Least Squares Learning? Evidence from the Laboratory[*]

Te Bao[†] Yun Dai[‡] John Duffy[§]

28th December 2022

Abstract

We report on an experiment testing the empirical relevance of least squares (LS) learning, a common way of modelling how individuals learn a rational expectations equilibrium (REE). Subjects are endowed with the correct perceived law of motion (PLM) for a price level variable they are seeking to forecast, but do not know the true parameterization of that PLM. Instead, they must choose and can adjust the parameters of this PLM over 50 periods. Consistent with the E-stability of the REE in the model studied, 93.1% of subjects achieve convergence to the REE in terms of their price level predictions. However, only 20.3% of subjects can be characterized as least squares learners via the adjustments they make to the parameterization of the PLM over time. We also find that subjects' parameter estimates are more accurate when there is greater variance in the independent variable of the model. We consider several alternatives to least squares learning and find evidence that many subjects employ a simple satisficing approach.

JEL Classification: C53, C91, D83, D84

Keywords: Rational Expectations Equilibrium, Least Squares Learning, Experimental Economics, Learning-to-Forecast Experiment, Behavioral Macroeconomics.

[*]We benefited from comments by Syngjoo Choi, Annarita Colasante, Nobuyuki Hanaki, Janet Jiang, Yaron Lahav, Dmitry Shapiro, Joseph Tao-yi Wang, Wei Xiao and participants of various workshops and conferences. The authors acknowledge financial support from an AcRF Tier 1 Grant RG69/19 from the Ministry of Education of Singapore, the NTU-WebBank Joint Research Center (NWJ-2020-003) and the National Science Foundation of China (No. 71803201, 71773013, and 71873149). This study is approved by the IRB of NTU (IRB-2018-01-035). We thank Jiaoying Pei for her excellent research assistance.

[†]School of Social Sciences and NTU-WeBank Joint Research Center, Nanyang Technological University. Email: baote@ntu.edu.sg.

[‡]Department of Finance, Lingnan College, Sun Yat-Sen University. Email: daiy28@mail.sysu.edu.cn.

[§]Department of Economics, University of California, Irvine, CA, 92697 USA Email: duffy@uci.edu.

This is our key bounded rational assumption: we back away from the RE assumption, replacing it with the assumption that, in forecasting prices, firms act like econometricians.

– Evans and Honkapohja, 2001: page 28

1 Introduction

The rational expectations hypothesis (REH) (Muth, 1961; Lucas Jr, 1972) has been a dominant paradigm in macroeconomics since the 1970s. Nevertheless, many researchers remain interested in finding an microfoundation justifying the REH. As pointed out by Sargent (2008), an important approach is to assume adaptive learning in combination with the"self-confirming equilibrium" (SCE) approach where the rational expectations equilibrium (REE) is considered as the possible destination of active learning by agents.

Adaptive learning models, including least squares learning models (Marcet and Sargent, 1988; Guesnerie and Woodford, 1991; Sargent, 1993; Evans and Honkapohja, 1999, 2001, 2003, 2009; Preston, 2006; Branch et al., 2013; Branch and McGough, 2016) usually assume that agents do not know the actual law of motion (ALM) of the economy. Instead, these learning agents use a *perceived* law of motion (PLM) and update the parameters of this PLM as new information arises. In the case of least squares learning, they minimize the sum of squared errors according to the least squares updating rule, just as econometricians do with their data.[1] Researchers in this field show that convergence to an REE can arise under certain conditions regarding the mapping between the perceived law of motion and the actual law of motion (i.e., E-stability).

In this paper, we take the theoretical prediction of least squares learning quite seriously and evaluate its predictions using an experiment based on the model of Bray and Savin (1986) and Fourgeaud et al. (1986). Differently from the many "learning-to-forecast" experiments, e.g., (Marimon et al., 1993; Hommes et al., 2005; Hommes, 2011; Assenza et al., 2014; Duffy, 2016; Arifovic and Duffy, 2018; Anufriev et al., 2019; Assenza et al., 2019; Kryvtsov and Petersen, 2020) where subjects make

[1] Also, as in econometrics, the specification of the perceived law of motion can be correct, including the REE as a special case, or incorrectly specified. Here we concentrate on the simpler case where the PLM is correctly specified.

only *point predictions*, subjects in our experiment submit parameterizations for the PLM directly. This design enables us to conduct the cleanest and most direct test of the *structural form* of the adaptive learning model that agents are using when learning. In particular, we can directly test whether individuals are adjusting the parameterization of their PLM *as if* they were running least squares regressions in their own minds, albeit without the assistance of computers and statistical software. In addition to least squares learning, we also consider the performance of several other learning models for explaining our results: past averaging models, constant gain learning, stochastic gradient descent learning and a satisficing model.

Our experiment involves two main treatments that alter the variance of the independent variable of the model that agents are seeking to learn. As is well known (see, e.g., Greene (2000)), least squares estimates tend to be more accurate (that is, they have lower variance) the larger is the variance in the independent variables. To address this feature of least squares learning, in *Treatment A* of our experiment, the exogenous independent variable of our model, which we refer to as "weather", follows a simple AR(1) process. By contrast, in *Treatment U*, this same independent variable follows an i.i.d. uniform distribution. The variance in realizations of the independent variable are therefore greater in treatment U as compared with treatment A. Since the variance of the independent variable is the *only* change made to the model between treatments A and U, it follows from econometric theory that learning and convergence to the REE should be faster in Treatment U as compared with Treatment A.

We find that at the aggregate level, subjects' forecasts in *Treatment U* do indeed converge faster than subjects' forecasts in *Treatment A*. By the end of the 50 periods of the experiment, the average forecast in *Treatment U* has converged to the REE while the average forecast in *Treatment A* fails to do so within this timeframe. At the individual level, around 97% of the expectations satisfy our criterion for *weak convergence* to the REE. The fraction of individual expectations that satisfy our criterion of *strong convergence* is 54.4% in *Treatment U* and 18.6% in *Treatment A*. Finally, just 12 out of 29 subjects (41.4%) in Treatment U, and 0 (0%) in Treatment A, for a grand total of 20.3% of all subjects in our experiment, can be categorized as least squares learners in terms of the adjustment of their parameterization of the PLM over time. Still, some alternative models such as constant gain learning model and "learning by averaging" perform *worse* than least squares learning in terms of their fitness to the experimental data as measured by the mean squared error.

Our findings suggest that while the E-stability condition works very well as a description of the stability under learning or "learnability" of rational expectations equilibrium at the aggregate level, individual subjects may update the parameters of their PLM in a heterogeneous way and deviate from the least-squares learning specification. Our results suggest that instead of searching for the least-squares minimizing combination of the two PLM parameters (a and b), many subjects seems to apply a "satisficing" heuristic (Simon (1955, 1956)) and stick with the "wrong" pair of parameters if that combination generates approximately the same point predictions as the true but unknown parameters. In other words, when faced with an unfamiliar and complex parameter search and updating problem in 2 dimensional (2-D) space, many subjects in our experiment appear to have reduced the problem to a simpler and more familiar single point prediction problem. This behavioral tendency to reduce a 2-D decision problem to its projection in 1-D space may also be found in theoretical models of "misspecified equilibrium" (Grandmont (1998)) and "(stochastic) consistent expectations equilibrium" (Hommes and Sorger (1998); Hommes and Zhu (2014)). Note that the subjects in our experiment did not have access to statistical software or computational resources that would enable them to run the regressions associated with least square learning. We did not provide such access since we interpret the notion of adaptive learners-as-econometricians in the "as if" sense of Friedman (1953).[2] Still, we find that 20.3% of subjects do form and adjust their forecasts according to the predictions of the least squares learning model. However, the majority have to apply some simplification method to make the problem (seemingly) more tractable for them.

Overall, this paper makes three main contributions to the literature.

First, to our knowledge, this is the first experiment where subjects submit structural expectations (model parameterizations) instead of simple point predictions of the variable they are learning about. This design allows us to observe precisely *how* individuals update the parameters of their PLM in real time. This is a particularly useful method for comparing competing models that predict the same qualitative outcome in terms of convergence, but which may differ in the way that individuals update the parameters of their forecasting models. Most surveys on expectation formation, like the Michigan Survey, only elicit point predictions or subject-

[2]Friedman (1953), p.21 argued that while expert billiard players might not know the complicated mathematical formulas underlying optimal play, they nevertheless behaved *as if* they knew those formulas. Here we are not supposing that subjects *optimally* form expectations but ask instead whether they form them in the manner prescribed by least squares learning in favorable conditions, i.e. given a PLM and the possibility to adjust the parameters of that PLM as new information arises. We would further add that it is unlikely that most members of the general public would have access to statistical software or be familiar with regression analysis.

ive probability distributions[3]. Data from our laboratory experiment are therefore particularly useful in answering questions regarding the structural path by which individuals update their expectations in real time and their weighting of different factors in forming those expectations. In the learning-to-forecast experiment literature, one study by Hommes et al. (2005) also asks for forecasting strategies, instead of point predictions in each period. But the strategies they elicited were regarding how participants made their *point* predictions, not how they searched for and updated parameters of the perceived DGP as in our study.

Second, this paper presents the first experimental test specifically evaluating least squares learning as a behavioral primitive process. Bao and Duffy (2016) run an experiment to test differences in theoretical predictions between adaptive and eductive learning (Binmore (1987); Guesnerie (1992); Evans and Guesnerie (2005); Evans et al. (2019)) models. But the adaptive learning model in that paper is a reduced form, point prediction version where the adaptive learning expectation degenerates to the sample average of past realizations of the prices. Therefore, those results do not reveal *how* people update the *parameters* in their perceived law of motion for the economy.

Third, our experiment also serves as a test on the capacity of humans to confront complex tasks without the help of computers. To this end, we also contribute to the literature on how the complexity of decision-making influences the accuracy of forecasting behavior (Mirdamadi and Petersen, 2018; Arifovic et al., 2019; He and Kucinskas, 2019) and bounded rationality in expectation formation in macroeconomics in general (Honkapohja, 1995; Branch, 2004; Woodford, 2013).

The rest of the paper is organized as follows: Section 2 presents the experimental design, Section 3 reports on the experimental results, and finally, Section 5 provides a summary and conclusions.

[3] For studies using this survey dataset, see Branch (2004), for studies that compare laboratory and field data on expectations, see Cornand and Hubert (2020); Landier et al. (2019). For evidence on how information rigidity leads to deviation from RE from survey data, see Coibion et al. (2018). For studies using Randomized Controlled Trials or field experiments, see e.g., Binder and Rodrigue (2018), Armona et al. (2019), Coibion et al. (2020b), Coibion et al. (2020a), Coibion et al. (2022).

2 Experimental Design

2.1 The Cobweb Model

Consider the cobweb model in Bray and Savin (1986), and Fourgeaud et al. (1986). There is a single market for a product that has a time lag in production (e.g., an agricultural product). The demand for this product depends negatively on the prevailing market price, p_t. The supply of the product is assumed to depend on both the average expectation across the homogeneous firms of the prices that will prevail in the current period, p_t^e as well as the weather in the current period in the form of an observable shock, w_t.[4] The demand d_t, and supply s_t equations are given by:

$$d_t = m_I - m_p p_t + v_{1t}, \quad m_p > 0$$

$$s_t = r_I + r_p p_t^e + r_w w_t + v_{2t}, \quad r_p > 0$$

where m_I, m_p and r_I, r_p are the intercept and slope coefficients, respectively of the demand and supply functions, while v_{1t} and v_{2t} are random noise terms. Thus, in equilibrium, when $d_t = s_t$, the true law of motion for the price of the product is given by:

$$p_t = \mu + \alpha p_t^e + \delta w_t + \eta_t \tag{1}$$

In the above equation, $\mu = m_p^{-1}(m_I - r_I)$, $\alpha = -m_p^{-1} r_p < 0$, $\delta = -m_p^{-1} r_w$, $\eta_t = m_p^{-1}(v_{1t} - v_{2t})$, and $\eta_t \sim iid(0, \sigma_\eta^2)$. The distribution of the weather variable w_t is an i.i.d. process in Bray and Savin (1986). Alternatively, it may also follow a stationary exogenous VAR (vector autoregression) process driven by a multivariate white noise shock with bounded moments as assumed by Evans and Honkapohja (2001). In our paper, we experimented with both cases, that is, the weather follows an i.i.d. distribution in one of the treatments, and an autocorrelated distribution in the other treatment.

Under adaptive learning, it is typically assumed that agents have a perceived law of motion (PLM) for prices that nests the REE as a special case. That is, their perceived law of motion is not misspecified as it implicitly takes account of the expectation feedback term, αp_t^e in the true law of motion.[5] For the model we

[4]Note that in the original Bray and Savin (1986) model, the current price level, p_t is assumed to depend on the lagged weather variable, w_{t-1}, as the supply in the current period will depend on the observable shock due to weather in the last period. In our experiment, we change this term to w_t in order to help subjects understand the setting more easily. This is a nominal change only and does not alter the results from the model because in the experiment, w_t is also realized and revealed to subjects *before* they make their decisions.

[5]Misspecified PLMs are also considered as discussed in Evans and Honkapohja (2001). Here we focus on the case where the PLM is *correctly* specified since our aim is to understand how agents

consider, this perceived law of motion (PLM) is given by:

$$p_t = a + bw_t + \eta_t \tag{2}$$

According to the least squares principle, prediction of the estimators of a simple linear regression model will be more precise (i.e. have lower variance) when there is a larger variation in the independent variables[6]. Therefore, theoretically the variance of the estimated coefficients a, b should be smaller in the treatment where the exogenous weather variable has the larger variance.

The unique REE prediction for prices in the Cobweb model is as follows:[7]

$$p_t = \bar{a} + \bar{b}w_t + \eta_t,\ \bar{a} = (1-\alpha)^{-1}\mu,\ \bar{b} = (1-\alpha)^{-1}\delta$$

Given the PLM (2), the REE of the system is learnable only if the parameters of the model satisfy the expectational stability (or E-stability) criterion. In this case, E-stability requires that $0 < \alpha < 1$.[8]

2.2 Least Squares Learning in the Cobweb Model

A common way of modeling the learning of REE is to assume that agents are least squares learners. Under a least squares learning (LSL) assumption, agents are assumed to start with some initial estimates for the parameters a and b of their PLM, e.g. $\hat{a}_0$, $\hat{b}_0$, and adjust these estimates over time so as to minimize the mean of the sum of squared errors between the linear PLM model predictions and actual realizations for prices, p. In our setting, agents regress p_i on x_i where

$$x_i' = (1\ w_i').$$

Thus, if agents are least squares learners, in each period t they will update their

update the parameters of a PLM that actually enables learning of the REE.

[6]In the simple linear regression model $y_i = \beta_1 + \beta_2 x_i + e_i$, an estimated model $\hat{y}_i = b_1 + b_2 x_i$ can be formed following the least squares principle, where $y_i = \hat{y}_i + \hat{e}_i$. $Var(b_1) = \frac{\sigma^2 N^{-1}\sum x_i^2}{\sum(x_i-\bar{x})^2}$, $Var(b_2) = \frac{\sigma^2}{\sum(x_i-\bar{x})^2}$. Thus, the wider the spread of the independent variable x, (i.e., a larger $\sum(x_i-\bar{x})^2$) will lead to a more precise estimate (i.e., smaller variance) of both of the parameters. Note that the spread of the independent variable does not affect the accuracy of the estimator because the expectation of the estimates derived using the least squares principle should always be unbiased, i.e. $E(b_1) = \beta_1, \quad E(b_2) = \beta_2$.

[7]A detailed derivation can be found in Appendix G).

[8]See Evans and Honkapohja (2001) Theorem 2.1.

parameter estimates $\hat{a}_t$ and $\hat{b}_t$ for the PLM, (2) like econometricians so that

$$\begin{pmatrix} \hat{a}_t \\ \hat{b}_t \end{pmatrix} = \left(\sum_{i=1}^{t} x_i x_i' \right)^{-1} \left(\sum_{i=1}^{t} x_i p_i \right) \tag{3}$$

Using the LS estimates, $\hat{\theta}_t = \binom{\hat{a}_t}{\hat{b}_t}$, the learning agent forecasts the price level for period t:

$$p_t^e = \hat{\theta}_t' x_t \tag{4}$$

This forecast (4) is substituted into equation (1) to determine the actual value for p_t. The formula for determining the least squares estimates can be written recursively as:

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{1}{t} R_t^{-1} x_t (p_t - \hat{\theta}_{t-1}' x_t)$$

$$R_t = R_{t-1} + \frac{1}{t}(x_t x_t' - R_{t-1})$$

where R is the variance-covariance matrix.

A simple alternative to least squares learning that we will also consider is constant gain learning. In this case the gain term on the coefficient vector $\hat{\theta}$ and the moment matrix R is not $1/t$ (decreasing) as it is under least squares learning but is instead a constant value, $\lambda \in (0, 1)$, that bests fits the data.

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \lambda R_t^{-1} x_t (p_t - \hat{\theta}_{t-1}' x_t)$$

$$R_t = R_{t-1} + \lambda(x_t x_t' - R_{t-1})$$

Note that under constant gain learning, the parameter vector is updated according to the prediction error in the last period. Therefore, the weight of the most recent past error will not decrease with t, and this algorithm exhibits more volatile dynamics. Indeed, if there is any source of noise in the model (as there is in our system), the constant algorithm will never quite settle down to the REE. Nevertheless, constant gain learning systems have been used by researchers to study learning dynamics, particularly in systems (unlike ours) that are subject to potential structural breaks in the variables being forecast, and so we also consider this specification.

2.3 Parameterization and Treatments

For the experiment, we chose to set $\mu = 9$, $\alpha = -0.5$ and $\delta = 0.9$, so that the market price is given by:

$$p_t = 9 - 0.5 p_t^e + 0.9 w_t + \eta_t, \quad \eta_t \sim i.i.d N(0, 1)$$

For each parameter tuple (a, b) submitted by subjects, the price expectation in each time period t is:

$$p_t^e = a + b \times w_t$$

Assuming that agents have rational expectations, i.e., $p_t^e = p_t$, the REE path for prices is given by[9]:

$$p_t = 6 + 0.6w_t + \eta_t.$$

Our experiment consists of two main treatments that vary the process for the weather term, w_t. Under both treatments, the long-run, expected value of the weather variable, $E_t(w_t) = 10$. The two treatments differ *only* in the variance and persistence of the independent weather variable, w_t.

***Treatment U* (Uniform Noise)**: In this treatment, the time t realization of w_t is an i.i.d. uniform random draw over the interval $[0, 20]$, i.e. $w_t \sim U(10, 20)$. Thus, $E_t(w_t) = 10$ and the variance of weather in this treatment is given by:

$$\sigma_w^2 = \frac{(20-0)^2}{12} = \frac{100}{3} \approx 33.33$$

In *treatment U*, the expected value of the market price is $E(p_t) = 6+0.6\times 10 = 12$, which is the REE value for the market price in this treatment.

***Treatment A* (Autoregressive Noise)**: In this treatment, we suppose that w_t follows the auto-regressive process:

$$w_t = 2 + 0.8w_{t-1} + \epsilon_t,\ \epsilon_t \sim N(0, 1).$$

For *treatment A*, we introduce a constant term into the DGP for w_t to ensure that the long-run equilibrium expected value $E(w_t) = 10$ is the same as in *treatment U* to facilitate comparisons.[10] Therefore, the REE value of the market price in *Treatment A* is the same as in *Treatment U*: $E(p_t) = 6 + 0.6 \times 10 = 12$. According to the definition of the AR(1) process, the variance of the weather variable in this

[9]In our experiment, the subjects are told that $a \in [0, 10], b \in [0, 1]$. Our experience with previous forecasting experiments suggests that subjects are very likely to start with the midpoint of the interval, i.e. (5, 0.5). To test whether least square learning will result in convergence to REE, we should choose a pair of a, b that are not (5, 0.5) but not too far from these values. Meanwhile, by running simulations, we learned that submitting (5, 0.5) will also generate high payoffs for (a, b) pairs like (6, 0.4) or (4, 0.6), namely, one of a, b is larger than and the other is smaller than the midpoint value. We therefore choose (6, 0.6) so that the REE is learnable and subjects have sufficient incentives to learn.

[10]We are aware that $E(w_t) = 2 + 0.8w_{t-1}$ is not a constant anymore, therefore, the REE in this system is no longer a point like in Treatment U. Detailed data on w_t and ϵ_t can be found in Table F.1.

treatment is given by:

$$\sigma_w^2 = \frac{\sigma_\eta^2}{1-0.8^2} = \frac{25}{9} \approx 2.78$$

Figure 1 (top panel) plots the time series of the realizations of w_t for the two treatments A and U. Below the plot of w_t for Treatment A, we also show a plot of the η_t noise term realizations used in all treatments. We used the same 50 realizations for w_t for all subjects who participated in either Treatment A or Treatment U of our experiment to facilitate comparisons across subjects and not add further noise across treatments. As shown in Figure 1, the variation in w_t is much greater in Treatment U as compared with Treatment A.

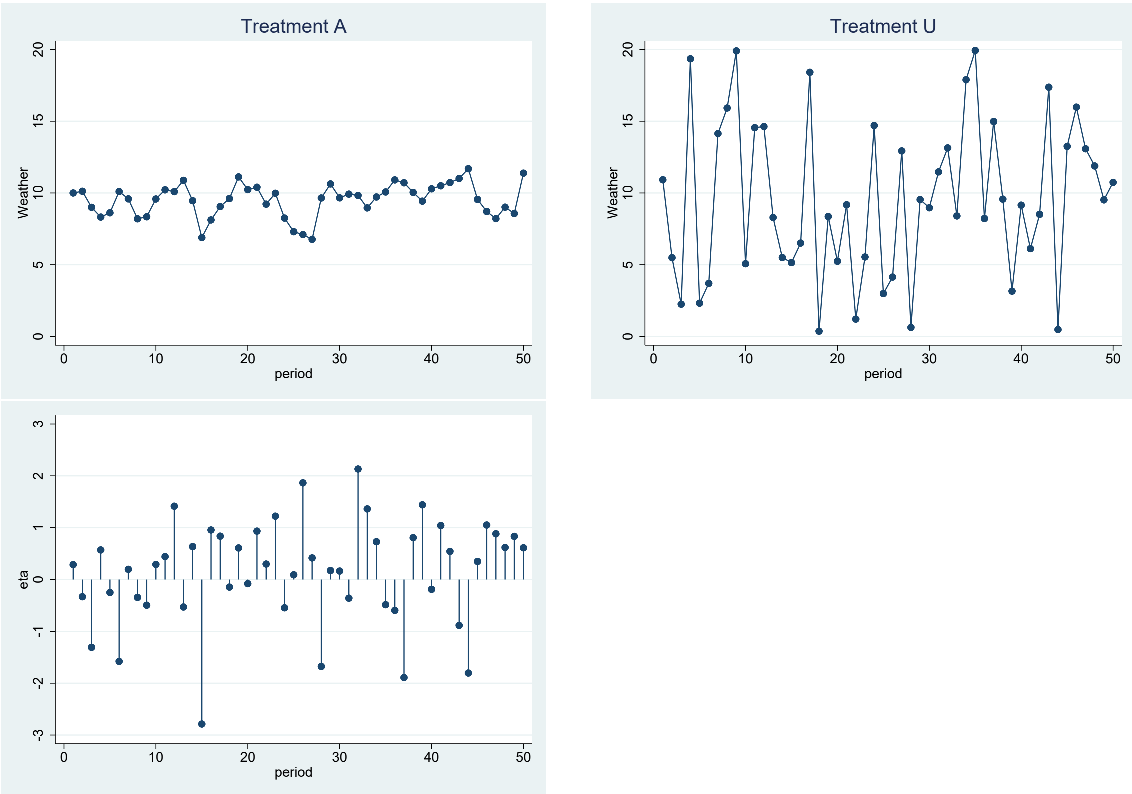


Figure 1. The time series realizations for w_t (top panel) and η_t (bottom panel) as used in the experiment. Note that w_t follows a uniform distribution in Treatment U and an AR(1) distribution in Treatment A, while η_t follows a standard normal distribution in both treatments.

2.4 Testable Hypothesis

Figure 2 shows the simulated time series for the estimates of a, b assuming that individuals follow least squares learning. Since individuals choose the values from $a \in [0, 10], b \in [0, 1]$, a natural guess would be that most of them would start from the midpoints of the intervals, i.e., $a_1 = 5, b_1 = 0.5$, and so we start the simulation at these points. The model updates the estimates $\hat{a}_t, \hat{b}_t$ using the realized p_t, w_t, in exactly the same way that the least squares learning model does.

The simulated dynamics suggest that while a, b have a tendency to converge to the REE in both treatments, the convergence is quicker and more reliable in Treatment U as compared with Treatment A. Indeed, because the variation of w_t in Treatment A is too small, the simulated least squares estimates for a, b can at times depart from the REE, for example, near the end of the 50 periods as illustrated in Figure 2 due to the variance of a and b being too large.[11]

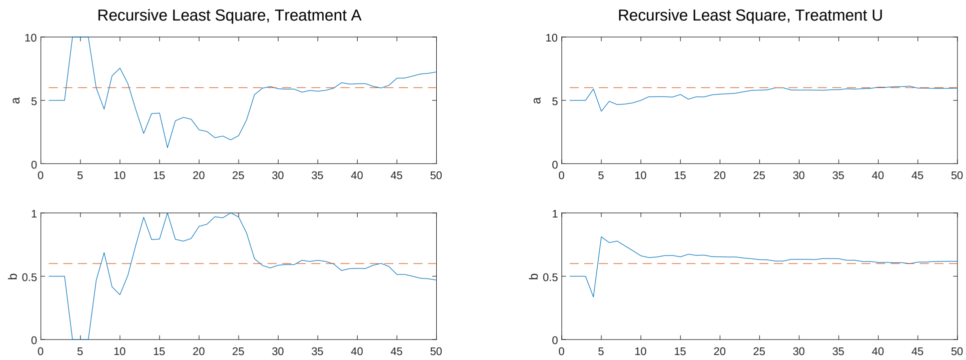


Figure 2. Least squares learning predictions for the paths of the parameter estimates a (top panels) and b (bottom panels) against the REE in each treatment. We initialize each simulation by setting $a = 5$, and $b = 0.5$, and we use the same realizations for w_t and η_t that were used in the experiment.

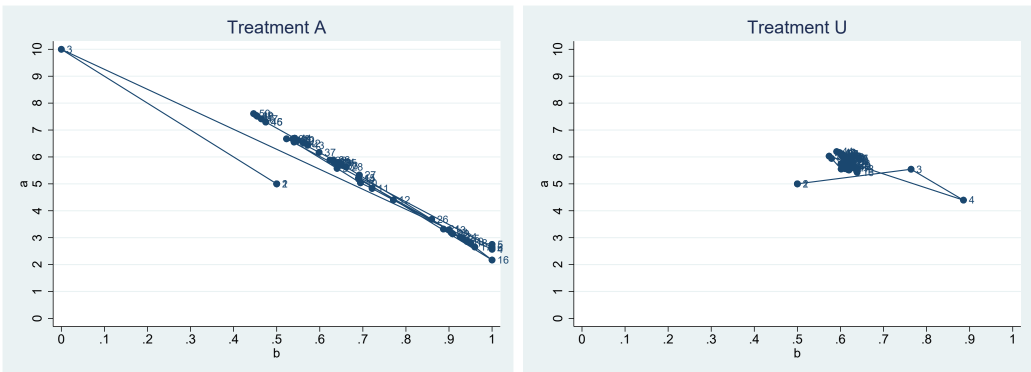


Figure 3. A scatter plot of the least squares learning prediction for the path of the parameters a and b against the REE in Treatment A (left panel) and Treatment U (right panel). The labels near the points indicate the period number. We initialize the simulation by setting $a = 5$, $b = 0.5$, and use the same realizations of w_t and η_t that were used in the experiment.

Figure 3 shows a scatter plot of the value of the estimated parameter pairs a, b over time in a 2-D plane. The labels (which are admittedly hard to read after

[11]Some may argue that the apparent non-convergence in Treatment A is due to the short horizon of the experiment. Subjects will be able to learn the REE if the experiment is run for more periods. We run an AR(1) regression on the dynamic adjustment of a and b in the 50 periods ahead least square model, and report the estimated parameters in Table B.7. It turns out that a and b do satisfy the *strong convergence* defined in Section 3.1.3, namely, we cannot reject that the long run equilibrium values of the parameters are equal to the REE levels in both treatments.

the first few periods) indicate the period numbers. This figure shows that while the parameters seem to make large movements along a downward-sloping line in Treatment A they do not quite reach the REE by the end of the 50 period horizon. By contrast, in Treatment U, the parameter estimates follow a more compact spiral that does yield convergence to the REE within 50 periods.

We also generated the simulated dynamics for a, b using the constant gain learning model. As suggested by Branch and Evans (2006); Pfajfar and Santoro (2010), the constant gain learning model that best fits the data is usually one with a small gain parameter, λ, e.g., between 0.01 and 0.02. We performed a grid search over λ values between 0.01 and 1 with a step length of 0.01, and selected the λ that minimizes the MSE between the model's 1 period ahead forecast and each individual's forecast. The results suggest that the mean of the optimal values for λ is 0.0148 in Treatment A and 0.0132 in Treatment U. Figure 4 shows the simulated a, b estimates over time. As the figure reveals, the simulated paths for a, b under constant gain learning model are not that different from those under least squares learning. This is because when λ is small, the "learning speed" of the constant gain learning model is not very high. The constant gain learning model also suggests that agents are able to learn the REE within 50 periods in Treatment U, but not within 50 periods in Treatment A.

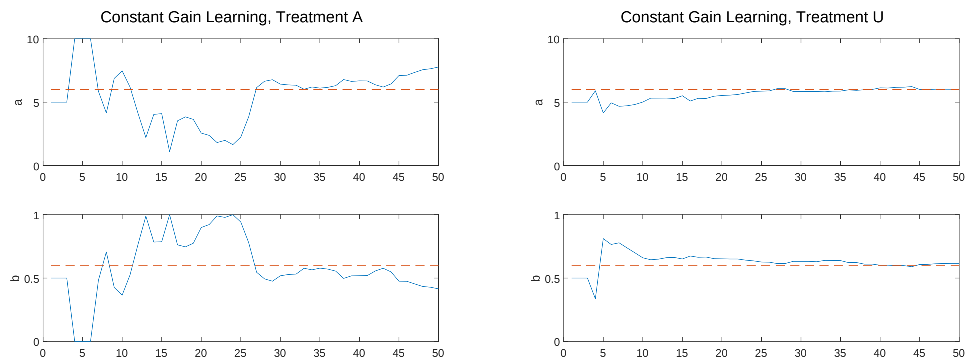


Figure 4. Constant gain learning predictions for the path of the parameters a and b against the REE in each treatment. We initialize the simulation by setting $a = 5$, $b = 0.5$, and we use the same realizations for w_t and η_t as used in the experiment.

Based on these theoretical predictions, we formulate the following main testable hypotheses:

Hypothesis 1 *(E-stability): Subjects can learn the REE, $\bar{a}, \bar{b}$ by the end of the experiment.*

Hypothesis 2 *(Least-squares learning): Subjects update the parameterization of the PLM (2) following the least squares principle, i.e., their estimates for $\hat{a}_t$ and $\hat{b}_t$ follow the predictions of (3), given the complete history of $\{p_s, w_s\}_{s=1}^{t}$*

Hypothesis 3 *(Learning and Variation in the Exogenous Variable): Subjects' estimates of the parameters are more precise when there is greater variation in the exogenous weather variable.*

Recall that the latter hypothesis is a property of the OLS estimator. Hypothesis 3 may not be supported if subjects use other forms of learning. For example, if subjects apply sample average learning (discussed later on) their performance may be worse when the exogenous weather variable is more volatile and has no auto-correlation.

2.5 Experimental Details

The experiment uses a between-subjects design in which individual subjects are placed either in Treatment A or U, but not both. They then repeatedly choose parameter vectors (a, b) for the PLM, equation (2) over 50 periods. Since this is an *individual decision-making* experiment where subjects do not interact with other subjects, we regard each subject as an *independent observation*. In total, 59 subjects were recruited from Nanyang Technological University (NTU, Singapore) to participate in our experiment, which was conduced in five experimental sessions. No participant took part in more than one session. We assigned 30 participants to *Treatment U*, and 29 participants to *Treatment A*. A summary of the number of observations in the different sessions and treatments is given in Table A.1 of Appendix A. After completing 50 periods of parameterizing the PLM, subjects were asked to complete a survey.[12]

The experiment was computerized. At the start of each session, subjects were given written instructions explaining the decisions they would make, the computerized decision screens they would use in making those decisions, and how they earned money from their participation in the experiment. A copy of the experimental instructions is found in Appendix E and screenshots of the experimental interface are found in Appendix F. Before subjects could proceed on to the experiment, they had to correctly answer several control questions testing their understanding of the instructions. These questions are also found in Appendix E.

Subjects earned points during the experiment based on the accuracy of their price predictions. The payoff function (in points) is a decreasing function of the price prediction *error*, and is denoted by:

$$\text{Payoff} = \frac{100}{1 + |p_t^e - p_t|}$$

[12]The survey asked them about their age, sex, and how many times they participated in prior economic experiments. The survey also asked them to provide the strategies they used throughout the experiment. A copy of the survey can be found in Appendix F.

Subjects were told that at the end of the experiment, points earned over all 50 periods would be converted into money earnings at a fixed and known rate (200 points = 1 SGD).

Note that we did *not* incentivize subjects to choose pairs (a, b) to be as close as possible to the values that least squares learning would predict at any moment in time, as our interest was in *whether* subjects would in fact choose their parameter estimates in the LSL fashion. Incentivizing subjects to update the PLM parameters in the LSL manner would only *bias* behavior in the direction of the LSL model since the necessary incentivization scheme would require disclosing to subjects the LSL updating rule by way of explaining their payoff function.[13]

Subjects chose a and b using two slider bars on their decision screen with a parameter range of [0,10] for a and a parameter range of [0,1] for b. (See screenshots in Appendix F). Note that these ranges include the REE values, $\overline{a} = 0.6$ and $\overline{b} = 6$.[14] As subjects moved the sliders for either parameter, the computer program showed both the value of a and b and the implied price forecast, p_t^e that would result from their choices for a, b, and by moving one slider at a time, they could see how a change in a or b affected their price forecast p_t^e. Subjects had unlimited time to move these sliders around and see what they implied for price forecasts *before* clicking on a submit button that finalized their choice for a and b in each period t. Thus, subjects were incentivized to think about their choices for the two parameters a and b of the PLM and what those choices implied for their price forecast, p_t^e.[15] Following each period, subjects received feedback in the form of an updated plot of all past prices together with their predictions. They also saw a table containing the history of all their prior period estimates for a, b, realizations of the weather variable w, their implied price forecast p^e the realized price, p their prediction error, $|p^e - p|$, and both their period and cumulative point totals.

Notice that the maximum payoff for a perfect forecast is 100 points per period. Subjects' final payoff is the sum of their 3 SGD show-up fee, and the money value of the points earned over all 50 periods of the experiment. The experiment takes around two hours on average to complete, and the average total payment (including show-up fee) is 20.83 SGD for *Treatment A*, and 20.70 SGD for *Treatment U*. The total average payoff of the experiment is 20.77 SGD.

[13]Similarly, we did not incentivize subjects to choose values for (a, b) to be as close as possible to the REE values $(\overline{a}, \overline{b})$ since subjects would have been able to discover these REE values by looking at their payoff point discrepancies alone.

[14]The midpoints of these parameter ranges, (0.5 and 5, respectively) are a natural first period guess for subjects and are not too far away from REE values. This choice of interval ranges was by design since most learning analyses (see, e.g. Evans and Honkapohja (2001)) study how agents learn in response to very small perturbations of expectations away from REE values.

[15]This design is similar in spirit to the "strategy method" of Selten (1965) that is used to elicit *strategies* as opposed to *actions* alone in game theory experiments.

3 Experimental Results

3.1 Convergence to REE

3.1.1 Convergence of the Market Price

Figure 5 shows the average deviation and the average absolute deviation of the market price from the REE in Treatments U and A. Note that since the realized weather variable, w_t, is different in the two treatments (recall Figure 1), the time series paths for the REE $p_t^* = 6 + 0.6w_t$ will also be different in the two treatments. As Figure 5 reveals, on average, the deviation from the REE price is small in both treatments. The difference between the average market price and the REE is usually less than 1. The results of a t-test suggest that the absolute difference between the market price and the REE is significant at the 5% level in both Treatment U ($t = 11.626, p - value = 0.000$) and Treatment A ($t = 10.257, p - value = 0.000$). On the other hand, the average difference between the market price and the REE is not significantly different from zero at the 5% level for either Treatment U ($t = -0.105, p - value = 0.381$) or Treatment A ($t = -0.884, p - value = 0.299$).

We also performed a t-test on whether the difference between the actual market price and the REE price is significantly different from 0 at the 5% significance level for each *individual* subject and we report these results in Table A.2 in Appendix A. It turns out that we cannot reject the null hypothesis of no difference for all but one subject each in both Treatments A and U. That is, we cannot reject the null of no difference for 29 out of 30 subjects in Treatment A, and 28 out of 29 subjects in Treatment U.

This result shows that when the economy satisfies E-stability ($\alpha < 1$), the market price indeed converges to the REE. But it is important to remain aware that for the same realized w_t, there are infinitely many pairs of values of a and b that satisfy the equation $a + bw_t = 6 + 0.6w_t$. Therefore, we cannot rule out the possibility that individuals successfully predict the REE but are using a model that differs from the REE values for a and b or from what least squares learning would predict for the estimates of those parameter values at any point in time. In the next section, we will consider in more detail whether individuals indeed learn to choose the right combination of values for a and b.

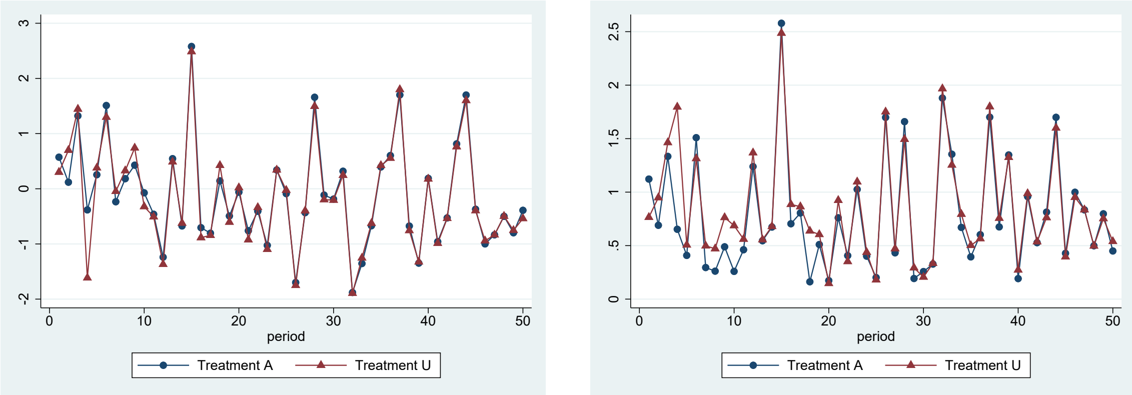


Figure 5. The average deviation (left panel) and average absolute deviation (right panel) of market price from the REE in Treatment A (triangles) and Treatment U (circles).

3.1.2 Sample Means for a, b

In this section, we investigate whether the mean parameter estimates for a, b converge to the REE values, $\bar{a}$, $\bar{b}$. We test two important characteristics, namely whether the parameter estimates are biased and whether they exhibit excess volatility, by comparing the mean and the variance of the coefficient estimates with the REE values in both treatments.

Figure 6 plots the average of all subject predictions of the parameters a, b in each of the 50 periods of the experiment against the REE values which are represented by the horizontal lines, where $\bar{a} = 6$, and $\bar{b} = 0.6$. The Figure reveals the rapid and dampened adjustment over time toward the REE in *Treatment U*. By contrast, in Treatment A, we observe a persistent upward bias in the average estimate for a and a corresponding downward bias in the average estimate for b, relative to REE predictions.[16]

[16]We also conduct tests on the speed of convergence, where following the analysis in Figure 1, subjects in *Treatment U* are able to reach the REE much faster than subjects in *Treatment A*. Details of this analysis can be found in Appendix A.

Figure 6. Average predictions for the parameters a and b for treatment A (upper panel) and treatment U (lower panel) against the REE values.

We next make a more direct comparison between the mean values for a_t, b_t and the REE values. Table 1 shows the sample means and standard deviations of the parameter estimates for a and b in the first 25 periods, the last 25 periods, and the full sample of 50 periods for the two treatments. The means for a, b in *Treatment U* are closer to the REE values, $a = 6$, $b = 0.6$ for all three intervals. The standard deviations for *Treatment U* are also lower than for *Treatment A* in all subsamples.

Table 1. Sample means and standard deviations for a, b, over the first 25 periods, the second 25 periods, and the full sample of 50 periods for the two treatments.

	Periods 1-25		Periods 26-50		Periods 1-50	
	sample average	s.d	sample average	s.d	sample average	s.d
			Treatment A			
a	6.170	1.453	6.188	1.404	6.179	1.428
b	0.574	0.152	0.580	0.148	0.577	0.152
			Treatment U			
a	5.988	1.220	6.018	1.390	6.003	1.120
b	0.598	0.134	0.603	0.114	0.600	0.126

Table 2 shows results from a t-test of the null hypothesis that the sample means of subjects' choices for a, b differ from the REE values over the first 25 periods, and the last 25 periods, and the full sample of 50 periods of both treatments. For *Treatment U* the t-tests reveal that the null hypothesis that the sample mean of the

Table 2. Test of whether the means of the predicted parameters a, b, are different from the REE values over the first 25, and last 25 periods, and all 50 periods for both treatments.

	Periods 1-25				Periods 26-50				Periods 1-50			
	$a = 6$		$b = 0.6$		$a = 6$		$b = 0.6$		$a = 6$		$b = 0.6$	
Treat	t-stat	p	t-stat	p	t-stat	p	t-stat	p	t-stat	p	t-stat	p
A	3.213	0.001	-4.676	0.000	3.433	0.000	-3.882	0.000	4.702	0.000	-8.790	0.000
U	-0.262	0.794	-0.437	0.662	0.660	0.509	0.626	0.531	0.229	0.819	0.152	0.880

parameters equals the REE values *cannot be rejected* at the 5% significance level over each of the three time windows. However, for *Treatment A*, the t-tests lead to *rejection* of the null that the sample means of both parameters are equal to the REE values over all three time windows. These results imply that both parameters in *Treatment U* converge to REE on average over all three samples, while the parameters fail to converge to the REE in *Treatment A* in any time sample.

Tables B.1 and B.2 show the 95% confidence intervals for a and b for each individual subject in Treatments A and U. If the confidence interval of a (b) contains 6 (0.6), it should imply that we cannot reject the null hypothesis that $a = 6$ ($b = 0.6$) for this individual. We count the number of subjects for whom we can reject neither $a = 6$ or $b = 0.6$ in each treatment, and find that we cannot reject that the means of a and b are equal to their REE level for 5 out of 30 subjects (16.67%) in Treatment A, and 14 out of 29 subjects (48.28%) in Treatment U.

Finally, we conduct a Siegel-Tukey test to determine whether *Treatment A* tends to have more widely dispersed predictions than *Treatment U*, using the average prediction of both parameters, their deviation from the REE, as well as the squared value of the deviation, respectively. The null hypothesis states that the variance and median of the predictions in the two treatments are the same, while the alternative hypothesis states that there is a higher variance in Treatment A. As reported in Table 3, we do not find a statistically significant difference in the deviation at the 5% significance level, indicating an equal dispersion in the predictions between two treatments.

Table 3. Siegel-Tukey tests on the dispersion of parameter values across treatments.

	Average prediction by period		Deviation from REE		Squared Deviation from REE	
	a	b	$a-6$	$b-0.6$	$(a-6)^2$	$(b-0.6)^2$
Treatment A	51.18	53.24	51.18	53.21	53.42	47.68
Treatment U	49.82	47.76	49.82	47.79	47.58	53.32
p-value	0.817	0.347	0.817	0.352	0.316	0.333

Note: The data in the rows "*Treatment A*" and "*Treatment U*" are the mean Siegel-Tukey ranks;p-values from tests of the null of no difference are shown in the third row.

3.1.3 Individual-level Analysis of the Convergence of a, b

The last section focused on whether the sample mean values for a, b converged to the REE values. In this section, we examine the development of the elicited a, b *over time*, i.e., whether they converge to the REE values of $a = 6, b = 0.6$ at the *individual subject* level.

We use a very intuitive criterion: a subjects' estimates for a, b are said to converge to the REE if they lie in a very small neighborhood ($\pm .3$ or $.03$) of the REE levels, i.e., $a \in [5.7, 6.3]$, $b \in [0.57, 0.63]$ *and* do not leave that interval following the first period the interval is entered (a consistency requirement). We chose these intervals because they correspond to the $REE \pm 5\% \times REE$. This measure has also been used in previous learning-to-forecast experiments e.g., Bao et al. (2013).

Table 4 reports the distribution of individuals in terms of the number of periods required for convergence to the REE. In treatment A, most subjects never learn the REE; only 3 out of 20 subjects (10%) learn the REE within the 50 periods allowed in the experiment. By contrast, in Treatment U, 6 out of 29 subjects managed to learn the REE within 10 periods, and 10 more managed to learn it within 50 periods. Thus, by the end of the experiment, more than half of subjects in Treatment U, 16 out of 29 (55%), have learned the REE.

Table 4. Distribution of the number of periods it takes for subjects to converge to both REE parameter values, $a \in [5.7, 6.3]$, $b \in [0.57, 0.63]$.

	Treatment			
No. of Periods before Convergence	*A*		*U*	
	No. of subjects	Percentage	No. of subjects	Percentage
$T = 1$	0	0.0%	1	3.4%
$T \in [2, 5]$	0	0.0%	4	13.8%
$T \in [6, 10]$	0	0.0%	1	3.4%
$T \in [11, 25]$	1	3.3%	4	13.8%
$T \in [26, 49]$	2	6.7%	6	20.7%
$T \geq 50$	27	90.0%	13	44.8%
Total	30	100%	29	100%

In addition to this simple convergence criterion, we also examine convergence using regression analysis. We use the convergence formula suggested by Bao et al.

(2013) to find the number and percentage of subjects who successfully achieve convergence to the REE in each treatment.

The linear equation we use for testing whether convergence obtains is as follows. We assume the updating of the parameters follows an AR(1) process. For the parameter submitted by each subject i in period t, we test how that parameter depends on last period's $(t-1)$ submitted parameter, where ρ stands for the coefficient of that relationship, μ is a constant term, and ϵ is the error term.

$$a_{i,t} = \rho_{a_i} a_{i,t-1} + \mu_{a_i} + \varepsilon_{a_i}$$

$$b_{i,t} = \rho_{b_i} b_{i,t-1} + \mu_{b_i} + \varepsilon_{b_i}$$

We say there is *weak convergence* if the parameter submitted by subject i has an estimated value for ρ that is significantly smaller than 1, i.e., if

$$|\hat{\rho}_i| < 1$$

We say there is *strong convergence* if the estimate of the long-run expected value of the parameters a_i, b_i, $\frac{\hat{\mu}_i}{1-\hat{\rho}_i}$ are not significantly different from the REE predicted values, a^*, b^*, where $\hat{\mu}$ stands for the i's estimated values of constant term[17].

In this model, *strong convergence* implies *weak convergence*, and not the reverse. Mathematically,

$$E(a_i) = a^* = 6 = \frac{\mu_{a_i}}{1-\rho_{a_i}}$$

$$E(b_i) = b^* = 0.6 = \frac{\mu_{b_i}}{1-\rho_{b_i}}$$

The result of this estimation is reported in Appendix B. We use robust standard errors in all regressions to ensure correct t-statistics, and to avoid heteroskedasticity. We find that the null hypothesis that $|\hat{\rho}_i| = 1$ *is rejected* for 97% of our sample (or 112 out of 116 predictions,[18] at the 5% significance level in favor of the alternative that $|\hat{\rho}_i| < 1$, implying that our sample exhibits some overall *weak convergence* when predicting the parameters. Comparing the Durbin-Watson d-statistics with the thresholds $(dL = 1.285, dU = 1.445)$ for our sample size $(n = 50, k' = 2)$, the null hypothesis, of no positive serial correlation (if $d < 1.285$) *is rejected* for just 7% of our sample (or 8 among a total of 116 predictions)[19], and the null hypothesis,

[17]Taking parameter a as an example, if there is convergence, then $E(a_{i,t}) = E(a_{i,t-1})$. Thus, taking the expectation form of the function $a_{i,t} = \rho_{a_i} a_{i,t-1} + \mu_{a_i} + \varepsilon_{a_i}$, it becomes: $E(a_{i,t}) = \frac{\mu_{a_i}}{1-\rho_{a_i}}$. Subsequently, if the prediction converges to the *REE, then* $E(a_{i,t}) = a^* = 6 = \frac{\mu_{a_i}}{1-\rho_{a_i}}$.

[18]Note that there are two parameters a, b, so the total number of equations is $59 \times 2 = 118$.

[19]Two of the predictions are omitted because of a collinearity problem, i.e., the two subjects

of no negative serial correlation (if $d > 2.715$) *is rejected* in less than 2% of our sample (or 2 among a total of 116 predictions). We regress the predictions with serial correlation problems using Newey-West standard errors to ensure correct t-statistics. Using a Wald-test and a 5% significance level to assess the predictions with statistically significant $|\hat{\rho}_i| < 1$, we find that 18.64% of such predictions (11 out of 60 predictions) also satisfy the *strong convergence* criterion in *Treatment A*. This percentage increases to 54.39% (31 among out of 58 predictions) in *Treatment U*.

We summarize our results to this point as follows:

Result 1 (E-stability): On average, subjects' predictions for both parameters converge to the REE in treatment U but not in treatment A. At the individual level, 96.55% of all parameter choices (112 out of 118) satisfy a *weak* form of convergence to the REE values. The fraction of individuals who satisfy *strong convergence* to REE parameter values is approximately 54.39% in *Treatment U* (31 out of 58) and 18.64% in *Treatment A* (11 out of 60).

3.2 Fit of the Least Squares Learning Model to the Data

3.2.1 Aggregate Level

In this section, we test whether subjects update their parameter estimates in each of the periods precisely in the manner predicted by least squares learning (3).

The least squares learning model states that subject i will update their parameter estimates for a, b in the current period t, based on the new realization for the weather variable w and past realized price information i.e., prices for periods 1 to $t-1$. For each subject i, in period t, the simple mathematical expression of these least squares learning estimates is given by:

$$\hat{b}_{i,t} = \frac{\sum_{s=1}^{t-1}(w_{i,s} - \overline{w_i})(p_{i,s} - \overline{p_i})}{\sum_{s=1}^{t-1}(w_{i,s} - \overline{w_i})^2}, \hat{a}_{i,t} = \overline{p_i} - \hat{b}_{i,t}\overline{w_i}$$

where $\overline{w_i} = \frac{\sum_{s=1}^{t-1} w_{i,s}}{t-1}, \overline{p_i} = \frac{\sum_{s=1}^{t-1} p_{i,s}}{p-1}$. Thus, the parameter estimates $\hat{a}, \hat{b}$ are the ones that subject i should submit in time period t if he or she follows the LS learning rule.

In each period, the LS learning model uses the same information set as subjects had available to them in the experiment and makes a one period ahead forecast for

submitted the exact same parameter values for all 50 periods.

subjects' choices, $\hat{a}_{i,t}$, $\hat{b}_{i,t}$. Note that this is different from the simulation we did in Figure 2 of Section 2 where the model makes 50 periods ahead forecasts for subject's choices for a, b in all 50 periods after we initialized the model using $a_1 = 5, b_1 = 0.5$. That is, here we are conditioning on the history of the weather (as we did before) but now on the past prices that each subject actually faced when deciding how to update their estimates of a and b (and not on the past prices generated by the LS learning algorithm).

We ran the iterated LS regression for both of the treatments, and recorded the predicted parameter estimates; the detailed values for $\hat{a}, \hat{b}$ can be found in Appendix C. Note that unless otherwise stated, the results we present in this section start from period $t = 3$ ($T \in [1, 2]$). This is due to the sample size being too small in period $t = 2$ ($T \in [1, 1]$). The sample also ends at period $t = 50$ as we do not have data on subjects' submitted parameters for period 51.

Figure 7 plots the average estimated $\hat{a}, \hat{b}$ for the LS learning model in each treatment against the average a, b from the experimental data, and the REE. From The figure reveals a striking difference between subjects' choices for a b and the least squares learning predictions in Treatment A. The estimated $\hat{a}$ ($\hat{b}$) is downward (upward) biased while the experimental data is upward (downward) biased relative to the REE! Meanwhile, the least squares learning model tracks subjects' choices for a, b considerably better in Treatment U. In both treatments, the human subject estimates are more volatile than the least squares learning estimates for both parameters.

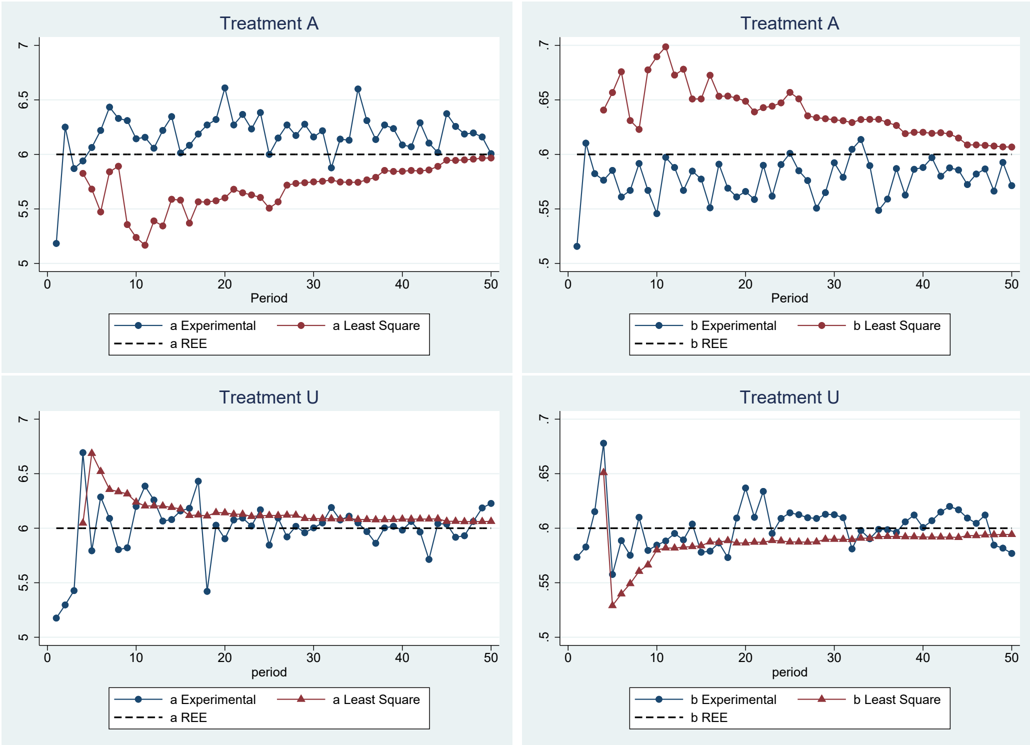


Figure 7. Mean experimental data versus the mean prediction of the 1 period least squares learning model for the parameters a (upper panel) and b (lower panel) against the REE in each treatment.

Table 5 reports the average mean squared error (MSE) of the least squares model relative to the experimental data and the average (square) root MSE (σ) for both treatments over all 50 periods, the first 25 periods and the second 25 periods. In general, the theoretical prediction of the least squares learning model is not far from the experimental data. The average root MSE is usually between 0.8 to 2 for parameter a, and between 0.08 and 0.2 for parameter b, corresponding to approximately $13.33\% - 33.33\%$ of the value of the REE. Not surprisingly, the MSE for the LS model higher in Treatment A than in Treatment U in all intervals except for parameter a in periods 26-50. Generally we find that the least squares model fits the data better in Treatment U than in Treatment A.

Table 5. MSE and root MSE (σ) of the least squares learning model relative to the experimental data in the first 25 periods, the second 25 periods, and the full sample of 50 periods for both treatments.

	Period 1-50		Period 1-25		Period 26-50	
	MSE	σ	MSE	σ	MSE	σ
			Treatment A			
a	4.536	1.984	6.733	2.479	2.428	1.219
b	0.050	0.207	0.073	0.257	0.055	0.132
			Treatment U			
a	1.837	1.121	2.470	1.343	1.228	0.774
b	0.020	0.117	0.025	0.136	0.016	0.088

We further investigate whether the null hypothesis that $a_{i,t} = \hat{a}_{i,t}$, $b_{i,t} = \hat{b}_{i,t}$, holds on average over the aggregate level. We claim a successful adoption of the LS learning rule if the null hypothesis that agent update parameter estimates according to least squares learning predictions *cannot be rejected* at the 5% significance level. The test results using t-tests, can be found in Table 6. It turns out that the estimates differ significantly from the LS learning rule at the aggregate level when we use the data over all 50 periods. The null hypothesis is also *rejected* even if we restrict the sample to last 25 periods (periods 26-50) with the sole exception of parameter a in Treatment U. In summary, we find almost no supportive evidence that subjects update their parameter estimates following the LS learning model at the aggregate level.

Table 6. Results of t-tests of the null hypothesis that $a_{i,t} = \hat{a}_{i,t}, b_{i,t} = \hat{b}_{i,t}$ holds on average over the full sample of subjects.

Treatment A				*Treatment U*			
a		b		a		b	
			All 50 Periods				
z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value
7.237	0.0000	-8.891	0.0000	-3.424	0.0006	3.6058	0.0003
			Second 25 Periods				
z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value
6.347	0.0000	-6.969	0.0000	-1.526	0.1274	2.423	0.0156

3.2.2 Individual Predictions

In this section, we examine how many subjects follow least squares learning at the *individual level*. When we conduct the estimation for each individual, we follow the "1 period ahead" method, namely, in each period, the individual updates the parameters a and b following the least squares learning rule as specified on page 9, and we load the realized values of p_i from the experimental data. We only show the fitted values a and b after Period 4 because we need at least 3 data points (periods) to run a meaningful OLS estimation.

We calculate the mean squared error (MSE) between the least squares learning model prediction and each subject's choice for a, b in Treatments A and U. We consider the person a user of the least squares learning model if their MSE is sufficiently small, i.e., if $MSE < 0.36$ (i.e. a Root MSE less than 0.6, or 10% of the REE) for a and a $MSE < 0.036$ for b (a Root MSE less than 0.06, or 10% of REE) for b. The results can be found in Table D.1 in Appendix D.

Our results reveal that while 12 out of 29 (41.4%) subjects in Treatment U (Subjects 1, 2, 6, 12, 15, 16, 17, 18, 20, 21, 23 and 28) can be categorized as least squares learners, there are *no subjects* in Treatment A who can be categorized as a least squares learners using the same approach.

Result 2 (Least Squares Learning): We *reject* the hypothesis that subjects update the parameters of the PLM following the LS learning rule on average in the aggregate. Yet at the individual level, around 20.3% of the subjects (41.4% of subjects in Treatment U and no subject in Treatment A) appear to update their beliefs following the LS learning rule (3).

A further implication of least square learning is Hypothesis 3 concerning the effect of variation in the exogenous variable on learning. From Table 5 we have following result regarding that hypotheses:

Result 3 (Variation in the Exogenous Variable): Subjects in *Treatment U* are able to estimate the parameters more precisely, and present a smaller standard deviation from the REE compared with subjects in *Treatment A*, though the dispersion of the predictions between two treatments is not statistically significant according to the result of the Siegel-Tukey test as reported in Table 3.

4 Other Learning Models

Since least squares learning does not seem to characterize very well what most subjects were doing in terms of parameterizing the PLM (2) over time, in this section we ask whether other models might do a better job of rationalizing the behavior of the subjects in our experiment. Specifically, we consider four alternatives to least squares learning: 1) a past averaging model, 2) a constant gain learning model,3) the least mean squares (or stochastic gradient) learning model, and finally 4) a model of satisficing.

4.1 Past Averaging Rule

Since subjects seem to simplify the parameter updating task to a point prediction task, we first consider whether they are applying some "learning by averaging" rule which is also found in the learning literature, e.g., Bray (1982), Hommes et al. (2007). Under such a rule, one considers all past actual prices –information that was available to the subjects in our experiment as of each period t – and use the average of those past realized prices as their period t forecast:

$$p_t^e = \overline{p} = \frac{1}{t}\sum_{i=1}^{t-1} p_i$$

Of course, to submit such a forecast, subjects would have to choose values for a and b that would implement that past averaging forecast, but as with satisficing (discussed later) the focus would be on the point prediction, and here we look at the performance of past averaging models in terms of their fit to subjects' implied forecasts for p alone.

Figure 8 shows the average point prediction from the experimental data (circles) and a simulation using the "learning by averaging" heuristic (triangles) in Treatment A (top panel) and in Treatment U (lower panel). Both time series seem to converge to the REE point prediction, $p^e = 12$, but "learning by averaging" seems to converge at a much *slower* speed as compared with the subjects in the experiment.

Table D.2 in Appendix D reports on the MSE between the past averaging heuristic and each subject's prediction in Treatments A and U. We can see that the fit of the learning by averaging rule is better in Treatment A than in Treatment U. The average MSE is 0.5645 in Treatment A and is 2.2342 in Treatment U. If we consider a subject a user of the learning by averaging heuristic if their MSE is smaller than 1.2 (10% of the REE), then the number of users of the learning by averaging rule is 28 out of 30 in Treatment A, but only 12 out of 29 in Treatment U. This is likely due to the fact that subjects in Treatment U learn the REE faster than do subjects in Treatment A.

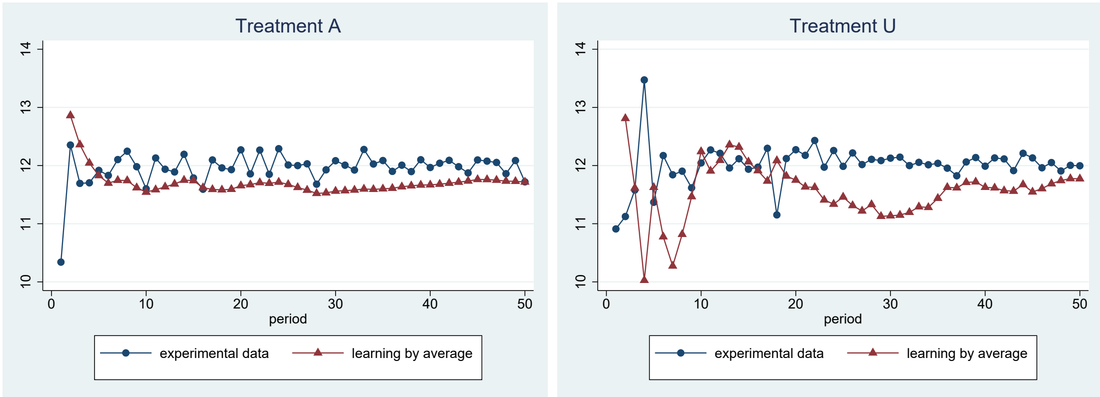


Figure 8. The average point prediction from experimental data (circles) and simulation using "learning by averaging" heuristic (triangles) in Treatment A (left panel) and U (right panel).

4.2 Constant Gain Learning

We also estimate the constant gain learning model (discussed earlier in section 2.2) for each individual and report the results in Table D.3 in Appendix D. Similar to the exercise reported on in the previous section, we consider the person a user of the constant gain learning model if the MSE between their parameterization of the PLM and the constant gain learning model predictions are sufficiently small, i.e., if $MSE < 0.36$ (Root of MSE less than 0.6) for a and $MSE < 0.036$ for b (Root of MSE less than 0.06) for b. It turns out that there are only 8 out of 29 (27.6%) subjects in Treatment U (Subjects 1, 2, 12, 15, 18, 20, 23 and 28) who can be categorized as users of the constant gain learning model, and no subject in Treatment A who can be categorized as a user of constant gain learning model. For the 8 subjects in Treatment U, the mean squared error of the constant gain learning model is larger than for the least squares learning model. In general, though the constant gain learning model is usually assumed to converge "faster" to an REE than least squares learning and is more suitable in the context where the price dynamics are more volatile, we do not find stronger evidence for constant gain learning in our individual-level analysis even though the constant gain learning model has a free parameter γ estimated for each subject that helps to best fit the experimental data.

4.3 Least Mean Squares Learning

The time and memory complexities of RLS and constant gain learning are both $O(m)^2$, where m is the dimension of x. A much simpler learning algorithm is the least mean squares (LMS) learning model which is also known as stochastic gradient descent learning (e.g. Evans et al. (2010)). In this case, only the parameter vector $\hat{\theta}$ is updated according to the gradient of the error term; the variance covariance matrix is not used.

This algorithm is also derived from the objective of minimizing the mean of squared errors, but it does not rely on cross-correlations or auto-correlations, i.e., on the variance-covariance matrix R. Thus, the time and memory complexities of LMS learning are $O(m)$. On the other hand, convergence to the global minimum is not assured under least mean squares learning unless the gain parameter λ is gradually reduced over time as in RLS.

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \lambda x_t (p_t - \hat{\theta}'_{t-1} x_t) \tag{5}$$

We estimate the least means squares learning model for each subject and report the MSE between those predictions and subjects' parameter choices in Table 7 in Section 4.6 and D.4 of Appendix D. The MSE of the least mean squares learning model turns out to be larger than for the least squares and constant gain learning models. If we consider a subject to be a user of the least mean squares learning model if the MSE is sufficiently small, i.e., if $MSE < 0.36$ (root MSE less than 0.6) for a and $MSE < 0.036$ for b (root MSE less than 0.06) for b, then it turns out that there are only 2 out of 29 (6.9%) subjects in Treatment U (Subjects 15 and 21) and 2 out of 30 (Subjects 2 and 20) subjects in Treatment U (6.7%) who can be categorized as users of the least mean square learning model. We think the main reason is that the stochastic nature of this learning process has difficulty in capturing the convergence to REE that we observe among most of our subjects.

4.4 A Satisficing Model

A final plausible explanation for the behavior of subjects in our experiment is that they were using some type of "simple satisfying heuristic" or satisficing rule as suggested by Simon (Simon, 1955, 1956). In particular, subjects might stay with a prediction rule, or a specific combination of a, b so long as those parameter choices kept their prediction error small, or reached a close enough neighborhood of the REE. Thereafter, they do not engage in any further updating of the parameter vector $(a,\ b)$.

In our experiment, the unique REE of the economy is $p^e = 6 + 0.6w_t$. If we ignore the variation in w_t and simply use the expected value, $E[w_t] = 10$, the numerical value of the price point prediction associated with the REE is $6+0.6E(w_t) = 6 + 0.6 \times 10 = 12$.

If the variation in w_t is small, then *any* combination of a, b that satisfies the equation $a + 10b = 12$ should generate a point prediction that is not very far from the REE of the economy, and hence yield only a small prediction error. If subjects learn via experimenting with different combinations of a, b and adjust their choices to minimize the prediction errors, this process may lead them to choose any pair of values for a, b that are not too far away from $a = 6, b = 0.6$ but which also satisfy the equation $a + 10b = 12$, for example, $a = 7$ and $b = 0.5$ or $a = 4.8$ and $b = 0.72$ would work.[20]

Figure 9 shows the dynamics of $a_i + 10b_i$ for each subject i in Treatments A and U. Indeed, though many subjects fail to learn the REE values for a and b, most of them are able to choose a combination of the a, b parameters that satisfies the equation $a + 10b = 12$.

Table D.5 in Appendix D reports on a 95% confidence interval for $a + 10b$ in both treatments. It turns out that this confidence interval includes the REE value of 12 for 30 out of 30 subjects in Treatment A, and 25 out of 29 subjects (that is, all subjects except Subjects 3, 6, 13, 26) in Treatment U. In other words, according to a t-test, we cannot reject the notion that subjects chose a and b so as to satisfy the equation $a + 10b = 12$ at the 95% level for most subjects in both treatments. In other words, subjects are able to come up with a point prediction that is close to the REE point prediction, even without learning the true REE values for a and b.

Figure 10 shows the scatter-plot of a and b for each individual in Treatment A (top panel) and Treatment U (lower panel). There seems to be a substantial level of heterogeneity in the way people learn over time. While the behavior of some subjects (Subjects 1, 8, 14, 16, 24 in Treatment A and Subjects 16 and 24 in Treatment U) seem to behave in a similar manner to the simulated path of a, b from the least squares learning model as shown in Figure 3, other subjects behave very differently. For example, some subjects (Subjects 11 17 and 21 in Treatment A, and Subjects 12, 18 and 20 in Treatment U) seem to experiment with different values of b while keeping the value of a fixed. Some subjects (Subjects 11, 20, 27 and 29 in Treatment A) are also able to reach a small neighborhood of the REE fairly quickly. Some subjects (Subjects 3, 6, 8, 10, 25 and 26 in Treatment U) explored a large range of values of the parameters before they settled down in a region that was usually not far from the REE values.

[20]Indeed, if a subject starts from either the midpoint of the domain of a or b and only updates the other parameter, we should observe many subjects choosing $a = 5, b = 0.7$ or $a = 7, b = 0.5$. It turns out we cannot reject this type of behavior for 6 subjects in Treatment A (Subject 10,12,13,15,21 and 30), and 2 subjects in Treatment U (Subject 6 and 9).

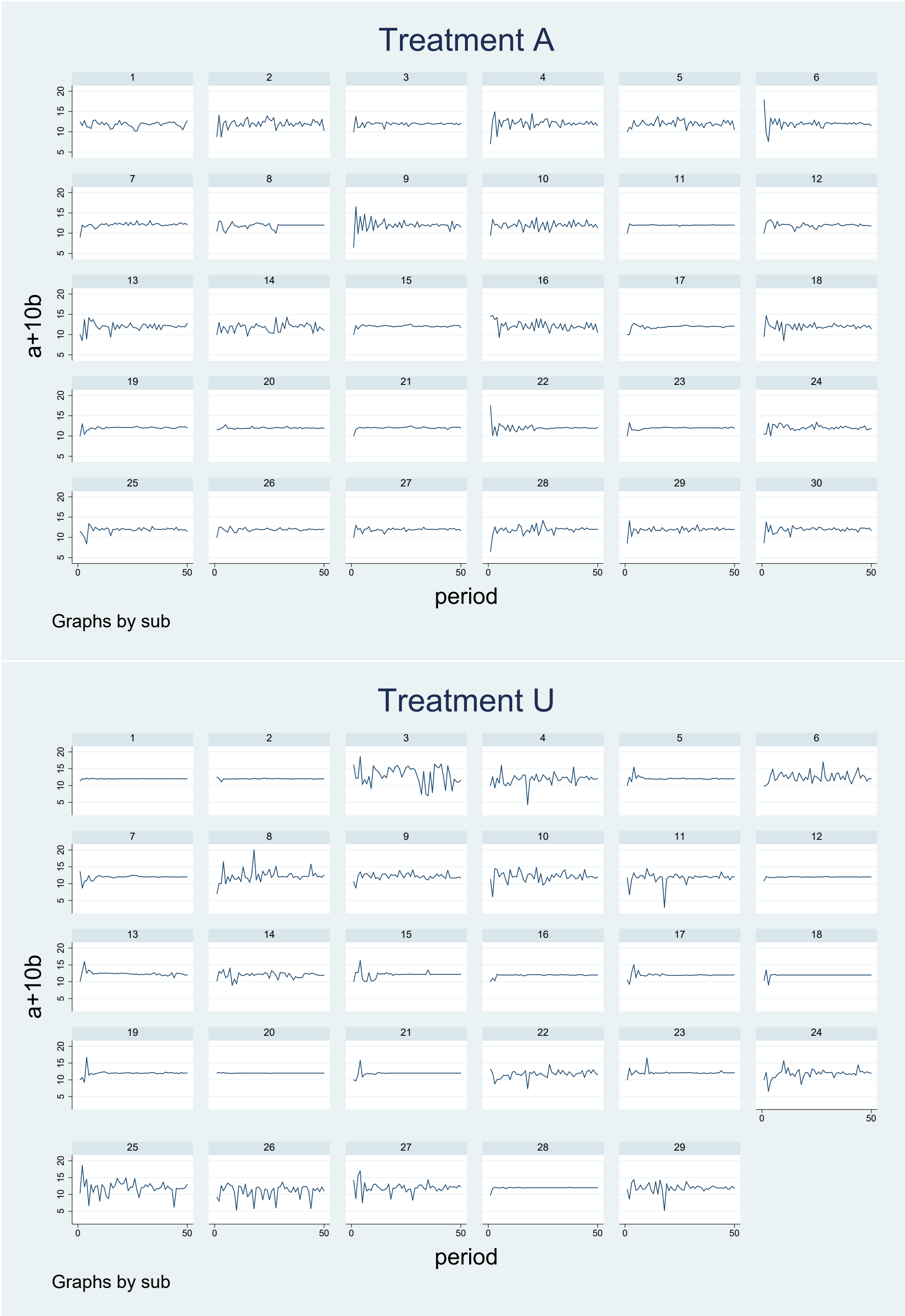


Figure 9. The value of $a + 10b$ for each individual in Treatment A (top panel) and U (lower panel). We report the 95% confidence intervals of $a + 10b$ for each subject in Table D.5. The confidence interval includes 12 for all subjects except Subject 3, 6, 13 and 26 in Treatment U.

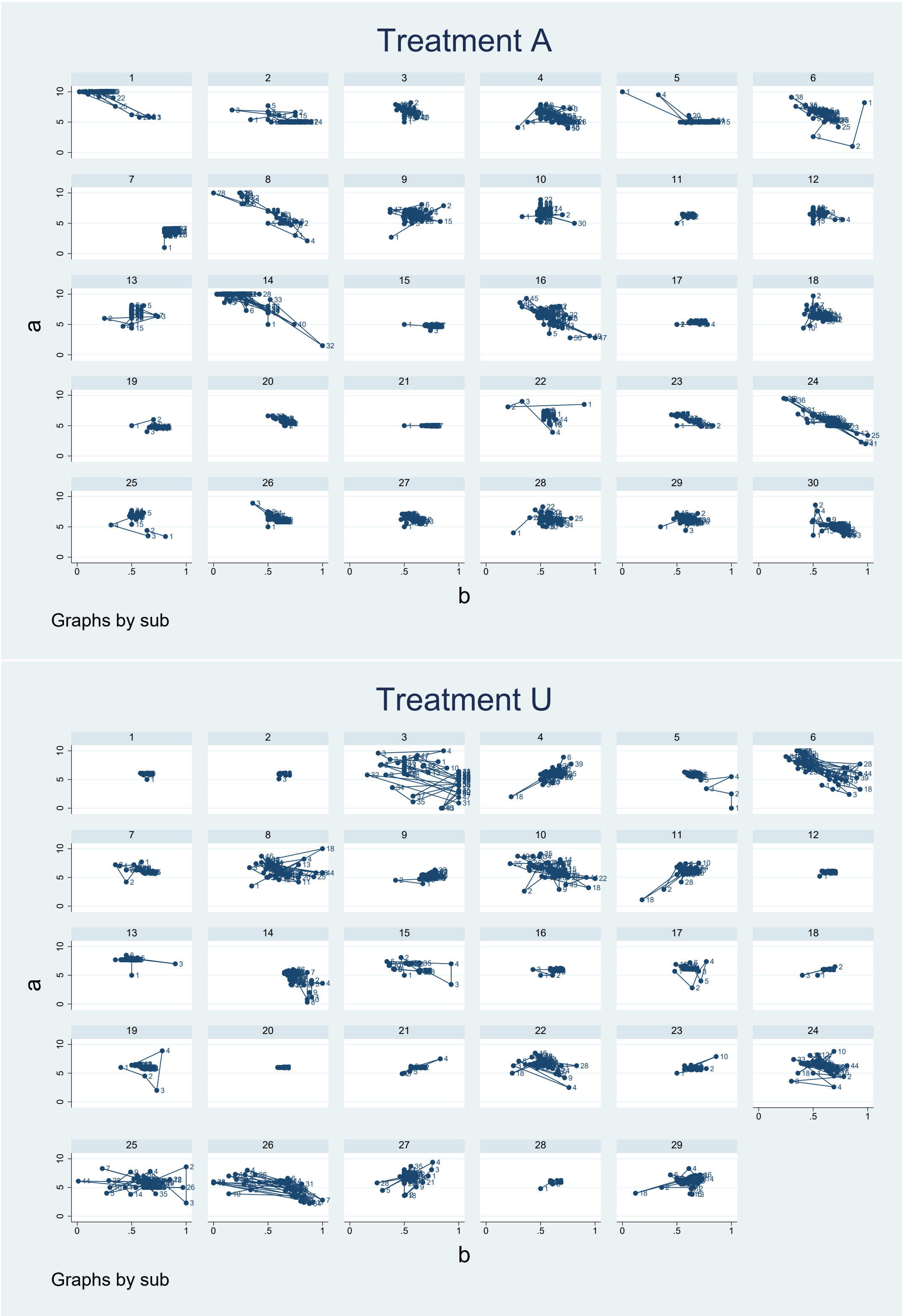


Figure 10. A scatterplot of a and b for each individual subject in Treatment A (top panel) and U (lower panel).

4.5 Comparison of Model Fits to the Experimental Data

Table 7 summarizes the average mean squared errors across the different learning models relative to the experimental data. MSE_a refers to the mean squared error between the experimental data and the model for parameter a, while MSE_b refers to the mean squared error between the experimental data and the model for parameter b and finally MSE_p refers to the mean squared error between the data and the model price forecast p. Note that the sample averaging (SA) model only provides a prediction for the price level, p. Similarly, the satisficing (SF) model assumes that agents choose a combination of a, b that satisfies $a + 10b = 12$. Thus it is not possible to calculate MSE_a, MSE_b for those two models, and the MSE_p of the the SF model will be the same as the RE model.

Table 7. The mean squared error for different learning models in terms of fitting experimental data. MSE_a is the squared error between the data and model prediction for parameter a, MSE_b is the mean squared error between the data and the model for parameter b and MSE_p is the mean squared error between the data and the model for the price forecast, p. The models we consider include RE (rational expectations), RLS (recursive least square), SA (sample averaging), CGL (constant gain learning), LMS (least mean square learning) and SF (a satisficing rule).

	Treatment A			Treatment U			Both Treatments		
	MSE_a	MSE_b	MSE_p	MSE_a	MSE_b	MSE_p	MSE_a	MSE_b	MSE_p
RE	2.069	0.023	0.969	1.254	0.016	1.168	1.662	0.019	1.069
RLS	3.707	0.044	1.345	1.517	0.018	0.979	2.612	0.031	1.162
SA	NA	NA	0.564	NA	NA	2.234	NA	NA	1.399
CGL	5.076	0.061	0.989	1.779	0.019	1.280	3.428	0.040	1.135
LMS	1.767	0.210	5.934	1.521	0.169	5.284	1.642	0.190	5.610
SF	NA	NA	0.969	NA	NA	1.168	NA	NA	1.069

In general, there is not a large difference between the average MSEs of the different learning models (least squares, sample averaging and constant gain learning) for both treatments, and the average MSEs for most models are generally greater than that of the rational expectations model where people constantly choose $a = 6, b = 0.6$. A likely explanation for this finding is that most of the learning models have long memory and put heavy weight on past observations. These models therefore predict slower learning speeds than the subjects' actual learning speed in the experiment, and therefore underperform relative to the RE model/satisficing rule.

If we consider a subject to be a user of the least mean squares learning model if the MSE_p is sufficiently small, i.e., if $MSE_p < 1.44$ (root MSE less than 1.2, 10% of the REE price prediction), then the number of subjects who can be categorized as users of different learning models can be summarized by Table 8.

Table 8. The number of subjects who can be categorized as users of difference learning models based on the size of MSE_p. The models we consider include RE (rational expectations), RLS (recursive least square), SA (sample averaging), CGL (constant gain learning), LMS (least mean square learning) and SF (a satisficing rule).

Model	Treatment A	Percentage	Treatment U	Percentage
RE	30	100.0%	29	100.0%
RLS	19	63.3%	8	27.6%
SA	29	96.7%	16	55.2%
CGL	29	96.7%	23	79.3%
LMS	5	16.7%	3	10.3%
SF	30	100.0%	29	100.0%

Among the learning models, the sample average learning model generates the smallest MSE in terms of fitting the price data in Treatment A, and the recursive least squares learning model generates the smallest MSE in terms of fitting the price data in Treatment U. This finding provides some support for the notion that more people use sample average learning/least squares learning in Treatments A/U. Still, as we have seen, the satisficing rule provides the best description of the overall pattern of subjects' prediction behavior in both treatments, and like the RE prediction it has the lowest overall MSE across both treatments as well as the largest number of users according to the criteria of sufficiently small MSE_p.

5 Conclusion

In this paper, we have conducted the first ever structural test of the seminal least squares learning model using a simple Cobweb model economy. The subjects in our experiment submit predictions for two unknown parameters in a linear PLM that nests the REE as a special case.

We observe how subjects update these parameters over time. Since the slope coefficient on the expectations term, α, is less than 1, our experimental economy satisfies the E-stability condition, and so learning agents should converge to the REE.

In general, all of our markets converge to a neighborhood of the REE, which is supportive of the E-stability prediction. We find that around 97% of the individual predictions satisfy a *weak convergence* criterion. On average, the predictions by subjects in *Treatment U* converge faster than the predictions of subjects in *Treatment A*. At the aggregate level, we observe convergence to the REE in *Treatment U* but not in *Treatment A* and there are almost three times as many subjects satisfying our *strong convergence* criterion in *Treatment U* as compared with *Treatment A*.

Our results suggest that the least squares learning model yields correct predictions at the aggregate level in terms of convergence or near convergence to the REE. However, at the individual level, it does not seem to be a good descriptor of how individual agents update their expectations over time. We find that overall, just 20.3% of our subjects can be categorised as following the least squares learning rule across both treatments. Least squares learners are found only in treatment U where they comprise 41.4% of the subjects in that treatment; we find no subject employing least squares learning in treatment A.

For those who deviate from least squares learning, many of them seem to adopt some kind of dimension reducing strategy focusing on price point prediction accuracy alone. This behavior is consistent with the "satisficing" approach of Simon (Simon, 1955, 1956), and a simple satisficing heuristic appears to explain our experimental data better than does least squares learning.

The environment we have studied is a very simple individual-decision making environment. In future research, it would be of interest to study how agents update the parameters of their PLM in settings where there is a group of n subjects whose forecasts matter for realizations of the variables being forecasted as in an n-player learning–to–forecast experiments. It would further be of interest to elicit subjects' PLMs, rather than giving them one that explicitly nests the REE solution and asking how they would parameterize that particular PLM. Finally it would also be of interest to give subjects access to statistical software or to provide them with a choice of forecasting models to parameterize their PLM or form forecasts of future prices. We view the present study as a first, *small step* in the direction of developing a more structural approach to understanding the manner in which agents form expectations and so we leave the study of these more complex environments to future research.

References

Anufriev, M., Hommes, C., and Makarewicz, T. (2019). Simple forecasting heuristics that make us smart: Evidence from different market experiments. *Journal of the European Economic Association*, 17(5):1538–1584.

Arifovic, J. and Duffy, J. (2018). Heterogeneous agent modeling: experimental evidence. In *Handbook of Computational Economics*, volume 4, pages 491–540. Elsevier.

Arifovic, J., Hommes, C., and Salle, I. (2019). Learning to believe in simple equilibria in a complex olg economy-evidence from the lab. *Journal of Economic Theory*, 183:106–182.

Armona, L., Fuster, A., and Zafar, B. (2019). Home price expectations and behaviour: Evidence from a randomized information experiment. *The Review of Economic Studies*, 86(4):1371–1410.

Assenza, T., Bao, T., Hommes, C., Massaro, D., et al. (2014). Experiments on expectations in macroeconomics and finance. *Experiments in Macroeconomics*, 17(2014):11–70.

Assenza, T., Heemeijer, P., Hommes, C. H., and Massaro, D. (2019). Managing self-organization of expectations through monetary policy: a macro experiment. *Journal of Monetary Economics*.

Bao, T. and Duffy, J. (2016). Adaptive versus eductive learning: Theory and evidence. *European Economic Review*, 83:64–89.

Bao, T., Duffy, J., and Hommes, C. (2013). Learning, forecasting and optimizing: An experimental study. *European Economic Review*, 61:186–204.

Binder, C. and Rodrigue, A. (2018). Household informedness and long-run inflation expectations: Experimental evidence. *Southern Economic Journal*, 85(2):580–598.

Binmore, K. (1987). Modeling rational players: Part i. *Economics & Philosophy*, 3(2):179–214.

Branch, W. and McGough, B. (2016). Heterogeneous beliefs and trading inefficiencies. *Journal of Economic Theory*, 163:786–818.

Branch, W. A. (2004). The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. *The Economic Journal*, 114(497):592–621.

Branch, W. A., Davig, T., and McGough, B. (2013). Adaptive learning in regime-switching models. *Macroeconomic Dynamics*, 17(5):998–1022.

Branch, W. A. and Evans, G. W. (2006). A simple recursive forecasting model. *Economics Letters*, 91(2):158–166.

Bray, M. (1982). Learning, estimation, and the stability of rational expectations. *Journal of Economic Theory*, 26(2):318–339.

Bray, M. M. and Savin, N. E. (1986). Rational expectations equilibria, learning, and model specification. *Econometrica: Journal of the Econometric Society*, 54(5):1129–1160.

Coibion, O., Gorodnichenko, Y., and Kamdar, R. (2018). The formation of expectations, inflation, and the phillips curve. *Journal of Economic Literature*, 56(4):1447–91.

Coibion, O., Gorodnichenko, Y., and Ropele, T. (2020a). Inflation expectations and firm decisions: New causal evidence. *The Quarterly Journal of Economics*, 135(1):165–219.

Coibion, O., Gorodnichenko, Y., and Weber, M. (2020b). Does policy communication during covid work? Technical report, National Bureau of Economic Research.

Coibion, O., Gorodnichenko, Y., and Weber, M. (2022). Monetary policy communications and their effects on household inflation expectations. *Journal of Political Economy*, 130(6):000–000.

Cornand, C. and Hubert, P. (2020). On the external validity of experimental inflation forecasts: A comparison with five categories of field expectations. *Journal of Economic Dynamics and Control*, 110:103746.

Duffy, J. (2016). Macroeconomics: A survey of laboratory research. *The Handbook of Experimental Economics*, 2:1.

Evans, G. W. and Guesnerie, R. (2005). Coordination on saddle-path solutions: the eductive viewpoint—linear multivariate models. *Journal of Economic Theory*, 124(2):202–229.

Evans, G. W., Guesnerie, R., and McGough, B. (2019). Eductive stability in real business cycle models. *The Economic Journal*, 129(618):821–852.

Evans, G. W. and Honkapohja, S. (1999). Learning dynamics. *Handbook of Macroeconomics*, 1:449–542.

Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press.

Evans, G. W. and Honkapohja, S. (2003). Adaptive learning and monetary policy design. *Journal of Money, Credit and Banking*, pages 1045–1072.

Evans, G. W. and Honkapohja, S. (2009). Learning and macroeconomics. *Annual Review of Economics*, 1(1):421–449.

Evans, G. W., Honkapohja, S., and Williams, N. (2010). Generalized stochastic gradient learning. *International Economic Review*, 51(1):237–262.

Fourgeaud, C., Gourieroux, C., and Pradel, J. (1986). Learning procedures and convergence to rationality. *Econometrica: Journal of the Econometric Society*, 54(4):845–868.

Friedman, M. (1953). *Essays in positive economics*. University of Chicago press.

Grandmont, J.-M. (1998). Expectations formation and stability of large socioeconomic systems. *Econometrica*, 66(4):741–781.

Greene, W. H. (2000). *Econometric Analysis.* New Jersey: Prentice Hall.

Guesnerie, R. (1992). An exploration of the eductive justifications of the rational-expectations hypothesis. *The American Economic Review*, 82(5):1254–1278.

Guesnerie, R. and Woodford, M. (1991). Stability of cycles with adaptive learning rules. In *Equilibrium theory and applications: Proceedings of the sixth international symposium in economic theory and econometrics*, pages 111–134. Cambridge University Press Cambridge.

He, S. and Kucinskas, S. (2019). Complexity and expectation formation. *Available at SSRN 3450207.*

Hommes, C. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, 35(1):1–24.

Hommes, C., Sonnemans, J., Tuinstra, J., and Van de Velden, H. (2005). Coordination of expectations in asset pricing experiments. *The Review of Financial Studies*, 18(3):955–980.

Hommes, C., Sonnemans, J., Tuinstra, J., and van de Velden, H. (2005). A strategy experiment in dynamic asset pricing. *Journal of Economic Dynamics and Control*, 29(4):823–843.

Hommes, C., Sonnemans, J., Tuinstra, J., and Van De Velden, H. (2007). Learning in cobweb experiments. *Macroeconomic Dynamics*, 11(S1):8–33.

Hommes, C. and Sorger, G. (1998). Consistent expectations equilibria. *Macroeconomic Dynamics*, 2(3):287–321.

Hommes, C. and Zhu, M. (2014). Behavioral learning equilibria. *Journal of Economic Theory*, 150:778–814.

Honkapohja, S. (1995). Bounded rationality in macroeconomics a review essay. *Journal of Monetary Economics*, 35(3):509–518.

Kryvtsov, O. and Petersen, L. (2020). Central bank communication that works: Lessons from lab experiments. *Journal of Monetary Economics.*

Landier, A., Ma, Y., and Thesmar, D. (2019). Biases in expectations: Experimental evidence. *Available at SSRN 3046955.*

Lucas Jr, R. E. (1972). Expectations and the neutrality of money. *Journal of Economic Theory*, 4(2):103–124.

Marcet, A. and Sargent, T. J. (1988). The fate of systems with" adaptive" expectations. *The American Economic Review*, 78(2):168–172.

Marimon, R., Spear, S. E., and Sunder, S. (1993). Expectationally driven market volatility: an experimental study. *Journal of Economic Theory*, 61(1):74–103.

Mirdamadi, M. and Petersen, L. (2018). Macroeconomic literacy and expectations. *Available at SSRN 3187131*.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica: Journal of the Econometric Society*, 29(3):315–335.

Pfajfar, D. and Santoro, E. (2010). Heterogeneity, learning and information stickiness in inflation expectations. *Journal of Economic Behavior & Organization*, 75(3):426–444.

Preston, B. (2006). Adaptive learning, forecast-based instrument rules and monetary policy. *Journal of Monetary Economics*, 53(3):507–535.

Sargent, T. J. (1993). Bounded rationality in macroeconomics: The arne ryde memorial lectures. *OUP Catalogue*, 35(3):509–518.

Sargent, T. J. (2008). Evolution and intelligent design. *American Economic Review*, 98(1):5–37.

Selten, R. (1965). Die strategiemethode zur erforschung des eingeschränkt rationalen verhaltens im rahmen eines oligopolexperimentes. Seminar für Mathemat. Wirtschaftsforschung u. Ökonometrie.

Simon, H. A. (1955). A behavioral model of rational choice. *The Quarterly Journal of Economics*, 69(1):99–118.

Simon, H. A. (1956). Rational choice and the structure of the environment. *Psychological Review*, 63(2):129.

Woodford, M. (2013). Macroeconomic analysis without the rational expectations hypothesis. *Annual Review of Economics*, 5(1):303–346.

Appendices

Appendix A: Supplementary Information

Appendix A.1: Session Information

Table A.1. Characteristics of Experimental Design.

Session Number	Treatment Type	Number of participants *(N)*
1	*A*	13
2	*A*	17
3	*U*	14
4	*U*	10
5	*U*	5

Appendix A.2: t-test on the Price Level

We perform a t-test on the equality between the realized market price and the REE price level and report the results in Table A.2.

Table A.2. t-test on the price level. The null hypothesis is that the average price is equal to the REE, i.e. $p_t^* = 6 + 0.6w_t$.

	Treatment A		***Treatment U***	
Sub	t-statistic	p-value	t-statistic	p-value
1	0.050	0.971	1.150	0.256
2	1.000	0.316	1.100	0.280
3	0.650	0.521	-1.750	0.086
4	0.150	0.873	1.150	0.247
5	1.350	0.189	-0.200	0.840
6	0.150	0.886	1.800	0.078
7	0.950	0.353	0.850	0.411
8	1.500	0.135	-0.050	0.951
9	0.250	0.823	0.300	0.782
10	0.300	0.769	0.000	0.985
11	1.200	0.236	1.600	0.116
12	0.900	0.368	1.750	0.084
13	0.400	0.690	-2.800	0.007
14	0.250	0.790	0.400	0.702
15	1.250	0.213	-0.600	0.550
16	-0.900	0.385	1.350	0.185
17	2.250	0.030	0.300	0.752
18	0.150	0.881	1.300	0.198
19	1.650	0.105	0.100	0.911
20	1.200	0.238	0.950	0.358
21	1.750	0.083	0.250	0.803
22	-0.200	0.840	0.350	0.729
23	1.500	0.138	-0.450	0.656
24	0.800	0.416	0.850	0.400
25	1.200	0.235	0.450	0.666
26	1.350	0.185	1.250	0.212
27	1.600	0.114	-0.600	0.556
28	1.200	0.237	1.400	0.171
29	1.100	0.276	0.650	0.517
30	1.300	0.205		

Appendix A.3: Comparison of the Demographic Characteristics of Participants in the Two Treatments

As a balance check to rule out the possibility of selection bias, we conduct a regression analysis of differences in demographic characteristics between the group of subjects assigned to Treatment A and the group assigned to Treatment U. Two sample t-tests are used to compare the demographic characteristics and participation experience between the two groups. The results indicate that there is no statistically significant difference at the 5% significance level between the groups on the recorded factors. It confirms that our randomization was successful and gives us more freedom to conclude that the observed differences with predictions are brought about by differences in the treatment conditions alone.

Appendix B: Testing Convergence Using Linear Estimation

Table B.1. Mean, standard error and 95% confidence interval (CI) of a in Treatment A and U.

	Treatment A				***Treatment U***			
Sub	Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI	
1	9.333	0.242	8.847	9.819	5.976	0.020	5.936	6.017
2	5.339	0.126	5.087	5.592	5.894	0.018	5.857	5.931
3	6.849	0.086	6.676	7.022	5.557	0.340	4.874	6.240
4	5.798	0.144	5.509	6.087	5.902	0.141	5.618	6.186
5	5.320	0.162	4.995	5.644	5.573	0.140	5.290	5.855
6	6.269	0.182	5.902	6.635	7.133	0.269	6.594	7.673
7	3.700	0.068	3.563	3.837	6.124	0.070	5.984	6.263
8	6.804	0.236	6.329	7.278	6.335	0.171	5.991	6.679
9	6.194	0.121	5.950	6.438	5.124	0.061	5.001	5.246
10	6.892	0.125	6.640	7.144	6.204	0.203	5.797	6.611
11	6.304	0.032	6.240	6.368	6.041	0.142	5.757	6.326
12	6.827	0.097	6.632	7.023	5.939	0.018	5.903	5.976
13	6.875	0.107	6.659	7.090	7.692	0.059	7.574	7.811
14	8.920	0.278	8.361	9.478	4.598	0.179	4.238	4.958
15	4.800	0.045	4.710	4.890	6.075	0.103	5.869	6.280
16	6.412	0.211	5.987	6.837	5.931	0.029	5.873	5.990
17	5.322	0.019	5.284	5.359	6.090	0.089	5.912	6.269
18	6.563	0.119	6.324	6.801	5.971	0.029	5.912	6.030
19	4.729	0.034	4.662	4.797	6.000	0.107	5.785	6.215
20	5.835	0.061	5.712	5.958	6.000	0.000	6.000	6.000
21	5.000	0.000	5.000	5.000	5.976	0.044	5.888	6.065
22	6.645	0.105	6.434	6.856	6.578	0.143	6.291	6.865
23	6.147	0.094	5.959	6.335	5.835	0.049	5.737	5.933
24	5.592	0.195	5.201	5.983	6.159	0.148	5.862	6.456
25	6.725	0.121	6.483	6.968	5.735	0.155	5.424	6.046
26	6.247	0.081	6.084	6.410	5.002	0.201	4.597	5.407
27	6.418	0.071	6.275	6.560	6.757	0.146	6.463	7.051
28	6.235	0.104	6.027	6.444	5.976	0.026	5.925	6.028
29	6.131	0.130	5.870	6.393	6.016	0.110	5.796	6.236
30	5.022	0.125	4.771	5.272				

Table B.2. Mean, standard error and 95% confidence interval (CI) of b in Treatment A and U.

	Treatment A				***Treatment U***			
Sub	Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI	
1	0.227	0.021	0.185	0.269	0.601	0.001	0.599	0.604
2	0.679	0.019	0.641	0.717	0.610	0.001	0.609	0.611
3	0.506	0.005	0.495	0.516	0.750	0.038	0.674	0.826
4	0.622	0.017	0.589	0.656	0.593	0.011	0.570	0.615
5	0.683	0.020	0.643	0.722	0.649	0.013	0.622	0.676
6	0.564	0.017	0.530	0.598	0.537	0.027	0.483	0.592
7	0.841	0.005	0.831	0.851	0.580	0.009	0.562	0.597
8	0.504	0.023	0.459	0.550	0.595	0.021	0.552	0.638
9	0.573	0.014	0.545	0.600	0.705	0.009	0.688	0.722
10	0.519	0.011	0.497	0.540	0.593	0.022	0.550	0.636
11	0.565	0.002	0.561	0.570	0.569	0.010	0.549	0.590
12	0.527	0.012	0.503	0.551	0.602	0.001	0.599	0.605
13	0.495	0.013	0.469	0.521	0.469	0.010	0.450	0.488
14	0.264	0.027	0.209	0.319	0.735	0.011	0.713	0.756
15	0.714	0.015	0.684	0.745	0.602	0.016	0.571	0.634
16	0.571	0.021	0.528	0.613	0.599	0.004	0.591	0.608
17	0.659	0.006	0.647	0.671	0.590	0.006	0.578	0.603
18	0.542	0.008	0.525	0.559	0.597	0.005	0.588	0.606
19	0.721	0.007	0.707	0.736	0.599	0.007	0.585	0.613
20	0.611	0.007	0.597	0.624	0.600	0.001	0.599	0.602
21	0.700	0.005	0.691	0.709	0.595	0.006	0.584	0.607
22	0.537	0.011	0.515	0.559	0.512	0.015	0.481	0.543
23	0.578	0.012	0.555	0.601	0.630	0.006	0.617	0.643
24	0.642	0.022	0.597	0.687	0.568	0.015	0.537	0.598
25	0.514	0.009	0.497	0.531	0.626	0.025	0.575	0.677
26	0.567	0.007	0.553	0.581	0.592	0.036	0.520	0.664
27	0.538	0.013	0.512	0.564	0.525	0.012	0.502	0.549
28	0.561	0.011	0.539	0.582	0.598	0.002	0.594	0.602
29	0.564	0.009	0.546	0.581	0.585	0.013	0.560	0.611
30	0.694	0.012	0.669	0.719				

Table B.3. Linear Estimation on parameter a in *Treatment A*.

Number	$\hat{\rho}_a$	p-value ($\|\hat{\rho}_a\| \geq 1$)	μ_a	R^2	Root MSE	Durbin Watson	Equilibrium	Wald test p-value
1	0.5466	0.0128	4.3842	0.3543	0.9517	2.3364	9.6777	0.0000
2	0.4304	0.0039	2.9828	0.1849	0.5606	2.3223	5.2333	0.0000
3	-0.1747	0.0000	8.1052	0.0366	0.5417	1.5705	6.8979	0.0000
4	-0.3225	0.0000	7.7630	0.1032	0.9402	1.8359	5.8677	0.2001
5	-0.0306	0.0000	5.2887	0.0020	0.6678	2.0131	5.1312	0.0000
6	0.1060	0.0005	5.5985	0.0117	1.2809	1.3254	6.2617	0.2007
7	0.0808	0.0000	3.4494	0.0173	0.3027	1.3677	3.7522	0.0000
8	0.7153	0.0017	2.0166	0.5706	1.0794	2.4043	7.0772	0.0488
9	-0.4139	0.0000	8.8678	0.2615	0.6072	1.7516	6.2716	0.0000
10	-0.2500	0.0000	8.5551	0.0630	0.7695	1.8870	6.8441	0.0000
11	0.2287	0.0000	4.8879	0.1569	0.1242	1.3604	6.3398	0.0000
12	0.3446	0.0000	4.4695	0.1542	0.4376	1.9248	6.8244	0.0000
13	-0.1271	0.0000	7.7926	0.0179	0.7325	2.0651	6.9148	0.0000
14	0.0083	0.0000	9.0764	0.0001	1.6261	2.0598	9.1515	0.0000
15	0.2406	0.0000	3.6101	0.0614	0.1371	1.6955	4.7563	0.0000
16	-0.0897	0.0000	6.8856	0.0072	1.4441	1.8947	6.3192	0.0947
17	0.435	0.0053	3.0136	0.2129	0.1158	2.2905	5.3345	0.0000
18	-0.0959	0.0000	7.2044	0.0098	0.8156	1.3068	6.5736	0.0000
19	-0.1225	0.0002	5.2974	0.0155	0.2384	1.6636	4.7168	0.0000
20	0.4822	0.0000	3.0389	0.2509	0.3745	2.0254	5.8668	0.2072
21				Omitted because of collinearity				
22	-0.0630	0.0004	7.0351	0.0045	0.7205	2.1160	6.6181	0.0000
23	0.5436	0.0002	2.8251	0.3116	0.5551	2.0980	6.1952	0.2744
24	0.1987	0.0000	4.4617	0.0400	1.3907	1.9089	5.5705	0.0818
25	0.5415	0.0028	3.1490	0.4200	0.5665	2.4158	6.8756	0.0000
26	0.4306	0.0081	3.5829	0.2045	0.5081	1.2646	6.2970	0.0263
27	0.1050	0.0000	5.7847	0.0132	0.4664	2.0765	6.4637	0.0000
28	0.0094	0.0000	6.1970	0.0001	0.6662	1.9393	6.2556	0.0087
29	-0.1564	0.0000	7.2344	0.0264	0.5354	1.7212	6.2578	0.0001
30	0.0495	0.0000	4.8307	0.0025	0.8623	1.3318	5.0826	0.0000

Table B.4. Linear estimation on parameter b in *Treatment A*. μ is the intercept of the regression while ρ is the slope.

Number	$\hat{\rho}_b$	p-value ($\lvert\hat{\rho}_b\rvert \geq 1$)	μ_b	R^2	Root MSE	Durbin Watson	Equilibrium	Wald test p-value
1	0.4512	0.0015	0.1188	0.2443	0.1194	2.2080	0.2168	0.0000
2	0.0503	0.0000	0.6452	0.0029	0.1204	1.9117	0.6792	0.0000
3	0.1641	0.0000	0.4227	0.0253	0.0384	1.9796	0.5060	0.0000
4	-0.2733	0.0000	0.7973	0.0869	0.1064	2.1886	0.6261	0.0267
5	0.1110	0.0000	0.6146	0.0260	0.0918	2.0267	0.6918	0.0000
6	0.2191	0.0000	0.4358	0.0629	0.0996	2.2610	0.5583	0.0156
7	0.3970	0.0004	0.5060	0.1576	0.0256	2.2699	0.8391	0.0000
8	0.7118	0.0560	0.1403	0.5312	0.1123	2.5591	0.4861	0.0572
9	-0.3416	0.0000	0.7795	0.1291	0.0835	1.4754	0.5805	0.0321
10	-0.1863	0.0000	0.6114	0.0428	0.0545	1.4801	0.5152	0.0000
11	0.2933	0.0116	0.4007	0.1307	0.0124	0.7740	0.5672	0.0000
12	0.6334	0.0949	0.1897	0.4013	0.0419	1.7072	0.5177	0.0000
13	-0.4799	0.0000	0.7472	0.2303	0.0552	1.3843	0.5047	0.0000
14	0.1943	0.0000	0.2117	0.0384	0.1909	2.0552	0.2630	0.0000
15	0.0551	0.0000	0.693	0.0133	0.0179	0.9173	0.7334	0.0000
16	-0.1969	0.0000	0.6941	0.0374	0.1296	1.9066	0.5798	0.1911
17	0.3991	0.0247	0.3992	0.2293	0.0309	2.0119	0.6639	0.0000
18	0.2261	0.0000	0.4220	0.0521	0.0586	2.1020	0.5452	0.0000
19	0.2941	0.0000	0.5170	0.2679	0.0197	1.7159	0.7323	0.0000
20	0.5458	0.0003	0.2777	0.3017	0.0395	2.0957	0.6123	0.3659
21	0.2621	0.0000	0.5207	0.3197	0.0127	1.4197	0.7060	0.0000
22	-0.1632	0.0042	0.6175	0.0462	0.0609	0.6189	0.5305	0.0000
23	0.3711	0.0004	0.3658	0.1396	0.0780	1.7425	0.5819	0.3005
24	0.1422	0.0000	0.5569	0.0216	0.1568	2.0167	0.6492	0.0598
25	0.1140	0.0014	0.4493	0.0250	0.0449	2.5970	0.5068	0.0000
26	0.6788	0.0376	0.1838	0.4738	0.0367	2.6100	0.5732	0.0513
27	-0.0552	0.0000	0.5799	0.0031	0.0532	1.7854	0.5497	0.0000
28	0.3800	0.0000	0.3551	0.2181	0.0565	2.3680	0.5726	0.0330
29	-0.2709	0.0000	0.7230	0.0962	0.0522	1.7406	0.5688	0.0000
30	0.6067	0.0022	0.2753	0.4106	0.0578	2.4363	0.6997	0.0000

Table B.5. Linear estimation on parameter a in *Treatment U*. μ is the intercept of the regression while ρ is the slope.

Number	$\hat{\rho}_a$	p-value ($\lvert\hat{\rho}_a\rvert \geq 1$)	μ_a	R^2	Root MSE	Durbin Watson	Equilibrium	Wald test p-value
1	-0.0237	0.0000	6.1375	0.0075	0.0409	2.4749	5.9959	0.4337
2	-0.0496	0.0000	6.1821	0.0026	0.1323	2.0565	5.8899	0.0000
3	0.5121	0.0004	2.6377	0.2677	2.0786	2.2056	5.4057	0.3332
4	-0.2320	0.0000	7.2833	0.0546	1.0047	2.0583	5.9115	0.4478
5	0.5536	0.0000	2.6036	0.8234	0.2628	3.1260	5.8386	0.0131
6	0.2782	0.0000	5.2089	0.0817	1.8439	2.0442	7.2147	0.0009
7	-0.0384	0.0011	6.3338	0.0018	0.4546	1.0051	6.0998	0.0863
8	0.0946	0.0000	5.8208	0.0099	1.1660	2.0604	6.4292	0.0240
9	0.0534	0.0000	4.8789	0.0034	0.4078	2.1459	5.1542	0.0000
10	0.1949	0.0000	4.9551	0.0359	1.4184	1.7531	6.1553	0.5380
11	0.0302	0.0000	5.8421	0.0009	1.0370	1.7025	6.0239	0.8743
12	0.2452	0.0000	4.4972	0.1778	0.0702	1.4111	5.9563	0.0011
13	-0.0792	0.0000	8.3560	0.0380	0.17230	1.5879	7.7428	0.0000
14	0.3589	0.0003	3.0141	0.1499	1.1252	2.1378	4.7020	0.0000
15	-0.0252	0.0001	6.2595	0.0007	0.7344	1.6560	6.1052	0.3105
16	0.4701	0.0467	3.1623	0.3711	0.1320	2.2418	5.9660	0.1869
17	-0.1087	0.0000	6.7600	0.0119	0.6486	1.5826	6.0956	0.2438
18	-0.4606	0.0000	8.7396	0.3757	0.1286	1.2381	5.9822	0.2128
19	-0.0751	0.0078	6.4503	0.0056	0.7857	1.9375	5.9994	1.0000
20	Omitted because of collinearity							
21	0.0010	0.0012	5.9899	0.0000	0.2917	2.2448	5.9962	0.9209
22	0.1406	0.0000	5.6320	0.0198	1.0177	1.9632	6.5565	0.0010
23	0.0453	0.0000	5.5884	0.0023	0.3367	2.0007	5.8531	0.0040
24	0.2191	0.0001	4.8327	0.0486	1.0494	2.0843	6.1882	0.3139
25	-0.3772	0.0000	7.9336	0.1436	1.0459	1.6751	5.7618	0.0279
26	-0.0171	0.0000	5.1070	0.0003	1.4408	1.9802	5.0211	0.0000
27	-0.2352	0.0000	8.3085	0.0553	1.0321	1.8578	6.7271	0.0000
28	0.0599	0.0000	5.6423	0.0230	0.0735	2.2513	6.0014	0.8964
29	-0.0518	0.0000	6.3510	0.0028	0.7914	2.0328	6.0382	0.7230

Table B.6. Linear estimation on parameter b in *Treatment U*. μ is the intercept of the regression while ρ is the slope.

Number	$\hat{\rho}_b$	p-value ($\lvert\hat{\rho}_b\rvert \geq 1$)	μ_b	R^2	Root MSE	Durbin Watson	Equilibrium	Wald test p-value
1	0.0190	0.0000	0.5892	0.0007	0.0056	1.9180	0.6004	0.4584
2	0.0833	0.0000	0.5586	0.0296	0.0024	1.3747	0.6098	0.0000
3	0.1628	0.0000	0.6303	0.0260	0.2725	2.0578	0.7527	0.0010
4	-0.1746	0.0000	0.6991	0.0314	0.0791	2.0426	0.5949	0.6213
5	0.6541	0.0402	0.2168	0.5824	0.0539	3.1359	0.6272	0.0349
6	0.0894	0.0000	0.4894	0.0000	0.1991	1.9727	0.5370	0.0461
7	0.8239	0.1050	0.1024	0.6732	0.0369	1.3821	0.5795	0.5115
8	-0.1858	0.0000	0.7092	0.0361	0.1507	2.0003	0.5978	0.9158
9	0.0166	0.0000	0.6946	0.0003	0.0624	1.6591	0.7067	0.0000
10	0.1822	0.0000	0.4891	0.0319	0.1531	2.0199	0.5978	0.9415
11	0.1144	0.0000	0.5050	0.0133	0.0753	1.9414	0.5700	0.0145
12	0.1509	0.0001	0.5120	0.0341	0.0086	1.2802	0.6031	0.0298
13	0.0480	0.0000	0.4452	0.0023	0.0705	2.0236	0.4674	0.0000
14	0.7078	0.1019	0.2110	0.5456	0.0515	2.4339	0.7226	0.0000
15	0.3533	0.0118	0.3914	0.1266	0.1071	1.8612	0.6043	0.8280
16	-0.0989	0.0000	0.6605	0.0127	0.0267	1.8985	0.6006	0.7735
17	0.5675	0.0243	0.2575	0.3692	0.0335	1.0479	0.5972	0.6980
18	-0.4875	0.0012	0.8895	0.2534	0.0283	1.7150	0.5974	0.4583
19	0.2392	0.0060	0.4595	0.0840	0.0409	1.4409	0.6045	0.6586
20	-0.0019	0.0000	0.6013	0.0000	0.0048	1.6340	0.5999	0.7671
21	-0.0047	0.0000	0.6002	0.0000	0.0406	2.1201	0.5972	0.6301
22	-0.1820	0.0000	0.6026	0.0350	0.1068	2.0488	0.5102	0.0000
23	-0.4326	0.0000	0.9052	0.2244	0.0378	1.7775	0.6315	0.0000
24	-0.1815	0.0000	0.6717	0.0329	0.1093	1.8828	0.5690	0.0174
25	0.1130	0.0000	0.5572	0.0128	0.1852	1.8554	0.6280	0.3460
26	-0.1335	0.0000	0.6720	0.0183	0.2570	2.0159	0.5926	0.8260
27	0.0571	0.0001	0.4920	0.0037	0.0806	1.9832	0.5218	0.0000
28	0.1020	0.0000	0.5390	0.1811	0.0032	1.4189	0.6002	0.7206
29	-0.0090	0.0000	0.5887	0.0001	0.0938	1.8079	0.5837	0.2150

Table B.7. Linear estimation on parameters a, b for the 50 periods ahead simulation of least square learning model.

	$\hat{\rho}$	p-value	μ	p-value	R^2	Root MSE	Equilibrium	Wald Test
Treatment A								
a	0.627	0.000	1.992	0.003	0.379	1.429	5.336	0.277
b	0.578	0.000	0.288	0.002	0.330	0.169	0.683	0.191
Treatment U								
a	0.495	0.000	2.951	0.000	0.277	0.271	5.845	0.091
b	0.239	0.080	0.472	0.000	0.064	0.048	0.620	0.129

Appendix C: Estimation of the Parameter using OLS

Table C1. Results of iterated regression in estimating $p_{i,T} = \hat{a}_{i,T} + \hat{b}_{i,T} \times w_T$ using OLS in *Treatment A* for Subjects 1-10.

Period(t)	$\hat{a}_1$	$\hat{b}_1$	$\hat{a}_2$	$\hat{b}_2$	$\hat{a}_3$	$\hat{b}_3$	$\hat{a}_4$	$\hat{b}_4$	$\hat{a}_5$	$\hat{b}_5$	$\hat{a}_6$	$\hat{b}_6$	$\hat{a}_7$	$\hat{b}_7$	$\hat{a}_8$	$\hat{b}_8$	$\hat{a}_9$	$\hat{b}_9$	$\hat{a}_{10}$	$\hat{b}_{10}$
3	-30.72	4.250	234.5	-22.08	170.5	-15.75	259.5	-24.50	58.03	-4.500	-335.9	34.50	138.5	-12.50	116.9	-10.42	432.3	-41.75	180.0	-16.67
4	1.157	1.082	18.06	-0.579	10.11	0.191	-14.39	2.720	5.183	0.751	29.31	-1.793	3.027	0.965	1.222	1.083	16.09	-0.393	3.621	0.860
5	6.806	0.511	7.581	0.480	8.572	0.346	5.418	0.716	0.436	1.231	10.68	0.0913	4.499	0.816	9.120	0.284	3.465	0.884	4.161	0.805
6	6.938	0.498	5.331	0.703	7.192	0.483	3.575	0.899	1.295	1.146	10.12	0.146	3.856	0.880	10.28	0.170	5.028	0.729	4.589	0.763
7	5.85	0.624	4.793	0.765	6.535	0.559	4.237	0.822	2.601	0.995	10.70	0.0789	5.698	0.666	9.582	0.250	8.589	0.316	4.557	0.767
8	6.067	0.593	4.890	0.752	6.658	0.542	4.554	0.776	2.725	0.977	10.52	0.106	5.724	0.662	9.62	0.245	8.233	0.367	4.720	0.743
9	4.348	0.769	4.258	0.816	5.544	0.655	3.950	0.838	2.441	1.006	8.192	0.343	6.673	0.565	7.283	0.483	7.810	0.410	4.177	0.799
10	4.081	0.796	3.713	0.871	5.38	0.672	2.933	0.941	2.692	0.981	8.966	0.265	6.781	0.554	6.931	0.519	5.885	0.605	4.824	0.733
11	4.033	0.802	3.713	0.871	5.384	0.671	2.648	0.978	2.774	0.97	8.922	0.270	6.928	0.536	6.934	0.519	5.448	0.661	4.789	0.738
12	4.358	0.764	3.900	0.85	5.581	0.649	3.883	0.835	3.176	0.924	8.360	0.335	7.433	0.477	6.639	0.553	6.307	0.562	4.898	0.725
13	4.097	0.795	4.200	0.815	5.605	0.646	3.935	0.829	3.497	0.886	7.633	0.420	7.53	0.466	6.519	0.567	5.833	0.617	5.045	0.708
14	4.379	0.763	4.549	0.776	5.829	0.621	4.752	0.738	4.101	0.819	7.446	0.441	8.12	0.4	6.625	0.555	6.034	0.595	5.067	0.705
15	4.396	0.763	4.503	0.776	5.827	0.621	4.733	0.738	4.066	0.819	7.452	0.441	8.119	0.4	6.618	0.555	6.025	0.595	5.016	0.706
16	4.586	0.744	3.653	0.864	6.274	0.575	4.084	0.805	3.581	0.869	7.24	0.463	7.064	0.509	6.432	0.574	4.945	0.706	4.441	0.765
17	4.891	0.714	4.166	0.813	6.091	0.593	4.655	0.749	4.086	0.819	7.148	0.472	7.027	0.512	6.375	0.58	5.403	0.661	5.17	0.693
18	4.894	0.714	4.158	0.814	6.085	0.593	4.634	0.75	4.041	0.822	7.167	0.471	6.986	0.515	6.352	0.581	5.408	0.66	5.162	0.694
19	4.891	0.714	4.189	0.809	6.071	0.595	4.653	0.747	4.075	0.817	7.193	0.467	7.021	0.51	6.397	0.575	5.403	0.661	5.154	0.695
20	5.138	0.686	4.285	0.798	6.073	0.595	4.207	0.798	4.301	0.791	6.794	0.512	7.283	0.48	6.48	0.565	5.176	0.687	4.994	0.713
21	5.079	0.693	4.48	0.774	6.097	0.592	4.704	0.737	4.422	0.777	6.766	0.516	7.346	0.472	6.593	0.551	5.247	0.678	5.226	0.685
22	5.089	0.692	4.425	0.781	6.044	0.599	4.53	0.758	4.469	0.771	6.685	0.526	7.351	0.471	6.622	0.548	5.141	0.691	5.055	0.705
23	5.068	0.693	4.386	0.783	6.053	0.598	4.547	0.757	4.425	0.774	6.673	0.526	7.304	0.475	6.574	0.551	5.104	0.694	4.927	0.714
24	5.02	0.7	4.44	0.776	6.064	0.597	4.509	0.762	4.358	0.783	6.612	0.534	7.282	0.478	6.549	0.554	5.024	0.704	4.76	0.736
25	5.024	0.699	4.122	0.807	6.052	0.598	4.502	0.763	4.104	0.807	6.345	0.56	7.079	0.497	6.291	0.58	4.761	0.73	4.848	0.727
26	5.288	0.672	4.225	0.797	5.863	0.617	4.602	0.752	4.268	0.791	6.611	0.533	7.101	0.495	5.708	0.639	5.067	0.699	4.665	0.746
27	5.575	0.643	4.517	0.767	6.059	0.597	4.765	0.736	4.516	0.766	6.586	0.536	7.238	0.481	5.859	0.623	5.324	0.673	5.265	0.685
28	5.653	0.635	4.541	0.765	5.993	0.604	4.779	0.734	4.589	0.758	6.565	0.538	6.959	0.51	5.902	0.619	5.427	0.662	5.067	0.705
29	5.656	0.635	4.496	0.772	6.016	0.6	4.763	0.737	4.543	0.765	6.533	0.543	6.977	0.507	5.848	0.627	5.447	0.659	5.015	0.713
30	5.654	0.635	4.533	0.767	6.037	0.598	4.9	0.72	4.62	0.756	6.518	0.545	7.034	0.5	5.891	0.622	5.384	0.667	4.994	0.716
31	5.659	0.634	4.553	0.764	6.038	0.597	4.918	0.717	4.645	0.752	6.526	0.544	7.044	0.498	5.897	0.621	5.411	0.662	5.034	0.709
32	5.665	0.633	4.543	0.765	6.039	0.597	4.955	0.712	4.637	0.753	6.532	0.543	7.046	0.498	5.915	0.619	5.39	0.665	5.03	0.71
33	5.638	0.637	4.518	0.769	6.011	0.601	4.922	0.717	4.652	0.751	6.503	0.547	7.033	0.5	5.905	0.62	5.377	0.667	5.035	0.709
34	5.646	0.637	4.488	0.771	6.021	0.601	4.98	0.713	4.661	0.75	6.501	0.547	6.996	0.503	5.905	0.62	5.39	0.666	5.023	0.71
35	5.645	0.637	4.478	0.773	6.018	0.601	5.015	0.707	4.643	0.753	6.498	0.548	6.992	0.503	5.908	0.62	5.381	0.667	5.015	0.711
36	5.652	0.636	4.523	0.767	6.043	0.598	5.013	0.708	4.676	0.749	6.507	0.546	7.017	0.5	5.929	0.617	5.404	0.664	5.054	0.706
37	5.693	0.631	4.543	0.764	6.047	0.597	5.085	0.699	4.632	0.754	6.509	0.546	7.104	0.49	5.967	0.612	5.392	0.666	5.063	0.705
38	5.736	0.626	4.67	0.749	6.079	0.593	5.184	0.687	4.753	0.739	6.531	0.544	7.188	0.48	6.029	0.605	5.484	0.655	5.253	0.683
39	5.702	0.631	4.674	0.749	6.067	0.595	5.162	0.69	4.771	0.737	6.523	0.545	7.173	0.482	6.028	0.605	5.485	0.655	5.222	0.687
40	5.702	0.631	4.672	0.749	6.069	0.595	5.162	0.69	4.77	0.737	6.526	0.545	7.178	0.482	6.029	0.605	5.485	0.655	5.212	0.686
41	5.707	0.63	4.653	0.751	6.075	0.594	5.156	0.691	4.756	0.739	6.535	0.544	7.18	0.482	6.041	0.604	5.473	0.656	5.205	0.687
42	5.693	0.632	4.692	0.746	6.073	0.595	5.162	0.69	4.77	0.737	6.5	0.548	7.152	0.485	6.032	0.605	5.472	0.656	5.257	0.681
43	5.702	0.631	4.732	0.741	6.046	0.598	5.196	0.686	4.806	0.733	6.503	0.548	7.152	0.485	6.031	0.605	5.484	0.655	5.285	0.678
44	5.723	0.628	4.824	0.731	6.055	0.597	5.253	0.68	4.865	0.726	6.496	0.548	7.185	0.481	6.061	0.601	5.521	0.651	5.291	0.677
45	5.764	0.624	4.932	0.719	6.109	0.591	5.339	0.67	4.947	0.717	6.508	0.547	7.287	0.47	6.117	0.595	5.599	0.642	5.351	0.67
46	5.766	0.624	4.92	0.719	6.107	0.591	5.333	0.67	4.937	0.717	6.505	0.547	7.288	0.47	6.116	0.595	5.617	0.642	5.336	0.67
47	5.78	0.622	4.926	0.718	6.096	0.592	5.375	0.667	4.964	0.714	6.51	0.547	7.28	0.471	6.11	0.596	5.557	0.647	5.365	0.668
48	5.801	0.62	4.943	0.717	6.11	0.591	5.357	0.668	4.951	0.716	6.515	0.546	7.235	0.475	6.092	0.597	5.656	0.638	5.36	0.668
49	5.855	0.616	4.977	0.714	6.101	0.591	5.389	0.666	4.976	0.713	6.518	0.546	7.231	0.475	6.087	0.598	5.655	0.638	5.388	0.666
50	5.838	0.617	4.935	0.718	6.12	0.59	5.387	0.666	4.962	0.715	6.511	0.547	7.213	0.477	6.077	0.599	5.667	0.637	5.381	0.666

Table C2. Results of iterated regression in estimating $p_{i,T} = \hat{a}_{i,T} + \hat{b}_{i,T} \times w_T$ using OLS in *Treatment A* for Subjects 11-20.

Period(t)	$\hat{a}_{11}$	$\hat{b}_{11}$	$\hat{a}_{12}$	$\hat{b}_{12}$	$\hat{a}_{13}$	$\hat{b}_{13}$	$\hat{a}_{14}$	$\hat{b}_{14}$	$\hat{a}_{15}$	$\hat{b}_{15}$	$\hat{a}_{16}$	$\hat{b}_{16}$	$\hat{a}_{17}$	$\hat{b}_{17}$	$\hat{a}_{18}$	$\hat{b}_{18}$	$\hat{a}_{19}$	$\hat{b}_{19}$	$\hat{a}_{20}$	$\hat{b}_{20}$
3	108.0	-9.500	116.4	-10.33	-51.97	6.500	137.2	-12.42	104.7	-9.167	19.11	-0.833	12.20	0.0833	229.1	-21.58	138.0	-12.50	16.45	-0.417
4	2.148	1.022	-2.433	1.471	-14.73	2.8	9.154	0.307	5.428	0.698	8.644	0.207	-3.570	1.65	3.495	0.836	11.65	0.0585	3.783	0.842
5	4.268	0.807	0.984	1.126	3.460	0.960	4.908	0.736	5.182	0.723	6.837	0.39	-0.569	1.347	5.911	0.592	8.442	0.383	4.784	0.741
6	4.074	0.826	1.485	1.076	-0.362	1.338	4.873	0.740	4.251	0.815	12.02	-0.123	-0.747	1.364	6.286	0.555	7.780	0.448	3.928	0.825
7	5.081	0.709	1.573	1.066	3.209	0.924	4.050	0.835	5.567	0.662	10.52	0.0512	1.637	1.088	5.571	0.638	8.57	0.357	4.534	0.755
8	5.128	0.703	1.770	1.038	3.670	0.858	4.219	0.811	5.635	0.652	10.23	0.0927	1.735	1.074	5.851	0.598	8.628	0.348	4.507	0.759
9	5.083	0.707	3.262	0.885	3.327	0.893	3.373	0.897	5.382	0.678	8.918	0.226	2.363	1.009	7.027	0.478	7.793	0.434	4.888	0.72
10	5.02	0.714	3.506	0.861	3.696	0.855	4.228	0.811	5.282	0.688	9.657	0.151	3.078	0.937	6.129	0.569	7.032	0.511	5.154	0.693
11	5.069	0.707	3.557	0.854	3.574	0.871	4.399	0.789	5.352	0.679	9.215	0.208	3.204	0.921	5.060	0.706	7.124	0.499	5.139	0.695
12	5.218	0.69	3.743	0.833	3.998	0.822	5.284	0.687	5.583	0.653	8.161	0.330	3.639	0.871	5.416	0.665	7.076	0.504	5.152	0.693
13	5.247	0.687	3.784	0.828	4.172	0.802	5.25	0.691	5.517	0.66	7.612	0.394	3.57	0.879	5.571	0.646	6.964	0.517	5.016	0.709
14	5.631	0.644	4.159	0.786	4.689	0.744	5.71	0.639	5.972	0.61	6.554	0.512	4.216	0.807	5.854	0.615	7.369	0.472	5.199	0.689
15	5.632	0.644	4.183	0.786	4.691	0.744	5.693	0.639	5.971	0.61	6.53	0.512	4.228	0.806	5.884	0.615	7.363	0.472	5.204	0.689
16	5.209	0.687	5.55	0.645	6.483	0.56	5.967	0.611	5.636	0.644	5.422	0.626	4.6	0.768	4.656	0.741	6.456	0.566	5.037	0.706
17	5.336	0.675	5.738	0.627	6.042	0.603	5.926	0.615	5.733	0.634	6.034	0.566	4.81	0.748	5.089	0.698	6.456	0.566	5.218	0.688
18	5.343	0.674	5.798	0.623	6.096	0.6	5.905	0.617	5.741	0.634	6.03	0.566	4.819	0.747	5.042	0.702	6.449	0.566	5.201	0.689
19	5.356	0.673	5.784	0.625	6.146	0.592	5.907	0.616	5.757	0.632	5.967	0.576	4.84	0.744	4.976	0.711	6.481	0.561	5.206	0.689
20	5.383	0.67	5.972	0.604	5.908	0.619	6.055	0.6	5.803	0.626	5.496	0.63	5.119	0.712	5.282	0.676	6.577	0.55	5.332	0.674
21	5.416	0.665	5.974	0.603	6.026	0.605	6.077	0.597	5.837	0.622	5.597	0.617	5.212	0.7	5.285	0.676	6.626	0.544	5.351	0.672
22	5.415	0.666	5.955	0.606	6.033	0.604	6.049	0.6	5.836	0.622	5.267	0.657	5.277	0.693	5.239	0.681	6.637	0.543	5.358	0.671
23	5.415	0.666	5.985	0.603	6.041	0.604	6.043	0.601	5.835	0.622	5.182	0.663	5.268	0.693	5.21	0.683	6.627	0.544	5.348	0.672
24	5.405	0.667	5.957	0.607	6.033	0.605	6.032	0.602	5.823	0.624	5.096	0.674	5.282	0.691	5.177	0.688	6.626	0.544	5.346	0.672
25	5.392	0.668	6.102	0.593	5.706	0.637	5.993	0.606	5.779	0.628	4.748	0.708	5.269	0.693	4.974	0.708	6.555	0.551	5.361	0.67
26	5.541	0.653	6.364	0.566	5.669	0.64	5.982	0.607	5.839	0.622	5.213	0.661	5.343	0.685	5.23	0.682	6.522	0.554	5.443	0.662
27	5.677	0.639	6.341	0.569	5.818	0.625	6.077	0.597	5.982	0.608	5.281	0.654	5.529	0.666	5.429	0.661	6.594	0.547	5.625	0.644
28	5.729	0.634	6.31	0.572	5.872	0.62	6.023	0.603	5.967	0.609	5.438	0.638	5.541	0.665	5.539	0.65	6.5	0.557	5.636	0.642
29	5.748	0.631	6.329	0.569	5.851	0.623	6.134	0.585	5.987	0.606	5.367	0.649	5.581	0.659	5.54	0.65	6.523	0.553	5.657	0.639
30	5.749	0.631	6.345	0.567	5.852	0.623	5.874	0.617	6.01	0.603	5.35	0.651	5.671	0.648	5.525	0.652	6.537	0.551	5.692	0.635
31	5.748	0.631	6.35	0.566	5.877	0.619	5.835	0.623	6.01	0.603	5.363	0.649	5.678	0.647	5.522	0.652	6.539	0.551	5.692	0.635
32	5.759	0.629	6.363	0.564	5.919	0.613	5.903	0.614	6.024	0.601	5.331	0.653	5.699	0.644	5.537	0.65	6.549	0.55	5.701	0.634
33	5.744	0.632	6.34	0.568	5.885	0.618	5.859	0.62	6.007	0.604	5.279	0.661	5.69	0.645	5.514	0.653	6.533	0.552	5.683	0.636
34	5.752	0.631	6.339	0.568	5.865	0.619	5.756	0.628	6.018	0.603	5.272	0.661	5.694	0.645	5.491	0.655	6.534	0.552	5.678	0.637
35	5.751	0.631	6.344	0.567	5.851	0.622	5.772	0.626	6.022	0.602	5.226	0.668	5.7	0.644	5.499	0.654	6.539	0.551	5.674	0.637
36	5.767	0.629	6.38	0.562	5.916	0.613	5.803	0.621	6.043	0.6	5.263	0.663	5.735	0.639	5.511	0.652	6.56	0.548	5.682	0.636
37	5.803	0.625	6.361	0.565	5.819	0.624	5.762	0.626	6.071	0.596	5.102	0.682	5.812	0.63	5.593	0.643	6.594	0.544	5.727	0.631
38	5.861	0.618	6.37	0.564	5.923	0.612	5.822	0.619	6.121	0.59	5.025	0.691	5.885	0.622	5.623	0.639	6.638	0.539	5.79	0.623
39	5.857	0.618	6.361	0.565	5.878	0.618	5.804	0.622	6.11	0.592	5.032	0.69	5.883	0.622	5.606	0.641	6.623	0.541	5.78	0.625
40	5.858	0.619	6.362	0.565	5.877	0.618	5.802	0.621	6.112	0.592	5.039	0.691	5.884	0.622	5.606	0.641	6.624	0.541	5.782	0.625
41	5.875	0.617	6.37	0.564	5.908	0.614	5.839	0.617	6.121	0.591	5.012	0.694	5.911	0.619	5.62	0.64	6.649	0.538	5.775	0.626
42	5.864	0.618	6.363	0.565	5.913	0.614	5.798	0.622	6.104	0.593	5.007	0.695	5.916	0.618	5.608	0.641	6.645	0.539	5.767	0.627
43	5.861	0.618	6.351	0.566	5.927	0.612	5.863	0.614	6.106	0.593	4.993	0.696	5.933	0.616	5.654	0.636	6.641	0.539	5.772	0.626
44	5.892	0.615	6.354	0.566	5.963	0.608	5.905	0.609	6.146	0.588	5.015	0.694	5.991	0.609	5.606	0.641	6.66	0.537	5.815	0.621
45	5.96	0.607	6.32	0.569	5.995	0.604	5.973	0.602	6.198	0.582	5.04	0.691	6.085	0.599	5.688	0.632	6.696	0.533	5.885	0.613
46	5.96	0.607	6.317	0.569	5.997	0.604	5.99	0.602	6.198	0.582	5.03	0.691	6.087	0.599	5.693	0.632	6.696	0.533	5.886	0.613
47	5.968	0.606	6.322	0.569	5.982	0.606	5.917	0.608	6.213	0.581	5.068	0.688	6.097	0.598	5.702	0.631	6.693	0.533	5.905	0.612
48	5.976	0.605	6.317	0.569	5.986	0.605	5.971	0.603	6.218	0.58	5.125	0.682	6.114	0.596	5.707	0.631	6.677	0.535	5.929	0.609
49	5.978	0.605	6.319	0.569	5.984	0.605	5.963	0.604	6.217	0.58	5.171	0.678	6.111	0.597	5.725	0.629	6.671	0.535	5.941	0.608
50	5.983	0.605	6.327	0.569	5.988	0.605	5.973	0.603	6.215	0.581	5.202	0.676	6.112	0.597	5.73	0.629	6.659	0.536	5.949	0.608

Table C3. Results of iterated regression in estimating $p_{i,T} = \hat{a}_{i,T} + \hat{b}_{i,T} \times w_T$ using OLS in *Treatment A* for Subjects 21-30.

Period(t)	$\hat{a}_{21}$	$\hat{b}_{21}$	$\hat{a}_{22}$	$\hat{b}_{22}$	$\hat{a}_{23}$	$\hat{b}_{23}$	$\hat{a}_{24}$	$\hat{b}_{24}$	$\hat{a}_{25}$	$\hat{b}_{25}$	$\hat{a}_{26}$	$\hat{b}_{26}$	$\hat{a}_{27}$	$\hat{b}_{27}$	$\hat{a}_{28}$	$\hat{b}_{28}$	$\hat{a}_{29}$	$\hat{b}_{29}$	$\hat{a}_{30}$	$\hat{b}_{30}$
3	79.70	-6.667	-301.6	31.08	151.4	-13.83	11.11	0.167	-17.72	3.000	116.4	-10.33	138.0	-12.50	179.8	-16.50	247.1	-23.33	229.6	-21.58
4	0.928	1.161	6.721	0.450	6.614	0.550	-6.596	1.926	10.58	0.187	-0.834	1.313	4.596	0.759	-11.39	2.497	12.47	-0.0166	6.766	0.556
5	3.295	0.921	14.18	-0.304	6.738	0.538	5.043	0.749	12.77	-0.0333	3.345	0.890	4.813	0.737	-1.620	1.509	6.154	0.622	2.592	0.978
6	3.236	0.927	11.94	-0.0823	6.592	0.552	3.425	0.910	8.162	0.422	3.943	0.831	5.812	0.638	-2.037	1.550	5.440	0.693	4.172	0.822
7	4.590	0.77	10.89	0.0397	6.419	0.572	5.481	0.671	9.845	0.227	4.086	0.814	6.511	0.557	1.558	1.133	5.391	0.698	4.310	0.806
8	4.682	0.757	10.74	0.0605	6.381	0.578	5.541	0.662	9.872	0.223	4.302	0.783	6.559	0.551	1.772	1.102	5.559	0.674	4.243	0.815
9	4.8	0.745	10.5	0.0851	5.981	0.619	4.220	0.797	7.389	0.477	4.47	0.766	5.348	0.674	1.307	1.149	5.030	0.728	4.023	0.838
10	4.844	0.74	9.514	0.185	5.651	0.652	3.682	0.852	6.632	0.554	5.085	0.704	5.646	0.644	1.795	1.1	4.876	0.744	3.544	0.886
11	4.947	0.727	8.992	0.252	5.672	0.649	3.728	0.846	6.646	0.552	4.899	0.728	5.587	0.652	1.898	1.087	4.955	0.734	3.625	0.876
12	5.271	0.69	8.661	0.290	5.759	0.639	4.425	0.765	6.892	0.523	5.221	0.69	5.482	0.664	2.875	0.974	5.488	0.672	3.823	0.853
13	5.306	0.685	7.842	0.386	5.768	0.638	4.71	0.732	6.822	0.532	5.255	0.686	5.462	0.666	3.147	0.942	5.28	0.696	4.129	0.817
14	5.765	0.634	6.551	0.530	6.047	0.607	4.648	0.739	7.145	0.495	5.577	0.651	5.625	0.648	4.162	0.829	6.321	0.58	3.566	0.88
15	5.769	0.634	6.55	0.530	6.04	0.607	4.66	0.739	7.128	0.496	5.557	0.651	5.63	0.648	4.095	0.829	6.324	0.58	3.523	0.88
16	5.574	0.654	6.169	0.569	5.26	0.688	5.417	0.661	7.063	0.502	5.344	0.673	5.989	0.611	3.488	0.892	5.577	0.657	3.889	0.843
17	5.663	0.645	6.41	0.546	5.285	0.685	5.709	0.632	6.881	0.52	5.601	0.647	6.077	0.602	4.164	0.825	5.716	0.643	4.062	0.826
18	5.658	0.646	6.392	0.547	5.29	0.685	5.762	0.629	6.871	0.521	5.619	0.646	6.06	0.603	4.193	0.824	5.685	0.645	4.045	0.827
19	5.68	0.643	6.363	0.551	5.321	0.68	5.773	0.627	6.908	0.515	5.625	0.645	6.072	0.602	4.233	0.818	5.691	0.645	4.114	0.817
20	5.8	0.629	6.301	0.558	5.486	0.661	5.848	0.619	6.676	0.542	5.71	0.636	6.172	0.59	4.368	0.802	5.598	0.655	4.411	0.783
21	5.837	0.624	6.374	0.549	5.543	0.654	5.81	0.623	6.735	0.535	5.737	0.632	6.178	0.589	4.697	0.762	5.638	0.65	4.492	0.773
22	5.844	0.623	6.147	0.576	5.568	0.651	5.815	0.623	6.733	0.535	5.712	0.635	6.181	0.589	4.704	0.761	5.665	0.647	4.587	0.761
23	5.845	0.623	6.162	0.575	5.567	0.651	5.8	0.624	6.721	0.536	5.716	0.635	6.176	0.589	4.583	0.769	5.666	0.647	4.597	0.761
24	5.847	0.623	6.124	0.58	5.56	0.652	5.858	0.616	6.713	0.537	5.706	0.636	6.166	0.591	4.473	0.784	5.659	0.648	4.648	0.754
25	5.828	0.625	6.089	0.584	5.549	0.653	5.961	0.606	6.55	0.553	5.609	0.646	6.014	0.605	4.414	0.789	5.548	0.659	4.63	0.756
26	5.87	0.621	6.082	0.584	5.677	0.64	5.851	0.617	6.419	0.566	5.678	0.639	6.111	0.596	3.955	0.836	5.798	0.633	4.959	0.722
27	6.006	0.607	6.222	0.57	5.841	0.624	6.015	0.6	6.44	0.564	5.868	0.619	6.124	0.594	4.076	0.824	5.756	0.638	5.133	0.705
28	5.991	0.608	6.257	0.567	5.916	0.616	6.003	0.602	6.292	0.579	5.969	0.609	6.107	0.596	4.464	0.784	5.748	0.639	5.239	0.694
29	6.022	0.604	6.262	0.566	5.936	0.613	6.017	0.6	6.332	0.573	5.975	0.608	6.12	0.594	4.49	0.78	5.785	0.633	5.259	0.69
30	6.045	0.601	6.209	0.572	5.953	0.611	6.031	0.598	6.289	0.578	6.004	0.605	6.115	0.595	4.617	0.764	5.714	0.641	5.305	0.685
31	6.042	0.601	6.201	0.573	5.952	0.611	6.024	0.599	6.307	0.575	6.018	0.602	6.118	0.594	4.579	0.77	5.713	0.642	5.311	0.684
32	6.053	0.6	6.199	0.574	5.963	0.609	6.013	0.6	6.325	0.573	6.015	0.603	6.136	0.592	4.639	0.762	5.751	0.636	5.338	0.68
33	6.036	0.602	6.173	0.577	5.949	0.612	5.998	0.603	6.301	0.576	5.994	0.606	6.117	0.594	4.646	0.761	5.731	0.639	5.323	0.682
34	6.036	0.602	6.186	0.577	5.958	0.611	6.04	0.6	6.325	0.574	6.013	0.604	6.104	0.595	4.645	0.761	5.69	0.642	5.321	0.683
35	6.038	0.602	6.178	0.578	5.958	0.611	6.043	0.599	6.351	0.57	6.022	0.603	6.104	0.595	4.66	0.759	5.678	0.644	5.319	0.683
36	6.053	0.6	6.186	0.577	5.974	0.609	6.04	0.599	6.364	0.569	6.043	0.6	6.118	0.594	4.705	0.753	5.682	0.644	5.355	0.678
37	6.085	0.596	6.188	0.576	6	0.606	6.09	0.594	6.375	0.567	6.088	0.595	6.139	0.591	4.884	0.732	5.79	0.631	5.371	0.676
38	6.138	0.59	6.212	0.574	6.059	0.599	6.15	0.586	6.418	0.562	6.165	0.586	6.192	0.585	4.978	0.721	5.839	0.625	5.47	0.664
39	6.124	0.592	6.193	0.576	6.056	0.599	6.164	0.585	6.408	0.563	6.159	0.587	6.187	0.585	4.99	0.719	5.846	0.624	5.449	0.667
40	6.126	0.592	6.195	0.576	6.058	0.599	6.165	0.585	6.408	0.563	6.164	0.587	6.19	0.586	4.996	0.72	5.846	0.624	5.447	0.667
41	6.144	0.59	6.196	0.576	6.066	0.598	6.178	0.583	6.406	0.564	6.149	0.589	6.205	0.584	5.04	0.714	5.875	0.621	5.459	0.666
42	6.136	0.591	6.165	0.58	6.053	0.6	6.159	0.585	6.415	0.563	6.127	0.591	6.198	0.584	5.056	0.712	5.855	0.623	5.483	0.663
43	6.14	0.59	6.139	0.583	6.045	0.601	6.122	0.59	6.41	0.563	6.102	0.594	6.197	0.585	5.111	0.706	5.914	0.616	5.501	0.661
44	6.181	0.585	6.141	0.583	6.081	0.597	6.133	0.588	6.444	0.559	6.144	0.589	6.225	0.581	5.248	0.69	5.911	0.616	5.581	0.652
45	6.195	0.584	6.178	0.579	6.142	0.59	6.143	0.587	6.449	0.559	6.208	0.582	6.235	0.58	5.388	0.674	6.015	0.605	5.636	0.645
46	6.196	0.584	6.179	0.579	6.141	0.59	6.143	0.587	6.442	0.559	6.208	0.582	6.232	0.58	5.383	0.674	6.01	0.605	5.632	0.645
47	6.207	0.583	6.196	0.577	6.143	0.59	6.157	0.586	6.458	0.557	6.224	0.581	6.23	0.58	5.394	0.673	6.022	0.604	5.633	0.645
48	6.213	0.582	6.211	0.576	6.142	0.59	6.142	0.587	6.434	0.56	6.239	0.579	6.21	0.582	5.414	0.672	6.008	0.605	5.644	0.644
49	6.215	0.582	6.22	0.575	6.142	0.59	6.172	0.585	6.438	0.559	6.24	0.579	6.217	0.582	5.414	0.671	6.009	0.605	5.647	0.644
50	6.216	0.582	6.233	0.574	6.143	0.59	6.211	0.581	6.433	0.56	6.252	0.578	6.216	0.582	5.424	0.671	6.014	0.604	5.649	0.644

Table C4. Results of iterated regression in estimating $p_{i,T} = \hat{a}_{i,T} + \hat{b}_{i,T} \times w_T$ using OLS in Treatment U for Subjects 1-10.

Period(t)	$\hat{a}_1$	$\hat{b}_1$	$\hat{a}_2$	$\hat{b}_2$	$\hat{a}_3$	$\hat{b}_3$	$\hat{a}_4$	$\hat{b}_4$	$\hat{a}_5$	$\hat{b}_5$	$\hat{a}_6$	$\hat{b}_6$	$\hat{a}_7$	$\hat{b}_7$	$\hat{a}_8$	$\hat{b}_8$	$\hat{a}_9$	$\hat{b}_9$	$\hat{a}_{10}$	$\hat{b}_{10}$
3	5.62	0.663	6.116	0.567	6.871	0.322	4.279	0.856	6.381	0.643	7.054	0.608	9.337	0.225	4.823	0.95	7.74	0.505	10.3	0.247
4	5.736	0.651	6.387	0.538	5.117	0.508	6.285	0.644	6.862	0.592	7.385	0.573	6.463	0.529	5.027	0.929	6.543	0.631	5.9	0.712
5	6	0.6	6.137	0.586	5.67	0.402	6.705	0.564	8.176	0.341	7.627	0.527	5.693	0.676	7.694	0.42	7.372	0.473	7.019	0.499
6	6.028	0.599	6.121	0.587	5.833	0.392	6.725	0.562	7.675	0.374	6.683	0.588	5.524	0.687	7.407	0.439	7.148	0.488	6.759	0.516
7	5.97	0.602	6.054	0.591	5.995	0.383	6.041	0.601	7.347	0.393	6.572	0.594	5.559	0.685	7.162	0.453	6.882	0.503	6.448	0.534
8	5.972	0.601	6.049	0.594	5.814	0.482	6.013	0.616	7.3	0.418	6.571	0.595	5.553	0.689	7.095	0.489	6.892	0.497	6.451	0.532
9	5.976	0.6	6.05	0.594	5.779	0.492	5.949	0.636	7.229	0.439	6.613	0.582	5.6	0.674	7.078	0.495	6.872	0.503	6.379	0.553
10	5.959	0.603	6.053	0.593	5.133	0.608	5.906	0.643	7.017	0.478	6.666	0.573	5.724	0.652	6.904	0.526	6.769	0.522	6.208	0.584
11	5.971	0.602	6.072	0.592	5.054	0.613	6.058	0.635	6.967	0.48	6.52	0.58	5.676	0.655	6.765	0.533	6.629	0.529	5.993	0.596
12	5.971	0.602	6.072	0.593	5.057	0.607	6.059	0.633	6.965	0.487	6.522	0.577	5.677	0.652	6.765	0.532	6.627	0.534	5.992	0.597
13	5.97	0.604	6.071	0.594	5.058	0.604	6.064	0.625	6.961	0.493	6.522	0.576	5.678	0.65	6.76	0.54	6.625	0.537	5.992	0.597
14	5.963	0.604	6.063	0.594	5.09	0.603	6.02	0.627	6.955	0.494	6.446	0.579	5.643	0.651	6.551	0.548	6.665	0.536	5.985	0.598
15	5.976	0.603	6.073	0.594	5.203	0.597	6.021	0.627	6.92	0.496	6.308	0.587	5.65	0.651	6.563	0.547	6.625	0.538	5.791	0.608
16	5.934	0.606	6.028	0.596	5.171	0.598	5.943	0.631	6.834	0.5	6.29	0.588	5.607	0.654	6.47	0.553	6.526	0.544	5.696	0.614
17	5.952	0.605	6.047	0.595	5.231	0.595	5.869	0.635	6.829	0.501	6.268	0.589	5.628	0.653	6.526	0.55	6.524	0.544	5.779	0.61
18	5.949	0.605	6.041	0.596	5.296	0.584	5.954	0.62	6.751	0.514	6.257	0.591	5.644	0.65	6.495	0.555	6.481	0.551	5.687	0.626
19	5.955	0.605	6.029	0.597	5.499	0.569	6.332	0.593	6.653	0.521	6.441	0.577	5.676	0.648	6.03	0.589	6.478	0.552	5.981	0.604
20	5.959	0.605	6.033	0.597	5.49	0.57	6.331	0.593	6.662	0.521	6.455	0.577	5.674	0.648	6.067	0.588	6.467	0.552	5.956	0.605
21	5.96	0.605	6.038	0.597	5.476	0.57	6.272	0.596	6.64	0.522	6.421	0.579	5.681	0.647	6.068	0.588	6.45	0.553	5.968	0.604
22	5.964	0.605	6.043	0.597	5.43	0.571	6.276	0.596	6.644	0.522	6.415	0.579	5.68	0.647	6.11	0.588	6.455	0.553	5.983	0.604
23	5.971	0.604	6.041	0.597	5.555	0.561	6.191	0.602	6.586	0.527	6.284	0.589	5.718	0.644	6.105	0.588	6.39	0.558	6.036	0.6
24	5.982	0.604	6.042	0.597	5.579	0.56	6.207	0.601	6.581	0.527	6.302	0.588	5.728	0.644	6.109	0.588	6.386	0.558	6.094	0.597
25	5.983	0.603	6.044	0.596	5.598	0.554	6.207	0.602	6.574	0.529	6.306	0.586	5.741	0.639	6.114	0.586	6.383	0.559	6.104	0.594
26	5.985	0.603	6.04	0.597	5.623	0.552	6.169	0.604	6.546	0.531	6.307	0.586	5.752	0.639	6.106	0.587	6.377	0.559	6.083	0.595
27	6	0.602	6.056	0.596	5.626	0.552	6.15	0.606	6.536	0.532	6.297	0.587	5.77	0.637	6.118	0.586	6.372	0.559	6.118	0.593
28	6	0.602	6.056	0.596	5.629	0.547	6.15	0.606	6.535	0.533	6.294	0.59	5.772	0.635	6.118	0.585	6.372	0.559	6.118	0.593
29	5.982	0.603	6.032	0.598	5.687	0.543	6.144	0.606	6.481	0.537	6.146	0.602	5.775	0.635	6.088	0.588	6.296	0.565	6.053	0.598
30	5.981	0.603	6.031	0.598	5.678	0.542	6.163	0.607	6.483	0.537	6.138	0.602	5.771	0.635	6.088	0.588	6.298	0.565	6.045	0.598
31	5.981	0.603	6.031	0.598	5.695	0.543	6.127	0.606	6.487	0.537	6.138	0.602	5.769	0.635	6.089	0.588	6.295	0.565	6.048	0.598
32	5.981	0.603	6.03	0.598	5.704	0.546	6.122	0.604	6.488	0.538	6.14	0.603	5.768	0.634	6.09	0.588	6.296	0.566	6.043	0.596
33	5.98	0.604	6.029	0.598	5.673	0.565	6.119	0.606	6.484	0.54	6.144	0.6	5.768	0.634	6.088	0.589	6.294	0.567	6.043	0.596
34	5.984	0.604	6.033	0.598	5.673	0.565	6.124	0.606	6.49	0.54	6.147	0.6	5.771	0.634	6.091	0.589	6.294	0.567	6.048	0.596
35	5.983	0.604	6.032	0.599	5.489	0.598	6.149	0.601	6.466	0.545	6.14	0.601	5.788	0.631	6.089	0.59	6.272	0.571	6.058	0.594
36	5.989	0.603	6.035	0.598	5.318	0.626	6.166	0.598	6.442	0.548	6.141	0.601	5.816	0.626	6.088	0.59	6.245	0.575	6.088	0.589
37	5.986	0.603	6.032	0.598	5.307	0.627	6.15	0.599	6.442	0.548	6.131	0.601	5.812	0.627	6.086	0.59	6.24	0.575	6.088	0.589
38	5.99	0.602	6.035	0.597	5.273	0.637	6.148	0.599	6.439	0.549	6.135	0.6	5.818	0.625	6.087	0.589	6.241	0.575	6.087	0.589
39	5.992	0.602	6.037	0.597	5.231	0.636	6.162	0.599	6.445	0.549	6.136	0.6	5.819	0.625	6.089	0.589	6.244	0.575	6.081	0.589
40	6.002	0.601	6.048	0.597	5.257	0.635	6.081	0.605	6.443	0.549	6.128	0.601	5.836	0.624	6.095	0.589	6.242	0.575	6.086	0.589
41	6.001	0.601	6.047	0.597	5.229	0.635	6.104	0.605	6.442	0.549	6.125	0.601	5.833	0.624	6.094	0.589	6.239	0.575	6.092	0.589
42	6.006	0.601	6.052	0.596	5.193	0.637	6.105	0.605	6.44	0.549	6.137	0.6	5.839	0.623	6.096	0.589	6.24	0.575	6.094	0.589
43	6.007	0.601	6.053	0.596	5.206	0.636	6.096	0.605	6.439	0.549	6.131	0.6	5.84	0.623	6.095	0.589	6.241	0.575	6.087	0.589
44	6.011	0.601	6.057	0.596	5.177	0.642	6.114	0.601	6.425	0.552	6.146	0.598	5.854	0.621	6.093	0.589	6.238	0.576	6.086	0.589
45	5.996	0.602	6.038	0.597	5.222	0.638	6.105	0.602	6.379	0.556	6.114	0.6	5.851	0.621	6.072	0.591	6.198	0.579	6.099	0.588
46	5.996	0.602	6.038	0.597	5.225	0.636	6.107	0.601	6.378	0.556	6.113	0.6	5.851	0.621	6.072	0.591	6.198	0.579	6.099	0.588
47	5.994	0.602	6.036	0.598	5.197	0.642	6.102	0.602	6.37	0.558	6.114	0.6	5.854	0.62	6.07	0.592	6.193	0.58	6.099	0.588
48	5.994	0.602	6.035	0.598	5.198	0.641	6.102	0.601	6.369	0.559	6.114	0.6	5.854	0.62	6.07	0.592	6.193	0.58	6.098	0.589
49	5.994	0.602	6.036	0.598	5.203	0.643	6.103	0.601	6.37	0.559	6.117	0.601	5.854	0.62	6.07	0.592	6.194	0.581	6.099	0.589
50	5.996	0.602	6.037	0.598	5.222	0.643	6.103	0.601	6.372	0.559	6.114	0.601	5.855	0.62	6.073	0.592	6.196	0.581	6.103	0.589

Table C5. Results of iterated regression in estimating $p_{i,T} = \hat{a}_{i,T} + \hat{b}_{i,T} \times w_T$ using OLS in Treatment U for Subjects 11-20.

Period(t)	$\hat{a}_{11}$	$\hat{b}_{11}$	$\hat{a}_{12}$	$\hat{b}_{12}$	$\hat{a}_{13}$	$\hat{b}_{13}$	$\hat{a}_{14}$	$\hat{b}_{14}$	$\hat{a}_{15}$	$\hat{b}_{15}$	$\hat{a}_{16}$	$\hat{b}_{16}$	$\hat{a}_{17}$	$\hat{b}_{17}$	$\hat{a}_{18}$	$\hat{b}_{18}$	$\hat{a}_{19}$	$\hat{b}_{19}$	$\hat{a}_{20}$	$\hat{b}_{20}$
3	10.1	0.241	5.276	0.726	3.394	0.937	5.386	0.729	3.454	0.932	5.787	0.718	8.081	0.486	4.018	0.86	6.209	0.683	5.803	0.621
4	6.991	0.57	5.629	0.688	4.25	0.847	6.463	0.615	5.983	0.664	5.794	0.718	6.201	0.685	5.959	0.655	7.502	0.546	5.823	0.618
5	7.256	0.519	5.979	0.621	5.025	0.699	7.501	0.417	7.481	0.379	6.474	0.588	7.477	0.442	6.253	0.599	8.612	0.335	5.922	0.6
6	6.767	0.551	6	0.62	4.929	0.705	7.366	0.426	6.895	0.417	6.277	0.601	7.362	0.45	6.134	0.606	7.723	0.393	5.946	0.598
7	6.579	0.562	5.961	0.622	4.801	0.713	7.38	0.425	6.887	0.417	6.151	0.608	6.904	0.476	6.032	0.612	7.373	0.413	5.918	0.6
8	6.582	0.56	5.973	0.616	4.81	0.708	7.399	0.415	6.763	0.485	6.161	0.602	6.864	0.497	6.037	0.609	7.306	0.449	5.911	0.603
9	6.581	0.561	5.99	0.611	4.832	0.701	7.237	0.463	6.738	0.493	6.177	0.598	6.796	0.518	6.052	0.605	7.21	0.478	5.911	0.604
10	6.524	0.571	6.034	0.603	4.882	0.692	7.154	0.478	6.335	0.565	6.2	0.593	6.651	0.544	6.077	0.6	7.03	0.51	5.902	0.605
11	6.243	0.586	6.046	0.602	4.901	0.691	7.453	0.463	6.362	0.564	6.171	0.595	6.546	0.549	6.067	0.601	6.88	0.518	5.923	0.604
12	6.243	0.584	6.046	0.603	4.901	0.69	7.453	0.462	6.357	0.576	6.171	0.594	6.545	0.55	6.067	0.601	6.88	0.52	5.923	0.605
13	6.243	0.586	6.046	0.603	4.901	0.69	7.45	0.467	6.357	0.576	6.171	0.595	6.544	0.552	6.067	0.601	6.878	0.522	5.922	0.606
14	6.161	0.589	6.027	0.603	4.888	0.69	7.417	0.468	6.308	0.578	6.143	0.596	6.514	0.554	6.05	0.602	6.826	0.524	5.919	0.606
15	6.242	0.584	6.026	0.604	4.913	0.689	7.331	0.473	6.23	0.582	6.138	0.596	6.456	0.557	6.05	0.602	6.767	0.528	5.937	0.605
16	6.21	0.586	5.976	0.606	4.881	0.691	7.18	0.482	6.114	0.589	6.074	0.6	6.367	0.562	5.999	0.605	6.656	0.534	5.9	0.607
17	6.201	0.587	5.982	0.606	4.9	0.69	7.171	0.482	6.073	0.591	6.092	0.599	6.374	0.562	6.006	0.604	6.638	0.535	5.917	0.606
18	6.184	0.59	5.99	0.605	4.909	0.688	7.133	0.489	6.051	0.595	6.083	0.6	6.35	0.566	6.006	0.604	6.595	0.542	5.913	0.607
19	6.605	0.559	5.989	0.605	4.939	0.686	6.985	0.5	5.959	0.601	6.064	0.602	6.278	0.571	6.003	0.605	6.489	0.55	5.926	0.606
20	6.608	0.559	5.994	0.605	4.945	0.686	6.977	0.5	5.971	0.601	6.06	0.602	6.279	0.571	6.004	0.604	6.491	0.55	5.931	0.606
21	6.57	0.561	5.991	0.605	4.949	0.686	6.934	0.502	5.972	0.601	6.049	0.603	6.265	0.572	5.999	0.605	6.466	0.551	5.934	0.606
22	6.559	0.561	5.994	0.605	4.951	0.686	6.931	0.502	5.973	0.601	6.048	0.603	6.272	0.571	6.002	0.605	6.47	0.551	5.939	0.606
23	6.514	0.564	5.992	0.605	4.971	0.684	6.873	0.506	5.992	0.599	6.044	0.603	6.239	0.574	6.004	0.605	6.423	0.555	5.95	0.605
24	6.474	0.567	6	0.605	4.987	0.683	6.843	0.508	6.001	0.599	6.047	0.603	6.242	0.574	6.012	0.604	6.417	0.555	5.961	0.604
25	6.477	0.565	6.001	0.604	4.989	0.683	6.842	0.508	6.006	0.597	6.052	0.601	6.238	0.575	6.014	0.603	6.415	0.556	5.963	0.604
26	6.45	0.567	6	0.604	5.002	0.682	6.881	0.506	6.011	0.597	6.049	0.601	6.22	0.576	6.012	0.603	6.393	0.557	5.968	0.603
27	6.428	0.569	6.019	0.603	5.027	0.68	6.834	0.508	6.028	0.596	6.061	0.601	6.225	0.576	6.025	0.603	6.39	0.558	5.985	0.602
28	6.427	0.57	6.019	0.603	5.026	0.68	6.835	0.508	6.028	0.595	6.061	0.6	6.224	0.577	6.026	0.603	6.39	0.558	5.985	0.603
29	6.463	0.567	6.003	0.604	5.022	0.681	6.8	0.51	6.016	0.596	6.027	0.603	6.174	0.581	6.005	0.604	6.334	0.562	5.968	0.604
30	6.459	0.567	6.001	0.604	5.024	0.681	6.822	0.511	6.015	0.596	6.025	0.603	6.176	0.581	6.004	0.604	6.336	0.563	5.969	0.604
31	6.454	0.567	6	0.604	5.028	0.681	6.811	0.511	6.014	0.596	6.024	0.603	6.178	0.581	6.004	0.604	6.337	0.563	5.969	0.604
32	6.455	0.567	5.999	0.604	5.028	0.681	6.81	0.51	6.013	0.596	6.023	0.603	6.178	0.581	6.003	0.604	6.337	0.563	5.969	0.604
33	6.455	0.567	5.997	0.605	5.026	0.682	6.809	0.511	6.012	0.596	6.022	0.603	6.176	0.582	6.002	0.605	6.335	0.564	5.968	0.605
34	6.453	0.567	6.001	0.605	5.035	0.682	6.824	0.511	6.015	0.596	6.024	0.603	6.18	0.582	6.005	0.604	6.337	0.564	5.972	0.604
35	6.444	0.569	6.001	0.605	5.039	0.681	6.817	0.512	6.021	0.595	6.03	0.602	6.172	0.584	6.006	0.604	6.332	0.565	5.971	0.605
36	6.424	0.572	6.009	0.603	5.069	0.676	6.788	0.517	6.071	0.587	6.034	0.601	6.169	0.584	6.013	0.603	6.326	0.566	5.977	0.604
37	6.412	0.573	6.005	0.604	5.073	0.676	6.813	0.516	6.068	0.587	6.028	0.602	6.165	0.584	6.01	0.603	6.325	0.566	5.974	0.604
38	6.417	0.571	6.01	0.602	5.077	0.675	6.81	0.517	6.072	0.586	6.032	0.601	6.167	0.584	6.014	0.602	6.325	0.566	5.978	0.603
39	6.415	0.571	6.011	0.602	5.083	0.675	6.823	0.517	6.073	0.586	6.034	0.601	6.168	0.584	6.015	0.602	6.329	0.566	5.98	0.603
40	6.396	0.573	6.02	0.602	5.103	0.673	6.853	0.515	6.084	0.585	6.042	0.6	6.17	0.584	6.023	0.602	6.329	0.566	5.991	0.602
41	6.391	0.573	6.018	0.602	5.106	0.673	6.861	0.515	6.083	0.585	6.042	0.6	6.169	0.584	6.022	0.602	6.324	0.566	5.99	0.602
42	6.38	0.573	6.022	0.601	5.121	0.672	6.854	0.515	6.087	0.585	6.043	0.6	6.17	0.584	6.026	0.601	6.323	0.566	5.995	0.602
43	6.375	0.573	6.023	0.601	5.128	0.672	6.847	0.515	6.088	0.585	6.043	0.6	6.17	0.584	6.026	0.601	6.323	0.566	5.996	0.602
44	6.367	0.575	6.027	0.601	5.143	0.67	6.851	0.515	6.093	0.584	6.05	0.599	6.168	0.584	6.031	0.601	6.323	0.566	6	0.601
45	6.296	0.58	6.011	0.602	5.131	0.67	6.802	0.518	6.08	0.585	6.032	0.6	6.138	0.586	6.014	0.602	6.284	0.569	5.986	0.602
46	6.296	0.58	6.011	0.602	5.131	0.67	6.802	0.518	6.08	0.585	6.032	0.6	6.137	0.587	6.014	0.602	6.284	0.57	5.986	0.602
47	6.289	0.582	6.01	0.602	5.132	0.67	6.797	0.519	6.079	0.585	6.031	0.6	6.133	0.588	6.013	0.602	6.275	0.572	5.984	0.603
48	6.289	0.582	6.01	0.602	5.132	0.67	6.797	0.52	6.079	0.585	6.031	0.601	6.133	0.588	6.013	0.602	6.275	0.572	5.984	0.603
49	6.292	0.582	6.01	0.602	5.133	0.67	6.798	0.52	6.079	0.585	6.031	0.601	6.133	0.588	6.013	0.602	6.276	0.572	5.984	0.603
50	6.29	0.582	6.011	0.602	5.138	0.67	6.801	0.52	6.08	0.585	6.032	0.601	6.135	0.588	6.014	0.602	6.278	0.572	5.986	0.603

Table C6. Results of iterated regression in estimating $p_{i,T} = \hat{a}_{i,T} + \hat{b}_{i,T} \times w_T$ using OLS in Treatment U for Subjects 21-29.

Period(t)	$\hat{a}_{21}$	$\hat{b}_{21}$	$\hat{a}_{22}$	$\hat{b}_{22}$	$\hat{a}_{23}$	$\hat{b}_{23}$	$\hat{a}_{24}$	$\hat{b}_{24}$	$\hat{a}_{25}$	$\hat{b}_{25}$	$\hat{a}_{26}$	$\hat{b}_{26}$	$\hat{a}_{27}$	$\hat{b}_{27}$	$\hat{a}_{28}$	$\hat{b}_{28}$	$\hat{a}_{29}$	$\hat{b}_{29}$
3	6.611	0.643	5.746	0.567	4.118	0.871	5.465	0.748	0.238	1.206	7.182	0.635	9.881	0.142	4.901	0.808	8.004	0.44
4	5.881	0.72	6.174	0.522	5.409	0.734	7.03	0.582	5.774	0.62	5.715	0.791	5.779	0.576	5.38	0.758	5.91	0.662
5	7.46	0.419	5.682	0.616	5.966	0.628	6.966	0.594	6.291	0.521	6.191	0.7	6.552	0.429	6.111	0.618	6.617	0.527
6	7.181	0.437	5.703	0.614	5.889	0.633	6.521	0.623	6.818	0.487	6.265	0.695	6.937	0.404	6.058	0.622	6.423	0.54
7	6.993	0.448	5.761	0.611	5.854	0.635	6.314	0.635	6.63	0.498	5.915	0.715	6.617	0.422	5.973	0.627	6.235	0.55
8	6.942	0.475	5.751	0.617	5.864	0.63	6.34	0.62	6.537	0.548	6.025	0.654	6.511	0.48	5.988	0.618	6.232	0.552
9	6.856	0.501	5.738	0.621	5.898	0.62	6.391	0.605	6.535	0.549	6.117	0.627	6.39	0.516	6.002	0.614	6.162	0.573
10	6.709	0.528	5.799	0.609	5.909	0.618	6.454	0.594	6.449	0.564	6.205	0.611	6.19	0.552	6.044	0.607	6.051	0.593
11	6.661	0.53	5.912	0.604	5.58	0.635	6.031	0.616	6.662	0.553	6.619	0.59	6.12	0.556	6.023	0.608	6.018	0.594
12	6.658	0.536	5.912	0.604	5.58	0.635	6.032	0.615	6.664	0.549	6.622	0.582	6.119	0.557	6.023	0.607	6.021	0.588
13	6.657	0.539	5.912	0.604	5.581	0.634	6.036	0.609	6.662	0.551	6.624	0.579	6.116	0.563	6.023	0.607	6.018	0.593
14	6.627	0.54	5.923	0.604	5.605	0.633	6.043	0.608	6.768	0.547	6.661	0.578	6.135	0.562	6.006	0.608	6.132	0.589
15	6.588	0.542	5.957	0.602	5.654	0.63	6.04	0.609	6.931	0.538	6.53	0.585	6.183	0.559	5.997	0.608	6.032	0.595
16	6.501	0.547	5.92	0.604	5.648	0.63	6.009	0.61	6.732	0.55	6.476	0.588	6.153	0.561	5.945	0.611	6.141	0.588
17	6.498	0.547	5.931	0.604	5.679	0.629	6.018	0.61	6.721	0.55	6.488	0.587	6.173	0.56	5.958	0.611	6.032	0.594
18	6.454	0.555	5.925	0.605	5.699	0.625	6.056	0.603	6.706	0.553	6.584	0.571	6.132	0.567	5.963	0.61	5.973	0.604
19	6.372	0.56	6.037	0.596	5.766	0.621	6.142	0.597	6.543	0.564	6.516	0.576	6.328	0.553	5.967	0.609	6.172	0.589
20	6.373	0.56	6.029	0.597	5.766	0.621	6.165	0.596	6.485	0.566	6.499	0.576	6.334	0.553	5.967	0.609	6.131	0.59
21	6.355	0.561	6.023	0.597	5.782	0.62	6.155	0.597	6.404	0.57	6.462	0.578	6.325	0.553	5.964	0.609	6.148	0.589
22	6.359	0.561	6.015	0.597	5.787	0.62	6.151	0.597	6.371	0.57	6.467	0.578	6.321	0.553	5.966	0.609	6.143	0.589
23	6.321	0.564	5.994	0.599	5.827	0.617	6.121	0.599	6.283	0.577	6.382	0.585	6.205	0.562	5.972	0.609	6.147	0.589
24	6.321	0.564	6.008	0.598	5.844	0.616	6.091	0.601	6.318	0.575	6.379	0.585	6.205	0.562	5.981	0.609	6.174	0.588
25	6.318	0.565	6.011	0.597	5.849	0.614	6.096	0.599	6.32	0.574	6.385	0.583	6.201	0.563	5.984	0.608	6.185	0.584
26	6.297	0.567	6	0.597	5.867	0.613	6.093	0.599	6.331	0.574	6.419	0.58	6.223	0.562	5.984	0.608	6.172	0.585
27	6.299	0.567	6.014	0.597	5.888	0.612	6.099	0.599	6.306	0.575	6.395	0.582	6.219	0.562	5.998	0.607	6.196	0.584
28	6.299	0.567	6.011	0.601	5.888	0.611	6.099	0.599	6.304	0.577	6.397	0.58	6.217	0.565	5.998	0.606	6.194	0.586
29	6.252	0.571	5.968	0.604	5.891	0.611	6.078	0.601	6.256	0.581	6.359	0.583	6.201	0.566	5.98	0.608	6.097	0.593
30	6.253	0.571	5.964	0.604	5.889	0.611	6.073	0.601	6.255	0.581	6.354	0.583	6.206	0.566	5.979	0.608	6.088	0.593
31	6.254	0.571	5.966	0.604	5.889	0.611	6.071	0.601	6.255	0.581	6.345	0.583	6.209	0.566	5.978	0.608	6.091	0.593
32	6.254	0.571	5.968	0.605	5.888	0.611	6.069	0.6	6.251	0.579	6.337	0.579	6.207	0.566	5.978	0.607	6.089	0.592
33	6.252	0.573	5.97	0.604	5.887	0.611	6.068	0.6	6.251	0.579	6.336	0.58	6.206	0.566	5.977	0.608	6.087	0.594
34	6.256	0.573	5.975	0.604	5.891	0.611	6.086	0.6	6.236	0.579	6.336	0.58	6.212	0.566	5.98	0.608	6.083	0.594
35	6.242	0.575	5.981	0.603	5.895	0.61	6.08	0.601	6.235	0.579	6.359	0.576	6.16	0.576	5.982	0.608	6.086	0.593
36	6.231	0.577	5.986	0.602	5.908	0.608	6.094	0.599	6.236	0.579	6.37	0.574	6.197	0.57	5.992	0.606	6.081	0.594
37	6.227	0.577	5.991	0.602	5.906	0.608	6.09	0.599	6.238	0.579	6.363	0.574	6.195	0.57	5.988	0.606	6.094	0.594
38	6.228	0.577	5.994	0.601	5.91	0.607	6.095	0.598	6.24	0.579	6.364	0.573	6.194	0.57	5.993	0.605	6.092	0.594
39	6.23	0.577	5.998	0.601	5.914	0.607	6.098	0.598	6.24	0.579	6.364	0.573	6.197	0.57	5.994	0.605	6.099	0.594
40	6.229	0.577	6.008	0.601	5.932	0.606	6.104	0.597	6.222	0.58	6.384	0.572	6.211	0.569	6.004	0.604	6.106	0.594
41	6.228	0.577	6.011	0.601	5.934	0.606	6.103	0.597	6.227	0.58	6.381	0.572	6.216	0.569	6.002	0.604	6.098	0.594
42	6.229	0.577	6.008	0.601	5.943	0.606	6.105	0.597	6.229	0.58	6.369	0.573	6.231	0.568	6.006	0.604	6.095	0.594
43	6.23	0.577	6.009	0.601	5.944	0.606	6.103	0.597	6.232	0.58	6.365	0.573	6.234	0.568	6.006	0.604	6.097	0.594
44	6.227	0.577	6.012	0.6	5.953	0.604	6.109	0.596	6.228	0.58	6.366	0.573	6.237	0.568	6.012	0.603	6.097	0.594
45	6.196	0.58	5.988	0.602	5.926	0.606	6.071	0.599	6.204	0.582	6.343	0.574	6.191	0.571	5.997	0.604	6.075	0.595
46	6.196	0.58	5.988	0.602	5.926	0.606	6.071	0.599	6.203	0.583	6.343	0.574	6.19	0.572	5.997	0.604	6.075	0.596
47	6.191	0.581	5.989	0.602	5.927	0.605	6.079	0.597	6.196	0.584	6.343	0.574	6.179	0.574	5.996	0.604	6.077	0.595
48	6.191	0.581	5.989	0.602	5.927	0.605	6.079	0.597	6.196	0.584	6.343	0.574	6.179	0.575	5.996	0.604	6.077	0.596
49	6.191	0.581	5.987	0.602	5.927	0.605	6.078	0.597	6.198	0.585	6.346	0.575	6.181	0.575	5.996	0.604	6.078	0.597
50	6.193	0.581	5.988	0.602	5.929	0.605	6.078	0.597	6.199	0.585	6.344	0.575	6.177	0.575	5.997	0.604	6.075	0.597

Appendix D: Modelling the Forecasting Strategy at the Individual Level

Table D.1. The mean squared error of the recursive least squares learning learning model for each subject in Treatment A and U.

Treatment A	MSE_a	MSE_b	Treatment U	MSE_a	MSE_b
1	20.2748	0.2272	1	0.0038	0.0001
2	1.4377	0.0234	2	0.0293	0.0003
3	1.4622	0.0138	3	5.5731	0.0993
4	2.6421	0.0361	4	0.9670	0.0074
5	1.6777	0.0277	5	1.1212	0.0211
6	3.5342	0.0475	6	4.1360	0.0437
7	10.8363	0.1156	7	0.2593	0.0063
8	4.7650	0.0498	8	1.8924	0.0238
9	2.0785	0.0288	9	1.8127	0.0272
10	4.1401	0.0440	10	1.7940	0.0226
11	1.0212	0.0129	11	0.9237	0.0054
12	3.6883	0.0383	12	0.0100	0.0003
13	3.9192	0.0437	13	7.7283	0.0537
14	15.2220	0.1851	14	6.8075	0.0571
15	1.2980	0.0138	15	0.2685	0.0064
16	4.0759	0.0513	16	0.0255	0.0003
17	1.5917	0.0229	17	0.3302	0.0029
18	1.7316	0.0173	18	0.0026	0.0001
19	5.7752	0.0532	19	0.4103	0.0050
20	0.6736	0.0084	20	0.0034	0.0000
21	1.0703	0.0121	21	0.3498	0.0028
22	2.9990	0.0392	22	1.3446	0.0183
23	0.4317	0.0061	23	0.1309	0.0022
24	3.0107	0.0370	24	1.0970	0.0112
25	1.9935	0.0158	25	1.2195	0.0292
26	1.5810	0.0198	26	3.8584	0.0668
27	0.4781	0.0080	27	1.2913	0.0077
28	5.1372	0.0770	28	0.0145	0.0006
29	1.3162	0.0164	29	0.5750	0.0073
30	1.3409	0.0194			

Table D.2. The mean squared error of the learning by averaging heuristic for each subject in Treatment A and U.

Treatment A	MSE	Treatment U	MSE
1	0.4623	1	0.4826
2	0.9803	2	0.4313
3	0.2500	3	12.9155
4	1.0926	4	3.1209
5	0.7714	5	1.1043
6	0.6954	6	2.6982
7	0.3960	7	0.4852
8	0.3095	8	4.4884
9	1.0730	9	1.7100
10	0.6949	10	3.0934
11	0.1305	11	3.2289
12	0.3410	12	0.3773
13	1.2440	13	1.8802
14	1.0216	14	1.3178
15	0.2451	15	1.5661
16	1.8393	16	0.5327
17	0.3047	17	1.1930
18	0.6630	18	0.4510
19	0.2313	19	1.3047
20	0.1672	20	0.4854
21	0.2053	21	1.2381
22	0.4540	22	1.3235
23	0.1316	23	0.8392
24	0.6092	24	1.8006
25	0.7027	25	4.4112
26	0.1583	26	5.6637
27	0.1684	27	3.4084
28	0.9507	28	0.4250
29	0.3060	29	2.8162
30	0.3348		

Table D.3. The mean squared error of the constant gain learning model for each subject in Treatment A and U.

Treatment A	MSE_a	MSE_b	γ	Treatment U	MSE_a	MSE_b	γ
1	25.1347	0.2460	0.01	1	0.0384	0.0002	0.27
2	2.4421	0.0369	0.01	2	0.0729	0.0002	0.01
3	3.1365	0.0340	0.3	3	7.9946	0.0577	0.15
4	3.5402	0.0502	0.26	4	1.5429	0.0086	0.22
5	2.5643	0.0398	0.32	5	0.5393	0.0195	0.32
6	4.1357	0.0445	0.5	6	5.9779	0.0528	0.2
7	12.1870	0.1652	0.01	7	0.5267	0.0128	0.31
8	6.4276	0.0793	0.31	8	2.2658	0.0274	0.62
9	4.1499	0.0576	0.3	9	0.8746	0.0210	0.3
10	6.0884	0.0674	0.28	10	2.2412	0.0216	0.17
11	2.1186	0.0318	0.37	11	1.3765	0.0047	0.08
12	2.9955	0.0459	0.33	12	0.0352	0.0006	0.31
13	3.3969	0.0427	0.37	13	6.8487	0.0457	0.41
14	13.7778	0.1480	0.22	14	5.9884	0.0551	0.29
15	4.9380	0.0680	0.39	15	1.0784	0.0267	0.37
16	6.3307	0.0558	0.21	16	0.0579	0.0005	0.32
17	3.7499	0.0543	0.39	17	0.5174	0.0072	0.44
18	4.5664	0.0514	0.3	18	0.1464	0.0007	0.25
19	3.5210	0.0524	0.01	19	1.0154	0.0130	0.34
20	1.3215	0.0161	0.32	20	0.0014	0.0000	0.02
21	4.1122	0.0571	0.37	21	0.1139	0.0050	0.49
22	3.9143	0.0472	0.38	22	1.4359	0.0226	0.36
23	2.4223	0.0310	0.01	23	0.4680	0.0046	0.28
24	6.1098	0.0748	0.5	24	1.6112	0.0116	0.19
25	3.5249	0.0370	0.45	25	2.1439	0.0415	0.26
26	2.7143	0.0301	0.32	26	3.9461	0.0669	0.3
27	2.2466	0.0316	0.37	27	1.9689	0.0141	0.31
28	5.3577	0.0572	0.34	28	0.0965	0.0017	0.34
29	2.4703	0.0360	0.29	29	0.6744	0.0090	0.01
30	2.8794	0.0327	0.01				

Table D.4. The mean squared error of the least mean square learning model for each subject in Treatment A and U.

Treatment A	MSE_a	MSE_b	λ	Treatment U	MSE_a	MSE_b	λ
1	8.2152	0.3560	0.21	1	0.2004	0.1436	0.03
2	0.3995	0.0234	0.01	2	0.0520	0.0137	0.01
3	0.7293	0.2409	0.14	3	7.4191	0.3794	0.18
4	2.0970	0.2693	0.11	4	1.0352	0.1852	0.06
5	5.4008	0.2731	0.25	5	3.2267	0.2507	0.15
6	2.7783	0.2374	0.17	6	4.8819	0.2525	0.13
7	2.0770	0.1385	0.06	7	2.5854	0.2315	0.14
8	2.1135	0.2537	0.17	8	2.6754	0.3079	0.16
9	1.8980	0.2597	0.15	9	0.4979	0.1606	0.03
10	0.6168	0.2511	0.08	10	2.1554	0.1417	0.02
11	0.2400	0.2457	0.06	11	1.4309	0.1534	0.04
12	0.4615	0.2401	0.12	12	0.1391	0.1718	0.04
13	0.9391	0.2307	0.13	13	2.4645	0.1985	0.1
14	8.3568	0.3701	0.27	14	3.1031	0.2396	0.08
15	0.0898	0.0023	0.01	15	0.9297	0.1530	0.03
16	2.6797	0.2681	0.11	16	0.2568	0.1596	0.04
17	0.1063	0.0032	0.01	17	0.3953	0.1181	0.03
18	1.4588	0.2394	0.13	18	0.2022	0.1634	0.04
19	0.1457	0.0128	0.02	19	0.5795	0.0218	0.01
20	0.5006	0.2298	0.04	20	0.0003	0.0030	0.01
21	0.0006	0.0008	0.01	21	0.2596	0.1495	0.03
22	1.0577	0.2457	0.13	22	0.8757	0.2029	0.05
23	0.8560	0.2364	0.04	23	0.3875	0.1692	0.03
24	3.6128	0.2627	0.03	24	1.0726	0.2220	0.05
25	0.9682	0.2456	0.2	25	1.3857	0.1760	0.02
26	1.0451	0.2336	0.04	26	3.4554	0.1029	0.01
27	0.3987	0.2337	0.07	27	1.1324	0.0595	0.01
28	1.4380	0.2433	0.11	28	0.3684	0.2021	0.07
29	0.4659	0.2496	0.05	29	0.9378	0.1781	0.03
30	1.9773	0.1055	0.03				

Table D.5. Mean, standard error and 95% confidence interval (CI) of $a + 10b$ in Treatments A and U.

	Treatment A				***Treatment U***			
Sub	Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI	
1	11.81	0.09	11.63	12.00	11.99	0.01	11.96	12.02
2	11.97	0.15	11.66	12.28	11.99	0.02	11.95	12.04
3	11.93	0.07	11.78	12.08	13.04	0.39	12.26	13.83
4	12.05	0.17	11.70	12.40	11.83	0.24	11.36	12.31
5	11.99	0.11	11.76	12.22	12.07	0.09	11.89	12.24
6	12.00	0.17	11.65	12.34	12.51	0.20	12.11	12.91
7	12.07	0.08	11.91	12.24	11.92	0.09	11.74	12.10
8	11.84	0.09	11.67	12.02	12.30	0.27	11.76	12.84
9	12.00	0.19	11.61	12.39	12.18	0.13	11.93	12.44
10	11.95	0.13	11.70	12.21	12.11	0.22	11.68	12.55
11	11.95	0.04	11.87	12.04	11.73	0.23	11.26	12.19
12	11.94	0.08	11.77	12.10	11.96	0.03	11.91	12.01
13	11.93	0.15	11.63	12.23	12.38	0.10	12.17	12.58
14	11.76	0.14	11.47	12.05	11.94	0.14	11.67	12.22
15	12.05	0.05	11.95	12.15	12.10	0.14	11.83	12.37
16	12.16	0.15	11.85	12.47	11.92	0.06	11.81	12.04
17	11.91	0.07	11.78	12.04	11.99	0.10	11.79	12.19
18	11.97	0.13	11.71	12.23	11.94	0.07	11.79	12.09
19	11.98	0.06	11.86	12.11	11.99	0.12	11.74	12.23
20	11.97	0.03	11.91	12.02	12.00	0.01	11.99	12.02
21	12.00	0.05	11.91	12.09	11.93	0.10	11.72	12.14
22	12.03	0.14	11.75	12.30	11.68	0.16	11.35	12.00
23	11.94	0.05	11.83	12.05	12.14	0.11	11.92	12.35
24	12.03	0.09	11.84	12.21	11.83	0.19	11.44	12.21
25	11.87	0.10	11.66	12.07	12.01	0.29	11.43	12.59
26	11.92	0.06	11.80	12.03	10.92	0.28	10.35	11.49
27	11.92	0.06	11.79	12.04	11.99	0.22	11.54	12.44
28	11.83	0.15	11.53	12.13	11.95	0.04	11.86	12.04
29	11.89	0.10	11.68	12.10	11.87	0.20	11.46	12.27
30	11.93	0.11	11.72	12.15				

Table D.6. The mean squared error of the learning by REE or the satisficing rule for each subject in Treatment A and U.

Treatment A	MSE	Treatment U	MSE
1	1.1678	1	0.8700
2	1.1804	2	1.1339
3	1.1930	3	0.9752
4	1.1112	4	0.9592
5	1.1018	5	1.0387
6	1.0945	6	1.3506
7	1.0930	7	0.8944
8	1.1017	8	0.9949
9	1.0912	9	1.0806
10	1.0946	10	1.0315
11	1.0912	11	0.8232
12	1.0824	12	0.9767
13	1.0980	13	1.1182
14	1.1111	14	1.0963
15	1.0827	15	0.8329
16	1.0571	16	1.1173
17	1.0739	17	0.8391
18	1.0765	18	0.9648
19	1.0964	19	0.8823
20	1.0775	20	0.8348
21	1.1064	21	0.8500
22	1.1059	22	1.0253
23	1.1325	23	0.8478
24	1.1662	24	0.8899
25	1.1777	25	0.9673
26	1.1307	26	0.8259
27	1.0961	27	0.9214
28	1.0038	28	0.9248
29	1.1821	29	0.9820
		30	1.0278

Appendix E: Experimental Instructions and Quiz

Appendix E.1: Experimental Instructions

Welcome to this experiment in economic decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. There is no talking for the duration of this session. If you have a question at any time, please raise your hand and your question will be answered in private.

General information

Imagine you are an advisor to a farm that is the only supplier of a product in a local market. In each time period the owner of the farm needs to decide how many units of the product he will produce. To make an optimal decision each period, the owner requires a good prediction of the market price of the product in each period. As the advisor to the farm owner, you will be asked to predict the market price, p_t of the product during 50 successive time periods, t=1,2,...,50. Your earnings from this experiment will depend on the accuracy of your price predictions alone. The smaller are your prediction errors, the greater will be your earnings.

About the prediction of the market price

Your firm will use the following model to predict the market price for the product in each time period, t.

$$p_t^e = a + b \times w_t$$

where a is a positive number that is usually between 0 and 10, w_t is the a measure of how good the weather is for producing the agricultural product, and b is the coefficient that measures how sensitive the product is to the change of weather.

The weather variable is randomly drawn in each period, and you will see the realisation of it at the beginning of each period. Suppose in one period, w_t=8, you estimates are a=3, b=0.5, your implied prediction will be:

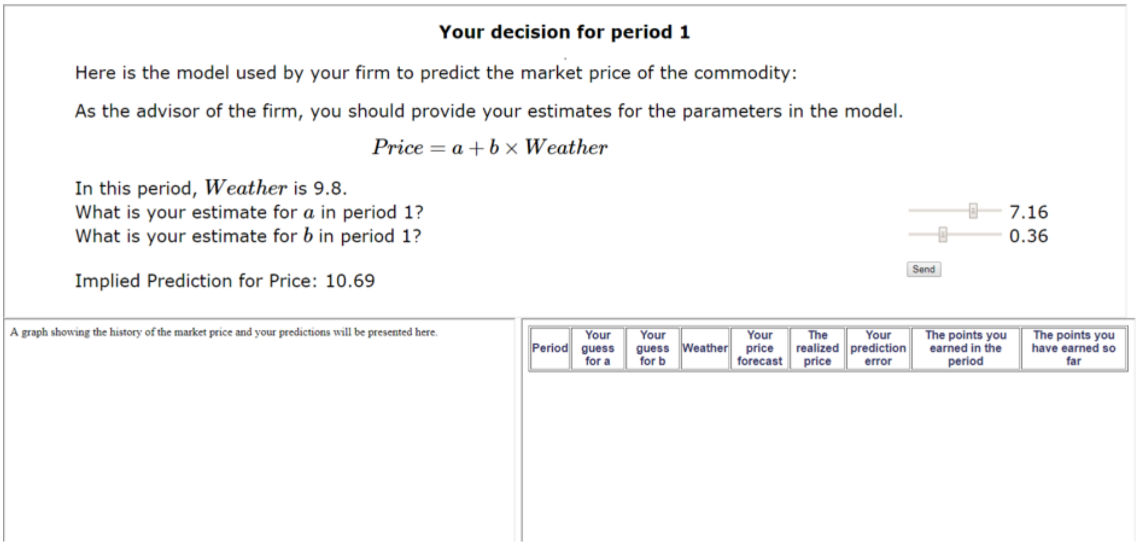
$$p_t^e = 3 + 0.5 \times 8 = 7$$

Suppose the market price in this period turns out to be $p_t = 4.9$. Your forecast error, $|p_t - p_t^e| = |7 - 4.9| = 2.1$. This forecast error of 2.1would determine your points for the period as discussed below.

Please also note that this example is for illustration purposes only. The value of the weather in your local market may be different from 8. The price determination function in this example may also be different from the price determination function in your local market. The precise value of weather in your market in each period will be given on your decision page.

Your task

Your only task in this experiment is to correctly predict the market price in each time period as accurately as possible. You need to choose the value of a and b using the slider bar on the computer screen. The value of a is between 0 and 10, and the value of b is between 0 and 1. The slider bar starts at the midpoint of the interval, and you can feel free to move it to any value that you want to choose. You can see your implied prediction $p_t^e = a + b \times w_t$ in real time in the line below. When you have decided on your choice of the parameters, you can press "send" to submit your decision.



At the beginning of the experiment you are asked to give a prediction for the price of your farm's product in period 1. Note that, while there are several farms being advised by a forecaster like you in each period, these different local markets are totally separate from your own so what happens in other markets does not have any influence on your market. After all forecasters have submitted their choice of parameters (and hence implied predictions) for the first period, the local market price for period 1 will be determined and will be revealed to you. Based the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all forecasters have submitted their predictions for the second period, the market price for that period in your local market will be revealed to you and your earnings will be calculated, and so on, for all 50 consecutive periods.

Information

Following the first period, you will see information on your computer screen that consists of 1) a plot of all past prices together with your predictions and 2) a table containing the history of the past prices, your past estimates of a, b, the implied price forecasts and payoffs.

About your payoff

Your payoff depends on the accuracy of your price forecast. The earnings shown on the computer screen will be in terms of points. When your prediction is p_t^e and the market price p_t your payoff is a decreasing function in your prediction error, namely the distance between your forecast and the realised price.

$$\text{payoff} = \frac{100}{1 + |p_t^e - p_t|}$$

Recalling the example above, if your forecast error for the period, $|p_t - p_t^e|$, was 3.2, then according to the payoff function you would earn 100/4.2=23.81 points for the period.

Notice that the maximum possible payoff in points you can earn from the forecasting task is 100 for each period, and the larger is your prediction error, $|p_t^e - p_t|$, the fewer points you earn. There is a Payoff Table on your desk, which shows the points you can earn for various different prediction errors.

At the end of the experiment your total points earned from all 50 periods will be converted into Euros at the rate of 1 dollar for every 200 points that you earned. Thus, the more points you earn, the greater are your Euro earnings.

Questions?

If you have questions about any part of these instructions at any time, please raise your hand and an experimenter will come to you and answer your question in private.

Appendix E.2: Quiz

We want to make sure that you understand the instructions. Therefore, we ask a few questions. You can only go to the decision page after you have answered all the questions correctly.

Question 1: Suppose in one period, the Weather is equal to 6, your estimates for the parameters in the model are a=3.6, b=0.5. What is your implied prediction for the price (a+b*Weather) in this period? (Answer: 6.6)

Question 2: If your forecast error for a period is 1, what is your payoff in this period? (Answer: 50)

Question 3: Is the price in our market influenced by other participants' price forecasts? (Answer: No)

Appendix F: Experimental Interface, Respondent Questionnaire, and Weather List

Your decision for period 1

Here is the model used by your firm to predict the market price of the commodity:

As the advisor of the firm, you should provide your estimates for the parameters in the model.

$$Price = a + b \times Weather$$

In this period, $Weather$ is 9.8.
What is your estimate for a in period 1? 5
What is your estimate for b in period 1? 0.5

Send

Implied Prediction for Price: To be computed based on the values of a and b

A graph showing the history of the market price and your predictions will be presented here.

Period	Your guess for a	Your guess for b	Weather	Your price forecast	The realized price	Your prediction error	The points you earned in the period	The points you have earned so far

Your decision for period 9

Here is the model used by your firm to predict the market price of the commodity:

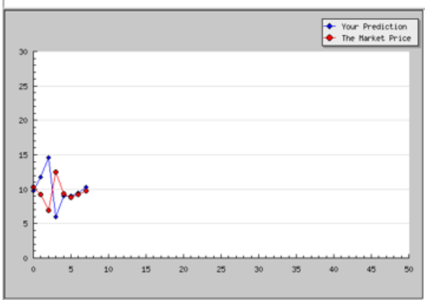
As the advisor of the firm, you should provide your estimates for the parameters in the model.

$$Price = a + b \times Weather$$

In this period, $Weather$ is 9.69.
What is your estimate for a in period 9? 5
What is your estimate for b in period 9? 0.5

Send

Implied Prediction for Price: To be computed based on the values of a and b



Period	Your guess for a	Your guess for b	Weather	Your price forecast	The realized price	Your prediction error	The points you earned in the period	The points you have earned so far
8	**6.23**	**0.39**	**10.47**	**10.31**	**9.78**	**0.53**	**2585.1**	**15581.5**
7	5	0.56	7.97	9.46	9.25	0.21	2597.66	12996.4
6	4.11	0.5	9.84	9.03	8.81	0.22	2597.43	10398.7
5	5.97	0.36	8.53	9.04	9.35	0.31	2594.9	7801.26
4	2.92	0.31	9.77	5.95	12.46	6.51	351.26	5206.36
3	6.48	0.75	10.79	14.57	6.88	7.69	0	4855.1
2	1.91	0.92	10.66	11.71	9.21	2.5	2268.37	4855.1
1	6.74	0.31	9.8	9.78	10.28	0.5	2586.73	2586.73

Figure F.1. Experimental interface.

Respondent Questionnaire

You have made your prediction for all periods! Here is a questionnaire to complete on your backgrounds. Please answer the questions and press "send" to submit. After that you will see the payment page.

1. Age: ____________

2. Gender:
☐ Male
☐ Female

3. Study Program: choose from list
☐ Faculty of Economics and Econometrics
☐ Faculty of Social and Behavioural Sciences: Psychology
☐ Faculty of Social and Behavioural Sciences: other than Psychology
☐ Faculty of Science, Mathematics and Computer Science
☐ Faculty of Law
☐ Faculty of Humanities
☐ Faculty of Medicine
☐ Another University
☐ Others

4. Have you come to an economics experiment before?
☐ No
☐ Yes, only once
☐ Yes, more than once

5. How do you describe your strategy in this experiment?

____

Table F.1. Distribution of weather w_t in Treatments A and U and $\epsilon_t \sim N(0,1)$.

period	Weather A	Weather U	ϵ_t
1	10	10.92	0.2872
2	10.12	5.49	-0.3316
3	9	2.25	-1.3085
4	8.32	19.35	0.5709
5	8.62	2.32	-0.2499
6	10.1	3.7	-1.5791
7	9.59	14.14	0.1971
8	8.2	15.92	-0.3451
9	8.34	19.9	-0.4954
10	9.58	5.07	0.2918
11	10.22	14.55	0.442
12	10.09	14.63	1.4143
13	10.88	8.29	-0.5298
14	9.46	5.5	0.6355
15	6.89	5.15	-2.7869
16	8.12	6.51	0.9556
17	9.05	18.41	0.8365
18	9.61	0.37	-0.1459
19	11.12	8.36	0.6092
20	10.23	5.24	-0.0798
21	10.4	9.18	0.9335
22	9.22	1.21	0.2988
23	9.98	5.54	1.2221
24	8.25	14.7	-0.5452
25	7.3	2.99	0.0912
26	7.1	4.14	1.8649
27	6.77	12.93	0.4169
28	9.65	0.63	-1.6766
29	10.63	9.54	0.1737
30	9.66	8.97	0.1636
31	9.92	11.47	-0.3594
32	9.83	13.14	2.132
33	8.96	8.4	1.3624
34	9.72	17.89	0.7295
35	10.07	19.93	-0.4855
36	10.91	8.22	-0.5949
37	10.71	14.98	-1.891
38	10.04	9.57	0.8072
39	9.43	3.16	1.4417
40	10.29	9.15	-0.189
41	10.5	6.12	1.0415
42	10.72	8.51	0.5437
43	11.01	17.37	-0.8844
44	11.69	0.48	-1.8038
45	9.55	13.25	0.3485
46	8.71	15.98	1.0516
47	8.21	13.08	0.8825
48	9.01	11.88	0.6185
49	8.57	9.52	0.8332
50	11.38	10.74	0.6123

Appendix G: Omitted Proof on Cobweb Model

For completeness of explanation, we repeat what we have mentioned in Section 2.1.

Consider the cobweb model in Evans and Honkapohja (2001) based on the analysis of Bray and Savin (1986), and Fourgeaud et al. (1986). It consists of a single competitive market with a time lag in production (e.g. agricultural product), where demand depends negatively on the prevailing market price; supply is assumed to depend on both the average expectation across the homogeneous firms of the price of the product in the current period, as well as the weather in the current period in the form of an observable shock. [21], denoted as:

$$d_t = m_I - m_p p_t + v_{1t}, \;\; m_p > 0$$

$$s_t = r_I + r_p p_t^e + r_w w_t + v_{2t}, \;\; r_p > 0$$

d_t, s_t represents the demand and supply of the product, m_I and r_I denotes the intercept, v_{1t} and v_{2t} are the random variables of unobserved random noise. Thus, at the market clearing price where $d_t = s_t$, the reduced form of price determination function is:

$$d_t = s_t$$

$$m_I - m_p p_t + v_{1t} = r_I + r_p p_t^e + r_w w_t + v_{2t}$$

$$p_t = \frac{m_I - r_I}{m_p} + \left(\frac{-r_p}{m_p}\right) p_t^e + \left(\frac{-r_w}{m_p}\right) w_t + \left(\frac{v_{1t} - v_{2t}}{m_p}\right)$$

Thus,

$$p_t = \mu + \alpha p_t^e + \delta w_t + \eta_t$$

In the above equation, $\mu = m_p^{-1}(m_I - r_I)$, $\alpha = -m_p^{-1} r_p < 0$, $\delta = -m_p^{-1} r_w < 0$, $\eta_t = m_p^{-1}(v_{1t} - v_{2t})$, and η_t $iid(0, \sigma_\eta^2)$. The distribution of the weather w_t can be followed by an iid process as was assumed in Bray and Savin (1986). Alternatively, it can follow a stationary exogenous VAR (vector autoregression) process driven by a multivariate white noise shock with bounded moments as the setting in Evans and Honkapohja (2001).

According to the least squares principle, prediction of estimators of a simple linear regression model will be more precise (i.e. with lower variance) when there is a larger variation of independent variables[22]. Therefore theoretically, the variance

[21] Note that the weather in the original setting is assumed to based on the weather in the previous term w_{t-1}, assuming that the supply in the current period will depend on the observable shock brought from weather in the last period. However, we change the source of this observable shock into w_t. This is to help the subjects understand the setting more easily, and the change in the term will not change the quantitative results from the model.

[22] In the simple linear regression model $y_i = \beta_1 + \beta_2 x_i + e_i$, an estimated model $\hat{y} = b_1 + b_2 x_i$ can be formed using least squares principle, where $y_i = \hat{y}_i + \hat{e}_i$. $Var\,(b_1) = \frac{\sigma^2 N^{-1} \sum x_i^2}{\sum (x_i - \overline{x})^2}$, $Var\,(b_2) = \frac{\sigma^2}{\sum (x_i - \overline{x})^2}$. Thus, the wider spread of independent variable weather (i.e. a larger $\sum (x_i - \overline{x})^2$) will lead to a more precise estimate (i.e. smaller variance) on both of the parameters. Note that the spread of the independent variable does not affect the accuracy on the estimator because the expectation of the estimates following least squares principle should always be unbiased, i.e.

of the estimates from Bray and Savin (1986) should be smaller than in the setting of Evans and Honkapohja (2001). We design two separate treatments to verify this hypothesis.

If we assume that subjects form a rational belief following the adaptive expectation or any other fixed-weight distributed lag formula, that is, the expected price in the current terms is to be based on (or conditional on) the information of information available in the previous term, then the expectation price in the current term can be written as:

$$p_t^e = E_{t-1}p_t$$

Operating with E_{t-1} on both sides and solve for $E_{t-1}p_t$, and combining with the equation of $p_t = E_{t-1}p_t + \eta_t$ we have:

$$E(p_t) = E(\mu + \alpha p_t^e + \delta w_t + \eta_t)$$
$$E_{t-1}p_t = \mu + \alpha E_{t-1}p_t + \delta w_t$$
$$E_{t-1}p_t = \frac{\mu}{1-\alpha} + \frac{\delta}{1-\alpha}w_t$$
$$p_t = \frac{\mu}{1-\alpha} + \frac{\delta}{1-\alpha}w_t + \eta_t$$

Thus,

$$p_t = \bar{a} + \bar{b}w_t + \eta_t, \quad \bar{a} = (1-\alpha)^{-1}\mu, \ \bar{b} = (1-\alpha)^{-1}\delta$$

The equation above states the unique REE of the cobweb model, and it is said to have unique REE because p_t does not depend on the expected future prices.

Though the firms may have difficulty in obtaining the real value of REE, the process is still learnable using LS learning according to Evans and Honkapohja (2012) since LS learning assumes that firm to have a subjective model of the relationship between p_t and the observable shock, namely the perceived law of motion, denoted as:

$$p_t = a + bw_t + \eta_t$$

Subsequently, under the assumption that firms have data on the evolution of the economy from periods $i = 0,\ldots,t-1$, they will update their belief on the parameters of a, b repeatedly in each period, using the information from the past. Letting (a_{t-1}, b_{t-1}) denote the estimation through time $t-1$, using the information set $\{p_i,\ w_i\}_{i=0}^{t-1}$. Thus, their prediction for period t would be the expectation of p_t using the price information from period 0 to period $t-1$:

$$E_{t-1}p_t = p_t - \eta_t = a + bw_t$$

$$p_t^e = a_{t-1} + b_{t-1}w_t$$

In this approach, the rationality is implied through the process of a continuous update on the parameters in the model instead of the immediate formation of

$E(b_1) = \beta_1, E(b_2) = \beta_2$.

expectation. Agents are to update the model like econometricians or statisticians using LS learning, with the formula denoted as the equation of:

$$\begin{pmatrix} a_{t-1} \\ b_{t-1} \end{pmatrix} = \left(\sum_{i=1}^{t-1} z_{i-1} z'_{i-1} \right)^{-1} \left(\sum_{i=1}^{t-1} z_{i-1} p_i \right)$$

where

$$z'_i = (1 \ w'_i)$$

The fully specified dynamic system is: at the beginning of time t, subjects form the expectation based on $p_t^e = a_{t-1} + b_{t-1} w_t$, and update their parameter according to $\binom{a_{t-1}}{b_{t-1}} = \left(\sum_{i=1}^{t-1} z_{i-1} z'_{i-1} \right)^{-1} \left(\sum_{i=1}^{t-1} z_{i-1} p_i \right)$, where $z'_i = (1 \ w'_i)$. On top of it, given the w_t and the random noise η_t, the time t price is determined by $p_t = \mu + \alpha p_t^e + \delta w_t + \eta_t$. This result could thus be used by the agent to update the parameters again, through adding $(p_t, \ w_t)$ to the data set and computing the revised estimates a_t and b_t, and subsequently to forecast p_{t+1}^e using w_{t+1} in the beginning of time $t+1$. This process continues repeatedly over time.

Meanwhile according to the E-stability principle (as the basic required concept governing the stability of equilibria that mapping from PLM to ALM from learning), in order for a_t and b_t to exhibit an asymptotic stability of an REE under LS learning (i.e. PLM is gradually converged towards ALM), the condition of $\alpha < 1$ must be satisfied to let $a_t \to \bar{a}, b_t \to \bar{b}$.

In other words, for a cobweb model, it must meet the condition of a downward-sloping demand curve as well as $|m_p| > r_p$, to reach an expectational stability or "E-stability" to let $\alpha = -m_p^{-1} r_p < 1$.