Signing-Out Confounding Shocks in Variance-Maximizing Methods

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Finding the drivers of the business cycle...

- Identifying the drivers of the business cycle has been a popular research topic, often through the lens of structural VARS and DSGE models (Galí, 1999; Smets and Wouters, 2007).

- Variance-maximizing SVARS in particular seek the shock that drives the majority of variation at business cycle frequencies or short-horizon forecast error variance.

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$$V = \left(\sum_{\tau=0}^{k-1} D^{\tau} \Sigma_u D^{\tau\prime} \right)$$

A lagrangian solves for the structural shocks that drive the majority of variance of the variable of interest using the reduced-form residual var-cov matrix Σ_u and MA-impact matrix D.

...can be problematic

- Variance-maximizing VAR methodologies offer an attractive methodology to identify the most important business-cycle drivers.
- However, as shown in Dieppe, Francis, and Kindberg-Hanlon (2021), variance maximizing methodologies are actually capturing a combination of shocks, not simply a dominant driver.
- In this paper, we propose a way to sharpen identification in the face of this problem: We combine variance maximizing methodologies with sign and magnitude restrictions to reduce the influence of shocks that are not of interest.

New Keynesian Example

Two variable (output gap and inflation) New Keynesian model with two shocks with unit variance (η, a demand-type shock, and ϑ a supply-type shock such as technology).

$$\begin{bmatrix} \tilde{y} \\ \pi \end{bmatrix} = \begin{bmatrix} \Psi_{y\eta} & \Psi_{y\vartheta} \\ \Psi_{\pi\eta} & \Psi_{\pi\vartheta} \end{bmatrix} \begin{bmatrix} \eta_t \\ \vartheta_t \end{bmatrix}$$

Applying a variance maximizing approach to identify the dominant driver of the output gap will not identify a single shock:

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Applying a variance maximizing approach to identify the dominant driver of the output gap will not identify a single shock:

$$\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}$$

Even if $\Psi_{y\eta} > \Psi_{y\vartheta}$, the impact will reflect both supply and demand fundamental shocks in proportion to their relative contribution to the variance of output.

New Keynesian Example

Effects on other variables are even more difficult to disentangle in this example. Impact of the dominant shock on π will be:

$$\frac{\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}}$$

- Even where the supply-side driver has a small impact on the output gap (small $\Psi_{y\vartheta}$), if it has a large impact on inflation ($\Psi_{\pi\vartheta}$), results will be biased in that direction.

- The initial IRF impact on inflation will be biased downward in the direct of the supply shock relative to the true dominant demand shock.

,

Our methodology

To sharpen identification, we propose a maximization procedure that imposes additional restrictions to reduce the influence of shocks that are of less interest to the researcher.

$$V(\alpha) = \alpha' V \alpha$$

subject to sign constraints a and magnitude constraints b

$$\alpha'\alpha = 1$$

$$C_R^{L\prime}\alpha \geq a$$

$$\frac{C_R^{NL1\prime}\alpha}{C_R^{NL2\prime}\alpha} \geq b$$

Theoretically-grounded restrictions can reduce bias

Theoretically consistent magnitude and sign restrictions can reduce IRF bias for multiple variables.

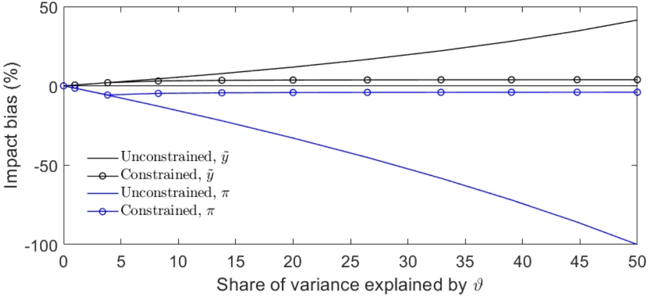


Figure: **Bias of the output gap and inflation impact response to the identified dominant driver of the output gap**

Note: Percent deviation of the identified shock from the true impact of η as the standard deviation of ϑ is increased to the point at which is explains half of the variance of $\tilde{y}$.

US data application

- 8 variable VAR applied to quarterly data since 1953.
- Spectral methodology of Dieppe, Francis, Kindberg-Hanlon (2021) to find dominant business cycle driver (6-32qs) of GDP per capita.
- Objective function to maximize $V = \left(\sum_{\tau=0}^{k-1} D^{\tau}(e^{-i\tau\omega})\Sigma_u D^{\tau}(e^{i\tau\omega})' \right)$
- Augmented with theoretically consistent magnitude restrictions. Dominant demand-side driver: ratio of inflation/GDP impact is $>$0.3. Dominant supply-side driver: ratio is $<$-0.3.

US data application

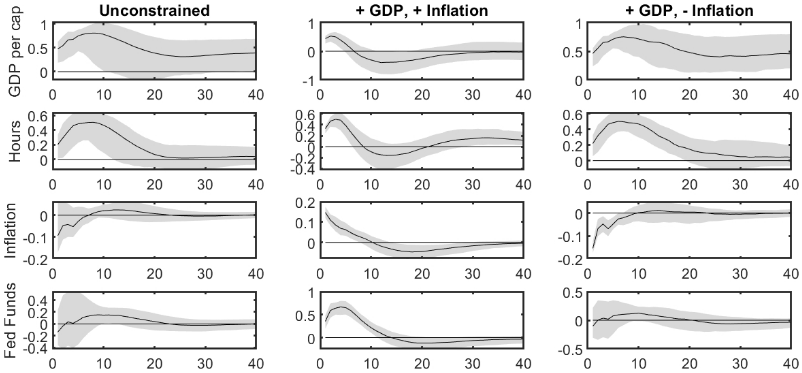
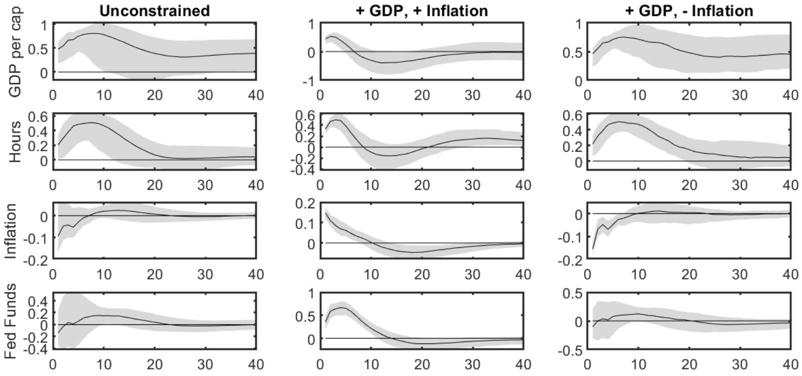


Figure: **Dominant driver of output at business-cycle frequencies, constrained and unconstrained: U.S Data**

Columns reflect the unconstrained eigenvalue-eigenvector solution; the maximizing shock where the impact on inflation is at least 0.3 times the GDP impact; and, the impact is constrained to be at least -0.3 times the GDP impact for inflation.

US data application



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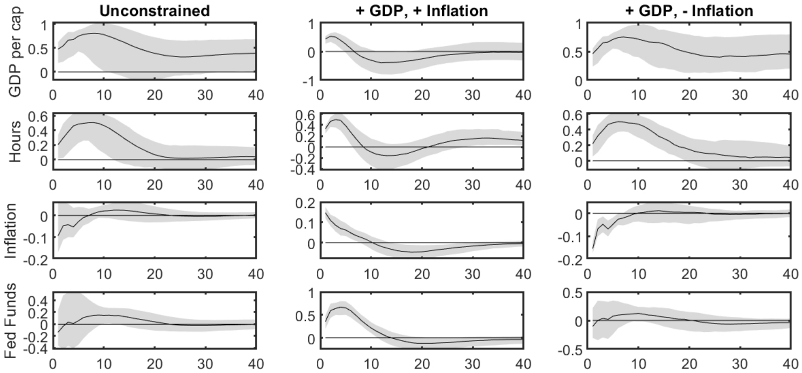
For GDP, more persistent supply shock, less persistent demand shock, hybrid unconstrained shock.

US data application



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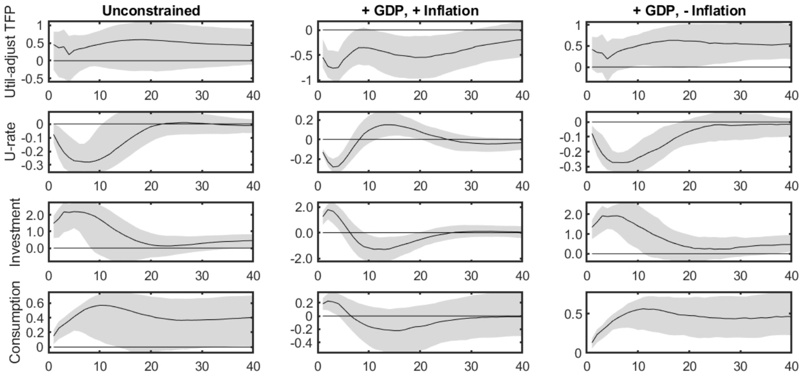
Demand shock: Rise in interest rates. Supply shock: neutral interest rate effect

US data application



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Demand shock - accompanied by negative TFP growth, while supply shock has positive TFP growth.

Business cycle implications

Constrained VARs explain a smaller share of business-cycle variation of GDP compared to unconstrained version (about 60%).

Table: **Contribution of identified shock to business-cycle and long-run variation of GDP**

Scale of restriction	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**
Positive inflation	53	53	50	48	46
	(47, 60)	(46, 60)	(43, 57)	(41, 56)	(38, 55)
Negative inflation	51	50	49	48	46
	(44, 59)	(43, 59)	(41, 58)	(40, 57)	(39, 55)

Is there a long/short-run disconnect between macro drivers?

- Using a main-business cycle shock targeting unemployment (6-32q) without constraints there a disconnect between the dominant business-cycle and long-run drivers of the macro-economy.
- Unconstrained U-targeting business-cycle shock explains 40 percent of business-cycle variation of GDP, but just 10 percent of long-run variation (40+ quarters).
- In contrast, we find supply components of main business cycle shock explain over 25 percent of both the business-cycle and long-run variation of GDP!

Is there a long/short-run disconnect between macro drivers?

Maximizing business cycle variation in unemployment: effect on GDP at business and long-run frequencies.

Business cycle (6-32q)					
Scale of restriction	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**
Positive inflation	40	40	39	38	38
	(32, 50)	(32, 50)	(30, 48)	(29, 48)	(28, 48)
Negative inflation	34	34	34	35	36
	(26, 44)	(24, 44)	(22, 44)	(20, 47)	(22, 47)

Long-run (40+q)					
Scale of restriction	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**
Positive inflation	10	9	8	9	12
	(4, 22)	(4, 20)	(3, 20)	(3, 22)	(4, 26)
Negative inflation	18	22	26	31	30
	(8, 33)	(9, 38)	(11, 44)	(13, 49)	(14, 50)

Conclusion

- We demonstrate a simple solution to reduce the "hybrid shock" problem associated with variance-maximizing SVAR identifications.
- Sign and magnitude constraints to the maximization reduce bias in identifying dominant shock in a theoretical NK model.
- However, relies on theoretical justifications, and does not fully remove bias.
- New methodology finds that US business cycle is broadly equally driven by supply and demand side factors. There is overlap between long-run and business-cycle drivers.

Appendix

What is captured by the maximization procedure in terms of fundamental shocks?

The reduced-form residuals are

$$\epsilon_{\mathbf{t}} = \begin{bmatrix} \epsilon_t^{\tilde{y}} \\ \epsilon_t^{\pi} \end{bmatrix} = \begin{bmatrix} \Psi_{y\eta} & \Psi_{y\vartheta} \\ \Psi_{\pi\eta} & \Psi_{\pi\vartheta} \end{bmatrix} \begin{bmatrix} \eta_t \\ \vartheta_t \end{bmatrix}$$

Assuming uncorrelated structural shocks with unit variance, the variance-covariance matrix of residuals is

$$= \begin{bmatrix} \Psi_{y\eta}^2 + \Psi_{y\vartheta}^2 & \Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta} \\ \Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta} & \Psi_{\pi\eta}^2 + \Psi_{\pi\vartheta}^2 \end{bmatrix}$$

What is captured by the maximization procedure in terms of fundamental shocks?

Let $\tilde{A}$ be the Cholesky decomposition of Σ, using the fact that:

$$= \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} a^2 & ab \\ ab & b^2 + c^2 \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} \sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2} & 0 \\ \frac{\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} & \frac{\Psi_{\pi\eta}\Psi_{y\vartheta} - \Psi_{\pi\vartheta}\Psi_{y\eta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} \end{bmatrix}$$

What is captured by the maximization procedure in terms of fundamental shocks?

$\tilde{A}$, can be combined with the selection matrix ($\alpha = \begin{bmatrix} 1 & 0 \end{bmatrix}$) to target the output gap, $\tilde{y}$, in order to form the matrix that is used to identify the dominant shock using the eigenvalue-eigenvector approach of Faust (1998).

$$V = \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2} & 0 \\ \frac{\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} & \frac{\Psi_{\pi\eta}\Psi_{y\vartheta} - \Psi_{\pi\vartheta}\Psi_{y\eta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} \end{bmatrix} \right]' \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2} & 0 \\ \frac{\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} & \frac{\Psi_{\pi\eta}\Psi_{y\vartheta} - \Psi_{\pi a}\Psi_{y\eta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} \end{bmatrix} \right]$$

The eigenvalues of V are the set $[\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2, 0]$, while the eigenvector corresponding to the largest eigenvalue is $\Gamma_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Full IRFs of constrained/unconstrained estimation

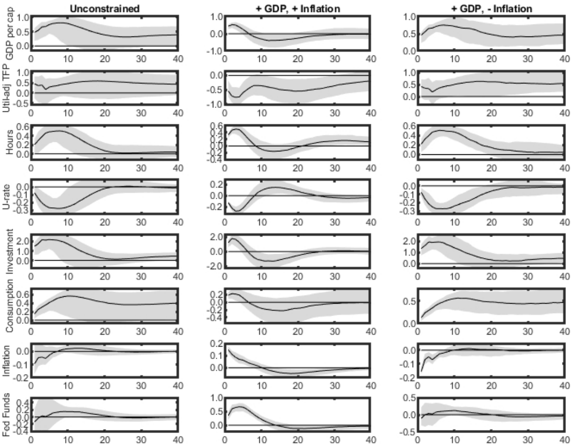

Figure: **Constrained/unconstrained estimation on US data**

Columns reflect the unconstrained eigenvalue-eigenvector solution; the maximizing shock where the impact on inflation is at least 0.3 times the GDP impact; and, the maximizing shock where the inflation impact is at least -0.3 times the GDP impact.

Applying method to larger models (S+W 2007)

- Model demonstration relied on a simple 2 shock NK model. Smets and Wouters (2007) contains 7 shocks. Three of the shocks have characteristics of a typical "demand" shock, three have "supply" shock characteristics. The model also contains an additional monetary policy shock.
- Unconstrained model clearly generates a "hybrid" shock, with supply side characteristics dominating as they do in the US data.
- The Phillips curve is very flat in the model in response to demand shocks and the output-inflation elasticity is estimated to be just 0.05. This minimum elasticity is imposed on impact to isolate demand-drivers of output.
- As some of the supply-type drivers do not affect output on impact, the positive output restriction is imposed after one year. The smallest inflation to output elasticity of the supply drivers in the model is 0.1, so this is imposed as a minimum constraint.

Applying method to larger models (S+W 2007)

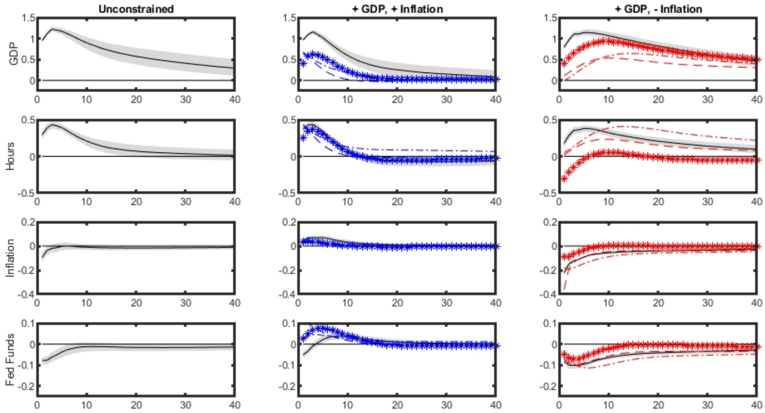

Figure: **Dominant driver of output in Medium-Scale New Keynesian model**

Note: Identified dominant driver of GDP in simulated data produced by the Smets-Wouters model. Blue lines show IRFs from "demand" type shocks in the model, red lines show shocks from supply-type shocks in the model.

Targeting dominant business-cycle drivers of unemployment

- Targeting unemployment rather than GDP yields IRFs which are more consistent with a "demand"-type shock.
- This may be because demand shocks drive a larger proportion of business-cycle frequency variation in unemployment than GDP. For example, in the Smets-Wouters model, the three demand shocks in the model account for about 60 percent of the business cycle variation of unemployment, but just 40 percent of the variation of GDP.

IRFs targeting business-cycle drivers of unemployment

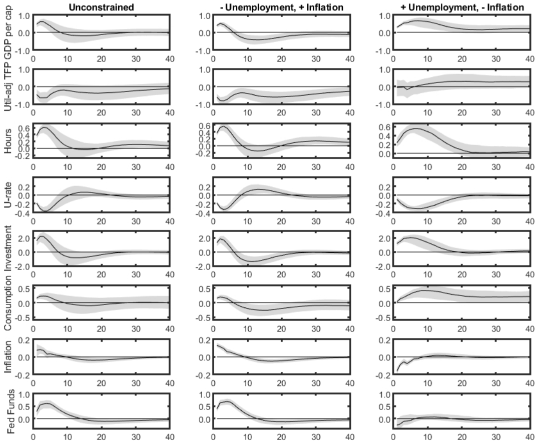

Figure: **Constrained/unconstrained estimation on US data targeting unemployment**

Columns reflect the unconstrained eigenvalue-eigenvector solution; the maximizing shock for U where the impact on inflation is at least +0.3 times the GDP impact; and, the maximizing shock where the inflation impact is at least -0.3 times the GDP impact.

Sign restrictions without maximization

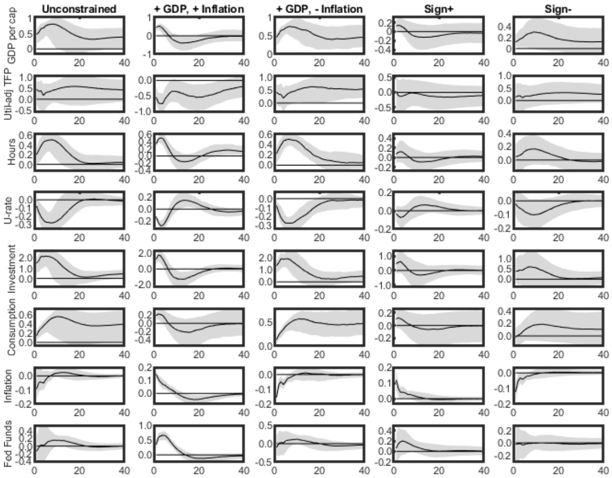

Figure: **Maximization with magnitude restrictions compared to simple sign and magnitude restrictions**