Monetary Policy and Endogenous Financial Crises

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Abstract

Should a central bank deviate from its price stability objective to promote financial stability? We study this question through the lens of a textbook New Keynesian model augmented with capital accumulation and search–for–yield behaviors that give rise to endogenous financial crises. We compare several interest rate rules, under which the central bank responds more or less forcefully to inflation and output. Our main findings are fourfold. First, monetary policy affects the probability of a crisis both in the short run (through aggregate demand) and in the medium run (through capital accumulation). Second, the central bank can both reduce the probability of a crisis and increase welfare by departing from strict inflation targeting and responding systematically to fluctuations in output. Third, using monetary policy as a backstop can prevent credit market collapses and further improves welfare. Fourth, financial crises may occur after a long period of unexpectedly loose monetary policy as the central bank abruptly reverses course.

Keywords: Monetary policy, financial crisis, search for yield.

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> "*While monetary policy may not be quite the right tool for the job, it has one important advantage relative to supervision and regulation —namely that it gets in all of the cracks.*" (Stein (2013))

> "*Swings in market sentiment, financial innovation, and regulatory failure are acknowledged sources of instability, but what about monetary policy? Can monetary policy create or amplify risks to the financial system? If so, should the conduct of monetary policy change? These questions are among the most difficult that central bankers face.*" (Bernanke (2022), page 367)

1 Introduction

The impact of monetary policy on financial stability remains a controversial subject. On the one hand, loose monetary policy can help stave off financial crises. In response to the 9/11 terrorist attacks and Covid–19 pandemic, for example, central banks swiftly lowered interest rates and acted as a backstop to the financial sector. These moves likely prevented a financial collapse that would otherwise have exacerbated the damage to the economy. On the other hand, empirical evidence shows that, by keeping their policy rates too low for too long, central banks may entice the financial sector to search for yield and feed macro–financial imbalances.[1] Loose monetary policy is thus sometimes regarded as one of the causes of the 2007–8 Great Financial Crisis (GFC). Taylor (2011), in particular, refers to the period 2003–2005 in the US as the "Great Deviation", which he characterises as one when monetary policy became less rule–based, less predictable, and excessively loose.

This ambivalence prompts the question of the adequate monetary policy in an environment where credit markets are fragile and financial stress may have varied causes.[2] *What are the channels through which monetary policy affects financial stability? Should central banks deviate from their objective of price stability to promote financial stability? To what extent may monetary policy itself brew financial vulnerabilities?* We study these questions through the lens of a New Keynesian (NK) model that features endogenous financial crises, *i.e.* crises that arise after an adverse non–financial shock or a protracted boom. The mechanics of financial crises present in our model are well–documented (see, among others, Gorton (2009), Brunnermeier (2009), Shin (2010), Griffin (2021), Mian and Sufi (2017)): when interest rates are low, borrowers tend to "search for yield", in the sense that they seek to boost their profits by leveraging up and investing in projects that are both socially inefficient and risky from the point of view of lenders. Beyond a certain point, default risk may be so high that prospective lenders refuse to lend, triggering a sudden collapse of credit markets.

[1] Empirical evidence on the potential adverse effects of loose monetary policy on financial stability can be found in *e.g.*, Maddaloni and Peydró (2011), Jiménez, Ongena, Peydró, and Saurina (2014).

[2] The Federal Reserve and European Central Bank (ECB)'s recent strategy reviews both emphasize that the importance of financial stability considerations in the conduct of monetary policy has increased since the GFC (Goldberg, Klee, Prescott, and Wood (2020), ECB (2021), Schnabel (2021b)).

As we focus on the effects of monetary policy we purposely abstract from other policies. Our intention is not to argue that other, *e.g.* macro–prudential, policies are not effective or should not be used to mitigate financial stability risks. Rather, it is to understand better how monetary policy can by itself create, amplify, or mitigate risks to the financial system.[3] Our model should therefore be taken as a benchmark, a first step toward richer models.

Our starting point is the textbook three–equation NK model, in which we introduce the possibility that firms search for yield and credit markets collapse. To do so, we depart from the textbook model in a few and straightforward ways. First, we assume that firms are subject to idiosyncratic productivity shocks —in addition to the usual aggregate ones. This heterogeneity gives rise to a credit market where productive firms borrow funds to buy capital from unproductive firms and the latter lend the proceeds of the sales of their capital goods. The credit market thus supports the reallocation of capital from unproductive to productive firms. Second, we assume two standard financial frictions that make this credit market prone to runs. The first friction is limited contract enforceability: prospective lenders may not be able to seize the wealth of a defaulting borrower, allowing firms to borrow and abscond. This possibility induces lenders to constrain the amount of funds that each firm can borrow. The second financial friction is that idiosyncratic productivities are private information. Together, these frictions imply that the loan rate must be above a minimum threshold to entice unproductive firms to sell their capital stock and lend the proceeds, rather than borrow and abscond in search for yield. When the marginal return on capital is too low, not even the most productive firms can afford paying this minimum loan rate and the credit market collapses. This is what we call a financial crisis. Crises are characterised by capital mis–allocation and a severe recession.[4] The third departure from the textbook NK model is that we allow for endogenous capital accumulation. As a consequence, the economy may deviate persistently from its steady state and expose itself to investment booms and capital overhang. Financial stress and crises have varied causes in our model: they may

[3] Despite the progress made since the GFC, macro–prudential policies are generally still perceived as not offering full protection against financial stability risks, not least due to the rise of market finance and non–bank financial intermediation (Woodford (2012), Stein (2013, 2021), Schnabel (2021a), Bernanke (2022)).

[4] Even though our model's financial block is cut to the bone, note that it still captures the essence of the role of the financial sector in the economy: (i) its usual role of transferring resources across periods and channelling savings to investment and (ii) its role of reallocating resources among agents, toward the most productive ones —as in Eisfeldt and Rampini (2006). Regarding the latter role, our narrative in terms of intra–period–inter–firm financial transactions should not be taken at face value. Rather, it should be interpreted more broadly as capturing the range of financial transactions and markets that help to reallocate initially mis–allocated resources, such as short term wholesale loan, commercial paper, or leasing markets. Two additional and related comments are in order. First, the financial frictions considered here (limited contract enforceability and asymmetric information) are standard —"textbook"— ones (see, *e.g.*, Freixas and Rochet (1997), Tirole (2006), Stiglitz and Weiss (1981), Mankiw (1986), Gertler and Rogoff (1990)) and not specific to a particular type of financial transaction or market. Second, even though our baseline narrative involves dis–intermediated financial transactions, one could add a layer of financial intermediaries standing in–between borrowers and lenders. For example, productive firms could borrow from banks to buy capital goods from unproductive firms, and unproductive firms could deposit the proceeds of the sales in the banks. As long as the latter face the same agency problem as unproductive firms, adding them would not change our results (see Section 7.1). Conferring banks an advantage over unproductive firms in lending activities (*e.g.* a better knowledge of borrowers) would amount to relaxing or removing financial frictions, and would eliminate the possibility of financial crises (see Sections 7.3 and 9.1). *E contrario*, adding financial frictions, for example a "double–decker" agency problem between depositors and banks (Holmstrom and Tirole (1997)), would certainly enrich our model and help refine the transmission channels that we uncover, but is not necessary to obtain financial crises. We therefore leave such extensions for future work.

occur after a large negative shock but also after a protracted investment boom without the economy experiencing a (large) shock —the latter cause being more common in our simulations.[5] Finally, we solve our model globally in order to capture the non–linearities embedded in the endogenous booms and busts of the credit market.[6]

We then study whether monetary policy can tame such booms and busts and, more generally, whether a central bank should deviate from its objective of price stability to promote financial stability. In the process, we compare the performance of the economy under simple linear interest rate rules, non–linear rules, and monetary policy discretion.

Our main findings are fourfold.

First, monetary policy affects the probability of a crisis not only in the short run through its usual effects on output and inflation, but also in the medium run through its effects on capital accumulation over the business cycle. In particular, policies that systematically dampen output fluctuations tend to slow down the accumulation of savings during booms. The lower saving rate stems excess capital accumulation and helps prevent financial crises. As these effects go through agents' expectations, they require that the central bank commit itself to a policy rule, and only materialize themselves in the medium run.

Second, a central bank can increase welfare by departing from strict inflation targeting (henceforth, SIT) and responding systematically to fluctuations in output. For example, we show that welfare may be higher under a Taylor–type rule, whereby the central bank responds to both inflation and output, than under SIT, even though the latter policy is optimal in the absence of financial frictions.

Third, we discuss the welfare gain of following more complex monetary policy rules, whereby the central bank commits itself to doing whatever it takes whenever needed to forestall crises. Such backstop policy requires to lower the policy rate below the level prescribed by the Taylor–type rule —and therefore to tolerate higher inflation— during periods of financial stress. We show that doing so significantly improves welfare. We also discuss the tradeoff between normalising monetary policy quickly —at the risk of triggering a crisis— and slowly —at the risk of unnecessarily high inflation, as well as the adequate speed of monetary policy normalisation. We show that the latter can be faster when the cause of financial stress is a short–lived exogenous negative shock than when it is a protracted investment boom.[7]

[5]In our model, the end of an investment boom may be associated with excess capital and a low marginal productivity of capital. Mian, Straub, and Sufi (2021) propose another mechanism that associates excess savings with low rates of return due to the difference in borrowers' and savers's marginal propensities to save out of permanent income.

[6]The presence of endogenous financial crises augments not only the richness of our model but also its complexity. This means that even though our model is parsimonious, we must still solve it both numerically and globally to compute the rational expectation dynamic general equilibrium. Put differently, we are able to solve our model numerically precisely because it is parsimonious. This in turn allows us to reveal the rich dynamics of financial crises. One example of such dynamics (that we will describe in more detail later) is the effect of the anticipation of a crisis in the near future on today's macro–economic outcome.

[7]One noticeable and novel feature of our model is that it accounts for the dual role of monetary policy (i) as a tool to achieve price stability and (ii) as a tool to restore financial market functioning. It also captures the

Fourth, we study the effects of discretionary monetary policy interventions, *i.e.* deviations from a Taylor–type rule, on financial stability. We show that financial crises may occur after a long period of unexpectedly loose monetary policy, and break out when the central bank abruptly reverses course by raising its policy rate.

The paper proceeds as follows. Section 2 sets our work in the literature. Section 3 describes our theoretical framework, with a focus on the microfoundations of endogenous financial crises, and describes the channels through which monetary policy affects financial stability. Section 4 presents the parametrization of the model as well as the average macroeconomic dynamics around financial crises. Section 5 revisits the "divine coincidence" result and analyses the tradeoff between price and financial stability. In Section 6, we study whether a central bank should commit itself to forestall crises, as well as the effect of monetary policy surprises on financial stability. In Section 7 we show that our results carry over to alternative versions of our model —including one with banks. A last section concludes.

2 Related Literature

Our main contribution is to study central banks' tradeoff between price and financial stability while taking into account the endogeneity and varied causes of financial distress.

As we do so, we bridge two strands of the literature. The first is on monetary policy and financial stability. Like Woodford (2012) and Gourio, Kashyap, and Sim (2018), we introduce endogenous crises in an otherwise standard NK framework.[8] The main difference is that they assume specific and reduced form relationships to describe how macro–financial variables (*e.g.* credit gap, credit growth, leverage) affect the likelihood of a crisis, whereas in our case financial crises —including their probability and size— are micro–founded and derived from first principles. This has important consequences in terms of our model's properties. One is that monetary policy influences not only the crisis probability but also the size of the recessions that typically follow crises, and therefore the associated welfare cost. Another is that, even though crises can be seen as credit booms "gone wrong", as documented in Schularick and Taylor (2012), not all booms are equally conducive to crises (Gorton and Ordoñez (2019), Sufi and Taylor (2021)) —a key element to determine how hard to lean against booms. More generally, our findings do not hinge on any postulated reduced functional form for the probability or size of a crisis. In this sense, ours can be seen as a fairly general framework that provides micro–foundations to the approaches in Woodford (2012), Gourio, Kashyap, and Sim (2018) and Svensson (2017).

The second strand of the literature relates to quantitative macro–financial models with

potential tensions between these two objectives. Examples of such tensions include the 2013 "taper tantrum" in the US and, more recently, the Bank of England's sudden purchases of government bonds to address market tumult amidst its monetary policy tightening cycle in September 2022 (see BoE press release of 29 September 2022).

[8]See Smets (2014) for a review of the literature as well as Bernanke and Gertler (2000), Galí (2014), Filardo and Rungcharoenkitkul (2016), Svensson (2017), Cairó and Sim (2018), Ajello, Laubach, López-Salido, and Nakata (2019).

micro–founded endogenous financial crises.[9] Ours complements existing work (*e.g.* Gertler and Kiyotaki (2015), Gertler, Kiyotaki, and Prestipino (2019), Fontanier (2022)) in that it focusses on the fragility of financial markets —as opposed to institutions— and emphasises the role of excess savings, low interest rates, and the resulting search for yield —as opposed to collateral constraints— as sources of financial fragility. In this respect, our work is closer to Martinez-Miera and Repullo (2017), who also propose a macroeconomic model that associates the search for yield in a low interest rate environment with moral hazard. In their case, banks are less likely to monitor firms as interest rates go down, whereas in ours firms are more likely to borrow and abscond. Both approaches are motivated by extensive anecdotal and empirical evidence of a rise in moral hazard (Ashcraft and Schuermann (2008), Brunnermeier (2009)) and various kinds of fraudulent behavior (Griffin (2021), Mian and Sufi (2017), Piskorski, Seru, and Witkin (2015)) in the run–up to the GFC.[10]

Our paper also belongs to the literature on the transmission of monetary policy in heterogeneous agent New Keynesian (HANK) models. Most existing HANK models focus on household heterogeneity and study the channels through which this heterogeneity shapes the effects of monetary policy on aggregate demand (Guerrieri and Lorenzoni (2017), Kaplan, Moll, and Violante (2018), Auclert (2019), Debortoli and Galí (2021)). In contrast, our model is on the effects of firm heterogeneity (as in Adam and Weber (2019), Manea (2020), Ottonello and Winberry (2020)) and the role of credit markets in channelling resources to the most productive firms.

Though in a more indirect way, our paper is also connected to recent works on how changes in monetary policy rules affect economic outcomes in the medium term (*e.g.* Borio, Disyatat, and Rungcharoenkitkul (2019), Beaudry and Meh (2021)) as well as to works on the link between firms' financing constraints and capital mis–allocation (Eisfeldt and Rampini (2006), Chen and Song (2013)). In particular, the notion that financial crises impair capital reallocation dovetails with the narrative of the GFC in the US and the literature that shows that a great deal of the recession that followed the GFC can be explained by capital mis–allocation (*e.g.* Campello, Graham, and Harvey (2010), Foster, Grim, and Haltiwanger (2016), Argente, Lee, and Moreira (2018), Duval, Hong, and Timmer (2019)).

3 Model

Our model is an extension of the textbook NK model (Galí (2015)), with sticky prices à la Rotemberg (1982) and capital accumulation, where financial frictions give rise to occasional

[9] See Boissay, Collard, and Smets (2016), Gertler, Kiyotaki, and Prestipino (2019), Benigno and Fornaro (2018), Paul (2020), Amador and Bianchi (2021), as well as Dou, Lo, Muley, and Uhlig (2020) for a recent review of the literature.

[10] Adiber and Kindleberger (2015) list the cases of mis–behaviors throughout the history of financial crises and make the point that moral hazard tends to increase toward the end of economic booms (Chapter 7). At the aggregate level, the core concern is not so much the existence of moral hazard in some segments of the financial system *per se* (*e.g.* in the subprime loan market before the GFC), but rather that the fear of being defrauded spread across markets and undermine confidence in —and potentially trigger a run on— the financial system as a whole. Our model captures this idea.

endogenous credit market collapses.

3.1 Agents

The economy is populated with a central bank, a representative household, a continuum of monopolistically competitive retailers, and a continuum of competitive intermediate goods producers (henceforth, "firms"). The only non–standard agents are the firms, which experience idiosyncratic productivity shocks that prompt them to resize their capital stock and participate in a credit market.

3.1.1 Central Bank

The central bank sets the nominal interest rate i_t on the risk–free bond according to the following simple policy rule:[11]

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y} \tag{1}$$

where π_t are Y_t and aggregate inflation and output in period t, and $\overline{Y}$ is the aggregate output in the deterministic steady state. The central bank implicitly targets a zero inflation rate. Throughout the paper, we experiment with different values of ϕ_π and ϕ_y —including a Taylor–type rule with Taylor (1993)'s original parameters (henceforth, TR93), as well as with a SIT rule whereby the central bank sets the policy rate so that $\pi_t = 0$ at all t.

3.1.2 Household Sector

The representative household is infinitely lived. In period t, the household supplies N_t hours of work at nominal wage W_t, consumes a Dixit–Stiglitz consumption basket of differentiated goods $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$, with $C_t(i)$ the consumption of good i purchased at price $P_t(i)$, and invests its savings in risk–free nominal bonds B_t and equity $Q_t(j)$ —in units of the consumption basket— issued by newborn firm j (with $j \in [0,1]$). The household can thus be seen as a venture capitalist providing startup equity funding to intermediate goods producers.

The household maximizes its expected lifetime utility:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \chi\frac{N_t^{1+\varphi}}{1+\varphi}\right)\right]$$

subject to the sequence of budget constraints

$$\int_0^1 P_t(i)C_t(i)\mathrm{d}i + B_t + P_t\int_0^1 Q_t(j)\mathrm{d}j \leq W_tN_t + (1+i_{t-1})B_{t-1} + P_t\int_0^1 D_t(j)\mathrm{d}j + P_t\int_0^1 \Pi_t(i)\mathrm{d}i$$

for $t = 0, 1, ..., +\infty$, where $\mathbb{E}_t(\cdot)$ denotes the expectation conditional on the information set available at the end of period t, $D_t(j)$ is firm j's dividend payout (expressed in final goods), and $\Pi_t(i)$ is retailer i's profit (see next section).[12] The conditions describing the household's optimal

[11] Given that there is no growth trend in our model, the term $Y_t/\overline{Y}$ corresponds to the GDP gap (or detrended GDP) as defined in Taylor (1993)'s seminal paper.

[12] Since firms live only one period, it should be clear that those that issue equity at the end of period t are not the same as those that pay dividends, and therefore that we use the same j index in $Q_t(j)$ and $D_t(j)$ only to economize on notations.

behavior are given by (in addition to a transversality condition):

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \quad \forall i \in [0,1]$$

$$\chi N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{2}$$

$$\beta(1+i_t)\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{1+\pi_{t+1}}\right] = 1 \tag{3}$$

$$\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}(1+r^q_{t+1}(j))\right] = 1 \quad \forall j \in [0,1] \tag{4}$$

where

$$1 + r^q_{t+1}(j) \equiv \frac{D_{t+1}(j)}{Q_t(j)} \tag{5}$$

is firm j's real rate of return on equity and $\pi_{t+1} \equiv P_{t+1}/P_t - 1$ is the inflation rate, with $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon}\mathrm{d}i\right)^{\frac{1}{1-\epsilon}}$ the price of the consumption basket. Since firms are born without resources and *ex ante* identical, the household ultimately invests the same amount Q_t in every firm:

$$Q_t(j) = Q_t \quad \forall j \in [0,1] \tag{6}$$

3.1.3 Retailers

A continuum of infinitely–lived retailers purchase intermediate goods at price p_t, differentiate them, and resell them in a monopolistically competitive environment subject to nominal price rigidities. Each retailer $i \in [0,1]$ sells $Y_t(i)$ units of the differentiated final good i and, following Rotemberg (1982), sets its price $P_t(i)$ subject to adjustment costs $\frac{\varrho}{2}P_tY_t\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2$, where $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$ denotes the aggregate output. The demand for final goods emanates from both the households (who consume), the firms (which use them as inputs to produce intermediate goods), and retailers (who face menu costs). Capital investment goods take the form of a basket of final goods similar to that of consumption goods, implying that firms' demand for final good i at the end of period t is

$$I_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} I_t \quad \forall i \in [0,1] \tag{7}$$

where I_t is aggregate capital investment. Since capital goods are homogenous to consumption goods, they also have the same price P_t. Accordingly, retailer i faces the demand schedule

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \quad \forall i \in [0,1] \tag{8}$$

Each period, retailer i chooses its price $P_t(i)$ so as to maximize its expected stream of future profits:

$$\max_{\{P_t(i)\}_{t=0,\dots,+\infty}} \mathbb{E}_0\left(\sum_{t=0}^{\infty}\Lambda_{0,t}\Pi_t(i)\right)$$

with

$$\Pi_t(i) \equiv \frac{P_t(i)}{P_t}Y_t(i) - \frac{(1-\tau)p_t}{P_t}Y_t(i) - \frac{\varrho}{2}Y_t\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 \tag{9}$$

subject to (8) for $t = 0, \ldots, +\infty$, where $\Lambda_{t,t+k} \equiv \beta^k \left(C_{t+k}/C_t\right)^{-\sigma}$ is the stochastic discount factor between period t and $t+k$ and $\tau = 1/\epsilon$ is a subsidy rate on the purchase of intermediate goods.[13] In the symmetric equilibrium, where $Y_t(i) = Y_t$ and $P_t(i) = P_t$, the optimal price setting behavior satisfies

$$(1+\pi_t)\pi_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} (1+\pi_{t+1})\pi_{t+1} \right) - \frac{\epsilon - 1}{\varrho} \left(1 - \frac{\mathscr{M}}{\mathscr{M}_t} \right) \tag{10}$$

where $\mathscr{M}_t$ is retailers' average markup given by

$$\mathscr{M}_t \equiv \frac{P_t}{(1-\tau)p_t} \tag{11}$$

and $\mathscr{M} \equiv \epsilon/(\epsilon - 1)$ is its (deterministic) steady state value.

3.1.4 Intermediate Goods Producers ("Firms")

The intermediate goods sector consists of overlapping generations of firms that live one period, are born at the end of period $t-1$ and die at the end of period t. Firms are perfectly competitive, and produce a homogeneous good, whose price p_t they take as given. They are identical *ex ante* but face idiosyncratic productivity shocks *ex post*, against which they hedge by borrowing or lending on short term (intra–period) credit markets. As in Bernanke and Gertler (1989), Fuerst (1995), Bernanke, Gertler, and Gilchrist (1999), "generations" in our model should be thought of as representing the entry and exit of firms from such credit markets, rather than as literal generations; a "period" in our model may therefore be interpreted as the length of a financial contract.[14]

Consider firm $j \in [0, 1]$ born at the end of period $t - 1$.

At birth, this firm receives $P_{t-1}Q_{t-1}$ startup equity funding, which it immediately uses to buy K_t units of capital goods. Among the latter, $(1-\delta)K_{t-1}$ are old capital goods that they purchase from the previous generation of firms, where δ is the rate of depreciation (or maintenance cost) of capital, and I_{t-1} are newly produced capital goods. New capital goods are produced instantaneously and one–for–one with final goods, and are homogenous to the old capital goods (net of the depreciation and maintenance cost).[15] All capital goods are therefore purchased at price P_{t-1}, implying that

$$K_t = Q_{t-1} \tag{12}$$

At the beginning of period t, firm j experiences an aggregate shock, A_t, as well as an idiosyncratic productivity shock, $\omega_t(j)$, and has access to a constant–return–to–scale technology

[13] This subsidy corrects for monopolistic market power distortions in the flexible–price version of the model.

[14] The overlapping generation approach is standard in macroeconomic models because it provides a tractable framework for dynamic general equilibrium analysis with firm heterogeneity. In the presence of agency costs, this approach is a way to ignore multi–period financial contracts contingent on past debt repayments (see *e.g.* Gertler (1992) for an example of multi–period contracts in a three–period model). Considering infinitely–lived firms with persistent idiosyncratic productivity shocks would raise the question of their reputation but not materially change our analysis and results (see Section 7.2).

[15] Hence, $K_t = (1-\delta)K_{t-1} + I_{t-1}$. Given that firms live only one period, the inter–temporal decisions regarding capital accumulation within the intermediate good sector are, in effect, taken by the households —their shareholders.

represented by the production function

$$X_t(j) = A_t(\omega_t(j)K_t(j))^\alpha N_t(j)^{1-\alpha} \tag{13}$$

where $K_t(j)$ and $N_t(j)$ denote the levels of capital and labor that firm j uses as inputs conditional on the realization of $\omega_t(j)$ and A_t, and $X_t(j)$ is the associated output. The idiosyncratic shock $\omega_t(j) \in \{0,1\}$ takes the value 0 for a fraction μ of the firms ("unproductive firms") and 1 for a fraction $1-\mu$ of the firms ("productive firms").[16] We denote the set of unproductive firms by $\Omega_t^u \equiv \{j \mid \omega_t(j) = 0\}$ and that of productive firms by $\Omega_t^p \equiv \{j \mid \omega_t(j) = 1\}$. The aggregate shock A_t evolves randomly according to a stationary AR(1) process $\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a$ with $|\rho_a| < 1$ and $\varepsilon \rightsquigarrow N(0, \sigma_a^2)$.

Upon observing $\omega_t(j)$, firm j may resize its capital stock by purchasing or selling capital goods on a secondary capital goods market.[17] To fill any gap between its desired capital stock $K_t(j)$ and its initial (and predetermined) one, K_t, firm j may borrow or lend on a credit market. The latter thus operates in lockstep with the secondary capital goods market. If $K_t(j) > K_t$, firm j borrows and uses the proceeds to buy capital goods. If $K_t(j) < K_t$, it instead sells capital goods and lends the proceeds to other firms.

Let r_t^c denote the real rate on the credit market, and consider a firm j that buys $K_t(j) - K_t$ (if $K_t(j) > K_t$) or sells $K_t - K_t(j)$ (if $K_t(j) < K_t$) capital goods, hires labor $N_t(j)$, and produces intermediate goods $X_t(j)$. Then, at the end of the period, this firm sells its production $X_t(j)$ to retailers at price p_t, pays workers the unit wage W_t, sells its un–depreciated capital $(1-\delta)K_t(j)$ at price P_t, and pays $P_t(1+r_t^c)(K_t(j) - K_t)$ to the lenders (or receives $P_t(1+r_t^c)(K_t - K_t(j))$ from borrowers if $K_t(j) < K_t$). Since firm j distributes its revenues as dividends, one obtains

$$P_t D_t(j) = p_t A_t(\omega_t(j)K_t(j))^\alpha N_t(j)^{1-\alpha} - W_t N_t(j) + (1-\delta)P_t K_t(j) - P_t(1+r_t^c)(K_t(j) - K_t) \tag{14}$$

for all $j \in [0,1]$. Implicit in (14) is the assumption that capital depreciates at the same rate δ (or must be maintained at the same cost) when firm j does not produce —*i.e.* keeps its capital stock idle— as when it does.[18] Using (5), (6), and (11)–(14), one can express firm j's real rate of return on equity as

$$r_t^q(j) \equiv \frac{D_t(j)}{K_t} - 1 = \frac{X_t(j)}{(1-\tau)\mathscr{M}_t K_t} - \frac{W_t}{P_t}\frac{N_t(j)}{K_t} - (r_t^c + \delta)\frac{K_t(j) - K_t}{K_t} - \delta \qquad \forall j \in [0,1] \tag{15}$$

The objective of firm j is to maximize $r_t^q(j)$ with respect to $N_t(j)$ and $K_t(j)$. We present the maximization problem of unproductive and productive firms in turn.

[16] Apart from its parsimony, one advantage of the Bernouilli distribution is that the effects of financial frictions on capital allocation only kick in during financial crises, not in normal times (as we show later). Outside of crisis times, the capital stock is therefore used efficiently. This property is appealing because it allows us to isolate the effects in normal times of agents' anticipation of a crisis and to pin down the externalities associated with excess precautionary savings (see Section 9.3). In earlier versions of the model, we considered a continuous distribution of $\omega_t(j)$ instead of a Bernouilli distribution. In that case, financial frictions also affect capital allocation in normal times but only marginally so, and our results are practically unchanged.

[17] Following Eisfeldt and Rampini (2006), we refer to this market as a secondary market because trades take place after the realization of the shocks.

[18] This assumption implies that the marginal return on capital of a productive firm is always strictly higher than that of an unproductive firm, as relation (19) shows.

Choices of an Unproductive Firm. It is easy to see that unproductive firms all take the same decisions and choose $N_t(j) = 0$, $X_t(j) = 0$, and $K_t(j) = K_t^u$, for all $j \in \Omega_t^u$, where the optimal adjusted capital stock K_t^u will be determined later, as we solve the equilibrium of the credit market (see Section 3.2). Using (15), firm j's maximization problem can be written as

$$\max_{K_t^u} r_t^q(j) = r_t^c - (r_t^c + \delta)\frac{K_t^u}{K_t} \qquad \forall j \in \Omega_t^u \tag{16}$$

where the first term is the return from selling capital and lending the proceeds and the second term is the opportunity cost of keeping capital idle.

Choices of a Productive Firm. Productive firms all take the same decisions, and choose $N_t(j) = N_t^p$, $X_t(j) = X_t^p$, and $K_t(j) = K_t^p$ for all $j \in \Omega_t^p$, where the optimal labour demand N_t^p satisfies the first order condition

$$\frac{W_t}{P_t} = \frac{(1-\alpha)X_t^p}{(1-\tau)\mathscr{M}_t N_t^p}$$

and will be determined later, along with the optimal adjusted capital stock K_t^p. Using (13), the above condition can be rewritten as

$$\Phi_t \equiv \frac{\alpha X_t^p}{K_t^p} = \alpha A_t^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{(1-\tau)\mathscr{M}_t\frac{W_t}{P_t}}\right)^{\frac{1-\alpha}{\alpha}} \tag{17}$$

where Φ_t denotes the marginal product of capital for a productive firm. The last term in relation (17) emphasizes that Φ_t is a function of the real wage W_t/P_t and retailers' markup $\mathscr{M}_t$ and is, therefore, taken as given by firm j. Using (17), firm j's maximization problem and real rate of return on equity in (15) can be written as

$$\max_{K_t^p} r_t^q(j) = r_t^c + \left(r_t^k - r_t^c\right)\frac{K_t^p}{K_t} \qquad \forall j \in \Omega_t^p \tag{18}$$

where

$$r_t^k \equiv \frac{\Phi_t}{(1-\tau)\mathscr{M}_t} - \delta > -\delta \tag{19}$$

denotes the marginal return on capital (net of depreciation) for a productive firm —and is also taken as given by firm j.

3.2 Market Clearing

We first consider the benchmark case of a frictionless credit market, where the idiosyncratic productivity shocks can be observed by all potential investors, and where financial contracts are fully enforceable, with no constraint on the amount that a firm can borrow. Then, we introduce financial frictions.

3.2.1 Frictionless Credit Market

Absent financial frictions, productive firms borrow and purchase capital as long as $r_t^c < r_t^k$ and until they break even. In equilibrium, one therefore obtains that $r_t^c = r_t^k > -\delta$, implying that $r_t^q(j) = r_t^k$ for all $j \in \Omega_t^p$ (see (18)). Since $r_t^c > -\delta$, the mass μ of unproductive firms sell their entire capital stock K_t to the mass $1-\mu$ of productive firms, implying that $K_t^u = 0$ and $r_t^q(j) = r_t^k$ for all $j \in \Omega_t^u$ (see (16)), and

$$K_t^p = \frac{K_t}{1-\mu} \tag{20}$$

In this economy, all capital goods are always perfectly reallocated and used productively (see Figure 9.1 in the appendix). The model then boils down to the textbook NK model with one representative intermediate goods firm.

3.2.2 Frictional Credit Market

Next, consider the case of financial frictions arising from limited debt contract enforceability and asymmetric information. We assume that a firm has the possibility to hide its idle capital from its creditors, to sell this hidden capital at the end of the period, and to abscond with the proceeds of the sale.[19] This possibility opens the door to moral hazard: the firm may boost its profit by borrowing, purchasing more capital, and absconding. It follows that no firm can credibly commit itself to paying back its debt. However, we assume that, when it defaults, the firm incurs a cost that is equal to a fraction $\theta \geq 0$ of the funds borrowed, where parameter θ reflects the cost of hiding from creditors.[20] Further, we assume that creditors do not observe a given firm j's productivity $\omega_t(j)$, and hence cannot assess its incentives to borrow and default. As Proposition 1 shows, these frictions put an upper bound on the leverage of any individual firm.[21]

Proposition 1. ***(Firms' Borrowing Limit)*** *A firm cannot borrow and purchase more than a fraction ψ_t of its initial capital stock:*

$$\frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \max\left\{\frac{r_t^c + \delta}{1-\delta-\theta}, 0\right\}$$

Proof. Suppose that an unproductive firm were to mimic a productive firm by borrowing and purchasing $K_t^p - K_t \geq 0$ capital goods, and then keep its capital stock K_t^p idle, resell it at the end of the period, and default. In this case, the firm would incur a hiding cost $P_t\theta(K_t^p - K_t)$ proportional to its loan, and its implied payoff would be $P_t(1-\delta)K_t^p - P_t\theta(K_t^p - K_t)$. That firm will not abscond as long as this payoff is smaller than the return, $P_t(1+r_t^c)K_t$, from selling its

[19] The assumption here is that the proceeds from the sales of capital goods at the end of period t can only be concealed if the capital goods have not been used for production. One can think of the firms that produce and sell intermediate goods as firms that operate transparently, and whose revenues can easily be seized by creditors. In contrast, the firms that keep their capital idle have the possibility to "go underground" and default, which limits the enforceability of financial contracts.

[20] The higher θ, the less stringent the contract enforcement problem. In Section 4.1, we parameterise μ and θ jointly so that simulations of the model can replicate both the size of and time spent in financial crises —and therefore the overall cost of crises— observed in the data.

[21] The opportunity cost of absconding is higher for productive than for unproductive firms, which therefore have more incentive to default. Since firm productivity is private information and unproductive firms may pretend they are productive, productive firms can only commit themselves to paying back their debt if they limit its amount. Such a combination of limited contract enforceability and asymmetric information is standard in the macro–finance literature (Gertler and Rogoff (1990), Azariadis and Smith (1998), Boissay, Collard, and Smets (2016)) and needed here to cause the credit market to occasionally collapse (see the discussion in Section 7.3).

entire capital stock and lending the proceeds —which is its best alternative option. Proposition 1 follows from the incentive compatibility condition $(1-\delta)K_t^p - \theta(K_t^p - K_t) \leq (1+r_t^c)K_t$. $\square$

As long as the condition in Proposition 1 is satisfied, unproductive firms will refrain from borrowing and absconding.[22] Importantly, the borrowing limit ψ_t *increases* with r_t^c: the higher the loan rate, the higher unproductive firms' opportunity cost of absconding, hence the higher the incentive–compatible leverage.

We are now in the position to construct the loan supply and demand schedules (see Figure 1). Unproductive firms are the natural lenders. Given relations (16) and Proposition 1, their aggregate credit supply, denoted $L^S(r_t^c)$, reads:

$$L^S(r_t^c) = \mu\left(K_t - K_t^u\right) = \begin{cases} \mu K_t & \text{for } r_t^c > -\delta \\ [0, \mu K_t] & \text{for } r_t^c = -\delta \\ 0 & \text{for } r_t^c < -\delta \end{cases} \tag{21}$$

When $r_t^c > -\delta$, the mass μ of unproductive firms sell their capital stock K_t and lend the proceeds on the credit market, implying $L^S(r_t^c) = \mu K_t$. When $r_t^c = -\delta$, they are indifferent between lending or keeping their capital idle, implying $L^S(r_t^c) \in [0, \mu K_t]$. When $r_t^c < -\delta$, they keep their capital stock K_t idle: $L^S(r_t^c) = 0$. Similarly, productive firms are the natural borrowers. Their aggregate credit demand, denoted $L^D(r_t^c)$, is given by (using (18) and Proposition 1):

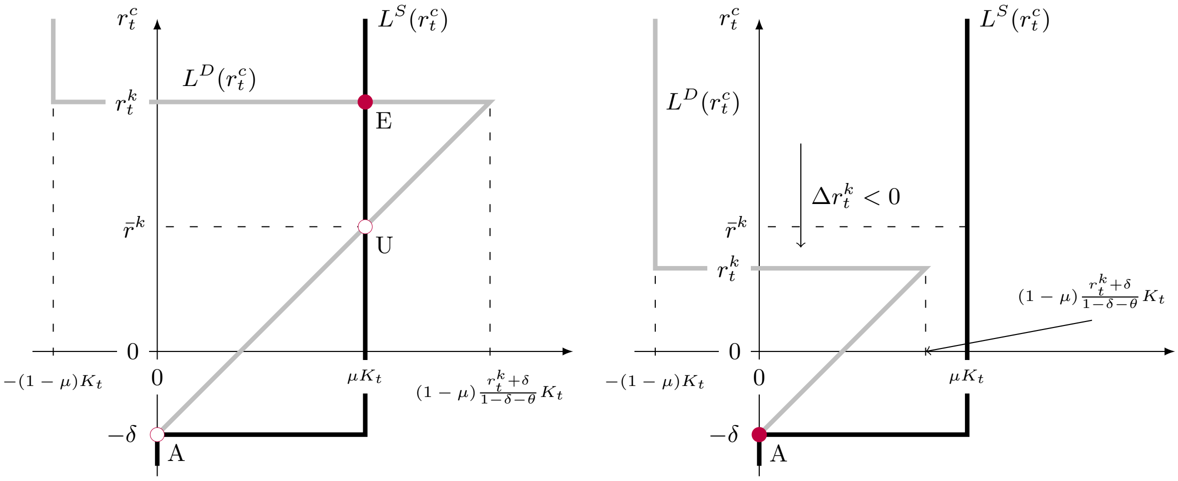
$$L^D(r_t^c) = (1-\mu)\left(K_t^p - K_t\right) = \begin{cases} -(1-\mu)K_t & \text{for } r_t^c > r_t^k \\ [-(1-\mu)K_t, (1-\mu)\psi_t K_t] & \text{for } r_t^c = r_t^k \\ (1-\mu)\psi_t K_t & \text{for } r_t^c < r_t^k \end{cases} \tag{22}$$

The $1-\mu$ productive firms borrow only if the net return from expanding their capital stock is positive. When $r_t^c > r_t^k$, they prefer to sell their capital and lend the proceeds rather than borrow: $L^D(r_t^c) = -(1-\mu)K_t$. When $r_t^c = r_t^k$, they are indifferent but may each borrow up to ψ_t as determined in Proposition 1, implying $L^D(r_t^c) \in [-(1-\mu)K_t, (1-\mu)\psi_t K_t]$. When $r_t^c < r_t^k$, they borrow up to the limit, so that $L^D(r_t^c) = (1-\mu)\psi_t K_t$.

[22] Even though default will be an out–of–equilibrium outcome, the mere possibility that firms abscond is the source of financial instability. This feature is consistent with the conventional wisdom that lenders' *fear* of being defrauded is more detrimental to the stability of the whole financial system than *actual* fraud and defaults *per se*, which often concern specific market segments (*e.g.* subprime mortgages) or players (*e.g.* rogue traders) and are typically small in the aggregate.

Figure 1: Credit Market Equilibrium

(a) Normal Times (b) Crisis Times



Note: This figure illustrates unproductive firms' aggregate supply on the credit market (black) and productive firms' incentive–compatible aggregate credit demand (gray) curves. In Panel (a), the demand curve is associated with a value of r_t^k strictly above $\bar{r}^k$ and multiple equilibria A, E, and U. In this case, U and A are ruled out on the ground that they are unstable (for U) and Pareto–dominated (for A). In Panel (b), the demand curve is associated with a value of r_t^k strictly below $\bar{r}^k$ and A as unique equilibrium. The threshold for the loan rate, $\bar{r}^k$, is constant and corresponds to the minimum incentive–compatible loan rate that is required to ensure that every unproductive firm sells its entire capital stock and lends the proceeds.

Proposition 2. ***(Credit Market Equilibrium)*** *An equilibrium with trade exists if and only if*

$$r_t^k \geq \bar{r}^k \equiv \frac{(1-\theta)\mu - \delta}{1-\mu}$$

Proof. From Panel (a), it is clear that an equilibrium with trade exists if and only if there is a range of interest rates for which demand exceeds supply, *i.e.* $\lim_{r_t^c \nearrow r_t^k} L^D(r_t^c) \geq \lim_{r_t^c \nearrow r_t^k} L^S(r_t^c)$. Proposition 2 follows. □

The interest rate threshold $\bar{r}^k$ is the minimum return on investment that guarantees the existence of an equilibrium with trade.[23] When condition in Proposition 2 holds, productive firms can afford paying the required loan rate, and there exist three possible equilibria, denoted by E, U, and A in Figure 1. In what follows, we focus on equilibria A and E which, unlike U, are stable under tatônnement.[24] When the condition in Proposition 2 does not hold, A is the only possible equilibrium. We describe equilibria A and E in turn.

Consider equilibrium A (for "Autarky"), where $r_t^c = -\delta$. At that rate, unproductive firms are indifferent between keeping their capital idle or selling it and lending the proceeds. Hence, any supply of funds within the interval $[0, \mu K_t]$ is consistent with optimal firm behavior. However,

[23] It can also be seen as the minimum loan rate that is required to permit and entice *every* unproductive firm to lend on the credit market —rather than borrow and default.

[24] We rule out equilibrium U because it is not tatônnement–stable. An equilibrium rate r_t^c is tatônnement–stable if, following any small perturbation to r_t^c, a standard adjustment process —whereby the loan rate goes up (down) whenever there is excess demand (supply) of credit — pulls r_t^c back to its equilibrium value (see Mas-Colell, Whinston, and Green (1995), Chapter 17). Since firms take r_t^c as given, tatônnement stability is the relevant concept of equilibrium stability. Note nonetheless that U and E yield the same aggregate outcome and overall rate of return on equity $\int_0^1 r_t^q(j)\mathrm{d}j$, and only differ in terms of the distribution of individual returns $r_t^q(j)$ across firms.

the incentive compatible amount of funds that can be borrowed at that rate is zero ($\psi_t = 0$). As a result, $L^D(-\delta) = L^S(-\delta) = 0$ and there is no trade and no capital reallocation, implying that $K_t^u = K_t^p = K_t$. In what follows, we refer to this autarkic equilibrium as a "financial crisis".

Equilibrium E, in contrast, features a loan rate $r_t^c = r_t^k \geq \bar{r}^k > -\delta$, at which every unproductive firm sells capital to productive firms, as if there were no financial frictions. In that case, there is perfect capital reallocation, with $K_t^u = 0$ and $K_t^p = K_t/(1-\mu)$ (as in relation (20)). We refer to this equilibrium as "normal times".

Finally, consider what happens when productive firms' return on capital, r_t^k, falls below the threshold $\bar{r}^k$, so that the condition in Proposition 2 is not satisfied anymore. This is illustrated in Panel (b) of Figure 1. In this case, the range of loan rates for which $L^D(r_t^c) > L^S(r_t^c)$ vanishes altogether, and only the autarkic equilibrium A survives.

In the rest of the paper, we assume that when equilibria A and E coexist, market participants coordinate on the most efficient one, namely, equilibrium E.[25] As a result, a crisis breaks out if and only if A is the only possible equilibrium, *i.e.* if and only if the condition in Proposition 2 does not hold.

3.2.3 Other Markets

As only productive firms hire labor and produce, the labor, and intermediate goods markets clear when

$$N_t = \int_{j \in \Omega_t^p} N_t(j)\mathrm{d}j = (1-\mu)N_t^p \tag{23}$$

$$Y_t = \int_{j \in \Omega_t^p} X_t(j)\mathrm{d}j = (1-\mu)X_t^p \tag{24}$$

and the final goods market clears when

$$Y_t = C_t + I_t + \frac{\varrho}{2}Y_t\pi_t^2$$

where the last term corresponds to aggregate menu costs.

3.3 Equilibrium Outcome

The level of aggregate output depends on the equilibrium of the credit market. In normal times, the entire capital stock of the economy is used productively and, given K_t and N_t, aggregate output is the same as in an economy without financial frictions (in our case, the textbook NK economy):

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \tag{25}$$

In crisis times, in contrast, unproductive firms keep their capital idle, only a fraction $1-\mu$ of the economy's aggregate capital stock is used productively, and aggregate productivity falls. For

[25]There are of course several —but less parsimonious— ways to select the equilibrium. For example, one could introduce a sunspot, *e.g.* assume that firms coordinate on equilibrium E (*i.e.* are "optimistic") with some constant and exogenous probability whenever this equilibrium exists. It should be clear, however, that the central element of our analysis is Proposition 2 for the existence of E, not the selection of E conditional on its existence. In other terms, our analysis does not hinge on the assumed equilibrium selection mechanism.

the same K_t and N_t, output is therefore lower than in normal times:

$$Y_t = A_t \left((1-\mu)K_t\right)^{\alpha} N_t^{1-\alpha} \tag{26}$$

Even though in normal times the aggregate production function is the same as in an economy with a frictionless credit market, note that N_t and K_t (and therefore output) will in general be higher. The reason is that households tend to accumulate precautionary savings and work more to compensate for the fall in consumption should a crisis break out. All else equal, the mere anticipation of a crisis induces the economy to accumulate more capital in normal times compared to a frictionless economy.

Corollary 1. ***(Monetary Policy and Financial Stability)*** *A crisis breaks out in period t if and only if*

$$\frac{Y_t}{\mathscr{M}_t K_t} < \frac{1-\tau}{\alpha}\left(\frac{(1-\theta)\mu-\delta}{1-\mu}+\delta\right)$$

Proof. Corollary 1 follows directly from Proposition 2 after combining relations (17), (19), (24), and the result that $K_t^p = K_t/(1-\mu)$ in normal times. □

The condition in Corollary 1 makes clear that crises may emerge through a fall in aggregate output (the "Y–channel"), a rise in retailers' markup (the "M–channel"), or excess capital accumulation (the "K–channel"). For example, given a (predetermined) capital stock K_t, a crisis is more likely to break out following a shock that lowers output and/or increases the markup. Such a shock does not need to be large to trigger a crisis, if the economy has accumulated a large enough capital stock. Indeed, when K_t is high, all other things equal, productive firms' marginal return on capital is low, and the credit market is fragile. As we show later, this may happen towards the end of an unusually long economic boom. In this case, even a modest change in Y_t or $\mathscr{M}_t$ may trigger a crisis.

As the above discussion suggests, the central bank can affect the probability of a crisis both in the short and in the medium run. In the short run, it does so through the effect of contemporaneous changes in its policy rate on output and inflation (the Y– and M–channels). For example, assume that the central bank unexpectedly raises its policy rate. On impact, all other things equal, the hike works to reduce aggregate demand and to increase retailers' markups. As a result, firms' marginal return on capital diminishes, which brings the economy closer to a crisis (as shown in Corollary 1). In the medium run, in contrast, monetary policy affects financial stability through its impact on the household's saving behavior and capital accumulation (the K–channel). For example, a central bank that commits itself to systematically and forcefully responding to fluctuations in output (*i.e.* to a high ϕ_y) will typically slow down capital accumulation during booms, and thereby improve the resilience of the credit market in the face of adverse shocks.

4 Anatomy of a Financial Crisis

Our model features various types of crises, whose origins may *a priori* range from an extreme adverse technology shock to a protracted investment boom.[26] The aim of this section is to describe the "average" dynamics around financial crises under a realistic parametrization of the model. As we shall see, the average crisis is in effect a mix of the above two polar types of crises.

4.1 Parametrization of the Model

We parameterize our model based on quarterly data (see Table 1) in the presence of aggregate TFP shocks and under Taylor (1993)'s original monetary policy rule (*i.e.* with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$).[27] The standard parameters of the model take the usual values. The utility function is logarithmic with respect to consumption ($\sigma = 1$). The parameters of labor dis–utility are set to $\chi = 0.814$ and $\varphi = 0.5$ so as to normalize hours to one in the deterministic steady state and to obtain an inverse Frish labor elasticity of 2 —this is in the ballpark of the calibrated values used in the literature. We set the discount factor to $\beta = 0.989$, which corresponds to an annualized average return on financial assets of about 4%. The elasticity of substitution between intermediate goods ϵ is set to 10, which generates a markup of 11% in the steady state. Given this, we set the capital elasticity parameter α to 0.36 in order to obtain a labor income share of 64% in the steady state. We assume that capital depreciates by 6% per year ($\delta = 0.015$). We set the price adjustment cost parameter to $\varrho = 105$, so that the model generates the same slope of the Phillips curve as in a Calvo pricing model with an average duration of prices of 4 quarters. The process of the technology shock is also standard, with $\rho_a = 0.95$ and $\sigma_a = 0.007$.

Compared to the textbook NK model, there are two additional parameters: the share of unproductive firms, μ, and the cost of absconding, θ. Parameter θ implicitly governs the degree of moral hazard and, given μ, the frequency of financial crises (see Proposition 2). We set $\theta = 0.71\%$ so that the economy spends 8% of the time in a crisis in the stochastic steady state.[28] Parameter μ directly affects the cost of financial crises in terms of productivity loss (see relation (26)). For $\alpha = 0.36$, we set $\mu = 1 - 0.97^{\frac{1}{0.36}} = 8.11\%$ so that capital mis–allocation entails a 3% fall in aggregate productivity during a financial crisis.[29]

[26]The stylized graphical representation of the optimal capital accumulation decision rule in Figure 9.2 in the appendix illustrates these polar cases.

[27]In addition to supply shocks, one could naturally also consider demand shocks. One can show that, with a standard parametrization, the presence of demand shocks would affect neither the main tradeoffs relating to financial stability nor our main results. For the sake of parsimony, we therefore work with productivity shocks only and relegate the analysis with both shocks to the appendix (see Section 9.5).

[28]Romer and Romer (2017) and Romer and Romer (2019) provide semiannual series on financial distress in 31 OECD countries and rank the level of financial distress at each point in time from 0 ("no stress") to 15 ("extreme crisis"). Using their data, we compute the average fraction of the time these countries spent in financial distress at or above level 4 ("minor crisis" or worse) over the period 1980-2017, and obtain 8.11%.

[29]Estimates of the fall in TFP during financial crises vary across studies, ranging from 0.8% in Oulton and Sebastiá-Barriel (2016), for a sample of 61 countries over the period 1954–2010, to about 5% in Fernald (2015) for the US during the GFC.

Table 1: Parametrization

Parameter	Target	Value
Preferences		
β	4% annual real interest rate	0.989
σ	Logarithmic utility on consumption	1.000
φ	Inverse Frish elasticity equals 2	0.500
χ	Steady state hours equal 1	0.814
Technology and price setting		
α	64% labor share	0.360
δ	6% annual capital depreciation rate	0.015
ϱ	Slope of the Phillips curve as with Calvo price setting	105.000
ϵ	11% markup rate	10.000
Aggregate TFP shocks		
ρ_a	Persistence	0.950
σ_a	Standard deviation of innovations (in %)	0.700
Interest rate rule		
ϕ_π	Standard quarterly Taylor rule (Taylor (1993))	1.500
ϕ_y		0.125
Financial frictions		
μ	A crisis entails a 3% fall in TFP	0.081
θ	The economy spends 8% of the time in a crisis	0.708

4.2 Average Dynamics Around Financial Crises

To derive the dynamics around the typical crisis, we proceed in two steps. First, we numerically solve our non–linear model using a global method.[30] Second, starting from the stochastic steady state, we feed the model with the productivity shocks, simulate it over 1,000,000 periods, and identify the crises' starting dates as well as the sequences of shocks around them. We then compute the average dynamics 20 quarters around these dates.[31]

For the sake of the exposition, our baseline economy only features TFP shocks as aggregate shocks. We present the results for the full model with both TFP and demand shocks in the appendix (Section 9.5). While the dynamics around crises may vary between the two versions of the model for some variables (*e.g.* prices), the mechanics and main insights of the model do not and are clearer in the baseline version. The average dynamics around crises are reported in Figure 2.

The main results from the analysis is that the average crisis occurs on the heels of a protracted economic boom (Figure 2, Panels (b) and (c)) driven by a long sequence of relatively small positive technology shocks (Panel (a)). Throughout the boom, the economy accumulates capital (Panel (b)), which over time gradually exerts downward pressures on productive firms' marginal

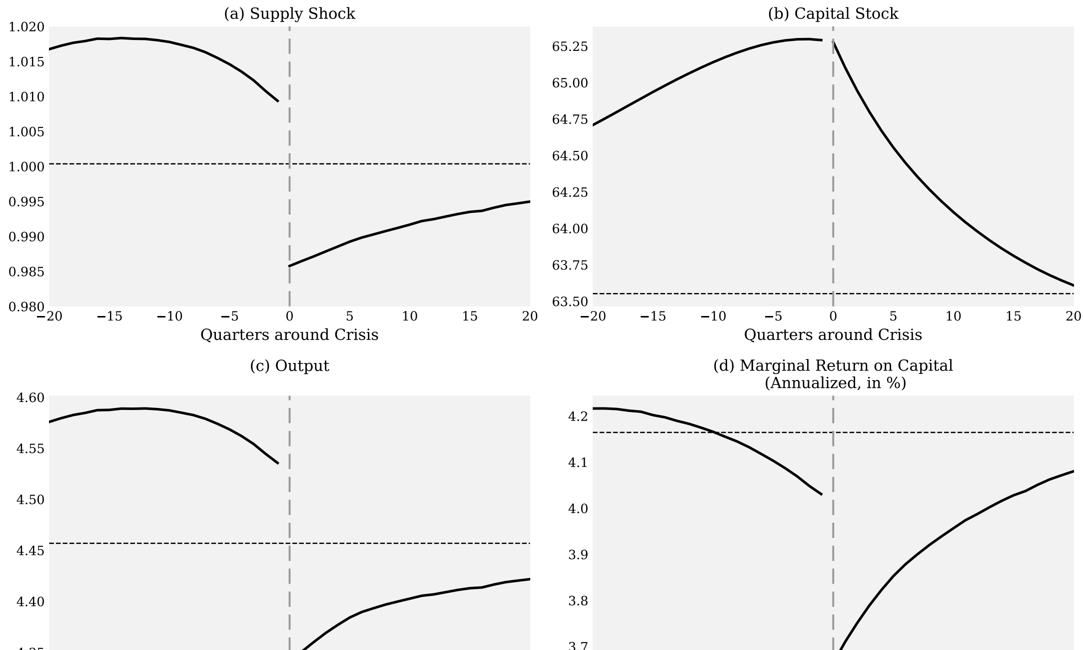
[30] Our model cannot be solved linearly because of discontinuities in the decision rules. It cannot be solved locally because crises may break out when the economy is far away from its steady state (*e.g.* when K_t is high). Details on the numerical solution method are provided in Section 9.7 in the appendix.

[31] Further note that the average dynamics mask the heterogeneity and varied causes of financial crises in our model. While some crises are due to large adverse shocks, others —most— follow a protracted investment boom. See Section 9.2 for a discussion.

return on capital (Panel (d)). At first, these pressures are more than compensated by the productivity gains, the credit market reallocates capital effectively to the most productive firms.

As the sequence of favorable TFP shocks runs its course, productivity recedes and output falls back toward its steady state, leaving firms with excess capital. As a result, firms' marginal return on capital goes down (Panel (d)), firms have greater incentives to borrow and abscond. The average crisis eventually breaks out in the face of an adverse shock (of about three standard deviations) that lowers TFP by around 1.5% (about two standard deviations) below its steady state (Panel (a)). Note that this shock is not the only cause of the crisis, in the sense that the same shock would not have led to a crisis, had the capital stock been lower in the first place. As Corollary 1 suggests, a capital overhang is indeed a pre–condition for a financial crisis to break out without an extreme shock. The crisis is characterised by the collapse of the credit market, capital mis–allocation, and a severe recession (Panel (c)).[32] On average, output falls by 4.94% during a crisis (Table 2).

Figure 2: Average Dynamics Around Crises



Note: Simulations for the TR93 economy. Average dynamics of the economy around the beginning of a crisis (in quarter 0). To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 20 quarters of normal times. The dashed line corresponds to the unconditional average across simulation.

[32]Absent aggregate demand shocks, the fall in TFP and capital mis–allocation sometimes entails inflationary pressures in a crisis (Figure 9.3, Panel (e), black plain line) —but not always (dashed line). This result is an artifact of TFP shocks being the only aggregate shocks in the economy. In an economy with both TFP and demand shocks, the average crisis is in addition associated with a fall in aggregate demand, which mitigates the inflationary pressures (see Figure 9.6). In this case, inflation is still positive but low during crises, which is consistent with the so–called "missing disinflation" during the GFC and, more generally, with Schularick and Taylor (2012)'s finding of small positive inflation during banking crises since WWII. For more details on the version of our model with both TFP and demand shocks, see Section 9.5.

The reason why crises break out even though they lead to an inefficient outcome is that neither the household nor retailers internalize the effects of their individual choices on financial fragility. To hedge against the future recession and smooth consumption, the household tends to accumulate savings, which contributes to increasing capital even further above what would be necessary to avert the crisis. Boissay, Collard, and Smets (2016) refer to this phenomenon as a "savings glut" externality.[33] The upshot is that anticipating the crisis paradoxically induces agents to precipitate —rather than avert— it. The externality calls for policy intervention, which we study next.

5 The "Divine Coincidence" Revisited

We now study whether central banks should account for financial stability risks when setting their policy rate. To do so, we compare welfare under different Taylor–type rules with that under SIT. We use SIT as benchmark because, in the absence of financial frictions, this rule concomitantly eliminates inefficient fluctuations in prices and the output gap and achieves the First Best allocation —the so–called "divine coincidence" (Blanchard and Galí (2007)).[34] We are interested in whether the central bank should depart from this benchmark in an environment where the credit market is fragile.

[33] A similar externality arises from the retail sector. In our baseline model, crises feature some inflation. As a result, retailers anticipate that productivity will fall and prices will rise if a crisis breaks out. To smooth their menu costs over time, they therefore tend to frontload their price increases ahead of a crisis, which contributes to maintaining high markups and further magnifies financial vulnerabilities. These "markup externalities" come on the top of the usual aggregate demand externalities (Blanchard and Kiyotaki (1987)) and are due to the presence of financial frictions. Markup externalities are less present in the full model with both TFP and demand shocks, however, to the extent that, in this version of the model, crises are less (or not) inflationary. Figure 9.4 illustrates the incidence of the savings glut and markup externalities under TR93 in our baseline model. The experiment consists in comparing the dynamics of capital and markups before the average crisis, *i.e.* between quarters -20 and -1, with their dynamics in an economy without financial frictions —and fed with the very same shocks. The latter counterfactual dynamics indicate how capital and markups would have evolved absent financial frictions. Since the credit market functions equally well in the two economies between quarters -20 and -1, the difference pins down the pure effect of crisis expectations. The bottom panels of Figure 9.4 show that the differences are positive, which means that capital and markups increase by more during the boom when the household and retailers anticipate a crisis.

[34] The comparison of welfare under the various policy rules in the absence of financial frictions in Table 2 (column "Frictionless") illustrates this established result.

Table 2: Economic Performance and Welfare Under Alternative Policy Rules

Rule	ϕ_y	Frictionless: Welfare loss (%)	Frictional credit market: Welfare loss (%)	Crisis time	Length (quarter)	Output loss (%)	$\mathbb{E}(\pi_t^2)$
SIT	–	0.0000	0.1114	9.85	5.91	-5.78	0.0000
Taylor rules ($\phi_\pi = 1.5$)	0.025	0.0000	0.1198	10.47	5.94	-5.75	0.0004
	0.050	0.0001	0.1137	9.87	5.80	-5.53	0.0012
	0.125	0.0009	0.0964	[8.00]	5.31	-4.94	0.0064
	0.250	0.0037	0.0706	5.00	4.58	-4.24	0.0200
	0.500	0.0116	0.0466	1.39	3.64	-3.16	0.0516
	0.750	0.0197	0.0467	0.45	4.49	-2.45	0.0817

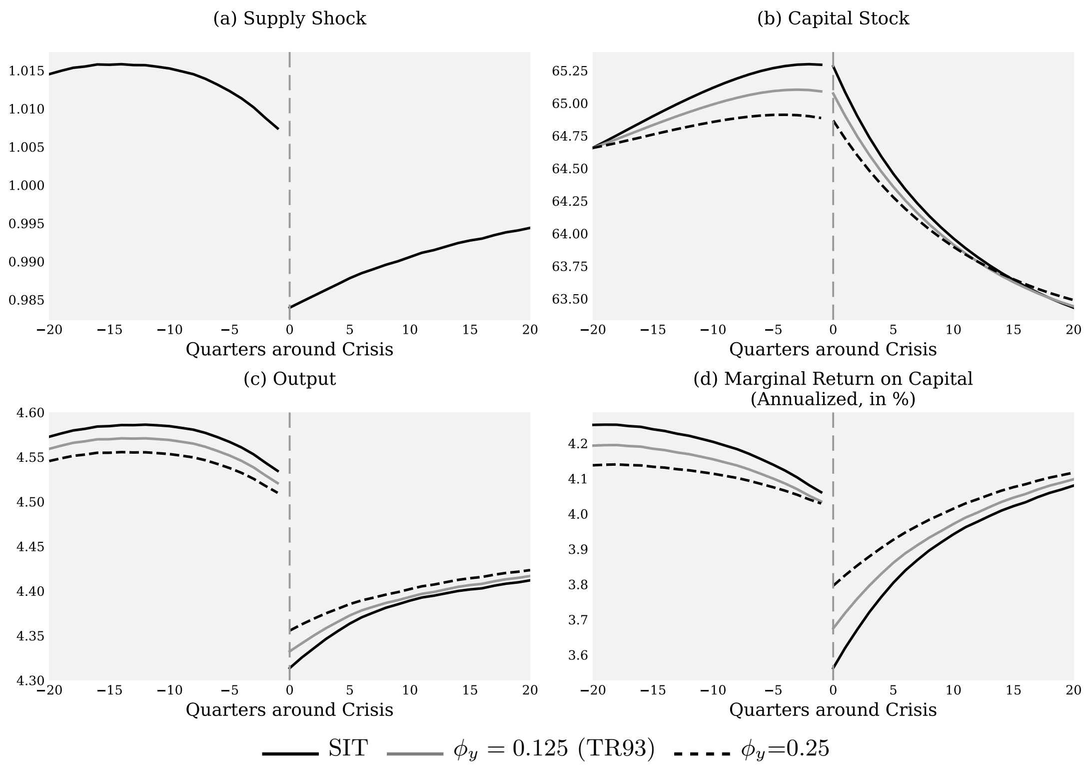
Note: Statistics of the stochastic steady state ergodic distribution. "Crisis time" is the percentage of the time that the economy spends in a crisis (in %). "Length" is the average duration of a crisis (in quarters). "Output loss" is the percentage fall in output from one quarter before the crisis until the trough of the crisis (in %). "Welfare loss" is the loss of welfare relative to the First Best economy, expressed in terms of consumption equivalent variation (in %), *i.e.* corresponds to the percentage of permanent consumption the household should be deprived of in the First Best economy to reach the same level of welfare as in our economy with nominal rigidities and financial frictions. In the case of the frictionless credit market economy (column "Frictionless"), the SIT economy reaches the First Best and there is no welfare loss in this case. In the case of the frictional credit market and the TR93 rule (case with $\phi_y = 0.125$), the economy spends by construction 8% of the time in a crisis (square brackets; see Section 4.1).

As a first step, we establish that SIT does not deliver the first best allocation in that environment, as the welfare loss under SIT amounts to 0.11% in terms of consumption equivalent variation, as shown in Table 2. Since the distortions (relative to the first best) due to sticky prices are fully neutralized under SIT, these welfare losses are entirely due to the cost of financial crises.

Can welfare be raised by following a monetary policy rule other than SIT? If so, this will necessarily come at the cost of deviating from price stability, implying a tradeoff between price and financial stability. To study this question, we report in Table 2 the statistics on the incidence of crises and price volatility in the stochastic steady state of the economy, when the central bank follows Taylor–type rules that differ in terms of the response to fluctuations in output, *i.e.* parameter ϕ_y in relation (1). On the one hand, responding more aggressively to output significantly reduces the time spent in a crisis, from 9.85% under SIT to 0.45% under a Taylor–type rule with $\phi_y = 0.75$ (column "Crisis time"). On the other hand, inflation volatility increases as ϕ_y goes up (last column). All in all, we find that responding to output improves welfare, provided that the central bank responds "enough". For example, the welfare loss is 0.015pp lower under TR93 than under SIT ($0.1114 - 0.0964 = 0.015$, fourth versus first row). This contrasts with the case of a frictionless credit market, where attaching a larger weight to output stabilization is associated with a reduction in welfare (column "Frictionless").

To gain intuition for the above results, we analyse the effects of monetary policy on financial stability. As already noted, these effects play out in both the medium and short run. In the medium run, leaning against investment booms helps reduce the probability of a crisis by reining in the build–up of macro–financial imbalances. Figure 3 illustrates this point by comparing the average dynamics around crises under SIT with the average dynamics of the economies under Taylor–type rules, when the latter are fed with the very same sequences of shocks as those leading to a crisis under SIT (Panel (a)). These counterfactual dynamics show that capital would have been accumulated more slowly had the central bank followed these rules instead of SIT (Panel (b)). The reason is twofold. First, there is the usual effect of the expected rate of return of capital on capital accumulation. As the central bank commits itself to curbing growth, it also lowers investors' expected returns during booms and thus makes capital investment less attractive. Second, by smoothing the business cycle, such a policy in effect provides the household with an insurance against future aggregate shocks, and helps them smooth consumption. This, in turn, reduces the need for precautionary savings and contributes to slowing down the accumulation of capital during expansions, making the economy more resilient. The upshot is that a monetary policy rule with a higher ϕ_y may lower the probability of a crisis. Because capital accumulation takes time, these effects only materialize over multiple years.

Figure 3: Medium Run Effects — Counterfactual Booms



Notes: For SIT: average dynamics around crises. For the two Taylor–type rules with $\phi_y = 0.125$ (TR93) or $\phi_y = 0.25$: counterfactual average dynamics, when the economy starts with the same capital stock in quarter −20 and is fed with the same technology shocks as the SIT economy (Panel (a)).

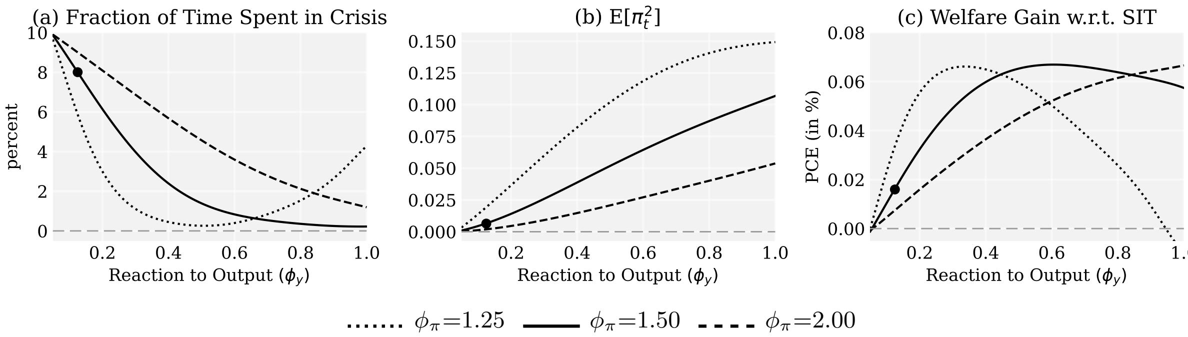
Monetary policy also affects the probability of a crisis through its short run macro–stabilization properties. Indeed, recall that the average crisis is triggered by a relatively mild adverse technology shock. Following such a shock, output falls by less and retailers' markups fall by more under a Taylor–type rule than under SIT, propping up firms' marginal return on capital.[35] All else equal,

[35]Recall that, around the steady state, our model essentially boils down to the textbook NK model with capital accumulation and, therefore, that the response of the economy to small shocks is standard. Figure 9.5 in the

the credit market is therefore more resilient when the central bank mitigates the fall in output.

Next, we are interested in how hard the central bank should lean. The comparison between the last two rows of Table 2 shows that the welfare loss is slightly higher with $\phi_y = 0.75$ than with $\phi_y = 0.5$, suggesting that the marginal welfare gain from leaning is decreasing. To understand this result, we present in Figure 4 the effects of raising ϕ_y on price and financial stability when the central bank follows a Taylor–type rule. For completeness, we also experiment with three different values of ϕ_π. Consider the baseline case with $\phi_\pi = 1.5$ first (plain line). As ϕ_y increases, the central bank stabilizes output and reduces the percentage of time spent in a crisis, which has a positive first order effect on welfare. At the same time, raising ϕ_y works to increase the volatility of markups, as reflected in the increase in the volatility of inflation (Panel (b)). For values of $\phi_y > 0.6$, the latter effect more than offsets the welfare gains associated with financial stability (Panel (c)).

Figure 4: The Price Versus Financial Stability Tradeoff



Notes: Panel (a): Time spent in a crisis (in %, y–axis), as parameter ϕ_y increases (x–axis), for three different loadings on inflation: $\phi_\pi = 1.25, 1.5, 2$. Panel (c): Welfare gain relative to SIT of following the Taylor–type rule considered, expressed in terms of consumption equivalent variation (in %). It corresponds to the percentage of permanent consumption the household should be given under SIT to reach the same level of welfare as under the Taylor–type rule considered. The black dot corresponds to our baseline calibration —where the economy spends 8% of the time in a crisis.

The price versus financial stability tradeoff is even starker when the central bank responds less forcefully to inflation.[36] For example, Panel (b) shows that inflation (and markup) volatility increases faster with ϕ_y when $\phi_\pi = 1.25$ (dotted line) than when $\phi_\pi = 1.50$ (plain line). Panel (a) further shows that, when $\phi_\pi = 1.25$, the percentage of time the economy spends in a crisis at first falls with ϕ_y until $\phi_y \approx 0.5$ and then increases with ϕ_y (dotted line). In those instances, the financial stability gain from slower capital accumulation (K–channel) is more than offset by the financial stability loss due to higher markup volatility (M–channel). This suggests that, beyond a certain threshold, responding more forcefully to output undermines both price and financial stability. Leaning "too hard" (here setting $\phi_y > 0.95$) can even reduce welfare compared to SIT (Panel (c), dotted line).

appendix, which compares the impulse responses to a negative one–standard deviation technology shock under SIT *versus* Taylor–type rules, illustrates this point.

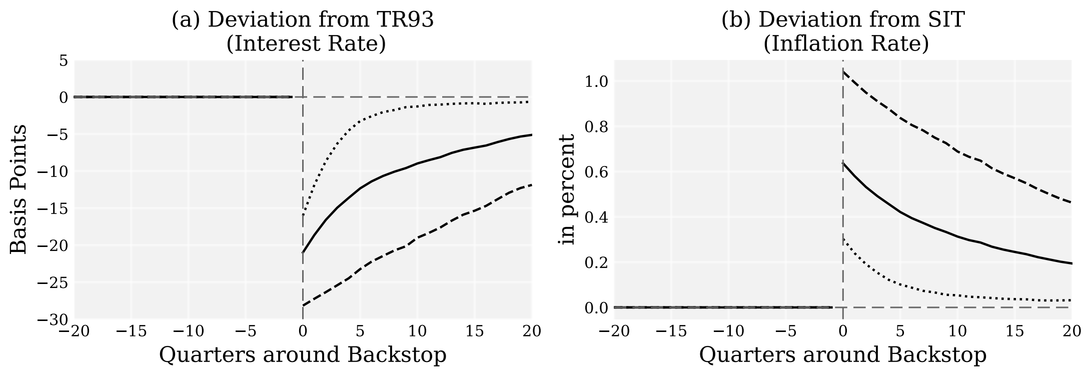
[36] Coimbra and Rey (2021) and Coimbra, Kim, and Rey (2021) present a related non–linear tradeoff in terms of economic activity versus financial stability.

6 Alternative Monetary Policy Strategies

6.1 Monetary Policy as a Backstop, and Normalisation Path

Should the central bank systematically backstop the credit market during financial distress episodes?[37] This section aims to evaluate the potential welfare gains from such policy. More precisely, we consider non–linear interest rate rules whereby the central bank commits itself to following SIT or a Taylor–type rule in normal times but also to doing *whatever it takes whenever needed* —and therefore exceptionally deviating from these rules— to forestall a crisis in periods of stress. In those instances, we assume that the central bank deviates "just enough" to avert the crisis, *i.e.* sets its policy rate so that $r_t^k = \bar{r}^k$ (see Proposition 2).[38] We refer to this contingent rule as a "backstop" rule.[39]

Figure 5: Backstop Required to Stave off a Crisis and Normalisation Path



Average stress - - - - Endogenous (predicted) stress ...... Exogenous (unpredicted) stress

Notes: Average deviations from the normal times' policy rule that the central bank must commit itself to and implement in order to forestall financial crises. Panel (a): deviation of the nominal policy rate, in basis points, when the central bank otherwise follows TR93. This deviation from the policy rate is akin to what Akinci, Benigno, Del Negro, and Queralto (2020) recently dubbed "$R^{\star\star}$". Panel (b): deviation of the inflation target from zero, in percentage point, when the central bank otherwise follows SIT. For the purpose of the exercise, financial stress is defined as a situation where there would have been a crisis absent the monetary policy backstop. A period of financial stress is classified as "endogenous" (respectively "exogenous") if the crisis probability in the quarter that precedes it (*i.e.* quarter −1) is in the top (respectively bottom) decile of its ergodic distribution (see Section 9.2 for a more detailed discussion).

As a first step, we show in Figure 5 the average deviations from TR93 (Panel (a)) and SIT (Panel (b)) that are needed in stress times to ward off the average crisis (plain line). These deviations are reported in terms of the policy rate (in basis points) for TR93 and in terms of the annualized inflation rate (in percentage points) for SIT. In both cases, the central bank must loosen its policy compared to normal times in order to avoid the crisis, which means it must

[37] For a recent discussion on the so–called "backstop principle", see BIS (2022).

[38] In the case of a Taylor–type rule $1 + i_t = (1 + \pi_t)^{1.5} \left(Y_t/\bar{Y}\right)^{0.125} \varsigma_t/\beta$, for example, this consists in setting the term $\varsigma_t = 1$ if $r_t^k \geq \bar{r}^k$, and setting ς_t such that $r_t^k = \bar{r}^k$ whenever (and only then) r_t^k would otherwise be lower than $\bar{r}^k$. Likewise, in the SIT case, the central bank tolerates deviations from strict inflation targeting just enough so that $r_t^k = \bar{r}^k$.

[39] Our notion of backstopping is related to, but different from, the notion of "cleaning", whereby the central bank mitigates the effects of a crisis only after it broke out —but does not forestall it.

temporarily lower its policy rate by 20bps below TR93 or temporarily tolerate a 0.6pp higher inflation rate under SIT.

Figure 5 also shows that the backstop policy must be unwound gradually, reflecting the time it takes for financial stress to dissipate. In our model, the adequate normalisation path is narrow. Tightening monetary policy more slowly would lead to unnecessary high inflation and associated costs. And tightening it too quickly would result in a financial crisis and a "hard landing". One important determinant of the speed of normalisation is the type of financial stress that is being addressed. When the stress is due to an exogenous adverse shock, the central bank can set its policy rate back to the TR93 rule already after about two years (panel (a), dotted line). When it is due to an excessive investment boom, in contrast, the normalisation takes longer and is only halfway after four years (panel (a), dashed line). The reason is clear. As the central bank intervenes to stem a crisis, it concomitantly slows down the adjustment that would be necessary to eliminate the capital overhang that causes financial stress. As a result, monetary policy must remain accommodative for longer to prevent a crisis.

Next, we study the economic performance and welfare gain of following a backstop rule. The welfare results, reported in Table 3, are directly comparable with those in Table 2. Two findings stand out. First, backstopping the economy improves welfare significantly and, depending on the policy rule, may almost entirely eliminate the welfare cost of credit market fragility. In the case of SIT (first row), in particular, the welfare loss falls to 0.001% with backstop, from 0.11% without backstop.[40] Second, the financial sector is —somehow paradoxically— more fragile when the central bank commits itself to backstopping the economy. Under SIT, for instance, the central bank has to backstop the economy —and therefore deviate from its normal times policy rule— slightly more than 15% of the time, whereas without backstop the economy would spend less than 10% of the time in a crisis (compare the column "BP time" with the column "Crisis time" in Table 2). This greater fragility is due to backstopping episodes being both more likely and more persistent (column "Length") than crisis episodes (column "Length" in Table 2). Indeed, when the central bank backstops the economy, it eliminates the negative wealth effects associated with financial crises. All else equal, the capital stock thus tends to be on average higher in that case, which makes the credit market more vulnerable than in an economy without backstop.[41]

[40]In the case of the Taylor–type rules, welfare decreases with ϕ_y because, when the central bank backstops the economy, there is little benefit from leaning against investment booms in normal times. In this case, it is desirable that, in normal times, the central bank focus on addressing the welfare cost of nominal rigidities —very much like in the standard NK model.

[41]Although backstop policies increase welfare, the result that the central bank will have to implement such policies relatively often admittedly raises the question of the central bank's ability to credibly commit to doing so.

Table 3: Economic Performance and Welfare Under Backstop Policies (BP)

Rule	ϕ_y	Welfare loss (%)	BP time (%)	Length (quarter)	$\mathbb{E}(\pi_t^2)$
SIT	–	0.0013	15.16	8.84	0.0019
Taylor rules ($\phi_\pi = 1.5$)	0.025	0.0012	17.99	9.17	0.0011
	0.050	0.0013	16.30	8.70	0.0017
	0.125	0.0019	11.81	7.45	0.0063
	0.250	0.0044	6.30	5.93	0.0196
	0.500	0.0117	1.38	4.43	0.0196
	0.750	0.0196	0.37	5.11	0.0821

Note: Statistics of the stochastic steady state ergodic distribution. "BP time" is the percentage of the time that central bank must backstop the economy (in %). The other statistics are the same as in Table 2.

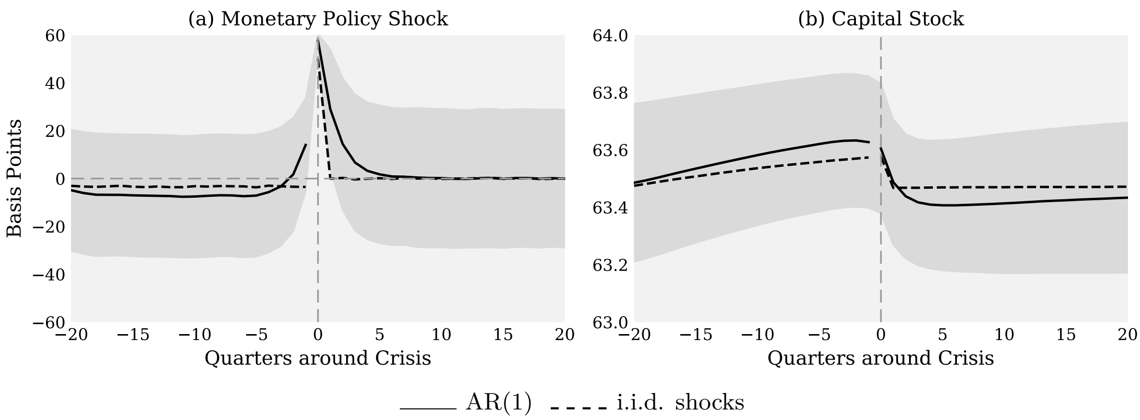
6.2 Monetary Policy Discretion and Financial Instability

In his narrative of the GFC, Taylor (2011) argues that discretionary and loose monetary policy may have exposed the economy to financial stability risks —the "Great Deviation" view. This section revisits this narrative and assesses the potential detrimental effects of monetary shocks —as opposed to rules— on financial stability. To do so, we consider a TR93 economy that experiences random deviations from the policy rule —"monetary policy shocks"— and where these shocks are the only source of aggregate uncertainty. More specifically, we consider a monetary policy rule of the form

$$1 + i_t = \frac{1}{\beta}(1+\pi_t)^{1.5}\left(\frac{Y_t}{\overline{\overline{Y}}}\right)^{0.125} \varsigma_t$$

with two alternatives exogenous processes for the monetary policy shock ς_t. One is an AR(1) process $\ln(\varsigma_t) = \rho_\varsigma \ln(\varsigma_{t-1}) + \epsilon_t^\varsigma$, with $\rho_\varsigma = 0.5$ and $\sigma_\varsigma = 0.0025$, as in Galí (2015). The other is a random independently and identically distributed (i.i.d.) shock that has the same volatility. We are interested in the dynamics of monetary policy shocks around crises in this new environment.

Figure 6: Rates too Low for too Long May Lead to a Crisis



Notes: Average dynamics around crises of the monetary policy shock (Panel (a)) and capital stock (Panel (b)), in an economy with only monetary policy shocks and where the central bank otherwise follows the TR93 rule.

The results, reported in Figure 6, show that the average crisis breaks out following a long period of unexpected monetary easing (Panel (a)) that feeds an investment boom (Panel (b)). In other words, by keeping the policy rate *too low for too long*, the central bank breeds macro–financial imbalances, leading the economy to a crisis. Moreover, the crisis is triggered by three consecutive, unexpected, and abrupt interest rate hikes toward the end of the boom in the case of the persistent shock and by a one–off 60 basis point jolt in the case of the i.i.d. shock. This latter finding is consistent with recent empirical evidence that unanticipated "last minute" interest rate hikes at the end of a boom are more likely to trigger a crisis than to avert it (Schularick, Ter Steege, and Ward (2021)). Overall, our analysis highlights that discretionary loose monetary policy may on its own be a source of financial instability.

7 Robustness

The aim of this section is to illustrate the robustness of our results by showing that they hold in two alternative versions of our model (i) with intermediated finance and (ii) with infinitely–lived heterogenous firms. In addition, we analyse the cases with only one financial friction —either limited contract enforceability or asymmetric information, and show that both frictions are necessary for our model to feature credit market collapses.

7.1 Intermediated Finance

We are interested in whether a financial intermediary can substitute for the credit market —especially when the latter has collapsed— without making a loss. For this, we consider a representative, competitive financial intermediary that purchases unproductive firms' K_t capital goods on credit at rate r_t^d ("deposits") and sells ℓ_t capital goods on credit to productive firms ("loans") at rate r_t^ℓ. We further allow the intermediary to keep $\mu K_t - (1-\mu)\ell_t \geq 0$ capital goods idle, and assume that idle capital depreciates at rate δ —like for the firms.

The intermediary faces the same financial frictions as the firms. It is not able to enforce contracts with borrowers and does not observe firms' idiosyncratic productivities. But it is not a source of financial frictions itself, in the sense that it can credibly commit itself to paying back its deposits —and always does so. The rest of the model is unchanged.

The intermediary's profit is the sum of the gross returns on the loans (first term) and idle capital (second term) minus the cost of deposits (last term):

$$\max_{\ell_t}(1-\mu)(1+r_t^\ell)\ell_t + (1-\delta)(\mu K_t - (1-\mu)\ell_t) - \mu(1+r_t^d)K_t \tag{27}$$

The intermediary's objective is to maximise its profit with respect to ℓ_t given r_t^ℓ and r_t^d, subject to productive firms' participation constraint $r_t^\ell \leq r_t^k$ and unproductive firms' incentive compatibility constraint

$$(1-\delta)(K_t + \ell_t) - \theta\ell_t \leq (1+r_t^d)K_t \tag{28}$$

The above constraint means that unproductive firms must be better–off when they deposit their funds with the intermediary (for a return r_t^d, on the right–hand side) than when they borrow ℓ_t and abscond (left–hand side). Since the profit increases with r_t^ℓ and decreases with r_t^d, a necessary condition for the intermediary to be active is that its profit be positive when $r_t^\ell = r_t^k$ and r_t^d satisfies (28) with equality, *i.e.*:

$$(1-\mu)(1+r_t^k)\ell_t + (1-\delta)(\mu K_t - (1-\mu)\ell_t) - \mu(1-\delta)(K_t + \ell_t) + \mu\theta\ell_t \geq 0$$

After re–arranging the terms, the above condition yields

$$r_t^k \geq \frac{(1-\theta)\mu - \delta}{1-\mu} = \bar{r}^k$$

which corresponds to the condition of existence of the credit market (see Proposition 2). This means that, when $r_t^k < \bar{r}^k$ and the credit market has collapsed, there is no room for financial intermediation either. When $r_t^k \geq \bar{r}^k$, financial intermediation may arise. But as unproductive firms can lend directly to productive ones at rate $r_t^c = r_t^k$ on the credit market in that case (see equilibrium E in Figure 1), the financial intermediary must offer the same conditions, with $r_t^\ell = r_t^d = r_t^k$, in order to be competitive —and makes zero profit.

It follows that our baseline model with dis–intermediated finance is isomorphic to a model with financial intermediaries. This result is intuitive. As long as intermediaries face the same agency problem as other lenders, whether financial transactions take place directly through a credit market, as in our baseline model, or indirectly through a loan market is irrelevant: these two markets rise and collapse in sync —and yield the same equilibrium outcome.[42]

7.2 Infinitely–lived Heterogenous Firms

In our baseline model, the household can freely re–balance its entire equity portfolio across firms at the end of every period. As a consequence, our model with one–period firms is isomorphic to a version where firms live infinitely and the idiosyncratic shocks $\omega_t(j)$ are independently and identically distributed across firms and time. Firms being *ex ante* equally productive, it is always optimal for the household to perfectly diversify its equity holdings by funding every firm with the same amount of equity. Even when firms live infinitely, they all enter period t with the same capital stock K_t. Assuming infinitely–lived firms is only relevant if firms are observationally heterogeneous *ex ante*.

The aim of this section is to show that our analysis goes through when firms live infinitely and are heterogenous *ex ante*. As an illustration, consider two observationally distinct sets of "high" (H) and "low" (L) quality firms of equal mass 1/2, characterised by probabilities μ^H and

[42]This equivalence result only emphasises that the key element of our model is the agency problem that lenders face, and not the type of the lender considered—*i.e.* whether an intermediary or a firm. In this respect, our approach wants itself general and close in spirit to Bernanke and Gertler (1989) —even though the agency problem considered here is different.

μ^L of being unproductive (*i.e.* of drawing $\omega_t(j) = 0$), with $\mu^H < \mu^L$.[43] The types H and L do not vary over time, and the household knows every firm's type. The rest of the model is unchanged.

In the presence of financial frictions, it is optimal for the household to hold more equity from the high quality firms than from the low quality ones. Hence, the former are larger than the latter. Let K_t^L and K_t^H denote low and high quality firms' respective initial capital stocks, with $K_t^L < K_t^H$ in equilibrium. The aggregate capital stock is $K_t = (K_t^H + K_t^L)/2$ and the share of K_t that is held by unproductive firms is[44]

$$\mu_t \equiv \frac{\mu^H K_t^H + \mu^L K_t^L}{K_t^H + K_t^L}$$

The constant returns to scale imply that productive firms have the same realized return on capital r_t^k, irrespective of their type L or H and initial capital stock, K_t^L or K_t^H. Moreover, Proposition 1 shows that their initial capital stock does not affect firms' borrowing limit either: $\psi_t = (r_t^c + \delta)/(1 - \delta - \theta)$ and is the same across high and low quality firms.[45] It follows that the aggregate credit supply and demand schedules in normal times are given by

$$L^S(r_t^c) = \mu_t K_t$$

and

$$L^D(r_t^c) \in [-(1-\mu_t)K_t, (1-\mu_t)\psi_t K_t]$$

and normal times arise in equilibrium only if there exists a credit market rate r_t^c such that $r_t^c \leq r_t^k$ and

$$\mu_t K_t \in \left[-(1-\mu_t)K_t, (1-\mu_t)\frac{r_t^c + \delta}{1-\delta-\theta}K_t\right]$$

which is the case if

$$\mu_t \leq (1-\mu_t)\frac{r_t^k + \delta}{1-\delta-\theta} \Leftrightarrow r_t^k \geq \frac{(1-\theta)\mu_t - \delta}{1-\mu_t} \tag{29}$$

The above condition is similar to that in Proposition 2, meaning that the Y–M–K transmission channels of monetary policy are still present and operate the same way as in our baseline model. The only difference is that μ_t is now endogenously determined at end of period $t-1$, *i.e.* that

[43] Another reason why infinitely–lived firms may be heterogenous *ex ante* is, for example, if they face convex equity issuance costs. However, adding such costs would require keeping track of the entire distribution of firm leverage over time, which —together with the embedded non–linearities— would likely make our model untractable.

[44] To see why $K_t^L < K_t^H$ and μ_t varies over time, first consider the case of a frictionless credit market. Absent financial frictions, firms perfectly hedge themselves against the idiosyncratic productivity shocks and all have the same return on equity: $r_t^q(j) = r_t^k$ for all j irrespective of the realization of the shock. As a consequence, firms' quality is irrelevant and the household does not discriminate across high and low quality firms, which thus all get the same equity funding: $K_t^H = K_t^L = K_t$. Hence, $\mu_t = (\mu_H + \mu_L)/2$ and is constant over time. In the presence of financial frictions, in contrast, the household understands that unproductive firms will distribute less dividends than productive firms if a crisis breaks out. It will invest in the equity of high and low quality firms until their marginal expected returns equate and no arbitrage is possible. Since low quality firms are less likely to be productive than high quality firms and the marginal return on equity decreases with the capital stock, it is optimal for the household to invest relatively more equity in high quality firms, especially so when the probability of a crisis goes up. It follows that $K_t^H > K_t^L$ and K_t^H/K_t^L increases with the crisis probability.

[45] Put differently, once the $\omega_t(j)$s are realized, what matters is whether a firm is productive, not its *ex ante* probability of being productive.

the share of capital in low versus high quality firms is yet another factor affecting financial stability. Insofar as μ_t is predetermined and does not affect r_t^k, the effect of this additional channel can only be of second order compared to the Y–M–K channels.

The upshot is that our results carry over to an economy with infinitely–lived and observationally *ex ante* heterogenous firms, provided that there remains some residual *ex post* heterogeneity (here in the form of the idiosyncratic productivity shocks $\omega_t(j)$s) and, therefore, a role for short term (intra–period) credit markets.

7.3 Only One Financial Friction

Our baseline model features two textbook financial frictions: limited contract enforceability and asymmetric information between lenders and borrowers. The aim of this section is to show that both frictions are needed, for the aggregate equilibrium outcome to depart from the first best outcome.

7.3.1 Asymmetric Information

Assume first that firms cannot abscond with the proceeds of the sales of idle capital goods. Then unproductive firms always prefer to sell their capital stock and lend the proceeds, and have no incentive to borrow. As a result, productive firms face no borrowing limit: they borrow until the marginal return on capital equals the cost of the loan and $r_t^\ell = r_t^k > -\delta$ in equilibrium.[46] No capital is ever kept idle, and the economy reaches the first best.

7.3.2 Limited Contract Enforceability

Assume next that firms' idiosyncratic productivities are perfectly observable at no cost. Then, lenders only lend to productive firms, which must nonetheless be dissuaded from borrowing $P_t(K_t^p - K_t)$ to purchase capital goods, keep them idle, and abscond. This will be the case if what they earn if they abscond, $P_t(1-\delta)K_t^p - P_t\theta(K_t^p - K_t)$ is less than what they earn if they use their capital stock in production, $P_t((1+r_t^c)K_t + (r_t^k - r_t^c)K_t^p)$ (from (18)), which implies:

$$(1-\delta)K_t^p - \theta(K_t^p - K_t) \leq (1+r_t^c)K_t + (r_t^k - r_t^c)K_t^p \Leftrightarrow \frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \frac{r_t^k + \delta}{1-\delta-\theta+r_t^c-r_t^k} \quad (30)$$

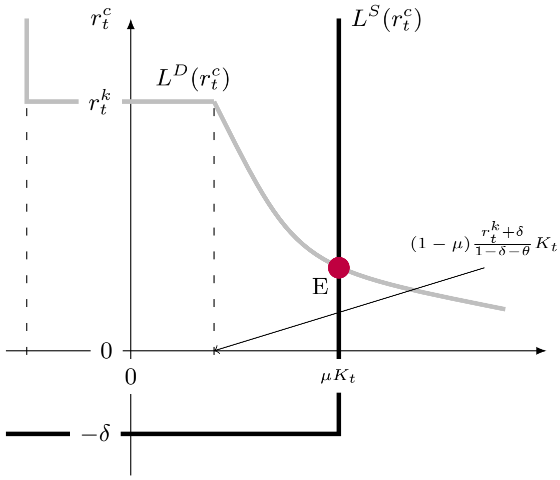
where the borrowing limit ψ_t now *decreases* with r_t^c: the higher the loan rate, the lower the productive firm's opportunity cost of borrowing and absconding, and hence the lower its incentive–compatible leverage. The aggregate loan supply and demand schedules take the same form as in (21) and (22), but with the borrowing limit ψ_t now given by (30) instead of Proposition 1. From Figure 7 it is easy to see that there is only one equilibrium outcome and the economy reaches the first best: no capital is ever kept idle. The only difference with the frictionless case is that, in equilibrium, unproductive firms' realised return on equity, r_t^c, is lower than that of productive firms, $(1-\delta)(1+\psi_t) - \theta\psi_t - 1$,[47] with $(1-\delta)(1+\psi_t) - \theta\psi_t - 1 > r_t^k > r_t^c$ (reflecting productive

[46] Note that, as firms' choice to lend or borrow perfectly reveals their type, the asymmetry of information dissipates and becomes irrelevant in that case.

[47] Since the incentive compatibility constraint (30) binds in equilibrium, the real gross return of a productive firm, $1 + r_t^c + (r_t^k - r_t^c)K_t^p/K_t$ is equal to $(1-\delta)K_t^p/K_t - \theta(K_t^p - K_t)/K_t = (1-\delta)(1+\psi_t) - \theta\psi_t$.

firms' excess return on leverage).

Figure 7: Credit Market Equilibrium Under Symmetric Information



Note: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate loan demand (gray) curves, when credit contracts are not enforceable but information is symmetric.

8 Conclusion

What are the channels through which monetary policy affects financial stability? Should central banks deviate from their objective of price stability to promote financial stability? To what extent may monetary policy itself brew financial vulnerabilities? To address these questions, we have extended the textbook NK model with capital accumulation, heterogeneous firms, and a credit market that allows the economy to reallocate capital across firms. Absent frictions on the credit market, the equilibrium outcome boils down to that of the standard model with a representative firm. With financial frictions, in contrast, there is an upper bound on the leverage ratio of any individual firm resulting from an incentive–compatibility constraint, which at times prevents capital from being fully reallocated to the most efficient firms. When the average return on capital is too low, possibly due to a capital overhang at the end of a long investment boom, the credit market collapses, triggering a financial crisis and a fall in activity due to capital mis–allocation.

We show that conventional monetary policy affects financial stability through three main channels: in the short run, through its effects on output and markups, and in the medium run, through its effects on capital accumulation. We also show that, by deviating from strict inflation targeting and systematically leaning against investment booms, the central bank may reduce the incidence of financial crises and, by so doing, improve welfare. Finally, we find that a systematic backstop policy that stem financial crises further improves welfare. The normalisation of monetary policy out of such backstop requires from the central bank to trade off price and financial stability, and its speed depends on the underlying source of financial stress. In contrast, discretionary monetary policy actions, such as keeping policy rates too low for too long and then unexpectedly and abruptly raising them toward the end of an investment boom, can be conducive to a financial crisis.

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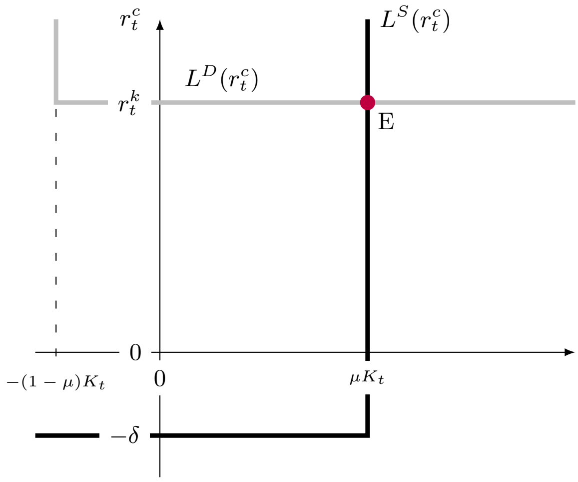
9 Appendix

9.1 Frictionless Credit Market Equilibrium

Figure 9.1 presents unproductive firms' aggregate credit supply (black) and productive firms' incentive–compatible aggregate demand (gray) curves in the absence of financial frictions (see Section 3.2.1).

Figure 9.1: Frictionless Credit Market Equilibrium



Note: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate loan demand (gray) curves, in the absence of financial frictions.

9.2 Financial Crises: Polar Types and Multiple Causes

Figure 9.2 is a stylized representation of the optimal capital accumulation decision rule, which expresses K_{t+1} as a function of state variables K_t and A_t. During a crisis, the household dis–saves to consume, which generates less investment and a fall in the capital stock, as captured by the discontinuous downward breaks in the decision rules. There are two polar types of crises. The first one can be characterised as "non–anticipated": for a given level of capital stock K_t^{average}, a crisis breaks out when productive firms' marginal return on capital, r_t^k, falls below the required incentive compatible loan rate, $\bar{r}^k$ (see Proposition 2). In Figure 9.2, this is the case in equilibrium $\text{A}_{\text{non–ant}}$, where aggregate productivity A_t falls from A_t^{average} to A_t^{low}. The other polar type of crisis can be characterised as "anticipated": following an unexpectedly long period of high productivity A_t^{high}, the household accumulates savings and feeds an investment boom that increases the stock of capital. All other things equal, the rise in the capital stock reduces productive firms' marginal return on capital until $r_t^k < \bar{r}^k$. The crisis then breaks out as K_t exceeds K_t^{high}, as in equilibrium A_{ant}. Accordingly, monetary policy can reduce the incidence of financial crises either by dampening the effects of shocks through a macro–economic stabilization policy (via the Y– or M–channel), or by improving the resilience of the economy by slowing down capital accumulation during booms (via notably the K–channel), or by doing both.

Figure 9.2: Optimal Decision Rules $K_{t+1}(K_t, A_t)$ and Two Polar Types of Crisis

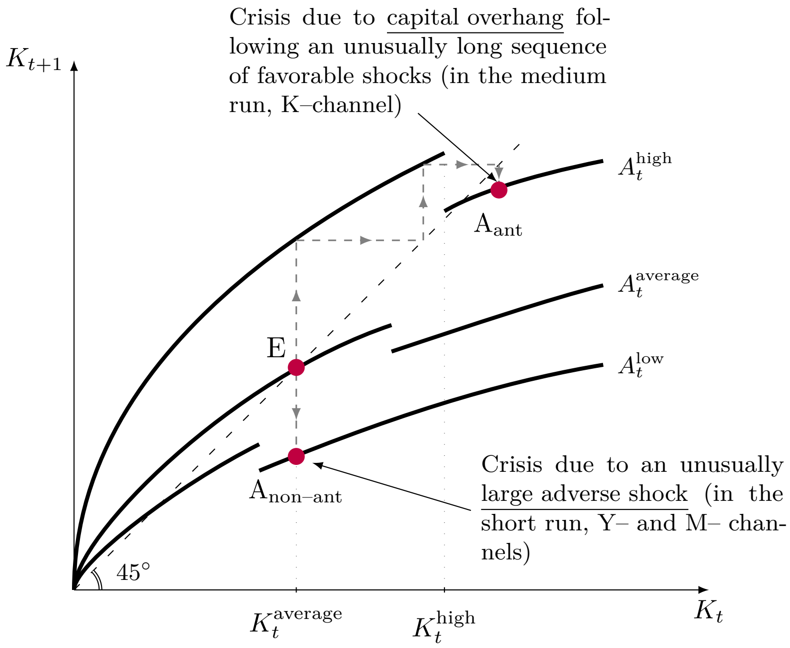
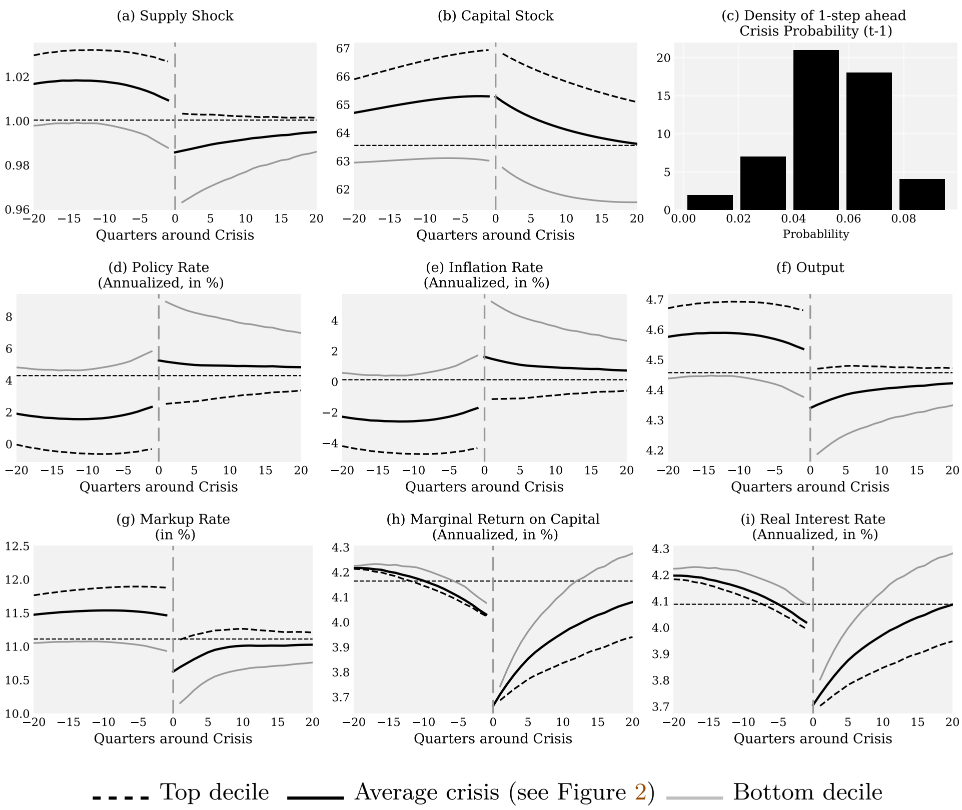


Figure 9.3: Predicted Versus Unpredicted Financial Crises



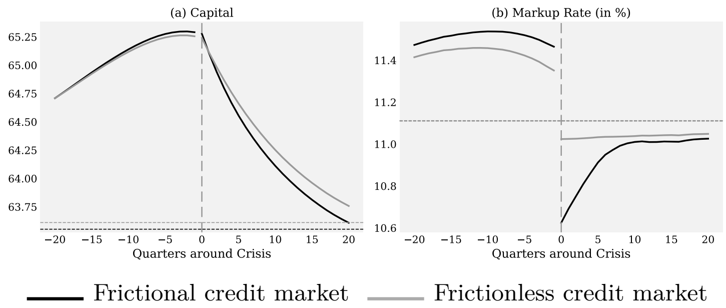
Note: Simulations for the TR93 economy. Average dynamics of the economy around the beginning of all (black line), predicted (dashed) and unpredicted (grey) crises (quarter 0), as in Figure 2. The subset of predicted (unpredicted) crises corresponds to the crises whose one–step–ahead probability in quarter -1 is in the top (bottom) decile of its distribution (Panel (c)). The 1–step ahead crisis probability (Panel (c)) is defined as $\mathbb{E}_{t-1}\left(\mathbb{1}\left(\frac{Y_t}{\mathscr{M}_t K_t} < \frac{1-\tau}{\alpha}\left(\frac{(1-\theta)\mu-\delta}{1-\mu}+\delta\right)\right)\right)$, where $\mathbb{1}\{\cdot\}$ is a dummy variable equal to one when the inequality inside the curly braces holds (*i.e.* there is a crisis) and to zero otherwise (see Corollary 1).

As the above discussion suggests, the average dynamics around crises reported in Figure 2 mask the heterogeneity of financial crises in our model. To document this heterogeneity, we contrast in Figure 9.3 the average dynamics around predicted (dashed line) and unpredicted (grey line) crises with those of the average crisis (black line and Figure 2). For the purpose of this exercise, we define a crisis as "predicted" (respectively "unpredicted") if the crisis probability in the quarter that precedes it (*i.e.* quarter -1) is in the top (respectively bottom) decile of its distribution (Panel (c)). Our prior is that endogenous crises are more predictable than exogenous ones and, therefore, that the crisis probability can be used as a reasonable measure of endogeneity. The main findings are twofold. First, in line with our prior, unpredicted crises occur when aggregate productivity is low (Panel (a), grey line), as in the case of crisis $\mathrm{A}_{\mathrm{exog}}$ in Figure 9.2, whereas predicted ones follow an investment boom (Panel (b), dashed line), and occur despite aggregate productivity being above average (Panel (a), dashed line), as in the case of crisis $\mathrm{A}_{\mathrm{endog}}$. Second, the distribution of the one–quarter–ahead crisis probability is left–skewed (Panel (c)), with means that the bulk of crises in our model are predicted/endogenous —albeit imperfectly. These findings are consistent with recent empirical evidence and the notion that financial crises are the byproduct of predictable boom–bust financial cycles (see Greenwood, Hanson, Schleifer, and Ahm Sørensen (2021), Sufi and Taylor (2021)).

9.3 The Role of Crisis Expectations: Savings Glut and Markup Externalities

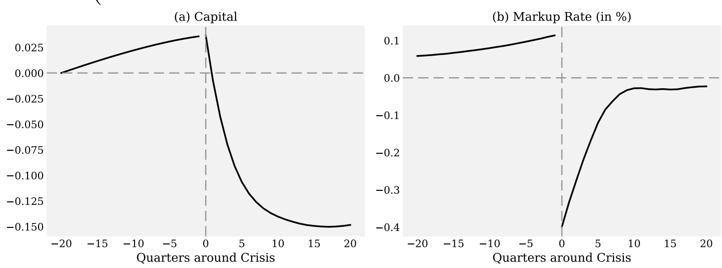
Figure 9.4: Savings Glut and Markup Externalities

Level



Note: The horizontal lines correspond to the unconditional average in each economy.

Difference (Frictional credit market − Frictionless credit market)

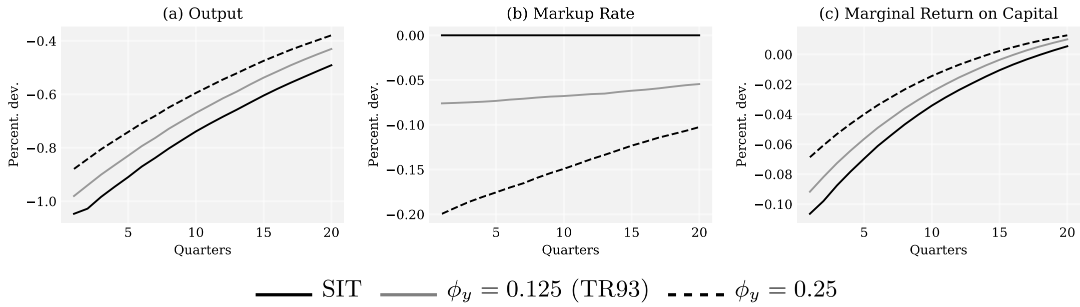


Notes: Comparison of two economies under TR93 with a frictional versus frictionless credit market. For the frictional credit market economy: same average dynamics as in Figure 2. For the frictionless credit market economy: counterfactual average dynamics, when the economy (*i.e.* the textbook NK economy) starts with the same capital stock in quarter −20 and is fed with the same technology shocks as the frictional credit market economy (Figure 2, Panel (a)).

Figure 9.4 illustrates the effects of crisis expectations during investment booms. It compares the dynamics of capital and markups during booms in an economy with a frictional credit market (black line) and a *counterfactual* economy with a frictionless credit market (grey line). Our focus is on the pre–crisis period, from quarter −20 to quarter −1. During this period, the credit market functions perfectly and the entire capital stock is used productively and efficiently in both economies. The *only* difference between the two economies over this period is that, in the frictional credit market one, the household and retailers anticipate that a crisis is forthcoming. These anticipations result in higher capital stock and markups (bottom panels), reflecting the excess accumulation of precautionary savings by the household (savings glut externality) and retailers' excess frontloading of price increases (markup externality) ahead of the crisis.

9.4 Standard Behavior of the Model Around Steady State

Figure 9.5: Short Run Effects: Impulse Response Functions



Note: Generalized impulse response functions following a negative technology under SIT, and the Taylor–type rule with $\phi_\pi = 1.5$ and $\phi_y = 0.125$ (TR93) or $\phi_y = 0.25$, around the average of the ergodic distribution in the stochastic steady state.

Figure 9.5 compares the effects of a negative one–standard deviation technology shock on output, markups, and productive firms' marginal return on capital, under SIT (black line), TR93 (grey line), and a Taylor–type rule with $\phi_y = 0.25$ (dashed line), at the stochastic steady state of the economy. Around this steady state, the economy is in normal times, the probability of a crisis is small, and the economy behaves like an economy with a frictionless credit market, which corresponds to the textbook NK economy. The differences across policy rules are well known. Following the negative shock, output falls by less while markups fall by more under Taylor–type rules than under SIT. As both effects limit the fall in productive firms' marginal return on capital, following a Taylor–type rule makes the economy more resilient than following SIT (see Proposition 2).

9.5 Model with Both Technology and Demand Shocks

The aim of this section is to briefly describe the dynamics of our model in the presence of both technology and demand (risk–premium) shocks à la Smets and Wouters (2007).[48] For the purpose of comparison with our baseline case with technology shocks only, we re–calibrate parameter θ so that the economy still spends 8% of the time in a crisis in the presence of demand shocks, all the other parameters being unchanged (see Tables 1 and 9.1).

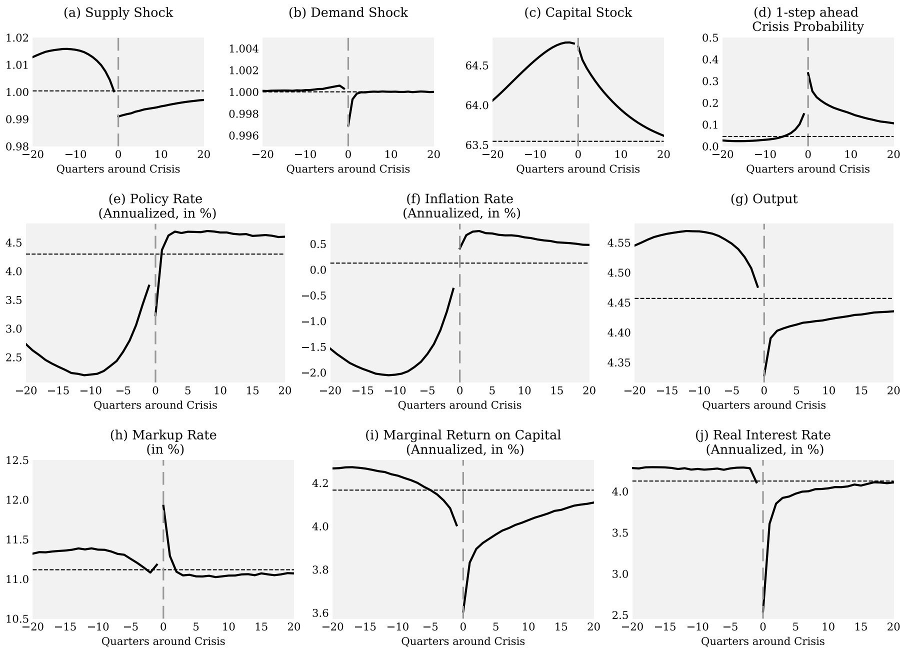
[48] Such a shock creates a wedge between the interest rate controlled by the central bank and the return on assets held by the household, and has the exact opposite effects of a demand shock. A positive risk–premium (hence a negative demand) shock typically increases the required return on assets and reduces current consumption. At the same time, it also increases firms' cost of capital and reduces the value of capital and investment. In a model with endogenous capital accumulation but without capital adjustment costs, like ours, this type of demand shock thus generates a positive correlation between consumption and investment —unlike a discount factor shock.

Table 9.1: Parametrization with Both Technology and Demand Shocks

Parameter	Target	Value
Aggregate risk–premium shocks		
ρ_z	As in Smets and Wouters (2007)	0.220
σ_z		0.230
Financial frictions		
θ	The economy spends 8% of the time in a crisis	0.730

Figure 9.6 reports the average dynamics around crises in this economy. The comparison with Figure 2 in the main text shows that the presence of demand shocks does not affect the main dynamics and, in particular, the way macro–financial imbalances build up ahead of crises.

Figure 9.6: Average Dynamics Around Crises with Both Technology and Demand Shocks



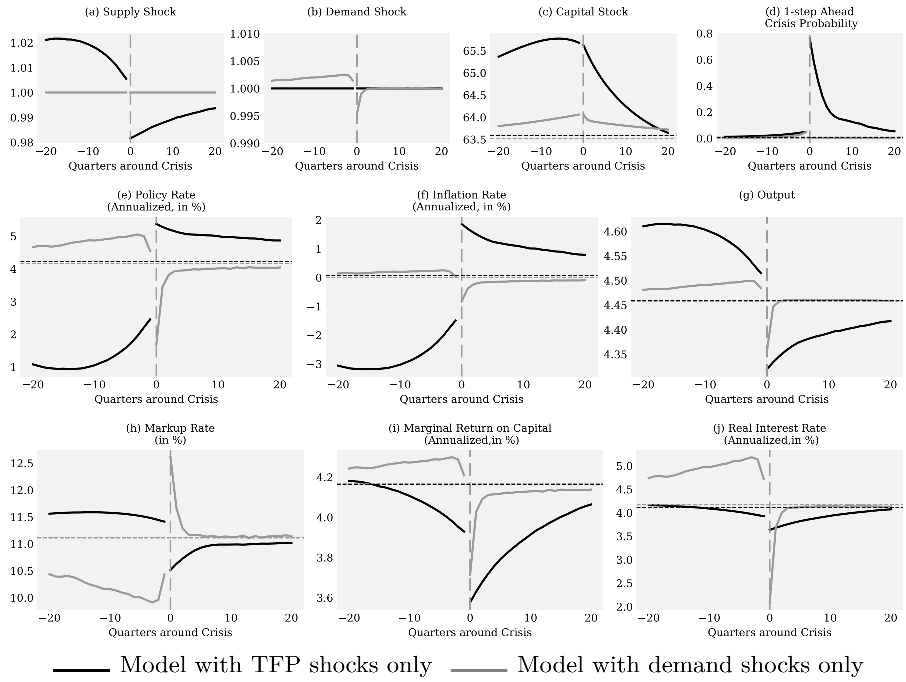
Note: Simulations for the TR93 economy, in the presence of both technology and risk–premium shocks à la Smets and Wouters (2007). Average dynamics of the economy around the beginning of a crisis (in quarter 0), as in Figure 2.

The main difference with our baseline economy with technology shock only is to be found in the economy's short run response to shocks. Under TR93, the central bank lets output decline in the face of negative technology and demand shocks, which explains why crises coincide with recessions —the Y–channel. At the same time, markups typically increase following negative demand shocks and decrease following negative technology shocks. These shocks therefore affect the probability of a crisis in opposite directions through the M–channel (see Figure 9.6).

Whether a crisis breaks out depends on the relative strengths of the Y– M– and K– channels as well as the size of the technology versus demand shocks. To get a sense of which type of shock is most conducive to a crisis, we solve and simulate our model separately for two "counterfactual" economies that experience either technology or demand shocks —not both—

and compute statistics on crises (see Table 9.2 as well as Figure 9.7). The main result is that crises are essentially due to technology shocks —either alone or in combination with demand shocks. Being relatively more persistent (see Table 1), such shocks are indeed more likely to give rise to protracted booms and capital overhang, which are pre-conditions for a crisis.[49] Technology–driven crises also last twice as long as the demand–driven ones and, accordingly, are associated with larger losses in output.

Figure 9.7: Average Dynamics Around Crises with Either Technology or Demand Shocks



Note: Simulations under TR93 for counterfactual economies with either technology (black line) or risk–premium shocks à la Smets and Wouters (2007) (grey line). Average dynamics of the economy around the beginning of a crisis (in quarter 0), as in Figure 2.

Table 9.2: Crisis Statistics and Origins with Both Technology and Demand Shocks

	Crisis time (%)	Output loss (%)
Economy with both shocks	[8.00]	-3.20
Economy with TFP shocks only	3.42	-4.76
Economy with demand shocks only	0.00	-2.90

Note: The first row reports statistics of the stochastic steady state ergodic distribution in the economy with both technology and demand shocks. The second and third rows report the same statistics, in counterfactual economies that experience either technology or demand shocks. In all cases, we assume that the central bank follows a TR93. "Crisis time" is the percentage of the time that the economy spends in a crisis (in %). By construction, it is equal to 8% under TR93 (square brackets). "Output loss" is the percentage fall in output from one quarter before the crisis until the trough of the crisis (in %).

[49]These results are conditional on the persistence of the demand shock taking the estimated value in Smets and Wouters (2007). With a higher persistence level, such as the one estimated with the Euro Area New Area-Wide model (Christoffel, Coenen, and Warne (2008)), demand shocks could be a substantial driver of financial instability under a standard Taylor rule.

Table 9.3: Economic Performance and Welfare Under Alternative Policy Rules with Both Supply and Demand Shocks

		Frictionless	Frictional credit market				
Rule	ϕ_y	Welfare loss (%)	Welfare loss (%)	Crisis time (%)	Length (quarter)	Output loss (%)	$\mathbb{E}(\pi_t^2)$
SIT	–	0.0000	0.0563	5.03	4.59	-5.60	0.0000
Taylor rules ($\phi_\pi = 1.5$)	0.025	0.0116	0.1575	13.11	1.75	-4.06	0.0006
	0.050	0.0093	0.1407	11.74	1.77	-3.77	0.0014
	0.125	0.0062	0.0994	[8.00]	1.78	-3.20	0.0065
	0.250	0.0064	0.0587	3.93	1.75	-2.71	0.0200
	0.500	0.0126	0.0297	0.46	1.46	-2.10	0.0524
	0.750	0.0203	0.0333	0.04	1.18	-1.53	0.0834

Note: Same statistics as in Table 2.

9.6 Equations of the Model

The differences between our model and the textbook NK model are highlighted in red.[50]

1. $1 = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1 + r_{t+1})\right\}$
2. $1 = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1 + r^q_{t+1})\right\}$
3. $\dfrac{W_t}{P_t} = \chi N_t^{\varphi} C_t^{\sigma}$
4. $Y_t = A_t\left(\omega_t K_t\right)^{\alpha} N_t^{1-\alpha}$
5. $\dfrac{W_t}{P_t} = \dfrac{\epsilon}{\epsilon - 1}\dfrac{(1-\alpha)Y_t}{\mathscr{M}_t N_t}$
6. $r^q_t + \delta = \dfrac{\epsilon}{\epsilon - 1}\dfrac{\alpha Y_t}{\mathscr{M}_t K_t}$
7. $(1 + \pi_t)\pi_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\dfrac{Y_{t+1}}{Y_t}(1 + \pi_{t+1})\pi_{t+1}\right) - \dfrac{\epsilon - 1}{\varrho}\left(1 - \dfrac{\epsilon}{\epsilon - 1}\cdot\dfrac{1}{\mathscr{M}_t}\right)$
8. $1 + i_t = \dfrac{1}{\beta}(1 + \pi_t)^{\phi_\pi}\left(\dfrac{Y_t}{Y}\right)^{\phi_y}$
9. $Y_t = C_t + I_t + \dfrac{\varrho}{2}Y_t\pi_t^2$
10. $\Lambda_{t,t+1} \equiv \beta\dfrac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$
11. $1 + r_t = \dfrac{1 + i_{t-1}}{1 + \pi_t}$
12. $K_{t+1} = I_t + (1 - \delta)K_t$
13. $\omega_t = \begin{cases} 1 & \text{if } r^q_t \geq \frac{(1-\theta)\mu - \delta}{1-\mu} \\ 1 - \mu & \text{otherwise} \end{cases}$

9.7 Global Solution Method

The model is solved by approximating expectations using a collocation technique (see Christiano and Fisher (2000)). We first discretize the distribution of the shocks using the approach proposed by Rouwenhorst (1995). This leads to a Markov chain representation of the shock, s_t, with $s_t \in \{a_1, \ldots, a_{n_a}\} \times \{z_1, \ldots, z_{n_z}\}$ and transition matrix $\mathbb{T} = (\varpi_{ij})_{i,j=1}^{n_a n_z}$ where $\varpi_{ij} = \mathbb{P}(s_{t+1} = s_j | s_t = s_i)$. In what follows, we use $n_a = 5$ and $n_z = 5$. We look for an approximate representation of consumption, gross inflation and the gross nominal interest rate as a function of the endogenous state variables in each regime, *e.g.* normal times and crisis times. More specifically, we use the approximation[51]

$$G_x(K_t; s) = \begin{cases} \sum_{j=0}^{p_x} \psi^x_j(n, s) T_j(\nu(K)) & \text{if } K \leqslant K^\star(s) \\ \sum_{j=0}^{p_x} \psi^x_j(c, s) T_j(\nu(K)) & \text{if } K > K^\star(s) \end{cases} \quad \text{for } x = \{c, \hat{\pi}, \hat{\imath}\}$$

[50] Relation 2 in the list of equations below is an other way to write relation (4) in the main text, using the definition of the average realized return on equity $r^q_{t+1} \equiv \int_0^1 r^q_{t+1}(j)\mathrm{d}j$. In turn, r^q_t can be re–written using (16) and (18) as $r^q_t = \mu\left(r^c_t - (r^c_t + \delta)K^u_t/K_t\right) + (1 - \mu)\left(r^c_t + (r^k_t - r^c_t)K^p_t/K_t\right)$. In normal times, $K^u_t = 0$ and $K^p_t = K_t/(1 - \mu)$, which implies that $r^q_t = r^k_t$. Using (17), (19), and (24), one further obtains $r^k_t + \delta = \alpha Y_t/((1 - \tau)\mathscr{M}_t K_t))$, and therefore, using $\tau = 1/\epsilon$, relation 6. In crisis times, $r^c_t = -\delta$ and $K^p_t = K_t$, which implies that $r^q_t + \delta = (1-\mu)(r^k_t + \delta)$. Using (17), (19), and (24), one obtains $r^k_t + \delta = \alpha Y_t/((1 - \mu)(1 - \tau)\mathscr{M}_t K_t))$, and therefore relation 6.

[51] Throughout this section, we denote $\hat{\pi} = 1 + \pi$ and $\hat{\imath} = 1 + i$.

where $T_j(\cdot)$ is the Chebychev polynomial of order j and $\nu(\cdot)$ maps $[\underline{K}; K^\star(s)]$ in the normal regime (respectively $[K^\star(s); \overline{K}]$ in the crisis regime into) [-1;1].[52] $\psi_j^x(r,s)$ denotes the coefficient of the Chebychev polynomial of order j is the approximation of variable x when the economy is in regime r and the shocks are $s = (a, z)$. p_x denotes the order of Chebychev polynomial we use for approximating variable x.

$K^\star(s)$ denotes the threshold in physical capital beyond which the economy falls in a crisis, defined as

$$r_t^k + \delta = \frac{\alpha Y_t}{(1-\tau)\mathscr{M}_t K_t} = \frac{\mu(1-\delta-\theta)}{1-\mu} \tag{31}$$

This value is unknown at the beginning of the algorithm as it depends on the decisions of the agents. We therefore also need to formulate a guess for this threshold.

9.7.1 Algorithm

The algorithm proceeds as follows.

1. Choose a domain $[K_m, K_s]$ of approximation for K_t and stopping criteria $\varepsilon > 0$ and $\varepsilon_k > 0$. The domain is chosen such that K_m and K_s are located 30% away from the deterministic steady state of the model (located in the normal regime). We chose $\varepsilon = \varepsilon_k = 1\text{e}^{-4}$.

2. Choose an order of approximation p_x (we chose $p_x = 9$) for $x = \{c, \hat{\pi}, \hat{\imath}\}$), compute the n_k roots of the Chebychev polynomial of order $n_k > p$ as

$$\zeta_\ell = \cos\left(\frac{(2\ell-1)\hat{\pi}}{2n_k}\right) \text{ for } \ell = 1, \dots, n_k$$

   and formulate an initial guess[53] for $\psi_j^x(n,s)$ for $x = \{c, \hat{\pi}, \hat{\imath}\}$ and $i = 1, \dots, n_a \times n_z$. Formulate a guess for the threshold $K^\star(s)$.

3. Compute K_ℓ, $\ell = 1, \dots, 2n_k$ as

$$K_\ell = \begin{cases} (\zeta_\ell + 1)\frac{K^\star(s) - K_m}{2} + K_m & \text{for } K \leqslant K^\star(s) \\ (\zeta_\ell + 1)\frac{K_s - K^\star(s)}{2} + K^\star(s) & \text{for } K > K^\star(s) \end{cases}$$

   for $\ell = 1, \dots, 2n_k$.

4. Using a candidate solution $\Psi = \{\psi_j^x(r, s_i); x = \{c, \hat{\pi}, \hat{\imath}\}, r = \{n, c\}, i = 0 \dots p_x\}$, compute approximate solutions $G_c(K; s_i)$, $G_{\hat{\pi}}(K; s_i)$ and $G_{\hat{\imath}}(K; s_i)$ for each level of K_ℓ, $\ell = 1, \dots, 2n_k$ and each possible realization of the shock vector s_i, $i = 1, \dots, n_a \times n_z$ and the over quantities of the model using the definition of the general equilibrium of the economy (see below). In particular, compute the next period capital $K'_{\ell,i} = G_K(K_\ell; z_i)$ for each $\ell = 1, \dots, 2n_k$ and $i = 1 \dots n_a \times n_z$.

[52] More precisely, $\nu(K)$ takes the form $\nu(K) = 2\frac{K - \underline{K}}{K^\star(s) - \underline{K}} - 1$ in the normal regime and $\nu(K) = 2\frac{K - K^\star(a,z)}{\overline{K} - K^\star(s)} - 1$ in the crisis regime.

[53] The initial guess is obtained from a first order approximation of the model around the deterministic steady state.

5. Using the next period capital and the candidate approximation, solve the general equilibrium to obtain next period quantities and prices entering the expectations in the household's Euler equations and in the price setting equation. Compute expectations

$$\widetilde{\mathscr{E}}_{c,t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(G_c(K'_{\ell,i}, z'_s))(1 + r^{k'}(K'_{\ell,i}, z'_s)) \right] \tag{32}$$

$$\widetilde{\mathscr{E}}_{\hat{\imath},t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[\frac{u'(G_c(K'_{\ell,i}, z'_s))}{G_{\hat{\pi}}(K'_{\ell,i}, z'_s)} \right] \tag{33}$$

$$\widetilde{\mathscr{E}}_{\hat{\pi},t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(G_c(K'_{\ell,i}, z'_s)) G_Y(K'_{\ell,i}, z'_s) G_{\hat{\pi}}(K'_{\ell,i}, z'_s)(G_{\hat{\pi}}(K'_{\ell,i}, z'_s) - 1) \right] \tag{34}$$

6. Use expectations to compute c, $\hat{\pi}$ and $\hat{\imath}$

$$\widetilde{c}_t = u'^{-1}\left(\widetilde{\mathscr{E}}_{c,t}\right) \tag{35}$$

$$\widetilde{\hat{\imath}}_t = z \frac{u'(G_c(K_\ell, z_i))}{\widetilde{\mathscr{E}}_{\hat{\imath},t}} \tag{36}$$

$$\widetilde{\hat{\pi}}_t = \frac{\sqrt{1 + 4\Delta_t} - 1}{2} \tag{37}$$

where

$$\Delta_t \equiv \frac{\widetilde{\mathscr{E}}_{\hat{\pi},t}}{G_c(K_\ell, z_i)^{-\sigma} G_y(K_\ell, z_i)} - \frac{\epsilon - 1}{\varrho}\left(1 - \frac{\epsilon}{\epsilon - 1} \frac{1}{G_{\mathscr{M}}(K_\ell, z_i)}\right) \tag{38}$$

7. Project $\widetilde{c}_t$, $\widetilde{\hat{\imath}}_t$, $\widetilde{\hat{\pi}}_t$ on the Chebychev polynomial $T_j(\cdot)$ to obtain a new candidate vector of approximation coefficients, $\widetilde{\Psi}$. If $\|\widetilde{\Psi} - \Psi\| < \varepsilon\xi$ then a solution was found and go to step 8, otherwise update the candidate solution as

$$\xi\widetilde{\Psi} + (1 - \xi)\Psi$$

where $\xi \in (0, 1]$ can be interpreted as a learning rate, and go back to step 3.

8. Upon convergence of Ψ, compute $\widetilde{K^\star}(s)$ that solves (31). If $\|\widetilde{K^\star}(s) - K^\star(s)\| < \varepsilon_k\xi_k$ then a solution was found, otherwise update the threshold as

$$\xi_k\widetilde{K^\star}(s) + (1 - \xi_k)K^\star(s)$$

where $\xi_k \in (0, 1]$ can be interpreted as a learning rate on the threshold, and go back to step 3.

9.7.2 Computing the General Equilibrium

In this section, we explain how the general equilibrium is solved. Given a candidate solution Ψ , we present the solution for a given level of the capital stock K, a particular realization of the shocks (a, z). For convenience, and to save on notation, we drop the time index.

For a given guess on the threshold, $K^\star(a, z)$, test the position of K.

If $K \leqslant K^{\star}(a,z)$, the economy is in normal times. Using the approximation guess, we get immediately

$$C = G^n_c(K,s),\ \hat{\pi} = G^n_{\hat{\pi}}(K,s),\ \hat{\imath} = G^n_{\hat{\imath}}(K,s)$$

and $\omega = 1$. If $K > K^{\star}(a,z)$, the economy is in crisis times. Using the approximation guess, we get immediately

$$C = G^c_c(K,s),\ \hat{\pi} = G^c_{\hat{\pi}}(K,s),\ \hat{\imath} = G^c_{\hat{\imath}}(K,s)$$

and $\omega = 1 - \mu$. Using the Taylor rule, we obtain aggregate output as

$$Y = \overline{Y}\left(\frac{\beta\hat{\imath}}{\hat{\pi}^{\phi_\pi}}\right)^{\frac{1}{\phi_y}}$$

and, from the production function, the level of hours required to produce it as

$$N = \left(\frac{Y}{a(\omega K^{\alpha})}\right)^{\frac{1}{1-\alpha}}$$

which leads to a markup rate of

$$\mathscr{M} = \frac{1-\alpha}{\chi(1-\tau)}\frac{Y}{N^{1+\varphi}}C^{-\sigma}$$

and a rate of return on capital of

$$r^k = \frac{\alpha}{1-\tau}\frac{Y}{\mathscr{M}K} - \delta$$

The investment level obtains directly from the resource constraint as

$$x = Y - C$$

implying a value for the next capital stock of

$$K' = x + (1-\delta)K$$

9.7.3 Accuracy

In order to asses the accuracy of the approach, we compute the relative errors an agent would makes if they used the approximate solution. In particular, we compute the quantities

$$\mathscr{R}_c(K,z) = \frac{C_t - \left(\beta\mathbb{E}_t\left[C_{t+1}^{-\sigma}(1+r^q_{t+1})\right]\right)^{-\frac{1}{\sigma}}}{C_t}$$

$$\mathscr{R}_{\hat{\imath}}(K,z) = \frac{C_t - \left(\beta\frac{\hat{\imath}_t}{z_t}\mathbb{E}_t\left[\frac{C_{t+1}^{-\sigma}}{\hat{\pi}_{t+1}}\right]\right)^{-1/\sigma}}{C_t}$$

$$\mathscr{R}_{\hat{\pi}}(K,z) = \hat{\pi}_t(\hat{\pi}_t - 1) - \beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\hat{\pi}_{t+1}(\hat{\pi}_{t+1}-1)\right) + \frac{\epsilon-1}{\varrho}\left(1 - \frac{\epsilon}{\epsilon-1}\cdot\frac{1}{\mathscr{M}_t}\right)$$

where $r^q_{t+1} \equiv \int_0^1 r^q_{t+1}(j)\mathrm{d}j$, $\mathscr{R}_c(K,z)$ and $\mathscr{R}_{\hat{\imath}}(K,z)$ denote the relative error in terms of consumption an agent would make by using the approximate expectation rather than the "true" rational expectation in the household's Euler equation. $\mathscr{R}_{\hat{\pi}}(K,z)$ corresponds to the error on inflation.

All these errors are evaluated for values for the capital stock that lie outside of the grid that was used to compute the solution. We used 1,000 values uniformly distributed between K_m and K_s. Table 9.4 reports the average of absolute errors, $E^x = \log_{10}(\frac{1}{n_k \times n_a \times n_z} \sum |\mathscr{R}_x(K,s)|)$, for $x \in \{c, \hat{\imath}, \hat{\pi}\}$.

Table 9.4: Accuracy Measures

	ϕ_y	Average absolute errors		
		E^c	$E^{\hat{\imath}}$	$E^{\hat{\pi}}$
SIT	–	-5.6088	–	–
Taylor rules ($\phi_\pi = 1.5$)	0.025	-5.5378	-5.5277	-5.2331
	0.050	-5.4967	-5.5603	-5.1380
	0.125	-5.3805	-5.1154	-4.9550
	0.250	-5.2889	-4.9216	-4.8640
	0.500	-5.5570	-4.9748	-5.0796
	0.750	-5.5235	-4.8875	-5.0160

Note: $E^x = \log_{10}(\frac{1}{n_k \times n_a \times n_z} \sum |\mathscr{R}_x(K,s)|)$ is the average of the absolute difference, in terms of the level of consumption, that is obtained if agents use the approximated expectation of variable x instead of its "true" rational expectation, for $x \in \{c, \hat{\imath}, \hat{\pi}\}$.

Concretely, $E^c = 10^{-5.6088}$ (first row, second column) means that the average error an agent makes in terms of consumption by using the approximated decision rule —rather than the true one— under SIT amounts to \$2.46 per \$1,000,000 spent. The largest approximation errors in the decision rules are made at the threshold values for the capital stock where the economy shifts from normal to crisis times. But even there, the maximal errors are relatively small, in the order of \$150 per \$1000,000 of consumption, and rare. By the usual standards, our approximation of agents' decision rules is therefore very accurate.