Non-linear Dependence and Portfolio Decisions over the life cycle

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Two Puzzles in Household Finance

Compared to model predictions, data indicates households are **not so interested in stock investment.**

1. Stock participation rate: < 50%
2. Risky share (conditional on participation): ≈ 55%

Why? A crucial element: labor income and its risk:

- Labor income process
- **Interplay between labor and financial markets** → main consideration in this paper

Model

1. Households' optimization problem with Epstein-Zin preferences:

$$V_{i,t} = \max_{\substack{\alpha_{i,t} \\ C_{i,t}}} \left\{ (1-\beta) C_{i,t}^{1-1/\psi} + \beta \left(E_t \left[p_{t+1} V_{i,t+1}^{1-\gamma} + b(1-p_{t+1}) X_{i,t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$$

Controls:

- $\alpha_{i,t}$: risky share -> controls the portfolio return $R_{i,t+1}^{p} = \alpha_{i,t} R_{i,t+1}^{p} + (1-\alpha_{i,t}) R_{i,t+1}^{p}$.
- $C_{i,t}$: consumption -> controls investment principal.

States:

- $X_{i,t}$: wealth $Y_{i,t}$: labor income R_t^S: stock return

Parameters:

- β: discount factor γ: risk aversion ψ: elasticity of intertemporal substitution
- p: survival probabilities b: bequest motive

Wealth Accumulation:

Investment participation cost income

$$X_{i,t+1} = \left(X_{i,t} - C_{i,t}\right) R_{i,t+1}^{p} - FI_p P_{i,t} + Y$$

2. Labor Income Process ($Y_{i,t}$)

$$\log Y_{i,t} = f(t, Z_{it}) + v_{it} + \varepsilon_{it} \text{ for } t \le K$$

$$v_{i,t} = v_{i,t-1} + u_{i,t}$$

$$u_{i,t} = \begin{cases} u_{i,t}^{(1)} \sim N(\mu_{u,1}, \sigma_{u,1}^2) \text{ with prob. } p_u \\ u_{i,t}^{(2)} \sim N(\mu_{u,2}, \sigma_{u,2}^2) \text{ with prob. } 1-p_u \end{cases}$$

3. Stock Returns Process (R_t^S)

$$R_t^S = R_f + \mu + \eta_t$$

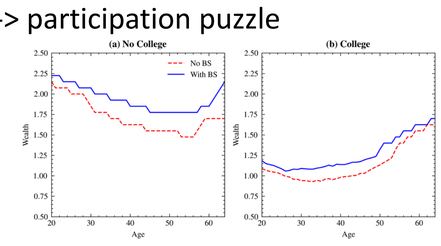
$$\eta_{i,t} = \begin{cases} \eta_{i,t}^{(1)} \sim N(\mu_{\eta,1}, \sigma_{u,1}^2) \text{ with prob. } p_\eta \\ \eta_{i,t}^{(2)} \sim N(\mu_{\eta,2}, \sigma_{u,2}^2) \text{ with prob. } 1-p_\eta \end{cases}$$

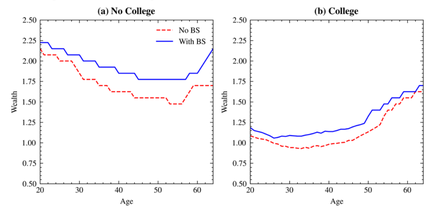
$\rho_{a,b} = corr(u_{i,t}^{(a)}, \eta_{i,t}^{(a)}),\ a=1,2,\ b=1,2.$
control the correlation and BS-Corr.

Effect of BS-Corr on investment decisions

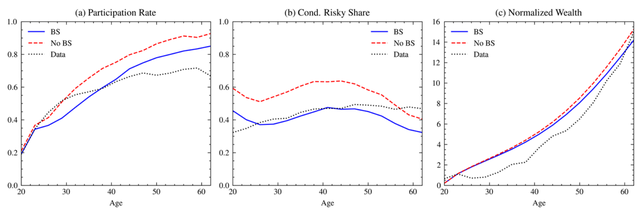
Including between-squares correlation **significantly**:

- Raise participation wealth threshold
  -> participation puzzle
- Lower the optimal risky asset shares
  -> risky share puzzle



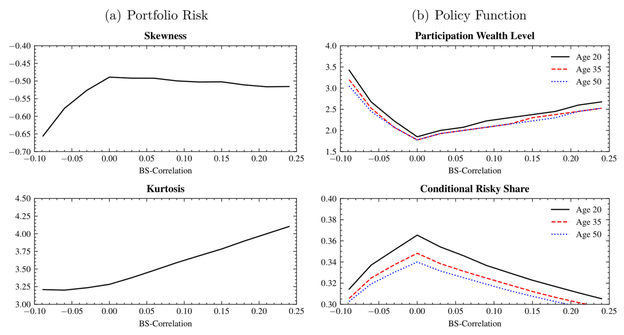


And thus, the model including BS-corr matches SCF data well.



Portfolio perspective of BS-Corr effect

Assume a **portfolio** including α share of stock and one unit of labor income flow.



Given other moments fixed (including correlation), between-squares correlation has nonlinear effect.

From panel (a) portfolio risk with changing BS-Corr

- (-)↗ 0: Skewness ↑, kurtosis →
- 0 ↗(+): Kurtosis ↑, skewness →
- |Between-squares correlation| ↑⇒ More risk

From panel (b) corresponding policy functions

- BS-Corr ≈ 0: more likely to enter the market
- BS-Corr deviates from 0: households reduce their risky asset holdings.

Between-squares Correlation (BS-Corr)

Definition: $Corr^{BS}(X,Y) \triangleq Corr\left((X-E[X])^2, (Y-E[Y])^2\right)$

Advantages:

1. Nonlinear feature -> A better measure of extreme co-movement
2. It is normalized co-kurtosis -> dependence through higher-order risks
3. More possible dependence patterns. Under mixture normal:

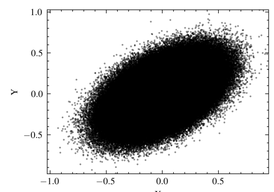


Figure 1. Scatterplot of linear correlation

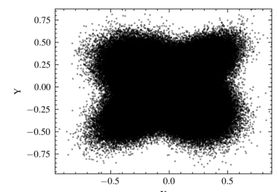


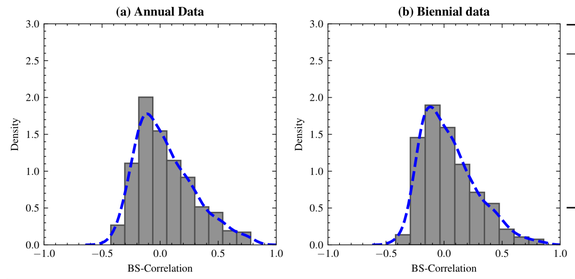
Figure 2. Scatterplot of BS-Corr

Stylized Facts of BS-Corr

Significant and positive BS-Corr between income and stock returns.

Data:

1. Ken French's data library: 1970-2017
2. PSID: annual 1970-1997 and biennially 1997-2017



	Full Sample	No College	College
Mean	0.044***	0.034**	0.070***
Median	0.006	0.007	0.005
25^{th} Percentile	−0.142	−0.152	−0.115
75^{th} Percentile	0.200	0.177	0.244
Fraction > 0	51.26%	51.63%	50.39%
Fraction > 0.1	34.71%	34.64%	34.88%
Fraction > 0.2	24.83%	22.55%	30.23%

Chart 1. Stylized Facts of Between-squares Correlation

- Positive across all groups
- 1/3 households > 0.1
- 1/5 households > 0.2

Calibration

Labor income

	No College	College
Mixture weight of u_{it} (p_u)	0.271	0.278
Normal distribution 1 mean ($\mu_{u,1}$)	−0.124	−0.156
Normal distribution 2 mean ($\mu_{u,2}$)	0.046	0.060
Normal distribution 1 standard deviation ($\sigma_{u,1}$)	0.172	0.231
Normal distribution 2 standard deviation ($\sigma_{u,2}$)	0.010	0.012
Standard deviation of transitory shock (σ_ϵ)	0.204	0.139
Dependence parameter 1 (ρ_1)	0.836	0.778
Dependence parameter 2 (ρ_2)	−0.164	−0.214

Stock Return

Mixture weight of η_t (p_η)	0.168
Normal distribution 1 mean ($\mu_{\eta,1}$)	−0.187
Normal distribution 2 mean ($\mu_{\eta,2}$)	0.038
Normal distribution 1 standard deviation ($\sigma_{\eta,1}$)	0.395
Normal distribution 2 standard deviation ($\sigma_{\eta,2}$)	0.127
Participation cost (F)	0.008

Dependence Structure:

	No College		College	
	Model	data	Model	data
Corr	0.038	0.038	0.033	0.034
BS-Corr	0.046	0.046	0.069	0.070

- Corr ≈ 0 (consistent with literatures).
- Small BS-Corr but significant effect.
- Precise calibration.

Preference:

	No College	College
Risk aversion (γ)	4.3	4.3
EIS (ψ)	0.9	0.3
Discount factor (β)	0.90	0.98
Bequest motive (b)	2.5	2.5

- Moderate risk aversion level.
- College group is more patient.
- Low fixed cost rate = 0.008.

Please refer to our working paper for more calibration details.

Empirical Evidence of BS-Corr

Data: PSID & CRSP (1997-2017)

BS-Corr has

- Significant effect
- **Nonlinear effect**

Models:

- Probit regression for participation rate
- Tobit for conditional risky share

	Probit participation model			Tobit investment model		
	Full sample	No College	College	Full sample	No College	College
|Between-squares Correlation|	−0.2506***	−0.2457***	−0.2469***	−0.1097***	−0.1309**	−0.0938**
Correlation	−0.0160	−0.0434	−0.0273	−0.0078	−0.0320	−0.0052
$\ln(y)$	0.6596***	0.5427***	0.5222***	0.4562***	0.3251***	0.2820***
$age/10$	−0.2821**	−0.0621	−0.3507	−0.5140	0.4663	0.3135
$age^2/100$	0.0265	0.0061	0.0388	1.3090*	−0.2331	0.3324
$Marriage$	−0.3363***	−0.1542**	−0.3243***	−0.1855***	−0.0893*	−0.1405***
$FWealth/10^5$	0.2438***	0.2529***	0.1672***	0.0077***	0.0758***	0.0041***
$Std(d\delta)$	−0.0151	−0.0370	0.04220	0.06335	−0.0198	0.1143*
$Skew(d\delta)$	−0.0221	−0.0163	−0.0246	−0.0119	−0.0180	−0.0035
$Kurt(d\delta)$	−0.0254*	−0.0296	−0.0352*	−0.003655	−0.0037	−0.0127*

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Conclusion

1. We document the existence of between-squares correlation in the data.
2. Introducing between-squares correlation lowers participation rates and risky asset shares, conditional on participation.
3. The perspective from portfolio helps understand between-squares correlation better and shows a nonlinear pattern.
4. Empirical evidence supports the model's prediction, and the nonlinear pattern of between-squares correlation's effect.

Reference

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