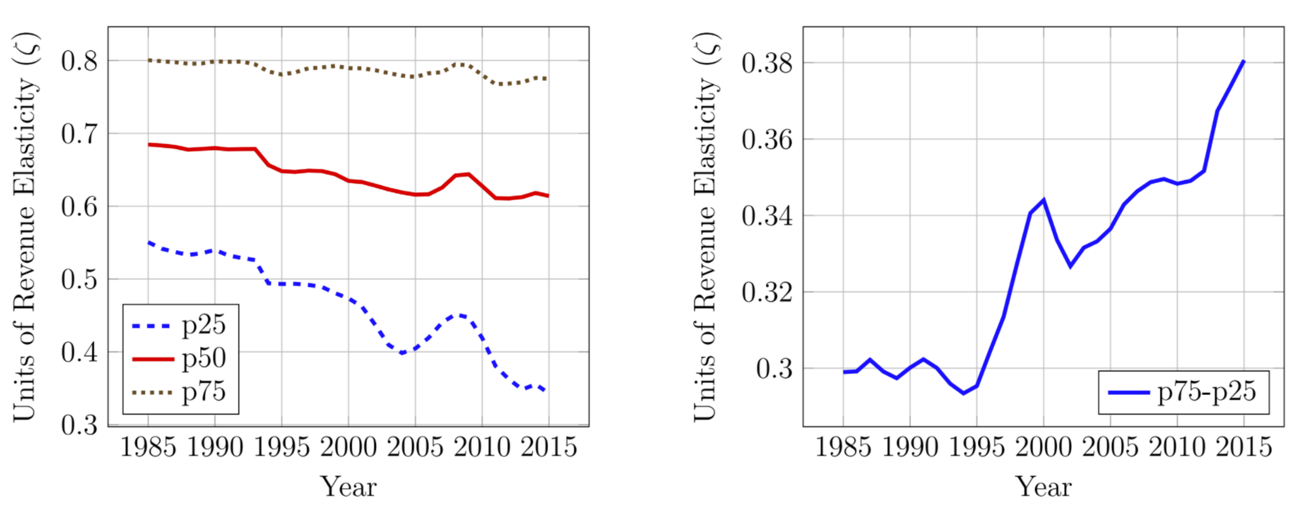
A firm's revenue response of the business cycle to demand and supply shocks depends on knowing any two of output elasticity, revenue elasticity or the markup. However,

- Revenue elasticity is sufficient to understand supply shocks ($\Delta \ln A$).
- Revenue elasticity is insufficient to understand demand shocks ($\Delta \ln \xi$). In order to understand demand shocks, we need information on any two of revenue elasticity, the markup or output elasticity.

$$\Delta \ln PQ \approx \frac{\zeta}{1-\zeta}(\Delta \ln A - \Delta \ln W + \Delta \ln \zeta) + \frac{1-1/\mu}{1-\zeta}\Delta \ln \xi,$$

where the factor price change ($\Delta \ln W$) accounts for general equilibrium channels in reacting to shocks. The key point is that with markups we could learn more, but in their absence revenue elasticity still has some uses.

Revenue Elasticity Measurement and Trends



(a) Quartiles of Revenue Elasticity Distribution (b) Inter-quartile Range

Figure 1: Revenue Elasticity Quartile Trends

Revenue elasticity (based on cost-share approach) is decreasing for upper quartile, lower quartile and median revenue elasticity firms. But, the decline among the high revenue elasticity firms is weaker than the decline among the low revenue elasticity firms.

Empirical Methodology

Our reduced-form model quantifies the effect of shocks on firm revenues conditional on firm revenue elasticity. In order to estimate the dynamics of differential responses across firms, we use local projection estimation. In particular, we interact productivity shocks with a firm's pre-existing traits.

$$\Delta^h \ln P_{j,t}Q_{j,t} = \beta_0^h \text{shock}_{j,t} + \beta_{1,\zeta}^h(\text{shock}_{j,t} \times \ln \zeta_{j,t}) + (\text{shock}_{j,t} \times \mathbf{traits}_{j,t}^\intercal)\mathbf{b}_1^h + \beta_2^h \ln \zeta_{j,t} + \mathbf{traits}_{j,t}^\intercal \mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h \quad (23)$$

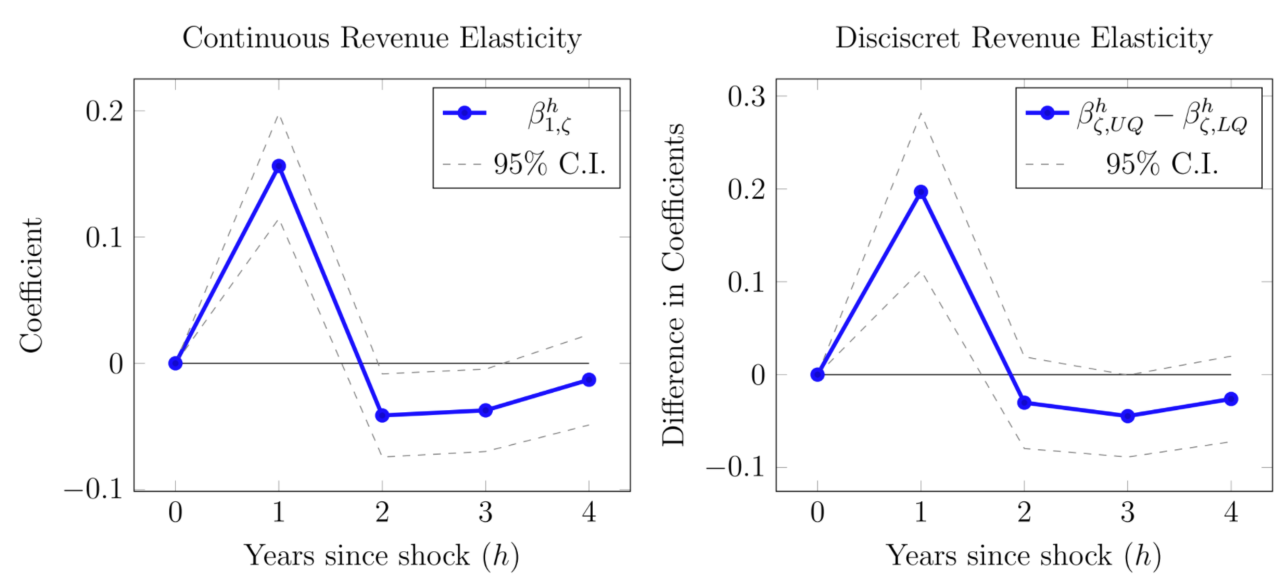
We index a firm with j and $h \geq 1$ represents the forecast horizon. The delta operator Δ^h represents the difference between $t+h$ and t, such that $\Delta^h \ln P_{j,t}Q_{j,t} \equiv \ln P_{j,t+h}Q_{j,t+h} - \ln P_{j,t}Q_{j,t}$ for h. Hence, the dependent variable is the difference between log revenue in period $t+h$ and log revenue in the current period t. The main coefficients of interest are $\beta_{1,\zeta}^h$ for $h = 1, 2, 3, 4$. A positive coefficient means a shock has a greater effect on revenue for firms with higher revenue elasticity.

As an alternative to regression, we consider a discrete measure of revenue elasticity. The dummy variable $UQ_{j,t}$ is 1 if firm j is in the upper quartile of revenue elasticities and is 0 otherwise. The dummy variable $LQ_{j,t}$ is 1 if firm j is in the lower quartile of revenue elasticities and is 0 otherwise. Our re-specified equation is

$$\Delta^h \ln P_{j,t}Q_{j,t} = \beta_0^h \text{shock}_{j,t} + \beta_{1,UQ}^h(\text{shock}_{j,t} \times UQ_{j,t}) + \beta_{1,LQ}^h(\text{shock}_{j,t} \times LQ_{j,t}) + \beta_{2,UQ}^h UQ_{j,t} + \beta_{2,LQ}^h LQ_{j,t} + \mathbf{traits}_{j,t}\mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h, \quad (24)$$

where the difference between the upper and lower quartile coefficients, $\beta_{1,UQ}^h - \beta_{1,LQ}^h$ for $h = 1, 2, 3, 4$, represents the difference in revenue response of high and low revenue elasticity firms to shocks. When the difference is positive, it implies that high revenue elasticity firms respond more to shocks than low revenue elasticity firms.

Empirical results



(a) Regression Equation (23) (b) Regression Equation (24)

Figure 4: Impulse Response Functions to Productivity Shock

Panel (a) represents the impulse response following a one percent productivity change. More specifically, the plots capture the effect of a productivity shock on revenue *conditional on a firm's revenue elasticity.* ***Firms with higher revenue elasticity adjust revenues more in response to a productivity shock than firms with lower revenue elasticity.*** The effect is large on impact, but dissipates after one year.

Panel (b) plots the differential response of productivity shocks on firms in the upper and lower quartiles. The upper quartile of revenue elasticity firms increase their revenue more than the lower quartile of revenue elasticity firms following a productivity shock.

Conclusion

We analyse the effect of firm-level revenue elasticities on business cycle behaviour.

- We focus on revenue elasticities because they are simple to obtain at the firm level, but are understudied relative to the related concepts of price markups and output elasticities.
- We present empirical results on the behaviour of revenue elasticities of U.S. firms over the last three decades.
- We present theory to show that higher revenue elasticities generate greater business cycle amplification.
- We test this theoretical relationship on U.S. data and find evidence in support of the theory.

References

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