Jump Bidding as a Signaling Game

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Abstract

This paper studies jump bidding in an ascending-bid auction with affiliated values using a multi-round signaling model. Bidders communicate their private information with one another via the sizes of jump bids. These signals are credible since bidders with lower private information incur a higher *ex ante* cost for choosing a jump bid with any given size. This prevents the bidders with lower private information from mimicking those with higher private information. In equilibrium, the signaling model predicts that the size of a jump bid placed each round is bounded above by a strategy that is equivalent to one in a first-price sealed-bid auction. The expected revenue to the seller is reduced due to bidders' abilities to send signals through jump bidding. Using data from a spectrum auction held by the Federal Communications Commission in the United States, the mean valuation estimated using the signaling model is higher compared to that of the "open exit" model. This implies that if bidders are indeed using jump bids as signals, ignoring it leads to estimates of the mean values that are biased downwards. This result is consistent with the prediction of the theoretical model that bidders pay lower prices with jump bidding than in an open exit auction. I estimate that if jump bidding was prohibited, the government could have had 8% higher revenues from the auction.

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1 Introduction

Jump bidding in an ascending-bid auction refers to the action of placing a bid in excess of what is required by the auctioneer for a particular round. It is a common phenomenon in art auctions, spectrum auctions and online auctions. It is also observed in auction-like settings such as takeover bids.

In the auction literature, an ascending-bid auction is often modeled as an "open exit" auction (also called a button auction, or a clock model), in which the price in each round increases by a small fixed amount set by the auctioneer. Bidders choose at which prices to drop out of the auction irrevocably. The open exit model leaves no scope to study the behavior of jump bidding, as bidders do not submit bids. In alternative models of the ascending-bid auction where bidders actively submit bids, jump bidding is weakly dominated by the strategy of bidding the minimum required price each round in the absence of two elements: transaction costs and signaling using bids between bidders. The two main theoretical strands of literature which explain jump bidding are concerned with these two elements.

This paper focuses on the informational role of jump bidding in an ascending-bid auction. There is substantial anecdotal evidence that bidders engage in jump bidding to intimidate their competitors so that they drop out of the auction sooner. In the earlier spectrum auctions organized by the Federal Communications Commission ("FCC"), jump bidding was pervasive. For example, in the very first FCC auction (the nationwide narrowband PCS auction) in 1994, 49% of all new high bids were jump bids that exceeded the high bid by more than two bid increments (Cramton, 1997). While the transaction cost theory could be applicable in this case, many economists believed that there were other factors at play given the large sizes of some of the jump bids and the fact that some were placed towards the end of the auctions. They suspected that bidders were using jump bids as signals of high valuation or "toughness" (McAfee and McMillan, 1996; Cramton, 1997).

In 1997, the FCC introduced click box bidding which limited the size of a jump bid. One of the motivations was arguably to reduce the potential anticompetitive effects of jump bidding. Before this change in auction rules, 15 spectrum auctions had been conducted by the FCC, bringing in a total revenue of $23 billion. Given the high stakes, if bidders were indeed using jump bids as signals and were successful in reducing the prices paid due to early withdrawal of competitors, the revenue effect could be significant. Any attempt to quantify this effect will require a theoretical framework that describes a signaling equilibrium in an ascending-bid auction.

Avery's 1998 paper provides an excellent starting point. The paper solves for equilibria of ascending-bid auctions with two symmetric bidders and affiliated values when jump bidding strategies may be employed as signals. In this model, each bidder receives a private observation, which is a noisy signal of her true valuation of the auctioned object. The private observations are strictly affiliated, meaning a large realized private observation for one bidder makes the other bidder more likely to have large realized private observations as well. The paper shows that if bidders are allowed to have a one-off opportunity to simultaneously choose from a discrete set of jump bids as their opening bids and continue according to prespecified asymmetric equilibrium in favor of the higher bidder, then there exists a unique symmetric signaling equilibrium where the size of the opening bid is weakly monotone in a bidder's private observation. In other words, the choices of jump bids are partially separating. The bidder with the higher private observation chooses a weakly larger jump bid. Each possibility of jump bidding provides a Pareto improvement for the bidders from the symmetric equilibrium of a second-price sealed-bid auction.

While Avery's paper provides a useful framework to empirically study the use of jump bidding as a signaling device, it has a number of stylized features that are incompatible with most empirical settings. It is a model limited to an auction with 2 bidders. More important, it only allows bidders a one-off opportunity to send signals through jump bidding at the beginning of the auction. Bidders' abilities to send signals in multiple rounds will change the predictions of the model.

This paper builds upon Avery's model and extends it to more than two players. By focusing on the equilibrium in which bidders are willing to drop out of the auctions as soon as they find out their private observations are not the highest, and extending the jump bid space from a bounded discrete set to a continuous set without an upper bound, the 2-player 2-stage game is transformed into a first-price sealed-bid auction.

One challenge of building a multi-round signaling model is the existence of multiple equilibria. Instead of focusing on equilibrium selection employing various refinement criteria, I instead construct a model that predicts characteristics of auction outcomes that are common to all equilibria. In particular, the model predicts the upper bounds for each bid placed depending on whether it is a jump bid. A jump bid is bounded above by a strategy that is equivalent to one in a first-price sealed-bid auction. On the other hand, the upper bound for a non-jump bid is the equilibrium strategy in an open exit model following Milgrom and Weber (1982). These model predictions suggest that the signaling model works like a hybrid model of a first-price sealed-bid auction and an open exit ascending-bid auction. To structurally estimate such a model involves estimating each of these two auction formats.

The data used in the structural estimation are from the FCC broadband PCS auction (C-block), or "Auction 5" that took place between December 1995 and May 1996. In this auction, the U.S. was divided into 493 regional markets and one license was offered for each market. The reason I chose this auction is two fold. First, Auction 5 brought in over \$10 billion of revenue, the highest among all 15 FCC spectrum auctions before jump bids were restricted by the introduction of click box bidding. Second, only small businesses (defined as those with annual revenues less than \$40 million) were eligible to participate. Compared to other auctions where the sizes of the participating firms are much more heterogeneous, Auction 5 is more suited to be described with the theoretical model of this paper which involves symmetric bidders.

The empirical context of a spectrum auction has an implication on the choice of information structure of the model (that is, a common value model vs. a private value model) . One would expect the value of a spectrum license to consist of both a common value component to reflect market attributes that are valued by all bidders, such as market size measured by population, and a private value component to allow values to differ across bidders. This suggests that a common value model, which is nested into the affiliated value framework of the theoretical model, is the most appropriate,.

Nonparametric identification of a common value model has been proven infeasible in both a first-price sealed-bid auction (Laffont and Vuong, 1996) and an ascending-bid auction (Athey and Haile, 2007). Given the challenges with nonparametric identification, I adopt a parametric approach by choosing a multiplicative specification and making distributional assumptions following Hong and Shum (2003).

The theoretical model places bounds on the bids observed. While a partial identification approach may be the most appealing, the implementation of such an approach is difficult. Haile and Tamer. (2003) point out that the lack of sufficient structure of an ascending-bid auction makes a mapping between the bids observed and the underlying demand structure challenging. They instead construct bounds on observed bids and partially identify the distribution functions using an independent private value model. However, it is infeasible to replicate this approach in a common value model due to the interdependence in distributions of bidder information and valuation. In this case, stronger assumptions become necessary. I follow the approach of Donald and Donald and Paarsch (1996), Paarsch (1997) and Hong and Shum (2003) where the last bid placed by each bidder (except the winner of the auction) is equal to the upper bound predicted by the theoretical model. This approach allows for point identification.

I estimate the distributional parameters of the signaling model using a simulated nonlinear least-squares approach following Laffont, Ossard and Vuong (1995) and Hong and Shum (2003). The estimation involves the computation of the equilibrium strategies for an open exit auction and a first-price sealed-bid auction respectively. The former has a closed form solution under the parametric assumptions. However, the equilibrium strategy of a first-price sealed-bid auction does not have a closed form solution. I compute it with a combination of simulation and numerical approximations.

In order to understand the impact of jump bidding on bidder valuation, I repeat the estimation using an open exit model. The mean valuation obtained is lower than the one estimated using the signaling model. This suggests that if bidders are indeed using jump bids as signals, ignoring it leads to an underestimation of the mean value. A counterfactual analysis using the estimation results from the signaling model suggests that if the FCC forbade the action of jump bidding and designed the auction following the open exit format, the total revenue would have been 8% higher. The result is consistent with the prediction of the theoretical model: by sharing information and coordinating among themselves using jump bids as signals, bidders are able to lower the prices they pay to the seller.

Understanding the revenue effect of jump bidding is imperative in designing an ascending-bid auction that maximizes the seller's revenue. This has direct welfare consequences if the seller is the government, as in the case of a spectrum auction. Theoretical models predict the direction of this revenue effect. It is up to the empirical research to quantify it. There is a growing body of empirical literature that studies the relation between auction revenue and jump bidding using reduced form methodologies, which I will discuss in more details in the next section. In short, these papers find a revenue effect in the opposite direction as predicted by the theoretical models potentially due to the endogenous effects of missing variables. The adoption of a structural approach in this case is particularly useful in shedding light on quantitative effect of jump bidding.

The remainder of the paper is organized as follows: Section 2 reviews the literature relevant to this paper; Section 3 introduces the theoretical model; Section 3 discusses the empirical strategy and identification; Section 4 describes the estimation methodology and presents the results; Section 5 discusses the counterfactual analysis; Section 6 concludes.

2 Literature Review

As briefly discussed earlier, there are mainly two strands of the theoretical literature that explain jump bidding. The first deals with transaction costs. If bidders incur a transaction cost every time a bid is placed, placing fewer but larger bids could reduce this cost. The transaction costs could be either pecuniary costs associated with revising and submitting a bid (Fishman, 1988; Daniel and Hirshleifer, 1998), or time costs due to bidders' impatience (Isaac, Salmon and Zillante, 2007; Kwasnica and Katok, 2009). This paper treats all costs of participating in an auction as sunk and assumes away any significant marginal transaction costs associated with placing an additional bid.

The second strand of literature is concerned with the informational role played by jump bids when bidders interact strategically. Jump bids are used as signals of private information either to deter the entry of new bidders (Easley and Tenorio, 2004) or to induce early withdrawal of existing bidders (Avery, 1998; Hörner and Sahuguet, 2007). This paper contributes to the theoretical literature by extending Avery's well-known signaling model to a form that is more compatible with auction settings in real life. It abstracts away from the effect of entry deterrence of an opening jump bid and treats entry as exogenous. Instead of information sharing, jump bids could also be used as tools to conceal information when some bidders have an information advantage over others (Ettinger and Michelucci, 2015, 2016).

The econometric literature on structural estimation of auctions with independent private values is vast. Hickman, Hubbard and Sağlam (2012) presents a comprehensive survey of structural econometric methods in auctions. In addition, Perrigne and Vuong (1999) survey on structural econometrics of first-price auctions and Athey and Haile (2007) discuss the nonparametric estimation methods of first-price auctions and ascending-bid auctions. However, when it comes to auctions with affiliated values, the literature becomes sparse due to considerable difficulties with identification. Laffont and Vuong (1996) show that a common value model in a first-price sealed-bid auction is unidentified nonparametrically. In fact, any affiliated value model is observationally equivalent to some affiliated private value model. Similarly in ascending-bid auctions, Athey and Haile (2002) prove that a common value model is generally not identified nonparametrically.

This paper is the first to analyze the revenue effect of jump bidding in ascending-bid auctions using a structural approach. The empirical research on jump bidding in spectrum auctions is limited and adopts a descriptive approach. McAfee and McMillan (1996) and Cramton (1997) analyze the first 3 and 6 FCC spectrum auctions respectively. Both papers document prevalent jump bidding behavior in these auctions but argue that it had little effect in deterring competition since most of the jump bids were eventually overtaken.

Based on the signaling model of this paper, whether an auction ends with a jump bid depends on the realization of the private observations of the top 2 bidders. If they are sufficiently dispersed, the auction ends with a jump bid with probability 1. Otherwise the auction ends with a jump bid with a positive probability. While it is true that if an auction does not end with a jump bid its revenue is the same as the one in an open exit model, the revenue reduction for those that do end with a jump bid is significant as demonstrated by the counterfactual analysis.

Jump bidding aside, there are two other papers that adopt a structural approach in estimating value distributions in an FCC spectrum auction. Hong and Shum (2003) develop an econometric model of ascending-bid auctions with affiliated values and bidder asymmetries. Fox and Bajari (2013) estimate the deterministic component of bidder valuations using a pairwise stability condition which results in a matching game.

There is a growing body of empirical literature on jump bidding in the Swedish and Norwegian housing markets, where houses are sold through broker-assisted ascending-bid auctions (Hungria-Gunnelin, 2018; Sommervoll, 2020; Khazal et al., 2020; Sønstebø, Olaussen and Oust., 2021). Using a reduced form approach, these papers investigate the motivation behind jump bidding and its effect on entry and withdrawal of bidders and the selling prices. The key findings are: 1) bidders' primary motive of jump bidding is to intimidate their competitors as indicated by the survey data; 2) jump bidding is effective in deterring entry and inducing early withdrawal of competitors; 3) auctions containing jump bids achieve a premium. While the first two findings are consistent with the signaling model of this paper, the third one appears to contradict its prediction. There are two potential explanations. First, the OLS regressions may not have controlled for all important attributes of the houses. It is even harder to control for the heterogeneity in individual tastes. Second, the participants in the housing markets are individuals, who are likely more susceptible to bounded rationality than the bidders in the spectrum auctions which are firms.

3 Theoretical Model

3.1 Timing

The timing of the auction game is as follows:

1. Before the auction, the auctioneer sets a reservations price $\rho^1 = 0$.

2. In round $r = 1$, all bidders submit bids $b_i^1 \geq \rho^1$ simultaneously.

3. At the end of round 1, all bidding information, including the value and identity of the bidder associated with each bid, is made public to all bidders. Each bidder (except the one(s) with the highest bid) publicly announces whether to drop out. The auction ends if only 1 bidder remains, who then becomes the winner and pays her bid.

4. If more than 1 bidder remains, the auctioneer sets the minimum required price for round $r = 2$ using the function $\rho^2 = P_{min}(p^1)$, where p^1 is the highest bid in round 1. $P_{min}(a) \geq a$, is strictly increasing, and is known to all bidders.

5. Repeat steps 2-4 until the auction ends.

3.2 Assumptions

There are $n \geq 2$ risk neutral bidders taking part in an ascending-bid auction of a single object. Each bidder i values the object at U_i, but does not observe U_i directly. Instead, each receives a scalar private observation $\tilde{X}_i$ about the object. $\tilde{X}_i$'s are identically distributed over the support $(0, \bar{X})$ and are strictly affiliated. Intuitively, strict affiliation means that large realized values for some of the variables make the other variables more likely to be large than small[1]. Formally, let z and z' be points in $\mathbb{R}^n$. Let $z \vee z'$ denote the element-wise maximum of z and z', and let $z \wedge z'$ denote the element-wise minimum. Strict affiliation requires that for all z and z',

$$f(z \vee z')f(z \wedge z) \geq f(z)f(z'). \tag{1}$$

[1]For a general definition of affiliation, see the Appendix of Milgrom and Weber (1982).

Bidder valuations are affiliated in the sense that bidder i's expected value of the object V_i is a function of private observations of all bidders $V_i = v(x_i, \{x_j\}_{j \neq i}) = E[U_i \mid x_i, \{x_j\}_{j \neq i}]$, where v is continuous and increasing in each argument. By assumption, the value function $v(\cdot)$ is the same for all bidders. This value function encompasses the common value case (which consists of both a private value component and a common value component) and two special cases: the private value case and the pure common value case. For the private value case, $V_i = v(x_i, \{x_j\}_{j \neq i}) = v(x_i)$, where bidder i's expected value only depends on her own private observation. For the pure common value case, $V_i = v(x_i, x_k, \{x_j\}_{j \neq i, j \neq k}) = v(x_k, x_i, \{x_j\}_{j \neq i, j \neq k}), \forall k \neq i$, that is, private observations of all bidders (including bidder i) enter into bidder i's value function symmetrically. In contrast, when both a private value component and a common value component are present (i.e. the common value model), a bidder's own private observation enters her value function differently from those of other bidders. Assume one places weakly more weight on her own private observation than her rival bidders', i.e. $V_i = v(x_i, x_j, \{x_k\}_{k \neq i, k \neq j}) \geq V_j = v(x_j, x_i, \{x_k\}_{k \neq i, k \neq j})$ iff $x_i \geq x_j$.

A jump bid is one that is "substantially" higher than the minimum required price set by the auctioneer for that round. Formally, a jump bid in round r is defined as $b \geq \rho^r + \bar{\kappa}(\rho^r)$, where $\bar{\kappa}(\rho^r) \gg 0$, and is known to all bidders.

3.3 A two-player, two-stage game by Avery

Placing a jump bid is not cheap talk because the bid may win the auction. Jump bids are costly to bidders since by placing a jump bid, bidders forego the possibility of winning at a lower price. However, if the cost of placing a jump bid (of the same size) differs across different "types" of bidders, it has the potential of being used as a signaling tool. In a symmetric equilibrium where the strategy is strictly increasing in the private observation x, a bidder prefers to lose if she can be convinced that there exists another bidder with a private observation higher than hers because winning requires her to pay above her expected value. If a jump bid of a given size is more costly to bidders with lower $x's$ than those with higher $x's$, then it can potentially be used as a signaling tool of the underlying value.

Before presenting the general model, I will first use a simple two-player, two-stage game taken from Avery's 1998 paper to illustrate the intuition behind using jump bids as signals for underlying values when bidder values are affiliated.[2] I will demonstrate

[2]See Avery (1998) Sections 3 and 4 for a more general discussion and more details.

that affiliation in private observations makes it more costly *ex ante* for the player with a lower x to place a jump bid than the one with a higher x.

The assumptions of this game follow the previous section. Additionally, assume $P_{min}(p^r) = p^r + \epsilon$, where ϵ is a small positive number, i.e. the auctioneer raises the minimum required price by a small amount from round to round. This is a standard assumption for an ascending-bid auction.

The game (referred to as the "metagame" hereafter) proceeds in two stages. In the first stage, the bidders simultaneously choose between two choices of bids: an ordinary bid 0 and a jump bid $K > 0$. The second stage resembles a standard ascending-bid auction, with the reservation price equal to the maximum bid from the first stage. In this stage, jump bids are not seen as signals by all bidders.

Let $S^*(x)$ denote a symmetric strategy played by both players at the second stage of the game, which represents the drop-out price in the ascending-bid auction. $S^*(x)$ is assumed to be increasing in x. Note that this strategy does not predict how a bidder behaves from round to round, but simply at which price the bidder will drop out. One example of such a symmetric strategy is $S^*(x) = v(x, x)$, derived by Milgrom and Weber (1982). There is an obvious subgame perfect equilibrium where both players bid 0 in the first stage, and follow $S^*(x)$ in the second stage. However, there exists another class of subgame perfect equilibria which deliver higher *ex ante* payoff to the bidders.

Let $S_a(x)$ denote a bidding strategy such that $S_a(x) < S^*(x)$ and $S^*(x) - S_a(x)$ is increasing. It can be shown that there exists a threshold $x^* \in (0, \bar{X})$ such that the following symmetric strategy is a subgame perfect equilibrium of the two-stage metagame: in the first stage, bid K if $x > x^*$, bid 0 otherwise; in the second stage, play $S_a(x)$ if outbid by the opponent in the first stage, play $S^*(x)$ otherwise. In this equilibrium, if the actions in the first stage are symmetric/asymmetric, the strategies played in the second stage are also symmetric/asymmetric. Table 1 summarizes the bidding functions of the second stage as a function of the first stage bidding. In addition, the threshold x^* is unique for a fixed two-tuple $\{S_a, K\}$.

Table 1: Bidding functions as a function of opening bids

Player 1's Bid	Player 2's Bid	
	K	0
K	$(S^*(x),\, S^*(x))$	$(S^*(x),\, S_a(x))$
0	$(S_a(x),\, S^*(x))$	$(S^*(x),\, S^*(x))$

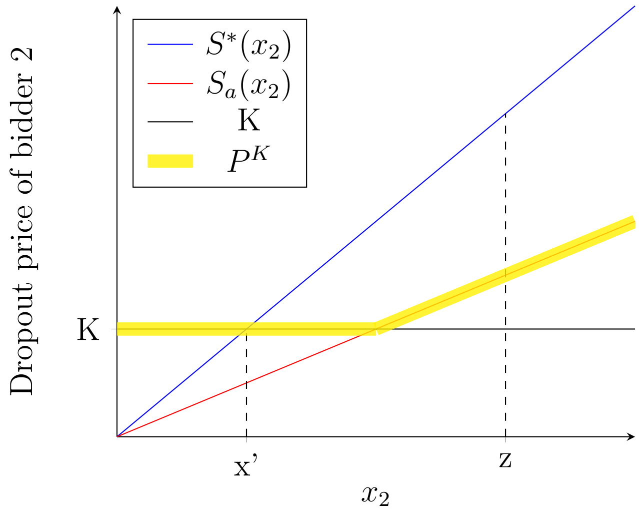

Figure 1: Symmetric equilibrium with jump bids

To prove this strategy is a subgame perfect equilibrium, I will start from the second stage of the metagame. Suppose it is common knowledge that $x_1 > x_2$, then the strategy pair $(S^*(x_1), S_a(x_2))$ form an equilibrium for the second stage. The reason is given $x_1 > x_2$, bidder 1 prefers to win since $V_1 = v(x_1, x_2) > v(x_2, x_2) = S^*(x_2) > S_a(x_2)$. Similarly, bidder 2 prefers to lose since $V_2 = v(x_2, x_1) < v(x_1, x_1) = S^*(x_1)$. Neither bidder has an incentive to deviate, the strategy pair is therefore an equilibrium of the second stage.

Given the equilibrium of the second stage, it remains to show that there exists a (partially) separating signaling equilibrium in the first stage with threshold x^*. Assume bidder 2 plays the following strategy: in the first stage, bid K if $x_2 > z$, z is fixed and is public knowledge; bid 0 otherwise. In the second stage, play $S_a(x_2)$ if outbid by bidder 1 in the first stage; play $S^*(x_2)$ otherwise. Conditional on $x_2 < z$, bidder 1 is able to induce bidder 2 to shift from a bidding strategy $S^*(x_2)$ to $S_a(x_2)$ in the second stage by placing a jump bid K. As shown in Figure 1, the yellow path $P^K(x_2) = \max(K, S_a(x_2))$ represents the price path faced by bidder 1 triggered by an unmatched jump bid. If $x_2 < x'$, bidder 1 is better off not placing a jump bid and face $S^*(x_2)$ since $S^*(x_2)$ lies below $P^K(x_2)$. If $x' < x_2 < z$, bidder i prefers to place a jump bid and faces the lower curve $P^K(x_2)$. Since $\tilde{X}_1$ and $\tilde{X}_2$ are strictly affiliated, then intuitively the larger the private observation x_1, the more likely that x_2 lies between x' and z, the more likely that bidder 1 chooses to place a jump bid.

Suppose that bidder 1 receives the private signal x and define

$$\phi(x,z) = E[S^*(x_2) - P^K(x_2) \mid x_1 = x, x_2 \leq z]. \tag{2}$$

$\phi(x,z)$ measures the expected reduction in price that bidder 1 has to pay conditional on placing a jump bid unmatched by bidder 2. $\phi(x,z) > 0$ implies that jump bidding by bidder 1 would be profitable conditional on bidder 1 winning. Since $S^*(x) - S_a(x)$ is increasing, $S^*(x) - P^K(x)$ is also increasing and continuous. As a result, $\phi(x,z)$ is continuous and strictly increasing in both arguments by the property of strict affiliation between x_1 and x_2. In addition, as depicted in Figure 1, $\phi(x,z)$ is negative for x near zero and positive for large x. By the intermediate value theorem, there is a unique value x^* such that $\phi(x^*,x^*) = 0$. Update bidder 2's strategy by replacing the threshold z with x^*. To complete the proof, it remains to show that player 1 does not want to deviate from a symmetric strategy that also uses x^* as the jump bid threshold in the first stage. There are 4 scenarios to consider: 1) $x_1 > x^* \geq x_2$; 2) $x_1 > x^*$, $x_2 > x^*$; 3) $x_1 \leq x^*$, $x_2 \leq x^*$; 4) $x_1 \leq x^* < x_2$. Scenario 1 is straight forward to show. Since $x_1 > x_2$, bidder 1 wins the auction regardless of her choice in the first stage. However, if she chooses to jump bid, the expected reduction in price is positive, i.e. $\phi(x_1, x^*) > 0$. Bidder 1 therefore does not deviate from the decision to jump bid. For the remaining 3 scenarios, Avery (1998) provides a detailed proof, which I will not repeat here.

The equilibrium in the first stage is partially separating as all bidders with $x's$ below threshold x^* ("low-value" types) choose one action and those with $x's$ above ("high-value" types) choose the other. Low-value types are not able to mimic high-value types because they face a higher *ex ante* cost attributed to the property of strict affiliation of private observations. If the outcome in the first stage is asymmetric, the bidder that chooses not to jump bid learns that her private observation is lower than her opponent's. As a result, she prefers to lose the auction than to win it. She is therefore indifferent between strategies S^* and S_a.[3] When $S_a \leq K$, she chooses to drop out right away at the end of the first stage. The willingness of lower types to drop out at prices lower than the equilibrium prices without signaling results in lower expected revenue to the auctioneer and higher expected profit to the bidders.

With independent private observations, there remains a unique signaling equilibrium with jump bids for each two-tuple $\{S_a, K\}$. However, this equilibrium is pooling,

[3] If a small transaction cost of ϵ is introduced every time a bid is placed, the bidder will strictly prefer S_a.

instead of partially separating. Both bidders choose the same action in the first stage, always leading to a symmetric outcome in the second stage. The expected revenue to the auctioneer is therefore equivalent to that under an ascending-bid auction without signaling. This result is in line with the what revenue equivalence theorem predicts.[4]

3.4 A symmetric equilibrium with one-round signaling

The simple 2-player 2-stage game has demonstrated that the existence of a partially separating signaling equilibrium is directly attributed to the property of strict affiliation. The same intuition applies when the auction is extended to allow for endogenous jump bids. Proposition 1 by Avery describes the conditions for the existence and the characteristics of a unique symmetric signaling equilibrium of a 2-player 2-stage game with endogenous jump bids in the first stage.

Proposition 1. *(Avery).*[5] *Suppose that the two bidders choose their opening bids endogenously and simultaneously from a fixed set of n possible bids and that they continue according to a prespecified asymmetric equilibrium in favor of the higher bidder. Then there is a unique symmetric signaling equilibrium, with strategies identical to those strategies for an n-stage descending signal game with the same set of possible jump bids.*

I will broadly outline the idea behind this proposition. For a detailed proof, see Avery (1998). First consider a fixed set of jump bids $\{K_1, K_2, ..., K_n\}$ where $K_1 > K_2 > ... > K_n > 0$. Unlike in the two-stage game, bidders may now signal in more than one round. In round 1, bidders face a set of $\{0, K_1\}$. If at least one bidder chooses K_1, no more signaling is allowed and bidders proceed to play S^* or S_a as in the simple game. If both bidders choose 0, then they proceed to round 2 and face the new choice set $\{0, K_2\}$. Bidders can now signal up to n rounds. The threshold x_r^* in round r is determined the same way as in the simple game and x_r^* is decreasing in r. The subgame that starts from the last round of signaling is identical to the simple game. Conditional on the existence of a unique symmetric equilibrium for this subgame, adding one more signaling round to the beginning of this subgame retains the structure of the simple game and the proof that a unique symmetric equilibrium exists for the bigger subgame

[4]The revenue equivalence theorem specifies a class of auctions that would generate the same expected revenue to the seller, in particular the independent private values model must apply. See Milgrom and Weber (1982), Theorem 0.

[5]This corresponds to Theorem 4.7 in Avery (1998)

is identical to the proof of the simple game. Hence by iteratively adding a signaling round to the beginning of the expanded subgame, it can be shown that the multi-round signaling game with descending jump bids has a unique symmetric equilibrium.

Next, suppose that bidders again can only signal in the first round, but are now allowed to choose their opening bids from the descending set $\{K_1, K_2, ..., K_n, 0\}$. Consider the choice of the lowest jump bid K_n in the simultaneous game of the first round. In equilibrium, the only bidders who will not make at least that bid are those with the lowest private observations. Suppose the threshold for such a bid is z_n. z_n is identified by finding the level of private observation which makes the receiver indifferent between bidding 0 and K_n, conditional on the opponent's private observation being below z_n. This condition is identical to the condition for the threshold in the n^{th} round of signaling in the multi-round signaling game. By inductive reasoning, the n thresholds in this single-round signaling game are equal to the thresholds in the multi-round signaling game with descending jump bids, thereby producing a unique equilibrium.

While Proposition 1 provides an excellent starting point for a structural model for estimation, there are two issues that need to be resolved. First, the equilibrium changes with the specification of S_a, the second-stage strategy played by the bidder with the lower jump bid from the first stage. S_a will need to be fixed to avoid multiple equilibria. The second is that the model needs to be generalized from a model with two bidders to a model with more than two.

Proposition 2. *The following symmetric strategy is a subgame perfect equilibrium of the multi-player ascending bid auction: in the first round of the auction, place a bid following a symmetric first-price sealed bid strategy $S_{1st}(x)$; if outbid by any bidder in the first round, drop out immediately, otherwise move on to the second round; from the second round onward, follow an ascending-bid strategy with drop-out price equal to $S^*(x \mid \hat{\Omega})$, where $\hat{\Omega}$ denotes all information available up to that point of the auction.*

Proof. This equilibrium can be written as $\{S_{1st}(x), (0, S^*(x \mid \hat{\Omega})\}$. Proposition 1 holds for any discrete set. Consider an equally spaced discrete set defined by $\{\underline{K}, \overline{K}, \delta\}$, where $\underline{K} > 0$ is the lower bound, $\overline{K} < \infty$ is the upper bound, and $\delta > 0$ is the equal spacing. Fix $S_a(x) = 0$. This implies that as long as the two bidders do not choose the same jump bid in the first round, the one with the lower jump bid drops out at the end of the first round. As $\underline{K} \to 0$, $\overline{K} \to \infty$, and $\delta \to 0$, the choice set approaches the set of non-negative (real) numbers. As the private observations are drawn from a continuous distribution, the probability that the two bidders draw the same private observations is zero. Hence the probability that they choose the same jump bid approaches 0 and the probability that the auction ends after the first round approaches 1. In the limit, the bidders are free to choose any nonnegative bid in the first (and only) round of the auction. The one with the lower bid drops out and the other one wins the auction and pays her own bid. This describes a first-price sealed bid auction. Let $S_{1st}(x)$ denote the strictly increasing symmetric equilibrium strategy for a first-price sealed bid auction, where $S_{1st}(x_i)$ maximizes $E[(U_i - b_i)\mathbb{1}(S_{1st}(x_j) \leq b_i) \mid x_i]$. This equilibrium strategy can be easily extended to when there are more than 2 bidders by redefining $x_j = \max_{k \neq i} x_k$.

In the limit of $\underline{K} \to 0$, $\overline{K} \to \infty$, and $\delta \to 0$, the auction beyond the first round becomes degenerate. Nevertheless, a strategy is needed for a subgame perfect equilibrium. I will adopt the strictly increasing symmetric strategy $S^*(x \mid \hat{\Omega})$ for 2 or more bidders by Milgrom and Weber (1982). Note that the key difference between an ascending auction with 2 bidders and one with more than 2 bidders is that more information is revealed in the course of the auction in the latter case. When a bidder drops out of the auction, if the auction does not end (i.e. when there are two or more bidders left), the remaining bidders are able to infer the private observation received by the bidder who drops out from her drop-out price. The remaining bidders can therefore update their expectations since the value function is assumed to be affiliated. $\hat{\Omega}$ denotes the set of drop-out prices observed up to a particular point of the ascending-bid auction. □

If either $\underline{K} \to 0$ or $\overline{K} \to \infty$ (or both) is relaxed, the first round can be interpreted as a first-price sealed bid auction with a price floor or a price ceiling (or both). In each case, there is a probability mass for the first-round bids at 0 or $\overline{K}$. The auction beyond the first round is no longer degenerate.

3.5 Multi-round signaling

This section explores an extension of the multi-player model with one round of signaling to one with multiple rounds of signaling.

Proposition 2 describes an equilibrium in which bidders send signals through jump bidding in the first round only. Beyond the first round, the implicit assumption is that any further jump bidding will not be interpreted as signals. If this assumption is relaxed, that is, jump bids in rounds beyond the first round are also seen as signals, then this provides further opportunities for bidders to increase the expected payoff.

Consider a simple case in which bidders can signal in the first 2 rounds. Again assume $\underline{K} \to 0$, $\overline{K} \to \infty$, and $\delta \to 0$. Consider the following strategy $\{S_{1st}(x), (0, S_{1st}(x)), (0, S^*(x \mid \hat{\Omega}))\}$. The difference between this strategy and the one in Proposition 2 is that in the second round, a bidder plays $S_{1st}(x)$ again if she is not outbid by any other bidders in the first round. If all bidders follow this strategy, the auction again ends after the first round. However, this strategy is no longer an equilibrium. Assume all bidders but i play this strategy. Consider an alternative strategy for bidder i where she places a reduced bid $\tilde{S} = S_{1st}(x_i) - \theta > 0$ in the first round. Let $S_{1st}(x_j)$ be the highest bid in the first round among the rest of the bidders. If $S_{1st}(x_j) > S_{1st}(x_i)$ (that is, $x_j > x_i$), then bidder i drops out right away and her payoff remains 0. If $\tilde{S} \leq S_{1st}(x_j) < S_{1st}(x_i)$, bidder i can move onto the second round with bidder j and bid $S_{1st}(x_i)$ to win the auction and receive the same payoff. In the last scenario when $\tilde{S} > S_{1st}(x_j)$, bidder i wins the auction at a lower price. Since the probability of the last scenario is non zero, the alternative strategy delivers a strictly higher *ex ante* payoff.

The two-round signaling example demonstrates that there is an incentive to place a bid lower than $S_{1st}(x_i)$ in the earlier round due to the possibility of winning at a lower price. The same argument applies when bidders can signal in more than 2 rounds, and in an unlimited number of rounds.

There are additional benefits for placing a bid lower than $S_{1st}(x_i)$ in earlier rounds when bidders can signal in multiple rounds. Bidders are able to infer more information about the private observations other bidders receive from their decisions to drop out or to stay in the auction using the inverse of the equilibrium strategy. This information affects both the expected value of the object as well as the expected probability of winning. In particular, having more information will allow bidders to form an expected value closer to the true value of the object, thus reducing the winner's curse typical to a sealed-bid auction. This ability to collect more information along the course of the auction and update one's expectations accordingly is what differentiates an ascending-bid auction from a second-price sealed bid auction. In this case, it also marks the difference between a multi-round signaling game and a one-round signaling game. I use $\tilde{S}_{1st}(x_i \mid \Omega^r)$ to denote the strategy that maximizes $E[(U_i - b_i)\mathbb{1}(\tilde{S}_{1st}(x_j \mid \Omega^r) \leq b_i) \mid x_i, \Omega^r]$, where Ω^r is the set of information revealed up to round $r - 1$.

3.6 General predictions for equilibria with multi-round signaling

When multi-round signaling is allowed and the decision to stop placing a jump bid becomes endogenous, it is complicated to specify a pure-strategy equilibrium. The very nature of a multi-round auction means the cost for bidders to make a mistake is low, in particular in earlier rounds. This suggests that there are likely multiple equilibria. It is complicated to predict which equilibrium is more likely to be played in reality. Instead of trying to argue which equilibrium prevails, I will focus on predicting some common characteristics shared by all the equilibria. Note that there is no limit on how many rounds a bidder can signal through jump bids. The only restriction is that once a bidder stops placing a jump bid in a round, she will not be allowed to do so in all future rounds. I will call the earlier rounds where jump bids are placed the signaling stage and the later rounds where non-jump bids are placed the ascending-bid stage.

Proposition 3. *For any symmetric (pure) strategy that constitutes a SPE of the multi-player ascending bid auction with multi-rounds of signaling, the following holds:*

1. *if b_i^r, the bid in round r by bidder i, is a jump bid, i.e. $b_i^r \geq \rho^r + \bar{\kappa}(\rho^r)$, then $\tilde{S}_{1st}(x_i \mid \Omega^r) \geq b_i^r$;*

2. *if b_i^r is not a jump bid, i.e. $\rho^r \leq b_i^r \leq \rho^r + \bar{\kappa}(\rho^r)$, then $\tilde{S}_{1st}(x_i \mid \Omega^r) \leq \rho^r + \bar{\kappa}(\rho^r)$ and $b_i^r < S^*(x_i \mid \hat{\Omega})$;*

3. *if a bidder i drops out at the end of round r, then either there is a high enough signal, i.e. $b_j^r \geq \rho^r + \bar{\kappa}(\rho^r)$ and $b_j^r \geq \tilde{S}_{1st}(x \mid \Omega^r)$, where b_j^r is the highest jump bid of round r; or the expected value is lower than the minimum required price of the next round, i.e. $S^*(x \mid \Omega^{r+1}) < \rho^{r+1}$.*

Proof. As discussed earlier, in round r, with additional information collected from the dropout decisions of other bidders, $\tilde{S}_{1st}(x_i \mid \Omega^r)$ gives the jump bid that makes bidder i indifferent between placing a jump bid and placing a bid equal to the minimum required amount. As a result, $\tilde{S}_{1st}(x_i \mid \Omega^r)$ serves as the upper bound of bidder i' jump bid in round r for all equilibria.

When bidder i places a non-jump bid, this means she has endogenously chosen to end the signaling stage as the smallest jump bid now exceeds the upper bound. She then moves on to follow the strategy in an ascending-bid auction, which specifies a drop out price at $S^*(x_i \mid \hat{\Omega})$.

A bidder decides to drop out at the end of round r as soon as she receives a signal that puts the sender's private observation above her own. This means there must be at least one jump bid that is higher than the upper bound of bidder i's jump bid in round r. There is another reason for the bidder to drop out. This happens when ρ^{r+1}, the minimum required price for the next round exceeds the drop-out price given by $S^*(x \mid \Omega^{r+1})$.

□

Proposition 3 places bounds on the observed bids based on whether a bidder places a jump bid and her dropout decision at the end of each round. These model predictions suggest that the signaling model works like a hybrid model of a first-price sealed-bid auction and an open exit ascending-bid auction. To structurally estimate such a model involves estimating each of these two auction formats.

4 Empirical Strategy

4.1 Auction background and data

In 1997, the FCC introduced "click box bidding" in Auction 16. Participants placed bids by simply clicking on the license numbers and all bids were exactly one increment above the standing high bid. It was believed that the change in auction rule was to primarily address the issues of jump bidding and code bidding[6] (Cramton and Schwartz, 2000; Bajari and Yeo, 2009). In later auctions, this rule was relaxed to allow bidders to place jump bids up to 9 bid increments. Nonetheless bidders' ability to signal using jump bids was restricted, making the later auctions less suitable for studying jump bidding.

Among the first 15 auctions where bidders faced no restriction on bid size, Auction 5 (broadband PCS auction (C-block)) was the largest in terms of total revenue raised (over \$10 billion). It was only open to small businesses with annual revenues less than \$40 million. Compared to other auctions, the bidders in Auction 5 are more homogeneous in size, therefore more appropriate to be described as symmetric, which

[6]Code bidding refers to the practice of attaching market numbers in the trailing digits of bids to signal key interest.

is a key feature of the theoretical model. In this auction, the U.S. was divided into 493 markets ("Basic Trading Areas"). One license was offered for each market. 255 small businesses took part in the auction.

In the empirical exercise that follows, I treat the auction of each of the 493 licenses as an independent auction and assume each auction follows the rules detailed in the theoretical section. However, in real life, these auctions ran in parallel following a simultaneous multiple round format. Each round, bidders simultaneously placed bids on all the licenses they were interested in. A key difference between the rules of the theoretical model and those of Auction 5 is that bidders did not need to announce whether they dropped out of the auction for a particular market at the end of each round. It is therefore unclear at which point the dropout took place. Following Donald and Paarsch (1996), Paarsch (1997), and Hong and Shum (2003), I assume this happened immediately after a bidder placed her last bid in a market and the other bidders made the same inference. While this assumption may not always hold, the eligibility rule of Auction 5 encouraged participants to actively bid in all markets they were interested in by making the maximum number of bids a bidder could place in future rounds dependent on the number of bids she placed in the current round. If a bidder adopted a "sit-and-wait" strategy in too many markets, she might eventually lose the eligibility to contest in some of the markets that she stood a chance to win. The eligibility rule could therefore be interpreted as a milder version of the irrevocable dropout assumption made in the theoretical model.

4.2 Define a jump bid

The theoretical model defines a jump bid in round r as $b \geq \rho^r + \bar{\kappa}(\rho^r)$, where $\bar{\kappa}(\rho^r) >> 0$, and is known to all bidders. In auctions where bidders can only place a bid that either equals the minimum required price, or exceeds it by whole multiples of a fixed increment set by the auctioneer (for example in FCC Auction 17 and some art auctions), the definition of a jump bid is unambiguous. However, in Auction 5, bidders faced no restrictions on the choice of bids, as long as they were not below the minimum required price. This begs the question: how should a jump bid be defined in this empirical context?

In the data, out of all the bids that exceed the minimum required price, over 20% exceed the minimum required price by \$100 or less, and over 40% exceed it by \$1000 or less (see Figure A in Appendix A). In a signaling model, jump bids are intended as tools to disclose some information about one's private observation. However, bidders could aim to achieve other goals with small jumps. For example, some may simply want to avoid a tie with another bidder and become the highest bidder of that round. In Auction 5, the highest bidder in a round does not need to raise her own bid in the following round. Further, since the FCC always sets the minimum required price in multiples of thousands, i.e. the last three digits are zeros, all code bids (bids with the market numbers attaching to the end) exceed the minimum required prices. Code bids send signals in a different way than jump bids.

It is entirely possible that bidders do not agree on how much a bid must exceed the minimum required price to be interpreted as a jump bid in the way described by the theoretical model. However, if the majority of them agree on a common definition, is there a way to identify it? Suppose a bidder believes a jump bid exceeds the minimum required price by at least $\bar{\kappa}$, then bidding $\rho^r + \bar{\kappa} - \epsilon$ is strictly dominated by bidding $\rho^r + \bar{\kappa}$ in most cases since the value of signaling exceeds ϵ when ϵ is small.[7] This suggests that we may observe "bunching", defined as a spike in the probability distribution over sizes of jumps, at the true $\bar{\kappa}$. Figure A is a histogram of the empirical distribution of jumps. Each bar correspond to a bin with width of 50. We observe distinct spikes for the following bins with lower bounds at 0, 100 (this bin contains jump sizes from 100 to 149), and whole thousands. Considering that numbers that are multiples of 100 and 1000 may contain additional informational value or associate with lower transaction cost, I remove bids with jump sizes equal to 100, 500, 1000 and 2000. The spikes with lower bounds at 100 and 1000 persist after the adjustment (see Figure A). The spike at 1000 become more prominent in the later rounds of the auction (see Figure A).

Based on the empirical evidence, I define $\bar{\kappa}$ at 1000. Compared to 100, 1000 has the added benefit of excluding all potential code bids (since there are 493 markets, a code bid exceeds the minimum required price by at most \$493). Further, a relatively stringent criterion for a jump bid will reduce the probability of over stating the revenue effect of jump bidding.

4.3 A parametric approach to estimation

As discussed earlier, the affiliated value framework of the theoretical model nests the private value model, the pure common value model and the common value model,

[7] $\rho^r + \bar{\kappa} - \epsilon$ is strictly dominated by $\rho^r + \bar{\kappa}$ if $\rho^r + \bar{\kappa} \leq \tilde{S}_{1st}(\cdot)$, and is strictly dominated by ρ^r if $\rho^r + \bar{\kappa} > \tilde{S}_{1st}(\cdot)$.

which includes a common value component and a private value component. In the context of a spectrum auction, one would expect the presence of both a common value component to reflect market attributes that are valued by all bidders, such as market size measured by population, and a private value component to allow values to differ across bidders. This suggests that a common value model is the most appropriate in this empirical context.

The difficulties with nonparametric identification of a common value model are well recognized in the econometric literature. Laffont and Vuong (1996) show that a common value model in a first-price sealed-bid auction is unidentified nonparametrically. In fact, any affiliated value model is observationally equivalent to some affiliated private value model. Similarly in ascending-bid auctions, Athey and Haile (2002) prove that a common value model is generally not identified nonparametrically. Given the challenges with nonparametric identification of a common value model, I adopt a parametric approach following Hong and Shum (2003).

I assume that U_i, the value of the object to bidder i, takes a multiplicative form $U_i = A_i V$, where A_i is a bidder-specific private value for i, and V is a common value component unknown to all bidders. V and A_i's are assumed to be independently log normally distributed. Let $v \equiv lnV$, $a_i \equiv lnA_i$, $u_i \equiv lnU_i$:

$$
\begin{aligned}
v &= m + \epsilon_v \sim N(m, r_0^2), \\
a_i &= \bar{a} + \epsilon_{a,i} \sim N(\bar{a}, t^2), \\
u_i &= m + \bar{a} + \epsilon_v + \epsilon_{a,i} \sim N(m + \bar{a}, r_0^2 + t^2).
\end{aligned}
$$

Each bidder receives private observation $X_i = U_i \cdot \exp(s\xi_i)$, where ξ_i is an unobserved error term with a standard normal distribution, and s is an unobserved parameter. Let $x_i \equiv lnX_i$, then conditional on u_i,

$$
x_i = u_i + s_i\xi_i \sim N(u_i, s^2).
$$

As a result, $(u_i, x_i, i = 1, ..., N)$ are jointly normally distributed. The joint distribution is fully characterized by parameters $\{m, r_0, \bar{a}, t, s\}$. These parameters are common knowledge among the bidders and are what I will estimate using the signaling model.

4.4 Identification

As pointed out by Haile and Tamer. (2003), the lack of sufficient structure of an ascending-bid auction makes a mapping between the bids observed and the underlying demand structure challenging. Point identification typically relies on observing at which price a bidder drops out. Haile and Tamer. (2003) instead construct bounds on observed bids and partially identify the distribution functions using an independent private value model.

As the theoretical model of this paper also places bounds on the bids observed, a partial identification approach could naturally follow. However, it is difficult to implement the approach by Haile and Tamer. (2003) in a common value model due to the interdependence in distributions of bidder information and valuation. In this case, stronger assumptions become necessary. I follow the approach of Donald and Paarsch (1996), Paarsch (1997) and Hong and Shum (2003) and assume that the last bid placed by each bidder (but the winner of the auction) is equal to the dropout prices predicted by the theoretical model. This assumption allows for point identification. As discussed in Section 4.1, whether this assumption is valid depends on whether the bidder indeed drops out immediately after placing the last bid. If instead the bidder stays in the auction for a few additional rounds without placing any bid, the last observed bid is below the real dropout price. While this scenario could happen in Auction 5 because a formal announcement of dropout was not required, an eligibility rule was put in place to discourage such a "sit and wait" strategy by reducing the maximum number of bids a bidder could place in each future round for bidders who did not place enough bids in a few consecutive rounds.

5 Structural Estimation

5.1 Simulated nonlinear least-squares (SNLS)

I use a simulated non-linear least squares estimator following the methodology of Laffont et al. (1995) and Hong and Shum (2003). This estimator minimizes the nonlinear least-squares objective function:

$$Q_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=2}^{N_t}(p_i^t - m_i^t(\theta))^2. \tag{3}$$

p_i^t is the observed log dropout price for bidder i in auction t. For tractability, I number the bidders in an auction in the reverse of the dropout order. For example, in auction t, the winner is labelled bidder 1, and the bidder who is the first to drop out is labelled bidder N_t, where N_t is the total number of bidders in this auction. Notice that the bidder number in Equation 3 starts from 2. This is because the dropout price for the winner is not observed since she never dropped out. $m_i^t(\theta)$ is the model-predicted counterpart of p_i^t, which is the mean of a multivariate truncated distribution. Due to the difficulties of computing $m_i^t(\theta)$ analytically, it is natural to replace it with a consistent simulation estimator $\tilde{m}_i^t(\theta)$.

$$\tilde{m}_i^t(\theta) = \frac{1}{S}\sum_{s=1}^{S}(b_{i,s}^t(\vec{x}_s;\theta)), \tag{4}$$

where $b_{i,s}^t(\vec{x}_s;\theta)$ is the model predicted dropout price for bidder i in auction t for a particular draw of $\vec{x}_s$ from the joint distribution.

The updated objective function is:

$$\bar{Q}_{S,T}(\theta) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=2}^{N_t}(p_i^t - \tilde{m}_i^t(\theta))^2. \tag{5}$$

Laffont et al. (1995) show that while $\tilde{m}_i^t(\theta)$ is a consistent estimator of $m_i^t(\theta)$, $\bar{Q}_{S,T}(\theta)$ is not a consistent estimator of $Q_T(\theta)$ for any fixed number of simulations S as T goes to infinity. It can be shown that the size of the bias for each auction t is:

$$\Delta_S(\theta) = \frac{1}{S(S-1)}\sum_{s=1}^{S}(b_{i,s}^t(\vec{x}_s;\theta) - \tilde{m}_i^t(\theta))^2. \tag{6}$$

As a result, to obtain a consistent simulation estimator for $Q_T(\theta)$, simply subtract Equation 6 from Equation 5:

$$\tilde{Q}_{S,T}(\theta) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=2}^{N_t}\{(p_i^t - \tilde{m}_i^t(\theta))^2 - \frac{1}{S(S-1)}\sum_{s=1}^{S}(b_i^t(\vec{x}_s;\theta) - \tilde{m}_i^t(\theta))^2\}. \tag{7}$$

As discussed in Section 4.4, $b_{i,s}^t(\vec{x}_s;\theta)$ in the signaling model is predicted by the equilibrium strategy of a first-price sealed-bid auction if the observed bid is a jump bid; and is predicted by the equilibrium strategy of an open exit auction if the observed bid is not a jump bid. In the rest of this section, I will discuss how each of the two is computed in auction t with simulated draw $\vec{x}_s$. For simplicity, I will suppress the superscript t.

5.2 Computing the open exit strategy

The equilibrium strategy of an open exit auction with symmetric affiliated value follows Milgrom and Weber (1982). In equilibrium, the price β_i at which bidder i drops out of the auction is defined as follows:

$$\beta_i(X_i) = E[U_i \mid X_i; X_j = X_i, j = 1, 2, ..., i-1; X_k, k = i+1, ..., N]. \tag{8}$$

Since bidder valuations are affiliated, the expected valuation of each bidder depends on the private observations of all bidders. For each bidder i, the private observations can be partitioned into 3 groups. The first group includes bidder i's own private observation X_i. The second group consists of X_j's in Equation 8, the private observations of the bidders that remain in the auction. Since these private observations are unobserved by bidder i, she needs to form expectations on them. Given the assumption of symmetric bidders, bidder i simply assumes $X_j = X_i$. The last group includes X_k's, the privates observations of the bidders that dropped out before bidder i. While bidder i does not observe these private observations at the beginning of the auction, she can infer them by inverting the bidding strategy after observing at which prices these bidders drop out.

Given the parametric assumptions per Section 4.3 , the log dropout price has a closed-form solution that can be easily computed with each simulated draw $\vec{x}_s \equiv (x_s^1, ..., x_s^N)'$. Let $\hat{x}_i = (\underbrace{x_s^i, ..., x_s^i}_{\text{first i elements}} , \underbrace{x_s^{i+1}, ..., x_s^N}_{\text{last N-i elements}})'$.

$$\begin{aligned} b_i(\vec{x}_s) =& ln\beta_i(X_i) \\ =& E[u_i \mid \hat{x}_i] + \frac{1}{2}Var[u_i \mid \hat{x}_i]. \end{aligned} \tag{9}$$

Denote the marginal mean-vector and variance-covariance matrix of $(u_i, x_1, ..., x_N)$ by $\mu_i \equiv (u_i, \mu^*)'$ and $\Sigma_i \equiv \begin{pmatrix} \sigma_i^2 & \sigma_i^{*\prime} \\ \sigma_i^* & \Sigma^* \end{pmatrix}$. Then using the formulas for conditional mean and variance of a multivariate normal distribution:

$$E[u_i \mid \hat{x}_i] = (u_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^*) + \hat{x}_i{}'\Sigma^{*-1}\sigma_i^*, \tag{10}$$

$$Var[u_i \mid \hat{x}_i] = \sigma_i^2 - \sigma_i^{*\prime}\Sigma^{*-1}\sigma_i^*. \tag{11}$$

5.3 Computing the first-price strategy

The equilibrium strategy of a first-price sealed-bid auction with symmetric affiliated value according to Milgrom and Weber (1982) is:

$$b_i(X_i) = \ln \int_0^{X_i} v(\alpha, \alpha) \frac{f_{Y_1}(\alpha \mid \alpha)}{F_{Y_1}(\alpha \mid \alpha)} L(\alpha \mid X_i) d\alpha, \tag{12}$$

where $Y_1 = \max\{X_j\}$, $j \neq i$, $L(\alpha \mid X_i) = \exp(-\int_\alpha^{X_i} \frac{f_{Y_1}(s|s)}{F_{Y_1}(s|s)} ds)$.

This strategy does not have a closed-form analytical solution. To compute it, I employ a combination of simulation and numerical approximation. First, I divide the integral into n bins and rewrite the it into a summation:

$$b_i(X_i) \approx \ln \sum_{s=1}^{n} \underbrace{v(\alpha_s, \alpha_s) \frac{f_{Y_1}(\alpha_s \mid \alpha_s)}{F_{Y_1}(\alpha_s \mid \alpha_s)} L(\alpha_s \mid X_i) \omega_s}_{\rho_{s,i}}, \tag{13}$$

where α_s is the mid-point of bin s and ω_s is the width of bin s. I use $\rho_{s,i}$ to denote the value of bin s conditional on X_i. Next, I estimate components of $\rho_{s,i}$ ($v(\alpha_s, \alpha_s)$, $\frac{f_{Y_1}(\alpha_s|\alpha_s)}{F_{Y_1}(\alpha_s|\alpha_s)}$ and $L(\alpha_s \mid X_i)$) separately for each bin s. Last, I sum up the values of all n bins and take the natural log to obtain an estimate for the equilibrium strategy of a first-price sealed-bid auction.

The number of bins I select for approximation is 10. I conduct tests for robustness using various bin numbers up to 50. The value estimated is not very sensitive to the choice of bin numbers. Further, as the number of bins increases, the estimation for $\frac{f_{Y_1}(\alpha_s|\alpha_s)}{F_{Y_1}(\alpha_s|\alpha_s)}$ tends to yield undefined values for bins with lower values since $F_{Y_1}(\alpha_s \mid \alpha_s)$ is more likely to be zero. The reason for this will become clear once I discuss how $F_{Y_1}(\alpha_s \mid \alpha_s)$ is estimated.

For a given number of bins, an intuitive way of defining the bins is to draw a large number of observations from the empirical distribution[8], and divide the distance between the minimum and maximum observations into equal intervals. However, given

[8]The empirical distribution changes in every iteration of the minimization algorithm due to changes in parameter values. To make sure that the definitions of bins are consistent across iterations, I make draws from the standard normal distribution to obtain the bounds for each bin, and rescale them based on the parameter values of a particular iteration. The number of observations drawn is 100 million.

the normal distribution, the majority of the observations drawn will fall into the middle bins, rendering estimations for $\frac{f_{Y_1}(\alpha_s|\alpha_s)}{F_{Y_1}(\alpha_s|\alpha_s)}$ and $L(\alpha_s \mid X_i)$ for the bins at the two ends inaccurate due to a lack of observations. As a result, I define the bins using percentiles of the observations drawn. In particular, when the number of bins is 10, the bins are defined by the deciles (i.e. the 10th, 20th, ... and 90th percentiles). This makes sure the probability of a randomly drawn observation falling into any of the bins is the same. I number these bins from 1 to 10 starting from the lowest decile.

$v(X_i = \alpha_s, Y_1 = \alpha_s)$ measures the expected value of bidder i when both bidder i and the bidder with the highest private observation among all other bidders receive a private observation equal to α_s. The analytical solution to $v(\cdot)$ is a multi-dimensional integral on the joint distribution of X's, which is difficult to compute. I therefore approximate $v(\vec{x}_s)$ by fitting a function on the largest two elements of 500 simulated draws. $v(\vec{x}_s)$ is evaluated using Equation 10. By assumption, $v(\vec{x}_s)$ is monotone in each element of vector $\vec{x}_s$ and the elements are strictly affiliated, $v(\vec{x}_s)$ is therefore likely to be monotone in the largest two elements, too. I experiment with a number of specifications up to a cubic term of each element. Higher level functions do not significantly improve the fitting than a linear specification, which is what I adopt.

The cumulative distribution function ("cdf") $F_{Y_1}(\alpha_s \mid \alpha_s)$ can be re-written as a conditional probability:

$$F_{Y_1}(Y_1 = \alpha_s \mid X_i = \alpha_s) = Pr(Y_1 \leq \alpha_s \mid X_i = \alpha_s). \tag{14}$$

To estimate F_{Y_1} using simulation, I again draw 500 observations of $\vec{x}_s$. I then proceed to identify all observations that contain at least an element which falls in bin s. Out of these observations, I calculate the share of observations which do not contain any element that falls in a bin higher than s. This share provides an approximation of $F_{Y_1}(\alpha_s \mid \alpha_s)$. It can be easily shown that F_{Y_1} is increasing in α_s. This pattern is generally observed for the estimates of F_{Y_1} for the higher bins. However, for the lower bins, the increasing pattern is often broken. This is because the probability of drawing an observation of $\vec{x}_s$ of which the largest element falls in bin s decreases as s decreases. The accuracy of the estimates therefore decreases as a result of a smaller number of observations for the lower bins. In some cases, the estimate falls to zero. Unfortunately this issue does not readily go away when the number of observations drawn increases. The probability distribution function $f_{Y_1}(\alpha_s \mid \alpha_s)$ is computed based on the estimates of the cdf's. Specifically,

$$f_{Y_1}(\alpha_s \mid \alpha_s) = (F_{Y_1}(\alpha_s \mid \alpha_s) - F_{Y_1}(\alpha_{s-1} \mid \alpha_s))/\omega_s \tag{15}$$

$L(\alpha_s \mid X_i) = \exp(-\int_{\alpha_s}^{X_i} \frac{f_{Y_1}(s|s)}{F_{Y_1}(s|s)} ds)$ is an integral. To approximate it, I again rewrite it into a summation:

$$L(\alpha_s \mid X_i) \approx \exp(-\sum_{m=s}^{N_i} \frac{f_{Y_1}(\alpha_m \mid \alpha_m)}{F_{Y_1}(\alpha_m \mid \alpha_m)} \omega_m), \tag{16}$$

where N_i is the number of the bin into which X_i falls. The computation of this summation for each bin s makes use of approximations of $\frac{f_{Y_1}(\alpha_s|\alpha_s)}{F_{Y_1}(\alpha_s|\alpha_s)}$. However, unlike $\frac{f_{Y_1}(\alpha_s|\alpha_s)}{F_{Y_1}(\alpha_s|\alpha_s)}$ which is independent of X_i, the approximation of $L(\alpha_s \mid X_i)$ depends on X_i through N_i. For $s > N_i$, define $L(\alpha_s \mid X_i)$ to equal zero.

To reflect the fact that X_i falls into bin N_i, but does not necessarily cover the entire bin, I adjust $\rho_{N_i,i}$ by multiplying it with the ratio $\frac{X_i - lb_{N_i}}{ub_{N_i} - lb_{N_i}}$, where lb_{N_i} and ub_{N_i} are the lower and upper bounds of bin N_i respectively.

5.4 Estimation results

The second panel of Table 2 shows the estimation results of the multi-round signaling model. The mean of the common value component m, is parameterized into 2 parts, a constant and a coefficient on population. I choose this parsimonious parameterization to reduce the burden of computation. In addition, simple regressions of the winning bids against a selection market attributes, including population, population density and area suggest that population is the best predictor. The mean of the private value component, $\bar{a}$ is not separately identified from the constant associated with the common value component.

In order to understand the impact on valuation from a potential model misspecification when bidders play according to the signaling model but the econometrician fits the data using an open exit model, I repeat the SNLS estimation using an open exit model with the same data. In this case, $b_{i,s}^t(\vec{x}_s; \theta)$, the model predicted dropout price for a random draw of private observations $\vec{x}_s$ as defined in Section 5.1 is simply equal to the closed-form analytical solution in Section 5.2. The estimation results are shown in the first panel of Table 2.

Table 2: Simulated nonlinear least-squares estimates

Coefficient	Open Exit	Signaling
Components of mean		
Constant[b]	11.57	13.18
	(0.22)	(0.14)
POP (mils)	0.91	0.95
	(0.29)	(0.27)
Standard deviations		
r0 (common value comp.)	12.75	2.66
	(5.96)	(0.29)
t (private value comp.)	2.84	3.58
	(0.13)	(0.26)
s (unobserved error)	3.95	0.65
	(0.27)	(0.05)
# auctions (T)	491	491

Note:

[a] Bootstrapped standard errors in brackets, computed from empirical distribution of parameter estimates from 100 parametric bootstrap resamples

[b] Not separately identified from $\bar{a}$

Comparing the estimates of the components of the m, the mean of the common value, the values for both parts are higher under the multi-round signaling model than the open exit model. Consider a market with a population of 200k (the median population of all 493 markets is 187k), the mean of the log value of the license estimated by the signaling model is 13.4, 14% higher than that estimated by the open exit model at 11.8. This suggests that if bidders are using jump bids as signals, ignoring them leads to underestimation of the mean.

6 Counterfactual Analysis

Table 3: Counterfactual mean log prices and total revenues

	"Jump bid" auctions		All auctions	
Mean actual log prices ($)				
Highest/winning bid	15.95			
Second highest bid (jump bid)	15.90			
Mean predicted log prices[a] ($)				
Multi-round signaling	**16.58**			
Single-round signaling	**16.90**			
Open exit auction (no signaling)	**17.18**			
Predicted total revenues ($bn)		% Δ		% Δ
Multi-round signaling	17		170	
Single-round signaling	23	33%	176	3%
Open exit auction (no signaling)	31	76%	183	**8%**
# auctions	81		491	

Note:
[a] Average model predicted log prices of 1000 simulated draws

In the counterfactual analysis, I assess the revenue impact of jump bidding using the estimates from the signaling model. In particular, I estimate the changes in revenues from an auction with multi-round signaling to one with single-round signaling, and to an open exit auction where no signaling is allowed.

If jump bids were prohibited, the auctions that would have different prices would be the ones where the winning bids were jump bids. However, the empirical model is not able to make any prediction on the winning bid because it is not the dropout price of the winner. To get around the limitation, I will instead focus on auctions with second highest bids being jump bids, and make counterfactual predictions on these bids to approximate a change in revenue.

In the data, 81 auctions out of 491 fit this description. I refer to these auctions as the "jump bid" auctions. As shown in Table 3, the mean log winning bid for the jump-bid auctions is at 15.95 and the mean log second highest bid is 15.90. The small difference between these two numbers suggest that the latter serves as a fairly close estimate to the former.

The multi-round signaling model predicts the mean log second highest bid to be at 16.58. If we only allow the bidders a one-off opportunity to signal using jump bid, the auction is transformed into a first-price sealed-bid auction as shown by the theoretical model. The mean log price is predicted to increase to 16.90. It continues to go up to 17.18 if all jump bids are forbidden, i.e. in an open exit auction.

The revenues from the 81 jump bid auctions are estimated to increase by 33% when the auction moves from one that allows multi-round signaling to one that only allows one round of signaling. The revenues increase by another 35% when signaling through jump bids are prohibited. The total increase in revenues amounts to 76% of those from the auction with multi-round signaling. Overall, this increase account for an 8% increase in total revenues across all 491 auctions.

7 Conclusions

The existing empirical research on jump bidding adopts either a descriptive or a reduced form approach. While both approaches help confirm the pervasiveness of jump bidding and identify its correlation with a number of economic factors, they are unable to quantify the revenue effect of jump bidding. This paper generalizes a theoretical signaling model on jump bidding and uses it as the basis for a structural estimation that uncovers the underlying bidder value distribution. A counterfactual analysis using these estimates finds an overall improvement of revenue by 8% for the FCC broadband PCS auction (C-block).

A Additional figures

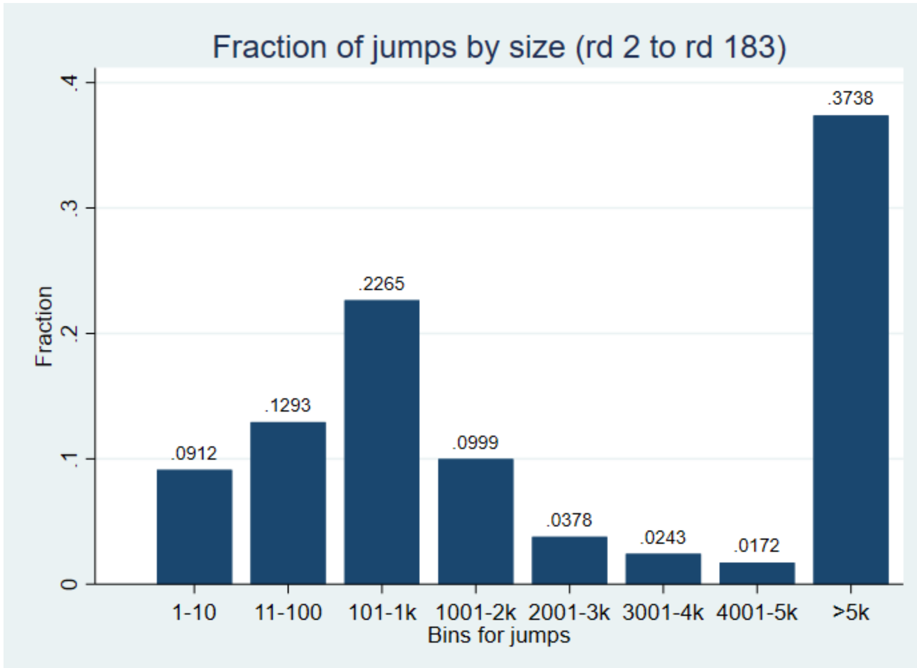

Figure 2: Density distribution of jump bids by size I

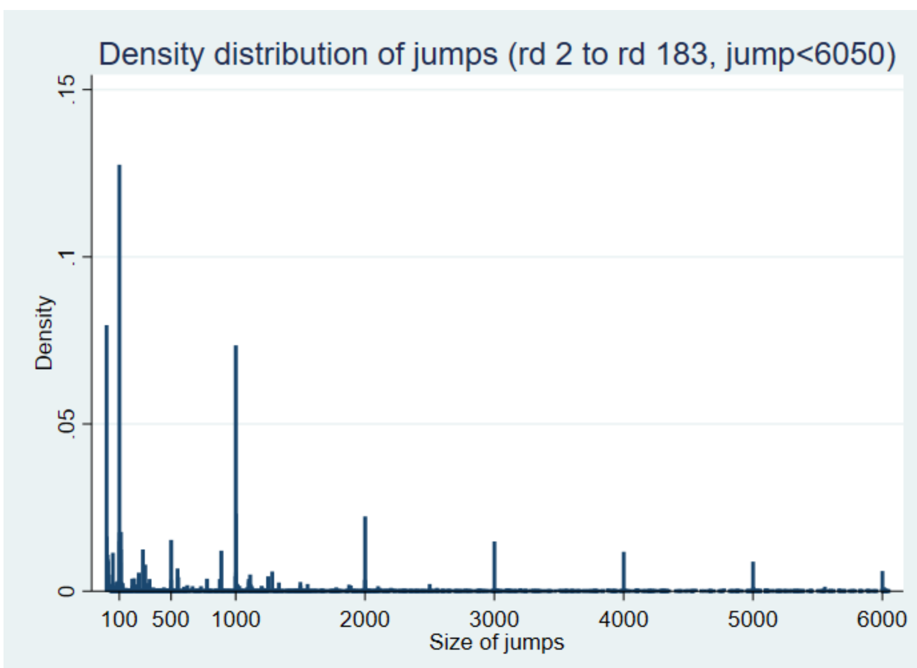

Figure 3: Density distribution of jump bids by size II

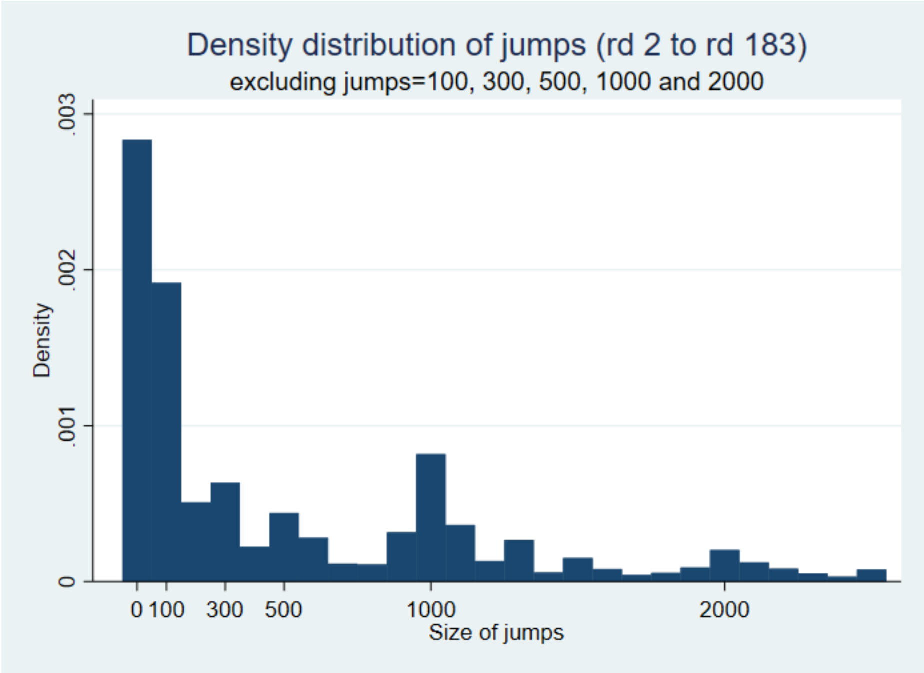

Figure 4: Density distribution of jump bids by size III

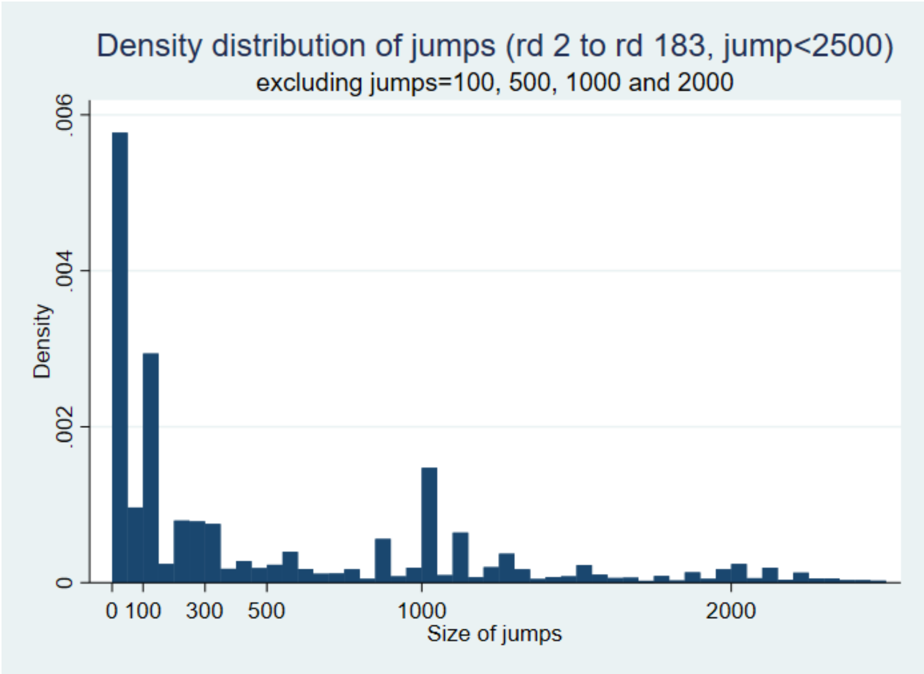

Figure 5: Density distribution of jump bids by size IV

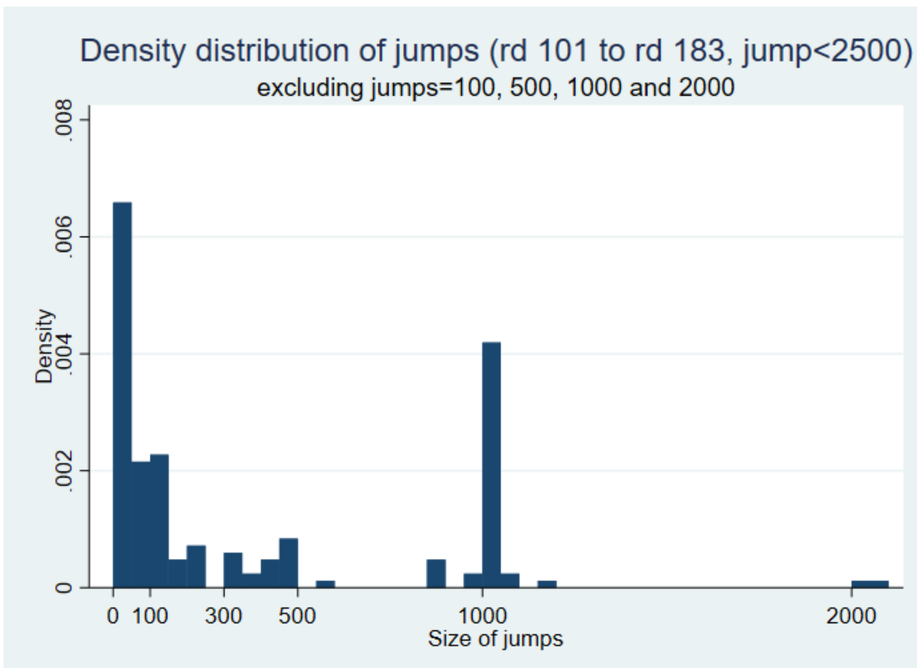

Figure 6: Density distribution of jump bids by size V

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