The **Fiscal Channel** of **Quantitative Easing**[1]

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2. UAB & BSE | 3. BoE & LSE.

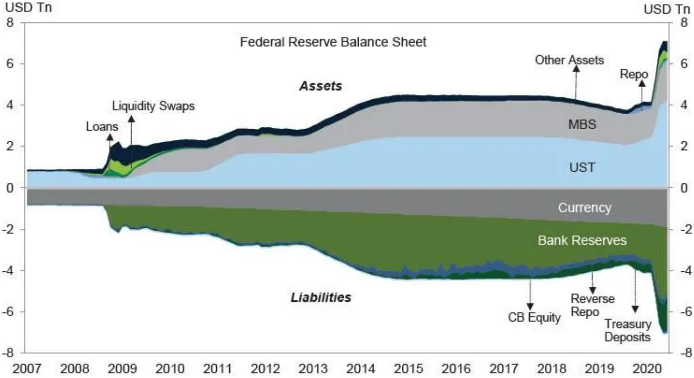
[1] DISCLAIMER: The views expressed here don't represent, in any way, those of the Bank of England, PRA, or any of its' committees.

What is QE?

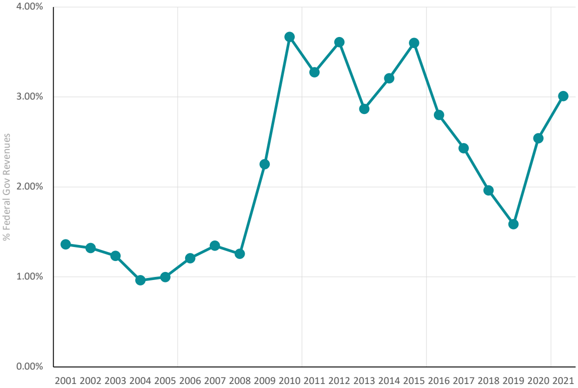
* Large scale **asset purchases** financed by **interest-paying reserves**.

* Goal: $\downarrow$ long term interest rates $\Rightarrow \uparrow$ (inflation, output).

* Massive expansion CB's **balance sheet**.

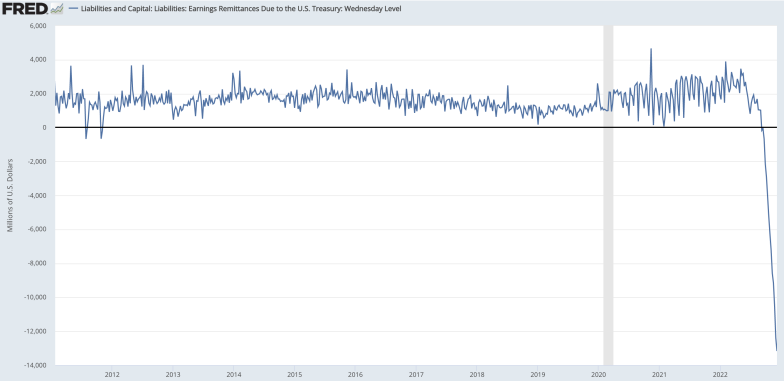


QE affects the Government's budget via CB's transfers



▸ Other fiscal consequences

QE can generate volatile remittances to the Government



This paper

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We study an economy where a government without commitment strategically decides what to do with the profits/losses of a large CB balance sheet, where:

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2. Transfers from households to the government are costly.

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$\Downarrow$

The size of the CB balance sheet performs risk-shifting from private to government expenditures.

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Central Bank's power to affect **asset prices** via QE depends on the **associated fiscal policy**.

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⇓

Key piece of QE's **Irrelevance theorems**.

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A rational Government would embark on **real resource reallocation**.

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   + Green corporate bonds programs (BoE, ECB).
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2. **New uses of QE** aimed explicitly at reallocating resources
   + Green corporate bonds programs (BoE, ECB).
   + Transmission Policy Instrument (ECB).

3. Evaluate fiscal effects of **raising interest** rates/ QT.

1.- QE with costly T and productive G

2.- QE with limited participation

The tool

The smallest possible model:

* A real and stochastic endowment economy.
* A representative investor.
* Rational Expectations.
* Incomplete markets: a risky asset $\{S, P, D\}$; a safe asset $\{B, 1/R, 1\}$.
* 2 periods t = 0, 1.
* Economic policy: $\{G, T, B, R, Q\}$.
* Tax cost function $H : T \to \mathbb{R}$, with $1 > H' > 0$ (Bohn, 1992).
* Welfare: $U = \mathbb{E}_0\{u(C_0) + \delta[yu(C_1) + (1 - y)v(G_1)]\}$.

Investor's problem

Equilibrium and Economic Policy

A **Competitive Equilibrium** is an asset price P, allocations $\{C_0, C_1, S, B^i\}$ and policies $\{G_0, G_1, T_0, T_1, B, R, QE\}$ that satisfy:

1. Investor's Euler Equations (2).
2. Investor's budget constraints (2).
3. Consolidated gov budget constraints (2).
4. Assets market clearing (2).

▸ Equations

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▸ Equations

* 12 endogenous variables; 8 equations.

$\Downarrow$

Economic policy needs to specify
4 variables out of $\{G_0, G_1, T_0, T_1, B, R, QE\}$.

* **Quantitative Easing**: $\{QE, B\} = \{Q, QPR\}$.

Unchanged fiscal policy

* **Institutional** framework: fiscal **support**; **passive** fiscal policy.
* Intertemporal Gov Budget Constraint:

$$\underbrace{Q\left(P - \frac{D_1}{R}\right)}_{\text{QE losses}} = \underbrace{\underbrace{T_0 + \frac{T_1}{R}}_{\mathcal{T}} - \underbrace{G_0 - \frac{G_1}{R}}_{\mathcal{G}}}_{\text{Primary Surplus } \mathcal{S}} \tag{1}$$

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* Common assumption: lump-sum T; exogenous G.

$$\mathcal{S}(Q, \cdot) = \mathcal{T}(Q, \cdot) \tag{2}$$

* WLOG suppose $\{G_0, G_1, T_0, \boldsymbol{T_1}\} = \{0, 0, 0, -\underbrace{\boldsymbol{Q(D_1 - PR)}}_{\text{QE gains}}\}$

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Is it the **best possible reaction**?

Exogenous G is not optimal

* Let $\{T_0, T_1, G_0, G_1\} = \{0, T, 0, G\}$... How should a **government** set (T, G)?

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$$\max_{\{T,G\}} U = \left[yu(C_1) + (1-y)v(G)\right] \tag{3}$$

$$\text{s.t. } C_1 = (1-Q)D_1 + QRP - T - H(T)$$
$$G = Q(D_1 - RP) + T$$

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* **Optimality** condition:

$$\underbrace{y[u'(D_1 - X - T - H)(1 + H'(T))]}_{\text{Tax Mg cost}} = \underbrace{(1-y)[v'(T+X)]}_{\text{Tax Mg gain}} \tag{4}$$

with $\boldsymbol{X} = Q(D_1 - RP)$ being **QE gains** (CB transfers).

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$$\Downarrow$$

$$\boldsymbol{G^* = g(Q)}$$

with $g'(Q) > 0$ if $X > 0$

$$\Downarrow$$

X has an income effect as it lower the tax distortions.

Does it matter?

QE becomes **effective**
with an **optimal** fiscal reaction.

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* Goods market clearing:

$$\{C_0, C_1\} = \{D_0, D_1 - H(T) - g(Q)\}$$

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Cases:

1. $H' = 0$ or $y = 1 \Rightarrow$ Irrelevance.
2. $H' > 0$ and $y < 1 \Rightarrow$ **Relevance**.

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Higher Q implies larger consumption volatility

$\Downarrow$

General Asset Price Inflation

Asset demand inelasticity makes QE relevant

$$g'(Q) = 0 \qquad \Rightarrow \quad \frac{\partial C_0^*}{\partial Q} = 0; \qquad \frac{\partial S^*}{\partial Q} = -1$$

$$g'(Q) > 0 \qquad \Rightarrow \quad \frac{\partial C_0^*}{\partial Q} < 0; \qquad 0 > \frac{\partial S^*}{\partial Q} > -1$$

▸ Some evidence ▸ Closed form

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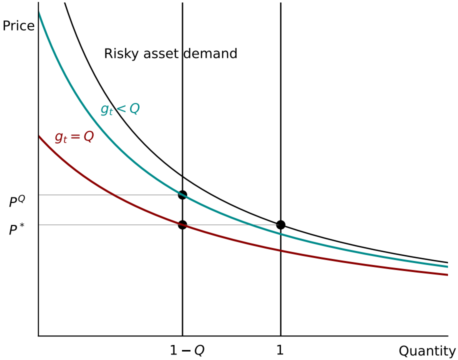

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Redistribution via QE

* 2 agents: investor & hand-to-mouth worker.
* Quantitative Easing: $\{QE, B\} = \{Q, QPR\}$
* Fiscal Policy: $\{T_0, T_1, G_0, G_1\} = \{0, QPR, 0, \boldsymbol{QD_1}\}$.
* Period 1 budget constraints:

$$\text{Investor:} \quad C_1 + T_1 = SD_1 + B \quad \Rightarrow \quad C_1 = D_1 - \boldsymbol{QD_1} \tag{6}$$

$$\text{Worker:} \quad C_1^w = W + G_1 \quad \Rightarrow \quad C_1^w = W_1 + \boldsymbol{QD_1} \tag{7}$$

* Asset price:

$$P^Q = \mathbb{E}_0 \left[\delta \frac{u'((1-Q)D_1)}{u'(D_0)} D_1 \right] \tag{8}$$

Redistribution via τ

* Quantitative Easing: $\{QE, B\} = \{0, 0\}$
* Fiscal Policy: $\{T_0, T_1, G_0, G_1\} = \{0, \tau D_1, 0, \tau D_1\}$.
* Period 1 budget constraints:

$$\text{Investor:} \quad C_1 + \tau D_1 = SD_1 + B \quad \Rightarrow \quad C_1 = D_1 - \boldsymbol{\tau D_1} \tag{9}$$

$$\text{Worker:} \quad C_1^w = W + G_1 \quad \Rightarrow \quad C_1^w = W_1 + \boldsymbol{\tau D_1} \tag{10}$$

* Asset price

$$P^\tau = \mathbb{E}_0\left[\delta \frac{u'\left((1-\tau)D_1\right)}{u'(D_0)}(1-\tau)D_1\right] \tag{11}$$

* Relative asset price policy-wise:

$$\frac{P^\tau}{P^Q} = (1-\tau)\frac{\mathbb{E}_0\left[\frac{u'\left((1-\tau)D_1\right)}{u'(D_0)}D_1\right]}{\mathbb{E}_0\left[\frac{u'\left((1-Q)D_1\right)}{u'(D_0)}D_1\right]} \quad \Rightarrow \quad \boldsymbol{\frac{P^\tau}{P^Q} = 1-\tau} \quad \text{for} \quad \tau = Q$$

$$\Downarrow$$

QE: **Redistribution** with **higher** to asset prices.

What would the optimal redistribution be?

* Fiscal authority problem:

$$\max_{\{T,G,G^W\}} \omega U^I + (1-\omega)U^W \tag{12}$$

s.t. Competitive Equilibrium, given asset prices

with

$$U^I = \mathbb{E}_0\Big\{u(C_0) + \delta\big[yu(C_1) + (1-y)v(G)\big]\Big\}$$

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$$\omega\mathbb{E}_0[v'(G)] = (1-\omega)\mathbb{E}_0[v'(G^W)] \tag{14}$$

An example

* Optimal taxes (from investor):

$$T^* = aD_1 + (b-1)X + yG^W \tag{15}$$

* Optimal investor preferred-G:

$$G^* = \bar{\omega}(aD_1 + bX) \tag{16}$$

* Optimal worker preferred-G:

$$G^{W*} = (\overline{1-\omega})(aD_1 + bX) \tag{17}$$

with

$$\bar{x} = \frac{x}{\omega + (1-y)(1-\omega)}$$

* Equivalent to a tax on dividends...

$$\tau^* = \frac{(\overline{1-\omega})bX}{D_1} \tag{18}$$

* ... except asset prices are higher under QE

$$\frac{P^{\tau*}}{P^{Q}} = 1 - \tau^* \tag{19}$$

Conclusions

1. Important effects of QE on the **fiscal space**.
2. How this additional fiscal space is managed is key to determine the overall QE effects.
3. Literature: "**unchanged** fiscal policy" but... **not optimal** in relevant environments.
4. With optimal fiscal reaction: **redistribute** real resources.

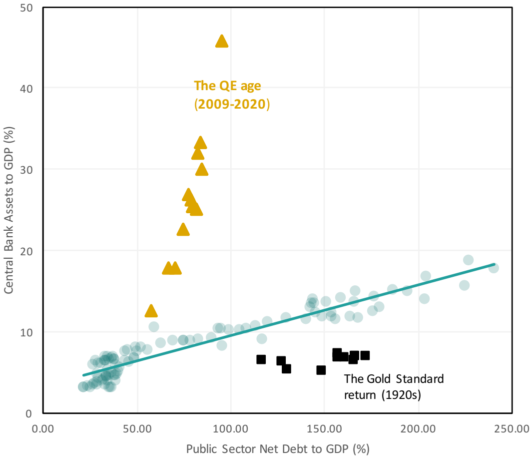
$$\Downarrow$$

The **Fiscal Channel**.

5. Looking backward: how relevant was it?
6. Looking forward: **new uses** of QE exploiting it?

Additional Info

New Monetary Policy



Back

QE influences the fiscal space decisively

Real government's budget constraint:

$$G_t + \sum_{m=1}^{M} B_{t-m,t} = \sum_{m=1}^{M} \frac{1+\pi_{t+m}}{1+i_{t+m}} B_{t+1,t+1+m} + T_t + X_t \tag{20}$$

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QE's Direct Effect:

$$G_t + \sum_{m=1}^{M} B_{t-m,t} = \sum_{m=1}^{M} \frac{1+\pi_{t+m}}{\underbrace{\downarrow 1+i_{t+m}}_{\text{Interests}}} B_{t+1,t+1+m} + T_t + \overbrace{\uparrow X_t}^{\text{CB's Remittances}} \tag{21}$$

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QE's Indirect Effect:

$$\underbrace{\downarrow G_t}_{\text{Spending}} + \overbrace{\downarrow \sum_{m=1}^{M} B_{t-m,t}}^{\text{debt real repayments}} = \sum_{m=1}^{M} \frac{\overbrace{\uparrow 1+\pi_{t+m}}^{\textit{Inflation}}}{1+i_{t+m}} B_{t+1,t+1+m} + \underbrace{\uparrow T_t}_{\text{Taxes}} + X_t \tag{22}$$

▸ Back

Investor's program

$$\max_{\{C_0, C_1, S, B\}} U = \mathbb{E}_0\{u(C_0) + \delta[yu(C_1) + (1-y)v(G)]\} \tag{23}$$

s.t.

$$C_0 + PS + \frac{B}{R} + T_0 + H(T_0) = (P + D_0)S_{-1}$$

$$C_1 + T_1 + H(T_1) = D_1 S + B$$

Back

Equilibrium

A **Competitive Equilibrium** is a vector of prices $\{P, R\}$, allocations $\{C_0, C_1, S, B^i\}$ and policies $\{G_0, G_1, T_0, T_1, B, Q\}$ such that:

1. Investor's Euler Equations are satisfied:

$$P = \mathbb{E}_0 \left[\delta y \frac{u'(C_1)}{u'(C_0)} D_1 \right] \tag{24}$$

$$\frac{1}{R} = \mathbb{E}_0 \left[\delta y \frac{u'(C_1)}{u'(C_0)} \right] \tag{25}$$

2. Investor's budget constraints:

$$C_0 + PS + \frac{B}{R} + T_0 + H(T_0) = (P + D_0)S_{-1} \tag{26}$$

$$C_1 + T_1 + H(T_1) = D_1 S + B \tag{27}$$

3. Consolidated gov budget constraints

$$G_0 + QP = \frac{B}{R} + T_0 \tag{28}$$

$$G_1 + B = T_1 + QD_1 \tag{29}$$

4. Assets market clearing:

$$S + Q = 1; \quad B^i = B \tag{30}$$

▸ Back

Exogenous G is not optimal: proof (I)

- **Optimality** condition:

$$\underbrace{y\mathbb{E}_0[u'(D_1 - X - T - H(T))(1 + H'(T))]}_{\text{Tax Mg cost}} = \underbrace{(1-y)\mathbb{E}_0[v'(T+X)]}_{\text{Tax Mg gain}} \tag{31}$$

- To simplify, assume H involves no uncertainty. Then, without QE (i.e. $X = 0$):

$$\frac{(1-y)}{y(1+H'(T))} = \frac{\mathbb{E}_0[u'(D_1 - T - H(T))]}{\mathbb{E}_0[v'(T)]} \tag{32}$$

- Now, consider $X > 0$. Call $\bar{T}$ the new tax level. If all the adjustment goes through taxes, $\bar{T} = T - X$. That implies $\uparrow C_1$ since $C_1 = D_1 - X - (T - X) - H(T - X)$ and $H' > 0$.

- $\uparrow C_1$ implies $\downarrow u'(\cdot)$ by the concavity of u. Then,

$$\frac{(1-y)}{y(1+H'(T))} = \frac{\mathbb{E}_0[u'(D_1 - T - H(T-X)]}{\mathbb{E}_0[v'(T)]} < \frac{\mathbb{E}_0[u'(D_1 - T - H(T))]}{\mathbb{E}_0[v'(T)]} = \frac{(1-y)}{y(1+H'(T))} \tag{33}$$

which is a contradiction. Hence, $\bar{T} = T - X$ cannot be optimal.

Exogenous G is not optimal: proof (II)

* Consider now all the adjustment going through G. Then, $\bar{T} = T$. By the concavity of u and v

$$\frac{(1-y)}{y(1+H'(T))} = \frac{\mathbb{E}_0[u'(D_1 - T - H(T) - X]}{\mathbb{E}_0[v'(T+X)]} > \frac{\mathbb{E}_0[u'(D_1 - T - H(T))]}{\mathbb{E}_0[v'(T)]} = \frac{(1-y)}{y(1+H'(T))} \tag{34}$$

  which is another contradiction. Then, no tax adjustment cannot be optimal either.

* Hence, the optimal T^* must lie somewhere in the middle, that is,

$$-1 < \frac{\partial T^*}{\partial X} < 0 \tag{35}$$

  and then,

$$0 < \frac{\partial G^*}{\partial X} < 1 \tag{36}$$

  since $\frac{\partial G^*}{\partial X} = 1 + \frac{\partial T^*}{\partial X}$. That completes the proof.

Back

Exogenous G is not optimal: an example

* No uncertainty.
* $u(\cdot) = v(\cdot) = ln(\cdot)$.
* Tax adjustment cost $H(T) = \alpha T$.
* Then,

$$\boldsymbol{G^* = aD_1 + bX} = \bar{g}(Q) \tag{37}$$

with $a = \frac{1-\gamma}{1+\alpha} > 0$ and $b = 1 - \frac{1+\alpha\gamma}{1+\alpha} > 0$.

* Two tax policies:

$$T = \begin{cases} aD_1 - \boldsymbol{X} & \text{if } \textbf{unchanged} \text{ FP} \\ aD_1 + \underbrace{(\boldsymbol{b-1})}_{<1} X & \text{if } \textbf{optimal} \text{ FP} \end{cases}$$

$$\Downarrow$$

With costly taxes it is **optimal**
to digest X with a **T-G combination**.

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Equilibrium taxes

Government:

$$G_0 + QP = \frac{B}{R} + T_0$$

$$0 + QP = QP + T_0 \quad \Rightarrow \quad \boldsymbol{T_0 = 0}$$

$$G_1 + B = T_1 + QD_1$$

$$G(Q) + QPR = T_1 + QD_1 \quad \Rightarrow \quad \boldsymbol{T_1 = G(Q) + Q(RP - D_1)}$$

Investor (taxes T_t^* that leave BC unchanged):

$$C_0 + PS + \frac{B}{R} + T_0 = (P + D_0)S_{-1}$$

$$C_0 + P(1 - Q) + QP + T_0 = (P + D_0) \quad \Rightarrow \quad \boldsymbol{T_0^* = 0}$$

$$C_1 + T_1 = D_1 S + B$$

$$C_1 + T_1 = D_1(1 - Q) + QPR \Rightarrow \quad \boldsymbol{T_1^* = Q(PR - D_1)}$$

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Closed form solutions

* A particular reaction function

$$G_t = G(Q, \cdot) = (Q - g)D_t$$

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* Stock equilibrium price

$$P^* = \delta \mathbb{E}_0 \left[\frac{C_0 + xG_0}{C_1 + xG_1} D_1 \right] = \frac{\delta D_0}{1 - (1 - x)(Q - g)} \tag{38}$$

since $C_1 + xG_1 = D_1[1 - (1 - x)(Q - g)]$.

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since $C_1 + xG_1 = D_1[1 - (1-x)(Q-g)]$.

* Stock demand

$$S^* = \frac{1}{P(1+\delta)} \Big(\delta D_0 + [\delta(1-Q) + x(g-Q) - g]P \Big) \tag{39}$$

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QE and precautionary savings (I)

* QE pass-through to taxes is a random variable $g_t \sim \mathcal{N}(Q, q)$.
* Expected full pass-through (irrelevance), but some fiscal risk.
* Bond's Euler Equation

$$u'(P_0(1 - S_0 - Q) + D_0) = \delta R \mathbb{E}_0[u'(D_1(S_0 + g_1)] \tag{40}$$

* With convex marginal utility

$$\mathbb{E}_0[u'(D_1(S_0 + g_1)] > \underbrace{\mathbb{E}_0[u'(D_1(S_0 + \mathbb{E}_0(g_1))]}_{\text{With QE}} = \underbrace{\mathbb{E}_0[u'(D_1 S_0)]}_{\text{Without QE}} \tag{41}$$

* Marginal benefits of savings go up for precautionary motives:

$$\text{QE:} \quad \uparrow \mathbb{E}_0[u'(D_1(S_0 + g_1)] \;\; \Rightarrow \;\; \uparrow S_0$$

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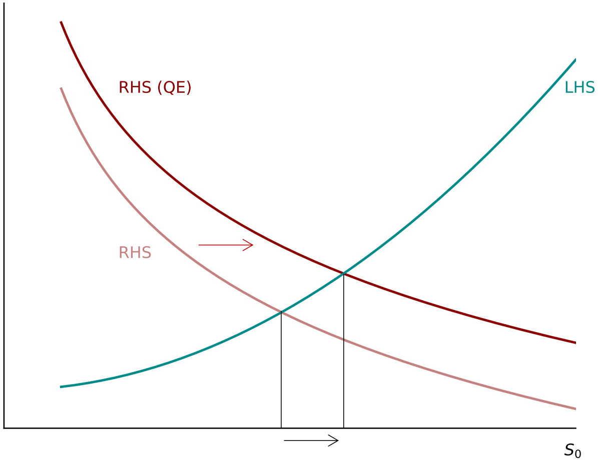
QE and precautionary savings (II)



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Euler Equations and asset pricing

* 2 readings of the Euler Equation
  1. Consumption theory: given interest rates $\Rightarrow$ use EE to determine $\{C_t, C_{t+1}\}$
  2. Asset pricing: given a consumption path $\Rightarrow$ use EE to determine P_t.

* Most QE literature, goes via 1. E.g. Harrison, 2017:

  Long rate equation

$$\mathbb{E}_t R^1_{L,t+1} = \hat{R}_t - \tau_t$$

  Consumption Euler Equation

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \sigma \left[\frac{1}{1+\delta} \hat{R}_t + \frac{\delta}{1+\delta} \mathbb{E}_t R^1_{L,t+1} - \mathbb{E}_t \pi_{t+1} \right] \tag{42}$$

* QE: $\uparrow \tau_t \Rightarrow \downarrow \mathbb{E}_t R^1_{L,t+1} \Rightarrow \uparrow \hat{c}_t$
* We take the Asset pricing reading of EEs.
* Question: $\tau_t = 0 \Rightarrow \mathbb{E}_t R^1_{L,t+1} = \hat{R}_t$. However, without log-lin

$$\mathbb{E}_t R^1_{L,t+1} = R_t - \frac{\mathbb{C}ov_t[u'(C_{t+1}), R^1_{L,t+1}]}{\mathbb{E}_t[u'(C_{t+1})]}$$

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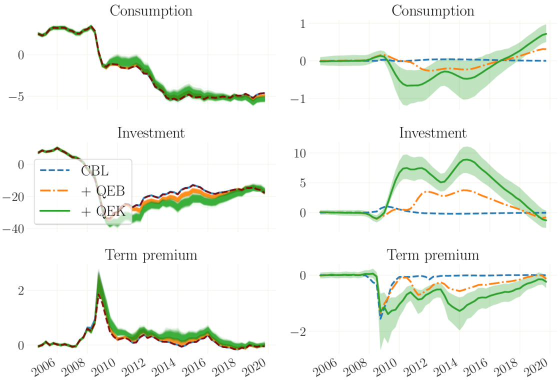
QE effects on consumption

* Boehl et al., 2021: ↓ aggregate consumption = 0.7%.



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2 agents economy

* Investor's problem:

$$\max_{\{C_0, C_1, S_0\}} \mathbb{E}_0[log(C_0) + \delta log(C_1)] \tag{43}$$

s.t.

$$C_0 + P_0 S_0 = (P_0 + D_0) S_{-1} \tag{44}$$

$$C_1 = (1 - \tau) D_1 S_0 \tag{45}$$

* Worker's problem

$$\max_{\{C_0^w, C_1^w\}} \mathbb{E}_0[log(C_0^w) + \delta log(C_1^w)] \tag{46}$$

s.t.

$$C_0^w = W_0 \tag{47}$$

$$C_1^w = W_1 + M \tag{48}$$

* Government: $M = \tau D_1$
* Market clearing: $C_t + C_t^w = D_t + W_t$; $S_t = S_{-1} = 1$.

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