A Smooth Shadow-Rate Dynamic Nelson-Siegel Model for Yields at the Zero Lower Bound

Daan Opschoor
Erasmus University Rotterdam
opschoor@ese.eur.nl

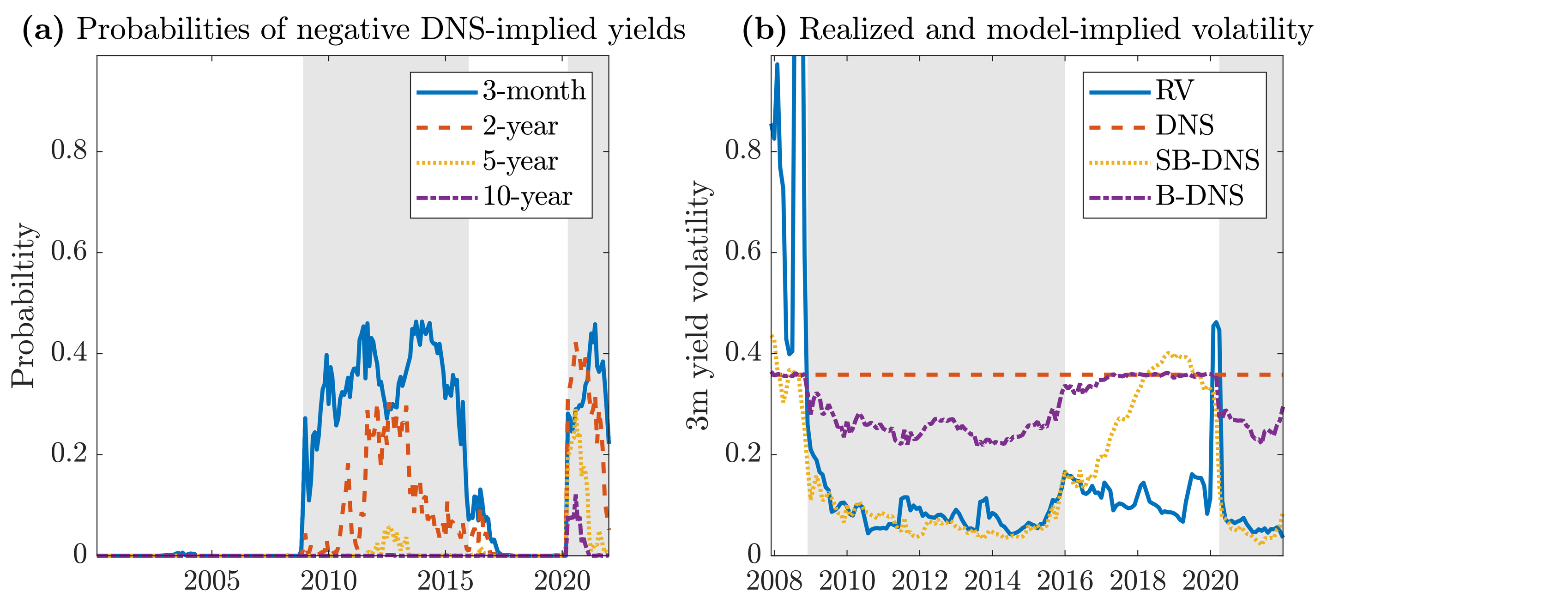
Michel van der Wel
Erasmus University Rotterdam
vanderwel@ese.eur.nl

Key Takeaways

- Traditional term structure models ignore zero lower bound (ZLB).
  $\Longrightarrow$ Not able to capture **changed dynamics** at ZLB!
- This paper proposes smooth shadow-rate version of Dynamic Nelson-Siegel (DNS) model that **softly imposes ZLB** onto yield curve.
  $\Longrightarrow$ By relaxing no-arbitrage restrictions, we obtain a highly tractable and flexible model with closed-form yield curve expression!
- Improves in-sample fit and out-of-sample performance relative to benchmarks, as well as provides shadow rate and lift-off horizon estimates!

Problems at the ZLB

(a) Traditional models generate **implausible negative yield** scenarios.
(b) Traditional models **ignore volatility compression** at ZLB.
$\Longrightarrow$ Yield forecasts revert too quickly to their long-term means!



Smooth Shadow-Rate DNS

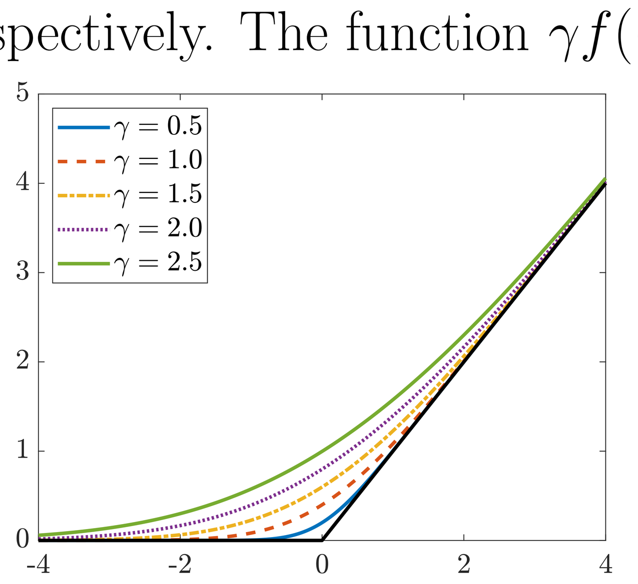
- Yield curve can be parameterized via Nelson-Siegel (NS) factor structure:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right),$$

  with VAR(1) dynamics for level, slope and curvature factors in $\boldsymbol{\beta}_t$.
  $\Longrightarrow$ Dynamic NS model does not restrict yields to be non-negative!
- Impose shadow-rate framework of Black (1995) in two ways on DNS:

Hard lower bound (B-DNS)	**Soft lower bound (SB-DNS)**
$\underline{y}_t(\tau) = \max\left(r_{LB}, y_t(\tau)\right)$	$\underline{y}_t(\tau) = r_{LB} + \gamma f\left(\frac{y_t(\tau)-r_{LB}}{\gamma}\right)$

  where $\gamma > 0$ governs the smoothness of the approximation.
- We adopt the function $f(x) = x\Phi(x) + \phi(x)$ with $\Phi(\cdot)$ and $\phi(\cdot)$ being the normal cdf and pdf, respectively. The function $\gamma f(\cdot/\gamma)$ is given by:



- Estimation proceeds with either (nonlinear) least squares or with the Extended Kalman filter in combination with maximum likelihood.

Pros of Smooth Shadow-Rate DNS

- Highly tractable model with **closed-form** ZLB yield curve expression.
  $\Longrightarrow$ No (numerical) approximations needed for estimation as in Wu and Xia (2016) and Christensen and Rudebusch (2015)!
- Flexibly allows for a **smooth transition** into and out of ZLB state.
- Can **easily be extended** with readily available DNS extensions.
  $\Longrightarrow$ Illustrate this with time-varying loadings and shifting endpoints!

Contribution to Literature

	Structural form (arbitrage-free)	Reduced form (model flexibility)
Traditional model (ignores ZLB)	✓	✓
Shadow-rate model (respects ZLB)	✓	**This paper**

Empirical Results

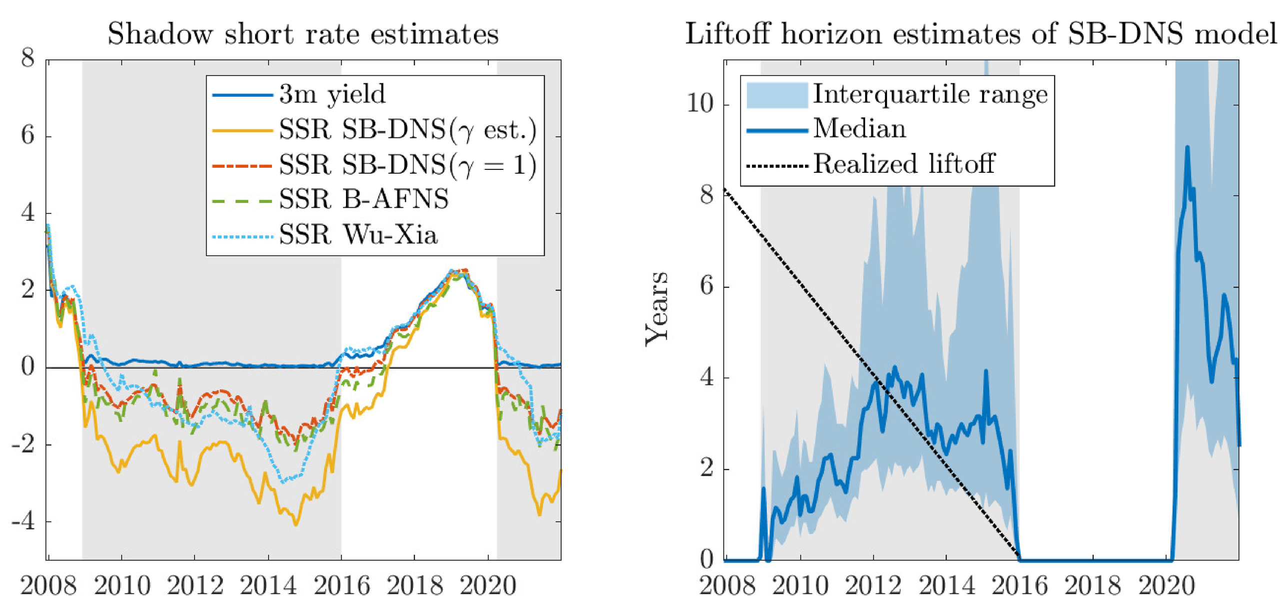
In-sample fit

Consider U.S. Treasury zero-coupon bond yields for eight maturities:

	Log-likelihood	**#Θ**	**AIC**	**BIC**
DNS	3010.4	27	-13.8	-13.6
B-DNS	3010.5	27	-13.8	-13.6
SB-DNS	3366.2	28	-15.5	-15.2
AFNS	2623.6	27	-12.0	-11.8
B-AFNS	3073.3	27	-14.1	-13.8

- Imposition of smooth shadow-rate framework leads to **improvements in fit** compared to (B-)DNS and (shadow-rate) arbitrage-free NS.
- Estimate of γ in SB-DNS model is 1.942 with standard error of 0.124.
  $\Longrightarrow$ Strong evidence of a **smooth transition** towards ZLB state!

Policy insights at the ZLB



Relative out-of-sample performance

Construct expanding-window based forecasts with $r_{LB} = 0\%$ and $\gamma = 1$:

	Maturities (in months)							
	3	**6**	**12**	**24**	**36**	**60**	**84**	**120**
Panel A: Six-month-ahead forecasts (h = 6)								
RW	1.01	1.02	0.99	0.91	0.88	0.89	0.92	0.98
DNS	1.01	1.01	1.01	1.02	1.03	1.02	1.03	1.05
AFNS	1.38	1.38	1.29	1.15	1.06	0.97	0.98	1.04
B-AFNS	0.93	0.98	1.02	1.04	1.06	1.04	1.05	1.06
Panel B: Two-year-ahead forecasts (h = 24)								
RW	1.03	1.02	0.98	0.89	0.83	0.80	0.80	0.83
DNS	1.02	1.02	1.02	1.02	1.02	1.02	1.03	1.04
AFNS	1.12	1.12	1.09	1.04	0.99	0.92	0.90	0.90
B-AFNS	1.06	1.08	1.09	1.08	1.06	1.01	0.99	0.97

Notes: Relative Root Mean Squared Forecast Errors (RMSFE) compared to SB-DNS model. Green cell indicates that SB-DNS has lower RMSFE.

- SB-DNS model **outperforms** DNS for all maturities and horizons.
- Random walk only outperformed for short maturities and arbitrage-free models for most maturities, but depends on horizon which ones

References

Black, F. (1995): "Interest Rates as Options," *The Journal of Finance*, 50, 1371–1376.

Christensen, J. H. E. and G. D. Rudebusch (2015): "Estimating Shadow-Rate Term Structure Models with Near-Zero Yields," *Journal of Financial Econometrics*, 13, 226–259.

Wu, J. C. and F. D. Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit and Banking*, 48, 253–291.