A Free and Fair Economy: A Game of Justice and Inclusion

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Introduction

How do basic principles of distributive justice affect equilibrium existence and efficiency in a non-cooperative economy? Consider an economy where agents freely and non-cooperatively choose their actions, and the surplus resulting from these action choices is shared following four principles (**ALUM**):

1. **Anonymity**: Your pay should not depend on your *name*.
2. **Local efficiency**: No portion of the surplus generated at any profile of action choices should be wasted.
3. **Unproductivity**: An unproductive agent should earn nothing.
4. **Marginality**: A more productive agent should not earn less.

It is generally agreed that **ALUM** form the core principles of *market justice*. However, a number of empirical observations have suggested that discrimination based on name, race, gender, culture, religion, and academic affiliation is prevalent in several contexts. We study how **ALUM** guarantee equilibrium existence (or stability) and efficiency through the lens of a model of a free and fair economy.

A Free and Fair Economy

A **free economy** is a list $\mathcal{E} = (N, X, o, f, \boldsymbol{\phi}, u)$:

- N: nonempty and finite set of agents, $n = |N|$;
- $X = \prod_{i=1}^{n} X_j$: X_i is agent i's action set, $|X_i| < \infty$;
- $o = (o_i)_{i\in N} : o_i \in X_i$ is agent i's initial point (e.g., unproduced endowments of goods);
- $f : X \to \mathbb{R}$ is a technology or production function, with $f(o) = 0$; $f(x)$ is the surplus at $x \in X$; $P(X) = \{f : X \to \mathbb{R},$ with $f(o) = 0\}$;
- $\boldsymbol{\phi} : P(X) \times X \to \mathbb{R}^n$, distribution or pay scheme, with $\sum_{i\in N} \boldsymbol{\phi}_i(f, x) \le f(x), \ \forall (f, x) \in P(X) \times X$;
- $u = (u_i)_{i\in N}$: $u_i : X \to \mathbb{R}$ is agent i's utility function; $u_i(x) = \boldsymbol{\phi}_i(f, x)$, for $f \in P(X)$ and $x \in X$.

Definition 1. *A free economy $\mathcal{E} = (N, X, o, f, \boldsymbol{\phi}, u)$ is **fair** if $\boldsymbol{\phi}$ satisfies **ALUM**.*

Let $x \in X$. An outcome $x' \in \Delta(x) \subseteq X$ is a *sub-profile* of x if $x' = x$ or $[x'_i \neq x_i \Longrightarrow x'_i = o_i]$, for $i \in N$.

Let $i \in N$. We define the relation Δ_o^i on X:

$$[x' \ \Delta_o^i \ x] \Leftrightarrow [x' \in \Delta(x) \text{ and } x'_i = o_i].$$

Denote $\Delta_o^i(x) = \{x' \in X : x' \ \Delta_o^i \ x\}$, $N^x = \{i \in N : x_i \neq o_i\}$, and $|x| = |N^x|$ the cardinality of N^x. For $x, x' \in X$, the profile of actions $(x'_{-i}, x_i) \in X$ is the outcome in which agent i chooses x_i, and every other agent j chooses x'_j.

Proposition 1. *There exists a unique scheme, denoted $\boldsymbol{Sh}$, that satisfies **ALUM**. For any $(f, x) \in P(X) \times X$, and $i \in N$:*

$$\boldsymbol{Sh}_i(f, x) = \sum_{x'\in\Delta_o^i(x)} \frac{(|x'|)!(|x| - |x'| - 1)!}{(|x|)!} [f(x'_{-i}, x_i) - f(x')].$$

Equilibrium and Efficiency

A free economy $\mathcal{E} = (N, X, o, f, \boldsymbol{\phi}, u)$ generates a **strategic form game** $G^{\mathcal{E}} = (N, X, u^{\mathcal{E}})$, where for each $x \in X$ and each $i \in N$, $u_i^{\mathcal{E}}(x) = u_i(f, x) = \boldsymbol{\phi}_i(f, x)$.

Definition 2. *Let $\mathcal{E} = (N, X, o, f, \boldsymbol{\phi}, u)$ be a free economy.*

1. *$x^* \in X$ is an equilibrium if and only if x^* is a pure strategy Nash equilibrium in $G^{\mathcal{E}}$.*
2. *$\mathcal{E}$ is weakly (resp. strictly) monotonic if f is weakly (resp. strictly) monotonic.*

Theorem 1. *Any free and fair economy admits an equilibrium.*

Theorem 2. *A weakly monotonic free and fair economy $\mathcal{E}$ admits an equilibrium that is Pareto-efficient. If $\mathcal{E}$ is strictly monotonic, then, the equilibrium is unique and Pareto-efficient.*

Social Justice and Inclusion

Definition 3. *Let $\mathcal{E} = (N, X, o, f, \boldsymbol{\phi}, u)$ be a free economy.*

1. *$\boldsymbol{\phi}$ is an egalitarian Shapley value if there exists $\alpha \in [0, 1]$ such that for all $(f, x) \in P(X) \times X$, and $i \in N$,*

$$\boldsymbol{\phi}_i(f, x) = \boldsymbol{ES}^{\alpha}(f, x) = \alpha \cdot \boldsymbol{Sh}_i(f, x) + (1 - \alpha) \cdot \frac{f(x)}{n}.$$

2. *$\mathcal{E}$ is a **free economy with social justice** if there exists $\alpha \in [0, 1]$ such that $\phi = \boldsymbol{ES}^{\alpha}$.*

Under $\boldsymbol{ES}^{\alpha}$, at each $x \in X$, a fraction $1-\alpha$ of $f(x)$ is shared equally among agents. $\boldsymbol{ES}^{\alpha}$ satisfies anonymity and local efficiency, but it violates unproductivity and marginality for $\alpha \in [0, 1)$. **Theorems 1 and 2** remain valid under any free economy with social justice $\mathcal{E}^{\alpha} = (N, X, o, f, \boldsymbol{ES}^{\alpha}, u)$, $\alpha \in [0, 1]$.

Conclusion

Basic principles of market justice guarantee equilibrium existence and efficiency in a free economy. We generalize our findings to economies with social justice and inclusion, implemented in progressive taxation and redistribution, and guaranteeing a basic income to unproductive agents. Our analysis uncovers a new class of strategic form games by incorporating normative principles into non-cooperative game theory.

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