# Crypto-CAPM: The Role of Speculative and Fundamental Demand in Cryptocurrency Pricing

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# Overview:

- What drives the price of cryptocurrencies?
- The answer is not straightforward:
  - There is no tangible fundamental supporting cryptocurrencies
  - Traditional asset pricing models (such as FF 5- and 3-factor models) fail to explain the price dynamics of cryptocurrencies (Liu and Tsyvinsky (2021)).
- Two strands of literature try to address the above question.
- One strand determines heterogenous belief and speculation as the driver of cryptocurrency prices (Sockin and Xiong (2020))
- Another strand identifies transactional benefits (convenience yield) as a determinant of cryptocurrency prices (Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2020))

# Literature Gap:

- In this paper, we offer a general equilibrium model which brings these two strands under the same roof.
- Why we need a new approach?
  - Previous studies focus on either belief heterogeneity or transactional benefit separately
  - Their approaches overlook an important channel as described below:
    - Optimism (positive sentiment) toward a crypto asset induces investment.
    - As more investors join the platform, it’s easier to find a transaction counterparty (network effect).
    - Transactional benefits pin down the impact of belief heterogeneity through affecting demand.
    - Thus, there is a bilateral relationship between optimism and transactional benefits.
- In recent studies, transactional benefits (and users’ type) are exogenous.

# Contribution-1:

- We endogenize transactional benefits

- We identify three priced components in each crypto asset:
  i) Systematic exposure to the crypto market portfolio.
  ii) Belief heterogeneity.
  iii) Transactional benefits.

- Our framework provides several interesting insights:
  - We derive an ”optimism coefficient”, that quantifies the magnitude of belief heterogeneity in each crypto asset.
  - We find a bilateral relationship between belief dispersion and transactional benefits.
  - We demonstrate that in boom episodes, over-optimism destabilizes crypto market and might lead to crash.
  - We show that cryptocurrency market specific factors such as “momentum” and “attention” can be explained through the lens of belief dispersion.

# Contribution-2:

- We show that optimism has two effects:
  - **Direct effect**: it leads to price inflation and consequently ***lowers the expected return.***
  - **Indirect effect**: price inflation makes the realization of transactional benefit more expensive. Thus, investors seek ***higher expected return*** to join a crypto-platform.
  - The net effect of optimism depends on multiple factors such as:
    - Level of belief dispersion
    - Productivity of the crypto asset
    - Investors' motive to utilize blockchain technology and transactional benefit
- Our theory explains the mechanism behind high volatility and repetitive crashes in crypto market.
- Motive for utilizing a crypto asset for transactional purposes depends on:
  1. Volatility-adjusted productivity normalized by platform user base
  2. Co-movement of productivity with transaction fee
  3. Co-movement of productivity with the expected return

# Empirical Perspective:

- We provide empirical support for our theory
  - We use PCA to capture transactional benefits
  - The CAPM-like pricing relation generates zero intercept in almost all leading cryptocurrencies.
  - We also generate more than 40% R-Square in our one-year sample
  - We show that the proposed optimism coefficient explains the observed trends in crypto market.

- To avoid selection bias, we consider Elon Musk's tweet on May 12th of 2021 as an unanticipated shock which results in a crash in crypto market
  - His tweet was associated with little changes in fundamentals of crypto market (i.e. productivity)
  - It's reasonable to believe that his tweet has altered the belief of investors.
  - We show that our proposed optimism coefficient captures the dramatic change in belief which results in a crash

# Model-1:

- We model an infinite horizon economy with discrete time.
- There is an infinitely lived representative miner as well as overlapping generations of investors (users) with dispersed belief
- Miners obtain reward by collecting newly generated tokens and transaction fees.
- Investors face a two-stage decision problem
  - In stage one: an investor decides whether to invest a specific amount of wealth in cryptocurrencies or take an alternative investment opportunity.
  - If so, in stage two, she decides on the creation of an optimal portfolio of crypto assets to maximize her utility
- Investors' utility is driven by capital gain and convenience yield (transactional benefit) in crypto market
- Overlapping generations of cryptocurrency investors lives for two dates: t, t+1.
  - At date t: they are called young
  - At date t+1: they are called old

# Model-2:

- There are N crypto assets in the economy
- New coins are generated based on a pre-determined rate in the form of block reward
- The number of tokens (coins) at each date is given by:

$$\Gamma_{t+1} = D_{\mathbf{1}+\tau}\Gamma_t$$

- Platform service providers (e.g. miners) collect this fee. We assume the following process for fee:

$$f_t = \mu + \varepsilon_t \quad , \quad \varepsilon_t \sim N(\mathbf{0}, \Sigma_f)$$

- We assume the following process for productivity:

$$y_t = \lambda + e_t \quad , \quad e_t \sim N(\mathbf{0}, \Sigma_y)$$

- We define $b_i$ as the investors' **endogenous motive** to utilize $i^{th}$ crypto asset for transactional purposes.

# Model-3:

- Investors have different opinion about future state of crypto market.
- There are two states in each crypto asset:
  - “Up” state: upward trend in prices and the return is positive.
  - “Down” state: downward price trend and the return is negative.
- We assume that there are two types of investors,
  - Rational (type 1) which is indexed by k=R
  - Over-confident (type 2) which is indexed by k=C
- The utility of joining crypto market:

$$E[U(Crypto\ Market)] = E[-e^{-A(Terminal\ Wealth)-(Convenience\ Yield)}]$$

- Considering the alternative investment, each type optimizes:

$$Max\{I(W_{tk}), Max\ E_t^k[-e^{-AW_{(t+1)k}-(W_{tk}X_t'^k(D_{\mathrm{b}}\, y_t))}]\}$$

# Model-4:

- Let's express the return in Up and Down state as below:

$$\begin{cases} r_t^U : \text{Vector of returns in up state at date t} \\ r_t^D : \text{Vector of returns in down state at date t} \end{cases}$$

- Let's express the belief of rational investors as below:

$$\begin{cases} \theta_i^R : The\ probability\ of\ Up\ state\ for\ crypto\ asset\ i\ given\ rationals'\ belief \\ 1-\theta_i^R : The\ probability\ of\ Down\ state\ for\ crypto\ asset\ i\ given\ rationals'\ belief \end{cases}$$

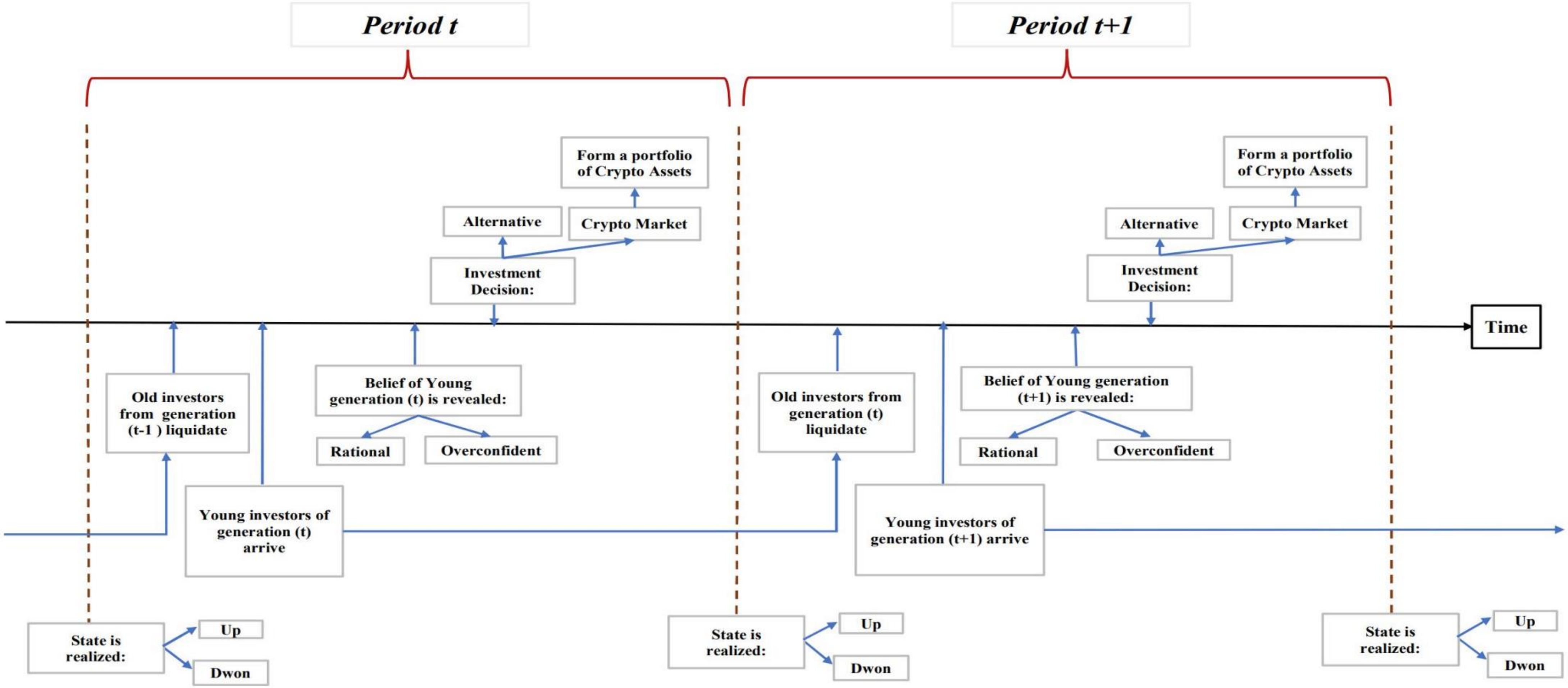
- Let's express the belief of over-confident investors as below:

$$\begin{cases} \theta_i^C : The\ probability\ of\ Up\ state\ for\ crypto\ asset\ i\ given\ Overconfidents'\ belief \\ 1-\theta_i^C : The\ probability\ of\ Down\ state\ for\ crypto\ asset\ i\ given\ Overconfidents'\ belief \end{cases}$$

- for each crypto asset, the following holds:

$$\begin{cases} \theta_i^C = \xi_{1i}\theta_i^R \\ 1-\theta_i^C = \xi_{2i}(1-\theta_i^R) \end{cases}$$

# Events' Timeline:



# Market Equilibrium:

- Optimal transactional motive:

$$D_b = \underbrace{N^{-1}\Sigma_y^{-1}\lambda}_{Term\ 1} + \underbrace{A\Sigma_y^{-1}\Sigma_{fy}}_{Term\ 2} - \underbrace{A\Sigma_y^{-1}\Sigma_{ry}}_{Term\ 3}$$

- Rational investors' optimal portfolio:

$$X_t^{R*} = \frac{1}{a_R}(\Sigma_T^R)^{-1}[E^R(r) - \mu + \frac{1}{A}\lambda' N^{-1}\Sigma_y^{-1}\lambda + \delta_f\lambda - \delta_r\lambda)]$$

Where $\Sigma_y^{-1}\Sigma_{fy} = \delta_f$ and $\Sigma_y^{-1}\Sigma_{ry} = \delta_r$

- Overconfident investors' optimal portfolio:

$$X_t^{C*} = \frac{1}{a_C}(\Sigma_T^C)^{-1}[E^C(r) - \mu + \frac{1}{A}\lambda' N^{-1}\Sigma_y^{-1}\lambda + \delta_f\lambda - \delta_r\lambda)]$$

- Miners' budget constraint:

$$\mathbf{W}_t^{Miner} = \underbrace{D_{f_t}\left(W_t^R X_t^R + W_t^C X_t^C\right)}_{Transaction\ fee} + \underbrace{D_{P_t}\left(\Gamma_t - \Gamma_{t-1}\right)}_{Block\ generation\ reward}$$

# Crypto-CAPM Relation-1:

- Market Clearing Condition:

$$W_t X_t^M = W_{tC} X_t^C + W_{tR} X_t^R + \mathbf{W_t^{Miner}}$$

- Equilibrium expected return:

$$E_t[r_{t+1}] = \frac{1}{\gamma+\xi_1} \cdot \frac{1}{1+f} \cdot \frac{1}{1+\tau} a \Sigma X_t^M + \frac{\xi_1 - 1}{\gamma+\xi_1} r_{t+1}^D + \frac{\gamma+1}{\gamma+\xi_1}(\mu - \frac{b}{A}\lambda)$$

$$Where \quad \Sigma = \Sigma_r + \Sigma_f + \frac{D_b D_b'}{A^2}\Sigma_y - 2\Sigma_{rf} - 2\frac{D_b}{A}\Sigma_{fy} + 2\frac{D_b}{A}\Sigma_{ry}$$

$$and \quad \Sigma_r = \theta(1-\theta)(r_{t+1}^U - r_{t+1}^D)^2$$

$$and \quad \frac{b}{A}\lambda = \frac{\lambda^2}{AN\sigma_y^2} + \delta_f \lambda - \delta_r \lambda$$

# Crypto-CAPM Relation-2:

- Return in UP and DOWN states:

$$r_{t+1}^{U} = \frac{1}{1+f}\cdot\frac{1}{1+\tau}\cdot\frac{\xi_1-\xi_2}{(\gamma+\xi_1)(1-\xi_2)}a\Sigma X_t^M + \frac{(\gamma+1)(\xi_1-\xi_2)}{(\gamma+\xi_1)(1-\xi_2)}(\mu-\frac{b}{A}\lambda) - \frac{\xi_1-1}{1-\xi_2}r_{t+1}^{D}$$

$$r_{t+1}^{D} = \frac{1}{1+f}\cdot\frac{1}{1+\tau}\cdot\frac{\xi_1-\xi_2}{(\gamma-\xi_2)(\xi_1-1)}a\Sigma X_t^M + \frac{(\gamma+1)(\xi_1-\xi_2)}{(\gamma-\xi_2)(\xi_1-1)}(\mu-\frac{b}{A}\lambda) - \frac{\gamma+\xi_1}{\gamma-\xi_2}\cdot\frac{1-\xi_2}{\xi_1-1}r_{t+1}^{U}$$

- Equilibrium expected return:

$$E_t(r_{t+1}) = \underbrace{\beta E_t(r_{M(t+1)})}_{Term\ 1} + \underbrace{\frac{\xi_1-1}{\gamma+\xi_1}r_{t+1}^{D}}_{Term\ 2} + \underbrace{\frac{\gamma+1}{\gamma+\xi_1}(\mu-\frac{b}{A}\lambda)}_{Term\ 3}$$

$$Where\ \beta = \frac{\sum X^M}{X'^M\sum X^M} = \frac{\sum X^M}{\sigma_M^2}$$

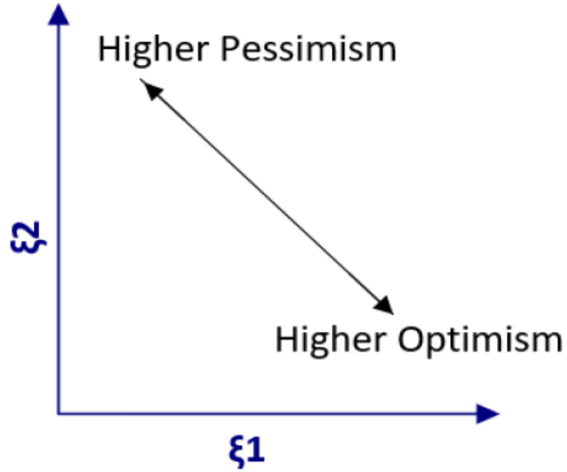
# Market Stability-1:

- Rational investors will participate if and only if, the following condition holds:

$$\frac{1-\xi_2}{\xi_1 - 1} \geq T$$

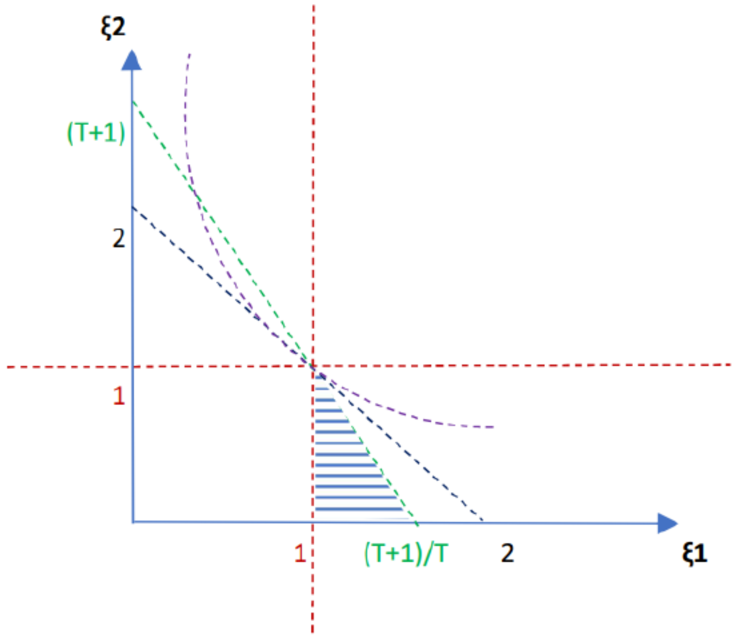
$$T = \frac{1}{\sigma^2}\left[(2(\sigma_T^2(-\ln(-I(W_{tR}) - a_R))^{0.5} + (\mu - \frac{b}{A}\lambda) - r^D\right]^2$$

- Consider the following belief structure:

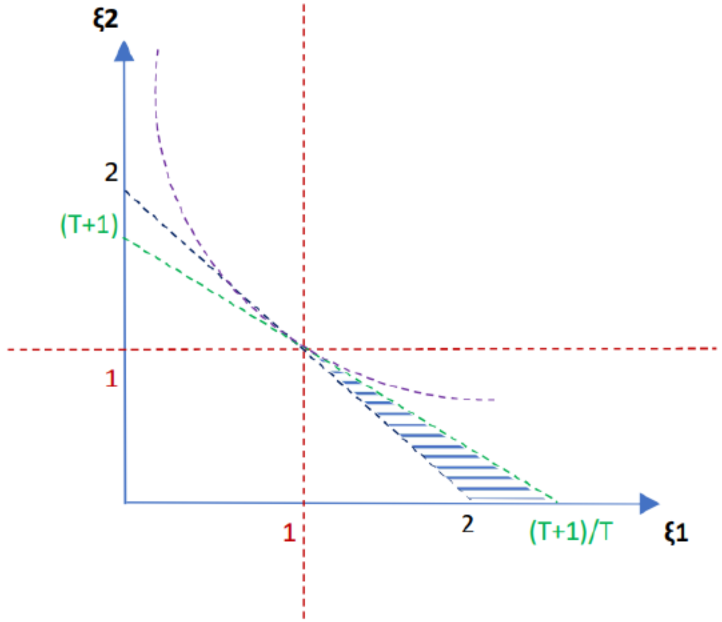


# Market Stability-2:

- **Case 1:** Over-confident investors are optimistic, and the crypto market is in boom

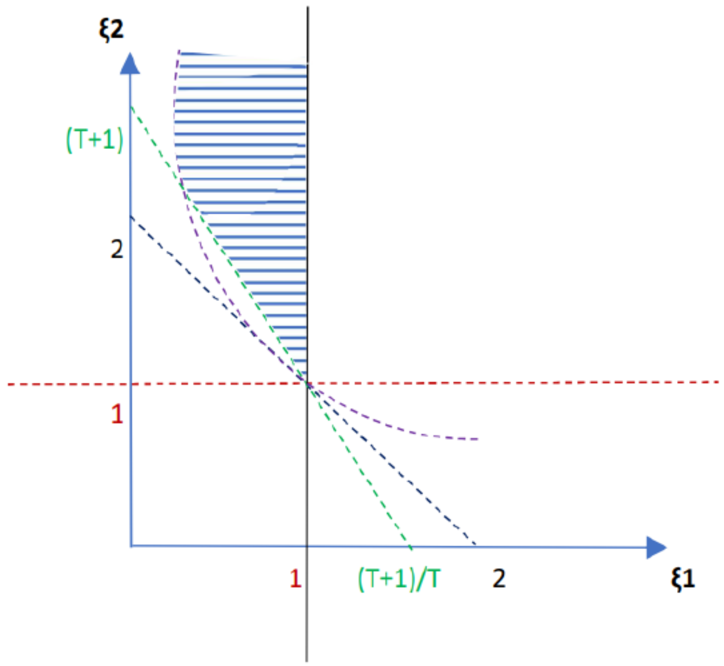



- **Case 2:** Over-confident investors are optimistic, and the crypto market is in downturn

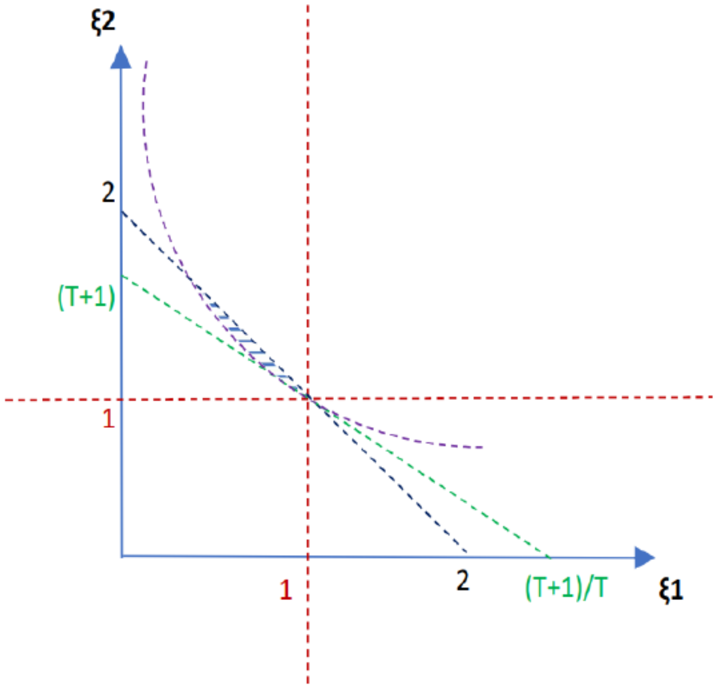


# Market Stability-3:

- **Case 3:** Over-confident investors are pessimistic, and the crypto market is in boom




- **Case 4:** Over-confident investors are pessimistic, and the crypto market is in downturn



# Cryptocurrency Market-specific Factors:

- Over-confident view can be interpreted as the outcome overreaction (under-reaction) to new information or surprises (Barberis, Shleifer, and Vishny (1998))
- Assume that Z is a vector of observable characteristics that investors can deduce information from them by observing the previous generation
- We can define belief dispersion as:

$$\xi_1 = e^{\omega'(Z-E[Z])}$$

Where the vector $\omega$ represents the appropriate weight to captures the mis-reaction to new information

- The market model return residuals follows

$$\ln(E[r] - \beta E[r_m]) = \ln(\gamma(\gamma+1)) + \ln(\mu - \frac{b}{A}\lambda) + B(Z - E[Z])$$

$$Where \quad B = \frac{\omega r^D}{(\gamma+1)(\mu - \frac{b}{A}\lambda)} + \frac{\omega}{1+\gamma}$$

# Empirical Analysis-1:

- We gather daily trading data on top 14 cryptocurrencies and 2 stablecoins from “marketcap.com” and “coinmetrics.io”
- We also gather data on platform characteristics from “coinmetrics.io”
- Our study period covers 2021
- The empirical implication of the crypto-CAPM can be summarized by the following cross-sectional regression:

$$r_{it} = \beta_i r_{Mt} + \gamma_i r_{it}^D + \kappa_i Benefit_{it} + \varepsilon_{it}$$

$$\begin{cases} r_{it} : The\ \ observed\ \ return\ \ of\ \ crypto - asset\ \ i \\ r_{Mt} : The\ observed\ return\ of\ crypto - market\ portfolio \\ r_{it}^D : The\ calculated\ Down - state\ return\ of\ crypto - asset\ i \\ Benefit_{it} : Net\ of\ transactional\ benefit \end{cases}$$

$$\varepsilon_{it}\ are\ i.i.d\ random\ noise\ in\ crypto - asset\ i$$

# Empirical Analysis-2:

- Return in Up and Down states are calculated as below:

$$r_{i,t}^{D} = \frac{low_{i,t} - Avg(closing_{i,t}, closing_{i,\ t+1})}{Avg(closing_{i,t}, closing_{i,\ t+1})}$$

$$r_{i,t}^{U} = \frac{high_{i,t} - Avg(closing_{i,t}, closing_{i,\ t+1})}{Avg(closing_{i,t}, closing_{i,\ t+1})}$$

- To calculate transactional benefits, we use PCA technique.
- We construct proxies for transactional benefits based on 22 observable characteristics in crypto market.
- We run the following regression:

$$r_{it} = \beta_i r_{mt} + \gamma_{it} r_{it}^{D} + \sum_j \kappa_i^j Comp_{it}^j + \varepsilon_{it}$$

# Empirical Analysis-3:

| | (1) BTC return | (2) BTC return | (3) BTC return |
|---|---|---|---|
| Market return | 1.019*** | 1.021*** | 1.060*** |
| | (72.62) | (73.38) | (45.93) |
| BTC Down return | -0.0685*** | -0.0821*** | -0.0998** |
| | (-3.22) | (-3.81) | (-2.60) |
| Control variable | | -0.000159*** | -0.00110* |
| | | (-2.87) | (-1.97) |
| btc tx benefit 1 | | | 0.00190** |
| | | | (2.06) |
| btc tx benefit 2 | | | 0.00234* |
| | | | (1.73) |
| btc tx benefit 3 | | | 0.000961 |
| | | | (1.41) |
| btc tx benefit 4 | | | 0.0005652 |
| | | | (-0.22) |
| Constant | -0.00400*** | -0.00290*** | 0.0138 |
| | (-4.33) | (-2.92) | (1.39) |
| Observations | 365 | 365 | 365 |
| $R^2$ | 0.938 | 0.939 | 0.960 |

$t$ statistics in parentheses
$^* p < 0.10$, $^{**} p < 0.05$, $^{***} p < 0.01$

| | (1) ETH return | (2) ETH return | (3) ETH return |
|---|---|---|---|
| Market return | 1.088*** | 1.090*** | 1.085*** |
| | (33.77) | (33.68) | (33.24) |
| ETH Down return | -0.103*** | -0.0973** | -0.0954** |
| | (-2.70) | (-2.52) | (-2.17) |
| Control variable | | 0.253 | -1.025 |
| | | (0.73) | (-1.35) |
| eth tx benefit 1 | | | -0.0005826*** |
| | | | (-3.52) |
| eth tx benefit 2 | | | -0.0016697 |
| | | | (-0.82) |
| eth tx benefit 3 | | | 0.00203 |
| | | | (0.60) |
| eth tx benefit 4 | | | 0.00102 |
| | | | (0.76) |
| Constant | -0.00269 | -0.00433 | 0.00519 |
| | (-1.26) | (-1.39) | (0.91) |
| Observations | 365 | 365 | 365 |
| $R^2$ | 0.764 | 0.764 | 0.771 |

$t$ statistics in parentheses
$^* p < 0.10$, $^{**} p < 0.05$, $^{***} p < 0.01$

| | (1) LTC return | (2) LTC return | (3) LTC return |
|---|---|---|---|
| Market return | 1.247*** | 1.227*** | 1.228*** |
| | (28.80) | (28.60) | (28.47) |
| LTC Down return | 0.135*** | 0.185*** | 0.197*** |
| | (2.99) | (3.99) | (3.90) |
| Control variable | | 0.474*** | 0.882*** |
| | | (3.71) | (2.94) |
| LTC tx benefit 1 | | | 0.00172 |
| | | | (0.50) |
| LTC tx benefit 2 | | | -0.00121 |
| | | | (-0.52) |
| LTC tx benefit 3 | | | -0.000205 |
| | | | (-0.19) |
| LTC tx benefit 4 | | | -0.000196*** |
| | | | (-3.35) |
| Constant | 0.00412 | -0.00742* | -0.0189 |
| | (1.49) | (-1.80) | (-0.86) |
| Observations | 365 | 365 | 365 |
| $R^2$ | 0.727 | 0.737 | 0.741 |

$t$ statistics in parentheses
$^* p < 0.10$, $^{**} p < 0.05$, $^{***} p < 0.01$

# May 2021 Crash Event Study:

- On May 12th, Elon Musk tweeted about the uncertain future of cryptocurrencies due to high Carbon emission of crypto mining.
- After his tweet, price dropped by 30%
- We show that the optimism coefficient represents the belief system correctly.

| | (1)<br>Crypto Return (Before) | (2)<br>Crypto Return (After) |
|---|---|---|
| Market Return | 0.9765** | 0.9842*** |
| | (2.21) | (3.84) |
| Down-State Return | -0.4838**** | 0.3587**** |
| | (-6.84) | (9.59) |
| TX Benefit | -1.02e-6*** | 0.0173*** |
| | (-2.67) | (3.80) |
| Constant | -0.0072 | 0.0151**** |
| | (-1.12) | (6.79) |
| Observations | 117 | 286 |
| $R^2$ | 0.092 | 0.096 |

*t* statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

# Empirical Derivation of UP and DOWN Return:

- We run a SEM regression based on our theory
- Endogenous variables: UP and DOWN states return

$$\begin{cases} r^U = \alpha_0 + \alpha_1 r_M + \alpha_2 Benefit + \alpha_3 Fee + \alpha_4 r^D \\ r^D = \beta_0 + \beta_1 r_M + \beta_2 Benefit + \beta_3 Fee + \beta_4 r^U \end{cases}$$

| | BTC | | ETH | |
|---|---|---|---|---|
| **Structural UP Return** | Coefficient | Z | Coefficient | Z |
| **Down Return** | -0.1559 | -6.01 | -0.1658 | -5.09 |
| **Market Return** | 0.153 | 0.47 | -0.0581 | -1.36 |
| **TX Benefit** | 0.0007 | 2.04 | -0.0029 | -6.81 |
| **Avg Fee** | 10.2653 | 1.78 | 0.1459 | 0.46 |
| **Constant** | 0.2192 | 14.56 | 0.0289 | 10.43 |
| **Structural Down Return** | Coefficient | Z | Coefficient | Z |
| **Up Return** | -0.3178 | -3.45 | -0.3083 | -3.88 |
| **Market Return** | 0.1421 | 2.36 | 0.162 | 2.42 |
| **TX Benefit** | 0.00005 | 0.1 | 0.0041 | 7.46 |
| **Avg Fee** | -36.098 | -3.63 | -1.0588 | -1.83 |
| **Constant** | -0.1788 | -6.13 | -0.0258 | -4.68 |

# Conclusion:

- We offer a CAPM-like equilibrium pricing for cryptocurrencies.
- We consider dual roles of cryptocurrencies:
  (1) Their role as investment assets
  (2) Their role in providing distinctive transactional benefits.
- We provide several interesting insights:
  - We find a bilateral relationship between transactional benefit and belief dispersion in crypto market
  - We show that over-optimism de-stabilizes crypto market and might lead to a crash
  - We show that cryptocurrency market specific factors such as momentum and attention can be explained by belief dispersion in this market.
- We evaluate the validity of our pricing relation with empirical data
  - We utilize PCA to quantify transactional benefit
  - We show that the observed trend in market cap and Sharpe ratio can be explained by our pricing relation
- Also, we show that our model can explain the observed trend in May 2021 crash

# Thank you

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