Removing Residual Seasonality from GDP

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Outline

1. Background: Removing residual seasonality (RS) from components of GDP
2. Hierarchical lattice structure of GDP
    a. Windows, muntins, hubs, spokes
    b. Panes and terraces
3. Top-down algorithm for seasonal adjustment (SA)
4. Illustration of the Top-down SA algorithm with U.S. national accounts data

Motivation

- Our recent study proposed three diagnostic tests for identifying residual seasonality (RS) in the estimates of U.S. GDP and its major aggregates. (JOS Vol. 38, No. 2, 2022, pp. 399-428)

- A solution method is needed to remove the RS once it is detected.

- This study proposes a ***Top-down*** seasonal adjustment algorithm which removes RS from indirectly seasonal adjusted series via frequency- or cross-aggregation in the ***hierarchical*** structured national accounting system and reconciles the adjusted series via an optimization framework according to the accounting constraints.

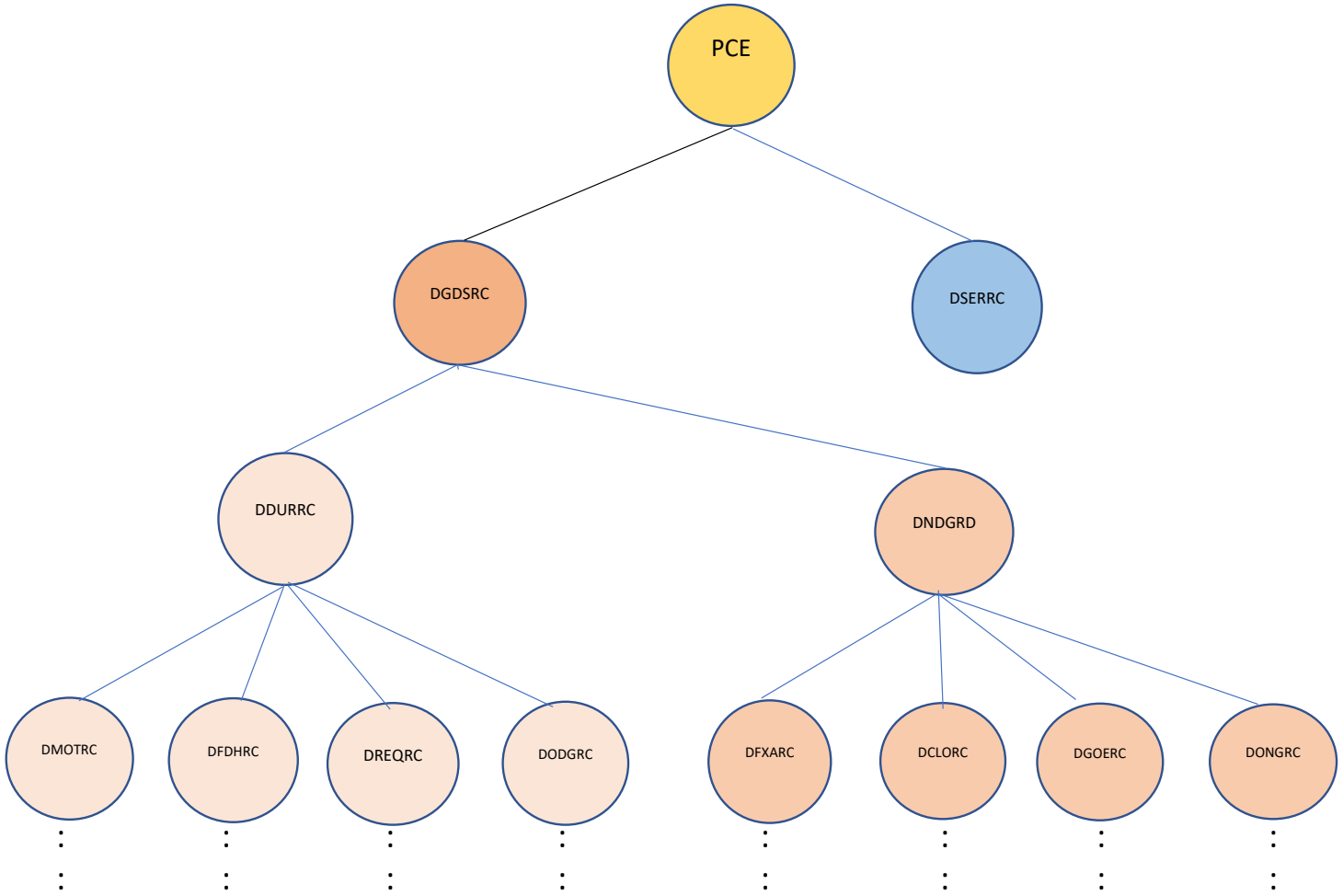
Objectives

- Components of GDP must satisfy many accounting relations.
- Direct seasonal adjustment of (***monthly*** or ***quarterly***) components results in violations of accounting relations, which is undesirable to the Bureau of Economic Analysis (BEA).
- We seek to modify existing seasonal adjustments as little as possible, and such that ***frequency*** (monthly to quarterly) and ***hierarchical*** (sub-aggregates to higher-aggregates) accounting relations are preserved.
- To do this, we must study the graph structure of GDP accounting relations. We take the largest component of GDP - personal consumption expenditure (PCE) to illustrate the hierarchical structure of the accounting system.

Background: Data Structure of PCE

- Personal Consumptions Expenditures (PCE) is published monthly and makes up about 2/3 of the components of GDP.
- PCE breaks into Goods (DGDSRC) and Services (DSERRC).
- Goods breaks into Durable Goods (DDURRC) and Nondurable Goods (DNDGRC), each with multiple tiers of disaggregated components.
- For example, Durable has four sub-aggregates: Motor Vehicles and Parts (DMOTRC), Furnishings and Durable Household Equipment (DFDHRC), Recreational Goods and Vehicles (DREQRC), and Other Durable Goods (DODGRC).

Figure 1: Hierarchical PCE Data Structure



Background: Accounting Relations

- **Accounting Relation:** Sub-aggregates add up to higher-aggregates (***hierarchical***), and three monthly values in a quarter average to the quarterly value (***frequency***).

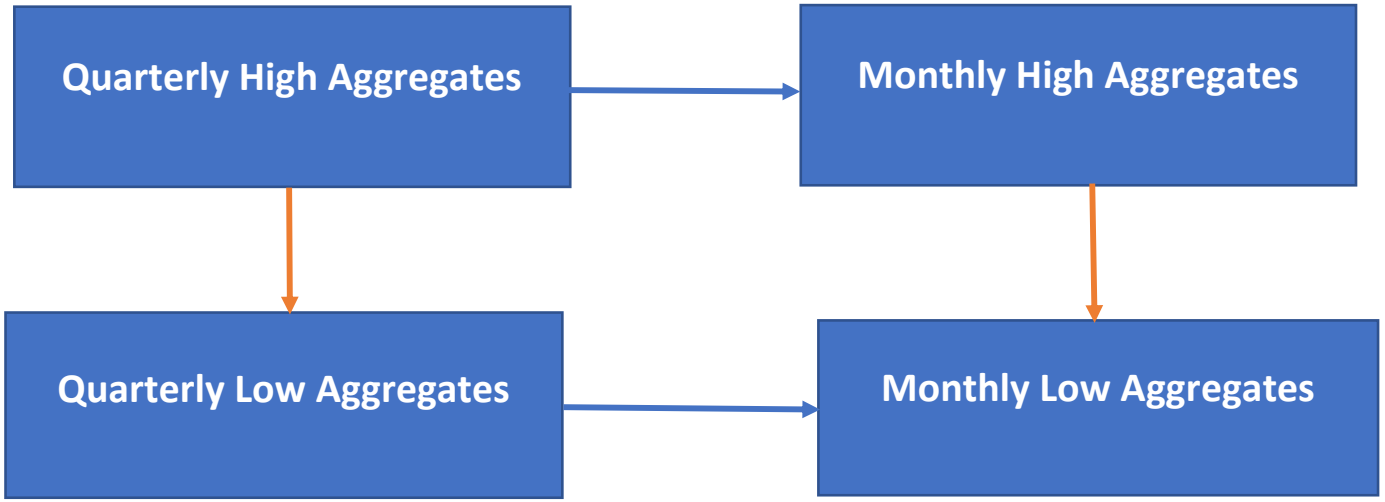
  There are *two parallel* ***hierarchies***, one for ***quarterly*** and one for ***monthly***.

- **Goal:** maintain accounting relations while modifying each time series to remove RS. (Many time series come to us already seasonally adjusted.)

  This framework is not unique to GDP. For example, NAICS uses a multi-tiered hierarchical structure.

Hierarchical Lattice Structure

Figure 2: Relations of sampling frequency and hierarchy.



Note: Higher aggregates are related to lower aggregates by the **orange** arrows, whereas sampling frequencies are related via **blue** arrows.

Hierarchical Lattice Structure

Goal: seasonally adjust all four variables in Figure 1, such that the accounting relations represented by the **orange** (hierarchical) and **blue** (Sampling frequency) arrows are preserved.

Repeat this for each cell of the twin hierarchical lattices.

Top-down Approach:

- first adjust higher aggregates, and then
- proceed downwards in a recursive fashion.

Hierarchical Lattice Structure

- Suppose Quarterly High Aggregates and Monthly High Aggregates are already adjusted

- Modify Monthly Low Aggregates such that:
  1. It has no RS
  2. It aggregates (via optimization procedure) to Monthly High Aggregates
  3. Its Quarterly Low Aggregate has no RS

Call such a procedure a Frequency Aggregation (FA) algorithm.

An Example of Hierarchical Lattice Structure

PCE Goods is the sum of durables and non-durables goods. The quarterly and monthly measures are

$$G^Q = DUR^Q + NDUR^Q \qquad \& \qquad G^M = DUR^M + NDUR^M.$$

Suppose quarterly and monthly series of PCE goods are seasonally adjusted, $\tilde{G}^Q$ and $\tilde{G}^M$, such that the *frequency constraint* is satisfied, $\tilde{G}^Q = \frac{1}{3}\sum_{M=1}^{3} \tilde{G}_Q^M$. Now we need to adjust monthly lower aggregates DUR^M and $NDUR^M$, such that *cross-aggregation* constraint is satisfied,

$$\tilde{G}^M = \widetilde{DUR}^M + \widetilde{NDUR}^M,$$

and frequency constraints are satisfied for both components, i.e.

$$\widetilde{DUR}^Q = \frac{1}{3}\sum_{M=1}^{3} \widetilde{DUR}_Q^M \quad \text{and} \quad \widetilde{NDUR}^Q = \frac{1}{3}\sum_{M=1}^{3} \widetilde{NDUR}_Q^M \ ,$$

Thus, cross-aggregation constraint for quarterly estimates is also satisfied,

$$\tilde{G}^Q = \widetilde{DUR}^Q + \widetilde{NDUR}^Q.$$

Hierarchical Lattice Structure

Reality Check: Figure 2 is far too simplistic.

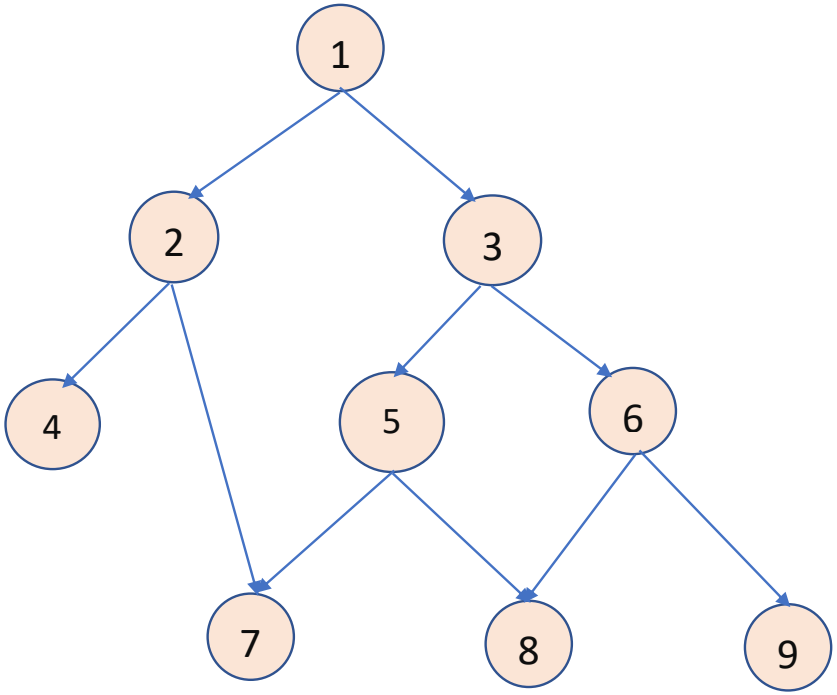
Property 1: Each variable (or node) can have many sub-aggregates.

Property 2: Each variable (or node) can be involved in many accounting relations.

Property 2 implies accounting relations is not a tree; instead, it is a lattice (with hierarchy).

Hierarchical Lattice Structure: Example

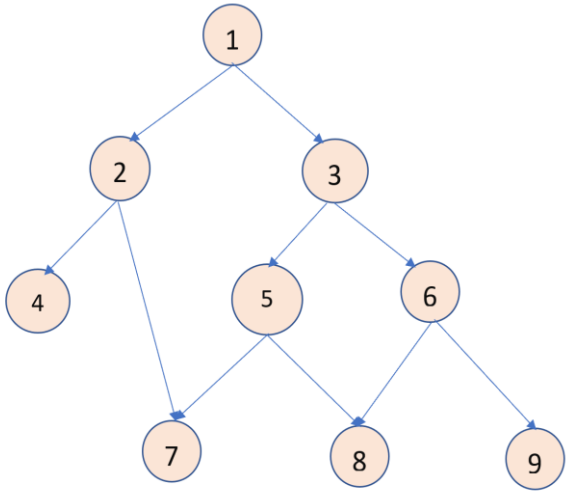
Figure 3: Example of a hierarchical lattice (from PCE services)



Note: Nodes 4, 7, 8 and 9 are related with nodes 2, 5 and 6. This situation occurs in the PCE services section of the data.

Hierarchical Lattice Structure: An Example

- Suppose node 2 in Figure 2 has already been adjusted; we want to adjust nodes 4 and 7 such that the accounting relation holds.
- But modifying node 7 affects accounting relation to node 5!
- Also, node 8 is involved with node 7 via accounting relation to node 5.
- And so forth: need to jointly adjust nodes 4, 7, 8 and 9, subject to three accounting relations!



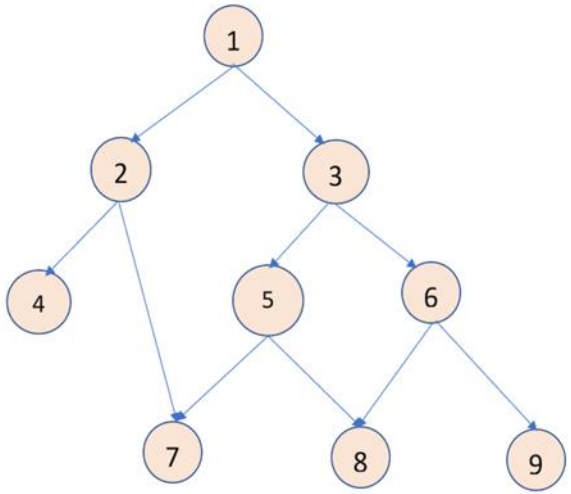
Need to determine batches of nodes that can be adjusted jointly without fear of causing an incoherency due to other neglected accounting relations.

Introducing terminologies for hierarchical lattice

~ Windows, Muntins, Hubs, and Spokes

Window: a hierarchical lattice, **W** which has ***n*** nodes connected by directed
edges, called **muntins**. Each muntin connects a **hub** node a **spoke** node.

- Each hub has a single accounting relation
- A hub's spokes are the variables subject to that accounting relation
- Each hub has at least 2 spokes



Terms related to hubs and spokes

- **Crest**: the initial node 1, which is a hub but not a spoke
- **Sill**: collection of the terminal nodes, which are spokes but not hubs.

Example: The spokes of node 2 are {4, 7}. The hubs of node 8 are {5, 6}. The sill is {4, 7, 8, 9}.

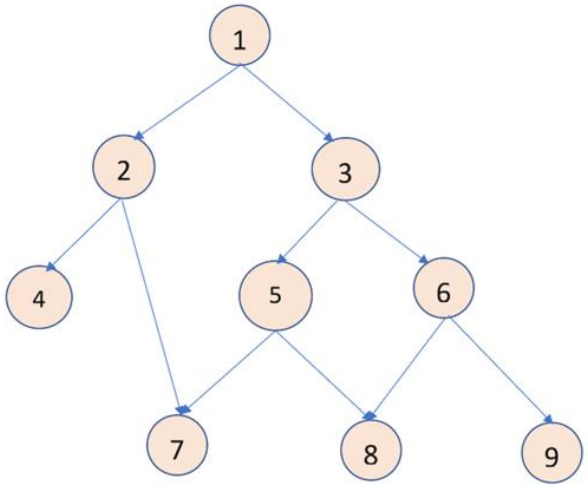
Terraces and Frequency Aggregation (FA) Algorithm

- **Terrace:** A subset of nodes K (outside of ending nodes) such that P(K) = K, the subset K contains all topologically adjacent nodes, i.e., there is no proper subset of K with this property.
- Terraces partition the entire system, except the ending nodes, and we can run the FA algorithm on each terrace.
- What order should be placed on terraces, to compute FA?
- Terraces in the top come first, and then the bottom terraces; this is the **Top-down** approach of the FA algorithm.

Terrace Example and Order of Top-down FA Algorithm

Example:

- The terraces are {1}, {3}, and {2, 5, 6}, with {4, 7, 8, 9} being the sill.
- To run **FA** on terrace {2, 5, 6} we must already have adjusted those hubs in a previous step.
- This requires that we already ran **FA** on terraces {1} (because this is the tier of node 2) and {3} (which is the tier of nodes 5 and 6).



Order of Top-down FA algorithm

- The Top-down approach of FA must run on terraces in the order of {1), {3} and {2, 5, 6}.

Illustration of the solution procedure

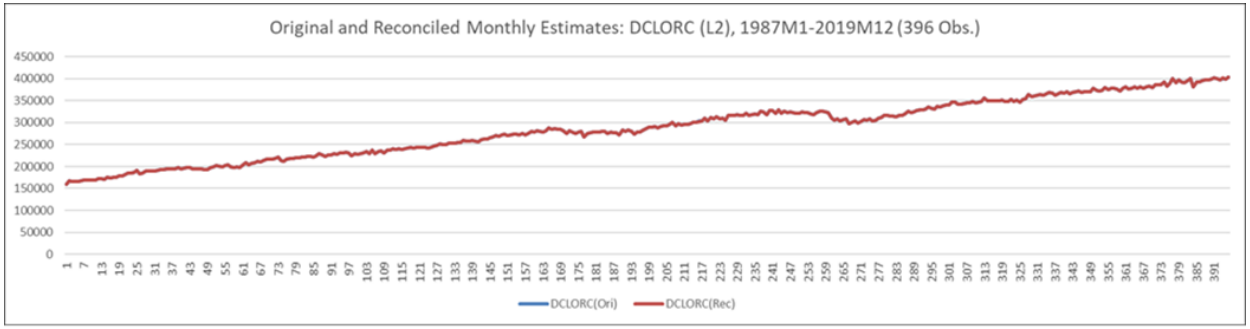
- **Data:**
  - **Variables:** 146 items from the PCE goods section of the ***hierarchical*** system.
  - **Sample:** 1987Q1-2019Q4.
  - **Frequency:** Monthly and quarterly.

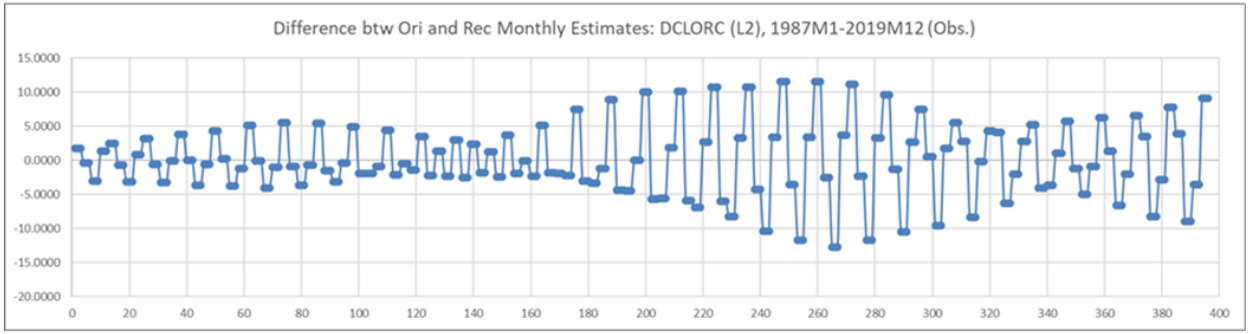
- **General observations of the results:**
  - RS was detected and removed from the 23 items (or 16%) of the PCE goods section of the accounts in both *monthly* and *quarterly* data
  - The percentage differences between the reconciled and the original estimates are generally quite small
  - RS was detected at the 2, 3, 4, and 5 disaggregated levels of the accounts, with PCE durable and nondurable goods being level 1.

Table 1: % Differences in Monthly Estimates Between Reconciled and Original Estimates ~ Showing results for 2019M01-2019M12

Line No.	PCE Code	Line Item Description	Agg. Level	2019M01	2019M02	2019M03	2019M04	2019M05	2019M06	2019M07	2019M08	2019M09	2019M10	2019M11	2019M12
6	DNDCRC	New domestic autos	5	0.83%	0.90%	0.88%	0.15%	0.18%	0.17%	-0.08%	-0.08%	-0.07%	-0.91%	-0.87%	-0.81%
7	DNFCRC	New foreign autos	5	-2.30%	-2.40%	-2.50%	-0.47%	-0.46%	-0.43%	0.20%	0.22%	0.21%	2.37%	2.45%	2.56%
70	DFXARC	Food and beverages purchased for off-premises	2	0.001%	0.001%	0.001%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	-0.001%	-0.001%	-0.001%
101	DCLORC	Clothing and footwear	2	0.001%	0.001%	0.001%	-0.002%	-0.002%	-0.002%	-0.001%	-0.001%	-0.001%	0.002%	0.002%	0.002%
110	DGOERC	Gasoline and other energy goods	2	0.012%	0.011%	0.011%	-0.007%	-0.007%	-0.007%	0.002%	0.002%	0.002%	-0.006%	-0.006%	-0.006%
111	DMFLRC	Motor vehicle fuels, lubricants, and fluids (59)	3	0.47%	0.45%	0.44%	-0.22%	-0.23%	-0.24%	0.04%	0.04%	0.05%	-0.24%	-0.23%	-0.24%
112	DGASRC	Gasoline and other motor fuel	4	0.46%	0.44%	0.43%	-0.21%	-0.22%	-0.23%	0.05%	0.05%	0.05%	-0.23%	-0.23%	-0.24%
113	DLUBRC	Lubricants and fluids	4	0.87%	0.91%	0.88%	-0.45%	-0.45%	-0.44%	-0.07%	-0.07%	-0.07%	-0.34%	-0.35%	-0.32%
114	DFULRC	Fuel oil and other fuels (29)	3	-6.46%	-6.42%	-6.14%	3.48%	3.50%	3.41%	-0.63%	-0.64%	-0.66%	3.65%	3.35%	3.69%
117	DONGRC	Other nondurable goods	2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
118	DPHMRC	Pharmaceutical and other medical products (40	3	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	0.04%	0.04%	0.04%	0.00%	0.00%	0.00%
120	DRXDRC	Prescription drugs	5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%
121	DNRDRC	Nonprescription drugs	5	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	-0.03%	-0.03%	-0.03%	0.02%	0.02%	0.02%
123	DREIRC	Recreational items (parts of 80, 92, and 93)	3	-0.01%	-0.01%	-0.01%	-0.14%	-0.14%	-0.14%	0.01%	0.01%	0.01%	0.16%	0.16%	0.16%
128	DHOURC	Household supplies (parts of 32 and 36)	3	0.03%	0.03%	0.03%	-0.04%	-0.04%	-0.04%	-0.03%	-0.03%	-0.03%	0.03%	0.03%	0.03%
134	DOPCRC	Personal care products (part of 118)	3	-0.03%	-0.03%	-0.03%	-0.05%	-0.05%	-0.05%	0.01%	0.01%	0.01%	0.06%	0.06%	0.06%
138	DTOBRC	Tobacco (127)	3	-0.07%	-0.07%	-0.08%	0.02%	0.02%	0.02%	0.04%	0.04%	0.04%	0.01%	0.01%	0.01%
139	DNEWRC	Magazines, newspapers, and stationery (part of	3	-0.01%	-0.01%	-0.01%	-0.11%	-0.11%	-0.11%	-0.03%	-0.03%	-0.03%	0.16%	0.16%	0.16%
142	DNFRRC	Net expenditures abroad by U.S. residents (131	3	1.42%	1.44%	1.41%	5.02%	5.02%	4.98%	-2.19%	-2.16%	-2.16%	-4.52%	-4.54%	-4.55%
143	DABDRC	Expenditures abroad by U.S. residents	4	0.32%	0.32%	0.32%	0.39%	0.39%	0.39%	-0.36%	-0.35%	-0.35%	-0.34%	-0.34%	-0.34%
144	DARTRC	Government employees' expenditures abroad	5	0.27%	0.27%	0.27%	0.34%	0.34%	0.34%	-0.30%	-0.30%	-0.29%	-0.30%	-0.30%	-0.30%
145	DARSRC	Private employees' expenditures abroad	5	0.66%	0.65%	0.65%	0.65%	0.70%	0.70%	-0.74%	-0.80%	-0.68%	-0.62%	-0.62%	-0.62%
146	DREMRC	Less: Personal remittances in kind to nonreside	4	-8.92%	-10.38%	-10.41%	-40.35%	-38.27%	-37.72%	15.28%	15.07%	14.59%	31.81%	30.68%	30.35%

Figure 3: Reconciled and Original **Monthly** Estimates, Level and % Difference: Clothing and Footwear (DCLORC) (L2)





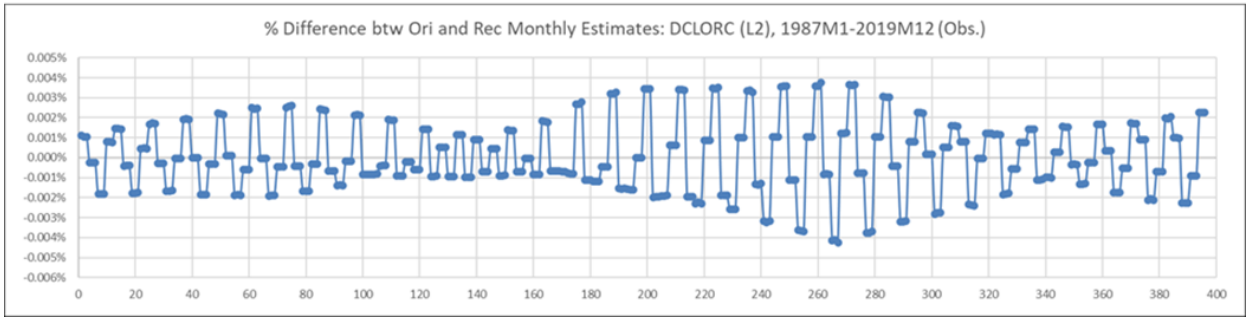
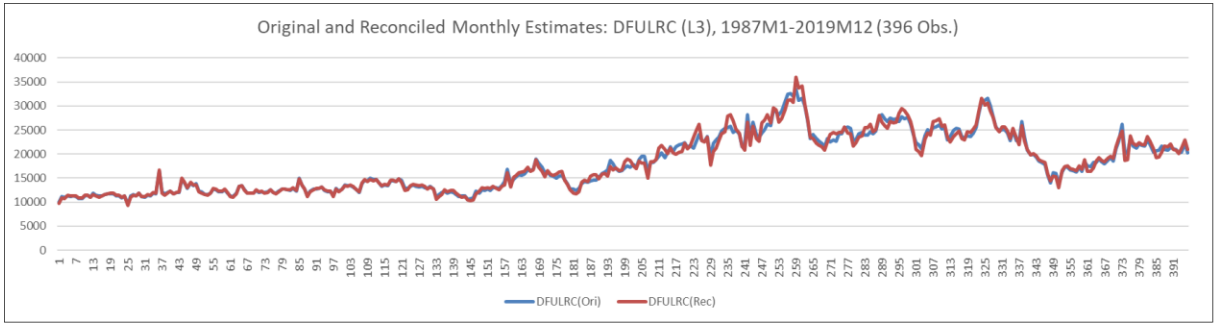
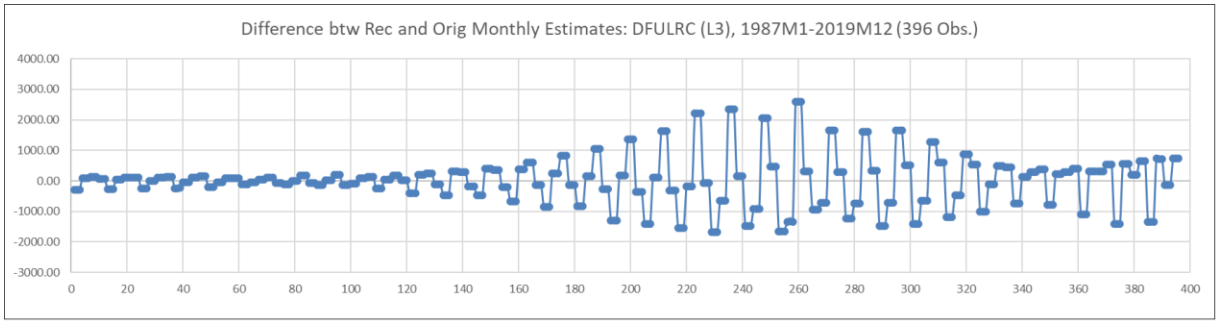


Figure 4: Reconciled and Original **Monthly** Estimates, Level and % Difference: Fuel oil and other fuels (DFULRC) (L3)





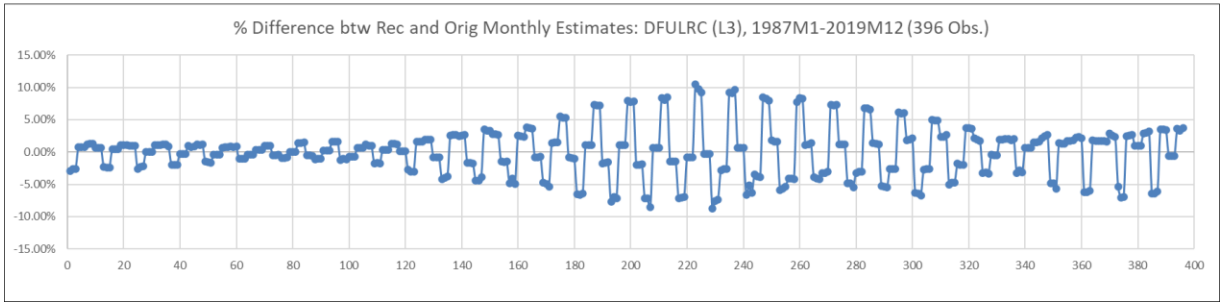


Figure 5: Reconciled and Original **Monthly** Estimates, Level and % Difference: Private employees' expenditures abroad (DARSRC) (L5)

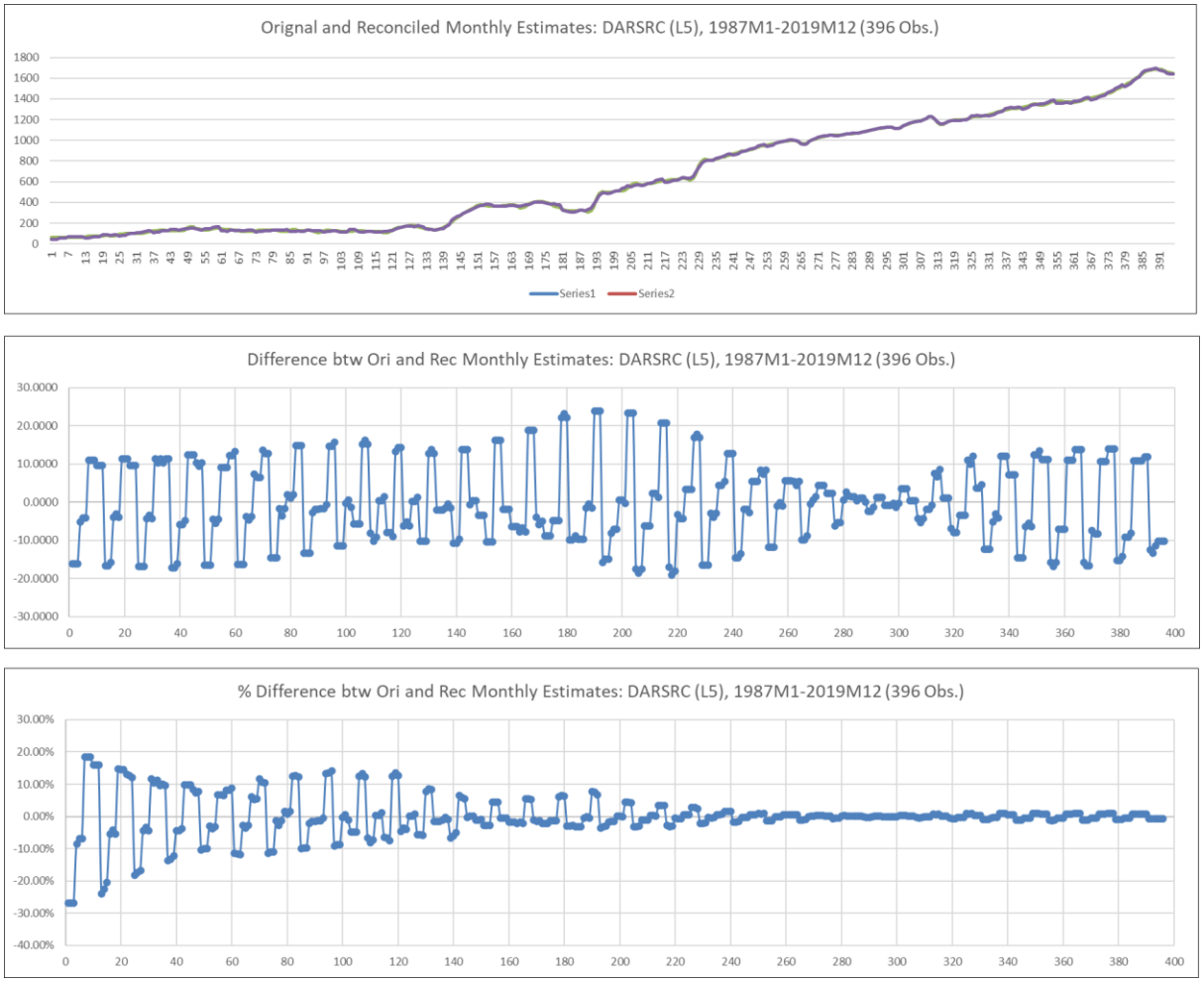
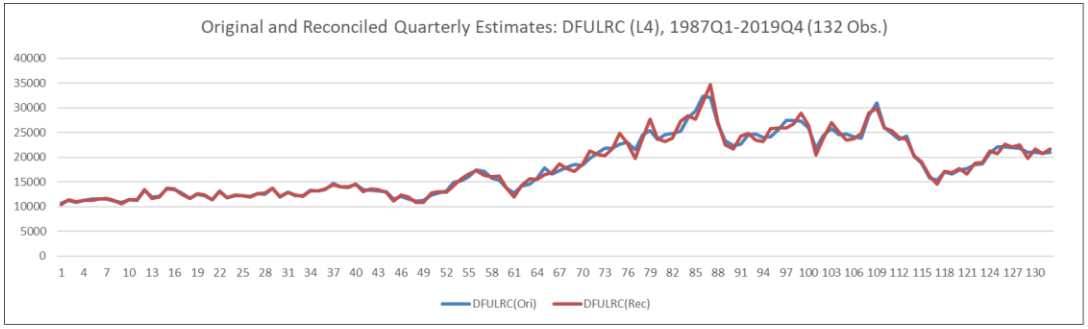
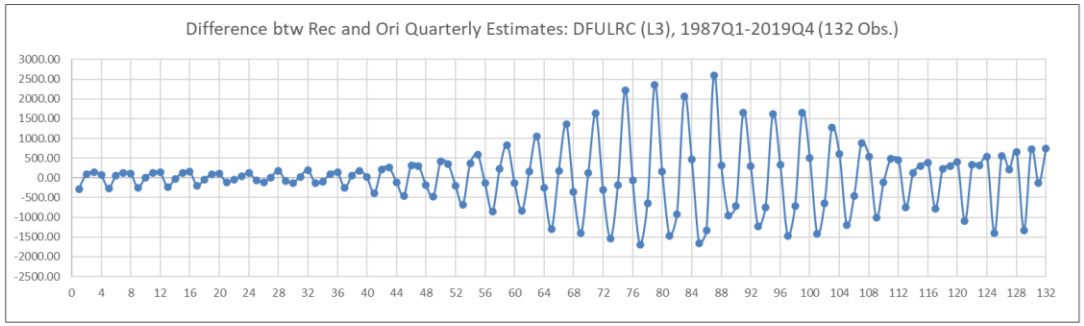


Figure 6: Reconciled and Original *Quarterly* Estimates, Level and % Difference: Fuel oil and other fuels (DFULRC) (L3)





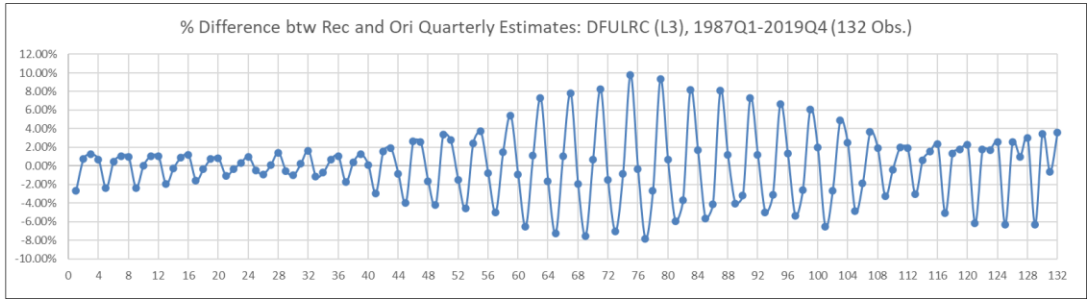
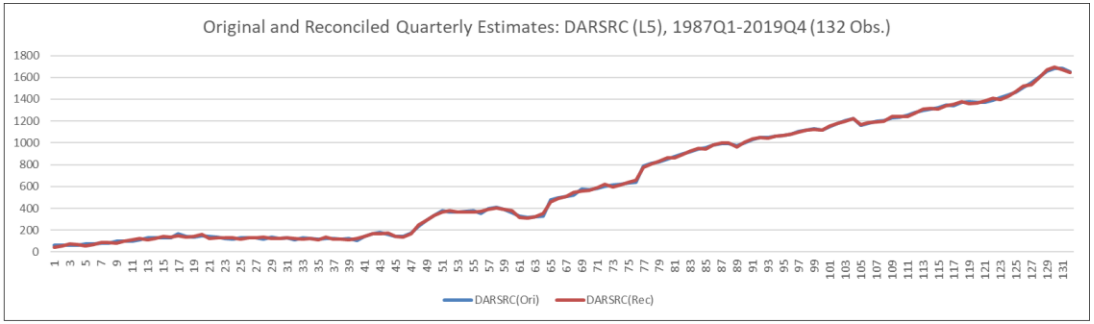
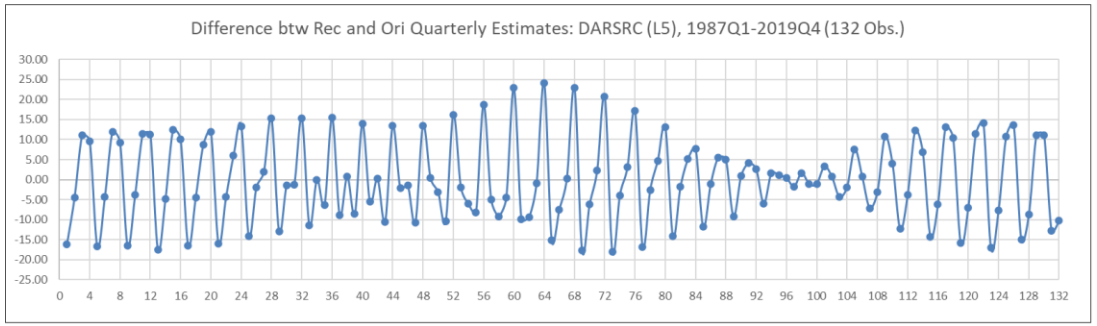
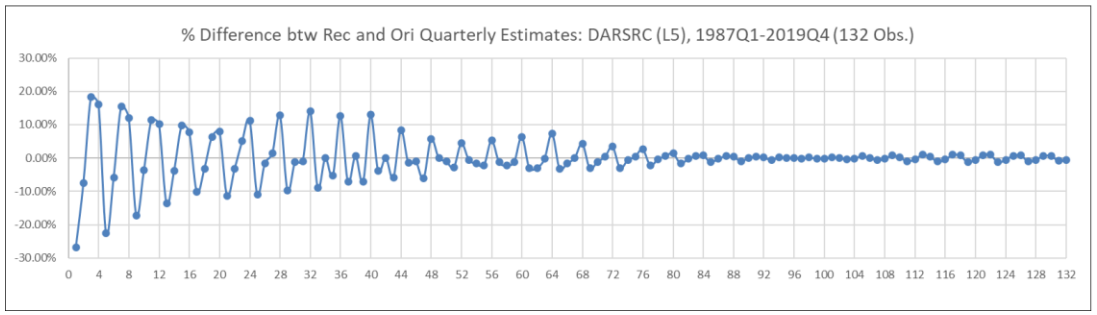


Figure 8: Reconciled and Original **Quarterly** Estimates, Level and % Difference: Private employees' expenditures abroad (DARSRC) (L5)







Concluding Remarks

- We presented a Top-down seasonal adjustment (SA) algorithm which removes RS in the *monthly* and *quarterly* series in a *hierarchical* structured accounting system and reconciles the adjusted series according to the accounting constraints.
- We illustrated the Top-down SA algorithm with data from the goods section of the PCE accounts
- The further work includes completing the program for the Top-down SA algorithm for the PCE services section of the accounts, which has the more complicated hierarchical structure as was represented in Figure 3.

Thank you for your attention!

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