An Explanation of Real US Interest Rates with an Exchange Economy

Matching evidence on US real bond rates

Tamas Csabafi, Max Gillman[1]

University of Missouri-St. Louis

December 29, 2021

[1]University of Missouri - St. Louis, Department of Economics, University of Missouri - St. Louis, 1 Uninversity Boulevard, St. Louis, MO 63121; gillmanm@umsl.edu

Explanation of Real US Rates with Exchange Economy

Matching evidence on US real bond rates using DSGE model

- Explanation of real 3-month US Tbill interest rates,
- 1975Q1-2020Q4. Offers solution to puzzle:
- Standard asset pricing fundamentals cannot explain rates.
- Model adds in a premium that can be viewed as reflecting liquidity
- Explains bus.cycle filtered real ex-post interest rates, & output, labor.
- Explains level of real interest rates by adding back HP filtered trend.
- 3 shock construction methods used to back out shock.
- Robustness & sensitivity analysis, eg. with money-only economy.
- Iterative convergence shocks: best correlation, rel. volatility near one

Motivation: Liquidity premium focus: post-2008 & Covid

Many Empirical and Theoretic explanations of real US government bond liquidity premium

- Liquidity premiums in wide array of monetary policy models.
- Think of as widening spread between equity return and bond return.
- Paper asks if augmented standard monetary real business cycle model
- can explain real bond interest rates, with small set of shocks:
- to goods sector productivity, bank sector productivity,
- money supply growth rate.
- In exchange economy, with money supply causing inflation,
- with bank sector producing alternative to money: "exchange credit."
- Explain also negative real interest rates as central bank "fix" rates.

Results with this Approach

- Model can explain rates accurately only if endogenous velocity;
- consumer earns interest on deposits in banks to avoid inflation tax.
- Bank-based exchange means: eg. credit card paid off end-period.
- Money is used at point of purchase: eg. debit card, currency.
- Historical inflation (not *iid*): "downward sloping" money demand.
- Optimal inflation tax avoidance induces asset pricing wedge:
- Expected change in the "user cost" of exchange.
- Results advance monetary real bus.cycle real bond interest rates
- even during negative real interest rates & historically 1975-2020.

Specific Results

- Show both HP filtered real bond rate; **Level** of rate in model & data.
- High correlation model predicted and data;
- higher correlation, better volatility with HP trend added in: Level.
- HP trend has content: inflation not random process; eg. War, Crises.
- Relative volatility near 1: stand. dev. of model divided by sd of data.
- Explains HP filtered Consumption, Labor, Invest., Nom. Rt, Inflation.
- Iterative convergence Shock construction method best (Benk);
- also Bayesian, Solow.

Three Alternative Methods to Back-out Shocks

- Solow residual is used for bank, goods sector productivity shocks,
- plus actual M1 money supply growth rate for money supply shock.
- Standard Bayesian estimation of all three shocks.
- "Benk" method extends Nolan-Thoenissen 2009;
- backs historical shock sequences using time series data
- & full DSGE solution for decision variables;
- each variable solution depends on state variable &3 shocks.

Iterative convergence enables matching variance-covariance matrices

- Pagan-Wickens 2019: DSGE model consistency
- with covariance matrix of shocks.
- Iterative convergence algorithm:
- same matrix in calibration, & in backed-out shocks.
- Shocks inputted back into model to give model
- predictions of real rate historically; compared to data.
- Overidentify shocks; estimate shocks
- using maximum likelihood estimation procedure.
- Solow method fairs poorly;
- both Bayes & Benk methods work well.

User cost premium, compared to standard asset pricing kernel

- Large literature: user cost = interest differential:
- Fried.-Schwartz1970, Barnett1980, Canz.-Diba2005, Lucas-Nicol.2015.
- Special case: cash-only economy gives fundamentals, user cost R.
- General user cost: $R - R^d$: nom.rate R minus dividend yield R^d.
- **Ratio of Expected change in $R - R^d$ to exp. ch in R :**
- is *Asset Pricing Wedge* as Liquidity Premium.
- Other wedges based on putting liquidity services in Utility function,
- but without explaining historical data.

Intuition

- Model: real business cycle (RBC) bank sector production function
- supported empirically in microeconomic banking literature.
- User cost equals per unit cost of banking that avoids inflation tax
- plus per unit inflation tax paid.
- Allows us only goods and leisure in utility function;
- explain historical real bond rates: econometrically constructed shocks,
- without ambiguous utility parameters or transaction cost functions.

Rest of Presentation

- Model: Cash-in-Advance extended exchange, endog. velocity.
- Show Results in Figures and Tables.
- Robustness: Special CIA case; *iid* shocks,
- Model extension with Interest on Reserves.
- Include Mixed model of both Benk shocks for productivity;
- + money supply growth rate.
- Mixed model does almost as well as Benk model.
- Discussion of policy, model, negative interest rates worldwide.
- Conclusion.

A Model with Exchange Credit

- $u(c_t, x_t) = \frac{[c_t x_t^{\psi}]^{1-\theta}}{1-\theta}$. Goods c_t, Leisure x_t
- $1 = x_t + l_{Gt} + l_{Qt}$. Time: labor in goods sector l_G; bank l_Q
- $w_t(l_{Gt} + l_{Qt}) + (1 + r_t - \delta_k)\, k_t$ income for labor and capital rental
- $\overline{R}_t^d d_t$ yield on bank deposits $\overline{R}_t^d$ is bank profit rate.
- $p_{Qt} q_t$ consumer cost: exchange credit;
- p_{Qt} is real compet. price; q quant.
- $w_t(l_{Gt} + l_{Qt}) + (1 + r_t - \delta_k)\, k_t + \overline{R}_t^d d_t - m_{t+1}\,(1 + \pi_{t+1}) + m_t - b_{t+1}\,(1 + \pi_{t+1}) + (1 + \overline{R}_t) b_t + \Gamma_t - c_t - p_{Qt} q_t \geq k_{t+1}$
- Next period capital stock after investment in money, bonds, capital.

Exchange Techology and Deposit Constraint

- Exchange for consumption c_t by real money + real lump-sum transfer,
- $m_t + \Gamma_t$, and by exchange credit q_t,
- as perfect substitutes in exchange.
- $m_t + \Gamma_t + q_t \geq c_t$.
- Consumption greater than or equal to deposited funds,
- $c_t \geq d_t$.

Bank Production of Exchange Credit

- exchange credit production is given by
- $q_t = A_Q e^{v_t} \left(l_{Qt}\right)^{\gamma} d_t^{1-\gamma}$.
- for $\gamma \in (0,1)$, marginal cost of $\frac{q_t}{d_t} = A_Q e^{v_t} \left(\frac{l_{Qt}}{d_t}\right)^{\gamma}$
- endogenously upward sloping in contrast to eg. Berk-Green 2004;
- lit. assumes exog. convex marginal cost of financial services.
- marginal cost per unit is convex upward sloping for $0 < \gamma < 0.5$,
- concave for $1 > \gamma > 0.5$, and $\rfloor$ - shaped at $\gamma = 0$.
- At $\gamma = 1$: no well-defined equilibrium; eg King-Plosser 1984
- Profit, $\Pi_{Qt} \equiv p_{Qt} q_t - w_t l_{Qt} - \overline{R}_t^d d_t$, with respect to l_{Qt} and d_t :
- $w_t = p_{Qt} \gamma A_Q e^{v_t} \left(\frac{l_{Qt}}{d_t}\right)^{\gamma-1}$; $\overline{R}_t^d = p_{Qt} (1-\gamma) A_Q e^{v_t} \left(\frac{l_{Qt}}{d_t}\right)^{\gamma}$.

Goods Producer and Government

- physical capital k_t and labor time l_{Gt} inputs to produce output y_t
- $y_t = A_G e^{z_t} \left(l_{Gt}\right)^{\alpha} k_t^{1-\alpha}$.
- First order conditions: maximizing profit, $y_t - w_t l_{Gt} - r_t k_t$,
- $w_t = \alpha A_G e^{z_t} \left(\frac{k_t}{l_{Gt}}\right)^{1-\alpha}$; $r_t = (1-\alpha) A_G e^{z_t} \left(\frac{k_t}{l_{Gt}}\right)^{-\alpha}$.
- Government: Nominal transfer
  $T_t = B_{t+1} - B_t \left(1 + \bar{R}_t\right) + M_{t+1} - M_t$.
- From new govt bonds B and new money M.
- Real terms: $\Gamma_t \equiv \frac{T_t}{P_t}$;
  $\Gamma_t = b_{t+1}\left(1 + \pi_{t+1}\right) - b_t\left(1 + \overline{R}_t\right) + m_{t+1}\left(1 + \pi_{t+1}\right) - m_t$.
- Let σ_t denote growth rate of money $\sigma : \Gamma_t = \sigma_t m_t = (\sigma + e^{u_t}) m_t$.

Equilibrium

- Shocks: matrix equation: $Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}$; shocks $Z_t = [z_t \;\; u_t \;\; v_t]'$
- [AR(1)] follow auto-regressive process of order one.
- Recursive representative consumer's optimization problem is
- $V(s) = \max_{c, x, l_G, l_Q, q, d, k', m', b'} \{U(c, x) + \beta E V(s')\}$,
- subject to time, budget and deposit constraints.

Asset Pricing Liquidity Premium based on User Cost

- User cost enters the intratemporal and intertemporal margins.
- User cost is $\overline{R}_t - \bar{R}_t^d$:
- weighted average of cost per unit of goods from using exchange credit
- plus cost per unit of goods from using money, $\hat{m} = m + \Gamma$.
- with the weights in equilibrium being $\frac{q_t}{c_t}$ and $\frac{\hat{m}_t}{c_t}$, $\frac{q_t}{c_t} + \frac{\hat{m}_t}{c_t} = 1$
- $\overline{R}_t - \bar{R}_t^d = \frac{q_t}{c_t}\gamma\overline{R}_t + \frac{\hat{m}_t}{c_t}\overline{R}_t$
- With $\frac{q_t}{c_t}\gamma\overline{R}_t = \frac{wl_{Qt}}{c_t}$, cost of labor used up in banking per unit of goods.
- $\frac{m_t}{c_t}\overline{R}_t$ is inflation tax paid to government.
- Average cost of exchange, $\overline{R}_t - \bar{R}_t^d$.

- Intratemporal marginal rate of substitution between goods and leisure,
- equals $\frac{1+\overline{R}_t-\bar{R}_t^d}{w_t}$, ratio of the shadow price of goods of 1 plus
- exchange cost $\overline{R}_t - \bar{R}_t^d$, to shadow price of leisure w.
- Intertemporal equilibrium conditions imply asset pricing kernel.
- $1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} \left(\frac{x_{t+1}}{x_t} \right)^{\psi(1-\theta)} \left(\frac{1+\overline{R}_{t+1}}{1+\pi_{t+1}} \frac{1+\overline{R}_t-\overline{R}_t^d}{1+\overline{R}_{t+1}-\overline{R}_{t+1}^d} \right) \right]$.
- Without exchange credit ($A_Q = 0$), $\overline{R}_t - \bar{R}_t^d = \overline{R}_t$, no avoidance of inflation tax:
- special case is $1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} \left(\frac{x_{t+1}}{x_t} \right)^{\psi(1-\theta)} \left(\frac{1+\overline{R}_t}{1+\pi_{t+1}} \right) \right]$.
- Write kernel as
  $$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} \left(\frac{x_{t+1}}{x_t} \right)^{\psi(1-\theta)} \left(\frac{1+\overline{R}_t}{1+\pi_{t+1}} \right) \left(\frac{1+\overline{R}_{t+1}}{1+\overline{R}_t} \frac{1+\overline{R}_t-\overline{R}_t^d}{1+\overline{R}_{t+1}-\overline{R}_{t+1}^d} \right) \right].$$
- Wedge, added factor, is $\left(\frac{1+\overline{R}_{t+1}}{1+\overline{R}_t} \frac{1+\overline{R}_t-\overline{R}_t^d}{1+\overline{R}_{t+1}-\overline{R}_{t+1}^d} \right)$ relative to fundamental kernel.

Interpretation of Asset Pricing Wedge Changes from Policy

- Random fluctuations in the expected user cost cause
- fluctuations in the real bond interest rate.
- Eg. wedge captures how decrease in expected % change in user cost
- by more than a decrease in the expected % change in nominal rate R
- causes current period real bond interest rate to fall.
- Eg. should excess reserves be expected to rise
- more seigniorage transferred from Fed to banks through IOER policy,
- cost of avoiding inflation tax would be expected to fall,
- and real bond rate would fall as in an indirect liquidity effect.
- For any macroprudential policy that affects liquidity in bank system
- user cost $\bar{R}_{t+1} - \bar{R}^d_{t+1}$ can change, gives kernel explanatory power.

Shocks

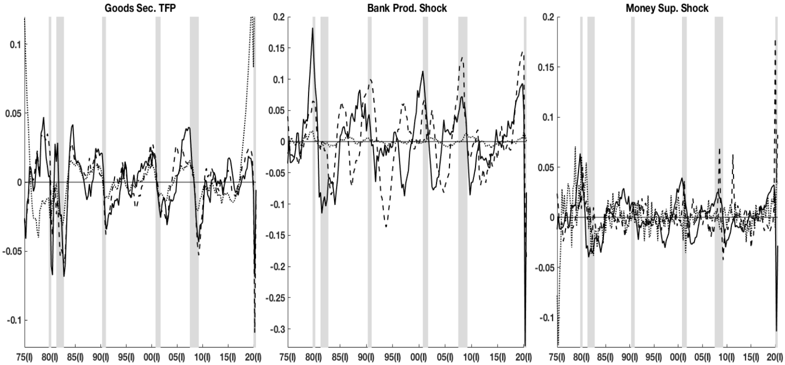

Figure: Benk (Solid), Bayes (Dots), Solow (Dashed) Shocks for TFP, Bank Productivity, and Money Supply Growth.

HP Filtered Variables: Model vs US Data

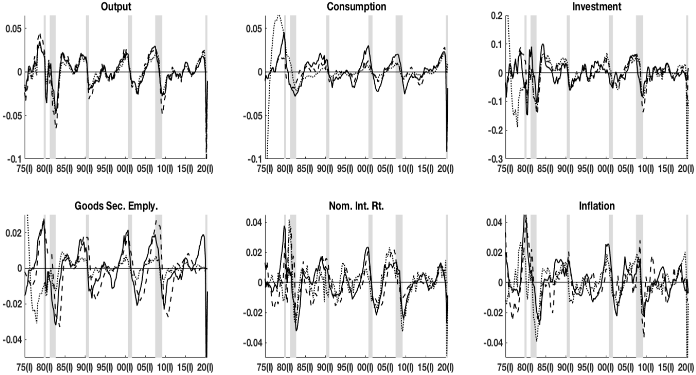

Figure: Key Variables: Benk and Bayesian Shock Implied Series vs. US Data.

2^{nd} Moments	1975Q1-2020Q4				1975Q1-2007Q4				2008Q1-2020C		
	Solow	Benk	Bayes	Mixed	Solow	Benk	Bayes	Mixed	Solow	Benk	Baye
$R-\pi$ Corr.	-0.05	**0.48**	0.16	0.41	-0.11	0.59	0.26	0.52	0.08	0.14	-0.2
Rel. Vol.	15.8	**1.09**	1.85	1.33	13.7	1.08	1.89	1.29	21.3	1.13	1.68
R Corr.	-0.39	0.62	0.50	0.62	-0.44	0.61	0.49	0.61	-0.42	0.79	0.73
Rel. Vol.	47.0	2.12	2.41	2.25	378	1.98	2.03	2.11	114	3.40	5.29
π Corr.	-0.34	0.73	0.34	0.77	-0.38	0.76	0.28	0.82	-0.27	0.66	0.48
Rel. Vol.	57.7	1.85	1.24	1.99	51.3	1.97	1.14	2.16	73.4	1.35	1.48
y Corr.	0.97	0.97	0.97	0.96	0.97	0.96	0.97	0.96	0.97	0.98	0.97
Rel. Vol.	1.52	0.75	0.71	0.82	1.49	0.78	0.68	0.83	1.57	0.67	0.78
c Corr.	0.79	0.87	0.32	0.78	0.82	0.90	0.42	0.81	0.86	0.93	0.33
Rel. Vol.	1.0	1.38	2.50	0.63	1.35	1.82	3.85	0.86	0.68	1.02	0.97
i Corr.	0.90	0.71	0.30	0.90	0.93	0.72	0.18	0.94	0.81	0.72	0.69
Rel. Vol.	2.21	0.89	1.85	1.12	2.14	0.98	1.94	1.11	2.42	0.61	1.60
I_G Corr.	0.29	0.60	0.05	0.51	0.41	0.72	0.03	0.60	0.01	0.33	0.12
Rel. Vol.	1.24	0.88	1.44	0.58	1.15	0.84	1.55	0.57	1.43	0.97	1.08

Table: Second Moments of Model Implied Variables, US data.

HP Filtered Ex post Real Interest Rate

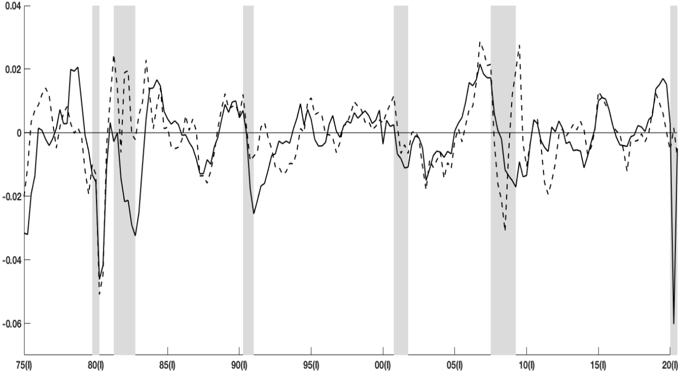

Figure: HP Filtered Ex post Real Interest Rate (Benk) vs. US Data.

HP Filter Trend Component

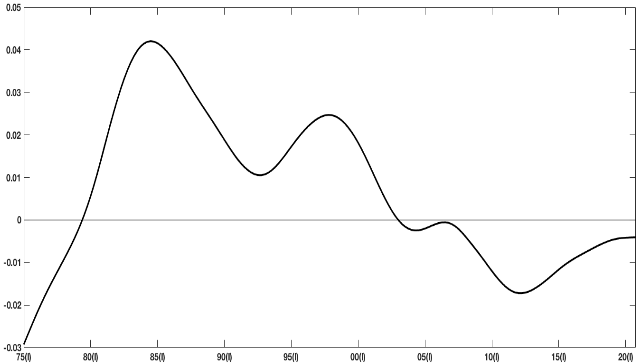

Figure: HP Filter Trend Component of the Ex-post Real Bond Interest Rate

Level of Ex-post Real Bond Interest Rate

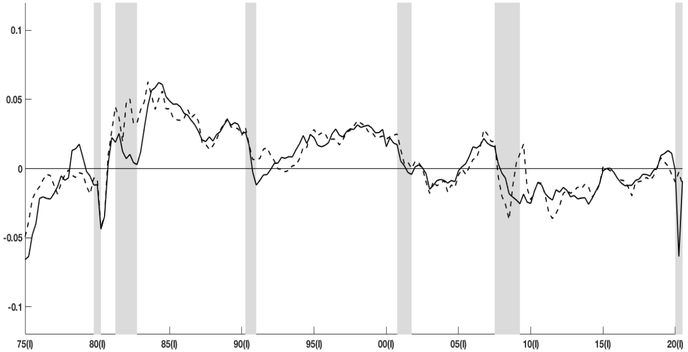

Figure: Level of Ex-post Real Bond Interest Rate: Benk Method (black line) vs. US Data (dashed).

2nd Moments of the Real Interest Rate in Levels with US Data				
Method	2nd Moments	1975Q1-2020Q4	1975Q1-2007Q4	2008Q1-2020Q4
Solow	Corr.	0.118	0.135	0.025
	Rel. Vol.	20.369	14.799	60.406
Benk	**Corr.**	**0.859**	**0.874**	**0.306**
	Rel. Vol.	**1.039**	**1.0636**	**1.106**
Bayes	Corr.	0.652	0.635	-0.147
	Rel. Vol.	1.354	1.406	1.495
Mixed	*Corr.*	*0.815*	*0.835*	*0.217*
	Rel. Vol.	*1.136*	*1.169*	*1.334*

Table: Correlation of Model Implied Unfiltered Real Interest Rate Series with US Data with relative standard deviation in parentheses.

Variance Decomposition of the Ex post Real Interest Rate											
Shock Order			1975Q1-2020Q4			1975Q1-2007Q4			2008Q1-2020Q4		
T	M	B	81.68%	8.65%	0.49%	84.27%	5.09%	1.57%	90.04%	7.46%	7.06%
T	B	M	81.69%	0.49%	8.65%	84.28%	1.56%	5.10%	90.06%	7.10%	7.42%
B	T	M	70.34%	16.26%	8.24%	82.68%	5.33%	5.10%	31.55%	67.91%	7.17%
B	M	T	70.33%	8.24%	16.28%	80.64%	5.09%	5.37%	32.52%	7.20%	67.92%
M	T	B	73.88%	24.69%	6.84%	74.03%	6.96%	10.29%	38.81%	65.78%	0.22%
M	B	T	73.88%	6.83%	24.70%	74.07%	10.28%	7.01%	38.81%	0.22%	65.75%
Average			T	B	M	T	B	M	T	B	M
			40.88%	25.89%	30.26%	32.20%	31.51%	28.08%	74.58%	12.95%	17.81%

Table: Variance Decomposition of the Real Interest Rate

Robustness: Three Experiments, Ex-post Level of Real Bond Interest Rate				
Method (Model)	2nd Moments	1975Q1-2020Q4	1975Q1-2007Q4	2008Q1-2020(
Baseline Benk	Corr.	0.859	0.874	0.306
	Rel. Vol.	1.039	1.064	1.106
Benk CIA	Corr.	0.546	0.521	-0.072
	Rel. Vol.	2.938	3.222	3.183
Benk i.i.d.	Corr.	0.736	0.758	-0.299
	Rel. Vol.	1.045	1.076	1.069
Benk: with IOER	Corr.	0.861	0.875	0.306
	Rel. Vol.	1.037	1.055	1.106

Table: Robustness: 3 Experiments for Level of Real Bond Interest Rate

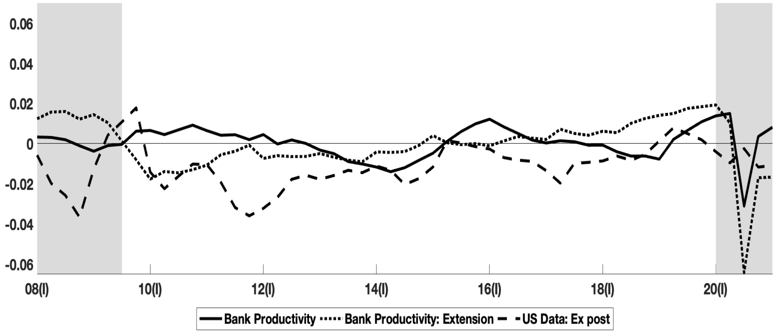

Figure: Post 2008Q1 Bank Shocks (Benk method) in Baseline (solid), IOER Extension (dotted), and with Real Interest Rate Data (dashed).

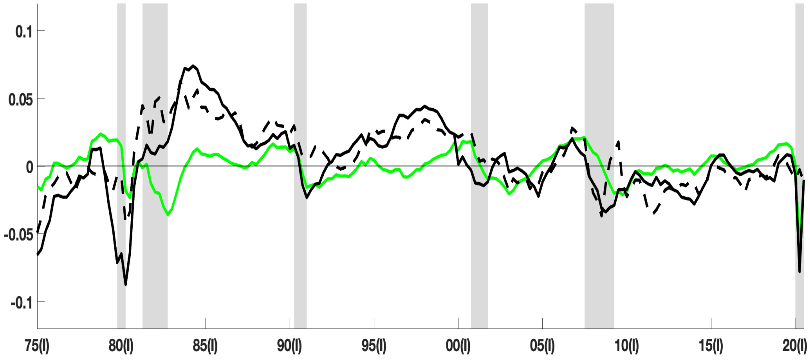

Figure: Goods TFP Productivity Shock vs. Model Predicted, Actual Data, Real T-bill Interest Rate.

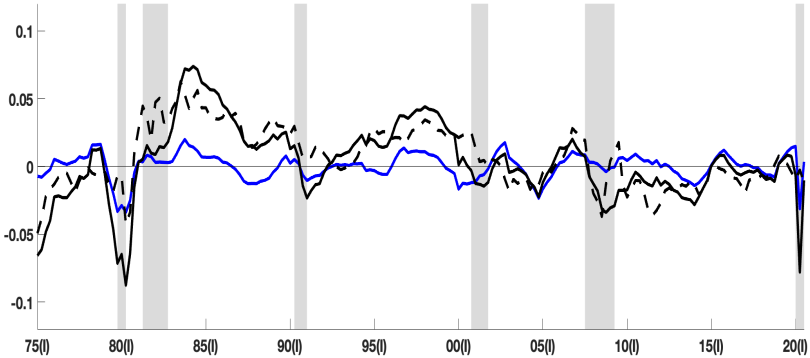

Figure: Bank Productivity Shock vs. Model Predicted, Actual Data, Real T-bill Interest Rate.

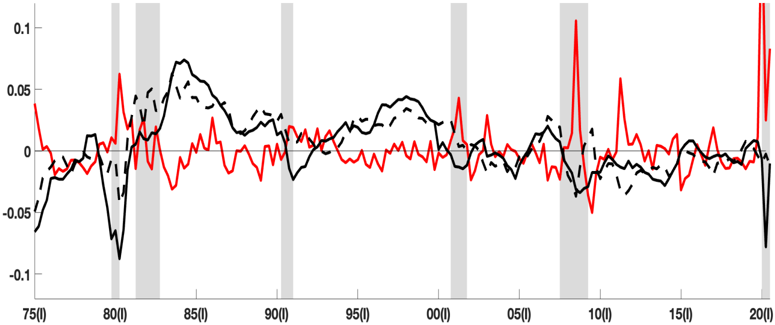

Figure: Money Supply Shock vs. Model Predicted, Actual Data, Real T-bill Interest Rate.

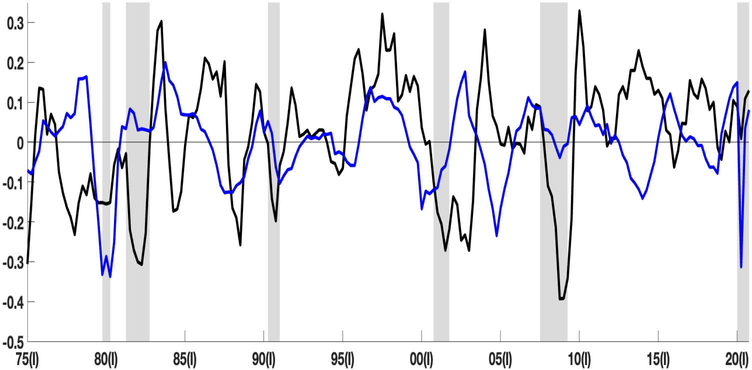

Figure: Real S&P 500 Yield versus Bank Productivity Shock.

Discussion 1: Model explains Data

- Results: model explains real US bond interest rates historically
- including "unconventional" times, negative real bond rates.
- Criticism: bank production tech. is simplistic, not bank behavior;
- rebutted by empirical evidence gathered for some 35 years
- estimating bank production with labor, capital & deposited funds;
- 3rd input solves dilemma of lack of equilibrium w/o deposits input.
- Left out capital as input in bank production function to simplify,
- while still fitting real interest rate data.

Discussion 2: Inflation Tax Wedge

- Our inflation tax wedge is related to other asset pricing wedges.
- Some posit wedge to be function of utility of holding credit good.
- We substitute use of utility for positing wedge by instead
- building exchange technology: "cash" & credit goods perfect subst.
- User cost: aver.cost of optimal mix money-exchange credit.
- Mixed model results , combined with impulse responses, imply
- money supply & bank productivity shocks are well identified.

Discussion 3: Price Theoretic Terms

- Ratio $\frac{\bar{R}^d}{\bar{R}-\bar{R}^d}$ is ratio of Red to Blue area with Marginal Cost MC
- Red: producer surplus R^d; Blue: User Cost $\bar{R} - \bar{R}^d$

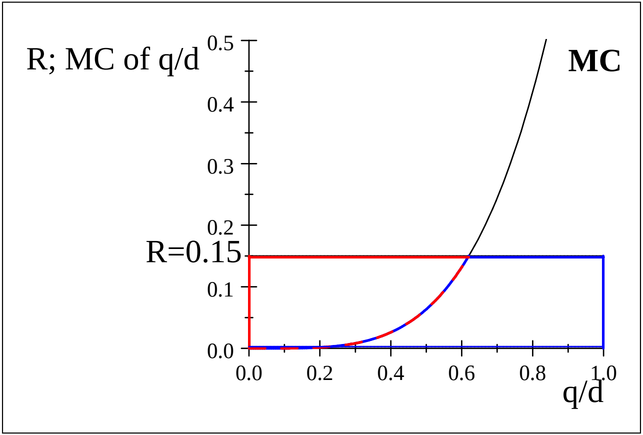

Figure: $\frac{R^d}{R-R^d}$: Inflation Tax Avoided (Red) to Inflation Tax Paid (Blue).

Conclusion

- Inflation tax avoidance causes wedge that determines asset pricing
- because markets must anticipate changes in R and R^d to price bonds.
- Paper offers rational expectations explanation of real bond rates.
- Inflation is caused by money supply growth:
- inflected upwards post-Bretton Woods, 2001, 2008, & 2020.
- Historical, 1975-2020, paper shows standard asset pricing puzzle
- can be solved by extending monetary real business cycle model
- through addition of exchange credit and endogenous velocity.
- Expected changes in user cost drive ex-post real bond interest rate,
- even into negative territory.