Option Liquidity and Gamma Imbalances

Leander Gayda, Thomas Grünthaler, Jan Harren
University of Münster
Germany

January 07, 2023

jan.harren@wiwi.uni-muenster.de

Outline

1. Motivation
2. Data
3. Methodology
4. Results
5. Conclusion

Outline

Motivation (1/3)

Option market making and liquidity provision.

- Market makers (MM) provide liquidity on option markets $\rightarrow$ take opposite side of a trade when counterparts' positions are not exactly met $\rightarrow$ zero net supply
- MM build up large inventories $\rightarrow$ might deviate from optimal MM inventory $\rightarrow$ hedge demand for (possible risky) inventory positions
- Hedging is costly and risky due to market imperfections (Figlewski, 1989)
- Deviations from optimal inventory, associated risks, and hedging costs should be reflected in **MM compensation for liquidity provision** $\rightarrow$ option spread

Motivation (2/3)

Three questions.

1. What is the relation between hedging needs and option liquidity?
2. When do market makers require more compensation for providing liquidity?
3. Which positions are associated with higher liquidity costs?

Motivation (3/3)

What we do.

- We compute the daily aggregated inventory
- We determine the magnitude of MM hedging activity by the aggregated gamma inventory (AGI)
- Gamma: Change in option's delta $\rightarrow$ good proxy for rebalancing activity of market makers inventory
- Gamma exposure approximates hedging costs of market makers (Gârleanu et al., 2009)
- We relate AGI to liquidity measures from intraday option trades

What do we find?

- Negative AGI is associated with wider spreads $\rightarrow$ higher compensation for providing liquidity
- Effect appears to be largest in magnitude and significance for OTM calls/puts
- MM manage their inventory in turbulent times $\rightarrow$ balanced gamma inventory (near zero) $\rightarrow$ especially when markets are volatile, illiquid, and intermediaries are especially constrained $\rightarrow$ rebalancing activity reduces to a minimum
- Balanced inventory $\rightarrow$ option expensiveness (variance risk premium) is high and liquidity risk premium is high

Mechanical trading to stay delta neutral

Hedging and trading.

- MM manage their book using delta heding $\rightarrow$ requires trading underlying and risk-free rate $\rightarrow$ non-informational channel why stock prices move
- Negative AGI: MM is **momentum** trader
- Positive AGI: MM is **reversal** trader

What could rationalize our findings?
E.g. MM is short gamma (negative AGI): $S \downarrow \rightarrow$ MM sells to stay delta neutral $\rightarrow$ trades in the same direction market $\rightarrow$ hard to find a counterpart $\rightarrow$ illiquid markets $\rightarrow$ AGI survives existing illiquditiy measures $\rightarrow$ MM appear to care about further risk sources

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Data

Focus on S&P 500 Options.

- C1 CBOE Open-Close database $\rightarrow$ signed trades
- OptionMetrics $\rightarrow$ Option mid-quotes, Δ, IVs $\rightarrow$ calculate Γ
- CBOE intraday option trades $\rightarrow$ liquidity measures

Sample period.

- January 01, 2004 - December 31, 2020
- Preceding years as a "burn-in period"

Outline

Construction. We follow Ni et al. (2021)

$$OI_{j,t}^{\text{buy},y} = OI_{j,t-1}^{\text{buy},y} + Volume_{j,t}^{\text{Open buy},y} - Volume_{j,t}^{\text{Close sell},y}$$

$$OI_{j,t}^{\text{sell},y} = OI_{j,t-1}^{\text{sell},y} + Volume_{j,t}^{\text{Open sell},y} - Volume_{j,t}^{\text{Close buy},y}$$

$$netOI_{j,t} = -1 \cdot \left[OI_{j,t}^{buy,cust} - OI_{j,t}^{sell,cust} + OI_{j,t}^{buy,firm} - OI_{j,t}^{sell,firm} \right]$$

Gamma weighting.

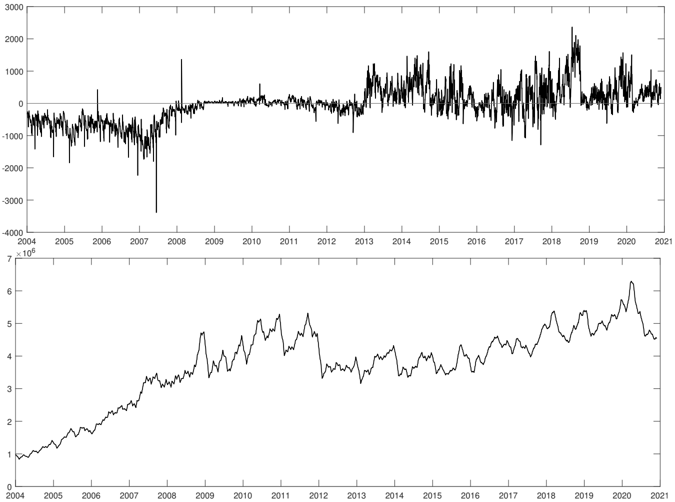
$$net\Gamma_t = S_t^2 \cdot \sum_{j=1}^{N} \left(netOI_{j,t} \cdot \Gamma_j \left(S_t, K, \tau, IV, r, d \right) \right), \tag{1}$$

where Γ_j is the Black and Scholes (1973) gamma for option j.

$$AGI_t = \frac{net\Gamma_t}{\frac{1}{M} \sum_{i=0}^{n-1} \text{Total Contracts}_{M-i}}, \tag{2}$$

where AGI_t is the aggregated dollar gamma exposure per unit of contract.

AGI and absolute number of contracts in inventory.



Liquidity Measures

Effective spreads. We follow Christoffersen et al. (2018)

$$ES_{k,j} = \frac{2 \cdot |O_{k,j}^P - O_{k,j}^M|}{O_{k,j}^M}$$
$$ES_j = \frac{\sum_k Vol_k ES_{k,j}}{\sum_k Vol_k}. \quad (3)$$

- 10 moneyness buckets $B \rightarrow$ DITM, ITM, ATM, OTM, DOTM for calls and puts
- compute the median ES_j^B within each bucket to obtain ES_t^B

Implied volatility effective spread.

- Chaudhury (2015) $\rightarrow$ effective relative and dollar spreads are biased towards lower priced options (illiquid even though most liquid in terms of speed and ease of execution)
- $IVES_t^B \rightarrow$ moneyness consistent liquidity measure

Outline

OLS WITH ES_t^B

$$ES_t = \alpha + \beta_1 AGI_t + \beta_2 VIX_t + \beta_3 ES_{t-1} + e_t. \tag{4}$$

Panel A: Calls.	DOTM	OTM	ATM	ITM	DITM
α	0.0375	0.0180	0.0043	0.0016	0.0010
	(14.23)	(13.26)	(9.84)	(5.28)	(5.48)
AGI_t	−0.0039	-0.0044	−0.0017	−0.0013	−0.0007
	(-5.40)	(-12.06)	(-10.54)	(-9.84)	(-13.16)
VIX_t	0.0015	−0.0013	0.0002	0.0009	0.0005
	(1.61)	(-4.05)	(1.32)	(3.57)	(4.80)
ES_{t-1}	0.0198	0.0202	0.0109	0.0033	0.0009
	(16.94)	(41.11)	(55.84)	(12.84)	(9.71)
adj. R^2	0.2194	0.6679	0.7528	0.5215	0.2784

Panel B: Puts.	DOTM	OTM	ATM	ITM	DITM
α	0.0179	0.0086	0.0039	0.0025	0.0031
	(11.95)	(5.82)	(9.23)	(6.50)	(10.25)
AGI_t	−0.0019	-0.0028	−0.0017	−0.0014	−0.0008
	(-5.07)	(-5.67)	(-10.40)	(-13.18)	(-10.24)
VIX_t	−0.0013	−0.0006	0.0001	0.0007	0.0001
	(-3.60)	(-2.96)	(0.95)	(3.43)	(1.29)
ES_{t-1}^B	0.0302	0.0193	0.0117	0.0039	0.0008
	(58.23)	(25.96)	(61.67)	(23.49)	(3.85)
adj. R^2	0.6626	0.7919	0.7871	0.5073	0.0907

- Effect is strongest for OTM options
- High R^2 for ATM options $\rightarrow$ highest Γ risk
- A one standard deviation decrease in AGI_t increases ES_t by 0.44% for OTM calls

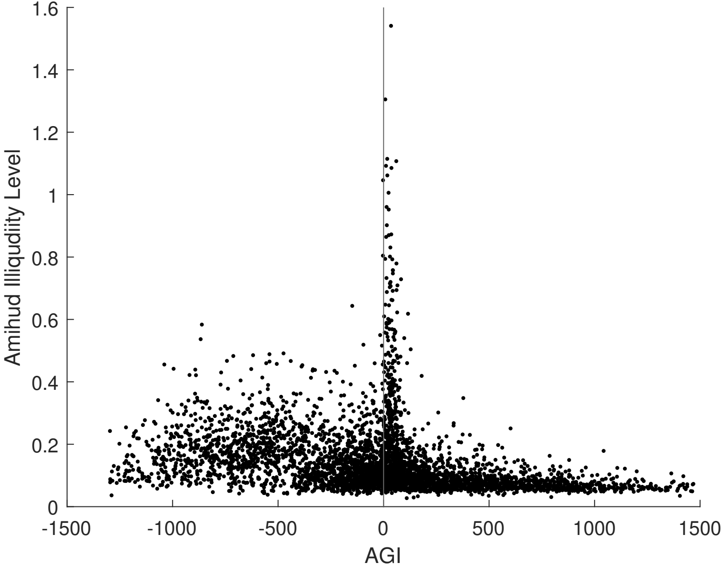
Panel regressions Illiquidity

$$ES_t^B = \alpha^B + \beta_1 AGI_t^B + \beta_2 ES_{t-1}^B + \beta_3 \mathrm{MI}_t^B + \beta_4 \mathrm{FI}_t^B + e_t^B \quad (5)$$

	ES_t			QS_t			$IVES_t$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
AGI_t	−0.0018	−0.0018	−0.0018	−0.0026	−0.0029	−0.0029	-0.0039	-0.0036	-0.0036
	(-2.50)	(-2.81)	(-2.67)	(-2.91)	(-3.14)	(-3.39)	(-3.57)	(-3.50)	(-3.58)
ES_{t-1}^B	0.0128	0.0127	0.0128						
	(3.84)	(3.80)	(3.81)						
QS_{t-1}^B				0.0258	0.0259	0.0259			
				(4.07)	(4.07)	(4.08)			
$IVES_{t-1}^B$							0.0073	0.0071	0.0071
							(8.67)	(8.40)	(8.46)
MI	−0.0003		−0.0003	−0.0006		0.0001	0.0007		0.0000
	(-0.94)		(-1.30)	(-0.99)		(0.19)	(3.77)		(-0.06)
FI		−0.0004	0.0000		−0.0017	−0.0018		0.0017	0.0018
		(-0.60)	(0.03)		(-1.62)	(-1.89)		(3.50)	(2.61)
within R^2	0.3380	0.3380	0.3380	0.4480	0.4490	0.4490	0.2240	0.2260	0.2260
Fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

- We include market and funding illiquidity (Amihud, 2002; Hu et al., 2013)
- $IVES_t$ outperform in terms of magnitude
- Not a phenomenon of illiquidity spillovers from underlying

Market Illiquidity vs. AGI_t



Probit model

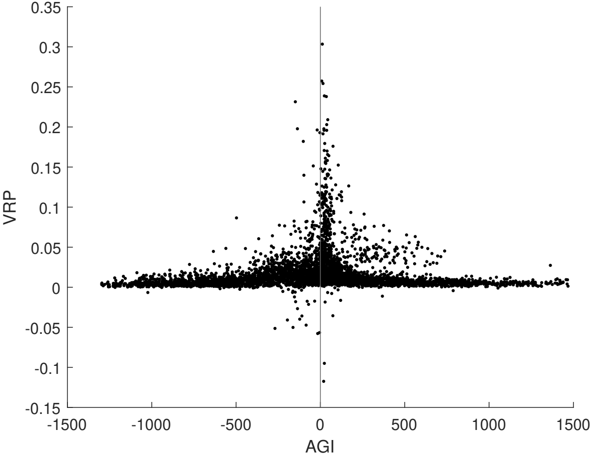
$$\mathbb{1}_t^{20} = \alpha + \beta_1 \text{MI}_t + \beta_2 \text{RV}_t + \beta_3 \text{HKM}_t + e_t \quad (6)$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	−1.2670	−1.5440	1.7640	−1.6200	1.0800	1.0970	0.8820
	(-31.73)	(-27.02)	(16.28)	(-29.89)	(10.40)	(8.05)	(7.37)
MI	3.0050			1.0330	2.2020		1.4800
	(15.38)			(4.08)	(7.93)		(4.57)
RV		5.4950		4.8880		2.2030	1.4200
		(13.52)		(10.69)		(7.14)	(4.00)
HKM			−4.5970		−3.8650	−3.9140	-3.6800
			(-21.42)		(-22.74)	(-18.15)	(-20.14)
adj. R^2	0.0608	0.1250	0.2690	0.1290	0.2840	0.2830	0.2880

Higher probability to end up in 20^{th} quantile of abs(AGI_t) if

- ... markets are more illiquid (Amihud, 2002)
- ... realized volatility is higher
- ... intermediaries are more constrained (they have lower financial health) (He et al., 2017)

VRP vs. AGI_t



- High VRP $\rightarrow$ high expensiveness $\rightarrow$ investor has incentive to sell existing positions (high prices) $\rightarrow$ no incentive to buy new long positions $\rightarrow$ leads to balanced inventory

OLS Option expensiveness

$$VRP_{t+1} = \alpha + \beta_1 \mathbb{1}_t^{20} + \beta_2 VRP_t + e_t \qquad (7)$$

	VRP_t	VRP_{t+1}	VRP_{t+1}	VIX_t	VIX_{t+1}	VIX_{t+1}
α	0.0124	0.0125	0.0021	0.1671	0.1672	0.0043
	(11.12)	(11.13)	(4.46)	(30.79)	(30.53)	(4.52)
$\mathbb{1}_t^{20}$	0.0261	0.0258	0.0038	0.1064	0.1058	0.0021
	(4.27)	(4.27)	(3.38)	(4.51)	(4.49)	(2.39)
VRP_t			0.8384			
			(23.82)			
VIX_t						0.9749
						(159.57)
adj. R^2	0.1657	0.1610	0.7473	0.2119	0.2097	0.9586

- Balanced abs(AGI_t) states are significantly related to increasing option expensiveness and VIX_t
- MM actively reduces its rebalancing needs in such states by obtaining a gamma-neutral inventory

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Conclusion

Signed gamma inventory matters.

- Large markups in spreads for states with negative $AGI_t \rightarrow$ option markets are more illiquid
- We rule out illiquidity spirals from the underlying as the economic force that drives our results
- Negative AGI_t represents sharp deviations from optimal inventory for which the MM wants compensation $\rightarrow$ higher spreads
- AGI of MM is balanced during turbulent times as indicated by higher illiquidity, elevated realized volatility, and low intermediary health

MM actively adjust option expensiveness to either...

- increase their compensation or
- to balance their gamma inventory in the desired direction

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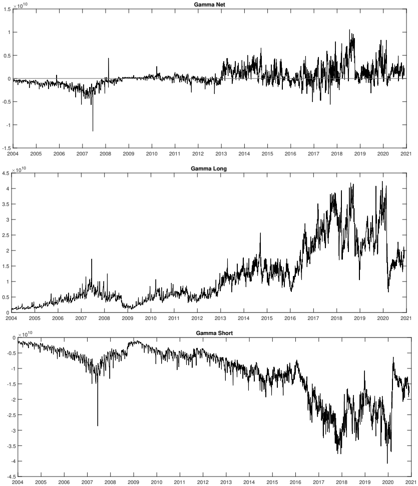
AGI (Long/short) in $ terms



Summary statistics net gamma and AGI_t

	Mean	Median	Min.	Max.	Std.	Skew.	ρ
Net gamma/$1e^{10}$	0.0294	0.0021	−1.1358	1.0571	0.1894	1.0897	0.8896
Net gamma long/$1e^{10}$	1.1462	0.7636	0.0839	4.2356	0.9353	1.0737	0.9892
Net gamma short/$1e^{10}$	−1.1169	−0.8527	−4.0788	−0.1001	0.8313	−1.0232	0.9870
$AGI_t/1e^3$	−0.0268	0.0056	−3.3875	2.3681	0.5481	0.1001	0.9192

Panel regressions

$$ES_t^B = \alpha^B + \beta_1 AGI_t^B + \beta_2 VIX_t^B + \beta_3 ES_{t-1}^B + e_t^B \qquad (8)$$

	ES_t			QS_t			$IVES_t$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
AGI_t	-0.0073	-0.0073	-0.0017	-0.0098	-0.0098	-0.0025	-0.0073	-0.0073	-0.0038
	(-4.31)	(-4.32)	(-2.52)	(-3.40)	(-3.41)	(-2.92)	(-6.71)	(-6.72)	(-3.48)
VIX_t		-0.0004	-0.0012		-0.0001	-0.0026		0.0002	-0.0002
		(-0.33)	(-1.98)		(-0.05)	(-2.34)		(0.19)	(-0.46)
ES_{t-1}^B			0.0128						
			(3.92)						
QS_{t-1}^B						0.0260			
						(4.17)			
$IVES_{t-1}^B$									0.0075
									(8.73)
within R^2	0.0976	0.0979	0.3400	0.0614	0.0614	0.4520	0.1120	0.1120	0.2220
Fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

- $IVES_t$ best in terms of magnitude & R^2 relative to controls
- $IVES_t \rightarrow$ moneyness consistent
- QS_t outperforms in terms of absolute R^2 $\rightarrow$ driven QS_{t-1}
- Results are unchanged when using predictive regressions

PREDICTIVE PANEL REGRESSIONS ES_t^B

$$ES_{t+1}^B = \alpha^B + \beta_1 AGI_t^B + \beta_2 VIX_t^B + \beta_3 ES_t^B + e_t^B \qquad (9)$$

	ES_{t+1}			QS_{t+1}			$IVES_{t+1}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
AGI_t	-0.0072	-0.0072	-0.0017	-0.0098	-0.0098	-0.0026	-0.0073	-0.0073	-0.0038
	(-4.29)	(-4.30)	(-2.44)	(-3.42)	(-3.43)	(-3.05)	(-6.72)	(-6.72)	(-3.41)
VIX_t		-0.0004	-0.0012		-0.0001	-0.0026		0.0002	-0.0002
		(-0.33)	(-1.98)		(-0.05)	(-2.49)		(0.18)	(-0.47)
ES_t^B			0.0128						
			(3.93)						
QS_t^B						0.0259			
						(4.18)			
$IVES_t^B$									0.0075
									(8.67)
within R^2	0.0962	0.0966	0.3400	0.0619	0.0619	0.4520	0.1120	0.1120	0.2220
Fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

- Same results as above

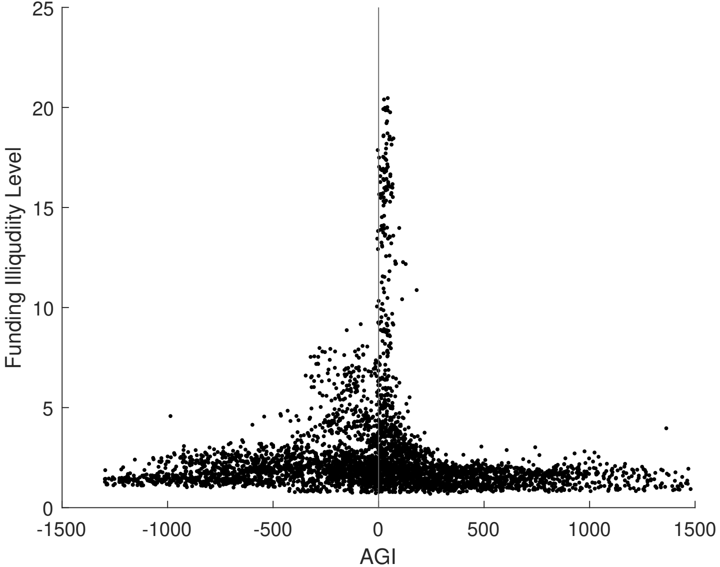
Predictive panel regressions Illiq

$$ES^B_{t+1} = \alpha^B + \beta_1 AGI^B_t + \beta_2 ES^B_t + \beta_3 \text{MI}^B_t + \beta_4 \text{FI}^B_t + e^B_t \quad (10)$$

	ES_{t+1}			QS_{t+1}			$IVES_{t+1}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
AGI_t	-0.0017	-0.0017	-0.0016	-0.0026	-0.0028	-0.0027	-0.0039	-0.0036	-0.0035
	(-2.43)	(-2.71)	(-2.54)	(-3.04)	(-3.09)	(-3.19)	(-3.51)	(-3.45)	(-3.53)
ES^B_t	0.0128	0.0127	0.0128						
	(3.85)	(3.79)	(3.81)						
QS^B_t				0.0258	0.0258	0.0258			
				(4.07)	(4.07)	(4.08)			
$IVES^B_t$							0.0073	0.0070	0.0071
							(8.61)	(8.32)	(8.38)
Market Illiq.	-0.0003		-0.0005	-0.0006		-0.0004	0.0007		-0.0003
	(-0.96)		(-2.25)	(-0.96)		(-0.69)	(3.89)		(-0.98)
Funding Illiq.		-0.0001	0.0005		-0.0010	-0.0005		0.0021	0.0024
		(-0.13)	(0.91)		(-1.11)	(-0.98)		(3.50)	(2.92)
adj. R^2	0.3370	0.3370	0.3380	0.4480	0.4480	0.4480	0.2230	0.2270	0.2270
Fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

- Same results as above

Funding Illiquidity vs. AGI_t



Robustness

	$IVES_t$				
	(1)	(2)	(3)	(4)	(5)
AGI_t	-0.0005	-0.0029	-0.0030	-0.0029	-0.0029
	(-2.09)	(-3.20)	(-3.28)	(-3.23)	(-3.27)
$IVES^B_{t-1}$		0.0049	0.0048	0.0047	0.0047
		(8.90)	(8.67)	(8.49)	(8.50)
$IVES^B_{t-2}$		0.0044	0.0043	0.0042	0.0043
		(9.49)	(9.31)	(9.40)	(9.41)
VIX_t		-0.0003			
		(-0.90)			
Market Illiq.			0.0004		0.0000
			(3.49)		(-0.16)
Funding Illiq.				0.0012	0.0012
				(3.16)	(2.37)
within R^2	0.0002	0.2570	0.2580	0.2590	0.2590
Entity FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	No	No	No	No