A Dynamic Model of Governmental Venture Capital

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- VC Industry: The incubator of innovation
  - The way of picking & growing early-stage R&D projects
  - Occasionally non-economic & socially pervasive impacts

- To amplify the effect of success, governments join VC industries
  - Governmental VC (GVC): a public entity as a VC itself
  - Sponsor or collaborate with private VCs (PVC)
  - Or support startups that do not receive capital from PVCs.

- Government as an equity investor is relatively rare in the US:
  - A policy tool that remains relatively understudied;
  - In contrast, China: a growth rate of 25% per annum (Li; 2022)

Previous Questions on GVC efficiency

- Previous questions in the literature
  1. Compared to PVCs, are they **efficient** in generating innovations?
  2. Do they **crowd in/out** private investments in the industry?

- The literature on GVC efficiency & outcomes
  1. Empirics: Conflicting observations & Lack of consensus
  2. Theories: Yet to provide explanations on empirical discrepancies

Lack of Consensus in Empirical Findings Decision

	Underperform	No evidence/Outperform
Sales growth	Grilli & Murtinu(2014a)	Lerner(1999) Grilli & Murtinu(2014b)
Inv.size	Cumming & McIntosh(2006) Brander et al.,(2010)	Brander et al.,(2015)
Exit rate/val	Brander et al.,(2010) Li (2022)	Brander et al.,(2015)
Innovation	Bertoni & Tykova(2015) (Under sole GVC)	Bertoni & Tykova(2015) (Under PVC syndication)

	Crowd Out	Crowd In/Augment
Private inv.	Brander et al.,(2010)	Guerini & Quas(2016) Lerner(1999), Howell(2017)

Research Questions

Introduction

1. How to understand the discrepancies & build the mechanism beneath?

2. If inefficiencies exist, then under which conditions do they arise?

Roadmap & Takeaways

1. The black-box model of general VC-financing:
   - VC-financing maximizes the sum of its **ongoing**/active entities' values;
   - No matter how intertwined & conflicting interests they have.
2. The individual startups under different private-public partnerships:
   - Under **active** GVC roles ⇒ Social optimum
   - Under **passive** GVC roles ⇒ Private optimum (as under PVC)
3. The industry-level effects of GVC through the two channels:
   - The lowered **entry** hurdle
   - The relaxed project **termination** threshold ⇒ Prolonged lifespan

- Inefficiencies are driven by the **misalignment of incentives**.

1. Admati and Pfleiderer (1994)
    - Entrepreneurs (want to prolong the project lifespan) $\neq$ Investors
    - Can be aligned through a **fixed** ownership allocation contract.
2. Inderst and Muller (2004)
    - VCs' bargaining powers $\Rightarrow$ Imbalanced ownership allocation/contract
    - A **fixed** contract: No room for renegotiation
    - Determines investment decisions afterwards $\Rightarrow$ Suboptimal outcomes

Different incentives $\rightarrow$ Conflicting investment decisions $\rightarrow$ Inefficiency

Resolving Misalignments: An Example

The model of general VC Financing

- Question: Does a startup operate in such ways?
  - Conflicting interests destroy values $\Rightarrow$ The agents would try to **resolve**
  - E.g., through **renegotiations** on their ownership allocations

An interview of a startup founder in South Korea

"(...) There were some moments we wanted to ***abandon*** *this project. At those times, the primary* ***investor*** *of our business ever since its launching encouraged us to push it further, promising to* ***yield more shares*** *(...)"*

Dynamic Adjustment of Misalignments Example

The model of general VC Financing

1. In a dynamic world:
   - Projects/startups have ups & downs (random states)
   - Conflicts/Misalignments emerge when the project state goes bad.
2. $\Rightarrow$ The entities negotiate whenever the conflicts are on the surface:
   - **Conceding** some rights/equity shares to another
3. $\Rightarrow$ **Gradual** adjustment over time:
   - Proceeds until both entities' incentives **align**.
4. $\therefore$ Misalignment $\nRightarrow$ Ultimate investment decisions $\Rightarrow$ **Optimum**
   - I.e., the startup's operation maximizes the sum of its entities' values.

What Does It Have to Do with GVCs?

A new phase with GVCs and PVCs

- When private & public entities are in a startup:
  - Different goals (economic vs social): potentially conflicting

- Misalignments are irrelevant ⇒ A startup's operation maximizes either
  1. The **private** (or **financial**) value
  2. The **social welfare** (financial + **non-financial**)

- I.e., the '**optimum**' a startup reaches may differ according to which participants comprise it.

- Question: Under which conditions do **GVC-backed** startups serve different optimum?

Different Outcomes under Different GVC Roles

A new phase with GVCs and PVCs

- Whether a GVC joins a startup's **ongoing** investment determines its investment choice and performance.

1. If GVC collaborates with PVC in a **passive** manner:

   GVC interests $\not\Rightarrow$ Firm operation $\Rightarrow$ Private value maximization

   $\Rightarrow$ Economic outcomes **equal** to PVC-funded cases.

2. If GVC joins as an **active** investor:

   GVC interests $\Rightarrow$ Firm operation $\Rightarrow$ Serves public/nonfinancial goals

   $\Rightarrow$ Relative to PVC-funded, **underperforming** financial outcomes.

Consistent Past Literature Intuitions

A new phase with GVCs and PVCs

Brander, Du, & Hellmann (2015)

"GVCs may be helpful in providing certain kinds of support, including financial support, but may become ***less useful*** *when they have actual control over* ***business decisions****."*

- Agents
  1. Entrepreneur (ENT): idea provider, private entity
  2. PVC: capital provider, private entity
  3. GVC: capital provider, public entity

- Key assumptions
  1. A project produces the **financial** (θ) & **non-financial** (ϕ) exit values.
  2. GVC is the only entity considering ϕ into its utility.
  3. PVC and GVC are identically efficient in their operation.

Three Cases of VC Financing
The model

1. Pure PVC financing (A benchmark) Pure PVC
2. Pure GVC financing with GVC as an **active** investor Pure GVC
3. GVC-PVC Syndication case with GVC as a **passive** seed investor Mixed

- R&D begins with an idea, not knowing when it will mature.
- Shaping the idea takes time, continuous effort & monitoring.
- When VC-backed, multiple entities contribute **complementary** efforts.
- These entities observe its progress every moment.

- The project's status ($X_t \leq 0$) evolves over time:

$$dX_t = \mu dt + \sigma dB_t, \quad X_0 = x < 0$$

and completes when $X_t \geq 0$ for the first time.

- A project is launched at $t = 0$ only if its $X_0 = x \geq h$:
  - The minimum initial quality to have VCs' NPVs ≥ 0.

Each Entity's Problems & Decisions Pure PVC Pure GVC Mixed

The model

1. The PV-maximizing **cutoff** for stop putting effort:

$$a\left(\frac{\text{The Exit Payoff}}{\text{The Rate of Cost Each Instant}}\right)$$

   - Abandon the project when $X_t < a$ for the first time
   - Not the same across entities (**Misalignment**)

2. The **renegotiation** policy over equity shares:
   - To prevent the pre-matured termination of the project
   - How much equity share it could concede under which conditions

Renegotiation Process in Nash Equilibrium Concessions

Whenever X_t falls to one party's current $a(\cdot)$, the other entity concedes its equity shares to him to incentivize not to abandon the project.

The Equilibrium Investment Policy

- The entities abandon only when everyone agrees to do so.
- Thus, the firm's $a(\cdot)$ ($\Rightarrow PV$) is either at private or social optimum

- I.e., the project terminates only when their misalignment is no more

$$a\left(\frac{v_e(\alpha_t)}{\gamma}\right) = a\left(\frac{v - v_e(\alpha_t)}{1-\gamma}\right) = a(v)$$

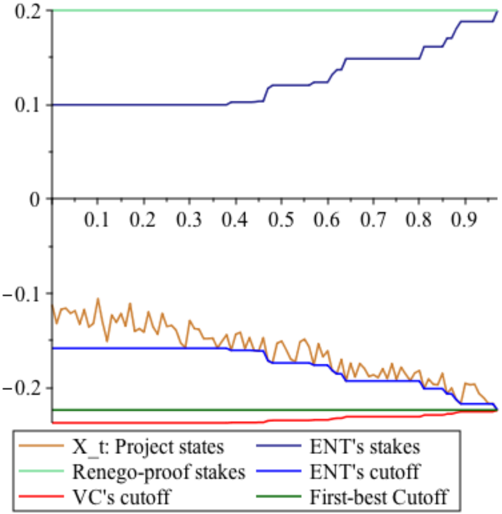
where

$$v \in \left\{ \underbrace{\theta}_{\text{Private Exit Payoff}}, \underbrace{\theta + \phi}_{\text{Social Exit Payoff}} \right\}$$

depending on who are the ongoing/active investors.

Example: The Gradual Adjustment of Misalignment

Black Box



1. Termination cutoffs:

$$\underbrace{a^{pvc} = a^{mix} = a(\theta)}_{\text{Private Optimum}} > a^{gvc} = \max\left\{ \underbrace{a(\theta + \phi)}_{\text{Social Optimum}}, a\left(\frac{\theta}{\gamma}\right) \right\}$$

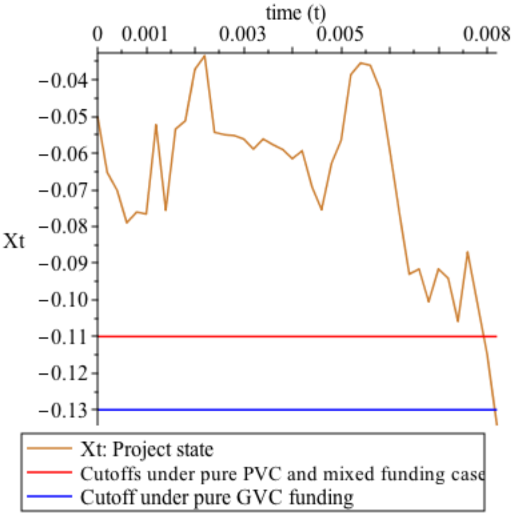
2. Project lifespans:

$$E\left[S\left[a^{pvc}\right] \wedge \tau\right] = E^x\left[S\left[a^{mix}\right] \wedge \tau\right] < E^x\left[S\left[a^{gvc}\right] \wedge \tau\right]$$

3. Financial NPVs and Innovation Literature :
   - **Maximum** private/financial value: pure-PVC & mixed funding Private (Lerner; 1999, GM; 2014b, BDH; 2015, BT; 2015)
   - **Lower** under pure-GVC funding Social (CM; 2006, GM; 2014a, BEH; 2010, Li; 2022, BT; 2015)

Example: Cutoffs & Lifespans

The Results



1. The minimum level of initial project quality [Funding Hurdle]

$$h^{gvc} < h^{mix} < h^{pvc}$$

2. The average initial quality of VC-backed firms in the market:

$$E\left(x|h^{gvc}\right) < E\left(x|h^{mix}\right) < E\left(x|h^{pvc}\right)$$

3. The average failure rates in the market (Brander et al., 2010, 2015):

$$E\left[\pi_a\left(x;a\right)|h\right]^{gvc} > E\left[\pi_a\left(x;a\right)|h\right]^{mix} > E\left[\pi_a\left(x;a\right)|h\right]^{pvc}$$

provided that the density of x is sufficiently right-skewed.

Industry-Level Qualities & Performances

The Results

1. The average return rates **conditional** on success (Pierrakis & Saridakis; 2017):

$$E\left[RoR_{\tau}(x;a)|\,h\right]^{gvc} < E\left[RoR_{\tau}(x;a)|\,h\right]^{mix} < E\left[RoR_{\tau}(x;a)|\,h\right]^{pvc}$$

   due to

   1. A longer lifespan ($\Rightarrow$ more costs);
   2. More relaxed entry condition ($\Rightarrow$ lower qualities)

2. Reverted orders for **individual unconditional** mean RoR:

$$RoR\left(x;a^{pvc}\right) = RoR\left(x;a^{mix}\right) < RoR\left(x;a^{gvc}\right)$$

   due to a longer lifespan ($\Rightarrow$ lower individual failure rate).

Equity Allocations

The Results

- Under pure PVC and mixed funding, ENT's share over time Equity Path

$$\alpha_t^{pvc},\ \alpha_t^{mix} \rightarrow \gamma$$

where γ is the fraction of ongoing costs to ENT.

- **Passive** GVCs' entry do not change the ownership allocations.
$\Rightarrow$ **Crowd in** PVC activities (Lerner; 1999)

Equity Allocations

The Results

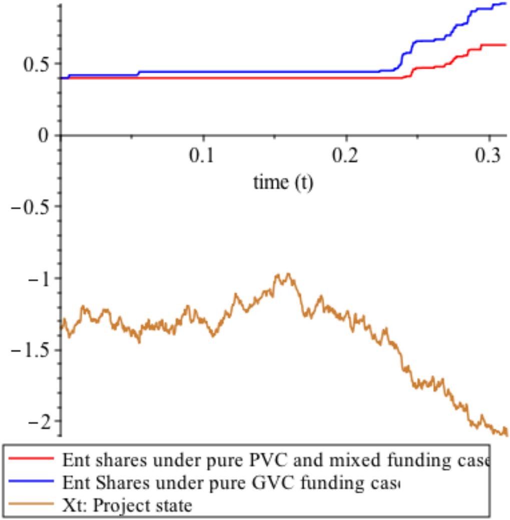
- Under pure GVC funding, ENT's share over time Equity Path

$$\alpha_t^{gvc} \rightarrow \min\left\{1, \gamma\left(1 + \frac{\phi}{\theta}\right)\right\} > \gamma$$

- **Active** GVCs provide more founder-friendly contract terms.
  $\Rightarrow$ **Crowd out** PVC activities (Brander, Egan, & Hellman; 2010)

Example: An Entrepreneur's Share over Time

The Results



1. How a general VC-financing operates:
   - Maximizes the sum of its ongoing investors' values;
   - Hence, the public-private partnership determines the rest outcomes
2. For individual startups:
   - Active GVC roles $\Rightarrow$ SW-maximization; economically suboptimal
   - Passive GVC roles $\Rightarrow$ PV-maximization; Identical to PVC-funding cases
3. GVC participation affects the industry through
   - A relaxed entry hurdle;
   - A prolonged project lifespan (only when it is actively involved)

Summary: The Paper's Contribution
Conclusion

1. Theoretical ground on the previous literature's insights:
    - Lerner (1999); BDH (2015); Bertoni & Tykova(2015)
2. Identifies the key source of the mixed outcomes in empirical studies:
    - The public-private partnership structures within startups
3. The dynamic model gives straightforward results:
    - The misaligning incentives are irrelevant to the firm's operation
      $\Rightarrow$ Straight access to analyzing its outcomes

$$\text{Ent:} \quad \max_{T^e, D_t} E^x \left[\underbrace{1_{\{\tau < T^e \wedge T^{pvc}\}} e^{-r\tau} \alpha_t \theta}_{\text{completion payoff}} - \underbrace{\int_0^{T^e \wedge T^{pvc} \wedge \tau} \gamma e^{-rt} dt}_{\text{required costs}} \right]$$

$$\text{PVC:} \quad \max_{T^{pvc}, U_t} E^x \left[\begin{array}{c} 1_{\{\tau < T^e \wedge T^{pvc}\}} e^{-r\tau} \left(1 - \alpha_t\right) \theta \\ - \int_0^{T^e \wedge T^{pvc} \wedge \tau} \left(1 - \gamma\right) e^{-rt} dt \end{array} \right] - I$$

subject to

$$\alpha_t = \kappa + U_t - D_t$$

$$T^j = S\left[a^j\right] := \inf\left\{t \,\middle|\, X_t < a^j\right\} \quad (j \in \{e, pvc\})$$

$$\text{Ent:} \quad \max_{T^e, D_t} E^x \left[\underbrace{1_{\{\tau < T^e \wedge T^{gvc}\}} e^{-r\tau} \alpha_t \theta}_{\text{completion payoff}} - \underbrace{\int_0^{T^e \wedge T^{gvc} \wedge \tau} \gamma e^{-rt} dt}_{\text{required costs}} \right]$$

$$\text{GVC:} \quad \max_{T^{gvc}, U_t} E^x \left[\begin{array}{c} 1_{\{\tau < T^e \wedge T^{gvc}\}} e^{-r\tau} \left[(1 - \alpha_t) \theta + \phi \right] \\ - \int_0^{T^e \wedge T^{gvc} \wedge \tau} (1 - \gamma) e^{-rt} dt \end{array} \right] - I$$

$$T^j = S\left[a^j\right] := \inf\left\{ t \,\middle|\, X_t < a^j \right\} \quad (j \in \{e, gvc\})$$

$$\text{Ent:} \quad \max_{T^e, D_t} E^x \left[\underbrace{1_{\{\tau < T^e \wedge T^{pvc}\}} e^{-r\tau} \alpha_t \theta}_{\text{completion payoff}} - \underbrace{\int_0^{T^e \wedge T^{pvc} \wedge \tau} \gamma e^{-rt} dt}_{\text{required costs}} \right]$$

$$\text{PVC:} \quad \max_{T^{pvc}, U_t} E^x \left[\begin{array}{c} 1_{\{\tau < T^e \wedge T^{pvc}\}} e^{-r\tau} \left(1 - \alpha_t\right) \theta \\ - \int_0^{T^e \wedge T^{pvc} \wedge \tau} \left(1 - \gamma\right) e^{-rt} dt \end{array} \right] - kI$$

$$\text{GVC:} \; E^x \left[1_{\tau < T^e \wedge T^{pvc}} e^{-r\tau} \phi \right] - \left(1 - k\right) I$$

1. Given the equity allocation at t, the misalignment:

$$\underbrace{a\left(\frac{v_e\left(\alpha_{t-}\right)}{\gamma}\right)}_{ENT's} > \underbrace{a\left(\frac{v - v_e\left(\alpha_{t-}\right)}{1-\gamma}\right)}_{VC's}$$

2. When the project state goes bad, conflict emerge onto the surface:

$$a\left(\frac{v - v_e\left(\alpha_{t-}\right)}{1-\gamma}\right) < X_t < a\left(\frac{v_e\left(\alpha_{t-}\right)}{\gamma}\right)$$

3. One party concedes its shares to the other up to

$$\alpha_t > \alpha_{t-} \Rightarrow X_t = a\left(\frac{v_e\left(\alpha_t\right)}{\gamma}\right)$$

4. A new contract term α_t remains until X_t hits one party's new cutoff.

$$X_{T^*} = S\left[a\left(\frac{(1-\alpha_{T^*})\,\theta}{1-\gamma}\right)\right] = S\left[a\left(\frac{\alpha_{T^*}\theta}{\gamma}\right)\right] = S\left[a\left(\theta\right)\right] \Longleftrightarrow \alpha_{T^*} = \gamma$$

$$\text{Financial: } E^x\left[1_{\tau<S[a(\theta)]}e^{-r\tau}\theta - \int_0^{S[a(\theta)]\wedge\tau} e^{-rt}dt\right] = PV\left(x,1,\theta\right)$$

$$\text{Social: } E^x\left[1_{\tau<S[a(\theta)]}e^{-r\tau}\left(\theta+\phi\right) - \int_0^{S[a(\theta)]\wedge\tau} e^{-rt}dt\right] < PV\left(x,1,\theta+\phi\right)$$

$$X_{T^*} = S\left[a\left(\frac{(1-\alpha_{T^*})\theta+\phi}{1-\gamma}\right)\right] = S\left[a\left(\frac{\alpha_{T^*}\theta}{\gamma}\right)\right] = S\left[a\left(\theta+\phi\right)\right]$$

$$\iff \alpha_{T^*} = \gamma\left(1+\frac{\phi}{\theta}\right)$$

$$\text{Financial: } E^x\left[1_{\tau<S[a(\theta+\phi)]}e^{-r\tau}\theta - \int_0^{S[a(\theta+\phi)]\wedge\tau} e^{-rt}dt\right] < PV\left(x,1,\theta\right)$$

$$\text{Social: } E^x\left[1_{\tau<S[a(\theta+\phi)]}e^{-r\tau}\left(\theta+\phi\right) - \int_0^{S[a(\theta+\phi)]\wedge\tau} e^{-rt}dt\right]$$
$$= PV\left(x,1,\theta+\phi\right)$$