Dynamic Patent Portfolio Management

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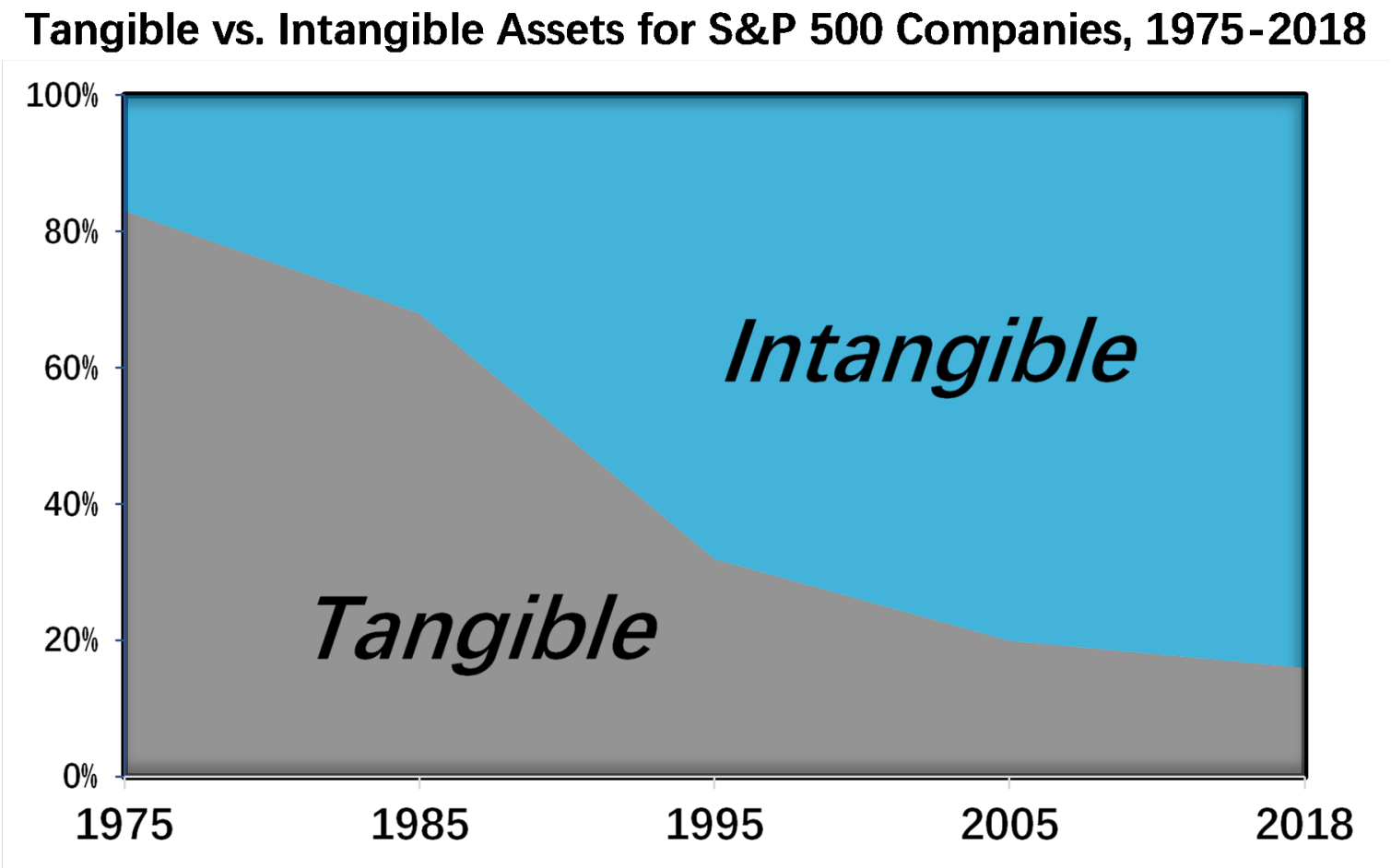
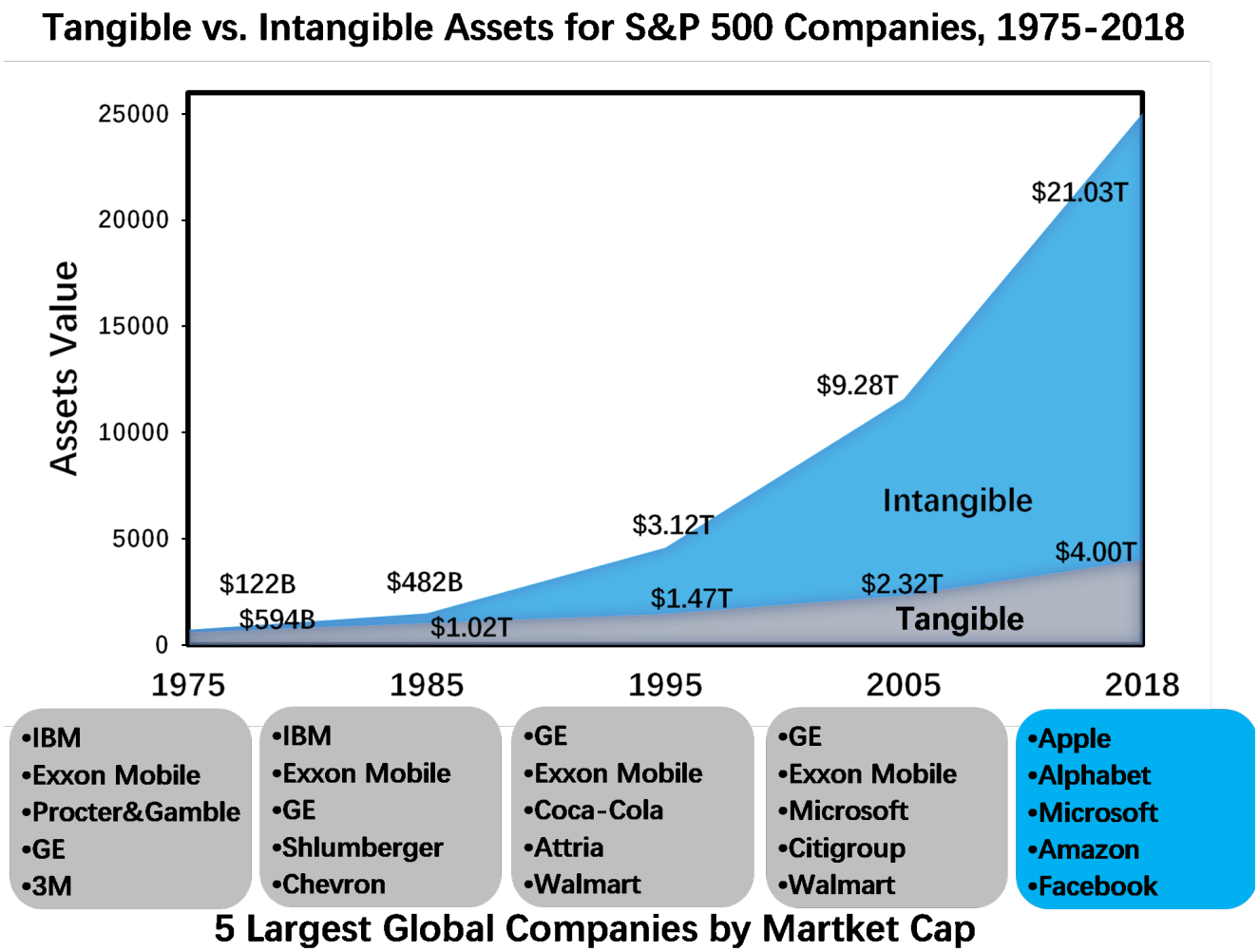
Content

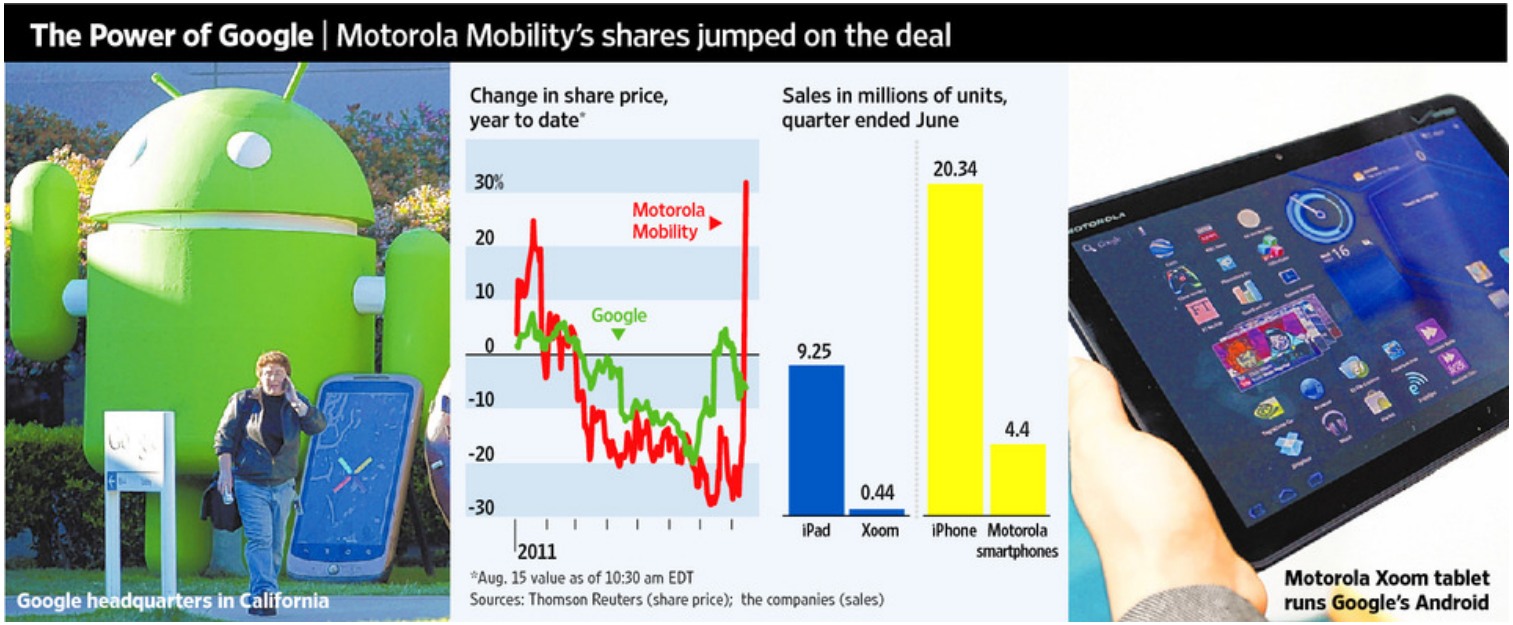
Motivation


Tangible vs. Intangible Assets for S&P 500 Companies, 1975-2018
100%
80%
60%
40%
20%
0%
1975
1985
1995
2005
2018
Intangible
Tangible

Motivation


Tangible vs. Intangible Assets for S&P 500 Companies, 1975-2018
Assets Value
25000
20000
15000
10000
5000
0
$122B
$594B
$482B
$1.02T
$3.12T
$1.47T
$9.28T
$2.32T
$21.03T
$4.00T
Intangible
Tangible
1975
1985
1995
2005
2018
•IBM
•Exxon Mobile
•Procter&Gamble
•GE
•3M
•IBM
•Exxon Mobile
•GE
•Shlumberger
•Chevron
•GE
•Exxon Mobile
•Coca-Cola
•Attria
•Walmart
•GE
•Exxon Mobile
•Microsoft
•Citigroup
•Walmart
•Apple
•Alphabet
•Microsoft
•Amazon
•Facebook
5 Largest Global Companies by Martket Cap

Motivation


The Power of Google | Motorola Mobility's shares jumped on the deal
Google headquarters in California
Change in share price, year to date*
30%
20
10
0
-10
-20
-30
Motorola Mobility
Google
2011
Sales in millions of units, quarter ended June
9.25
0.44
20.34
4.4
iPad
Xoom
iPhone
Motorola smartphones
*Aug. 15 value as of 10:30 am EDT
Sources: Thomson Reuters (share price); the companies (sales)
Motorola Xoom tablet runs Google's Android

Motivation

- What is the optimal capital allocation for long-term investors to think about patent portfolio management given illiquid intangible investment opportunities.
- What explains cross-sectional variation of capital allocation strategies and investment policies?
- How do protection of intangibles (patent protection) influence these policies?

Related Literature

- Portfolio Choice Literature (Merton, Samuelson, Markowitz)
- Intangibles and firm value (Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017) etc.)
- Internal capital reallocation (Dai, Giroud, Jiang, and Wang (2020) etc.)
- Patent protection, Intellectual property rights protection (Mezzanotti (2020) etc.)

Modern Portfolio Theory

- The classic Markowitz-Metron-Samuelson rule:

$$\frac{\text{Risk Premium}}{\text{Risk Aversion} * \text{Variance of Returns}} \tag{1}$$

Patent Portfolio Management

Managing patent portfolio is an ongoing process:

- asset allocation
- diversification
- scale
- accumulation
- asset illiquidity
- risk management

Capital Accumulation

- Following the total capital accumulation process with heterogeneous investments:

$$dO_t = (I_t + \Gamma_t - \delta O_t)dt + \sigma_\gamma \Gamma_t dZ_t^\gamma, \quad t \geq 0, \tag{2}$$

where I and Γ denote the level of physical investment and intangible investment respectively, $\delta \geq 0$ is the rate of depreciation of total capital.

Productivity Formation

$$A_t^{\gamma} = Q(O_t, \Gamma_t, \lambda), \tag{3}$$

This key assumption is consistent with the empirical findings in Doraszelski and Jaumandreu (2013), Kogan, Papanikolaou, Seru, and Stoffman (2017), Hsu (2009), Lin (2012), Hirsh- leifer, Hsu, and Li (2013), and Kumar and Li (2016).

Productivity Formation

- endogenous productivity

$$dA_t^{\gamma} = \lambda \gamma_t dt + \sigma_{\gamma} \gamma_t dZ_t^{\gamma}, \quad t \geq 0, \tag{4}$$

- exogenous productivity

$$dA_t = \mu dt + \sigma dZ_t, \quad t \geq 0, \tag{5}$$

Patent Protection

- Intangible capital accumulation with imitation shocks in the scenario with perfect patent protection

$$dX_t = (\Gamma_t - \delta X_t)dt + \sigma_\gamma \Gamma_t dZ_t^\gamma + \underbrace{\sigma_X O_t dB_t}_{\text{Stochastic Imitation}} . \quad (6)$$

This assumption is also consistent with the arguement that the patent system is characterized by a lot of uncertainty (Lemley and Shapiro (2005)), and enforcement tends to be a noisy process (Jaffe and Lerner (2011)).

Market Distortion

- Let

$$M_t = \exp\left(\int_0^t \alpha_s dB_s - \frac{1}{2} \int_0^t \alpha_s^2 ds \right) \tag{7}$$

is a $(P, \mathcal{F}_t)$-martingale. The rivals' strategy is a real process $\alpha = (\alpha_t)$ stisfying the Novikov condition

- $$\frac{dQ^\alpha}{dQ}|_{\mathcal{F}_t} = M_t, \tag{8}$$

and the process (B_t^α) defined by

- $$B_t^\alpha = B_t - x_t \int_0^t \alpha_s ds, \tag{9}$$

where $x = X/O$

Market Distortion

- Under measure Q^{α}, intangible capital accumulation follow dynamics:

$$dX_t = (\Gamma_t - \delta X_t)dt + \sigma_{\gamma}\Gamma_t dZ_t^{\gamma} + \sigma_X\left(\phi\alpha_t X_t dt + O_t dB_t^{\alpha}\right), \qquad (10)$$

In contrast to Helpman (1993), I use a dynamic zero-sum game to describe the "patent war" in the scenario of imperfect patent protection.

Operating Profit

- Therefore, the firm's incremental operating profit dY_t under imperfect patent protection over time increment dt as follows:

$$dY_t = O_t[dA_t^{\gamma} + dA_t] - I_t dt - \Gamma_t dt - G(I_t, \Gamma_t, O_t)dt, \quad t \geq 0, \quad (11)$$

  where the term $G(I_t, \Gamma_t, O_t)$ represents the total adjustment costs which the firm incurs in both the physical investment and intangible investment process as in Peters and Taylor (2017).
- So that the total adjustment costs are given as follows:

$$G(I, \Gamma, O) = g(i, \gamma) \cdot O = \frac{\theta i^2 + \xi \gamma^2}{2} O, \quad (12)$$

Equilibrium and Patent War

- I assume that the firm can liquidate its assets with liquidation value L_t in which L_t is proportional to the firm's capital. So, I write that $L_t = lO_t$, where $l > 0$ is a positive constant. The discounted expected value of the rivals future utility is given by

$$\mathbb{E}_t^{Q^\alpha}\left[\int_0^\tau e^{-rt}H(\alpha_t, X_t)dt\right]. \tag{13}$$

- By using the minimax theorem, at any Nash equilibrium, the firm value is given by

$$\sup_{I,\Gamma}\inf_{\alpha}\mathbb{E}^{Q^\alpha}\left[\int_0^\tau e^{-rt}dY_t + e^{-r\tau}lO_\tau + \frac{\zeta}{2}\int_0^\tau e^{-rt}H(\alpha_t, X_t)dt\right], \tag{14}$$

Model Solution

- Benchmark: Perfect Patent Protection

$$rV(O) = \max_{I,\Gamma}[(\lambda\gamma + \mu)O - I - \Gamma - G(I, \Gamma, O)] + (I + \Gamma - \delta O)V_O, \tag{15}$$

- $$q^{FB} = \max_{i,\gamma} \frac{(\lambda - 1)\gamma + \mu - i - g(i, \gamma)}{r - (i + \gamma - \delta)}, \tag{16}$$

First-best Investment

- The firm's two types of first-best investment policy are given by $\Gamma_t^{FB} = \gamma^{FB} O_t$ and $I_t^{FB} = i^{FB} O_t$ respectively, where

$$\gamma^{FB} = \frac{q^{FB} + \lambda - 1}{\xi} \quad \text{and} \quad i^{FB} = \frac{q^{FB} - 1}{\theta}. \tag{17}$$

Patent War under Imperfect Patent Protection

A. Reallocation Region

- The firm value $P(O,X)$ satisfies the following Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation:

$$\begin{aligned} rP(O,X) = &\sup_{I,\Gamma}\inf_{\alpha}[(\lambda\gamma+\mu)O - I - \Gamma - G(I,\Gamma,O)] \\ &+ (I+\Gamma-\delta O+\sigma_X\phi\alpha X)P_O \\ &+ (\Gamma-(\delta-\sigma_X\phi\alpha)X)P_X + \frac{\sigma_\gamma^2\Gamma^2+\sigma_X^2O^2}{2}P_{OO} \\ &+ (\sigma_\gamma^2\Gamma^2+\sigma_X^2O^2)P_{OX} \\ &+ \frac{\sigma_\gamma^2\Gamma^2+\sigma_X^2O^2}{2}P_{XX} + \frac{\zeta}{2}H(\alpha,X). \end{aligned} \tag{18}$$

Patent War under Imperfect Patent Protection

- For simplicity, I suppose a quadratic case of $H(\alpha, X)$ follows

$$H(\alpha, X) = \alpha^2 X. \tag{19}$$

- the rivals' optimal patent litigation strategy α^* is given by

$$\alpha^* = -\frac{\sigma_X \phi (P_X(O, X) + P_O(O, X))}{\zeta}. \tag{20}$$

- intangible investment-total capital ratio $\gamma = \Gamma / O$ then satisfies the following first-order condition:

$$\gamma^* = \frac{P_O(O, X) + P_X(O, X) + \lambda - 1}{\xi - \sigma_\gamma^2 O[P_{OO}(O, X) + 2P_{OX}(O, X) + P_{XX}(O, X)]}. \tag{21}$$

Patent War under Imperfect Patent Protection

- the first-order condition for physical investment-total capital ratio $i = I/O$ is given by

$$i^* = \frac{P_O(O, X) - 1}{\theta}, \tag{22}$$

- I rewrite the firm value as

$$P(O, X) = O \cdot p(x), \tag{23}$$

Patent War under Imperfect Patent Protection

B. Patent Wall Region

- For $x > \overline{x}$, we have the following equation for $p(x)$:

$$p(\overline{x}) = p(x) - xp'(x), \quad x > \overline{x}. \tag{24}$$

- The condition for the endogenous upper boundary $\overline{x}$ is given by:

$$p'(\overline{x}) = 0. \tag{25}$$

- The following "super contact" condition must be held at this point:

$$p''(\overline{x}) = 0. \tag{26}$$

Patent War under Imperfect Patent Protection

C. Liquidation Region

- The firm value can be written as

$$P(0)O = lO \tag{27}$$

- By exploiting homogeneity, we obtain:

$$p(0) = l, \tag{28}$$

Parameter Choice

Table: **Summary of Key Variables and Parameters**
This table summarizes the key variables and parameter values for our baseline analysis. Whenever applicable, parameter values are annualized.

Parameters	Symbol	Value
Risk-free rate	r	4%
Rate of depreciation for total capital	δ	10.07%
Patent protection parameter	ζ	10
Risk-neutral mean exogenous productivity shock	μ	9.8%
Volatility of exogenous imitation shock	σ_X	9%
Physical Investment adjustment cost parameter	θ	3
Intangible investment adjustment cost parameter	ξ	15
technology choice parameter	λ	1.192
Volatility of endogenous intangible investment shock	σ_γ	10%
Firm liquidation value	l	0.9
adjustment parameter to rivals' action	ϕ	1.61

Patent War, Misallocation, and Capital Reallocation

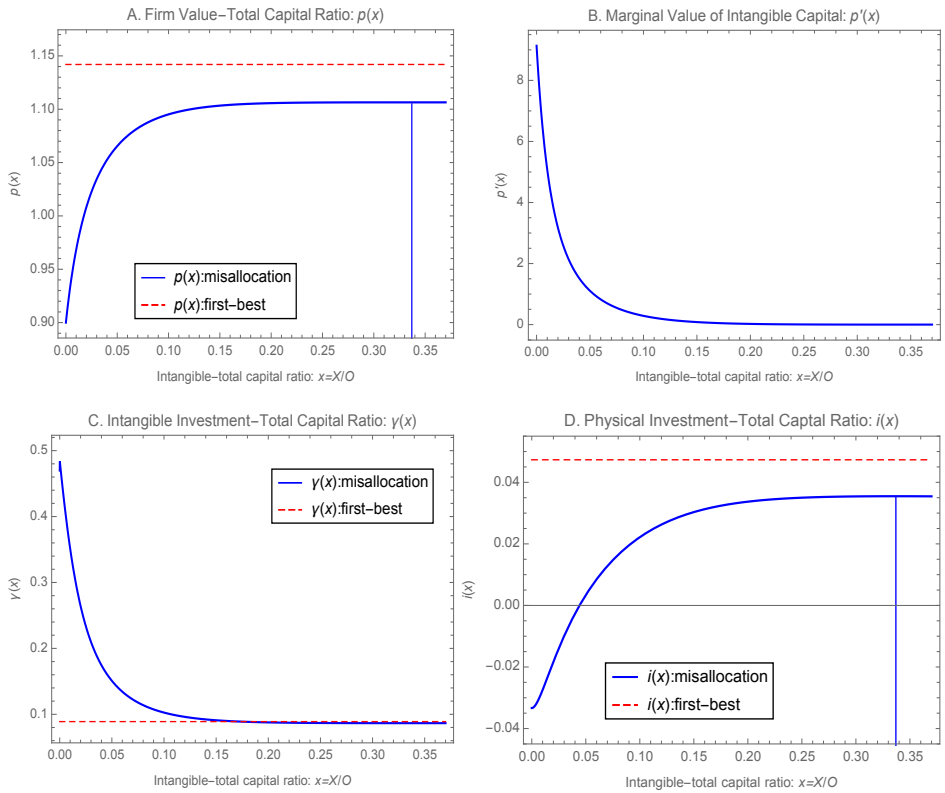


Figure: **Misallocation and Reallocation.** This figure plots the solution for the internal capital misallocation case in which the firm has to liquidate when it runs out of intangible capital.

Patent Risk Management

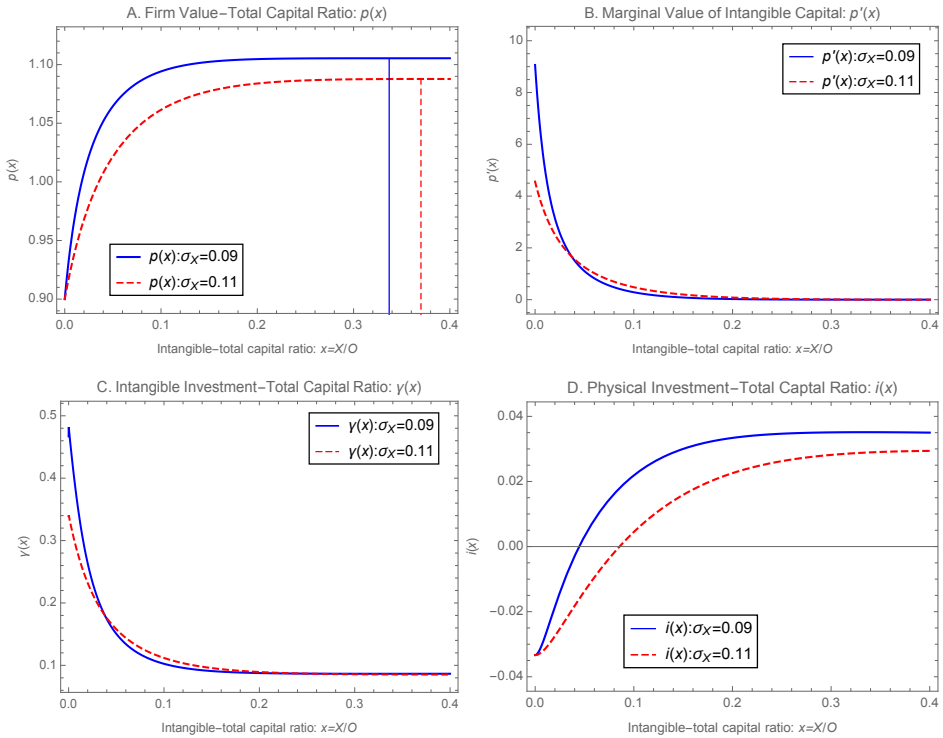


Figure: **Risk management effect.** This figure plots the solution for the internal capital misallocation case in which the firm makes different decisions for the various strength of imitation shock.

Patent Wall

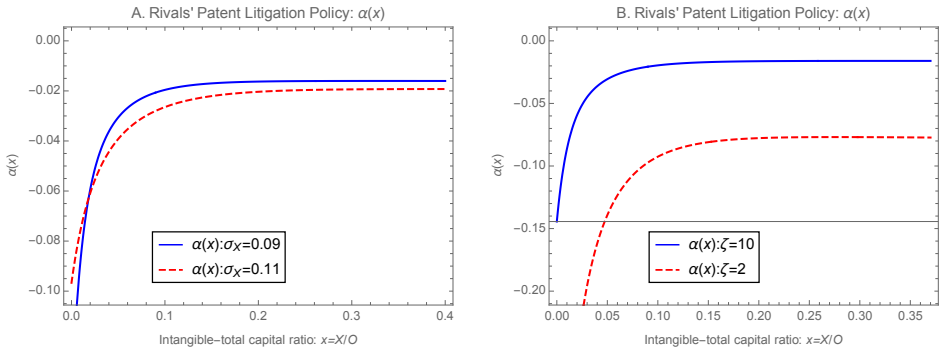


Figure: **Patent wall.** This figure plots the solution for the rivals' patent litigation policy in which the rivals make different decisions for the various strength of imitation shock and various imperfect degree of patent protection.

Conclusion

- Patent portfolio management naturally becomes a central issue for many firms
- Patent risk provides a novel rationale for corporate risk management policies.
- This paper helps explain in particular why imperfect patent systems can induce two opposite consequences of corporate innovation behavior simultaneously that broadly exist in the current empirical studies
- The central importance of the capital intangibility for corporate decisions when patent protection is imperfect