Democratic Aggregation: Issues and Implications for Consumer Price Indexes

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Outline

1. Introduction
2. Group Price Index Theory
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Overview

- Current Consumer Price Index (CPI) methodology geared to measure inflation from macroeconomic perspective
  - ▸ Prices and expenditure weights for a "representative consumer"
  - ▸ Budget shares matches the "average dollar" of expenditure
- Different from inflation experienced by the average household?
  - ▸ Among other potential differences, like consumption vs. payments
- Main finding: Aggregation differences are three times larger when using Tornqvist formula than when using modified Laspeyres (Lowe)

Selected Literature

- Price index theory: Prais (1958), Pollak (1989), ILO/IMF/OECD/UNECE/Eurostat/World Bank (2004), Ley (2005)

- Individual and subgroup heterogeneity: Garner, et. al. (1996), Cage, Garner, and Ruiz-Castillo (2002), McGranahan and Paulson (2005, Chicago Fed IBEX), Hobijn and Lagakos (2005), Hobijn, et. al. (2009), Jaravel (2018, 2021), Kaplan and Shulhofer-Wohl (2017), Cage, et. al. (2018)

- **Common finding: Plutocratic-Democratic gap tends to be small when using Lowe (a.k.a. modified Laspeyres) formula**

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Group Price Indexes

- Standard price index formulas use market-level prices and expenditures (i.e., a "representative consumer")
  - Formulas used by BLS
    - Lowe/modified Laspeyres: For CPI-U, fixed biennial weights
    - Tornqvist: For Chained CPI-U, updating monthly weights

- These implicitly give more weight to households with higher expenditure

- Democratic price indexes give equal weight to each household
  - Building blocks are household-specific price indexes

Household Price Indexes

- Items $i = 1, \ldots, N$; households $h = 1, \ldots, H$
- Time: months $t = 1, \ldots;$
  - For Lowe: b is biennial weight reference period, v is pivot month
- Quantities q_{ith}, prices p_{it}
  - <u>Data limitation</u>: assume households face common p_{it}
- Expenditure shares
  - For Lowe: $s_{i\{v,b\}h} = p_{iv}q_{ibh} / \sum_{i=1}^{N} p_{iv}q_{ibh}$
  - For Tornqvist: $s_{ith} = p_{it}q_{ith} / \sum_{i=1}^{N} p_{it}q_{ith}$

Household Price Indexes (2)

- Lowe: $P_{Lo,h} = \sum_{i=1}^{N} s_{i\{v,b\}h} \frac{p_{it}}{p_{iv}}$

- Tornqvist: $P_{T,h} = \prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}}\right)^{w_{ith}}, w_{ith} = .5\left(s_{i,t-1,h} + s_{ith}\right)$

- In application, p_{it}/p_{iv} and $p_{it}/p_{i,t-1}$ represent elementary CPI for 211 items and 32 geographic areas

- Household share of total expenditure

  - For Plut. Lowe: $S_{\{v,b\}h} = \sum_{i=1}^{N} p_{iv}q_{ibh} / \sum_{h=1}^{H} \sum_{i=1}^{N} p_{iv}q_{ibh}$

  - For Plut. Torn: $S_{th} = \sum_{i=1}^{N} p_{it}q_{ith} / \sum_{h=1}^{H} \sum_{i=1}^{N} p_{it}q_{ith}$

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Democratic and Plutocratic Price Indexes

	Lowe	Tornqvist
Plutocratic	$$P_{PLo} = \sum_{h=1}^{H} S_{\{v,b\}h} P_{Lo,h}$$ $$= \sum_{i=1}^{N} s_{i\{v,b\}} \frac{p_{it}}{p_{iv}},$$ $$s_{i\{v,b\}} = \sum_{h=1}^{H} S_{\{v,b\}h}\, s_{i\{v,b\}h}$$	$$P_{PT} = \prod_{h=1}^{H}\prod_{i=1}^{N}\left(\frac{p_{it}}{p_{i,t-1}}\right)^{w_{ith}^{*}}$$ $$= \prod_{i=1}^{N}\left(\frac{p_{it}}{p_{i,t-1}}\right)^{w_{it}},$$ $$w_{ith}^{*} = .5\left(S_{t-1,h}\,s_{i,t-1,h} + S_{th}\,s_{ith}\right), w_{it} = \sum_{h=1}^{H} w_{ith}^{*}$$
Democratic	$$P_{DLo} = \frac{1}{H}\sum_{h=1}^{H} P_{Lo,h}$$ $$= \sum_{i=1}^{N} \bar{s}_{i\{v,b\}} \frac{p_{it}}{p_{iv}},$$ $$\bar{s}_{i\{v,b\}} = \frac{1}{H}\sum_{h=1}^{H} s_{i\{v,b\}h}$$	$$P_{DT} = \frac{1}{H}\sum_{h=1}^{H} P_{T,h}$$

The Plutocratic Gap

■ Gaps derive from differences in how price changes are weighted

▶ Lowe Plut. Gap: $P_{PLo} - P_{DLo} = \sum_{i=1}^{N}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})\frac{p_{it}}{p_{iv}}$

▶ Torn. Plut. Gap: $\ln(P_{PT}) - \ln(P_{DT}) \approx \sum_{i=1}^{N}(w_{it} - \bar{w}_{it})\ln\left(\frac{p_{it}}{p_{i,t-1}}\right) - \zeta^2$

– $\bar{w}_{it} = \frac{1}{H}\sum_{h=1}^{H} w_{ith}$, ζ is the coeff. of var. of $P_{T,h}$ across h, captures use of arithmetic mean instead of geometric mean across h

■ Ley (2005): Gap determined by how spending patterns vary by household expenditure level and how they covary with price changes

Methods Overview

- Using CPI elementary indexes, Consumer Expenditure Survey (CE) for second-stage aggregation weights
  - Key limitation: same elementary price indexes (211 items) used for each household. Only variation is by 32 geographic areas
- CE = two independent surveys: Interview and Diary
- For Tornqvist, the democratic average is over the one-month index links, which are then chained together as in C-CPI-U
- For Lowe, to better mimic biennial weighting of CPI-U, using only households who completed four interviews

Interview-Diary Matching

- Interview households: Records for 3-12 months
  - **Roughly 75% of CPI expenditures:** most of Housing, Transportation, Medical Care, Education and Communication
- Diary households: Records for 1-2 weeks
  - **Roughly 25% of CPI expenditures:** Most of Food and Beverages, Apparel. Significant portion of Recreation and Other Goods
- For each Interview-month, select one Diary which has similar demographic characteristics, where similarity is based on predicted expenditures (similar to Hobijn, et. al. 2009)
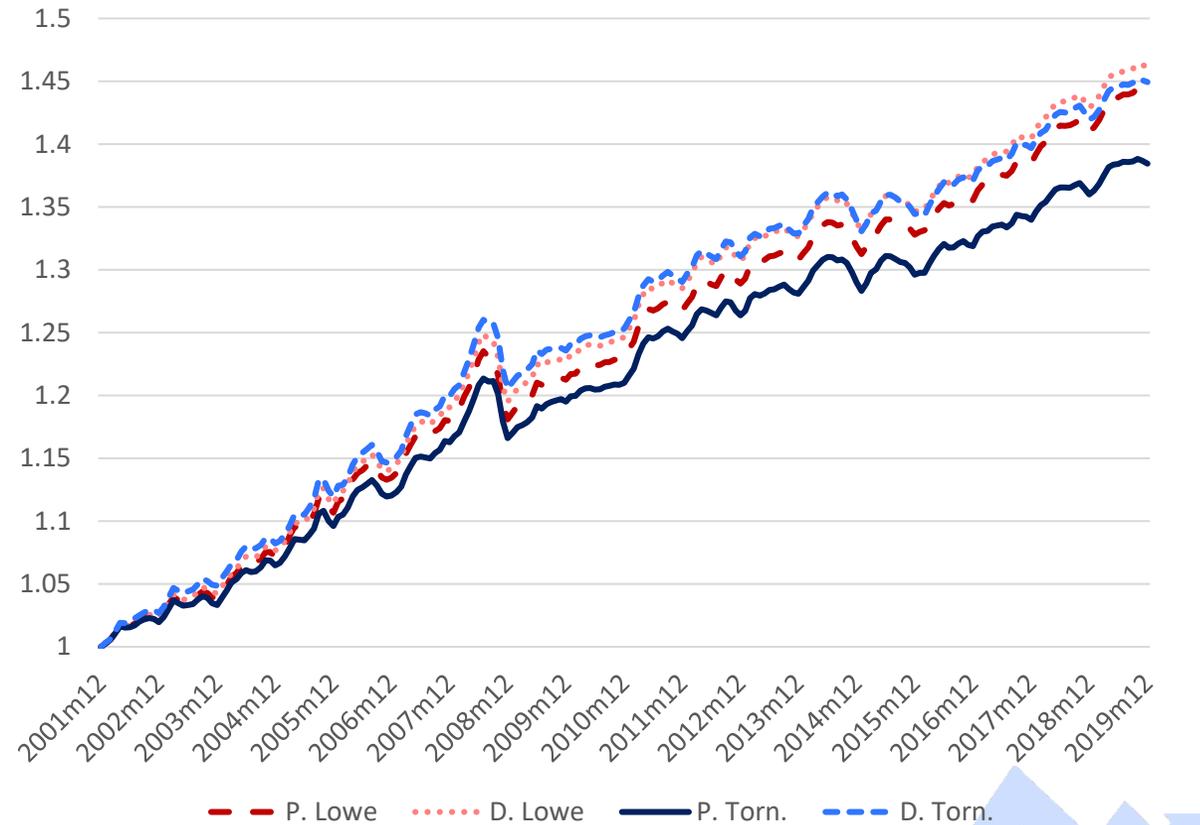
Results

Indexes for the Urban Population

Avg. 12 mo. Percent Change
(Dec. 2002-Dec. 2019)

Index	Estimate
Plutocratic Lowe	2.055%
Democratic Lowe	2.132%
Difference, Lowe	-0.077%
Plutocratic Tornqvist	1.831%
Democratic Tornqvist	2.073%
Difference, Tornqvist	-0.242%
Official Indexes	
CPI-U (Lowe)	2.099%
C-CPI-U (Torn.)	1.845%

Index Levels (Dec. 2001 = 1.0)

Legend: - - - P. Lowe · · · · · D. Lowe ——— P. Torn. - - - D. Torn.

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Tornqvist Aggregation Differences Over Time

Avg. 12 mo. % ch., Urban

	P. Torn.	D. Torn	Diff
2001-04	1.910%	2.389%	-0.479%
2005-08	3.016%	3.508%	-0.492%
2009-12	1.540%	1.568%	-0.028%
2013-16	0.941%	0.959%	-0.018%
2017-19	1.679%	1.916%	-0.237%

Gap in Tornqvist 12 mo. % ch., Urban

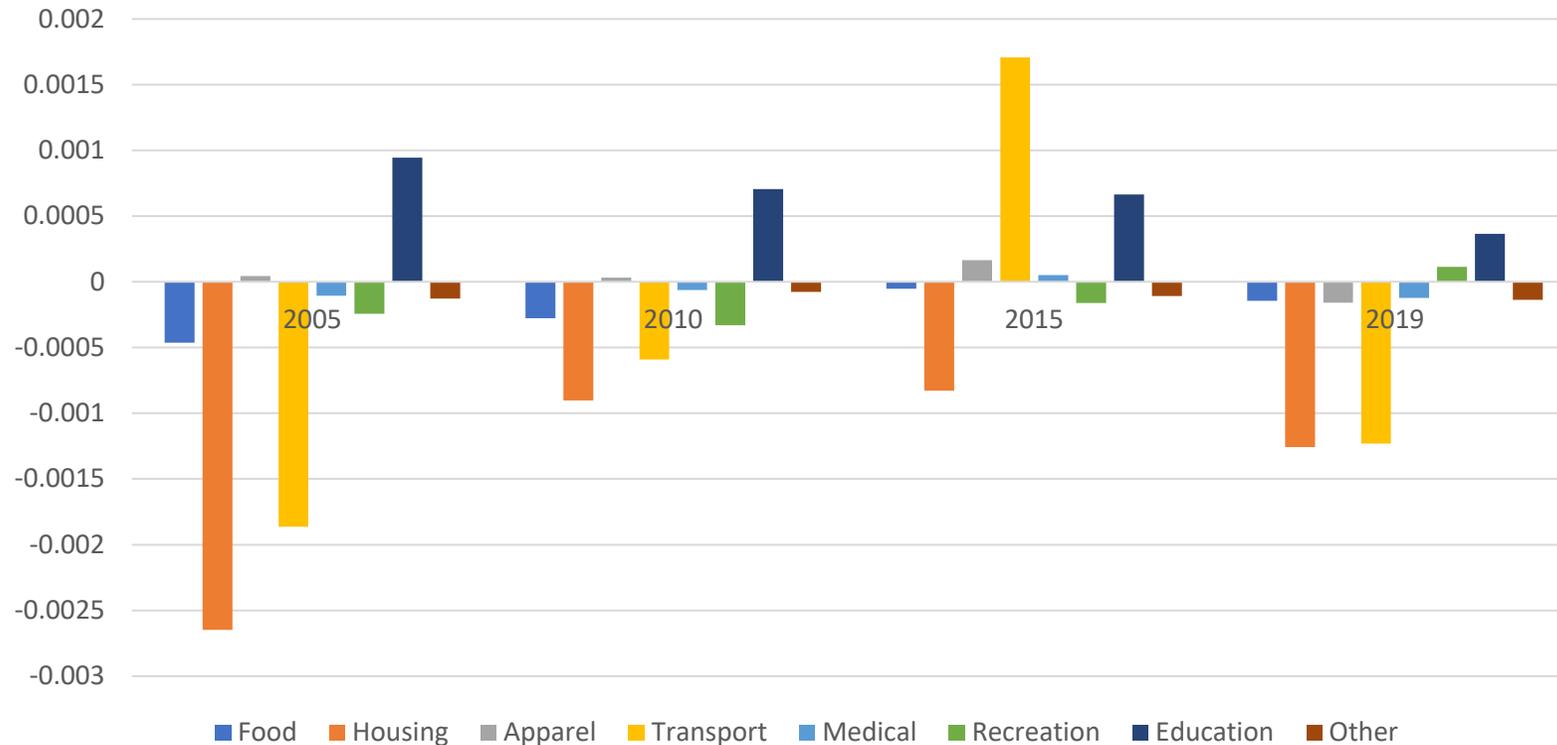P. Torn. - D. Torn.   95% CI (prelim.)
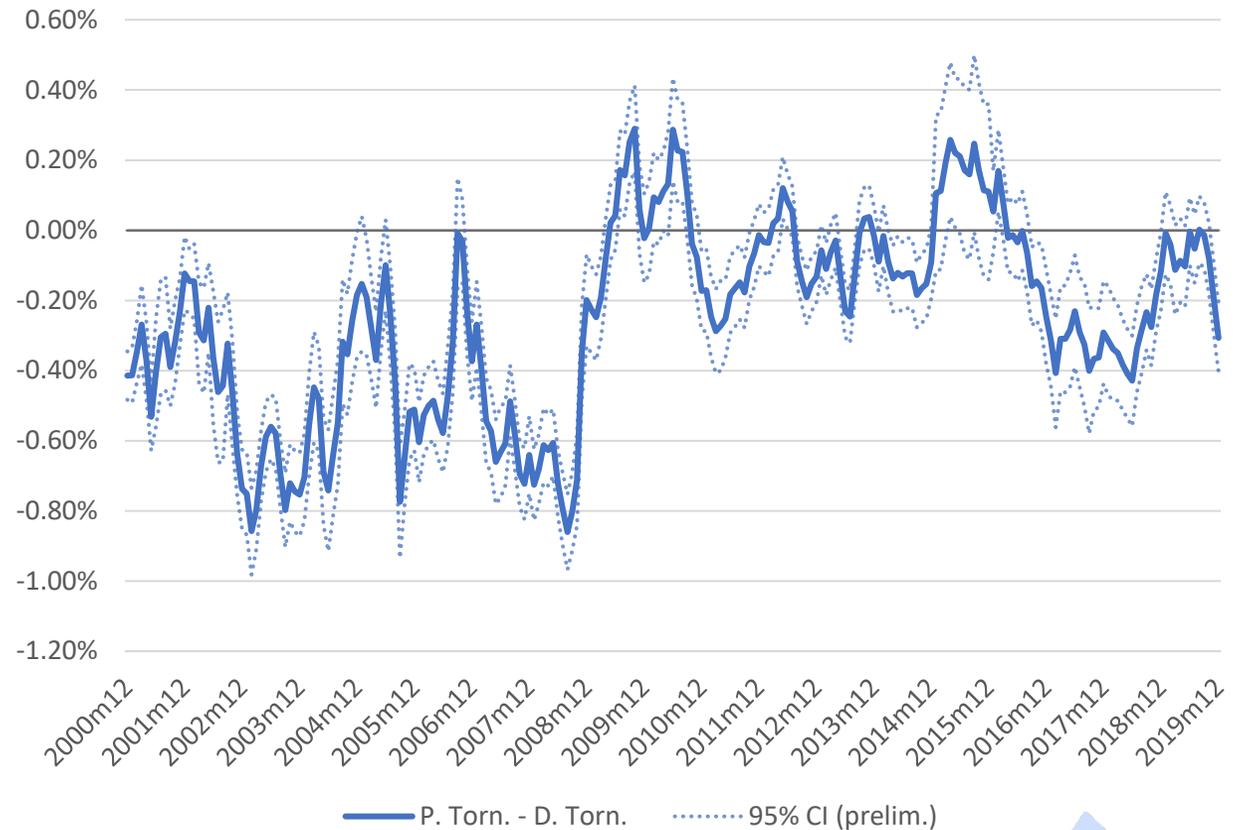
Note: Margins of error are preliminary and conditional on elementary item-area indexes.

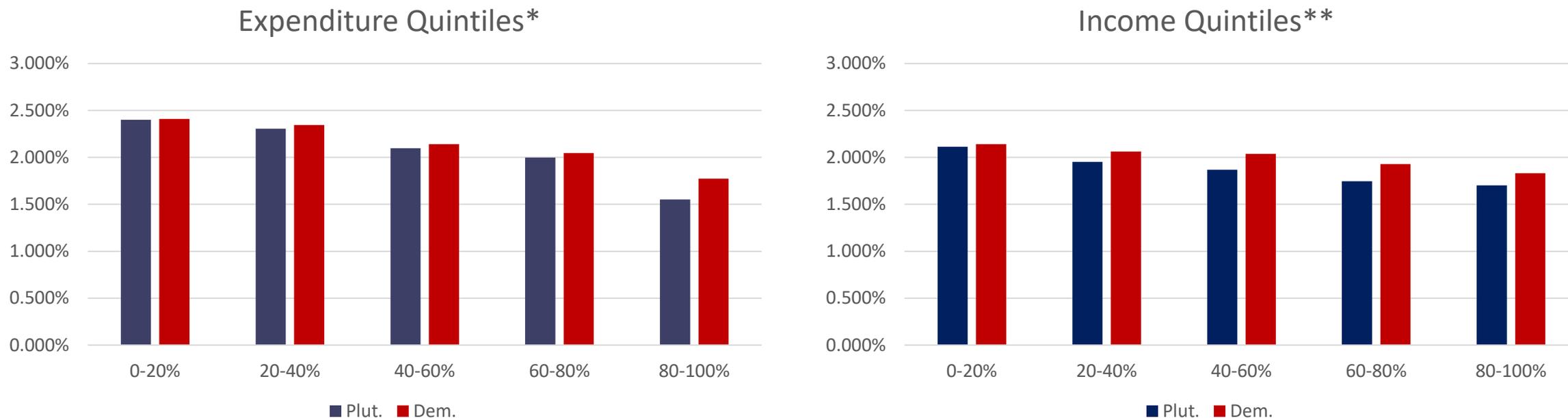# Contributions to Tornqvist Plutocratic Gap

By CPI Major Group (December 12 mo. log-change)

Bars represent $\sum_{t=Jan.}^{Dec.}\sum_{i \in MG}(w_{it} - \overline{w}_{it}) \ln\left(\frac{p_{it}}{p_{i,t-1}}\right)$ over items in the major group

Tornqvist Indexes for Subgroups

Average 12-month Percent Change

Expenditure Quintiles*

Income Quintiles**

*Averages cover Dec. 2000 – Dec. 2019. Quintiles based on total CPI-eligible expenditures of CE Interview households in the index month

**Averages cover Jan. 2005 – Dec. 2019. Quintiles based before-tax income of CE Interview households in the prior year. CE started imputing missing income in 2004.

Summary and Conclusions

- Equally-weighting household has (on average) three times the impact when using the Tornqvist formula versus the Lowe
  - ▶ Similar results using only Interview expenditures or geo. mean across h
  - ▶ The gap is still small relative to overall dispersion in $P_{T,h}$
- The Tornqvist gap has trended smaller over time, but has ticked up recently
- Housing and Transportation are the most important categories in determining the gap

Future Research

■ This is work in progress, so comments are welcome

■ Ongoing: more practical approximations to democratic index (i.e., an average of indexes by expenditure quintile)

■ Ongoing: impact of democratic aggregation on sampling variation

■ Long-term: missing "within-stratum" heterogeneity (e.g., in prices paid) is likely very important (Jaravel, 2021)

Thank you!

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