The Unattractiveness of Indeterminate Dynamic Equilibria

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Indeterminate dynamic equilibria

- Macroeconomic forces often generate multiple equilibria
- Non-convexity or complementarity only locally powerful
- Multiple steady states and regions of local indeterminacy
- Set of REE can be very large, especially in a stochastic environment
- Many papers focus on REE that are perturbations of perfect foresight paths
- We resolve multiplicity by appealing to learnability

Results

1. Such environments have a learnable REE
2. The properties of the learnable REE do not resemble simple perturbations around convergent perfect foresight paths
3. An econometrician could improperly infer that equilibrium is characterised by convergent behaviour perturbed by sunspot shocks

Related literature

Indeterminacy under rational expectations

Benhabib and Farmer 1994, Cazzavillan et al. 1998, Clarida et al. 2000, Lubik and Schorfheide 2004, Benhabib et al. 2001, Christiano et al 2018, Eggertson et al. 2019, Huo and Ríos-Rull 2013, Kaplan and Menzio 2016, Eeckhout and Lindenlaub 2019, Fernández-Villaverde et al. 2020, Galí 2015, Greiner and Bondarev 2017, Manzano and Vives 2011, Mäler et al. 2003, Krugman 1991, Angeletos et al 2019

Learning in macroeconomics

Lucas 1986, Woodford (1990), Evans and Honkapohja 2001, McCallum 2007, Ellison and Pearlman 2011, Hommes and Sorger 1998, Bullard 1994, Berardi and Duffy 2015

"In the recursive learning literature, it has generally been difficult to obtain expectational stability of sunspot equilibria", Arifovic et al. (2013)

Neural networks in macroeconomics

Maliar at al 2019, Azinovic et al. 2019, Fernández-Villaverde et al. 2019, Villa and Valaitis 2019

New Keynesian Framework

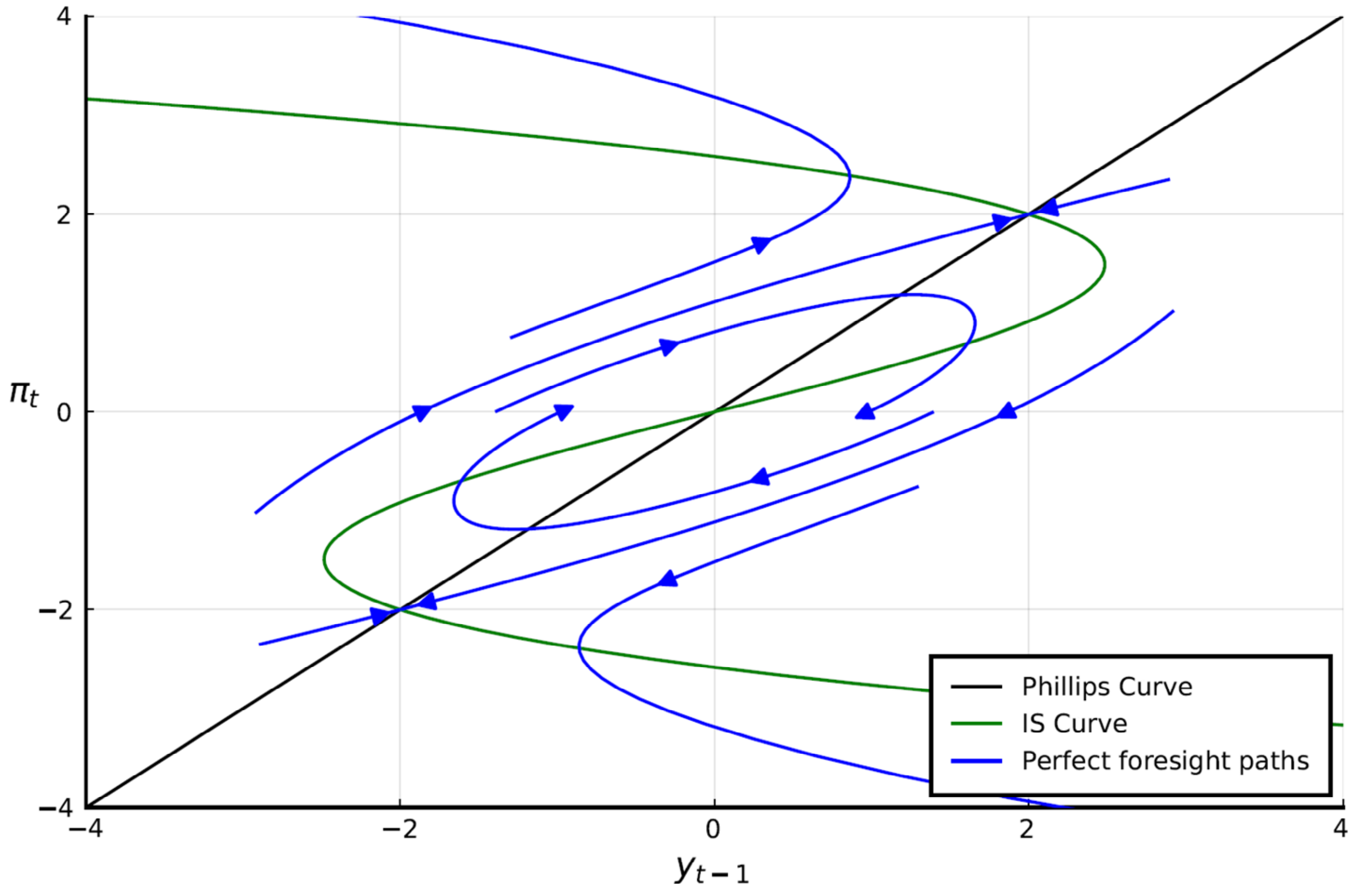
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \epsilon_{\pi,t}$$

$$y_t = \eta y_{t-1} - \sigma(r_t - E_t \pi_{t+1}) + \epsilon_{y,t}$$

$$r_t = \phi_\pi \pi_t + \alpha \pi_t^3$$

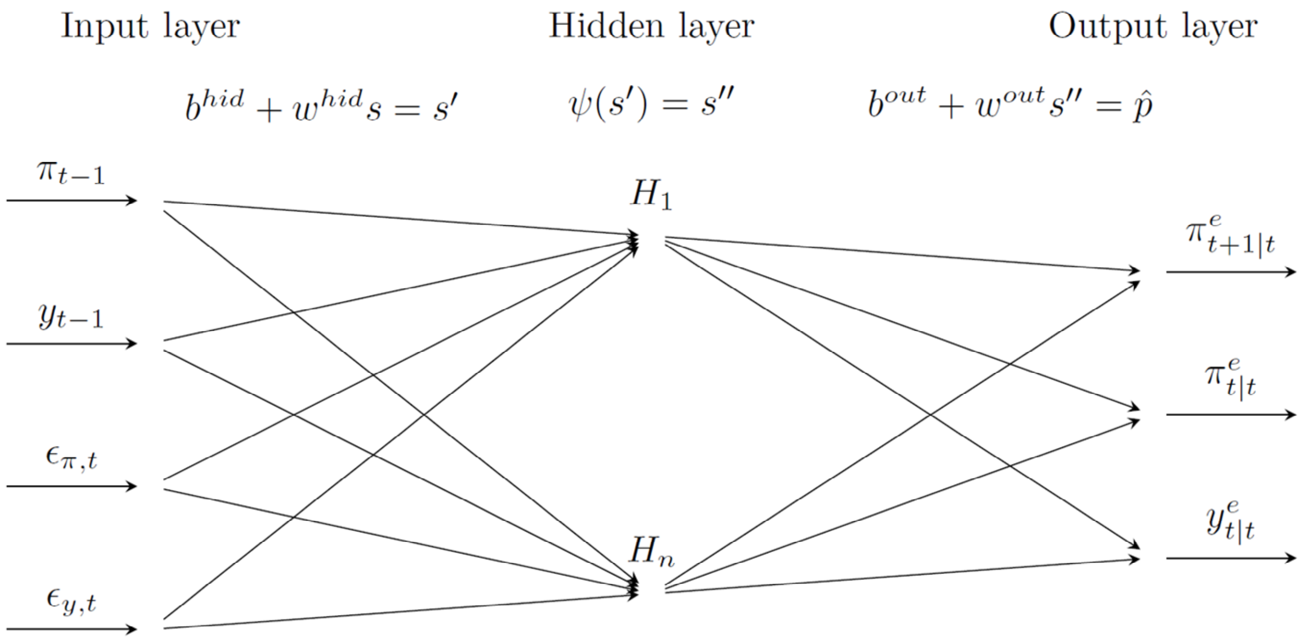
- Taylor Rule $\phi_\pi < 1$ and $\alpha > 0$
- Three deterministic steady states at (-2,-2), (0,0) and (2,2)
- (0,0) is complex sink with multiple convergent perfect-foresight paths

Perfect foresight solution



Neural network learning

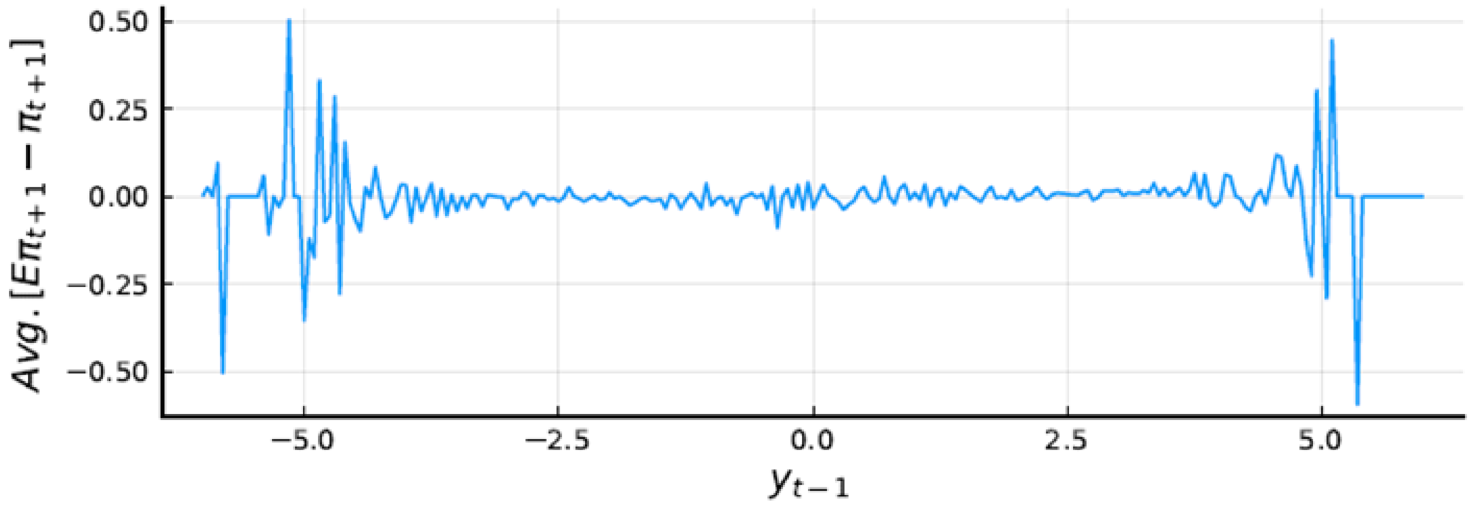
- Agents learn by updating a neural network



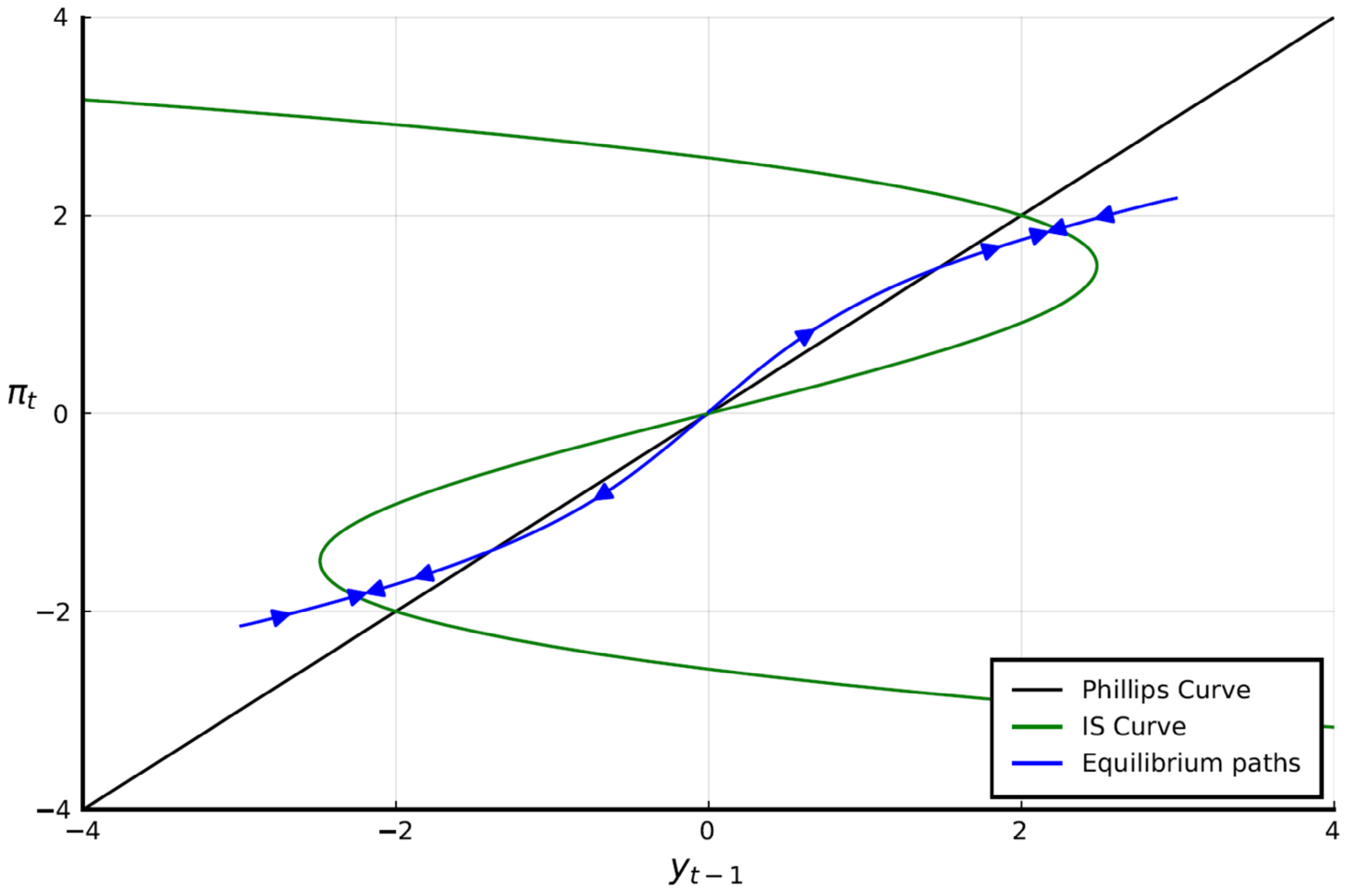
- Properties of REE verified by Den Haan and Marcet 1994 accuracy test

Verifying REE with neural network learning

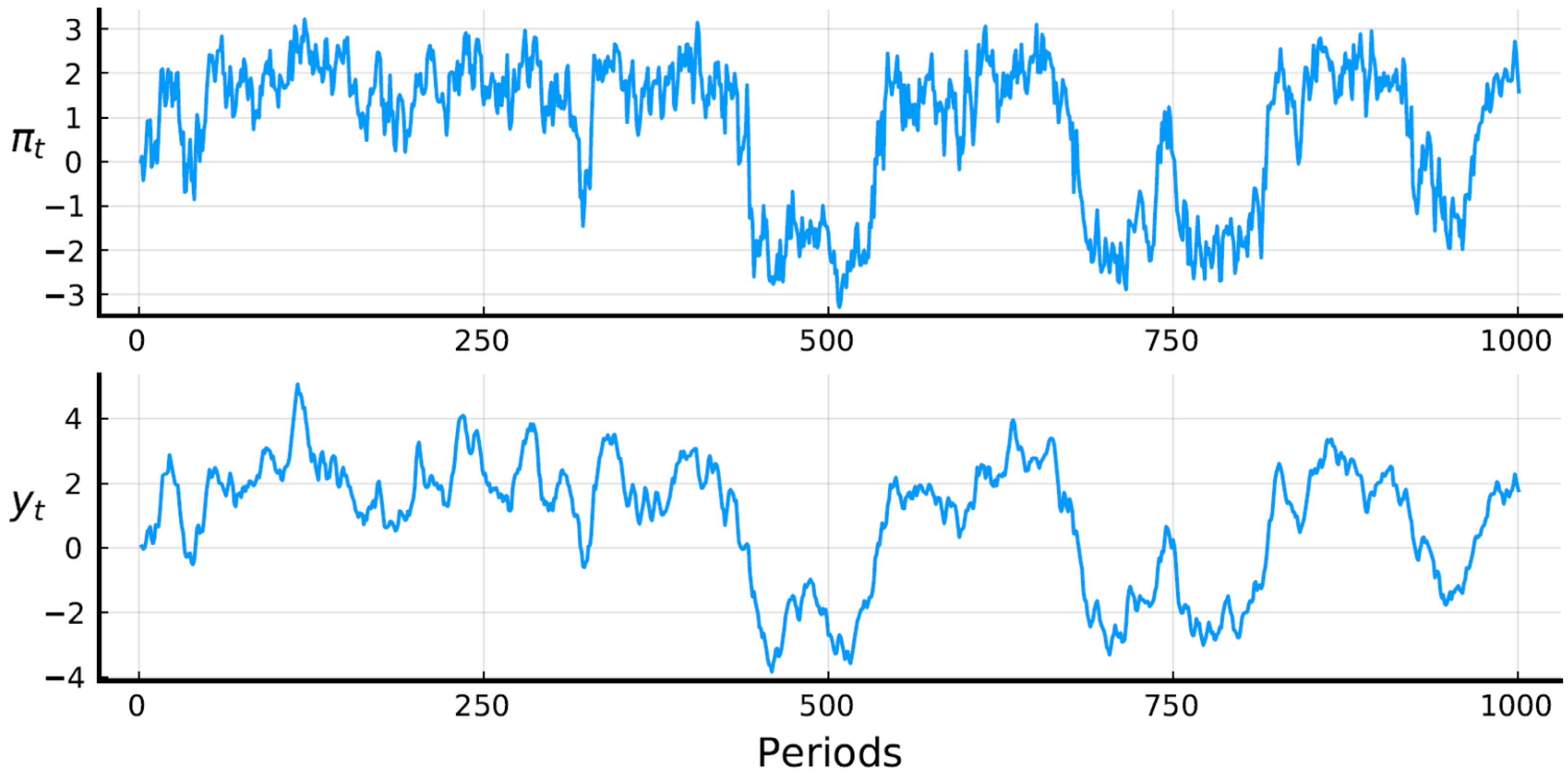
$h(s_t)$	Lower-tail	Upper-tail
Constant	0.0461	0.0520
Constant, y_t and π_t	0.0420	0.0635
Extensive	0.0384	0.0781



REE under neural network learning



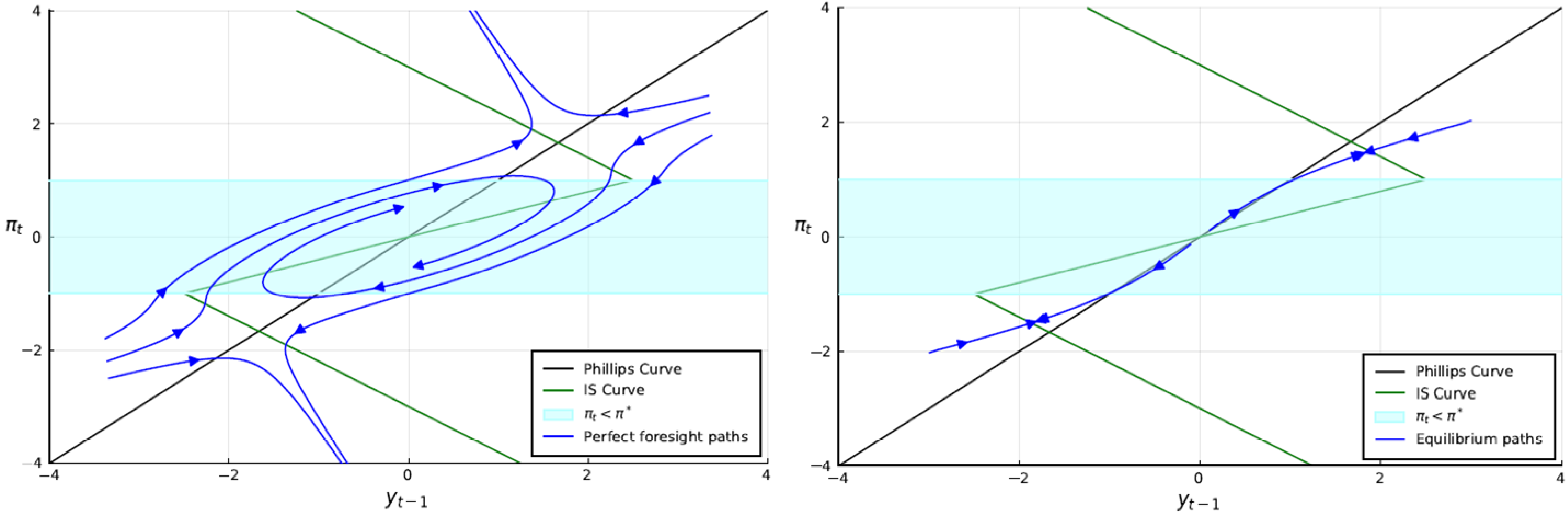
Characteristics of REE



Greater linearity

$$r_t = \begin{cases} \phi_\pi \pi_t & -\pi^* \geq \pi_t \geq \pi^* \\ \phi_\pi \pi_t + \alpha(\pi_t - \pi^*) & \pi_t > \pi^* \\ \phi_\pi \pi_t - \alpha(\pi_t - \pi^*) & \pi_t < -\pi^* \end{cases}$$

Greater linearity



(a) Perfect foresight

(b) Neural network learning

Adaptive learning

Perceived laws of motion (PLMs)

$$\pi_t = \beta_{\pi,1} + \beta_{\pi,2} y_{t-1} + \beta_{\pi,3} \pi_{t-1} + \beta_{\pi,4} y_{t-1}^2 + \beta_{\pi,5} y_{t-1}^3 + \upsilon_{\pi,t}$$

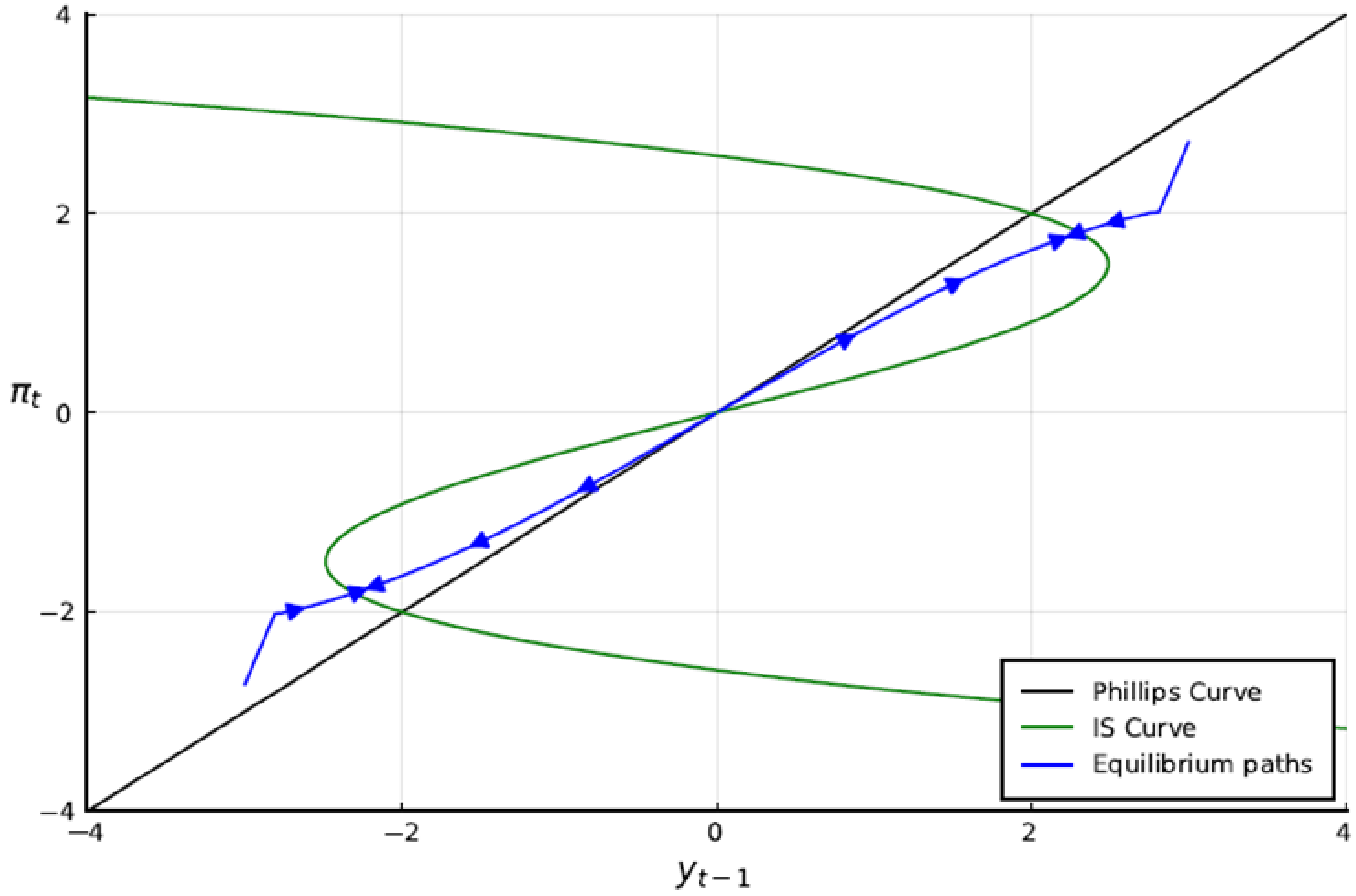
$$y_t = \beta_{y,1} + \beta_{y,2} y_{t-1} + \beta_{y,3} \pi_{t-1} + \beta_{y,4} y_{t-1}^2 + \beta_{y,5} y_{t-1}^3 + \upsilon_{y,t}$$

Least squares learning

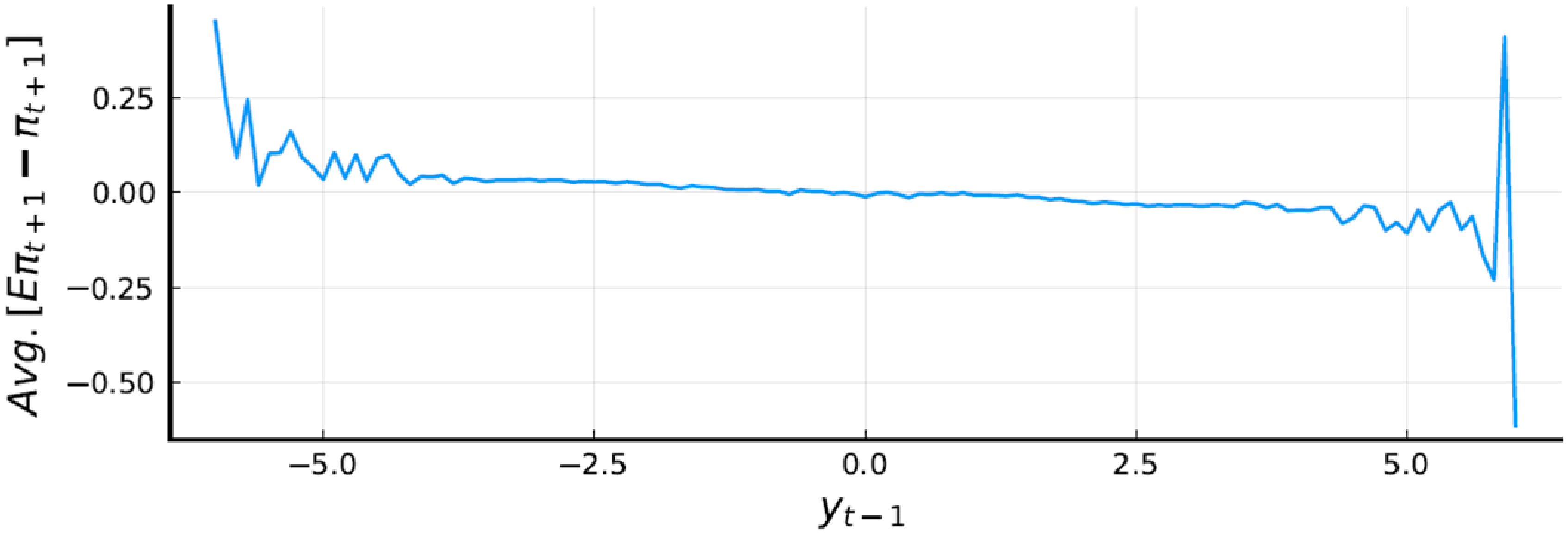
$$\beta_t = \beta_{t-1} + \frac{1}{t} R_{t-1}^{-1} X_t' (Y_t - X_t \beta_t)$$

$$R_t = R_{t-1} + \frac{1}{t} (X_t' X_t - R_t)$$

Adaptive learning



Not quite REE with adaptive learning



Testing for indeterminacy and sunspots

- Simulate data under converged neural network learning
- Is data generated by a locally determinate system?
- Or perturbations of perfect foresight paths in a locally indeterminate system?
- Lubik and Schorfheide 2004 test
- Based on log-linearised dynamics around a steady state
- Sunspots to perturbations of perfect foresight paths as in Farmer et al 2005

Results

Parameter	Prior mean	Posterior mean Determinate	Posterior mean Indeterminate
β	$\mathcal{N}(1,1)$	-0.34 $(-0.61,0.09)$	1.06 (0.97,1.12)
κ	$\mathcal{N}(0,1)$	1.02 (0.83,1.23)	-0.03 $(-0.08,0.04)$
η	$\mathcal{N}(1,1)$	0.95 (0.89,1.03)	0.85 (0.82,0.88)
σ	$\mathcal{N}(0,1)$	-0.03 $(-0.36,0.20)$	0.41 (0.32,0.52)
ϕ_π	$\mathcal{N}(2,1)$	0.57 $(-0.32,1.18)$	0.55 (0.42,0.70)
ρ_π	$\mathcal{N}(0,1)$	0.59 (0.58,0.60)	0.45 (0.41,0.49)
ρ_y	$\mathcal{N}(0,1)$	0.48 (0.46,0.50)	0.50 (0.48,0.51)
σ_π	$\mathcal{IG}(0.5,2)$	0.48 (0.42,0.55)	0.25 (0.21,0.28)
σ_y	$\mathcal{IG}(0.5,2)$	0.21 (0.18,0.26)	0.17 (0.17,0.18)
σ_ζ	$\mathcal{IG}(0.5,2)$	-	0.39 (0.37,0.42)
$\omega_{\pi,\zeta}$	$\mathcal{B}(0,0.3,-1,1)$	-	0.70 (0.61,0.82)
$\omega_{y,\zeta}$	$\mathcal{B}(0,0.3,-1,1)$	-	0.51 (0.43,0.60)
Log data density		-3927	-3889

Clear preference for indeterminate model

Results

1. Such environments have a learnable REE
2. The properties of the learnable REE do not resemble simple perturbations around convergent perfect foresight paths
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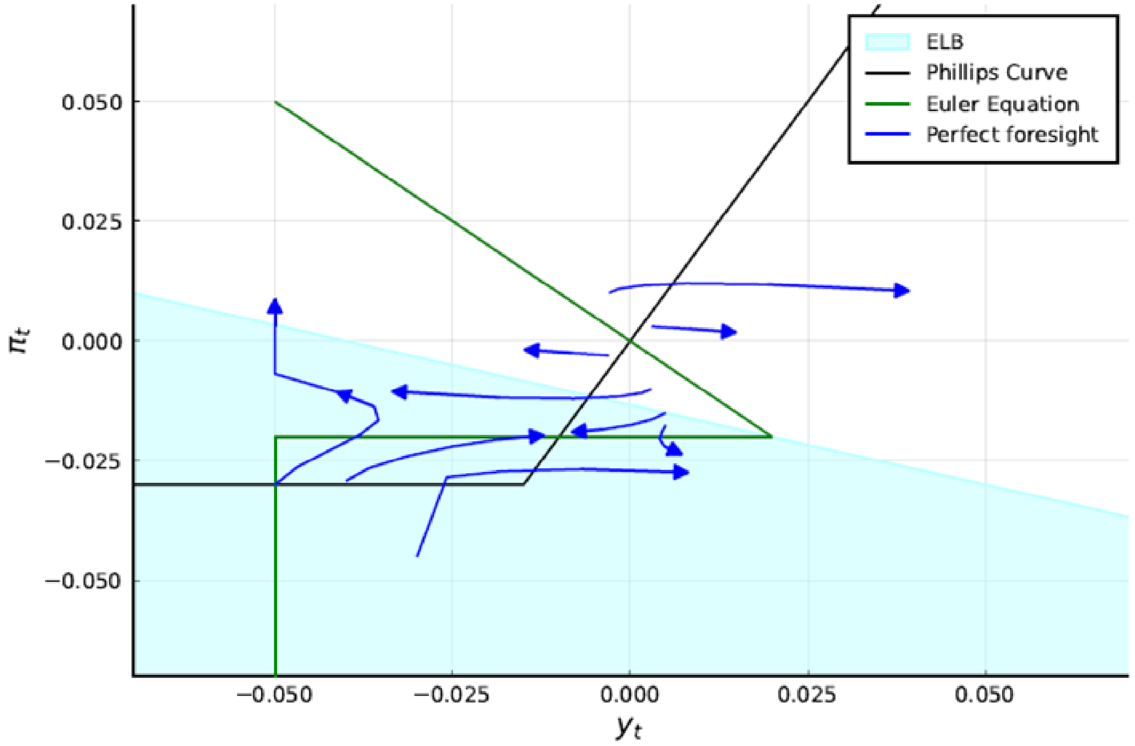
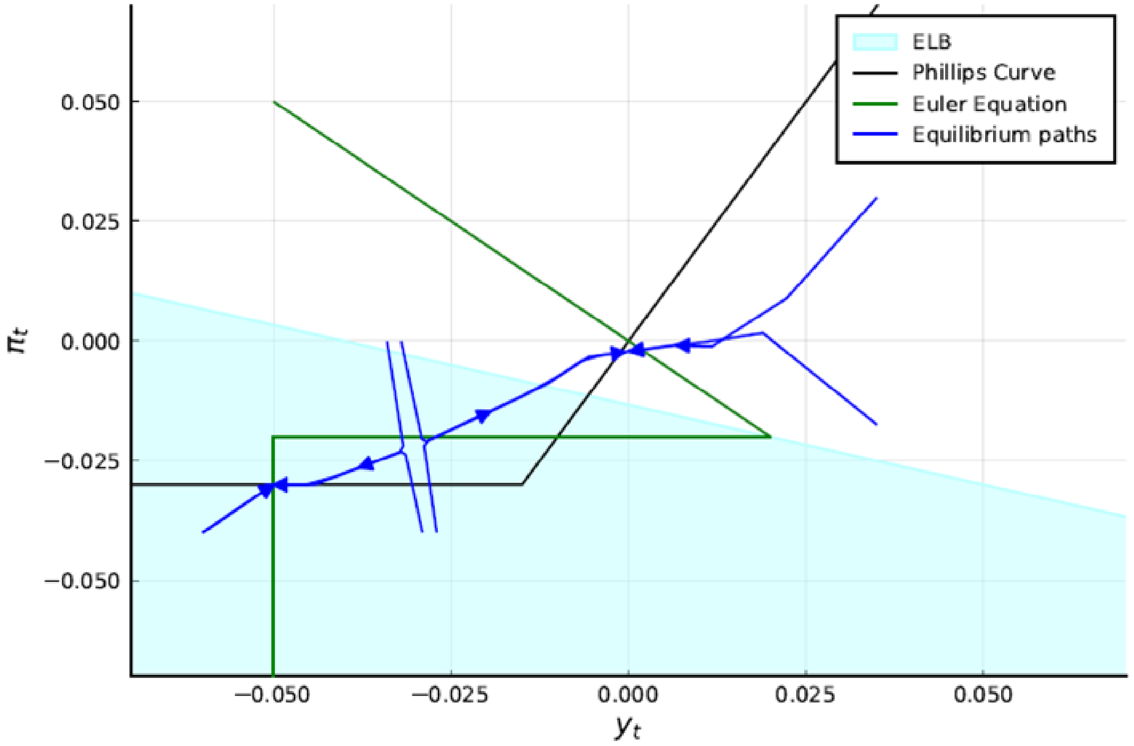
ZLB application of Evans, Honkapohja and Mitra 2020



(a) Perfect foresight



(b) Neural Network Learning